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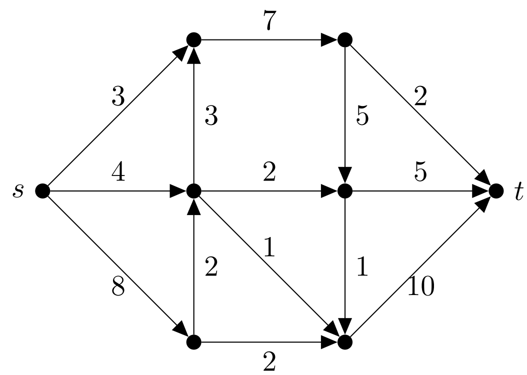
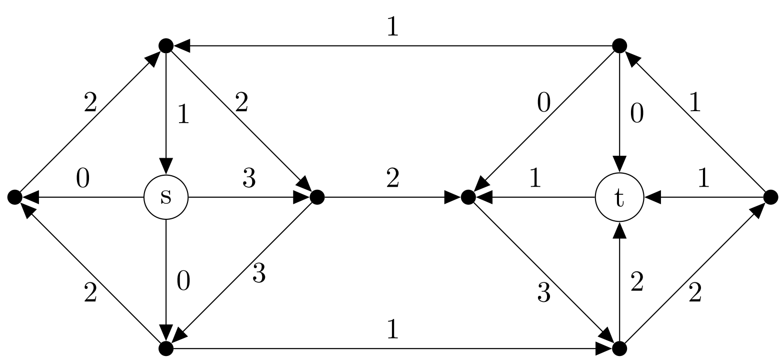
MATH 566 Discrete Optimization
Homework 6

1. Consider the graph below

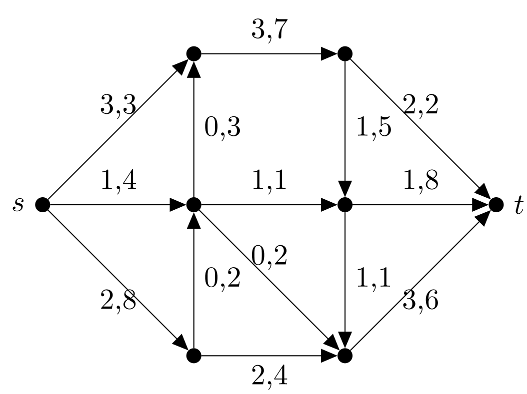


Find a shortest path and prove optimality using duality (find dual LP and its optimal solution)

2. Consider the network below with given edge values, forming an integer feasible flow. Find a list of path and cycle flows whose sum is this flow.

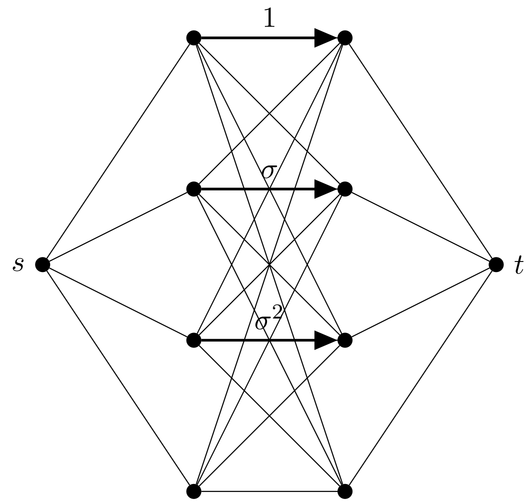


3. Consider the network below with given capacity and flow values. (The edge label f, u means flow-value f and capacity u.) Find augmenting paths and augment the flow to a maximum flow. Provide the list of residual graphs AND augmenting paths. It other words, run Ford-Fulkerson algorithm.



4. Let (G, u, s, t) be a network, and let $\delta^+(X)$ and $\delta^+(Y)$ be minimum s-t-cuts in (G, u). Show that $\delta^+(X \cap Y)$ and $\delta^+(X \cup Y)$ are also minimum s-t-cuts in (G, u).

5. Show that in case of irrational capacities, the Ford-Fulkerson algorithm may not terminate at all. Hint: See the Korte book (in particular exercises on page 199.). It contains the following network:



Where $\sigma = \frac{\sqrt{5}-1}{2}$. Note that σ satisfies $\sigma^n = \sigma^{n+1} + \sigma^{n+2}$. All other capacities are 1.

6. Red-Blue meta algorithm for MST. Let G be a graph and w be a weight assignment to $E(G)$. Assume that all weights are distinct. Start with all edges being uncolored. Apply the following rules as long as possible.

   - if $e \in E$ is in a cycle C where e is the heaviest edge, color e red
   - if there is a cut where $e \in E$ is the lightest edge, color e blue.

   Claim is that blue edges form a minimum spanning tree.

   - Show that red edge cannot be in MST.
   - Show that blue edge must be in MST.
   - Show that blue edges form a tree
   - Show that every edge gets colored.
   - Show that no edge satisfies both red and blue criteria. (i.e. every edge has one color).

7. Implement Edmons-Karp algorithm and run it on the network from question three. Print the sequence of augmenting paths used by your implementation. Print the flow and its value.