Math 562 **Spring 2016** Homework 2

(1) (Problem 6.1, page 47) Let P be an orthogonal projector.
 (a) Prove that $I - 2P$ is unitary
 (b) Describe the action of $I - 2P$ geometrically. (What can you say about the relationship between a point and its image?)

(2) (Problem 6.3, page 47) Suppose that $A \in \mathbb{C}^{m\times n}$, with $m \geq n$.
 (a) Show that A^*A is nonsingular if and only if A has full rank.
 (b) Show that if A has full rank then $P = A(A^*A)^{-1}A^*$ is an orthogonal projector onto the range of A.

(3) (Problem 7.5, page 55) Suppose that $A \in \mathbb{C}^{m\times n}$, with $m \geq n$, and let $A = \hat{Q}\hat{R}$ be the reduced QR factorization of A.
 (a) Show that A has full rank if and only if all the diagonal entries of $\hat{R}$ are nonzero.
 (b) Suppose that $\hat{R}$ has k nonzero diagonal entries and $n-k$ zero diagonal entries. What does this imply about the rank of A. Justify your answer.

(4) (Problem 8.1, page 61) Let A be an $(m \times n)$ matrix. Determine the exact number of floating point additions, substractions, multiplications, and divisions involved in computing the reduced QR factorization of A using Algorithm 8.1 on page 58.

(5) (Problem 10.1, page 76) Let $F = I - 2uu^T$ be a Householder reflector on $\mathbb{R}^m$. Determine the eigenvalues, the determinant, and the singular values of F. Give a geometric argument supporting your algebraic eigenvalue calculation.

(6) (Complex Reflectors) For convenience we write the 2-norm as $\|\cdot\|$.
 (a) Let $x, y \in \mathbb{C}^m$ satisfy i) $x \neq y$, ii) $\|x\| = \|y\|$, iii) x^*y is real-valued. Show that there is a reflector F (satisfying $F^* = F$, $F^2 = I$) such that $Fx = y$.
 (b) Let $x \in \mathbb{C}^m$, $x \neq 0$. The polar form of the first component of x is $x_1 = re^{i\theta}$. Set $y = \|x\|e^{i\theta}e_1$. Assuming x is not a multiple of e_1, show that x, y satisfy properties i), ii) and iii) above.

(7) Write a MATLAB function $[W, R] = \text{house}(A)$ that takes as input a $(m\times n)$ matrix A and returns an implicit representation of the full QR factorization of A. The matrix W should be the lower triangular matrix whose columns are the vectors $v_1, \ldots, v_n$ where $v_k = \|x_k\|e_1 - x_k$ is used to define the Householder reflector F_k at the k-th stage of the process. R should be the triangular factor in the factorization. Some test matrices will be supplied next week. You should turn in your code and the output from the test matrices for this problem.