MATH-566 HW 2

Due **Sep 14** before class. Just bring it before the class and it will be collected there.

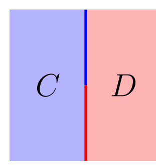
1: (*Separation of sets*)
(a) Give an example of bounded convex sets C and D in $\mathbb{R}^d$, where $C \cap D = \emptyset$ but there are no hyperplane **strictly** separating C and D. That is $\mathbf{a} \in \mathbb{R}^d$ and $b \in \mathbb{R}$ such that

$$\mathbf{a}^T\mathbf{x} > b, \forall \mathbf{x} \in C \qquad \text{and} \qquad \mathbf{a}^T\mathbf{x} < b, \forall \mathbf{x} \in D.$$

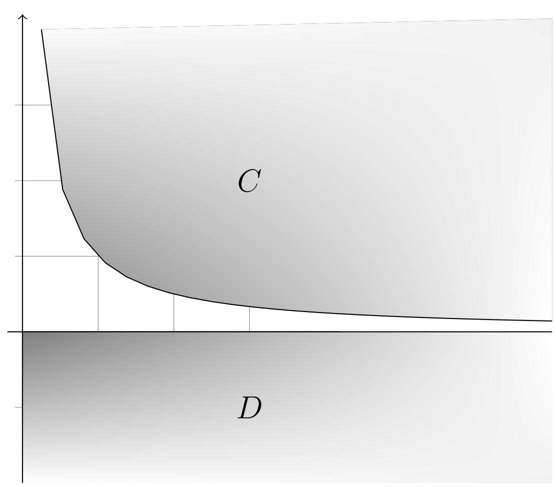
(b) Give an example of closed convex sets C and D, where $C \cap D = \emptyset$ that cannot be **strictly** separated.

Solution:
There are many examples.

(a) Let $C = \{(x,y) : -1 < x < 0, -1 < y < 1\} \cup \{(0,y) : 0 < y < 1\}$ and $D = \{(x,y) : 0 < x < 1, -1 < y < 1\} \cup \{(0,y) : -1 < y < 0\}$.



(b) Let $C = \{(x,y) : 0 < x, y \geq \frac{1}{x}\}$ and $D = \{(x,y) : 0 \leq x, 0 \geq y\}$.



2: (*Dualization*)
Dualize your diet problem. Take data of your diet problem from HW1 and create dual of the program and solve it. Answer should contain solution of the dual and interpretation of the result of the dual (what do the numbers mean).

Solution:

The matrix is transposed. The variables correspond to *prices* of different nutrient in the diet.

3: (*Integer program*)
Paper mill manufactures rolls of paper of a standard width 3 meters. But customers want to buy paper rolls of shorter width, and the mill has to cut such rolls from the 3 m rolls. One 3 m roll can be cut, for instance, into two rolls 93 cm wide, one roll of width 108 cm, and a rest of 6 cm (which goes to waste). Let us consider an order of

- 97 rolls of width 135 cm,
- 610 rolls of width 108 cm,
- 395 rolls of width 93 cm, and
- 211 rolls of width 42 cm.

What is the smallest number of 3 m rolls that have to be cut in order to satisfy this order, and how should they be cut?
(Hints: 1m = 100cm, you may need to consider that variables are integers instead of just real numbers. APMonitor can do it by prefix int in variable and Sage by using new_variable(integer=True)*).*

Solution:
We model this by creating variables, where each variable corresponds to the number of rolls cut in different ways. Say x_1 will represent cutting $300cm$ into twice $135cm$. Here are the options:

$$\begin{aligned}
&x_1 : 2\times 135 && x_7 : 108+93+2\times 42\\
&x_2 : 135+108+42 && x_8 : 108+4\times 42\\
&x_3 : 135+93+42 && x_9 : 3\times 93\\
&x_4 : 135+3\times 42 && x_{10} : 2\times 93+2\times 42\\
&x_5 : 2\times 108+2\times 42 && x_{11} : 93+4\times 42\\
&x_6 : 108+2\times 93 && x_{12} : 7\times 42
\end{aligned}$$

With these variables, we can express if we have enough rolls of width $135cm$.

$$2x_1 + x_2 + x_3 + x_4 \geq 97$$

The problem without int variables is

```
Model rolls
  Variables
    x[1:12] = 0 , >= 0
  End Variables
  Equations
    minimize x[1]+x[2]+x[3]+x[4]+x[5]+x[6]+x[7]+x[8]+x[9]+x[10]+x[11]+x[12]
```

```
      ! 135
    2*x[1] + x[2] + x[3] + x[4] >= 97
     ! 108
    x[2]+2*x[5]+x[6]+x[7]+x[8]  >= 610
     ! 93
    x[3]+2*x[6]+x[7]+3*x[9]+2*x[10]+x[11] >= 395
     ! 42
    x[2]+x[3]+3*x[4]+2*x[5]+2*x[7]+4*x[8]+2*x[10]+4*x[11]+7*x[12] >= 211
  End Equations
End Model
```

And with int variables it is (not fitting to the page)

```
Model rollsINT
  Variables
    intx[1:12] = 0 , >= 0
  End Variables
  Equations
    minimize intx[1]+intx[2]+intx[3]+intx[4]+intx[5]+intx[6]+intx[7]+intx[8]+intx[9]+int
      ! 135
    2*intx[1] + intx[2] + intx[3] + intx[4] >= 97
     ! 108
    intx[2]+2*intx[5]+intx[6]+intx[7]+intx[8]  >= 610
     ! 93
    intx[3]+2*intx[6]+intx[7]+3*intx[9]+2*intx[10]+intx[11] >= 395
     ! 42
    intx[2]+intx[3]+3*intx[4]+2*intx[5]+2*intx[7]+4*intx[8]+2*intx[10]+4*intx[11]+7*intx
  End Equations
End Model
```

Solution is $x_1 = 7, x_2 = 83, x_5 = 165$ and $x_6 = 198$. The solution is NOT unique. The value of the objective function is 453.0