

MATH 517: HOMEWORK 3

SPRING 2016

NOTE: For each homework assignment observe the following guidelines:

- Include a cover page.
- Always clearly label all plots (title, x -label, y -label, and legend).
- Use the `subplot` command from MATLAB when comparing 2 or more plots to make comparisons easier and to save paper.

Part 1: 9-point Laplacian

Consider Poisson's equation in 2D:

$$\begin{aligned} -u_{,x,x} - u_{,y,y} &= f(x, y) \text{ in } \Omega = [0, 1] \times [0, 1], \\ u &= g(x, y) \text{ on } \partial\Omega. \end{aligned}$$

1. Discretize this equation using the 9-point Laplacian on a uniform mesh $\Delta x = \Delta y = h$. Use the standard natural row-wise ordering.
2. Write a MATLAB code that constructs the sparse coefficient matrix A and the appropriate right-hand side vector \vec{F} . **NOTE:** you will need to modify the right-hand side vector to include the appropriate Laplacian of the right-hand side function (see lecture notes and/or pages 64-65 of the textbook).
3. Using your code, do a numerical convergence study for the following right-hand side forcing and exact solution:

$$f(x, y) = -1.25e^{x+0.5y} \quad \text{and} \quad u(x, y) = e^{x+0.5y}.$$

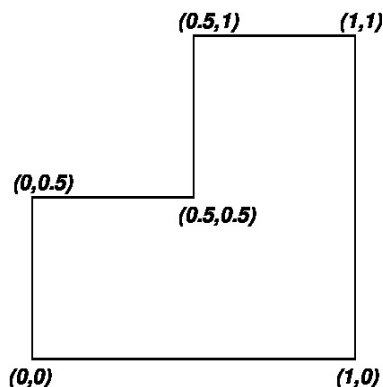
Just use the built-in backslash operator in MATLAB to solve the linear system (in this case the backslash operator will use a sparse LU decomposition + forward and backward substitution).

Part 2: L-shaped domain

Consider Poisson's equation in 2D:

$$\begin{aligned} -u_{,x,x} - u_{,y,y} &= f(x, y) \text{ in } \Omega, \\ u &= 0 \text{ on } \partial\Omega, \end{aligned}$$

where Ω is the L-shaped domain:



4. Discretize the above PDE using the standard 5-point Laplacian. Write a MATLAB code that generates the sparse coefficient matrix A for this discretization.
5. For $N = 19$ and $N = 39$ produce a `spy` plot of the matrix.
6. Solve the PDE using your code with the right-hand side

$$f(x, y) = 1.$$

7. Solve the PDE using your code with the right-hand side

$$f(x, y) = 2 \exp \left[-(10x - 5)^2 - (10y - 5)^2 \right].$$

8. Compute the **Cholesky factorization** of A in MATLAB:

$$R = \text{chol}(A);$$

where R is an upper triangular matrix such that $A = R^T R$ (i.e., the LU factorization). For $N = 19$ and $N = 39$ produce a `spy` plot of R . Create a table showing the number of non-zeros in R for $N = 9, 19, 39, 79, 159, 319$.

9. Permute the matrix A using the reverse Cuthill-McKee algorithm in MATLAB and call the permuted matrix B :

$$P = \text{symrcm}(A); \quad B = A(P, P);$$

Compute the **Cholesky factorization** of B in MATLAB:

$$R = \text{chol}(B);$$

For $N = 20$ and $N = 40$ produce a `spy` plot of R . Create a table showing the number of non-zeros in R for $N = 9, 19, 39, 79, 159, 319$.