Nonconservative discontinuous Galerkin methods for shallow water moment models on the sphere

by

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DOCTOR OF PHILOSOPHY

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The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation/thesis. The Graduate College will ensure this dissertation/thesis is globally accessible and will not permit alterations after a degree is conferred.

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ABSTRACT

We present a discontinuous Galerkin method for the generalized shallow water equations first introduced by Kowalski and Torrilhon. These generalized shallow water equations introduce vertical moments into the shallow flow's velocity profile. As a result of these additional moments a nonconservative term appears in these hyperbolic equations. We use the Dal Maso, Le Floch, and Murat theory of nonlinear hyperbolic systems in nonconservative form to correctly discretize this nonconservative term. Using this theory a high order discontinuous Galerkin method is presented for the generalized equations on the sphere. This work also updates some of the standard shallow water tests on the sphere for the generalized shallow water equations.

CHAPTER 1. INTRODUCTION

In this thesis, I present discontinuous Galerkin methods for

CHAPTER 2. The Models

2.1 Shallow Water Moment Models

The shallow water moment equations (SWME) were first introduced by Kowalski and Torrilhon

2.1.1 Derivation

We begin by considering the Navier-Stokes equations, $\div\breve{u} = 0$

$\breve{u}_t + \div * \breve{u}\breve{u} = -\frac{1}{\rho}p + \frac{1}{\rho} \div \sigma + \breve{g}$, where $\breve{u} = u, v, w^T$ is the vector of velocities, p is the pressure, ρ is the constant density, σ is the deviatoric stress tensor, and $\breve{g}$ is the gravitational force vector. We also have two boundaries, the bottom topography $h_b(t, x, y)$, and the free surface $h_s(t, x, y)$. At both of these boundaries the kinematic boundary conditions are in effect and can be expressed as $h_{st} + u(t, x, y, h_s), v(t, x, y, h_s)^T \cdot h_s = w(t, x, y, h_s)$

$h_{bt} + u(t, x, y, h_b), v(t, x, y, h_b)^T \cdot h_b = w(t, x, y, h_b)$. In practice the bottom topography is unchanging in time, but we express h_b with time dependence to allow for a symmetric representation of the boundary conditions.

2.1.1.1 Dimensional Analysis

Now we consider the characteristic scales of the problem. Let L be the characteristic horizontal length scale, and let H be the characteristic vertical length scale. For this problem we assume that $H << L$ and we denote the ratio of these lengths as $\varepsilon = H/L$. With these characteristic lengths we can scale the length variables to a nondimensional form

$$x = L\hat{x}, \quad y = L\hat{y}, \quad z = H\hat{z}. \tag{2.1}$$

Now let U be the characteristic horizontal velocity, then because of the shallowness the characteristic vertical velocity will be εU. Therefore the velocity variables can be scaled as follows,

$$u = U\hat{u}, \quad v = U\hat{v}, \quad w = \varepsilon U\hat{w}. \tag{2.2}$$

Now with the characteristic length and velocity, the time scaling can be described as

$$t = \frac{L}{U}\hat{t} \tag{2.3}$$

The pressure will be scaled by the characteristic height, H, and the stresses will be scaled by a characteristic stress, S. It is assumed that the basal shear stresses, σ_{xz} and σ_{yz} are of larger order than the lateral shear stress, σ_{xy}, and the normal stresses, σ_{xx}, σ_{yy}, and σ_{zz}, so that

$$p = \rho g H\hat{p}, \quad \sigma_{xz/yz} = S\hat{\sigma}_{xz/yz}, \quad \sigma_{xx/xy/yy/zz} = \varepsilon S\hat{\sigma}_{xx/xy/yy/zz}. \tag{2.4}$$

CHAPTER 3. METHODS AND PROCEDURES

This is the opening paragraph to my thesis which explains in general terms the concepts and hypothesis which will be used in my thesis.

With more general information given here than really necessary.

3.1 Introduction

Here initial concepts and conditions are explained and several hypothesis are mentioned in brief.

As can be seen in Table **??** it is truly obvious what I am saying is true.

Table 3.1 This table shows a standard empty table. Remove the square bracketed information to get longer captions in the LOT/ LOF **?**

3.1.1 Hypothesis

Here one particular hypothesis is explained in depth and is examined in the light of current literature.

This can also be seen in Figure **??** that the rest is obvious.

Figure 3.1 This table shows a standard empty figure. Remove the square bracketed information to get longer captions in the LOT/ LOF **?**

3.1.1.1 Parts of the hypothesis

Here one particular part of the hypothesis that is currently being explained is examined and particular elements of that part are given careful scrutiny.

3.1.2 Second Hypothesis

Here one particular hypothesis is explained in depth and is examined in the light of current literature.

3.1.2.1 Parts of the second hypothesis

Here one particular part of the hypothesis that is currently being explained is examined and particular elements of that part are given careful scrutiny.

3.2 Criteria Review

Here certain criteria are explained thus eventually leading to a foregone conclusion as can be seen in Table **??**.

Table 3.2 This table shows a standard empty table with a limited captionwidth

CHAPTER 4. RESULTS

This is the opening paragraph to my thesis which explains in general terms the concepts and hypothesis which will be used in my thesis.

With more general information given here than really necessary.

4.1 Introduction

Here initial concepts and conditions are explained and several hypothesis are mentioned in brief.

Of course, data on this as seen in Table **??** is few and far between.

Table 4.1 Moon Data

Element	Control	Experimental
Moon Rings	1.23	3.38
Moon Tides	2.26	3.12
Moon Walk	3.33	9.29

4.1.1 Hypothesis

Here one particular hypothesis is explained in depth and is examined in the light of current literature.

Or graphically as seen in Figure **??** it is certain that my hypothesis is true.

4.1.1.1 Parts of the hypothesis

Here one particular part of the hypothesis that is currently being explained is examined and particular elements of that part are given careful scrutiny.

Figure 4.1 Durham Centre

4.1.2 Second Hypothesis

Here one particular hypothesis is explained in depth and is examined in the light of current literature.

4.1.2.1 Parts of the second hypothesis

Here one particular part of the hypothesis that is currently being explained is examined and particular elements of that part are given careful scrutiny.

4.2 Criteria Review

Here certain criteria are explained thus eventually leading to a foregone conclusion.

CHAPTER 5. SUMMARY AND DISCUSSION

This is the opening paragraph to my thesis which explains in general terms the concepts and hypothesis which will be used in my thesis.

With more general information given here than really necessary.

5.1 Introduction

Here initial concepts and conditions are explained and several hypothesis are mentioned in brief.

Or graphically as seen in Figure **??** it is certain that my hypothesis is true.

Figure 5.1 Durham Centre— Another View

5.1.1 Hypothesis

Here one particular hypothesis is explained in depth and is examined in the light of current literature.

As can be seen in Table **??** it is truly obvious what I am saying is true.

5.1.1.1 Parts of the hypothesis

Here one particular part of the hypothesis that is currently being explained is examined and particular elements of that part are given careful scrutiny.

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5.2 Criteria Review

Here certain criteria are explained thus eventually leading to a foregone conclusion.

5.3 Results And Discussion

Here the results can be inserted

Table 5.1 This table shows almost nothing but is a sideways table and takes up a whole page by itself

Element	**Control**	**Experimental**
Moon Rings	1.23	3.38
Moon Tides	2.26	3.12
Moon Walk	3.33	9.29

APPENDIX A. ADDITIONAL MATERIAL

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More stuff

Supplemental material.

APPENDIX B. STATISTICAL RESULTS

This is now the same as any other chapter except that all sectioning levels below the chapter level must begin with the *-form of a sectioning command.

Supplemental Statistics

More stuff.