AerE 546 **Homework 7** 11/17/15

Due in two weeks

Solve Laplace's equation on a curvilinear grid. (See lecture notes on BlackBoard.)

Code development:

- Code the discrete Laplacian in a suitable form for curvilinear grids. The discrete form has a 9-point stencil.
- Read the grid and generate the metric terms by the routine written in the previous homework.
- Solve the discrete equations by Gauss-Seidel iterations, with SOR. In Δ form

$$\Delta\Psi_{jk} = \left(\omega_{jk} - \sum_{n=1,9} A_{jk}(n)\Psi_{jk}(n)\right) / A_{jk}(5)$$

  The right side is the residual. You don't have to create the coefficients, $\boldsymbol{A}(j,k,9)$, only $A_{jk}(5)$: see lecture notes.
- Print a norm of the residual so you can monitor convergence.

Applications:

Exercise 1, *O-mesh*

The stencil on the O-cut was described in class.

Solve for the flow over a circular cylinder of unit radius. This will test your code because there is an exact solution. Let the outer boundary be a circle of radius 10 and the inner be a circle of unit radius. Generate a suitable grid (about 201×201).

- The exact solution for potential flow over a cylinder with circulation is

$$\Psi = -y\left(1 - \frac{1}{r^2}\right) + \frac{\Gamma}{4\pi}\log r^2; \quad \text{where } r^2 = x^2 + y^2$$

- Solve numerically with $\Gamma = 0$. The oncoming velocity is uniform, from left to right: $\Psi = -y$ on the outer, circular, boundary. Compare to the exact solution. Provide plots of the exact and your numerical streamfunction.

- Solve numerically with $\Gamma = -\pi$. Use the solution cited above to prescribe the outer boundary condition. Again, provide plots of exact and numerical results.

- By Bernoulli's equation, the surface pressure coefficient (i.e., non-dimensional surface pressure) is

$$C_p \equiv \frac{P - P_\infty}{{}^1\!/_2\, U_\infty^2} = 1 - \frac{U^2 + V^2}{U_\infty^2}$$

  where the velocities are evaluated on the cylinder, $r = 1$. Plot the computed surface pressure distribution in the form $C_p(x)$ for $\Gamma = 0$ and $\Gamma = \pi$. (Note: at each x there are 2 values of C_p, for the top and the bottom of the cylinder.)

- The integrated force is

$$\boldsymbol{F} = \oint p\boldsymbol{n}dl$$

or, in Cartesian components

$$F_x = -\oint pdy, \qquad F_y = \oint pdx$$

What values do you get for F_x and F_y with $\Gamma = 0$ and π?

Exercise 2, *Compute the stream function for flow through the duct of the last homework*

Lower wall: $y = 0,\ -1 \le x \le 1$

Upper wall: $y = 1 - 0.4e^{-8(0.6-x)^2} \qquad -1 \le x \le 0.6$

$y = 0.6 \qquad 0.6 \le x \le 1.0$

using a 150×150 grid. That is, solve Laplace's equation on the curvilinear grid. Note that the inlet is 1 unit high and the exit is 0.6 units high. The inlet velocity is unity; the exit velocity is $1/0.6$ by mass conservation. What are the boundary conditions?

Hand in:

- A plot of streamlines (lines of $\Psi = const$).
- A plot the pressure distribution on the lower wall, relative to the inlet pressure ($P - P_{in}$ vs x). Bernoulli's equation is $P + |\boldsymbol{u}|^2/2 = P_{in} + |\boldsymbol{u}_{in}|^2/2$. By definition $u = -\partial_y \Psi$, $v = \partial_x \Psi$