MATH 561 Fall 2015 – Homework Set # 5

Last Name: ______________________ First Name: ______________________

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1. (30 points) (a) Consider an explicit multistep method of the form

$$\mathbf{u}_{n+2} - \mathbf{u}_{n-2} + \alpha(\mathbf{u}_{n+1} - \mathbf{u}_{n-1}) = h[\beta(\mathbf{f}_{n+1} + \mathbf{f}_{n-1}) + \gamma \mathbf{f}_n]$$

   Show that the parameters α, β, γ can be chosen uniquely so that the method has order $p = 6$. {Hint: to preserve symmetry, and thus algebraic simplicity, define the associated linear functional on the interval $[-2, 2]$ rather than $[0, 4]$ as in Sect. 6.1.2. }

   (b) Discuss the stability properties of the method obtained in (a).

2. (30 points) Construct a pair of four-step methods, one explicit, the other implicit, both having $\alpha(\xi) = \xi^4 - \xi^3$ and order $p = 4$, but global error constants that are equal in modulus and opposite in sign.

3. (40 points) Consider the (slightly modified) model problem

$$\frac{dy}{dx} = -\omega[y - a(x)],\ 0 \le x \le 1;\ y(0) = y_0,$$

   where $\omega > 0$ and (i) $a(x) = x^2,\ y_0 = 0$; and (ii) $a(x) = e^x,\ y(0) = 1$.

   (a) In each of the cases (i) and (ii), obtain the exact solution $y(x)$.
   (b) In each of the cases (i) and (ii), apply the kth-order Adams-Bashford method and kth-order Adams predictor/corrector method, for $k = 4$, using exact starting values and step lengths $h = \frac{1}{20}, \frac{1}{40}, \frac{1}{80}, \frac{1}{160}$. Print the exact values y_n and numerical solution u_n, and check the accuracy of the methods. Try $\omega = 1, 10, 50$. Summarize your results.