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AERE 504 Intelligent Air Systems
Final Take-Home Exam

#1 What is the size of the state space?

The state space is the product of the number of options of r, h and t. So the size of the state space is $2 \times 21 \times 11 = 462$.

#2 What is the size of the observation space?

After each action we know that t decreases by one, and we can't directly observe if aircraft B is responsive or not. We can only observe the new h. So the size of the observation space is 21.

#3 What is the dimensionality of our belief state?

The only unobservable part of the state space is the responsiveness r of aircraft B, so the dimensionality of the belief space is 1. The belief state will be a single parameter p which will represent the probability that $r = 0$, and thus $1 - p$ is the probability that $r = 1$.

#4 Assume our initial belief is uniform over all states with $t = 10$. After the first observation, how many components of the belief vector will be non-zero?

After the first observation we know h and t exactly, so the only unknown is r. If we started with an uniform belief that would mean that intially we assign each possibility probability 0.5. One observation won't change this probability to zero. This means that the only component of the belief vector will be non-zero.

#5 Suppose we have a belief b that assigns probability 1 to state $[1, 10, 1]^T$; what is $Q^*(b, a_{+1})$ (assume $= -0.5$)? Provide an exact numerical value and explain.

#6 Suppose we have a belief b that assigns probability 1 to state $[0, 10, 1]^T$; what is $U^*(b)$ (assume $\lambda \leq 0$)? Provide an exact numerical value and explain.

$$
\begin{aligned}
U^*(b) &= \max_a \left\{ \sum_s \left(b(s)R(s, a) \right) \right\} \\
&= \max_a \left\{ R([0, 10, 1]^T, a) \right\} \\
&= \max_a \left\{ R([0, 10, 1]^T, a) \right\}
\end{aligned}
$$

#7 Is it possible for $U^*\left([r, h, t]^T\right) \neq U^*\left([r, -h, t]^T\right)$ for some λ, r, h, and t? If so provide an example. If not, provide a simple explanation.

#8 As $\lambda \to -\infty$, what is $\min_s\{U^*(s)\}$? Why?

#9 Suppose we have a belief b that assigns probability 1 to state $[0, 9, 0]^T$. State an action that will maximize $Q^*(b, a)$ when $\lambda = 5$. Is it unique?

#10 Draw a two-step conditional plan from the state $[0, 1, 10]^T$ where the action associated with the root node is a_0. Only show the observation branches that have a non-zero probability of occuring.

#11 If we are using the fast informed bound (FIB) to approximate the optimal value function, how many alpha vectors will there be?

#12 If α_{QMDP} is an alpha vector generated by QMDP and α_{FIB} is an alpha vector generated by FIB, can there exist a b such that $b^T\alpha_{QMDP} \leq b^T\alpha_{FIB}$? Why or why not?

#13 Suppose we have a belief state b that assigned probability 0.5 to $[0, 0, 1]^T$ and probability 0.5 to $[1, 0, 1]^T$. What is the value for $U^*(b)$ in terms of λ (which may take on any negative value)?

#14 Why would you not use a particle filter to update your belief for this problem?

You would not use a particle filter to update your belief for this problem, because the state space is not particularly large or continuous. A particle filter is sampling approach that uses particle to sample the state space. In this case the state space is small enough to be enumerated and so sampling isn't necessary.

#15 Suppose your initial belief is uniform over the state space and then you observe that aircraft A is 3 units above aircraft b after executing a_0. What probability would an exact Bayesian update of your belief state assign to aircraft B being non-responsive? Why?

#16 Write a little paragraph about what you learned in this class.