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AER E 546 Fluid Mechanics and Heat Transfer I
Homework 1

#1

(a) How many 'data' points are needed to obtain a third order accurate polynomial approximation? Derive a finite difference formula for $\partial T/\partial x$ that is third order accurate in Δx. Use only the minimum number of points.

(b) Derive the second order accurate centered difference formula for $\dfrac{\partial^2 T}{\partial x^2}$.

#2

(a) The equation for a damped oscillator is

$$\ddot{Y} + \sigma\dot{Y} + \omega^2 Y = 0.$$

Let the non-dimensional frequency be $\omega = 1$. Consider the two damping rates $\sigma = 0.0$ and $\sigma = 0.5$. Solve this by RK2, out to $t = 32$, with the intial conditions $Y(0) = 1$ and $\dot{Y}(0) = 0$. The time-step can be $\Delta t = 32/N$, where N is the number of integration points. Plot solutions with $N = 21, 101, 301$. What is the analytical solution? Compare your numerical solutions to the exact result.

First I will compute the analytical solution to this differential equation. This can be done by finding the characteristic polynomial of the equation, which is

$$r^2 + \sigma r + 1 = 0.$$

Using the quadractic formula, we see that the roots of this polynomial are $r = -\frac{\sigma}{2} \pm \frac{\sqrt{\sigma^2-4}}{2}$. When $\sigma = 0.0$, the roots are $r = \pm i$. In the case of complex roots the general solution will be

$$Y(t) = c_1 \cos(t) + c_2 \sin(2).$$

Using the intial conditions we see that the exact solution is

$$Y(t) = \cos(t).$$

When $\sigma = 0.5$ the roots are $r = -\frac{1}{4} \pm \frac{\sqrt{15}}{4} i$. In this case the general solution is

$$Y(t) = e^{-\frac{1}{4}t}\left(c_1 \cos\left(\frac{\sqrt{15}}{4}t\right) + c_2 \sin\left(\frac{\sqrt{15}}{4}t\right)\right)$$

and the exact solution with boundary conditions is

$$Y(t) = e^{-\frac{1}{4}t} \cos\left(\frac{\sqrt{15}}{4}t\right).$$

(b) The equation for a nonlinear spring (without damping) is

$$\ddot{Y} + Y - BY^3 = 0.$$

Solve by RK2 out to $t = 32$ with the intial conditions $Y(0) = 1$ and $\dot{Y}(0) = 0$. Plot $Y(t)$ for $B = 0.2, 0.6, 0.9, 0.999$. Chose N large enough to get an accurate solution; that will depend on the value of B.

#3 Repeat the linear spring computation (ex. 2.a) with AB2. What does the solution for $\sigma = 0.0$ tell you about the stability of AB2?