COURSE SYLLABUS

AerE/ME 546 Computational Methods for Fluid Dynamics

Professor: Paul Durbin, Room 2237 Howe Hall, 294-7234, durbin@iastate.edu

Office Hours: 1:00-3:00 MW

Class Hours: 2:10 ~ 3:25 pm, TuTh
Classroom: Room 1226 Howe Hall

Textbook: Pletcher, R.H., Tannehill, J.C. and Anderson, D.A. Computational Fluid Mechanics and Heat Transfer, 3nd Edition, Taylor & Francis Book Company, Washington, DC, 2013.

Course Grade:	Mid-Term Exam	~15%
	Final Exam	~35%
	Homework	~50%

Students may consult with each other on homework, but **each student must hand in a complete, independent assignment**. It will include a listing of (relevant parts of) computer programs. This is an important part of the course and the student is encouraged to spend time debugging codes until they work. *Late homework will not be accepted*.

The physical motivation behind governing equations will be discussed for students with various backgrounds.

Homework assignments will be posted on **BlackBoard**.

Scope: approximately part 1: chapters 1-4.

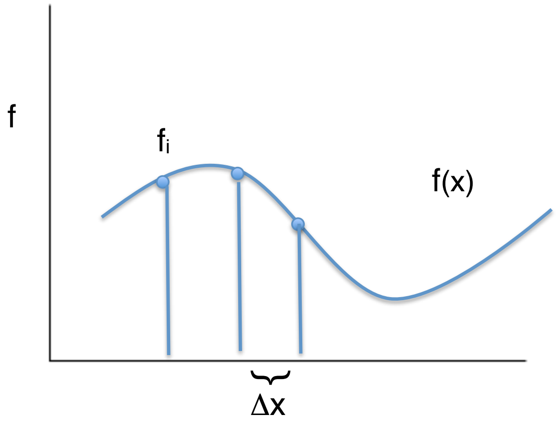
Numerical methods for p.d.e.'s (start with o.d.e.'s, but they are used in NS solvers). Not CFD, per se: that will be cited descriptively at times; but could be geophys, electromag., solid mech. (but not finite element -- finite diff & finite vol.)

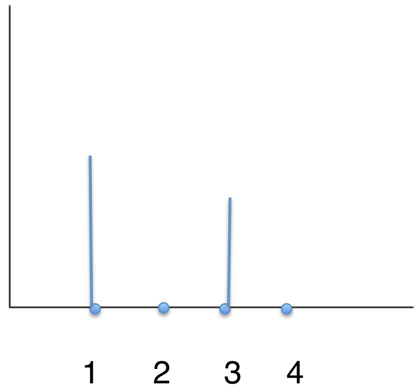
Lecture Topics

- Introduction to discrete analysis
- Introduction to Ordinary Differential Equations
  - Classification
  - Initial Value Problems
  - Boundary Value Problems
- Numerical methods for ODE's
  - Euler, Runge-Kutta, Adams-Bashforth
- Introduction to Partial Differential Equations
  - Classification and Mathematical Properties
  - Qualitative properties
  - Model Equations
- Parabolic equations
  - Heat conduction, 1 and 2-D
  - Temporal discretization
  - Factorization and integration methods
- Elliptic equations
  - Irrotational flow
  - Iteration methods, acceleration of convergence
  - Curvilinear geometry
- Hyperbolic equations
  - Linear and nonlinear waves
  - Explicit methods
  - Upwinding, shock capturing
- Grid generation
  - Computational and physical space
  - Algebraic methods
  - Change of coordinates
- Other Considerations (within previous topics)
  - Stability, and Convergence
  - Numerical errors; Order of accuracy
  - Computational complexity

A. Discretization. Calculus in reverse: p.d.e. → discrete

$f(x)$ -> in computer $f(x(i))=f_i$ Store f_i (& x_i) at $f(i)$





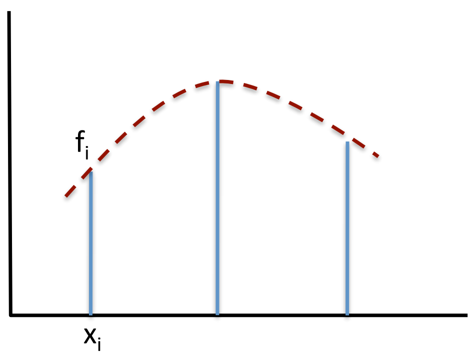
(*c.f. mean value*)

or

$$\frac{df}{dx} \approx \frac{\delta f}{\delta x}\Big|_2 = \frac{f_3 - f_1}{x_3 - x_1} \qquad\qquad \frac{f_2 - f_1}{x_2 - x_1}$$

Discrete formula is not unique, for finite Δx. Order of accuracy; stability; dispersive/dissipative; conservation/convection form; we will see.

B. Interpolation *or reconstruction*



given $f_i \leftrightarrow f(x)$

1) Polynomial interpolation

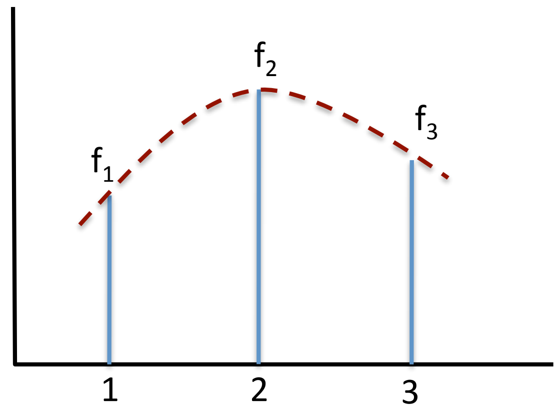
$f = a + bx + O(x^2)$ need two data points

aside on order of accuracy, NB x/L<<1

$f_1 = a + b\, x_1$
$f_2 = a + b\, x_2$
$b = (f_2 - f_1) / (x_2 - x_1)$ <= finite difference for $\delta f/\delta x$

$f = a + bx + c\, x^2 + O(x^3)$ need 3 points
$f_0 = a$
$f_1 = a + b\, \Delta x + c\, \Delta x^2$
$f_2 = a + 2b\, \Delta x + 4c\, \Delta x^2$
$b = (4f_1 - f_2 - 3\, f_0) / 2\Delta x$ <= one-sided formula for $\delta f/\delta x$ at $x=0$

2) Lagrange interpolation
   Get polynomial w/o solving equations
   Define interpolation polynomials



$L_1 = 1$ @ 1
$L_1 = 0$ @ 2,3

$$L_1 = \frac{(x_2 - x)(x_3 - x)}{(x_2 - x_1)(x_3 - x_1)}$$

Similarly for L_2 and L_3. Evaluate in subroutine, then

$$f(x) = f_1 L_1 + f_2 L_2 + f_3 L_3$$

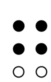
$$f(x) = \sum f_n L_n \qquad \text{order} = N-1$$

Linear case $$L_1 = \frac{(x_2 - x)}{(x_2 - x_1)}; \quad L_2 = \frac{(x - x_1)}{(x_2 - x_1)}$$

$$f = f_1 L_1 + f_2 L_2 = \frac{(x_2 - x)}{(x_2 - x_1)} f_1 + \frac{(x - x_1)}{(x_2 - x_1)} f_2$$

$$= \underbrace{\frac{f_2 - f_1}{\Delta x}}_{\delta f / \delta x} x + \frac{f_1 x_2 - f_2 x_1}{\Delta x}$$

Similarly $$\int_{x_1}^{x_2} f dx = \int_{x_1}^{x_2} f_1 L_1 + f_2 L_2 dx = \frac{f_1 + f_2}{2} \Delta x$$; Trapezoidal rule

(Bi-linear reconstruction in 2-D f=a+bx+cy+dxy )

3) Other interpolation methods (not covered)
   Fourier series/Tchebychev, rational polynomials (Pade), least squares...

C. Taylor series

1. Order of accuracy e.g. if error goes to zero like Δx^3 method is second order accurate.
   Consistency: error -> 0 as Δx -> 0 ($\Delta x/L$ -> 0; discuss)
   But, how fast? Answer, like Δx^n so method is accurate to n-1.

2. Taylor series with remainder:

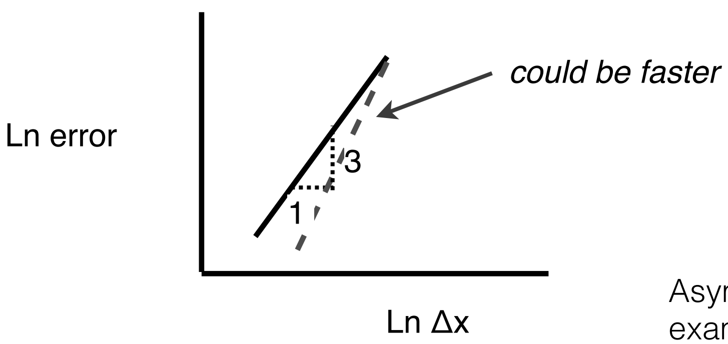
$$f(x) = f(0) + xf'(0) + \frac{1}{2}x^2 f''(0) + \frac{1}{3!}x^3 f'''(sx), \quad 0 \le s \le 1$$

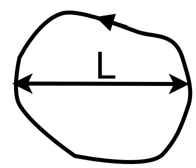
$||f'''||$ bounded

written as

$$f(\Delta x) = f(0) + \Delta x f'(0) + \frac{1}{2}\Delta x^2 f''(0) + O(\Delta x^3)$$

called second order accurate; means error goes to zero at least as Δx^3 : asymptotic error





$L \sim \Delta x$

Asymptotic error: $L/\Delta x$ -> 0. LES counter example

3. Discrete derivatives via Taylor series

$$f_1 = f_0 + f_0'\Delta x + \frac{1}{2}f_0''\Delta x^2 + O(\Delta x)^3$$
$$f_{-1} = f_0 - f_0'\Delta x + \frac{1}{2}f_0''\Delta x^2 + O(\Delta x)^3$$

Add:

$$f_1 + f_{-1} - 2f_0 = 0 + f_0''\Delta x^2 + O(\Delta x^3)$$
$$\frac{\delta^2 f_0}{\delta x} = \frac{f_1 - 2f_0 + f_{-1}}{\Delta x^2}$$

Second order accurate because f is. (Can be a point of confusion)

Subtract $\delta f_0/\delta x = (f_1 - f_{-1})/ 2\Delta x$