Review of G-S in residual form $(A = L + D + U)$ $(\tilde{A} = L+D)$

$(\mathbf{L}+\mathbf{D}) \cdot \boldsymbol{\psi}^{n+1} = \boldsymbol{\omega} - \mathbf{U} \cdot \boldsymbol{\psi}^{n}$ let $\boldsymbol{\psi}^{n+1} = \boldsymbol{\psi}^{n} + \Delta\boldsymbol{\psi}$ then

$(\mathbf{L}+\mathbf{D}) \cdot \Delta\boldsymbol{\psi} = \boldsymbol{\omega} - (\mathbf{L} + \mathbf{D} + \mathbf{U}) \cdot \boldsymbol{\psi}^{n} = \boldsymbol{\omega} - \mathbf{A} \cdot \boldsymbol{\psi}^{n} \equiv \mathbf{R}^{n}$; $\mathbf{D} \cdot \Delta\boldsymbol{\psi} = \mathbf{R}^{n} - \mathbf{L} \cdot \Delta\boldsymbol{\psi}$

If $\boldsymbol{\psi}^{n}$ is a solution, then R=0. Iteration = drive residual to zero.

In G-S, accelerated by SOR, this is solved as

$\mathbf{D} \cdot \Delta\boldsymbol{\psi} \equiv R^{n} - \lambda\mathbf{L} \cdot \Delta\boldsymbol{\psi}$ and $\boldsymbol{\psi}^{n+1} = \boldsymbol{\psi}^{n} + \lambda\Delta\boldsymbol{\psi}$

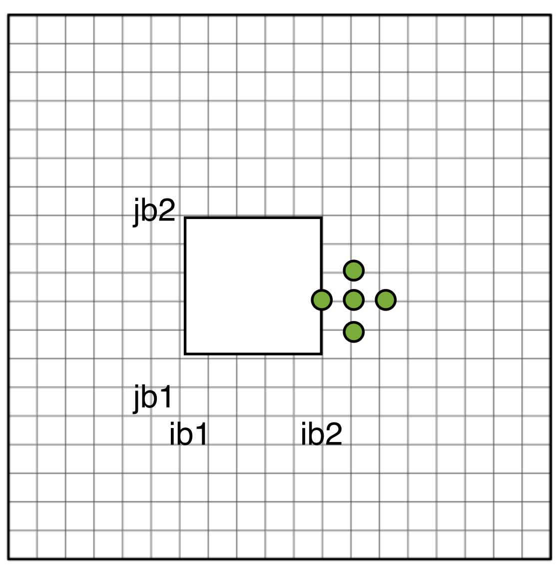
show Grids.pptx

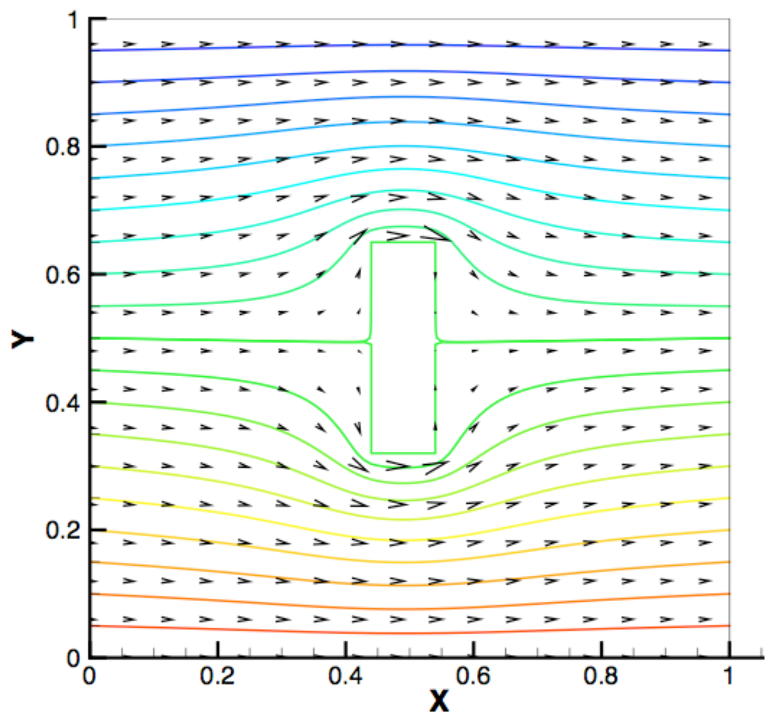
Toward geometry: i-blanking

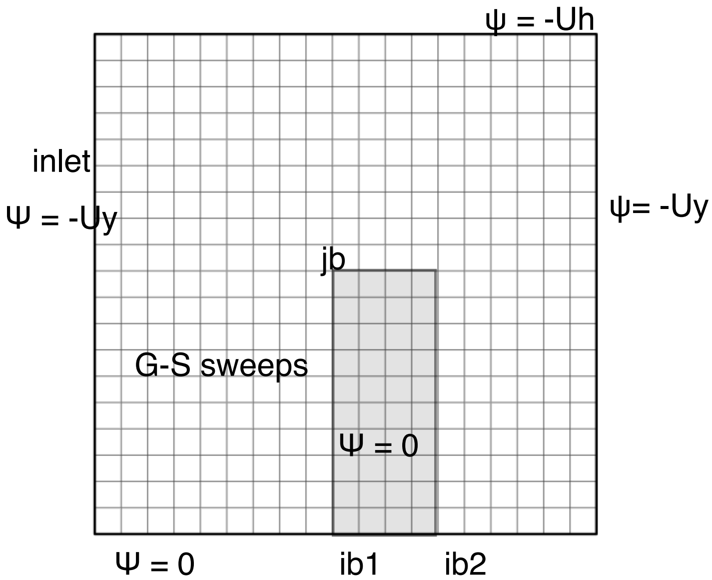
A. Consider a rectangular geometry and Cartesian grid -- nodes at intersections

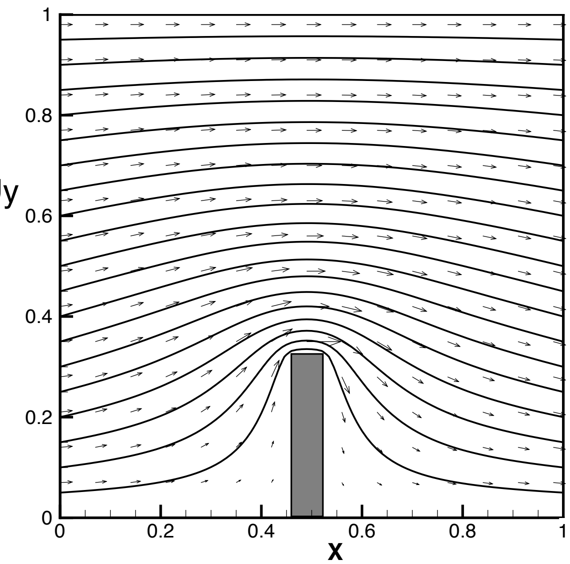
B. Inside rectangle solve Ψ_{ij} = const, so row of matrix is 1 on diagonal and b is 0. NB: Ψ can be any constant on rectangle -- e.g. circulation; or temperature

C. Linear algebra: $\mathbf{A} \cdot \boldsymbol{\Psi} = \boldsymbol{\omega}$. In rib $\mathbf{A} = \mathbf{I}$ so equation is $\boldsymbol{\Psi} = \boldsymbol{\omega} = \boldsymbol{\Psi}_B$.
Or $\mathbf{A} \cdot \Delta\boldsymbol{\Psi} = \mathbf{R}^{n} = 0$; actually, G-S is $\mathbf{D} \cdot \Delta\boldsymbol{\Psi} = \mathbf{R}^{n} - \mathbf{L} \cdot \Delta\boldsymbol{\Psi}$ and $\Delta\boldsymbol{\Psi}=0$ inside rectangle.



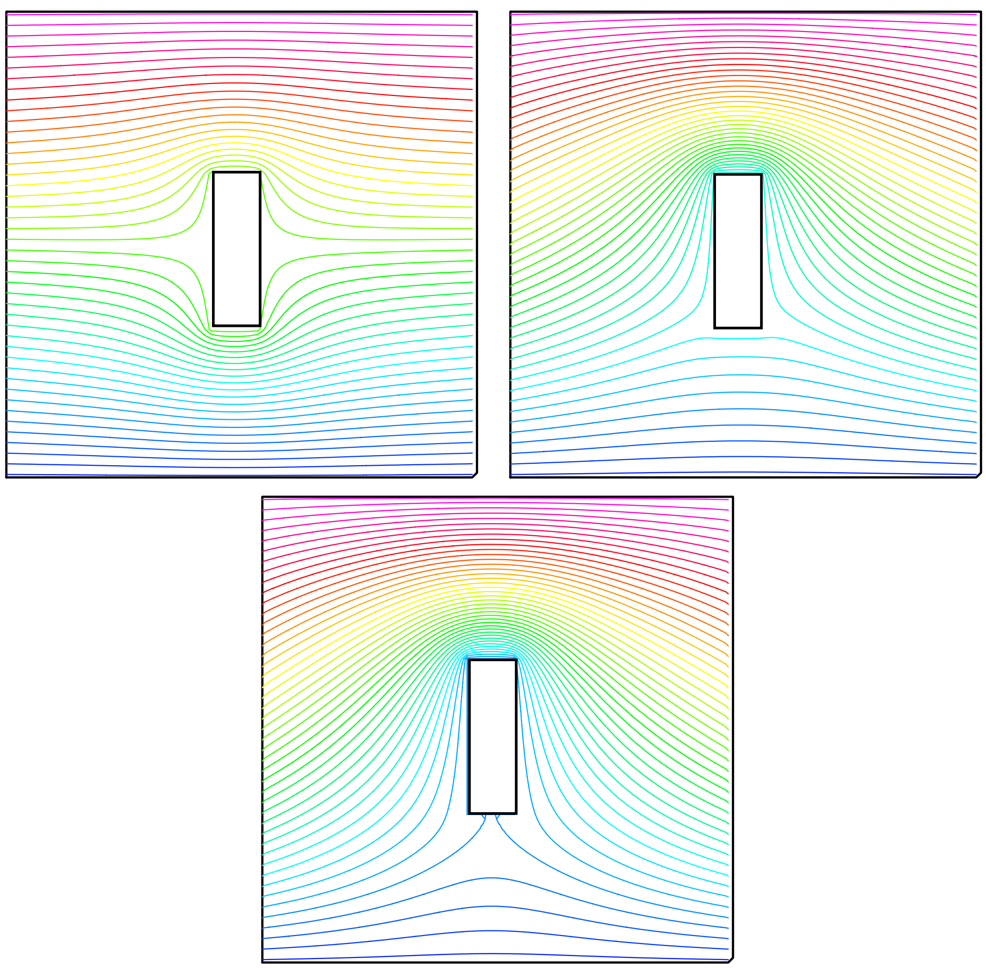


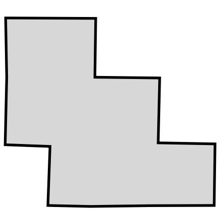




Non-dimensionalize: uniform flow becomes ψ= -y; lower wall, y=0, ψ=0 ; upper wall y=1, ψ=-1.

ψ rib = -0.5, -0.2 and -0.1





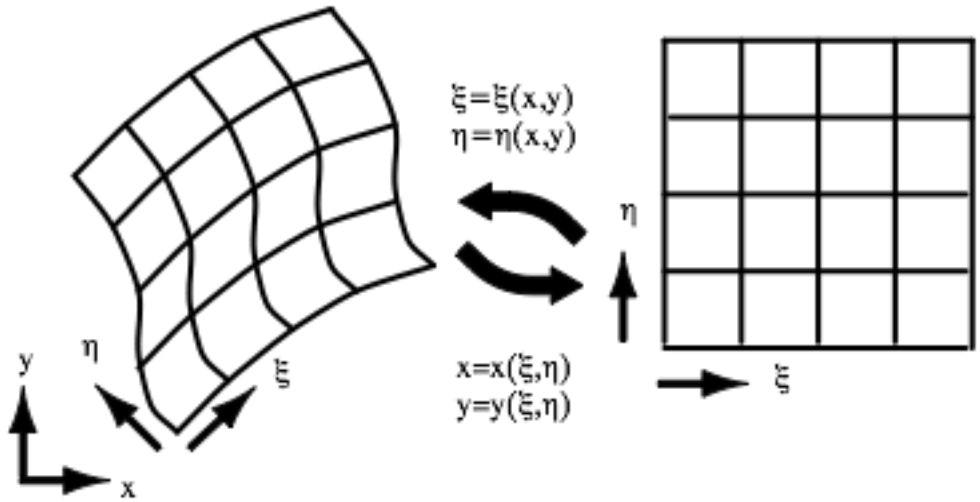
D. Pseudo-code for blanking; 2 options: change **A** or make use of **IB = 1** in fluid, 0 in geometry

```
      IB(:,:) = 1
      ψ(:,:)  = 0     ! initial guess
      dy = 1/(J-1) ; dx = 1/(I-1)
   DO j=1,J       !  B.C. ψ=-y
      ψ(1,j) = -(j-1)*dy
      ψ(I,j) = -(j-1)*dy
   ENDDO
      ψ(:,J)  = -1  ! Upper boundary
      ψ(:,1)  = 0   ! Lower boundary

   A(:,:,1) = A(:,:,5) = 1./dy**2     ! 5 point stencil
   A(:,:,2) = A(:,:,4) = 1./dx**2
   A(:,:,3) = -(A(:,:,1)+A(:,:,2)+A(:,:,4)+A(:,:,5))
   !----
   ! inside & surface of rib: the ONLY CHANGE to Poisson2D_GS.f90
   !----
   DO j=jb1,jb2
      DO i=ib1,ib2
         A(i,j,[1,2,4,5]) = 0
         A(i,j,3) = 1
         IB(i,j)  = 0      ! zero out in geometry
         ω(i,j,3) = 0
         ψ(i,j)   = ψrib  (0 if on lower wall)
      ENDDO
    ENDDO
```

! Gauss-Seidel with SOR (sweep LL-UR)

```
     L∞ = 0; L2 = 0
  DO j=2,J-1
     DO i=2,I-1
        R  = ω(i,j)
           -( A(i,j,1)*ψ(i,j-1)+A(i,j,2)*ψ(j-1,k)+A(i,j,3)*ψ(i,j) &
             +A(i,j,4)*ψ(j+1,k)+ A(i,j,5)*ψ(i,j+1) )
        Δψ = R/A(i,j,3)
        ψ(i,j) = ψ(i,j) + IB(i,j)*λ*Δψ

        L∞ = amax(Linf,abs[Del(i,j)])
        L2 = L2 + IB(i,j)*Del(i,j)^2
     ENDDO
  ENDDO
```

Introduction to structured grids:
Computational and physical space

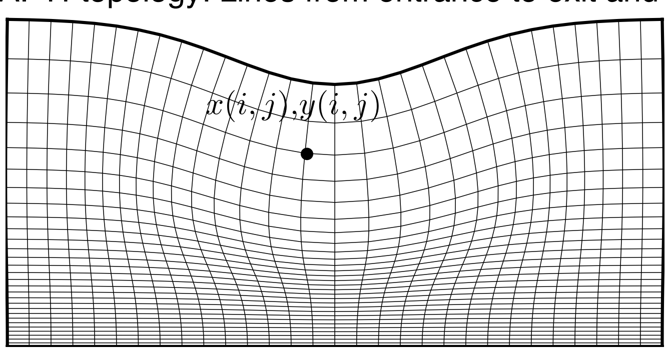


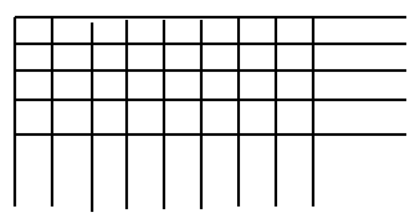
Physical space η Computational space

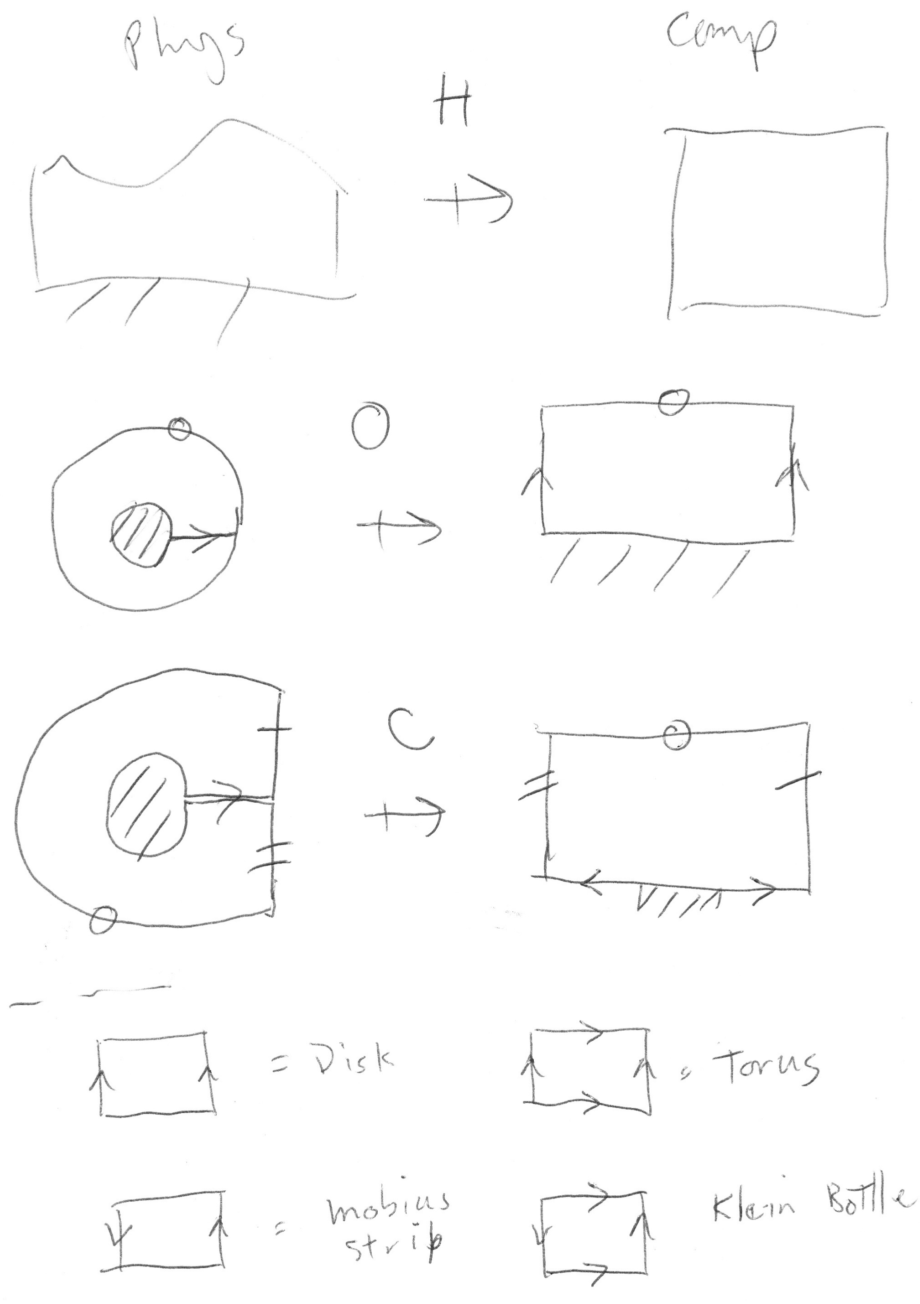
$\xi \longleftrightarrow$ j and $\eta \longleftrightarrow$ k Note $\Delta\eta = 1$. x(i,j), y(i,j) is mapping from computational to physical space --- but just arrays of coordinates.

Grid `topologies'

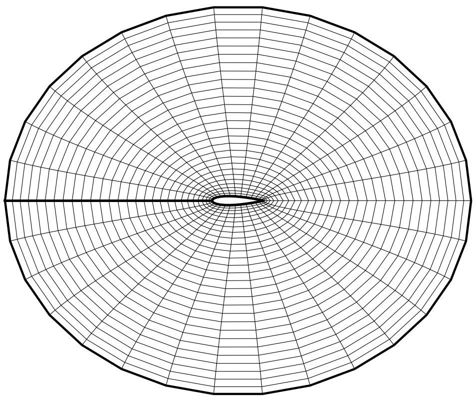
A. H-topology: Lines from entrance to exit and top to bottom. Maps to rectangular grid

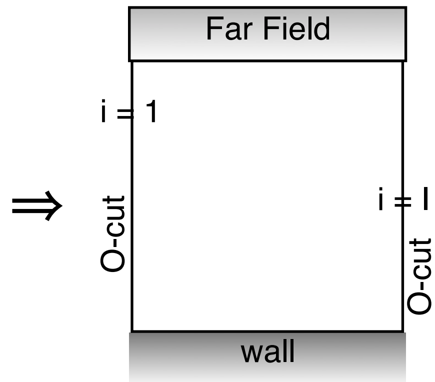


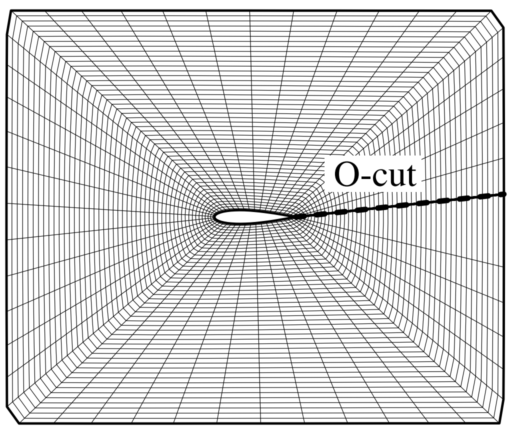



Phys
Comp
H
O
C
= Disk
= Torus
= mobius strip
Klein Bottle

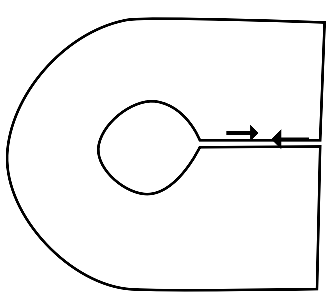
B. O-mesh topology Singly connected: i=1 ≡ i=I

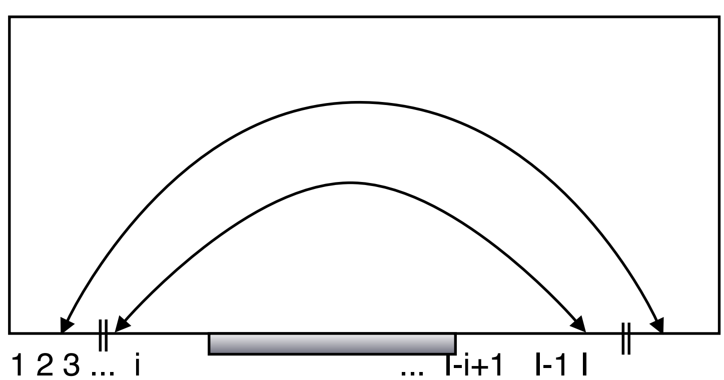


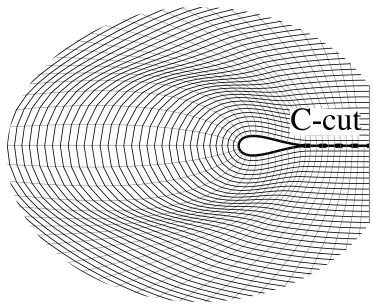


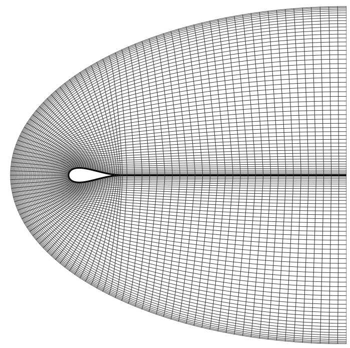


C. C-mesh



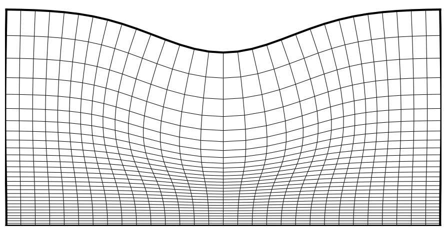




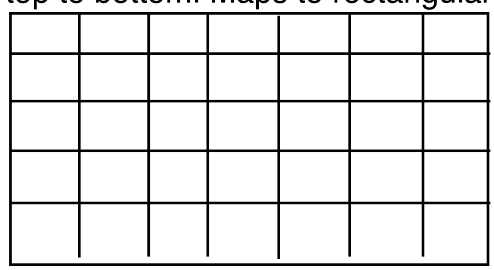


Implications for discretization

A. H-topology: Lines from entrance to exit and top to bottom. Maps to rectangular grid



⇔

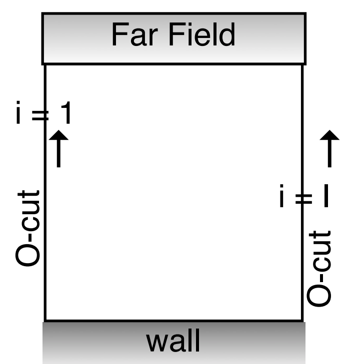


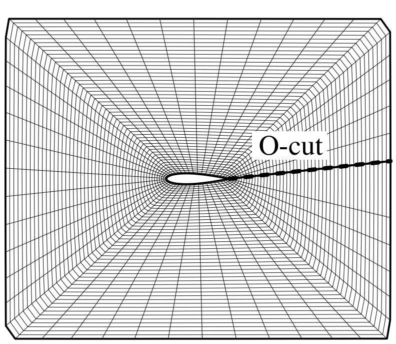
B. O-mesh topology Singly connected: $i=1 \equiv i=I$.

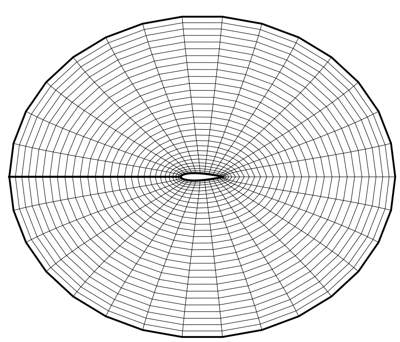
Periodic in i: Identify: $\psi(1,j) = \psi(I,j)$

For stencil: $\psi(0,j) = \psi(I\text{-}1,j)$

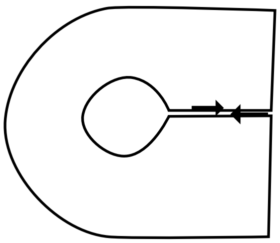
$\delta\psi/\delta x l_1 = \psi(2,j) - \psi(0,j) = \psi(2,j) - \psi(I\text{-}1,j)$

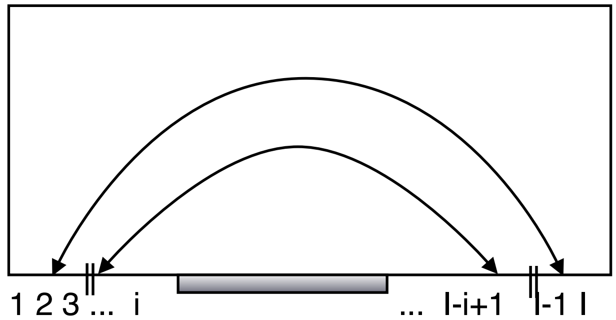






C. C-mesh





For C-cut on j=1, i=1 <-> i=I   i=2 <-> i=I-1

i <-> I-i+1 $\delta \psi/\delta x|_i = \psi(i,2) - \psi(i,0) = \psi(i,2) - \psi(I\text{-}i\text{+}1,2)$