Estimating the Covariance Matrix

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Summary

In this document, we do the following:

1. Verify numerically the true covariance matrix in the simple normal model.

Problem

In `20240202-estcovmat.pdf` and `20240202-gls_sim.pdf`, we ran simulations in which we used the *true* covariance matrix to get the best estimator. Unfortunately, further analysis suggests that the covariance matrix used previously was slightly incorrect. This report tests to see if this new result is correct.

The previously accepted true variance estimate[1] was the following:

$$\text{Cov}(\hat{f}_1, \hat{f}_2) = N^{-1}\pi^{-1}I(\delta_1 = \delta_2)\text{Cov}(f_1, f_2)$$

and the new variance estimate[2] is

$$\text{Cov}(\hat{f}_1, \hat{f}_2) = N^{-1}(I(\delta_1 \neq \delta_2)\text{Cov}(f_1, f_2) + \pi^{-1}I(\delta_1 = \delta_2)\text{Cov}(f_1, f_2)).$$

The only difference between the estimates is the covariance terms between segments. Are these independent or not?

[1] See Page 219 of notes
[2] See Page 221 of notes

Simulation

Actual	Exp1	Exp2
0.00409	0.004	0.0040
0.00402	0.004	0.0040
0.00402	0.004	0.0040
-0.00005	0.000	0.0010
-0.00031	0.000	0.0010
0.00002	0.000	0.0010
0.00015	0.000	0.0010
-0.00004	0.000	0.0010
0.00791	0.008	0.0080
0.00589	0.006	0.0060
-0.00011	0.000	0.0010
-0.00046	0.000	0.0020
-0.00018	0.000	0.0010
-0.00002	0.000	0.0015
-0.00010	0.000	0.0010
0.00801	0.008	0.0080
-0.00010	0.000	0.0010
-0.00041	0.000	0.0015
-0.00017	0.000	0.0010
-0.00012	0.000	0.0020
0.00002	0.000	0.0010
0.00395	0.004	0.0040
0.00389	0.004	0.0040
0.00003	0.000	0.0010
-0.00013	0.000	0.0010
0.00000	0.000	0.0010
0.00787	0.008	0.0080
0.00012	0.000	0.0010
-0.00017	0.000	0.0015

Actual	Exp1	Exp2
-0.00010	0.000	0.0010
0.00395	0.004	0.0040
0.00391	0.004	0.0040
0.00012	0.000	0.0010
0.00784	0.008	0.0080
0.00002	0.000	0.0010
0.00390	0.004	0.0040

Conclusion

Which is correct? It looks like the first derivation is correct. This does make sense as I view the different segments as independent. The question is then what mistake did I make in my own analysis?