Assigned: Tuesday March 23 2017
Due Date: Thursday April 6, 2017

MATH 565: Homework 4
Spring 2017

1. Let $\underline{\underline{A}} \in \mathbb{R}^{n \times n}$ by a symmetric positive definite matrix.

   (a) Show that the unit vectors $\underline{d}_1$, $\underline{d}_2$, …, $\underline{d}_n$ are $\underline{\underline{A}}$-conjugate vectors if and only if $\underline{\underline{D}}^T\, \underline{\underline{A}}\, \underline{\underline{D}} = \underline{\underline{I}}$ where $\underline{\underline{D}} = \begin{bmatrix} \underline{d}_1 & \underline{d}_2 & \cdots & \underline{d}_n \end{bmatrix}$.

   (b) If $\underline{\underline{Q}} \in \mathbb{R}^{n \times n}$ is an orthogonal matrix ($\underline{\underline{Q}}^T\, \underline{\underline{Q}} = \underline{\underline{I}}$), $\underline{d}_1$, $\underline{d}_2$, …, $\underline{d}_n$ are $\underline{\underline{A}}$-conjugate vectors, and $\underline{\underline{D}} = \begin{bmatrix} \underline{d}_1 & \underline{d}_2 & \cdots & \underline{d}_n \end{bmatrix}$, show that the columns of $\underline{\underline{D}}\,\underline{\underline{Q}}$ are also $\underline{\underline{A}}$-conjugate vectors.

2. Page 162: Problem 6.3.

3. Page 162: Problem 6.4.

4. Page 162: Problem 6.6.

5. Implement the BFGS Method (Algorithm 6.1 on Page 140, or lecture notes).

6. Apply the BFGS method to the following function:

$$f(x, y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2.$$

   Use $\underline{\underline{H}}_0 = \underline{\underline{I}}$ and an exact line search for each each step lengths. Do all of the following:

   - Plot this function using the `contour` or `contourf` commands and observe that there are 4 local minimums.
   - Pick 4 different initial guesses, each one that is reasonably close (not too close, but close enough to get convergence to the desired point) to one of the four local minima.
   - On a **SINGLE PLOT**, show the contours of f, and the 4 paths that each initial guess undergoes through the course of the BFGS method.
   - Report your initial guesses, your tolerance, your final positions, the number of iterations required in each case, and the value of the objective function at the final guess.