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MATH 565 Continuous Optimization
Final Exam

1.

2.

3.

4. (a) This problem can be written as a linear programming problem in the following way.

$$
\begin{aligned}
\max 3P + 4Q - \theta R & \\
2P + 2Q - R &= 0 \\
2P + Q &\leq 10P \qquad\qquad \leq 8 \\
P, Q, R &\geq 0
\end{aligned}
$$

The objective function describes the sale price of the products less the cost of the materials. The first constraint enforces the amount of materials required for the products. The second constraint enforces the labor restriction. The third constraint enforces the equipment restriction, and the final constraints enforce nonnegativity. In standard form, linear programs can be written as

$$
\begin{aligned}
\max \underline{c}^T \underline{x} & \\
\underline{\underline{A}}\underline{x} &= b \\
\underline{x} &\geq 0
\end{aligned}
$$

In order to put our linear program in this form I will introduce slack variables, s_1 and s_2. In this case the linear program can be expressed as

$$
\begin{aligned}
\max \underline{c}^T \underline{x} & \\
\underline{\underline{A}}\underline{x} &= b \\
\underline{x} &\geq 0
\end{aligned}
$$

where

$$
\underline{x} = \begin{bmatrix} P \\ Q \\ R \\ s_1 \\ s_2 \end{bmatrix} \qquad \underline{c} = \begin{bmatrix} 3 \\ 4 \\ -\theta \\ 0 \\ 0 \end{bmatrix} \qquad \underline{\underline{A}} = \begin{bmatrix} 2 & 2 & -1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 0 \\ 8 \\ 10 \end{bmatrix}
$$

(b)