Analog circuits

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Abstract

In this lab, an FM demodulator was constructed. Based on the components used, the RLC bandpass in the demodulator had a resonance frequency of 1.07 MHz, and a bandwidth of 219 kHz. The RC low-pass filter used to smooth the signal had a rolloff point at 106 kHz. The output of the FM demodulator was then plugged into a common-emitter amplifier, where the ratio of R_C to R_E was 2.32. The FM demodulator was able to successfully extract signals modulated in FM with a 1.045 MHz carrier frequency. Additionally, the speed of propagation of signals in a BNC cable was measured to be 1.9×10^8 m/s.

1 Introduction

Frequency demodulation and amplification contain many important concepts in the basics of analog circuits. A frequency-modulated (FM) signal has the mathematical form [1]

$$y(t) = A_c \cos\left(2\pi f_c t + 2\pi f_\Delta \int_0^t x_m(\tau)\, d\tau\right) \tag{1}$$

where A_c is the carrier amplitude, f_c is the carrier frequency, f_Δ is the maximum frequency deviation, and x_m is the signal being transmitted. Many radio stations are broadcast in FM, and electronic circuits that are able to convert an FM signal to the signal encoded in the FM, such as FM demodulators, are needed. While FM demodulators are useful for extracting signals encoded in FM, in order for signals to be audible, they must go through an amplifier. One common amplifier design, which will be discussed in detail later, is the common-emitter amplifier. The common-emitter amplifier uses a transistor circuit to amplify signals, where the gain is the ratio of the collector resistor to the emitter resistor.

Signals often must be sent from one physical location to another. In order to do this, a transmission line must be used. It is possible for one to point to many different types of cables and say that "this is a transmission line," but the simplest understanding of a transmission line is that it is a cable that transmits electrical energy from one location to another. One very useful property of transmission lines is that their impedance does not depend on the length of the cable. While transmission lines are very useful, one possible problem with them is that reflections on the cable can occur and distort a signal. The amount of wave that gets reflected in a transmission line is governed by the reflection coefficient [2]

$$\Gamma = \frac{Z_{load} - Z_0}{Z_{load} + Z_0} \tag{2}$$

where Z_{load} is the impedance of whatever the cable is plugged into and Z_0 is the impedance of the cable itself. There is also a related coefficient called the transmission coefficient, where $T = 1 + \Gamma$.

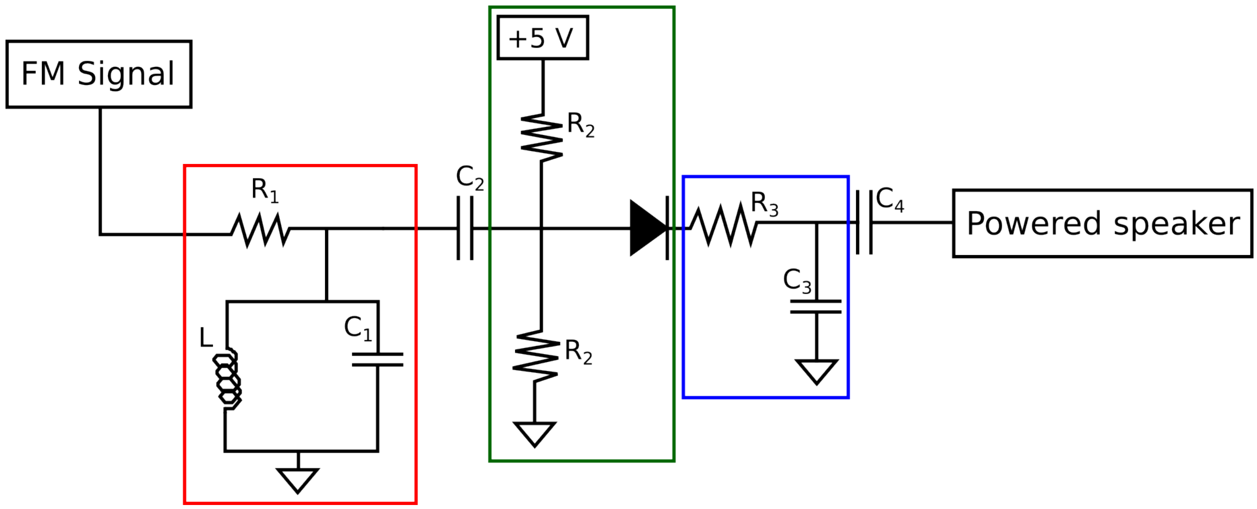

Figure 1: Circuit diagram of the FM demodulator. Different units in the circuit are highlighted with the colored boxes.

One thing that can be deduced from (2) is that if one wants to prevent reflections in a transmission line from occurring, then one must attach a load that is equal to the impedance of the cable being used to transmit a signal. Typical BNC and SMA cables have impedances of about $50\,\Omega$ and termination resistors for these cables are fairly common.

2 The FM Demodulator

The goal of the FM demodulator is to extract an signal from an FM radio wave. The FM demodulator used can be seen in Figure 1. First, an frequency modulated signal is provided by a function generator and an audio player. The signal travels though an RLC bandpass filter, which can be seen in the red box, which converts an FM signal to an AM signal. After that, a DC offset is applied to a signal before going through a diode, which is used to clip the bottom of the signal. Finally, the signal travels through a low-pass filter, denoted by the blue box, which removes high frequency oscillations from the signal, which allows the encoded signal to pass through. Finally, the signal runs though the capacitor C_4, which removes any remaining DC bias from the signal. After that, the signal is fed into either a speaker or an amplifier. In this lab, the carrier frequency of the FM signal is 1.045 MHz.

2.1 The RLC Filter

The RLC filter is the part of Figure 1 that is enclosed in the red box and the purpose of the filter was to convert an FM signal into an AM signal. The response of the RLC filter can be computed using the voltage divider equation as $\left|\frac{Z}{Z+R}\right|$, where Z is the equivalent impedance of the LC parallel circuit. This evaluates to

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\omega L}{\sqrt{R^2\left(\omega^2 LC - 1\right)^2 + \omega^2 L^2}} \tag{3}$$

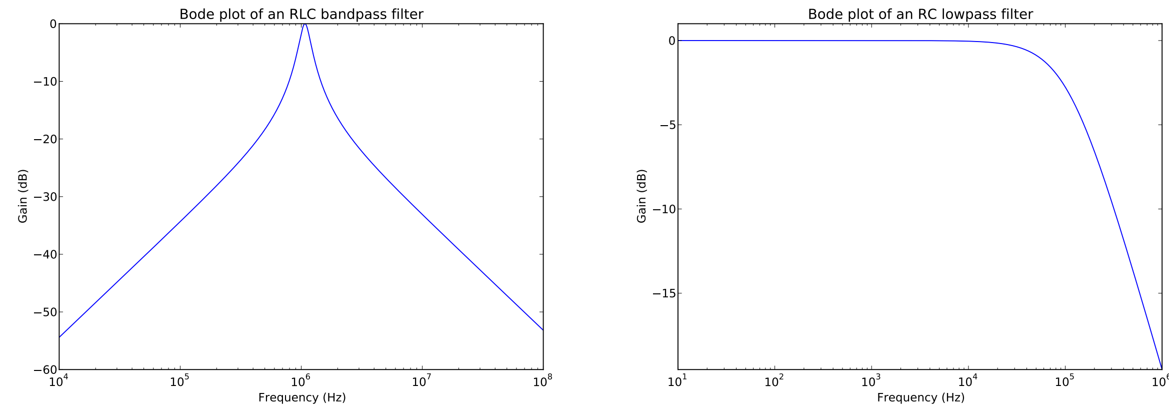

Figure 2: Gain can be expressed in decibels using the formula $20\log_{10}\left|\frac{V_{out}}{V_{in}}\right|$.

where $\omega = 2\pi f$. It can be seen from inspection that (3) has a maximum value of 1 at $f_0 = \frac{1}{2\pi\sqrt{LC}}$, which is the resonance frequency of the circuit. The transfer function for the RLC bandpass can be seen in Figure 2. Since the RLC circuit is a bandpass, there are two roll-off points where the transfer function goes below $-3\,\mathrm{dB}$, which are located at

$$f_{-3\,\mathrm{dB}} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} + \frac{1}{2R^2C^2} \pm \sqrt{\frac{1}{R^2C^3L} + \frac{1}{4R^4C^4}}} \tag{4}$$

The values of R, L, and C are determined by the specifications of the bandpass. The goal was to tune the LC parallel circuit it $1\,\mathrm{MHz}$. Since the only inductors available in the lab were $1\,\mu\mathrm{H}$, the capacitor required for the desired resonance frequency is about $25.3\,\mathrm{nF}$. Since the capacitor with the value closest to what was required for the $1\,\mathrm{MHz}$ resonance was $22\,\mathrm{pF}$, the RLC circuit theoretically had a resonance of $1.07\,\mathrm{MHz}$. The value of R was chosen by using a bandpass of $\Delta f_{-3\,\mathrm{dB}} = 200\,\mathrm{kHz}$ and using the appropriate Q factor to match that condition. Using the fact that $Q = \omega_0 RC = \frac{f_0}{\Delta f_{-3\,\mathrm{dB}}}$ and the values of L and C, the best resistor for the RLC circuit was determined to be a $36\,\Omega$ resistor. The resistor chosen for the circuit was a $33\,\Omega$. Using the actual values of R, L, and C used in the bandpass filter, both (4) and the Q factor equation give the same prediction that $\Delta f_{-3\,\mathrm{dB}} = 219\,\mathrm{kHz}$.

2.2 The Envelope Detector

The green and blue boxes in Figure 1 form an envelope detector. The goal of an envelope detector is to extract a signal from an amplitude-modulated (AM) wave. This can be seen in Figure 3, where the AM signal is in blue and the envelope, which is the signal that is getting extracted, is the red curve. The envelope detector consists of two main parts. The first is the diode, which is in the green box, and the second is the low-pass filter in the blue box.

The use of the diode in the envelope detector is pretty straightforward. Since current can only flow one way though a diode, the diode cuts off the bottom half of the signal. This is desirable, as the envelope to extract is only on the top half of the signal anyways. The part of Figure 1 enclosed in the green box involving a voltage source and the resistors R_2 ($150\,\Omega$ resistors were used in the

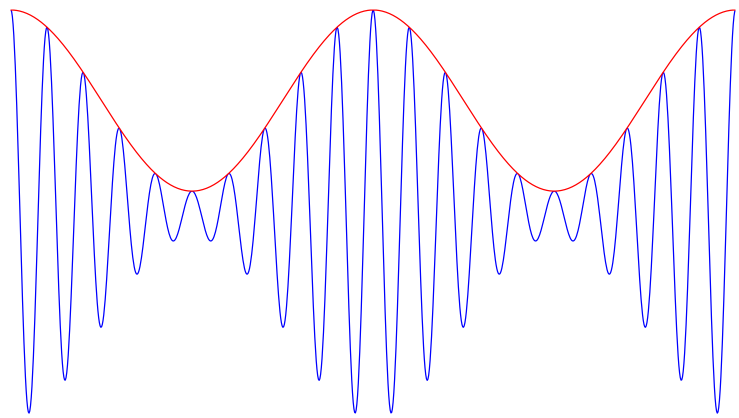

Figure 3: Example of an amplitude-modulated wave (blue) with its envelope (red).

circuit) adds a current bias. This is useful, because there needs to be a large enough voltage drop across the diode in order for current to flow. Typically, there needs to be a 0.6 V voltage drop across the diode in order for it to conduct properly.

The signal that makes it through the diode must then go through a low-pass filter, which can be seen in the blue box in Figure 1. The purpose of the low-pass filter is to remove high-frequency oscillations from the signal it gets in order to smooth out the envelope. The components used for the low-pass filter were a 100 Ω resistor for R_3 and a 10 nF capacitor for C_3. This gives a rolloff frequency of 106 kHz, which will make sure that the 1.045 MHz carrier frequency is absent from the output signal.

3 The Common-Emitter Amplifier

3.1 Amplifier Design

The signal coming out of the FM demodulator is very low, so in order to make it audible, it must be amplified. The amplifier used to amplify the signal before it went to the speaker can be seen in Figure 4. The gain of the amplifier is specified by $|R_C/Z_E|$, where $Z_E^{-1} = R_E^{-1} + X_{C_E}^{-1}$, and $X_{C_E} = \dfrac{1}{jwC_E}$. This evaluates to the gain of the amplifier being

$$g = \frac{R_C}{R_E}\sqrt{1 + \omega^2 R_E^2 C_E^2} \tag{5}$$

where $\omega = 2\pi f$. This gain is useful for amplifying high-frequency signals and is approximately linear for large frequencies. In order to achieve a gain of about 2 when there is no capacitor, R_C was set to 510 Ω and R_C was set to 220 Ω. Due to what was available in the lab, a 1 µF capacitor was used for C_E. Without the capacitor, the gain of the amplifier was 2.32, which is 7.3 dB. The response of the amplifier, in decibels, can be seen in Figure 5. At low frequencies, the response is flat, but starts to increase at frequencies of 500 Hz. This makes the amplifier sort-of suitable for music, where notes are typically between 30 Hz and 4 kHz.

The resistors R_1 and R_2 do not do the gain, but they are important for the amplifier, as they set the bias voltage at the base. The resistors R_1 and R_2 form a voltage divider, which sets the base

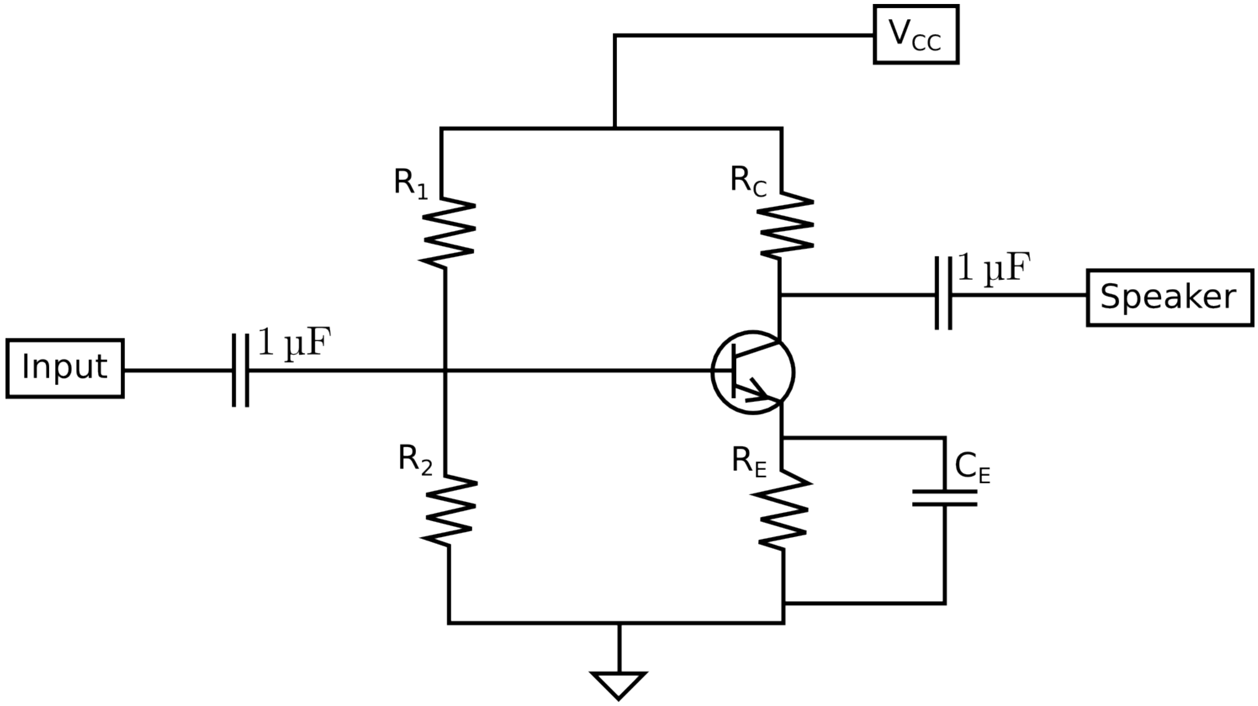

Figure 4: Circuit diagram of the common-emitter amplifier.

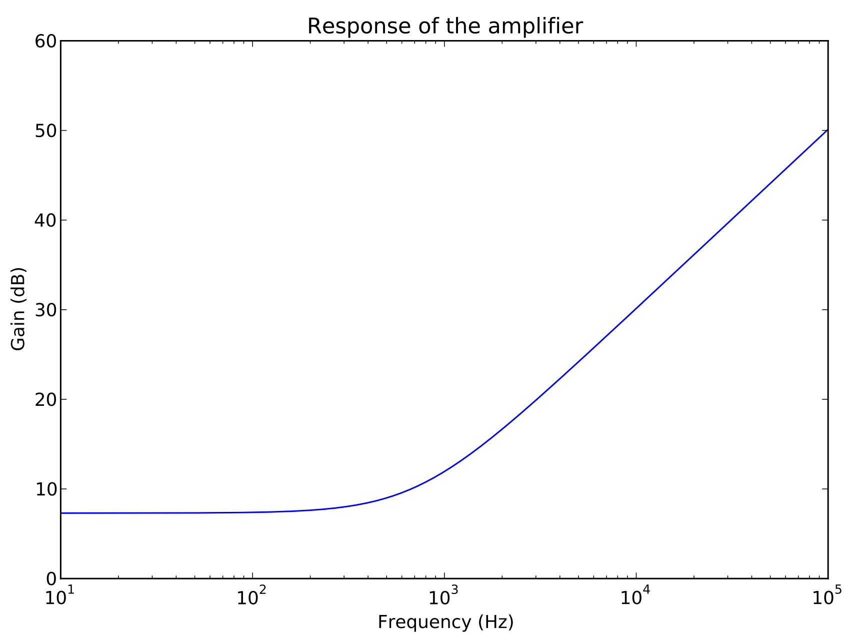

Figure 5: The gain of the amplifier increases as the frequency increases.

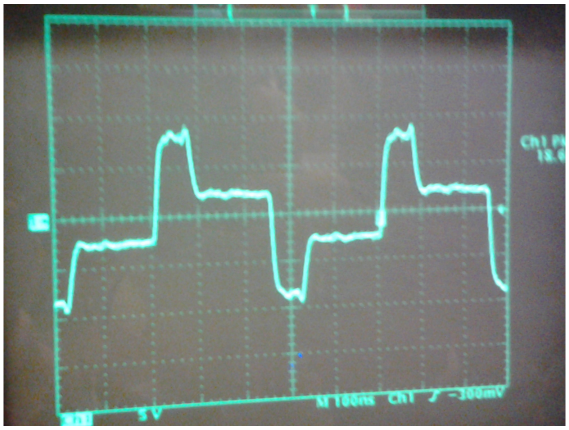

Figure 6: Reflected waves on a long BNC cable. The end of the cable is shorted.

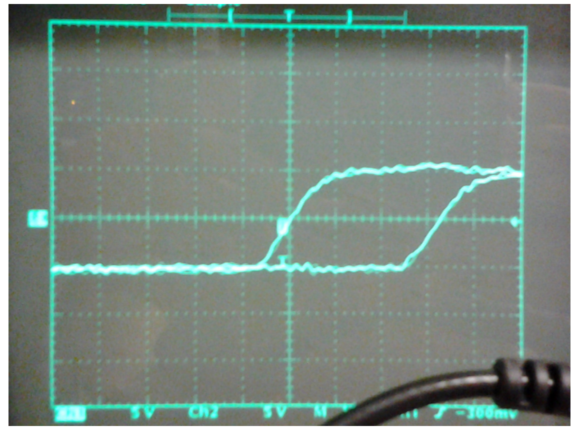

Figure 7: Directly measuring the time it takes for a signal to propagate along the BNC cable.

voltage bias at $\frac{R_2}{R_1+R_2}V_{CC}$, where V_{CC} is typically set at 5 V. Since V_{BE} was measured to be 0.73 V for the transistor used in the amplifier, the base voltage was decided to be given a bias of about 1 V. This was done by setting R_1 to 3900 Ω and R_2 to 996 Ω. R_2 was created from a series of resistors as no 1 kΩ resistors seemed to be available in the lab. Using these resistor values, the base of the transistor was biased to 1.01 V with V_{CC} set at 5 V. The maximum operating amplitude for a 10 kHz wave was measured to be 30 mV. Increasing V_{CC} up to 31.5 V increased the maximum operating amplitude at 10 kHz up to 800 mV.

3.2 Connection the amplifier to transmission lines

Suppose that the amplifier was connected to a very long BNC cable before it was connected to a speaker. When transmission lines are long, reflections of waves along them become more noticeable. In order to deal with this, transmission cables need to be terminated properly. According to (2), if the load impedance is equal to the impedance of the cable, then there will be no reflections along the cable. Audio speakers typically have impedances of 8 Ω, so adding a 42 Ω resistor in series with the speakers should properly terminate the cable so that no waves get reflected.

The effect of reflections can be seen in Figure 6. The cable used was made by combining several BNC cables until there was about 6.22 meters worth of cable. Since the end of the cable is shorted, then according to (2), $\Gamma = -1$, which means that a wave of equal magnitude, but inverted gets reflected back down the BNC cable. However, due to the length of the cable, the reflected wave does not align with the incident wave. There is a time delay due to the amount of time it takes for a signal to travel down a cable. This time delay does not depend on frequency and only depends on the length of the cable.

The following setup was employed to measure the speed of propagation along the BNC cable. First, a properly terminated cable was plugged so that one end was in Channel 1 of the scope and the other end, 6.22 feet of cable in between, was plugged into Channel 2 of the scope. The input waveform was a square wave. It was found that there was a delay of 32 ns between the two channels, which can be seen in Figure 7. Based on these measurements, the speed of propagation along the BNC

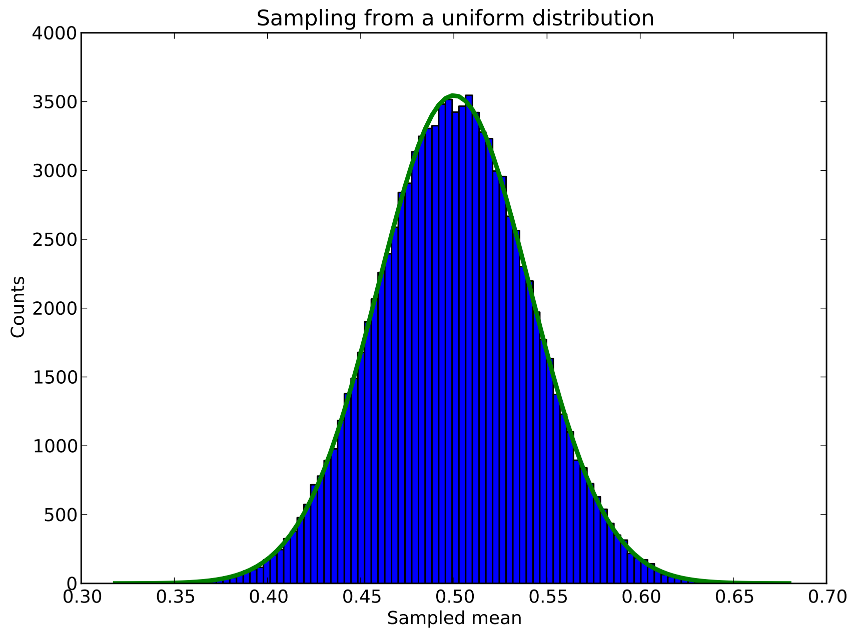

Figure 8: Demonstration of the central limit theorem. The distribution that the sampled means were taken from was a uniform distribution from 0 to 1. The sample size for each mean was 50 and 100,000 samples were taken. The green line is a Gaussian curve where the mean is the expected mean of a uniform distribution and the standard deviation is the expected standard deviation divided by the square root of the sample size.

cable is approximately $1.9 \times 10^8\,\mathrm{m/s}$. Since the speed of light in a material is

$$v = \frac{1}{\sqrt{\epsilon\mu}} \approx \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_0}} \tag{6}$$

it can be concluded that the relative permittivity, ϵ_r, for the BNC cable is about 2.4.

4 Noise

It is impossible to build a circuit without noise. The thermal motion of electrons produces noise, known as Johnson-Nyquist noise, which produces noise power of $P = 4k_BTB$ [3], where K_B is Boltzmann's constant, T is the temperature, and B is that bandwidth of the system. If one wanted to find the noise of the amplifier in Figure 4, then one would only need to calculate or measure the operating bandwidth of the circuit. In Figure 4, R_2 and the $1\,\mu\mathrm{F}$ capacitor form a high-pass filter. Since R_2 was $996\,\Omega$, the lower cutoff on the amplifier is $160\,\mathrm{Hz}$. The higher cutoff on the amplifier is probably determined by some frequency on the gain of the amplifier where V_{CC} cannot provide enough voltage to amplify the signal or possibly something with the physical limits of the transistor.

It is fairly straightforward to measure the noise of a resistor. The RMS voltage from Johnson-Nyquist noise in a resistor is $V_{rms} = \sqrt{RP} = \sqrt{4k_BTBR}$. The noise voltage for a $50\,\Omega$ at room temperature with a $100\,\mathrm{MHz}$ bandpass is $1.3\,\mu\mathrm{V}$. In order for this noise to be detected on an oscilloscope, it needs to be amplified above $1\,\mathrm{mV}$. This requires an amplification $58\,\mathrm{dB}$. Unfortunately,

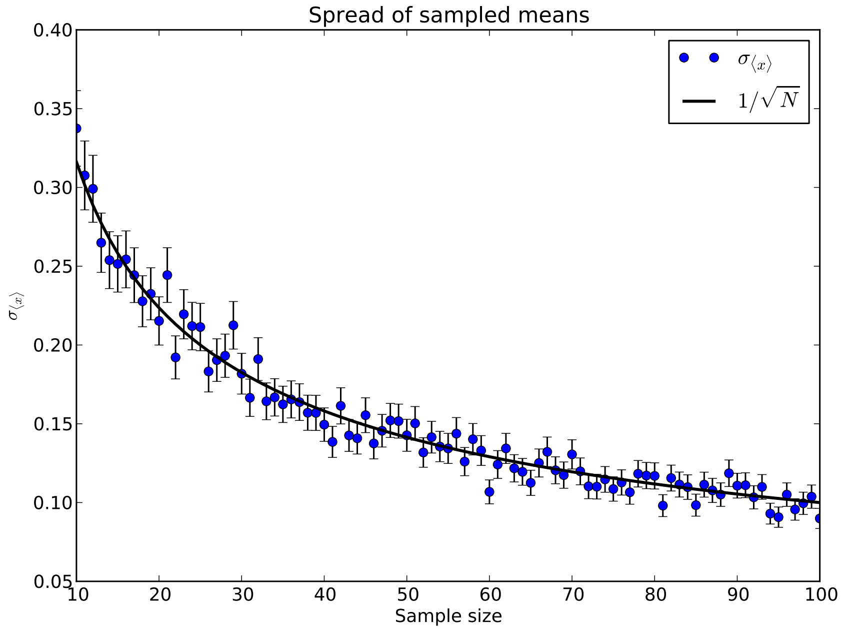

Figure 9: The standard deviation of a mean drops as $1/\sqrt{N}$. In this plot, the sample size used to sample the mean of a standard normal distribution is varied. The standard deviation of the sampled means is calculated from 100 sampled means for every sample size.

this is not currently achievable in the astronomy lab, since only one amplifier is currently working, and it has a gain that is in the ballpark of 25 dB. That amplifier, by itself, will only raise the noise level to about 20 mV. That voltage level cannot be measured with the oscilloscope.

Suppose that a noise source was placed in front of an amplifier, and suppose that the distribution of noise measurements is known well. In this example, a noise source with a uniform distribution between 0 and 1 in some system of units will be considered, although because of the Central Limit Theorem, the shape of the distribution doesn't matter. If the mean noise level is determined by sampling 50 times, then after computing 100,000 sampled means, the distribution of mean noise levels should look like Figure 8. If the sample size is increased, then the spread of the sampled means should decrease as $1/\sqrt{N}$, where N is the sample size used in computing the mean.

An example of how changing the sample size affects the standard deviation of the sampled means can be seen in Figure 9. In that figure, the distribution sampled from was a standard normal distribution, although because of the Central Limit Theorem, the standard deviation dropping as $1/\sqrt{N}$ does not depend on the distribution being sampled from. Figure 9 shows the expected results of varying the sample size when sampling a mean. The error bars in Figure 9 come from the fact that the percent error on standard deviation is $1/\sqrt{2N-2}$ [4], where in this case, N is the amount of sampled means used to compute the standard deviation, as opposed to the sample size used for computing the means.

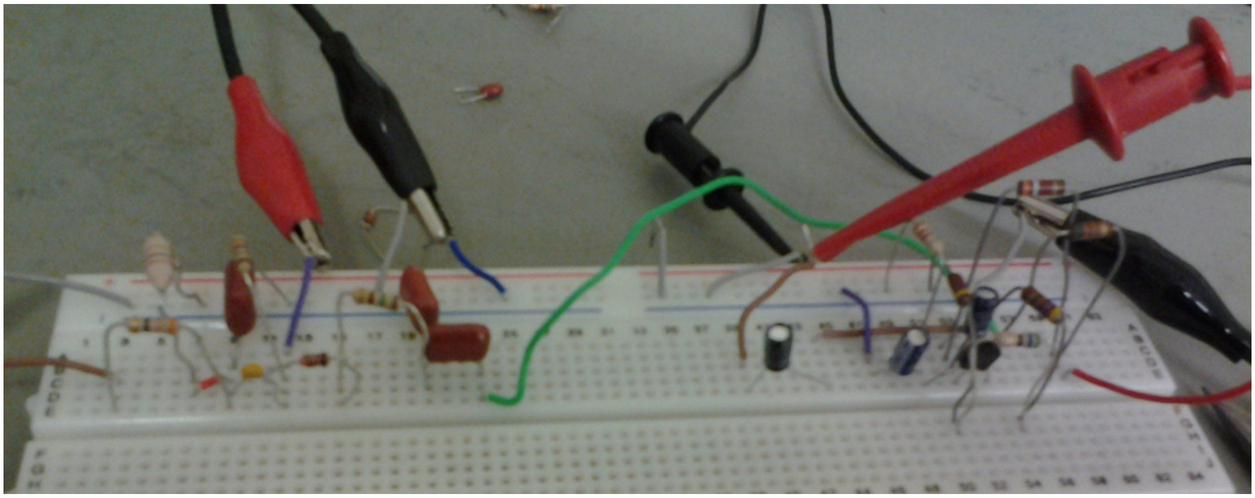

Figure 10: The FM demodulator and the amplifier together. The FM demodulator is the circuit on the left and the amplifier is on the right.

5 Conclusion

Building the FM demodulator and amplifier seemed to be a success. A picture of the full circuit can be seen in Figure 10. The demodulator was able to extract music from an FM signal at 1.045 MHz and the amplifier was able to amplify the extracted music so that it was audible when played through a speaker. Of course, there are ways in which the design of the circuit could be improved. Figure 5 shows that the gain of the amplifier is not uniform in frequency. It would be more desirable to have an amplifier where the gain is relatively flat in the range of frequencies that humans can hear. Another thing to possibly be resolved is that the frequency regime for which the amplifier works is not totally known. The lower bound of this regime can be deduced from a high-pass filter in the amplifier, but the upper bound on the frequency regime still needs to be worked out. Another interesting result demonstrated in this lab was the speed of propagation along the BNC cable, and thus the relative permittivity can be easily determined from measuring the time delay of waves along a long BNC cable. Finally, the quick study of noise shows that statistics works. Unfortunately, measurements of the noise of a resistor could not be made due to broken lab equipment. With the exception of the broken amplifiers, this lab seemed to be a success.

6 Acknowledgments and Notes

The lab group that built the FM demodulator and amplifier also consisted of Leonardo Sattler Cassará and David Galbraith. The code used to generate the plots found in this document can be found on Github at https://github.com/isaacdomagalski/astro121-radio-lab/ [5].

References

[1] Wikipedia. Frequency Modulation, 2014.
https://en.wikipedia.org/wiki/Frequency_modulation.

[2] AstroBaki. Termination, 2014.
https://casper.berkeley.edu/astrobaki/index.php/Termination.

[3] Wikipedia. Johnson-Nyquist Noise, 2014.
https://en.wikipedia.org/wiki/Johnson_noise.

[4] I.G. Hughes and T.P.A Hase. *Measurements and their Uncertainties*. Oxford University Press, 2010.

[5] I. Domagalski, 2014.
https://github.com/isaacdomagalski/astro121-radio-lab/tree/master/lab-analog.