Discontinuous Galerkin Method for solving a Thin-Film Equation

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Overview

1 Introduction

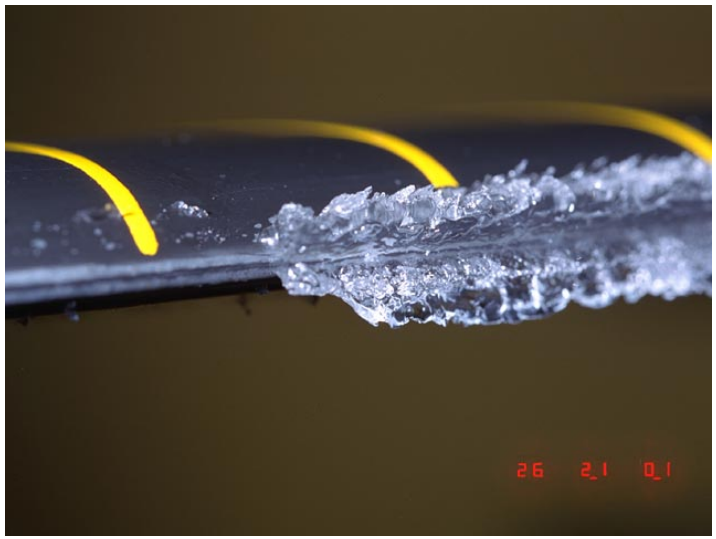
2 Convection

3 Diffusion

4 Conclusion

Motivation

- Aircraft Icing
- Runback



- Industrial Coating
- Paint Drying

Model Equations

- Navier-Stokes Equation

$$u_t + (uv)_x = 0$$
$$(uv)_t + =$$

- Asymptotic Limit, $u << L$
- Thin-Film Equation - 1D

$$u_t + \left(f(x,t)u^2 - g(x,t)u^3\right)_x = \left(h(x,t)u^3 u_{xxx}\right)_x$$

Current Model

- Simplified Expression

$$u_t + \left(u^2 - u^3\right)_x = \left(u^3 u_{xxx}\right)_x$$

- Operator Splitting

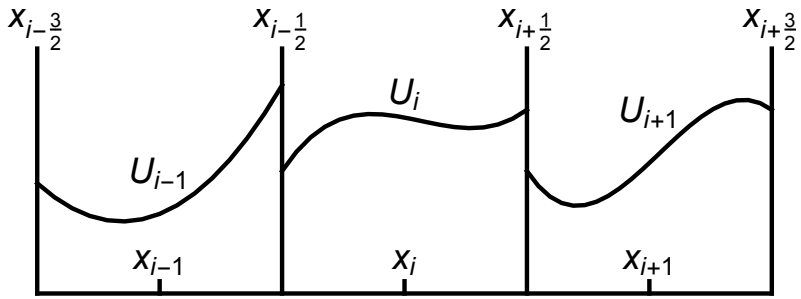
$$u_t + \left(u^2 - u^3\right)_x = 0$$
$$u_t - \left(u^3 u_{xxx}\right)_x = 0$$

Introduction to Discontinuous Galerkin

- Partition the domain, $[a, b]$ as

$$a = x_{1/2} < \cdots < x_{i-1/2} < x_{i+1/2} < \cdots < x_{N+1/2} = b$$

- Label the cells $V_i = \left[x_{i-1/2}, x_{i+1/2}\right]$
- $\Delta x = x_{i+1/2} - x_{i-1/2}$
- $x_i = \frac{x_{i+1/2} + x_{i-1/2}}{2}$.



- Solution on each cell

$$u|_{x \in V_i} \approx U_i = \sum_{k=1}^{M} U_i^k \phi^k(\xi)$$

- Legendre Basis - ϕ^k

Convection

- Convection Equation

$$u_t + \left(u^2 - u^3\right)_x = 0$$

- Runge-Kutta Discontinuous Galerkin

Numerical Results

Diffusion

- Diffusion Equation

$$u_t - \left(u^3 u_{xxx}\right)_x = 0$$

- Local Discontinuous Galerkin

Multigrid Solver

Numerical Results

Future Work

- Higher dimensions
- Curved surfaces
- Space and time dependent coefficients
- Incorporation with air flow models

Conclusion

- Questions?

Bibliography

[1] Y. Ha, Y.-J. Kim, and T.G. Myers. "On the numerical solution of a driven thin film equation". In: *J. Comp. Phys.* 227.15 (2008), pp. 7246–7263.

[2] J.A. Rossmanith. DoGPack. Available from `http://www.dogpack-code.org/`.