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AER E 546 Fluid Mechanics and Heat Transfer I
Homework 5

1. Solve the Laplace equation

$$\partial_x^2 \psi + \partial_y^2 \psi = 0$$

in the domain $0 \leq x, y \leq 1$, with

$$\begin{aligned} \psi &= \sin^2(3n\pi y) \text{ on } x = 0 \\ \psi &= \sin^2(3n\pi x) \text{ on } y = 0 \\ \psi &= 0 \text{ on } x = 1 \\ \psi &= 0 \text{ on } y = 1. \end{aligned}$$

Obtain numerical solutions with $n = 1$ and $n = 3$.

Use Gauss-Seidel with SOR. Stop either when the modulus of the residual either has dropped by a factor of 10^{-4} or after 2500 iterations.

- Provide the algorithm part of your code.
- Provide contour plots of streamfunctions for each of 15×15 and 151×101 grids for both n values.
- For the 151×101 grid and $n = 1$, on a single graph, plot residual versus iteration for each of the relaxation parameters $\lambda = 1$, $\lambda = 0.5$, $\lambda = 1.5$, and $\lambda = 1.95$. Plot as $\log_{10}(residual/residual_0)$ versus iteration number. Use the L_2 norm

$$residual = \|\Delta\Psi\|_{L_2} = \sqrt{\sum_{i=1}^{I} \left(\sum_{j=1}^{J} \left((\Delta\Psi_{ij}^2)/(I \times J) \right) \right)}$$

2. Solve the Poisson Equation

$$\partial_x^2 \psi + \partial_y^2 \psi = 70 e^{-32(x-0.5)^2 + (y-0.1)^2}$$

in $0 \leq x, y \leq 1$, with boundary conditions

$$\begin{aligned} \psi(0, y) &= -y \\ \psi(1, y) &= -y \\ \psi(x, 0) &= 0 \\ \psi(x, 1) &= -1 \end{aligned}$$

on the grid

$$\begin{aligned} x(j) &= \frac{e^{2(j-1)/(J-1)} - 1}{e^2 - 1} \quad 1 \leq j \leq J \\ y(j) &= \frac{e^{2(k-1)/(K-1)} - 1}{e^2 - 1} \quad 1 \leq k \leq K. \end{aligned}$$

- Provide the algorithm part of your code.
- Provide a plot of the grid and a line-contour plot of the converged solution.

3. Solve the Poisson equation

$$\partial_x^2\psi + \partial_y^2\psi = 2000(\sin(4\pi x)\sin(4\pi y))^5$$

on the domain $0 \le x \le 1$, $0 \le y \le 1$, with boundary conditions

$$\begin{aligned}
\psi(x,0) &= \sin^2(\pi x) + 5\sin^2(4\pi x)10\sin^2(8\pi x) \\
\psi(0,y) &= \sin^2(4\pi y) \\
\psi(x,1) &= 0 \\
\psi(1,y) &= 0
\end{aligned}$$

Use a 151×151 grid. Submit only the algorithm part of your code. Plot $\log_{10}()$ of the residual - defined at $\|\Delta\psi\|_{L_2}/\|\Delta\psi(0)\|_{L^2}$ - versus iteration. Stop when the residual goes below 10^{-5}. Provide a contour plot of the converged solution.