MATH-566 **HW 3**

Due **Sep 21** before class. Just bring it before the class and it will be collected there.

1: (*Farkas Lemma*)
Show that

$$A\mathbf{x} = \mathbf{b} \text{ has a non-negative solution iff } \forall \mathbf{y} \in \mathbb{R}^m \text{ with } \mathbf{y}^T A \geq \mathbf{0}^T \text{ implies } \mathbf{y}^T\mathbf{b} \geq 0$$

implies

$$A\mathbf{x} \leq \mathbf{b} \text{ has a non-negative solution iff } \forall \mathbf{y} \in \mathbb{R}^m, \mathbf{y} \geq \mathbf{0} \text{ with } \mathbf{y}^T A \geq \mathbf{0}^T \text{ implies } \mathbf{y}^T\mathbf{b} \geq \mathbf{0}.$$

Solution:
From book by Matoušek: We need an equivalent condition for $A\mathbf{x} \leq \mathbf{b}$ having a nonnegative solution. To this end, we form the matrix $A' = (A|I_m)$, where I_m is the identity matrix $m \times m$. We note that $A\mathbf{x} \leq \mathbf{b}$ has a nonnegative solution if and only if $A'\mathbf{x} = \mathbf{b}$ has a nonnegative solution. By the first equivalence, this is equivalent to the condition that all $\mathbf{y}$ with $\mathbf{y}^T A' \geq \mathbf{0}^T$ satisfy $\mathbf{y}^T\mathbf{b} \geq 0$. And finally, $\mathbf{y}^T A' \geq \mathbf{0}^T$ says exactly the same as $\mathbf{y}^T A \geq \mathbf{0}^T$ and $\mathbf{y} \geq 0$, and hence we have the desired equivalence.

2: (*Fitting line as linear program*)
Some university in Iowa was measuring the loudness of the fan's screaming during the first touchdown of the local team. The measurements contain loudness in dB and the number of people at the stadium in thousands.

# fans	53	55	59	61.5	61.5
dB	90	94	95	100	105

Find a line $y = ax + b$ *best* fitting the data. There are several different notions of best fitting. Commonly used is least squares that is minimizing $\sum_i (ax_i + b - y_i)^2$. But big outliers move the result a lot (and it is troublesome to do it using linear programming). Use the one that minimizes the sum of differences. That is

$$\sum_i |ax_i + b - y_i|.$$

Write a linear program that solves the problem and solve it for the "measured" data.

(Fun facts: The Seattle Seahawks, who boast that their fans caused a small earthquake after a 2011 touchdown, acclaimed their crowds record 136.6-decibel noise level this September after an effort orchestrated by the fan group Volume 12. The loudest crowd roar at a sports stadium is 142.2 db and was achieved by fans of the Kansas City Chiefs, at Arrowhead Stadium in Kansas City, Missouri, on 29 September 2014.)

Solution:
The problem is minimize $\sum_i |ax_i + b - y_i|$. Trouble is the absolute value. We can get around it by introducing new variables z_i for every point. The program will look like:

$$\begin{cases} \text{minimize} & \sum_i z_i \\ \text{subject to} & ax_i + b - y_i \leq z_i \text{ for all } i \\ & -ax_i - b + y_i \leq z_i \text{ for all } i \end{cases}$$

Now we write the data to APmonitor and it gives solution $a = 1.176\ldots$, $b = 27.647\ldots$ and the value of the objective function is $8.705\ldots$.

3: (*Scheduling participants*)
Suppose you are preparing a schedule for classes. You have fixed number of classes and students. Every student told you which classes (s)he wants to attend. However, you do not have enough time slots to run all classes sequentially so you need to make some classes run in parallel. Create a schedule and argue why it is the best schedule in the sense that people *as few conflicts as possible.* You should be able to justify the optimality of the schedule in some sense.

Make schedule with 3 and with 4 timeslots. Assume that there is no limit on how many classes can run in one timeslot.

Row corresponds to one class, columns corresponds to one students. 1 means the student wants to attend the class.

```
[[0,0,0,1,0,1,0,1,0,0,1,0,1,0,0,1,1,1,1,1,1,1,1,0,1,1,1,1,0,0,1,0,0,0,1,1,1,1,1,1,0,1,1,1,1,1,1,0,0,1,0,1,0,0],
[1,1,1,1,0,1,0,1,0,0,0,1,1,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,1,0,1,0,0,1,0,0,0,1,0,1,0,1,0,0,0,0,1,0,0,1,1,1,0],
[1,0,1,0,1,0,1,0,1,1,1,1,0,0,1,1,1,1,1,1,0,0,0,1,1,1,0,1,0,0,0,1,0,0,1,1,0,1,1,1,0,1,1,1,1,1,0,0,0,1,1,0,0,0],
[0,0,1,0,0,0,1,0,0,1,1,0,0,0,0,1,0,0,0,0,0,0,0,0,1,1,0,0,0,1,0,0,1,0,0,0,1,0,0,0,0,0,0,1,1,1,0,1,1,0,0,0,0],
[1,0,0,1,0,1,0,0,0,1,0,0,1,0,0,0,0,0,0,0,1,0,1,0,0,1,0,1,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0],
[0,1,1,1,1,0,0,0,1,1,1,1,1,0,1,0,1,1,0,1,0,0,0,0,1,0,0,0,1,1,0,0,1,1,0,1,1,1,1,0,0,0,1,1,1,1,0,0,1,0,1,0,1],
[0,0,1,1,0,1,1,0,0,1,1,1,0,1,1,0,1,1,0,1,0,1,1,0,0,1,1,0,0,0,1,0,0,0,0,0,1,0,0,0,0,1,1,0,1,1,0,0,1,0,1,1,0],
[1,0,0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,1,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0],
[0,1,0,0,0,1,1,0,1,0,1,0,0,0,0,1,1,1,0,1,0,1,1,1,1,1,0,0,0,0,1,0,1,0,0,0,1,0,1,1,0,0,1,1,1,0,1,0,0,0,0,1,0,1],
[1,1,1,1,0,1,0,1,0,0,0,1,1,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,1,0,1,0,0,1,0,0,0,1,0,1,0,1,0,0,0,0,1,0,0,1,1,1,0],
[1,0,1,0,1,0,1,0,1,1,1,1,0,0,1,1,1,1,1,1,0,0,0,0,1,1,0,1,0,0,0,1,0,0,1,1,0,1,1,1,0,1,1,1,1,1,0,0,0,1,1,0,0,0],
[0,0,1,0,0,0,1,0,0,1,1,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,1,1,1,0,1,1,0,0,0,0],
[1,0,0,1,0,1,0,0,0,1,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0]]
```

See the latex source on black board to copy the matrix easily. I suggest to use Sage (or Matlab) to deal with the data.

PS: These data are real - they are not made up.