MATH 676 Project for Caleb Fowler

Proposed project: FE simulation of heat transport with applications to metallic powder bed adhesive manufacturing processes.

Note: Most of this project idea is based on a Review article by Schoinochoritis et al. (`https://doi.org/10.1177/0954405414567522`)

Your project will center around solving the heat equation, the temperature T is determined by

$$\rho c_p \frac{\partial}{\partial t} T - \nabla \cdot \big(k \nabla T\big) = \dot{Q}_S(t).$$

Here, ρ denotes the density (either constant or a scalar field), c_p is the specific heat capacity at constant pressure, k is the thermal heat conducitivity and $\dot{Q}_S$ is a heat source term.

As a first step, I suggest you implement a solver for the above PDE, where ρ, c_p and k are constants and $\dot{Q}_S$ is given by a prescribed, time dependent Gaussian that models heating by an intense laser.

Then, you will need to decide on how to model the adhesion process. This is a step that I will discuss with you in more detail later—in particular, we will need to come up with a sensible model, which will require some discussion…

A minimal model I envision should contain of an indicator θ encoding the adhesion state (for example, ranging from `0` pure powder to 1 fully adhered) that changes with the temperature T (with a local ODE, or a PDE), and a model for the thermal heat conductivity $k(\theta)$. We will then discuss suitable coupling strategies for simulating the coupled system.

Process. I suggest to have a look at `step-26` first. It solve the time-dependent heat equation with adaptive mesh refinement. Modify the source code to solve the heat equation on a rectangular domain and introduce the parameters above to the equation. In particular, you will need to implement a time-dependent right hand side modeling a laser beam. Then, you will need to decide how to model the adhesion process and augment the system by the scalar indicator θ.

You will need to design a "benchmark configuration" to verify that your program is implemented correctly and demonstrate convergence under grid refinement.

Note: This project is subject to modification by mutual agreement between you and me. The goal is to have a project that fits your research interests. If you would like to deviate from the path outlined here, talk to me!

Relevant tutorial programs: For your project, you will want to read through tutorial programs `step-26` . You should consider starting the program you will write for your project as a variation of one of these. (Links to all tutorial programs can be found at `https://www.dealii.org/developer/doxygen/deal.II/Tutorial.html`.)

Relevant video lectures: In addition to the resources already posted on the course website (Google Drive), you could watch lectures **27, 29, 30, 30.25** for background material necessary for your project. Links to all videos can be found at `http://www.math.colostate.edu/~bangerth/videos.html`.

Keep notes on these resources and all background reads in your journal as on all other external resources you consult!

Project tasks: As part of your project, you will need to meet the following milestones:

- *Milestone 1 (April 1, 2025):*

  - Modify `step-26` to solve the heat equation above with the given parameters and with a time-dependent heat source modelling a laser beam.
  - Decide on how to model the adhesion process and start implementing the PDE.
  - You will need to prepare and give a 10 minute presentation in class on your progress so far.

- *Milestone 2 (May 6, 2025):*
  - Finish your implementation of the full adhesion model.
  - Verify your code with a chosen benchmark configuration and demonstrate convergence under grid refinement.
  - You will need to prepare and give a 15 minute presentation in class or during the final exam time on the results of your project.
  - You will also need to prepare a report on your project in the style of the tutorial program – i.e., including an introduction, a results section, and commented code that contains everything to run a simulation you show in your result section.

Deliverables: Your deliverables at the end of semester include the following items:

- Your final report (as discussed above in Milestone 2) as a PDF file, checked into a github repository to which you give me access.
- Your finished, documented code and all input files necessary to run it, checked into a github repository to which you give me access.
- Your finished online journal.

Grading: I will determine your final grade in this class based on the following criteria:

- Sophistication of the code beyond the program from which it was started.
- Extent of documentation in the code.
- Extent of the documentation surrounding the program, i.e., description of the equation and its properties, description of the principles used in the implementation, and documentation of worked-out examples computed with the program.
- Sophistication and realism of the testcases to which you apply your numerical scheme.
- quality and extend of entries in your journal,
- your three presentations in class.

As an example of how these reports should look like (though maybe not quite as extensive), take a look at the deal.II tutorial programs.

If you are interested, good projects may be published as part of the library and distributed with future versions (see for example the step-21, step-24, step-25, step-81 tutorial programs that were created by students of a prior class), or as part of the code gallery (see `https://dealii.org/code-gallery.html`). Of course, you will then also be credited publicly for your work.