AerE 546 **Homework 2** 9/8/15

Due in two weeks

Exercise 1, *Heat fin*

The heat fin equation is the linear o.d.e.

$$\frac{d^2T}{dx^2} = MT$$

where M is a sort of thermal mass. First write the finite difference equation in terms of a tridiagonal matrix. Solve that equation using the Thomas algorithm (Gaussian elimination).

a) Solve the heat fin equation with the boundary conditions $T(0) = 1,\ T(1) = 0$. This corresponds to a fin that is between a hot and a cold reservoir. In non-dimensional terms, the heat flux into the cold reservoir is $-dT/dx$ at $x = 1$. Obtain the heat flux at $x = 1$ for $M = 1,\ 5,\ 9$. Use enough grid points to obtain 1% accuracy. Provide your three numerical values of the heat flux.

b) Solve the heat fin equation with the boundary conditions $T(0) = 1,\ dT(1)/dx = 0$. This corresponds to a fin that is insulated at one end. Solve for the temperature, $T(1)$, at the insulated end for $M = 1,\ 5,\ 9$. Provide your three numerical values of $T(1)$.

Also plot $T(x)$ for $M = 9$ with each pair of boundary conditions and compare to the exact solution.

c) Add a distributed heat source: Compute and plot a solution of the non-homogeneous equation

$$\frac{d^2T}{dx^2} = MT - 100x^2(1-x)^2$$

with $M = 9$, $T(0) = 1,\ dT(1)/dx = 0$.

Exercise 2, *Equation types*

i) What type of p.d.e. is

$$\frac{\partial^2\phi}{\partial x\partial y} + \phi = 25 \quad ?$$

ii) What type of p.d.e. does the velocity potential, ϕ, satisfy if

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

with

$$u = \frac{\partial\phi}{\partial x}\ ; \quad v = \frac{\partial\phi}{\partial y} \quad ?$$

iii) The boundary layer momentum equation is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{Re}\frac{\partial^2 u}{\partial y^2}$$

where Re is the Reynolds number. What type is this equation?