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Convection

Diffusion

Conclusio

Discontinuous Galerkin Method for solving a Thin-Film Equation

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Overview

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- 1 Introduction
- 2 Convection
- 3 Diffusion
- 4 Conclusion

Motivation

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Aircraft Icing

Runback





- Industrial Coating
- Paint Drying

Model Equations

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■ Navier-Stokes Equation

$$\rho_t + (\rho u)_x = 0$$
$$(\rho u)_t + (\rho u^2 + p)_x =$$
$$E_t + (u(E + p))_x =$$

- **A**symptotic Limit, $\rho << L$
- Thin-Film Equation 1D with u as fluid height.

$$u_t + (f(x,t)u^2 - g(x,t)u^3)_x = (h(x,t)u^3u_{xxx})_x$$

Current Model

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Simplified Expression

$$u_t + \left(u^2 - u^3\right)_x = \left(u^3 u_{xxx}\right)_x$$

Operator Splitting

$$u_t + (u^2 - u^3)_x = 0$$

$$u_t - (u^3 u_{xxx})_x = 0$$

Introduction to Discontinuous Galerkin

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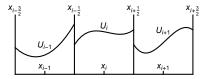
Introduction

Convection

Conclusio Reference ■ Partition the domain, [a, b] as

$$a = x_{1/2} < \cdots < x_{i-1/2} < x_{i+1/2} < \cdots < x_{N+1/2} = b$$

- $V_i = [x_{i-1/2}, x_{i+1/2}]$
- $\Delta x = x_{i+1/2} x_{i-1/2}$
- $x_i = \frac{x_{i+1/2} + x_{i-1/2}}{2}$.



■ Solution of order *M* on each cell

$$u|_{x\in V_i} \approx U_i = \sum_{k=1}^M U_i^k \phi^k(\xi)$$

Legendre Basis - ϕ^k

Numerical Solutions

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- Use canonical variable $\xi \in [-1,1]$
- Let $\{\phi^k(\xi)\}$ be the Legendre polynomials.
- Solution of order *M* on each cell

$$u|_{x\in V_i} \approx U_i = \sum_{k=1}^M U_i^k \phi^k(\xi)$$

Convection

Convection

Convection Equation

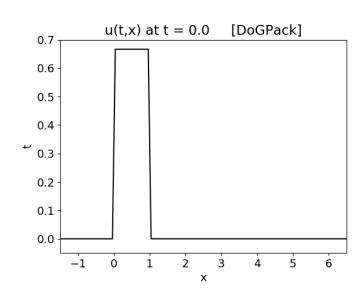
$$u_t + \left(u^2 - u^3\right)_x = 0$$

Weak Form

$$\int_{-1}^{1} u_{t} \phi(\xi) + \frac{2}{\Delta x} (u^{2} - u^{3})_{\xi} \phi \, d\xi$$

Runge-Kutta Discontinuous Galerkin

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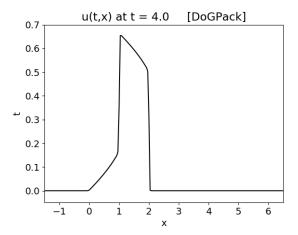


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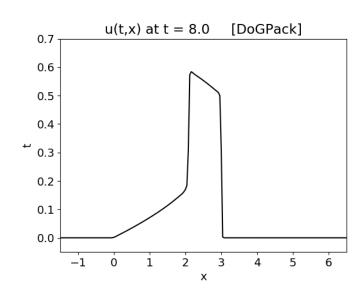
Convection

Diffusion

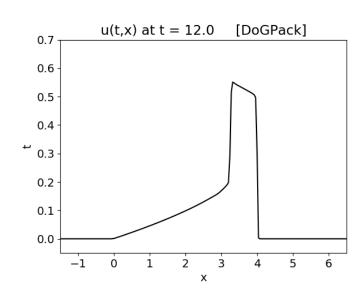
Conclusion



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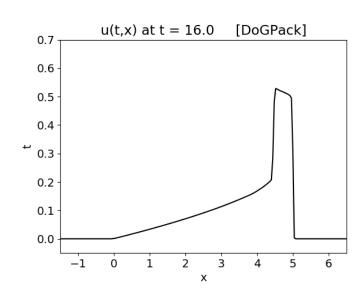


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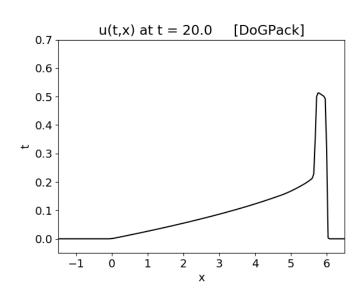


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■ Diffusion Equation

$$u_t - \left(u^3 u_{xxx}\right)_x = 0$$

■ Local Discontinuous Galerkin

Multigrid Solver

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Numerical Results

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References

Future Work

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Introduc

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Conclusion

- Higher dimensions
- Curved surfaces
- Space and time dependent coefficients
- Incorporation with air flow models

Conclusion

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Questions?

Bibliography

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References

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