Caleb Logemann
MATH 565 Continuous Optimization
Homework 1

1. Problem 6
   Consider the steepest descent method with exact linear searches applied to the convex quadractic function (3.24). Using the properties given in this chapter, show that if the initial point is such that $x_0 - x^*$ is parallel to an eigenvector of Q, then the steepest descent method will find the solution in one step.

2. Problem 8
   Let Q be a postive definite symmetric matrix. Prove that for any vector $\underline{X}$, we have

   $$\frac{\left(\underline{x}^T\underline{x}\right)^2}{\left(\underline{x}^T\underline{\underline{Q}}\underline{x}\right)\left(\underline{x}^T\underline{\underline{Q}}^{-1}\underline{x}\right)} \geq \frac{4\lambda_n\lambda_1}{(\lambda_n + \lambda_1)^2}$$

   where λ_n and λ_1 are, respectively, the largest and smallest eigenvalues of $\underline{\underline{Q}}$. (This relation, which is known as the Kantorovich inequality, can be used to deduce (3.29) from (3.28).)

   Proof. □

3. Problem 12
   Consider a block diagonal matrix B with 1×1 and 2×2 blocks. Show that the eigenvalues and eigenvectors of B can be found by computing the spectral decomposition of each diagonal block separately.

4. For the quadractic function

   $$f(\underline{x}) = \frac{1}{2}\underline{x}^T\underline{\underline{B}}\underline{x} - \underline{x}^T\underline{b}$$

   where $\underline{\underline{B}} \in \mathbb{R}^{n \times n}$ is symmetric positive definite, show that the Newton search direction with $\alpha = 1$ satisfies the sufficient decrease assumption (3.4) for any $c_1 \leq \frac{1}{2}$ and the curvature conditions (3.5) for any $c_2 > 0$.

5. Consider the function:

   $$f(\underline{x}) = 20(x_2 - x_1^2)^2 + (1 - x_1)^2$$

   Write a MATLAB steepest descent code to find the minimizer of this function. The function be in the form

   ```
   xstar = SteepestDescent(f, x0, TOL, MaxIters)
   ```

   Use $\underline{x_0} = (1.2, 1.2)^T$. Use an exact line search to find α. You can use your $1D$ rootfinding code from Homework 1 to compute the exact line search solution α. Plot the contour lines of f and superimport the various guesses (each connected to the next via a line segment) made by the steepest descent algorithm. Produce a table of your approximations and the errors (if there are many iterations, you can just show the first 10 iterations and the last 10 iterations).

6. Consider the function:

   $$f(\underline{x}) = 20(x_2 - x_1^2)^2 + (1 - x_1)^2$$

   Write a MATLAB Newton descent code to find the minimizer of this function. The function be in the form

```
xstar = NewtonDescent(f, x0, TOL, MaxIters)
```

Use $\underline{x_0} = (1.2, 1.2)^T$. Use $\alpha = 1$. Use the backslash operator to invert the appropriate matrices. Plot the contour lines of f and superimport the various guesses (each connected to the next via a line segment) made by the Newton descent algorithm. Produce a table of your approximations and the errors (if there are many iterations, you can just show the first 10 iterations and the last 10 iterations).