Assigned: Thursday Jan. 19, 2017
Due Date: Thursday Feb. 2, 2017

MATH 565: Homework 1
Spring 2017

1. Page 27: Problem 1.
2. Page 28: Problem 8.
3. Page 29: Problem 9.
4. Page 29: Problem 13.
5. Page 29: Problem 14.
6. Consider the following fixed point iteration scheme:

$$x_{k+1} = x_k - \frac{[g(x_k)]^2}{g(x_k + g(x_k)) - g(x_k)}.$$

Prove that if this method convergences to a root $x^\star$ of $g(x)$ such that $g'(x^\star) \neq 0$ and $g''(x^\star) \neq 0$, then the rate of convergence is quadratic: $p = 2$.

7. Implement the method from Problem 6 in MATLAB (or Python if you prefer). Use it to solve Problem 8 and 9.
8. The van der Waal equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

generalizes the ideal gas law $PV = nRT$. In each equation, P represents the pressure (atm), V represents the volume (liters), n is the number of moles of gas, and T represents the temperature (K). R is the universal gas constant and has the value

$$R = 0.08205 \, \frac{\text{liters} \cdot \text{atm}}{\text{mole} \cdot K}.$$

Determine the volume of 1 mole of isobutane at a temperature of $T = 313K$ and a pressure of $P = 2$ atm, given that, for isobutane, $a = 12.87 \text{ atm} \cdot \text{liters}^2$ and $b = 0.1142$ liters. Compare this to the value predicted from the ideal gas law. You may use any one of your methods (make clear in your writeup which one you are using, what initial guesses or intervals you are using, etc...).

9. According to Archimedes' law, when a solid of density σ is placed in a liquid of density ρ, it will sink to a depth h that displaces an amount of liquid whose weight equals the weight of the solid. For a sphere of radius r, Archimedes law is

$$\frac{1}{3}\pi\left(3rh^2 - h^3\right)\rho = \frac{4}{3}\pi r^3 \sigma.$$

Given $r = 5$, $\rho = 1$, and $\sigma = 0.6$, determine h. You may use any one of your methods (make clear in your writeup which one you are using, what initial guesses or intervals you are using, etc...).