Fall 2016, MATH-566

Appendix A - Linear programming

Discuss: class policies, hw, need for sage, outline of the class, books, graph theory

Optimization problem

$$(P)\begin{cases}\text{minimize} & f(\mathbf{x})\\ \text{s.t.} & g_1(\mathbf{x}) \le b_1\\ & \vdots \\ & g_m(\mathbf{x}) \le b_m,\end{cases}$$

where $\mathbf{x} \in \mathbb{R}^n$, $f, g_i : \mathbb{R}^n \to \mathbb{R}$, $b_i \in \mathbb{R}$. Program (P) is *linear* if f, g_i are linear functions. Reformulation:

$$(LP)\begin{cases}\text{minimize} & \mathbf{c}^T \cdot \mathbf{x}\\ \text{s.t.} & A\mathbf{x} \le \mathbf{b},\end{cases}$$

where $\mathbf{c} \in \mathbb{R}^n$, $\mathbf{b} \in \mathbb{R}^m$, $A \in \mathbb{R}^{m\times n}$. Also maximize, $\le$, $=$. Program (LP) if efficiently solvable.

History note: Dantzig and Kantorovich

Examples of linear programming:

Diet problem: How much apricots (x_1), bananas (x_2) and cucumbers (x_3) one has to eat to get enough of Vit A, B, C? Minimize cost.

Need to know: % of daily value and cost:

	A	C	K	\$
apricots	60	26	6	1.53
bananas	3	33	1	0.37
cucumbers	2	7	12	0.18

Does the table make sense?

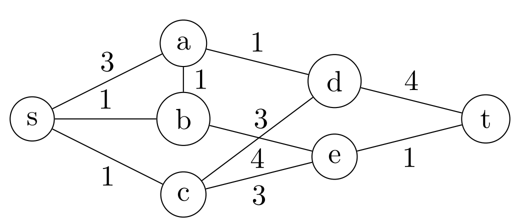
Solution: There should be units for apricots, bananas and cucumbers. It is one serving, which is 155g, 225g, and 133g respectively.

$$(LP)\begin{cases}\text{minimize} & 1.53x_1 + 0.37x_2 + 0.18x_3\\ \text{s.t.} & 60x_1 + 3x_2 + 2x_3 \ge 100\\ & 26x_1 + 33x_2 + 7x_3 \ge 100\\ & 6x_1 + 1x_2 + 12x_3 \ge 100\end{cases}$$

Solution: $(x_1, x_2, x_3) = (1.4, 0.3, 7.6)$ and the cost is \$3.62.

HW: Feed the professor!

Network flow: Firefighters in Washington need your help them calculate how much water they could use. Sketch gives, water source s, fire location t and scheme of network of pipes.



How to write a linear program?

Ropes: We are producing packages of two 15cm and one 20cm long ropes (say for some kid's game). What is the to maximize the number of packages if we have 400 times 50cm and 100 times 65cm ropes? (How to cut the ropes?)

Solution: #15 cm = A, #20 cm = B,

$$\begin{aligned}50cm &= 15 + 15 + 20 \ldots x_1 \ldots 2A + B\\ &= 20 + 20 \ldots x_2 \ldots 2B\\ &= 15 + 15 + 15 \ldots x_3 \ldots 3A\end{aligned}$$

$$\begin{aligned}65cm &= 20 + 20 + 20 \ldots y_1 \ldots 3B\\ &= 15 + 15 + 15 + 15 \ldots y_2 \ldots 4A\\ &= 20 + 15 + 15 + 15 \ldots y_3 \ldots B + 3A\\ &= 20 + 20 + 15 \ldots y_4 \ldots 2B + A\end{aligned}$$

$$(LP)\begin{cases}\text{maximize} & p\\ \text{s.t.} & p \le \frac{1}{2}A\\ & p \le B\\ & A = 2x_1 + 3x_3 + 4y_2 + 3y_3 + y_4\\ & B = x_1 + 2x_2 + 3y_1 + y_3 + 2y_4\\ & 400 \ge x_1 + x_2 + x_3\\ & 100 \ge y_1 + y_2 + y_3\end{cases}$$

Solution:

$$p = 528.5, x_1 = 400, x_2 = 0, x_3 = 0, y_1 = 14.28, y_2 = 0y_3 = 85.71, y_4 = 0$$

We are missing that x_i, y_j are actually integers!