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MATH 566 Discrete Optimization
Homework 2

1.

2. The original diet problem can be written as

$$\min \mathbf{c}^T\mathbf{x}$$
$$s.t. A\mathbf{x} \geq \mathbf{b}$$

where

$$A = \begin{bmatrix} 125 & 35 & 0 & 0 & 0.275 \\ 190 & 7 & 2 & 7 & 0.130 \\ 110 & 27 & 1 & 0 & 0 \\ 50 & 13 & 0 & 0 & 0 \\ 190 & 27 & 1 & 4 & 0.360 \\ 100 & 18 & 1 & 2 & 0.06 \\ 60 & 9 & 2 & 2 & 0.2 \\ 20 & 4 & 2 & 1 & 0.38 \\ 70 & 12 & 3 & 4 & 0.37 \\ 60 & 25 & 4 & 2 & 0.105 \\ 120 & 2 & 0 & 1 & 0.025 \\ 140 & 24 & 1 & 4 & 0.44 \\ 80 & 1 & 0 & 8 & 0.69 \\ 60 & 0 & 0 & 11 & 0.52 \\ 150 & 25 & 1 & 4 & 0.025 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 2779 \\ 383 \\ 38 \\ 60 \\ 1.5 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 0.75 \\ 0.13 \\ 0.34 \\ 0.13 \\ 0.48 \\ 0.17 \\ 0.21 \\ 0.21 \\ 0.21 \\ 0.25 \\ 0.42 \\ 0.26 \\ 0.50 \\ 1.15 \\ 0.29 \end{bmatrix}$$

The dual of the diet program can be written as follows

$$\max 2779n_1 + 383n_2 + 38n_3 + 60n_4 + 1.5n_5$$

3. A paper mill manufactures rolls of paper of a standard width, 3 meters. But customers want to buy rolls of shorter width, and the mill has to cut such rolls from the 3m rolls. Let us consider an order of

   - 97 rolls of width 135 cm,
   - 610 rolls of width 108 cm,
   - 395 rolls of width 93 cm, and
   - 211 rolls of width 42 cm.

   What is the smallest number of 3m rolls that have to be cut in order to satisfy this order?

   First we must enumerate all of the different ways that a 3m roll can be cut up into these lengths.

   (a) $c_1 = 2 \times 135$
   (b) $c_2 = 135 + 108 + 42$
   (c) $c_3 = 135 + 93 + 42$
   (d) $c_4 = 135 + 3 \times 42$

(e) $c_5 = 2 \times 108 + 2 \times 42$

(f) $c_6 = 108 + 2 \times 93$

(g) $c_7 = 108 + 93 + 2 \times 42$

(h) $c_8 = 108 + 4 \times 42$

(i) $c_9 = 3 \times 93.$

(j) $c_{10} = 2 \times 93 + 2 \times 42$

(k) $c_{11} = 93 + 4 \times 42$

(l) $c_{12} = 7 \times 42$

Any other cuts leave excess paper that could be used for the order so won't be necessary. Then we can create the integer linear program.

$$\begin{aligned} \min \sum_{i=1}^{12} (c_i) \\ s.t. 2c_1 + c_2 + c_3 + c_4 \geq 97 \\ c_2 + 2c_5 + c_6 + c_7 + c_8 \geq 610 \\ c_3 + 2c_6 + c_7 + 3c_9 + 2c_{10} + c_{11} \geq 395 \\ c_2 + c_3 + 3c_4 + 2c_5 + 2c_7 + 4c_8 + 2c_{10} + 4c_{11} + 7c_{12} \geq 211 \end{aligned}$$