AerE/ME 546 **Homework 3**

Due two weeks from 9/25/17

Exercise 1, *1-D Diffusion equation by Euler explicit*

A slab of metal is initially at uniform temperature. One end is suddenly raised to a high temperature, while the other end is kept cool. Compute the penetration of heat into the slab as a function of time.

In dimensional form the temperature diffusion equation, initial and boundary values are

$$\frac{\partial T^*}{\partial t_*} = \kappa \frac{\partial^2 T^*}{\partial x_*^2}; \qquad T^*(x,0) = 0; \qquad T^*(0,t) = 0;\ T^*(L,t) = T_w.$$

Non-dimensionalize temperature by T_w and length by L and time by L^2/κ.

Integrate by **Euler Explicit**, up to a non-dimensional time of 0.3. Use $N_x = 121$ grid points in x. Let $\Delta t = \alpha \Delta x^2$. Try a value of a value of $\alpha > 0.5$. What happens? Why?

How small must α be to obtain an accurate solution?

Provide a *single* figure with line plots of the solution at time intervals of 0.04. Note that the computational time-step will be smaller than 0.04.

The bulk heat transfer coefficient is defined as

$$h_T = \frac{Q}{T(1) - T(0)}$$

where $Q = \partial T/\partial x(1)$ is the heat flux into the slab. Plot h_T as a function of time **for $\boldsymbol{t > 0.01}$**.

Exercise 2, *1-D Diffusion equation by RK2*

A slab is heated by shining a laser on it. The laser is shut off and heat diffuses through out the slab. Its ends are insulated. This is modeled as the non-dimensional problem: solve

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\kappa \frac{\partial T}{\partial x}\right]$$

in the interval $0 \le x \le 1$, with the initial condition

$$T(x,0) = \frac{e^{-(x-0.5)^2/\sigma^2}}{\sigma\sqrt{\pi}},\ \sigma = 0.1$$

The slab length is normalized to unity and σ characterizes the region heated by the laser. Consider a material with variable diffusivity. Let

$$\kappa = 0.1 + 2.0e^{-5x}$$

(non-dimensional).

The no-flux boundary condition

$$\frac{\partial T}{\partial x}(0,t) = 0 = \frac{\partial T}{\partial x}(1,t)$$

is applied at the insulated ends. Use **second order Runge-Kutta**. Solve with about $N_x = 250$ grid points in x.

Choose a small time-step to obtain an accurate solution. Integrate up to a non-dimensional time of 0.05.

Provide a *single* plot containing the initial condition and curves showing the solution $T(x)$ at time intervals of 0.01. Plot $\int_0^1 T(x)dx$ versus time. What should the value of this integral be?

Exercise 3, *1-D Diffusion equation by implicit, Crank-Nicholson method*

Now consider the case where one end of the slab is insulated and the other is held at constant temperature:

$$\frac{\partial T}{\partial x}(0) = 0\,; \quad T(1) = 1$$

Solve the constant diffusivity, diffusion equation as in the first problem, but use **Crank-Nicholson**.

a) Set $\Delta t = \alpha \Delta x^2$. Try a couple of relatively large values of α and see whether your calculation converges, or blows up (should it?).

b) Provide a single figure with plots of $T(x)$ at intervals of 0.04 up to $t = 0.4$. Explain why your solution makes sense.

The bulk heat transfer coefficient is defined as

$$h_T = \frac{Q}{T(1) - T(0)}$$

where $Q = \partial T/\partial x(1)$. Plot h_T as a function of time for $t > 0.01$.

Exercise 4, *Von Neuman stability analysis*

a) Is the scheme

$$U_i^{n+1} = U_i^n - \frac{C}{2}(U_{i+1}^n - U_{i-1}^n)$$

stable, conditionally stable or absolutely unstable? C is a constant

b) Is the scheme

$$U_i^{n+1} = U_i^n - \frac{C}{2}(U_{i+1}^{n+1} - U_{i-1}^{n+1})$$

stable, conditionally stable or absolutely unstable? C is a constant

c) Which method would be called *implicit*?