Nonparametric Double Robustness

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Abstract

Use of nonparametric techniques (e.g., machine learning, kernel smoothing, stacking) are increasingly appealing because they do not require precise knowledge of the true underlying models that generated the data under study. Indeed, numerous authors have advocated for their use with standard methods (e.g., regression, inverse probability weighting) in epidemiology. However, when used in the context of such singly robust approaches, nonparametric methods can lead to suboptimal statistical properties, including inefficiency and no valid confidence intervals. Using extensive Monte Carlo simulations, we show how doubly robust methods offer improvements over singly robust approaches when implemented via nonparametric methods. We use 10,000 simulated samples and 50, 100, 200, 600, and 1200 observations to investigate the bias and mean squared error of singly robust (g Computation, inverse probability weighting) and doubly robust (augmented inverse probability weighting, targeted maximum likelihood estimation) estimators under four scenarios: correct and incorrect model specification; and parametric and nonparametric estimation. As expected, results show best performance with g computation under correctly specified parametric models. However, even when based on complex transformed covariates, double robust estimation performs better than singly robust estimators when nonparametric methods are used. Our results suggest that nonparametric methods should be used with doubly instead of singly robust estimation techniques.

Key Words: semiparametric theory; nonparametric methods; double-robust estimation; causal inference; epidemiologic methods.

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Introduction

Recent years have seen many developments in semiparametric theory and estimation, ^{1,2} including increasingly popular doubly robust methods. ^{3–5} Doubly robust methods have tremendous potential for improving the quality of inference and estimation in epidemiology. Under standard exchangeability assumptions, they can be used to adjust for missing data or confounding, provided that data on a sufficient set of covariates is collected.

Several authors have reviewed doubly robust estimation for applied ^{6–9} and technical ^{2,5,10–13} audiences. Doubly robust estimation is so named because these methods allow two chances for adjustment. In the case of confounding adjustment, these chances arise because the analyst must fit two models: a model for the outcome regressed against the exposure and all confounders (outcome model); and a model regressing the exposure against all confounders (the propensity model). These are combined into a "union model" to estimate the effect of interest. ¹²

Thus far, reviews of doubly robust estimation, as well as the growing number of applied examples, have fallen into two categories. On the one hand, several authors have implemented doubly robust estimators using *parametric* working models. e.g., ^{14,15} On the other, researchers have employed data-adaptive techniques, often in conjunction with a meta-learning approach such as stacking (e.g., super learner), to implement doubly robust estimators nonparametrically. e.g., ^{16,17}

Valid inference with parametric regression models relies on hard to verify modeling assumptions, such as the specification (or lack thereof) of key covariate interactions, or linearity assumptions. These assumptions are not necessarily met simply by resorting to doubly robust estimation. If both the exposure and outcome models are specified using an incorrect parametric form, bias may result. In contrast, nonparametric estimation does not rely on strong modeling assumptions, and can thus be used to minimize bias that results from incorrect specification of parametric models.

For this reason, numerous authors have advocated using nonparametric (i.e., data-adaptive or machine learning) techniques, for both singly and doubly robust estimation. However, when used in conjunction with singly robust approaches, nonparametric methods are subject to problems due to the "curse of dimensionality" and lack of regularity conditions needed to accurately quantify uncertainty for point estimates. Using nonparametric methods with doubly-robust estimators miti-

gates against this issue: under relatively mild conditions, doubly robust estimators remain root-*n*-consistent and asymptotically normal even when nonparametric data-adaptive methods are used to fit the exposure and outcome models. ^{18,19}

This little recognized feature of doubly robust estimators has important implications for applied researchers. Here, we examine these implications using Monte Carlo simulation. ²⁰ We start by illustrating the problems with using parametric singly robust approaches. We demonstrate how relying on mis-specified parametric models to implement doubly robust estimators does not improve upon consistency problems. Finally, we show how standard singly robust approaches perform poorly when data-adaptive methods are used, but that doubly robust estimators perform well with data-adaptive nonparametric approaches.

Observed Data & Causal Estimand

Consider the setting with a single binary exposure (X) and a set of continuous confounders $(\mathbf{C} = \{C_1, C_2, C_3, C_4\})$ measured at baseline, as well as a single continuous outcome (Y) measured at the end of follow-up. In an observational cohort study to estimate the effect of X on Y, \mathbf{C} would be assumed the minimally sufficient adjustment set, 21 and the exposure and outcome would be assumed generated according to some unknown models:

$$P(X=1 \mid \mathbf{C}) = f(\mathbf{C}) \tag{1}$$

$$E(Y \mid X, \mathbf{C}) = g(X, \mathbf{C}), \tag{2}$$

where $f(\bullet)$ and $g(\bullet)$ represent functions of C, and X and C, respectively. In an observational cohort study, the exact form of the exposure and outcome models is usually completely unknown.²² Without loss of generality, we focus here on the average causal effect:

$$\psi = E(Y^{x=1} - Y^{x=0})$$

where Y^x is the outcome that would be observed if X were set to x. This estimand is identified by

$$\psi = E\{g(X = 1, \mathbf{C}) - g(X = 0, \mathbf{C})\} = E\left\{ \left[\frac{XY}{f(\mathbf{C})} \right] - \left[\frac{(1 - X)Y}{1 - f(\mathbf{C})} \right] \right\}$$

under positivity, consistency, and exchangeability. ^{23,24} If these conditions hold, ψ can be estimated using a number of approaches. In the equations to follow, i refers to sample observations, and $\hat{f}_i(\mathbf{C})$ and $\hat{g}_i(X = x, \mathbf{C})$ are individual sample predictions for $P(X = 1 \mid \mathbf{C})$ and $E(Y \mid X = x, \mathbf{C})$, respectively.

Using an estimate of Model 1, ψ can be estimated via inverse probability weighting 25 as:

$$\hat{\psi}_{ipw} = \frac{1}{N} \sum_{i=1}^{N} \left\{ \left[\frac{X_i Y_i}{\hat{f}_i(\mathbf{C})} \right] - \left[\frac{(1 - X_i) Y_i}{1 - \hat{f}_i(\mathbf{C})} \right] \right\}. \tag{3}$$

With an estimate of Model 2, ψ can be estimated via g computation²⁴:

$$\hat{\psi}_{gComp} = \frac{1}{N} \sum_{i=1}^{N} \left\{ \hat{g}_i(X = 1, \mathbf{C}) - \hat{g}_i(X = 0, \mathbf{C}) \right\}.$$
 (4)

Both approaches 3 and 4 are "singly robust" in that they rely entirely on the correct specification of a single regression model. Alternatively, one may employ a "doubly robust" technique. Using predictions from both Models 1 and 2, ψ can be estimated as:

$$\hat{\psi}_{aipw} = \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{(2X_i - 1)[Y_i - \hat{g}_i(X, \mathbf{C})]}{(2X_i - 1)\hat{f}_i(\mathbf{C}) + (1 - X_i)} + \hat{g}_i(X = 1, \mathbf{C}) - \hat{g}_i(X = 0, \mathbf{C}) \right\}.$$
 (5)

Equation 5 is an augmented inverse probability weighted estimator, and will converge to the true value if either $f(\mathbf{C})$ or $g(X,\mathbf{C})$, but not necessarily both, are consistently estimated. The estimator 5 can be viewed as a bias-corrected version of the g computation estimator, where the correction is the term incorporating the propensity score.

Finally, model 1 can be used to "update" model 2 via targeted minimum loss-based estimation: $^{26(p72-3)}$

$$\hat{\psi}_{tmle} = \frac{1}{N} \sum_{i=1}^{N} \left\{ \hat{g}_{i}^{u}(X=1, \mathbf{C}) - \hat{g}_{i}^{u}(X=0, \mathbf{C}) \right\}, \tag{6}$$

where $\hat{g}_i^u(X=1,\mathbf{C})$ are predictions from an "updated" outcome model. This outcome model is up-

dated by first generating an inverse probability weight as:

$$H(X, \mathbf{C}) = \begin{cases} \frac{1}{\hat{f}_i(\mathbf{C})} & \text{if } X = 1\\ -\frac{1}{1 - \hat{f}_i(\mathbf{C})} & \text{otherwise} \end{cases}$$

and then including this inverse probability weight in a no-intercept logistic regression model for the outcome that includes the previous outcome predictions $\hat{g}_i(X, \mathbf{C})$ as an offset. The $\hat{g}_i^u(X=1, \mathbf{C})$ predictions are then generated from this model by setting X to 1 and then to 0 for all individuals in the sample. This approach is asymptotically equivalent to 5 but can have better finite-sample performance since the resulting estimate will be appropriately bounded by, e.g., the minimum and maximum empirical values of Y. ²⁷

Parametric Estimation

For binary *X* and continuous *Y*, it is customary to specify models 1 and 2 parametrically using logistic and linear regression:

$$P(X = 1 \mid \mathbf{C}) = \expit(\alpha_0 + \alpha_1 C_1 + \alpha_2 C_2 + \alpha_3 C_3 + \alpha_4 C_4),$$

$$\expit(\bullet) = 1/(1 + \exp[-\bullet])$$
(7)

$$E(Y \mid X, \mathbf{C}) = \beta_0 + \beta_1 X + \beta_2 C_1 + \beta_3 C_2 + \beta_4 C_3 + \beta_5 C_4,$$

$$Y \mid X, \mathbf{C} \sim \mathcal{N}\left(E(Y \mid X, \mathbf{C}), \sigma^2\right)$$
(8)

where we let $\beta_0 + \beta_1 X + \beta_2 C_1 + \beta_3 C_2 + \beta_4 C_3 + \beta_5 C_4 = \mu$, and we collectively refer to all the β 's in model 8 as β . Imposing these forms on $f(\mathbf{C})$ and $g(X, \mathbf{C})$ permits use of maximum likelihood for estimation and inference.²⁸

Estimation via Parametric Outcome Model

Model 8 imposes several parametric constraints on the form of $g(X, \mathbf{C})$: (i) Y follows a conditional normal distribution with constant variance not depending on X or \mathbf{C} ; and (ii) the mean μ is related to the covariates X and \mathbf{C} additively, as detailed in model 8. If these constraints on $g(X, \mathbf{C})$ are true, and other "regularity" conditions hold, $^{29(ch2)}$ the maximum likelihood estimates of β are

asymptotically efficient. $^{30(p144)}$ Relatedly, under the model constraints and regularity conditions, as the sample size increases the estimates of $g(X, \mathbf{C})$ and/or $f(\mathbf{C})$ will converge to the truth at an optimal (i.e., \sqrt{N}) rate, and their distribution will be such that confidence intervals can be easily derived.

If assumption (i) is violated, the maximum likelihood estimator is no longer the most efficient, but can still be used to estimate ψ consistently. If assumption (ii) is violated, then the maximum likelihood estimator is no longer consistent. Depending on the severity to which assumption (ii) is violated, the bias may be substantial. Unfortunately, in an observational study the true form of model 8 is almost never known. This means that such maximum likelihood estimates are almost always biased, with the degree of bias depending on the (unknown) extent to which the model is mis-specified. 31

Estimation via Parametric Exposure Model

One way to avoid relying on correct outcome model specification is to use a parametric model for exposure model 1, and estimate ψ via $\hat{\psi}_{ipw}$. Such an estimator is not as efficient as $\hat{\psi}_{gComp}$, and can be subject to important finite-sample biases when weights are highly variable. But as the sample size increases, the inverse probability weighted estimator converges at the same ideal rate as the g computation estimator. ³² Unfortunately, as with the outcome model, the true form of model 1 will almost never be known in an observational study. Mis-specification of model 7 will also lead to biased estimation of ψ , again with the degree of bias depending on the unknown extent of model mis-specification.

Parametric Doubly Robust Estimation

To mitigate against mis-specification of the exposure or outcome models, numerous authors have advocated for the use of estimators such as equation 5 or 6. These double robust estimators remain consistent even if either the exposure model or the outcome model is mis-specified, but not both. However, if it is unlikely that either model 7 or 8 is correct, then the doubly robust estimator will also likely be biased, and so not much better than the singly robust estimators. ^{10,19}

Nonparametric Singly Robust Estimation: The Curse of Dimensionality

Nonparametric methods are an alternative to parametric models. For example, nonparametric maximum likelihood estimation (NPMLE) for models 1 or 2 would entail fitting models 7 and 8, but with a parameter for each unique combination of values defined by the cross-classification of all covariates (i.e., saturating the model). However, the NPMLE will be undefined in any finite sample with a continuous confounder, since there will be no covariate patterns containing both treated and untreated subjects.

Alternatively, one could use nonparametric methods like kernel regression, splines, random forests, boosting, etc., which can exploit smoothness across covariate patterns to infer the regression function. Matching can be viewed as the estimator 4 with \hat{g} constructed from nearest neighbors. However, for nonparametric methods there is an explicit bias-variance trade-off that arises in the choice of tuning parameters; less smoothing yields smaller bias but larger variance, while more smoothing yields smaller variance but larger bias. In this framework, parametric models can be viewed as an extreme form of smoothing. A central challenge for nonparametric estimators involves efficiency, e.g., the rate at which the estimator converges to the truth as the sample size increases. In typical nonparametric settings, it is impossible to estimate regression functions at the fast \sqrt{N} rates attained by correctly specified parametric estimators. ³³ Convergence rates for nonparametric estimators depend on the smoothness and dimension of the regression function, becoming slower with less smoothness and more covariates ³⁴ (i.e., the curse of dimensionality ^{35,36}). Sometimes this is viewed as a disadvantage of nonparametric methods; however, this is just the cost of making weaker assumptions. If a parametric model is misspecified, it will be converging very quickly to the wrong answer. Another cost of nonparametric assumptions, in addition to slower convergence rates, is that inference and confidence intervals are harder to obtain. Specifically, even in the rare case where asymptotic distributions are tractable, it is typically not possible to construct centered confidence intervals (even via the bootstrap) without impractical undersmoothing.³⁶

These complications (slow rates and lack of inferential tools) are generally inherited by the singly robust estimators 3 and 4, apart from a few special cases (which require simple estimators, e.g., kernel methods, strong smoothness assumptions, and careful tuning parameter choices that are

suboptimal for estimating f or g). For general nonparametric estimators \hat{f} and \hat{g} , the estimators 3 and 4 will converge at slow rates, and confidence intervals will not be available.

Nonparametric Doubly Robust Estimation

Remarkably, doubly robust estimators that rely on nonparametric estimates of f and g do not suffer from the same limitations as the nonparametric versions of the singly robust estimators. In particular the doubly robust estimators f and g are converging at slower nonparametric rates. In other words, the doubly robust estimator is less susceptible to the curse of dimensionality. This is a result of the fact that the error of the doubly robust estimator depends on the *product* of the errors of \hat{f} and \hat{g} , which goes to zero as fast or faster than either error alone. In particular, as long as \hat{f} and \hat{g} are converging to their targets at faster than $n^{-1/4}$ rates (in L_2 norm), the doubly robust estimator will behave asymptotically just as if both f and g were estimated with correct parametric models. Importantly, $n^{-1/4}$ rates can be attained nonparametrically under relatively weak smoothness, sparsity, or other structural assumptions. 34,36 This improved performance of nonparametric methods when used with doubly robust techniques has major implications for applied research.

Simulation Study

Data Generating Mechanism: Correct Specification

To explore these implications, we carried out an extensive simulation study. We simulated 10,000 Monte Carlo samples, with sample sizes of $\{50, 100, 200, 600, 1200\}$ using data generating mechanisms that would lead to challenging conditions for estimation and inference in empirical settings. Specifically, we generated four independent standard normal confounders, denoted C. Both the exposure and outcome models included each of these confounders, and all of their 2-way interactions. The exposure was generated from a logistic model with:

$$P(X=1 \mid Z) = \expit(\boldsymbol{\theta}C), \tag{9}$$

with $\theta = \{-0.5, \log(2), \log(2.5), \log(5), \log(1.5), \log(1.75), \log(1.5), \log(1.25), \log(1$

$$Y = 120 + \psi X + \beta C + \epsilon, \tag{10}$$

where the <u>true</u> average causal effect $\psi = 6$, with ϵ drawn from a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 20$. The β 's were set to 120, 3.5, 2.5, -1, 5, 2, 2.5, 1.5, 1.5, 1.5, 1.

Data Generating Mechanism: Model Misspecification

To induce model misspecification, we followed previous research 10 and transformed each of the continuous confounders as follows:

$$Z_1 = \exp(C_1/2)$$

$$Z_2 = C_2/(1 + \exp(C_1)) + 10$$

$$Z_3 = (C_1C_3/25 + 0.6)^3$$

$$Z_4 = (C_2C_4 + 20)^2$$

Thus, while the true models generating the exposure and outcome variables included only the untransformed variables C, analyses conducted under model misspecification included only the transformed variables Z (without any interactions).

Simulation Analysis

In each Monte Carlo sample, we estimated the average treatment effect $\psi = E(Y^1 - Y^0) = 6$ using the inverse probability weighted estimator, the g computation estimator, augmented inverse probability weights, and targeted minimum loss-based estimation under two settings: (i) only the true confounder data C were available and used to specify all models (parametric and nonparametric), and (ii) only the transformed confounder data Z were available and used to specify all models (parametric and nonparametric).

Parametric models were implemented as generalized linear models, with a binomial distribution and logistic link for the exposure, and a Gaussian distribution and identity link for the outcome model. Nonparametric estimation was accomplished using the Super Learner algorithm with a diverse library of candidate algorithms listed in Table 1.

There were a total of two simulation scenarios (parametric, nonparametric) with five sample sizes: $\{50,100,200,600,1200\}$ and four estimators (g Computation, inverse probability weighted, augmented inverse probability weighted, targeted minimum loss-based). For each estimator in each scenario, we computed the bias: $B(\hat{\psi}) = E(\hat{\psi}) - \psi$, as well as the mean squared error: $MSE(\hat{\psi}) = E(\hat{\psi} - \psi)^2$. We also computed the Wald-type confidence interval coverage and width for each estimation approach. Standard errors for the g Computation estimator were obtained as the standard deviation of 100 bootstrap resample analyses. Standard errors for the inverse probability weighted approach were obtained using the robust variance estimator. Standard errors for both doubly robust approaches were obtained using the variance of the efficient influence function. Simulations were done in R version 3.3.3. All software code needed to reproduce results and figures is available on GitHub.

Simulation Results

eFigure 1 shows scatterplots and distributions of all four transformed continuous confounders and the outcome under a single simulation with N=1200. eFigure 2 shows the propensity score overlap when obtained using the true parametric models and the nonparametric SuperLearner. This Figure shows a relatively strong degree of non-overlapping propensity scores, making estimation via exposure modeling strategies difficult.³⁷ In contrast to the original study, 10,38 eFigure 3 shows that our outcome models were specified such that perfect predictions did not occur, irrespective of the manner in which the models were specified. Finally, eTables 1 and 2 show the crossvalidated SuperLearner risks for the outcome and propensity score models in the corresponding sample. While these tables suggest that extreme gradient boosting and GLM with stepwise interaction selection played the largest role in the superlearner algorithm, we generally observed large variation in which algorithms contributed to the ensemble predictor.

Figure 1 shows the estimated bias across all sample sizes. The bottom panel of Figure 1 shows

bias when all models use the correct set of confounders and are fit parametrically. With the exception of N=50, all methods are relatively unbiased. In contrast, when the transformed confounders are used and models are specified parametrically, all four estimators are subject to considerable bias which increases as the sample size increases 1. A different picture emerges when models are fit nonparametrically. Even when the correct set of confounders is used, using the Superlearner for each singly robust estimator results in considerable bias for sample sizes of 200 or less, and moderate bias for larger sample sizes. Similarly, with $N \le 200$, the augmented IPW is subject to moderate bias. However, TMLE returns estimates with little to no bias. When the transformed confounders are used via nonparametric estimation, all estimators are subject to bias. However, singly robust approaches are more biased than doubly robust approaches. Thus, the top two panels of Figure 1 demonstrate an important benefit of using nonparametric methods in the context of doubly robust estimators instead of with singly robust approaches.

Figure 2 shows the relation between each estimator when fit parametrically and nonparametrically for N = 1200 when the correct set of confounders are used. The density curves for both doubly robust estimators are centered on the true value, which is not the case for the singly robust approaches. Furthermore, the correlation between parametric and nonparametric estimators is lower for the IPW-based estimators relative to the regression based estimators. This is the consequence of the highly variable weights that result from estimating the propensity score.

Figure 3 shows the square root of the mean squared error (rMSE) for each simulation scenario. Several features of Figure 3 align with theoretical expectations. Most notably, when models were fit nonparametrically with the transformed covariates (top panel), the double robust estimators outperform both singly robust approaches. This lower rMSE is the result of the faster doubly robust estimator convergence relative to the singly robust g computation estimator. In contrast, when the correct set of confounders is used with parametric models, the g computation estimator outperforms the IPW estimator, as well as both doubly robust approaches.

Discussion

Doubly robust estimation is becoming increasingly popular in epidemiology. This popularity stems from the fact that these estimators offer two chances to adjust for confounding or missing

data, thereby offering the potential for additional protection against model misspecification. Model misspecification can occur for a number of reasons, including incorrect causal ordering of variables, incomplete confounder adjustment set, or incorrect functional form. ³⁹ This latter type of misspecification is specifically what doubly robust estimators protect against.

A misspecified functional form can occur if the analyst fails to correctly account for the manner in which exposure and confounders relate to the outcome. For a generalized linear model, this would occur if chosen link function is not compatible with how the data were actually generated, ⁴⁰ if the analyst fails to account for curvilinear relations between the covariates and the outcome, or fails to include important exposure-confounder or confounder-confounder interactions. Unfortunately, in an observational study the true nature of these relations is typically never known.

Nonparametric techniques based on data-adaptive machine learning algorithms offer a degree of protection against each of these functional form assumptions. This feature has motivated a growing body of work in which data-adaptive methods are used to estimate parameters of interest. In particular, a number of authors have advocated for use of machine learning methods to estimate propensity scores, ^{41–43} or to mitigate against the strict parametric assumptions required by the g computation algorithm. ^{44,45}

But, as we have shown, for singly robust estimators this protection may not always be worth the price. Under our chosen data generating mechanisms, implementing each estimator using correct parametric models resulted in unbiased estimation. However, when implemented nonparametrically using the correct set of confounders, both g computation and inverse probability weighting were considerably biased, while both doubly robust approaches were much less biased. Indeed, TMLE was unbiased when the sample size was 600 or larger when the correct set of confounders was used. These results suggest that researchers should carefully weigh the benefits tradeoffs of using nonparametric methods with singly robust approaches.

Nonparametric doubly robust estimators perform better than nonparametric singly robust approaches because of the faster rates at which these estimators converge to the truth. Convergence rates are faster for doubly robust estimators because combining both exposure and outcome models results in a smaller error term, depending on the *product* of errors from each approach. ¹⁹ This

means the doubly robust estimator can achieve parametric-type behavior (e.g., fast \sqrt{N} rates and valid confidence intervals) even under weak nonparametric assumptions. When \sqrt{N} rates are not achievable, modifications of the doubly robust estimator based on "higher order" influence functions can be shown to be optimal. However, at present, implementation of these higher-order estimators is considerably more complex. Nonparametric doubly robust-type methods have been extended to a wide variety of settings, including continuous 48,49 and time-varying exposures, 50 instrumental variables, 51 mediation, 52 and missing data. 53,54

We have shown that, when used with singly robust approaches, nonparametric estimation techniques can yield suboptimal statistical properties. However, this can be ameliorated by using nonparametric methods with doubly robust estimators. In general, the choice between estimators should be motivated by their statistical properties. ⁵⁵ Taking full advantage of the availability of modern nonparametric methods requires implementation of double robust estimation.

Table 1: Component algorithms of the Super Learner library to estimate f() and g() nonparametrically.

Algorithm	Description
ranger	Random forests with 500, 1000, 5000 trees, each with 2 or 3 predictors sampled for splitting at each node.
xgboost	Extreme gradient boosting with 100 and 500 trees, interaction depth of 1 and 2, and shrinkage parameters of 0.1, 0.01, and 0.001.
glmnet	Least absolute shrinkage and selection operator with elasticnet mixing parameter from 0 to 1 by 0.5.
rpart	Classification and regression trees with a complexity parameter of 0.01, and maximum tree depth of 30.
svm	Support vector machine with $v=0.5$, cost parameter = 1, and 3 degree polynomial.
gam	Generalized additive models with 2, 3, 4, and 5 knots.
glm	Generalized linear models with logit link for $f()$ and identity link for $g()$.
SL.Int	Stepwise selection of generalized linear model 2nd order interaction terms.
earth	Multivariate adaptive regression splines with degree $= 2$ and penalty parameter $= 3$.
bayesglm	Bayesian GLM with normally distributed coefficient priors, mean=0, sd = 2.5.
mean	standard mean estimator.

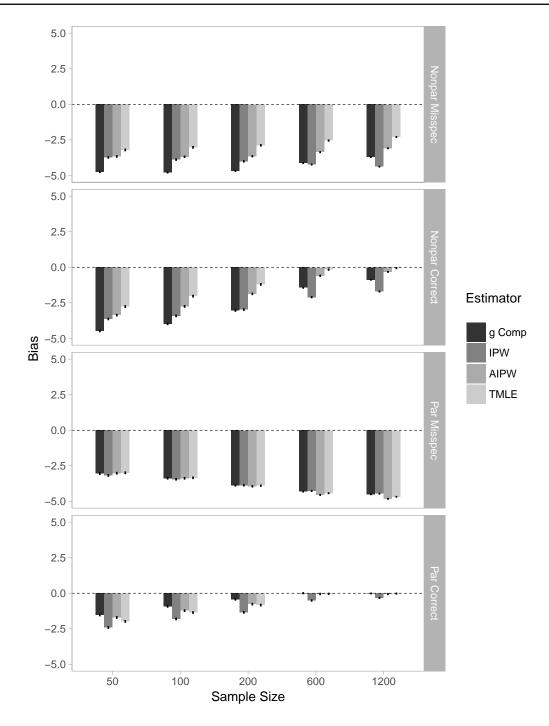


Figure 1: Estimated bias of g computation, inverse probability weighted, and doubly robust estimators for sample sizes ranging from N=50 to N=1,200 observations, when models for each estimator are specified parametrically (correct, mis-specified) using linear regression, and nonparametrically using classification and regression trees.

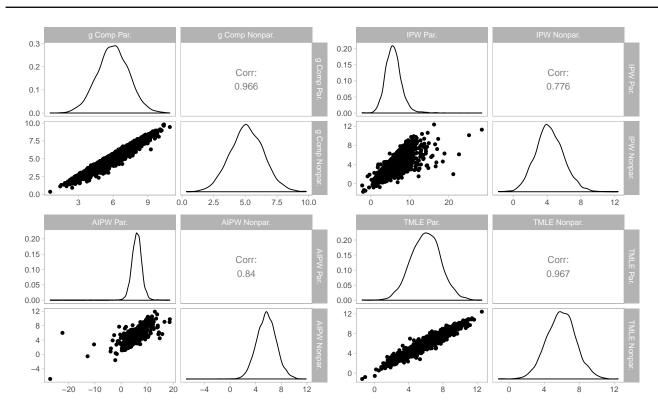


Figure 2: Distribution of point estimates and their correlation for all four estimators when fit parametrically and nonparametrically under N = 1200.

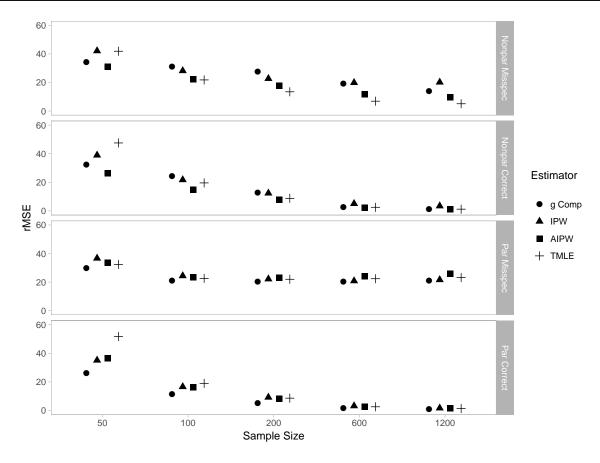


Figure 3: Root Mean Squared Error of g computation, inverse probability weighted, and doubly robust estimators for sample sizes ranging from N=50 to N=1,200 observations, when models for each estimator are specified parametrically (correct, mis-specified) using linear regression, and nonparametrically using classification and regression trees.

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