

MATH-566 Final Exam

Due **72 hours since your starting time**. Just bring it to the Emperor (office 422).

Help the Emperor to rule the Empire and crush the Rebels.



In every problem where you use some algorithm to solve the problem, describe the algorithm. Moreover, describe the steps of the algorithm as it is running on the problem instance you are solving or provide an implementation in Sage. You can reuse code you implemented while creating HW. APMonitor is also allowed.

May the SAGE be with you!

1: (Collect taxes for the Emperor)

Emperor has decided to build Death Star II. He needs money to pay for it. He sent you to collect taxes from several planets. Here are the coordinates of the planets assigned to you. You are starting at Corusant (0,0) and want to return there. The time of travel between the planets corresponds to their Euclidean distance. You have to visit all planets. Minimize the travel time.

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X = [ [0,0], [9,0], [1,1], [8,1], [0,2], [9,2], [4,3], [5,3], \
      [0,4], [1,4], [2,4], [3,4], [6,4], [7,4], [8,4], [9,4], \
      [0,5], [1,5], [3,5], [2,5], [6,5], [7,5], [8,5], [9,5], \
      [4,6], [5,6], [0,7], [9,7], [1,8], [8,8], [0,9], [9,9] ];
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Provide approximate solution and provide also a lower bound. Create map of your trip (means 2D plot). The bounds do not have to be perfect.

2: (Plan a defense against a surprise rebel attack)

You are leading a group of 4 imperial fighters that are protecting an imperial shuttle transporting secret plans to Death Star II. You were ambushed by 3 rebel ships. You must protect the secret plans! Do not repeat the same mistake that happened with Death Star.

Choose one fighter from your group to accompany the transport shuttle cover their retreat by fighting the rebel ship. The remaining three ships will each try to fight one rebel ship. You need to fight against each of the rebel ships to prevent pursuit of the transport shuttle and you do not want to leave the transport ship unguarded.

For the remaining 3 ships that fight the rebel ships, maximize the total damage (sum of damages) caused to the rebel ships. For every pair of imperial and rebel ship, the damage caused by imperial ships to rebel ships is described in the following table.

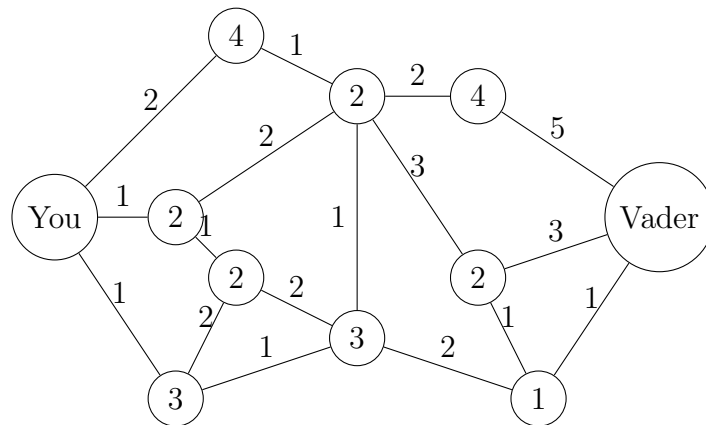


	r_1	r_2	r_3
i_1	3	2	3
i_2	2	3	1
i_3	4	2	2
i_4	1	5	1

Provide optimal solution.

3: (*Bring Vader's lightsaber*)

Darth Vader forgot his lightsaber. Bring it to him as fast as you can so the Emperor can watch a lightsaber fight between Darth Vader and Luke Skywalker. You have only a small ship and hence you need refueling. Here is a map of the space. Every planet has a number that corresponds to the time needed for refueling and every connection has a travel time associated to it.



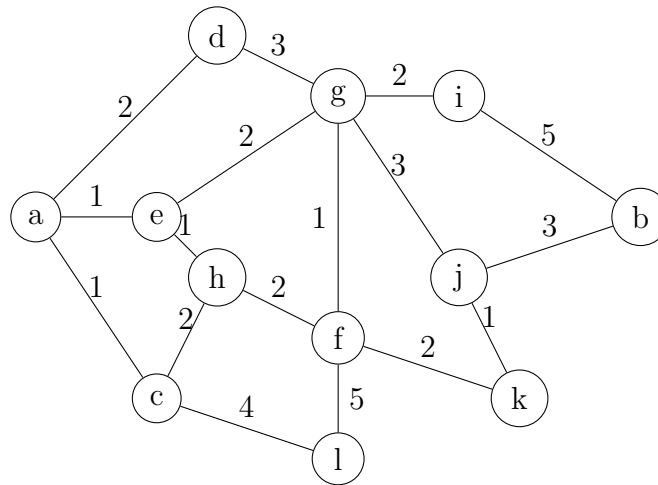
Provide optimal solution.



4: (*Promote Chaos*)

The Emperor, as any other good emperor, knows that it is best if his enemies are fighting among each other. Here is a map of Emperor's enemies and how much does it cost for each pair to start fighting each other. Find for the Emperor which pairs to trick into fighting each other such that every enemy is in exactly one fight. Provide also a certificate verifying the optimality of your solution.

Provide optimal solution and certificate of optimality.



Additional explanations

- APMonitor allowed
- MixedIntegerProgram allowed
- You need to explain what and why are you doing. Solving problems by staring at them is not acceptable.

Question 1: Reasonable bounds do not have to be the best possible bounds. They must be justified.