Assigned: Thursday Feb. 18, 2016
Due: Thursday March 3, 2016

MATH 517: Homework 3
Spring 2016

NOTE: For each homework assignment observe the following guidelines:

- Include a cover page.
- Always clearly label all plots (title, x-label, y-label, and legend).
- Use the `subplot` command from MATLAB when comparing 2 or more plots to make comparisons easier and to save paper.

Part 1: 9-point Laplacian

Consider Poisson's equation in 2D:

$$-u_{,x,x} - u_{,y,y} = f(x,y) \;\text{ in } \Omega = [0,1]\times[0,1],$$
$$u = g(x,y) \;\text{ on } \partial\Omega.$$

1. Discretize this equation using the 9-point Laplacian on a uniform mesh $\Delta x = \Delta y = h$. Use the standard natural row-wise ordering.
2. Write a MATLAB code that constructs the sparse coefficient matrix A and the appropriate right-hand side vector $\vec{F}$. **NOTE:** you will need to modify the right-hand side vector to include the appropriate Laplacian of the right-hand side function (see lecture notes and/or pages 64-65 of the textbook).
3. Using your code, do a numerical convergence study for the following right-hand side forcing and exact solution:

   $$f(x,y) = -1.25e^{x+0.5y} \quad\text{and}\quad u(x,y) = e^{x+0.5y}.$$

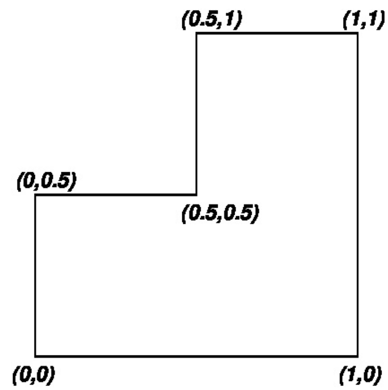
   Just use the built-in backslash operator in MATLAB to solve the linear system (in this case the backslash operator will use a sparse LU decomposition + forward and backward substitution).

Part 2: L-shaped domain

Consider Poisson's equation in 2D:

$$-u_{,x,x} - u_{,y,y} = f(x,y) \;\text{ in } \Omega,$$
$$u = 0 \;\text{ on } \partial\Omega,$$

where Ω is the L-shaped domain:



4. Discretize the above PDE using the standard 5-point Laplacian. Write a MATLAB code that generates the sparse coefficient matrix A for this discretization.

5. For $N = 20$ and $N = 40$ produce a `spy` plot of the matrix.

6. Solve the PDE using your code with the right-hand side

$$f(x, y) = 1.$$

7. Solve the PDE using your code with the right-hand side

$$f(x, y) = 2 \exp\left[-(10x - 5)^2 - (10y - 5)^2\right].$$

8. Compute the **Cholesky factorization** of A in MATLAB:

```
R = chol(A);
```

   where R is an upper triangular matrix such that $A = R^T R$ (i.e., the LU factorization). For $N = 20$ and $N = 40$ produce a `spy` plot of R. Create a table showing the number of non-zeros in R for $N = 10, 20, 40, 80, 160, 320$.

9. Permute the matrix A using the reverse Cuthill-Mckee algorithm in MATLAB and call the permuted matrix B:

```
P = symrcm(A);    B = A(P,P);
```

   Compute the **Cholesky factorization** of B in MATLAB:

```
R = chol(B);
```

   For $N = 20$ and $N = 40$ produce a `spy` plot of R. Create a table showing the number of non-zeros in R for $N = 10, 20, 40, 80, 160, 320$.