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MATH 520 Methods of Applied Math II
Homework 2

Section 10.9

#10 Let S_+ and S_- be the left and right shift operators on ℓ^2. Show that $S_- = S_+^*$ and $S_+ = S_-^*$.

Proof. □

#11 Let T be the Volterra integral operator $Tu = \int_0^x u(y)\,\mathrm{d}y$ considered as an operator on $L^2(0,1)$. Find T^* and $N(T^*)$.

#12 Suppose $T \in \mathcal{B}(\mathbf{H})$ is self-adjoint and there exists a constant $c > 0$ such that $\|Tu\| \geq c\|u\|$ for all $u \in \mathbf{H}$. Show that there exists a solution of $Tu = f$ for all $f \in \mathbf{H}$. Show by example that the conclusion may be false if the assumption of self-adjointedness is removed.

Proof. This conclusion may be false if that operator is not self-adjoint. Consider the operator S_+ on ℓ^2. □

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