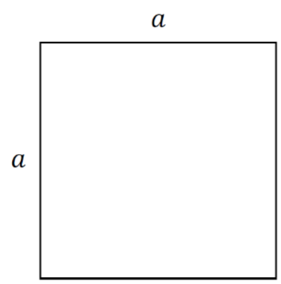
Student ID: 7809587 Caleb Princewill Nwokocha November 14, 2022

Faculty of Science

The University of Manitoba

I: Topic on p Root of $c^r - b^q$ and $g^r + b^q$

Consider below the square of side length a.



If a is an irrational number (i.e., a number with infinite decimal places), then the area a^2 of the above square cannot be calculated in finite time. Hence, a should be rational for finite calculation of a^2. Likewise, b and c in $a^p + b^q = c^r$ should be rationals.

If p is negative real, then it is possible that p is a negative decimal number. In this case of p being a negative decimal number, a^p is then a complex number with real part of 0. To avoid complex number arithmetic, I constrain p to positive real numbers; likewise, q and r in $a^p + b^q = c^r$ should be positive real numbers.

The equation $a^p + b^q = c^r$ is similar to the Fermat equation, except that a, b, c are rationals, and p, q, r are positive reals. Assuming $a^i + b^j = c^k$ where i, j, k are also positive real numbers, then

$$a^i = c^k - b^j \tag{1.1}$$

$$a(a^i) = a(c^k - b^j) \tag{1.2}$$

$$a^{i+1} = a(c^k - b^j) \tag{1.3}$$

$$a^{i+1} + b^{j+1} = a(c^k - b^j) + b^{j+1} \tag{1.4}$$

Let $p = i + 1$, $q = j + 1$, and $r = k + 1$

$$a^p + b^q = a(c^{r-1} - b^{q-1}) + b^q \tag{1.5}$$

If $a^p + b^q = c^r$, then $a^p = c^r - b^q$ and $a = \sqrt[p]{c^r - b^q}$

$$\therefore c^r - b^q + b^q = a(c^{r-1} - b^{q-1}) + b^q \tag{1.6}$$

$$c^r = a(c^{r-1} - b^{q-1}) + b^q \tag{1.7}$$

$$c^r - b^q = a(c^{r-1} - b^{q-1}) \tag{1.8}$$

$$\frac{c^r - b^q}{c^{r-1} - b^{q-1}} = a = \sqrt[p]{c^r - b^q} \tag{1.9}$$

$$\frac{c^r - b^q}{c^{r-1} - b^{q-1}} = \sqrt[p]{c^r - b^q} \tag{1.10}$$

Case 1: I noted earlier that b and c are rationals, so b and c can be integers within the set of rational numbers. Also, p, q, r can be positive integers since any $\mathbb{Z}^+$ is a subset of positive real numbers. It is true that if b and c are integers, and p, q, r are positive integers, and $p = q = r$, then $\sqrt[p]{c^r - b^q} = \frac{c^r - b^q}{c^{r-1} - b^{q-1}}$ leads to contradiction. [1] For instance, let $p = q = r = 2$, the following contradiction appear:

$$\sqrt{c^2 - b^2} = \frac{c^2 - b^2}{c - b} \tag{1.11}$$

$$\sqrt{c^2 - b^2} = \frac{(c + b)(c - b)}{c - b} \tag{1.12}$$

$$\sqrt{c^2 - b^2} = c + b \tag{1.13}$$

Thus, $\sqrt[p]{c^r - b^q} \neq \frac{c^r - b^q}{c^{r-1} - b^{q-1}}$ if b and c are integers, and p, q, r are positive integers, and $p = q = r$.

Case 2: $\sqrt[p]{c^r - b^q} \neq \frac{c^r - b^q}{c^{r-1} - b^{q-1}}$ if $p \neq q \neq r$, because p can be any positive real number and the p root of $c^r - b^q$ will remain the same.

Case 3: Since case 2 proves that arbitrary positive p in $\mathbb{R}$ cannot satisfy (1.10), then p must be either q or r to satisfy (1.10). By case 1, p cannot be q and r at the same time; thus, if $p = q \neq r$ or $p = r \neq q$, then $\sqrt[p]{c^r - b^q} = \frac{c^r - b^q}{c^{r-1} - b^{q-1}}$ is satisfied.

For the definition of $\sqrt[p]{g^r + b^q}$, assume again that $a^i - b^j = g^k$ where i, j, k are positive real numbers, and g is a rational number:

$$a^i = g^k + b^j \tag{1.14}$$

$$a(a^i) = a(g^k + b^j) \tag{1.15}$$

$$a^{i+1} = a(g^k + b^j) \tag{1.16}$$

$$a^{i+1} - b^{j+1} = a(g^k + b^j) - b^{j+1} \tag{1.17}$$

Let $p = i + 1$, $q = j + 1$, and $r = k + 1$

$$a^p - b^q = a(g^{r-1} + b^{q-1}) - b^q \tag{1.18}$$

If $a^p - b^q = g^r$, then $a^p = g^r + b^q$ and $a = \sqrt[p]{g^r + b^q}$

$$\therefore g^r + b^q - b^q = a(g^{r-1} + b^{q-1}) - b^q \tag{1.19}$$

$$g^r = a(g^{r-1} + b^{q-1}) - b^q \tag{1.20}$$

$$g^r + b^q = a(g^{r-1} + b^{q-1}) \tag{1.21}$$

$$\frac{g^r + b^q}{g^{r-1} + b^{q-1}} = a = \sqrt[p]{g^r + b^q} \tag{1.22}$$

$$\frac{g^r + b^q}{g^{r-1} + b^{q-1}} = \sqrt[p]{g^r + b^q} \tag{1.23}$$

Case 1, 2, and 3 can be modified to prove that if $p = q \neq r$ or $p = r \neq q$, then $\sqrt[p]{g^r + b^q} = \frac{g^r+b^q}{g^{r-1}+b^{q-1}}$ is satisfied.

Caution: Computers with limited calculative precision may incorrectly calculate $\sqrt[p]{g^r + b^q}$, $\sqrt[p]{c^r - b^q}$, $\frac{g^r+b^q}{g^{r-1}+b^{q-1}}$, and $\frac{c^r-b^q}{c^{r-1}-b^{q-1}}$.

Reference

[1] Nwokocha, Caleb. (2022). A Proof of Fermat's Last Theorem. 10.13140/RG.2.2.28604.31363.