Caleb Logemann
MATH 520 Methods of Applied Math II
Homework 3

Section 11.4

#2 Verify that $\mathbf{H} \times \mathbf{H}$ is a Hilbert space with the inner product given by (11.1.2), and prove Proposition 11.1.

Proof. □

#5 If $T : D(T) \subset \mathbf{H} \to \mathbf{H}$ is a densely defined linear operator, $v \in \mathbf{H}$ and the map $u \to \langle Tu, v \rangle$ is bounded on $D(T)$, show that there exists $v^* \in \mathbf{H}$ such that (v, v^*) is an admissible pair for T^*.

#8 Show that if T is self-adjoint and one-to-one then T^{-1} is also self-adjoint.

#11 Assume that T is closed and S is bounded

(a) Show that $S + T$ is closed

(b) Show that TS is closed, but that ST is not closed, in general.

#14 It T is closable, show that T and $\overline{T}$ have the same adjoint.