AerE 546 **Homework 5** 10/20/15

Due in two weeks

Exercise 1, *Laplace Equation by Gauss-Seidel*

Solve the Laplace equation

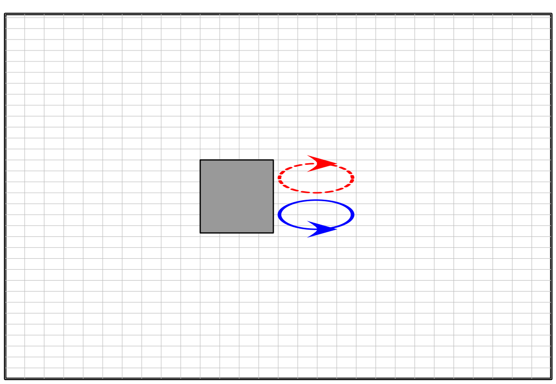
$$\partial_x^2 \psi + \partial_y^2 \psi = 0$$

with $\psi = 0$ on $x = 0$; $\psi = 0$ on $x = 1$; $\psi = 0$ on $y = 1$ and $\psi = \sin(3n\pi x)$ on $y = 0$. Obtain numerical solutions with $n = 1$ and $n = 3$.

Use Gauss-Seidel, with SOR. Stop either when the modulus of the residual either has dropped by a factor of 10^{-4}, or after 2,000 iterations.

- Provide the algorithm part of your code.
- Provide contour plots of streamfunction for each of 15×15 and 151×101 grids, for both n values.
- For the 151×101 grid and $n = 1$, on a single graph, plot residual versus iteration for each of the relaxation parameter values $\lambda = 1$, $\lambda = 0.5$, $\lambda = 1.25$ and $\lambda = 1.9$. Plot as $\log(residual/residual_0)$ versus iteration number. (Use the L_2 residual defined at the end of the next problem.)

Exercise 2, *Potential flow round a square*



Solve incompressible flow,

$$\nabla^2 \Psi = \omega$$

in the domain $0 \leq x \leq 2$, $0 \leq y \leq 1$. A 0.3×0.3 square lies in $0.35 \leq y \leq 0.65$, $0.7 \leq x \leq 1$. The vorticity is

$$\begin{aligned} \omega &= \ 50,\ 0.35 \leq y \leq 0.5,\ 1 \leq x \leq 1.3 \\ \omega &= \text{-}50,\ 0.5 \leq y \leq 0.65,\ 1 \leq x \leq 1.3 \end{aligned}$$

The boundary conditions are

$\Psi = -y + 0.5$	on $x = 0$ and $x = 2$ for $0 \leq y \leq 1$
$\Psi = \ 0$	on (and in) the rectangle
$\Psi = \ 0.5$	on the lower wall, y=0, and
$\Psi = -0.5$	on the upper wall, y=1

The inlet and exit condition, $\Psi = -y + 0.5$, corresponds to flow in the x-direction with unit velocity.

Use a 200×181 point, uniformly spaced, Cartesian grid. Represent the rectangle by *i-blanking*: that is, set the implicit matrix, $\boldsymbol{A}$, to the identity matrix and $\Delta\Psi = 0$ at all points inside and on the surface of the rectangle.

Solve by Gauss-Seidel with SOR. Iterate until $||\Delta\Psi|| < 10^{-4}||\Delta\Psi||_0$, where the L_2 norm is

$$||\Delta\Psi|| = \sqrt{\Sigma_{i=1,I}\Sigma_{j=1,J}(\Delta\Psi_{ij})^2/(I \times J)}$$

and $||\Delta\Psi||_0$ is the initial correction.

- Provide the algorithm part of your code.
- Provide contour line plots of Ψ