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Convection

Diffusion

Conclusio

Discontinuous Galerkin Method for solving a Thin-Film Equation

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Overview

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ntroduction Convection Diffusion Conclusion

- 1 Introduction
- 2 Convection
- 3 Diffusion
- 4 Conclusion

Motivation

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Introduction

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Diffusion

Conclusion

Aircraft Icing

Runback





- Industrial Coating
- Paint Drying

Model Equations

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Navier-Stokes Equation

$$u_t + (uv)_x = 0$$
$$(uv)_t + =$$

- Asymptotic Limit, *u* << *L*
- Thin-Film Equation 1D

$$u_t + (f(x,t)u^2 - g(x,t)u^3)_x = (h(x,t)u^3u_{xxx})_x$$

Current Model

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Simplified Expression

$$u_t + \left(u^2 - u^3\right)_x = \left(u^3 u_{xxx}\right)_x$$

Operator Splitting

$$u_t + (u^2 - u^3)_x = 0$$

$$u_t - (u^3 u_{xxx})_x = 0$$

Introduction to Discontinuous Galerkin

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Introduction

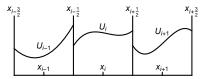
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■ Partition the domain, [a, b] as

$$a = x_{1/2} < \cdots < x_{i-1/2} < x_{i+1/2} < \cdots < x_{N+1/2} = b$$

- Label the cells $V_i = [x_{i-1/2}, x_{i+1/2}]$
- $\Delta x = x_{i+1/2} x_{i-1/2}$
- $x_i = \frac{x_{i+1/2} + x_{i-1/2}}{2}$.



■ Solution on each cell

$$u|_{x\in V_i} \approx U_i = \sum_{k=1}^M U_i^k \phi^k(\xi)$$

Legendre Basis - ϕ^k

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Convection Equation

$$u_t + \left(u^2 - u^3\right)_x = 0$$

■ Runge-Kutta Discontinuous Galerkin

Numerical Results

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Diffusion Equation

$$u_t - \left(u^3 u_{xxx}\right)_x = 0$$

■ Local Discontinuous Galerkin

Multigrid Solver

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Industrial

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Numerical Results

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References

Future Work

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Introduc

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Conclusion

- Higher dimensions
- Curved surfaces
- Space and time dependent coefficients
- Incorporation with air flow models

Conclusion

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Conclusion

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References

Questions?

Bibliography

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References

- [1] Y. Ha, Y.-J. Kim, and T.G. Myers. "On the numerical solution of a driven thin film equation". In: *J. Comp. Phys.* 227.15 (2008), pp. 7246–7263.
- [2] J.A. Rossmanith. DoGPACK. Available from http://www.dogpack-code.org/.