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AER E 546 Fluid Mechanics and Heat Transfer I
Homework 2

1. (a) First I will establish a some notation. I will discretize the fin into $N+1$ points. Let $x_i = \frac{i}{N}$, then $x_0 = 0$ and $x_N = 1$. Let the approximate solution at x_i be represented by T_i. Then a numerical solution consists of a set of values T_i for $i \in \mathbb{N}$, $0 \le i \le N$.

   Next I will discretize the partial differential equation into a discrete equation. The second order central finite difference for the second derivative is

$$\frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2}$$

   Plugging this into the partial differential equation gives the following difference equation

$$\frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} = MT_i$$

   Simplifying this gives

$$T_{i-1} + \left(-2 - M\Delta x^2\right)T_i + T_{i+1} = 0$$

   For this problem the boundary conditions are $T(0) = 1$ and $T(1) = 0$. This can be encoded into the numerical solution at $T_0 = 1$ and $T_N = 0$. Now only the values for T_i for $1 \le i \le N-1$ need to be found. These can be found by solving the equations

$$T_{i-1} + \left(-2 - M\Delta x^2\right)T_i + T_{i+1} = 0$$

   for $i = 1$, this become

$$\left(-2 - M\Delta x^2\right)T_1 + T_2 = -1$$

   and for $i = N-1$ the equation is

$$T_{N-2} + \left(-2 - M\Delta x^2\right)T_{N-1} = 0$$

   These equation can be written in matrix form as

$$\begin{bmatrix} -2-\Delta x^2 & 1 & & & 0 \\ 1 & -2-\Delta x^2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2-\Delta x^2 & 1 \\ 0 & & & 1 & -2-\Delta x^2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_{N-2} \\ T_{N-1} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

   (b)

   (c)

2.