Assigned: Tuesday April 11 2017
Due Date: Tuesday April 25, 2017

MATH 565: Homework 5
Spring 2017

1. Page 269: Problem 10.1.

2. Page 269: Problem 10.5.

3. Consider the **underdetermined** linear system $\underline{\underline{J}}\,\underline{x} = \underline{r}$, where $\underline{\underline{J}} \in \mathbb{R}^{m\times n}$, $\underline{x} \in \mathbb{R}^n$, $\underline{r} \in \mathbb{R}^m$, and $m < n$ (i.e., there are less equations than unknowns). Assume that the rank of $\underline{\underline{J}}$ is m (i.e., it has full rank). There will exist infinitely many solutions. The **minimum norm solution** of $\underline{\underline{J}}\,\underline{x} = \underline{r}$ is the solution closest to the origin, which may be regarded as the solution of the constrained optimization problem:

$$\min_{\underline{x}\in\mathbb{R}^n} \|\underline{x}\|^2 \qquad \text{subject to} \qquad \underline{\underline{J}}\,\underline{x} = \underline{r}.$$

   (a) Using the Lagrange multiplier method, derive the solution to this optimization problem:
$$\underline{x} = \underline{\underline{J}}^T \left(\underline{\underline{J}}\,\underline{\underline{J}}^T\right)^{-1} \underline{r}.$$

   (b) Find the minimum norm solution of the 3×5 system $\underline{\underline{J}}\underline{x} = \underline{r}$, where
$$\underline{\underline{J}} = \begin{pmatrix} 1 & 2 & 0 & 3 & 2 \\ -1 & -1 & 4 & 2 & 0 \\ 3 & -2 & 2 & 1 & 1 \end{pmatrix}, \qquad \underline{r} = \begin{pmatrix} 4 \\ 1 \\ -7 \end{pmatrix}.$$

4. An important problem in signal processing amounts to finding parameters $c_1, c_2, \ldots, c_n$ and $\lambda_1, \lambda_2, \ldots, \lambda_n$ such that
$$\sum_{k=1}^{n} c_k\, e^{-\lambda_k t} \approx f(t),$$
for a given *signal* function $f(t)$. One approach for solving this problem is to formulate a nonlinear least squares problems. Let
$$\underline{x} := (x_1, x_2, \ldots, x_{2n}) = (c_1, \ldots, c_n, \lambda_1, \ldots, \lambda_n)$$
be the vector of parameters to be determined. Let $s_j = f(t_j)$ be given samples of f for $j = 1, \ldots, 2n$ and set
$$r_j(\underline{x}) = \sum_{k=1}^{n} c_k e^{-\lambda_k t_j} - s_j = \sum_{k=1}^{n} x_k e^{-x_{n+k} t_j} - s_j.$$
We then obtain the parameters as the solution of the nonlinear least squares problem:
$$\min_{\underline{x}\in\mathbb{R}^{2n}} \|\underline{r}(\underline{x})\|^2.$$

   (a) Find the general expression for $\underline{\underline{J}}(\underline{x})$.

(b) Let $n = 2$, and

$$\underline{t} = (0.0,\, 0.3,\, 0.6,\, 0.9) \qquad \text{and} \qquad \underline{s} = (2.700,\, 1.480,\, 0.819,\, 0.458)\,.$$

In MATLAB or PYTHON, program a Gauss-Newton iteration scheme for this problem. Apply the scheme the following initial guess:

$$\underline{x}_0 = (1, 1, 1, 2)\,,$$

and run until convergence.

5. Page 352: Problem 12.4.

6. Page 353: Problem 12.13.

7. Page 354: Problem 12.18.

8. Page 354: Problem 12.21.