AerE 546 | **Homework 1** | 8/25/15

Due in two weeks

Exercise 1, *Discretization*

How many 'data' points are needed to obtain a third order accurate ploynomial approximation? Derive a finite difference formula for $\delta T/\delta x$ that is third order accurate in Δx. Derive it twice, first from a Taylor series and then from a Lagrange interpolation.

Write the second-order accurate, centered difference formula for d^2T/dx^2.

Exercise 2, *Runge Kutta*

a) The equation for a damped oscillator is

$$\ddot{Y} + \sigma\dot{Y} + \omega^2 Y = 0$$

Let the non-dimensional frequency be $\omega = 1$. Consider the two damping rates $\sigma = 0.0$ and 0.5. Solve this by RK2, out to $t = 32$, with the initial conditions $Y(0) = 1$, $\dot{Y}(0) = 0$. The time-step can be $\Delta t = 32/N$, where N is the number of integration points. Plot solutions with $N = 21,\ 101,\ 301$. What is the analytical solution? Compare your numerical solutions to the exact result.

b) The equation for a nonlinear spring (without damping) is

$$\ddot{Y} + Y - BY^3 = 0$$

Solve by RK2, out to $t = 32$, with the initial conditions $Y(0) = 1$, $\dot{Y}(0) = 0$. Plot $Y(t)$ for $B = 0.2,\ 0.6,\ 0.9,\ 0.999$. **Choose N large enough to get an accurate solution**; that will depend on the value of B.

Exercise 3, *Adams-Bashforth*

Repeat the linear spring computation (ex. 2.a) with AB2. What does the solution for $\sigma = 0.0$ tell you about the stability of AB2?