Student ID: 7809587 Caleb Princewill Nwokocha November 10, 2022
Faculty of Science
The University of Manitoba

Equality of Lower and Higher Degree Polynomials

Sometimes, we may want to change the degree of a polynomial for convenience. In this paper, I show that a polynomial $f(x)$ of lower degree $d \in \mathbb{Z}^+$ is equal to a polynomial $f'(x)$ of higher degree $d' \in \mathbb{R}$.

The general formula of the dth degree polynomial is

$$f(x) = w_1 x^d + w_2 x^{d-1} + w_3 x^{d-2} + \cdots + w_n x^{d-d}$$

where $x \in \mathbb{R}$ and $w_i \in \mathbb{R}$; $1 \leq i \leq n$

Suppose $f'(x) = w_1 x^{d'} + w_2 x + w_3 x + \cdots + w_n x$, and $f'(x) = f(x)$, then

$$w_1 x^{d'} + w_2 x + w_3 x + \cdots + w_n x = w_1 x^d + w_2 x^{d-1} + w_3 x^{d-2} + \cdots + w_n x^{d-d}$$

$$w_1 x^{d'} = w_1 x^d + w_2 x^{d-1} + w_3 x^{d-2} + \cdots + w_n x^{d-d} - w_2 x - w_3 x - \cdots - w_n x$$

$$w_1 x^{d'} = f(x) - w_2 x - w_3 x - \cdots - w_n x$$

$$\ln\left(w_1 x^{d'}\right) = \ln(f(x) - w_2 x - w_3 x - \cdots - w_n x)$$

$$\ln(w_1) + \ln\left(x^{d'}\right) = \ln(f(x) - w_2 x - w_3 x - \cdots - w_n x)$$

$$\ln\left(x^{d'}\right) = \ln(f(x) - w_2 x - w_3 x - \cdots - w_n x) - \ln(w_1)$$

$$d' \ln(x) = \ln(f(x) - w_2 x - w_3 x - \cdots - w_n x) - \ln(w_1)$$

$$d' = \frac{\ln(f(x) - w_2 x - w_3 x - \cdots - w_n x) - \ln(w_1)}{\ln(x)}$$

$$\therefore w_1 x^{\frac{\ln(f(x) - w_2 x - w_3 x - \cdots - w_n x) - \ln(w_1)}{\ln(x)}} + w_2 x + w_3 x + \cdots + w_n x = w_1 x^d + w_2 x^{d-1} + w_3 x^{d-2} + \cdots + w_n x^{d-d}$$

Rule 1: For any $d \in \mathbb{Z}^+$, $d' \in \mathbb{R}$, $x \in \mathbb{R}$, and $w_i \in \mathbb{R}$; where $1 \leq i \leq n$, the equality $f(x) = f'(x)$ is true if $d' = \frac{\ln(f(x) - w_2 x - w_3 x - \cdots - w_n x) - \ln(w_1)}{\ln(x)}$.

I now consider whether $d' > d$. If $d' < d$ and rule 1 are true, then $w_2 x + w_3 x + \cdots + w_n x$ is greater than $w_2 x^{d-1} + w_3 x^{d-2} + \cdots + w_n x^{d-d}$. It cannot be the case that $w_2 x + w_3 x + \cdots + w_n x$ is greater than $w_2 x^{d-1} + w_3 x^{d-2} + \cdots + w_n x^{d-d}$, because d is a positive integer; thus, $d' \geq d$. Further, if $d' = d$, then $w_2 x^{d-1} + w_3 x^{d-2} + \cdots + w_n x^{d-d}$ is equal to $w_2 x + w_3 x + \cdots + w_n x$, which is false. Therefore, since $d' < d$ and $d' = d$ are false, $d' > d$ is true.

Rule 2: If rule 1 is true, then the degree d' of $f'(x)$ is higher than degree d of $f(x)$.