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MATH 562 Numerical Analysis II
Homework 1

1. Problem 1.1 Let B be a 4×4 matrix to which the following operations are applied in the given order.

   1. double column 1
   2. halve row 3
   3. add row 3 to row 1
   4. interchange columns 1 and 4
   5. subtract row 2 from each other rows
   6. replace column 4 by column 3
   7. delete column 1

   The result can be written as a product of 8 matrices one of which is B.

   (a) What are the other 7 matrices and what order do they appear in the matrix?
   The matrix that doubles column 1 is

$$C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   when right multiplied. The following matrix halves row 3 when left multiplied.

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   The following matrix adds row 3 to the row 1 when left multiplied.

$$E = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   The following matrix interchanges columns 1 and 4 when right multiplied.

$$F = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The following matrix subtracts row 2 from every other row, when left multiplied.

$$G = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

The following matrix replaces column 4 with column 3 when right multiplied.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The following matrix deletes column 1 when right multiplied.

$$I = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The resulting matrix product is given by $GEDBCFHI$, where the matrices are given above.

(b) The result can also be written as a product ABC what are A and C?

In this case A and C are given by the product of the matrices to the left and the right of B in the part (a). Therefore

$$A = \begin{bmatrix} 1 & -1 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0.5 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2.

3. Let $A \in \mathbb{C}^{m\times m}$ be Hermitian, that is $A = A^*$. Suppose that $A\mathbf{x} = \lambda\mathbf{x}$, where $\mathbf{x} \in \mathbb{C}^{m\times m}$ and $\lambda \in \mathbb{C}$, so $\mathbf{x}$ is an eigenvector and λ is an eigenvalue.

(a) Prove that λ must be real.

Proof. Then consider $\mathbf{x}^* A\mathbf{x}$.

$$\begin{aligned}
(\mathbf{x}^* A)\mathbf{x} &= \mathbf{x}^*(A\mathbf{x}) \\
(A^*\mathbf{x})^*\mathbf{x} &= \mathbf{x}^*(A\mathbf{x}) \\
(\lambda\mathbf{v})^*\mathbf{v} &= \mathbf{v}^*(\lambda\mathbf{v}) \\
\overline{\lambda}\mathbf{v}^*\mathbf{v} &= \lambda\mathbf{v}^*\mathbf{v} \\
\overline{\lambda} &= \lambda
\end{aligned}$$

Since $\overline{\lambda} = \lambda$, λ must be real. □

4.

5.

6.

7.