

**Exercise 1, Heat fin**

The heat fin equation is the linear o.d.e.

$$\frac{d^2T}{dx^2} = MT$$

where  $M$  is a sort of thermal mass. First write the finite difference equation in terms of a tridiagonal matrix. Solve that equation using the Thomas algorithm (Gaussian elimination) for:

**a)** Compute a solution with the boundary conditions  $T(0) = 1$ ,  $T(1) = 0$ . This corresponds to a fin that is between a hot and a cold reservoir. In non-dimensional terms, the heat flux into the cold reservoir is  $-dT/dx$  at  $x = 1$ . Obtain the heat flux at  $x = 1$  for  $M = 1, 5, 9$ . Use enough grid points to obtain 1% accuracy. Provide your three numerical values of the heat flux. Provide a single graph with curves of  $T(x)$  for the 3 values of  $M$ .

**b)** Compute a solution with the boundary conditions  $T(0) = 1$ ,  $dT(1)/dx = 0$ . This corresponds to a fin that is insulated at one end. Solve for the temperature,  $T(1)$ , at the insulated end for  $M = 1, 5, 9$ . Provide your three numerical values of  $T(1)$ .

Also plot  $T(x)$  for  $M = 9$  with each pair of boundary conditions and compare to the exact solution.

**c)** Add a distributed heat source: Compute and plot a solution of the non-homogeneous equation

$$\frac{d^2T}{dx^2} = MT - 100x^2(1-x)^2$$

with  $M = 9$ ,  $T(0) = 1$ ,  $dT(1)/dx = 0$ .

**Exercise 2, Equation types**

i) What type of p.d.e. is

$$\frac{\partial^2 \phi}{\partial x \partial y} + \phi = 25 \quad ?$$

ii) What type of p.d.e. does the velocity potential,  $\phi$ , satisfy if

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

with

$$u = \frac{\partial \phi}{\partial x} ; \quad v = \frac{\partial \phi}{\partial y} \quad ?$$

iii) The boundary layer momentum equation is

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{Re} \frac{\partial^2 u}{\partial y^2}$$

where  $Re$  is the Reynolds number. What type is this equation?