

Caleb Logemann
MATH 562 Numerical Analysis II
Homework 1

1. Problem 1.1 Let B be a 4×4 matrix to which the following operations are applied in the given order.

1. double column 1
2. halve row 3
3. add row 3 to row 1
4. interchange columns 1 and 4
5. subtract row 2 from each other rows
6. replace column 4 by column 3
7. delete column 1

The result can be written as a product of 8 matrices one of which is B .

(a) What are the other 7 matrices and what order do they appear in the matrix?

The matrix that doubles column 1 is

$$C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

when right multiplied. The following matrix halves row 3 when left multiplied.

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The following matrix adds row 3 to the row 1 when left multiplied.

$$E = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The following matrix interchanges columns 1 and 4 when right multiplied.

$$F = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The following matrix subtracts row 2 from every other row, when left multiplied.

$$G = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

The following matrix replaces column 4 with column 3 when right multiplied.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The following matrix deletes column 1 when right multiplied.

$$I = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The resulting matrix product is given by $GEDBCFHI$, where the matrices are given above.

- (b) The result can also be written as a product ABC what are A and C ?

In this case A and C are given by the product of the matrices to the left and the right of B in the part (a). Therefore

$$A = \begin{bmatrix} 1 & -1 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0.5 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2.

3.

4.

5.

6.

7.