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AER E 546 Fluid Mechanics and Heat Transfer I
Homework 2

1. The heat fin equation is the linear o.d.e.

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = MT$$

where M is a sort of thermal mass. First write the finite difference equation in terms of a tridiagonal matrix. Solve that equation using the Thomas algorithm (Gaussian elimination) for:

(a) Compute a solution with the boundary conditions $T(0) = 1$ and $T(1) = 0$. This corresponds to a fin that is between a hot and a cold reservoir. In non-dimensional terms, the heat flux into the cold reservoir is $-\frac{\mathrm{d}T}{\mathrm{d}x}$ at $x = 1$. Obtain the heat flux as $x = 1$ for $M = 1, 5, 9$. Use enough grid points to obtain 1% accuracy. Provide your three numerical values of the heat flux. Provide a single graph with curves of $T(x)$ for the 3 values of M.

(b) Compute a solution with the boundary conditions $T(0) = 1$, $\frac{\mathrm{d}T(1)}{\mathrm{d}x} = 0$. This corresponds to a fin that is insulated at one end. Solve for the temperature, $T(1)$, at the insulated end for $M = 1, 5, 9$. Provide your three numerical values of $T(1)$. Also plot $T(x)$ for $M = 9$ with each pair of boundary conditions and compare to the exact solution.

(c) Add a distributed heat source: Compute and plot a solution of the non-homogeneous equation

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = MT - 100x^2(1-x)^2$$

with $M = 9$, $T(0) = 1$ and $\frac{\mathrm{d}T(1)}{\mathrm{d}x} = 0$.

2.