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MATH 566 Discrete Optimization
Homework 9

1. Implement Directed Minimum Mean Cycle in Sage. Before implementing the algorithm, show that it is possible to slightly modify the algorithm. Instead of adding an extra vertex s and edges from s to all other vertices, it is possible to simply assign $F_0(v) = 0$ for all $v \in V$ at the beginning. This avoids the hassle with adding an extra vertex. But it requires an argument that the algorithm is still correct.

2. Show that in integer program, it is possible to express the following constraint:

$$x \in [100, 200] \cup [300, 400]$$

in other words

$$100 \leq x \leq 200 \text{ or } 300 \leq x \leq 400$$

How to express the constraint *without* using *or*?

3. Determine which of the matrices below are (i) unimodular, (ii) totally unimodular, or (iii) neither. Be sure to explain your answer.

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

a. **b.** **c.**

(i) Since these are all square matrices, they will be unimodular if their determinant is ± 1. So I compute the determinants of these 3 matrices. First I will compute the determinant of (a).

$$\det(a) = \begin{vmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

Expand along first column

$$\det(a) = 1 \times \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix} - (-1) \times \begin{vmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix}$$

Expand along the first column for the first determinant and along the last column for the second determinant

$$\begin{aligned} &= 1 \times \left(-1 \times \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\right) - (-1) \times \left(1 \times \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix}\right) \\ &= 1 \times (-1 \times -1) - (-1) \times (1 \times -1) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

Since the determinant of (a) is 0 this matrix is not unimodular.
Second I will compute the determinant of (b).

$$\det(b)$$

(ii)

(iii)

4. Show that $A \in \mathbb{Z}^{m \times n}$ is totally unimodular iff $[A\ I]$ is unimodular (where I is $m \times m$ unit matrix).
5. Find a unimodular matrix A, that is not totally unimodular.