

FUNCTIONAL RULE EXTRACTION METHOD FOR ARTIFICIAL NEURAL NETWORKS

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ABSTRACT

The idea I propose in this paper is a method that is based on comprehensive functions for directed and undirected rule extraction from artificial neural network operations. Firstly, I defined comprehensive functions, then constructed a comprehensive multilayer network (denoted as N). Each activation function of N is parametrized to a comprehensive function. Following N construction, I extracted rules from the network by observing that the network output depends on probabilities of composite functions that are comprehensive functions. This functional rule extraction method applies to the perceptron and multilayer neural network. For any N model that is trained to predict some outcome given some event, that model behaviour can be expressed – using the functional rule extraction method – as a formal rule or informal rule obeyed by the network to predict that outcome. As example, figure 1 consist of a comprehensive physics function that is parameter for one of the network hidden activation functions. Using the functional rule extraction method, I deduced that the comprehensive multilayer network prediction depends on probability of that physics function and probabilities of other composite comprehensive functions in N . Additionally, functional rule extraction method can aid in applied settings for generation of equations of observed phenomena. This generation can be achieved by first training an N model toward predicting outcome of a phenomenon, then extracting the rules and assuming that probability values of the network comprehensive functions are constants. Finally, to simplify the generated equation, comprehensive functions with probability $p \sim 0$ may be omitted.

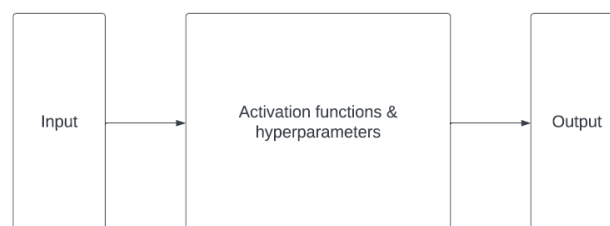
KEYWORDS

Rule extraction, comprehensive functions, neural network

1. INTRODUCTION

For decades, artificial neural network has been the most useful pattern recognition technique in various science and engineering areas like machine learning, computer vision, natural language processing, robotics, etcetera [3, 1, 2]. Real world applications of this technique span vast number of industries, with increasing utilization across domains [11, 10, 7]. This immense utilization of artificial neural network stem from explosion of data and predictive power of models.

A standard model consist of input, activation functions, hyperparameters, and output. The input include train set and testing set, activation functions may be sigmoid, rectified linear unit, or tanh, [4] hyperparameters are used to control activation functions, and output is the model prediction.



The model undergo a training procedure that involve optimizing hyperparameter values until the model learn to output accurate predictions when given further training set as input. After the training procedure, the model is tested with a testing set to measure generalization capacity of the model. If it past the test, the model is fully ready for use by its user.

Although, this model may serve its user adequately, the user lack concrete understanding of any rule obeyed by the model to predict an output. [9] This lack of concrete understanding of the model rules resulted to the term “black-box network”, meaning values of activation functions and hyperparameters are only numbers and they have no comprehensive relation to the probability distribution that the model learned. Without extracted rules, a user cannot prove that a model will make a right or wrong prediction at any given time. Before safely deploying a network model in a high-risk environment such as surgical robot in an operating room, autonomous vehicles on road highways, bots on trading system, etcetera, a user should be able to extract rules of the network model.

The functional rule extraction method explained in the following sections, is an efficient method for extracting rules from an N model consisting comprehensive functions. Rules extracted by this method can also be interpreted as equations of observed phenomena.

2. RELATED WORK

3. COMPREHENSIVE FUNCTION

A solution to problem of rule extraction is forming a comprehensive relationship between the learned probability distribution and the input. I will use the sigmoid function $A(X)$ to form a comprehensive relationship, although other activation function can be used.

$$A(X) = \frac{1}{1 + e^{-X}} = (1 + e^{-X})^{-1}$$

A **comprehensive function** is a function that has explanatory relationship within a set of composite functions.

3.1. Univariate Comprehensive Function

A **univariate comprehensive function** f_c (for the purpose of this paper) is a comprehensive function with parameter wx , such that $f_c: wx \rightarrow \mathbb{R}$.

Let X be a univariate comprehensive function $f_c(wx)$, then

$$A(f_c(wx)) = (1 + e^{-f_c(wx)})^{-1}$$

Some examples of univariate comprehensive function are:

1. Volume of a cube $V(s) = f_c(s) = s^3$
2. Trigonometry identity $T(a) = f_c(a) = \sin^2 a + \cos^2 a$

3.2. Multivariate Comprehensive Function

A **multivariate comprehensive function** f_c (for the purpose of this paper) is a comprehensive function with parameters w_1x_1, \dots, w_nx_n , such that $f_c: w_1x_1, \dots, w_nx_n \rightarrow \mathbb{R}$.

Some examples of multivariate comprehensive functions are:

1. Force $F(m, a) = f_c(m, a) = m \cdot a$
2. Quadratic formula $Q(a, b, c) = f_c(a, b, c) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = (-b \pm \sqrt{b^2 - 4ac})(2a)^{-1}$

By definitions of comprehensive function, I deduce that from example 1 of the univariate case, $f_c(s) = f_c(wx)$ if and only if $s = wx$. This means that the activation function

$$A(f_c(wx)) = (1 + e^{-f_c(wx)})^{-1}$$

can be treated as

$$A(V(wx)) = (1 + e^{-[(wx)^3]})^{-1} \text{ iff } V(s) = V(wx)$$

Likewise, the second univariate example is treated as

$$A(T(wx)) = (1 + e^{-\sin^2 wx - \cos^2 wx})^{-1} \text{ iff } T(a) = T(wx)$$

For the multivariate case, the comprehensive force sigmoid activation function is written as

$$A(F(w_1x_1, w_2x_2)) = (1 + e^{-(w_1x_1w_2x_2)})^{-1}$$

$$\text{iff } F(m, a) = F(w_1x_1, w_2x_2)$$

and the comprehensive quadratic sigmoid activation function is written as

$$A(Q(w_1x_1, w_2x_2, w_3x_3)) = \left(1 + e^{(w_2x_2 \pm \sqrt{(w_2x_2)^2 - 4w_1x_1w_3x_3})(2w_1x_1)^{-1}}\right)^{-1}$$

$$\text{iff } Q(a, b, c) = Q(w_1x_1, w_2x_2, w_3x_3)$$

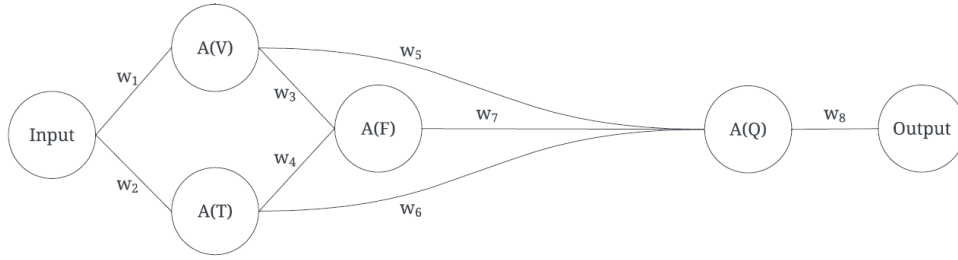


Figure 1: Comprehensive multilayer artificial neural network

4. RULE EXTRACTION

[6] According to M.Sato and H. Tsukimoto, rule extraction from neural networks is the task for obtaining comprehensible descriptions that approximate the predictive behavior of neural networks. I will extract the rule from the network model in figure 1. Firstly, I describe the model as a comprehensive multilayer artificial neural network because its hidden unit are activated by comprehensive and sigmoid functions introduced in earlier examples. The comprehensive model is made up of six units: one input unit i , one output unit o , and four hidden units. The output unit has no comprehensive function parameter for its sigmoid function. Algebraically, this model output can be expressed as the following:

$$o \left(A \left(Q \left(A(V(iw_1))w_5, A \left(F(A(V(iw_1))w_3, A(T(iw_2))w_4) \right) w_7, A(T(iw_2))w_6 \right) \right) w_8 \right)$$

$$= \left(1 + e^{-w_8 \left(\frac{\left(w_7 \left(1 + e^{-w_3w_4(1+e^{-iw_1})^{-1}(1+e^{-iw_2})^{-1}} \right)^{-1} \pm \sqrt{\left(w_7 \left(1 + e^{-w_3w_4(1+e^{-iw_1})^{-1}(1+e^{-iw_2})^{-1}} \right)^2 - 4w_5(1+e^{-iw_1})^{-1}w_6(1+e^{-iw_2})^{-1}} \right)^{-1}}{w_7 \left(1 + e^{-w_3w_4(1+e^{-iw_1})^{-1}(1+e^{-iw_2})^{-1}} \right)^{-1}} \right) \left(w_5(2+2e^{-iw_1})^{-1} \right)^{-1}} \right)^{-1} \right)^{-1}$$

Observe that each weight w_i (except w_1 and w_2 of the input) are weights of composite functions consisting of sigmoid functions and comprehensive functions. Since the extraction is concerned with rules compose of only comprehensive functions, the connection between the weights and sigmoid functions is not needed. Only connection between weights and comprehensive functions is needed in this rule extraction method. To disconnect weight from a sigmoid function and only reserve the weight connected to associated comprehensive function, consider the weighted sigmoid below:

$$A(V)' = A(V)(w_3)$$

Let weighted volume $V' = (V)(p)$ where p is probability of the volume V , and assuming that

$$V' = A(V)'$$

$$(V)(p) = A(V)(w_3)$$

$$p = \frac{A(V)(w_3)}{V}$$

To now extract rule from the model, I extract the composites $r_i \in r, \forall i : 1 \leq i \leq 4$ of comprehensive functions (together with associated weights and probabilities) from the above output expression:

$$r_1 = p_8 \left(-i^2 w_1 w_2 p_3 p_4 p_7 \pm \sqrt{(i^2 w_1 w_2 p_3 p_4)^2 p_7 - 4i^2 w_1 w_2 p_5 p_6} \right) (i w_1 p_5)^{-1}$$

$$r_2 = p_8 \left(-VT p_3 p_4 p_7 \pm \sqrt{(VT p_3 p_4)^2 p_7 - 4VT p_5 p_6} \right) (V p_5)^{-1}$$

$$r_3 = p_8 \left(-F p_7 \pm \sqrt{F^2 p_7 - 4VT p_5 p_6} \right) (V p_5)^{-1}$$

$$r_4 = Q p_8$$

The compositions r_1, r_2, r_3, r_4 are **formal rules** of the model in figure 1. An **informal rule** is worded statement of a formal rule. As shown, r_1 is a much complex formal rule than r_2, r_3 , and r_4 . This complexity in r_1 result from explicitly stating complete weight distribution across the input and comprehensive functions. Even probabilistic w_1, \dots, w_8 may not sum to 1 for simple explanation of the r_1 . Hence, I normalize w_1, \dots, w_8 sum to 1 by using softmax σ equation on all weights and mapping $\sigma(w_i)$ to p_i . Here p_i represent associated probabilite(ies) to an input or comprehensive function.

$$p_i \leftarrow \sigma(w_i) = \frac{e^{w_i}}{\sum_{i=1}^n e^{w_i}}, \forall i: 1 \leq i \leq 8$$

Then the formal rules r_1, \dots, r_4 can be noted as **normal formal rules** below.

$$r_1 = p_8 \left(-i^2 p_1 p_2 p_3 p_4 p_7 \pm \sqrt{(i^2 p_1 p_2 p_3 p_4)^2 w_7 - 4i^2 p_1 p_2 p_5 p_6} \right) (i p_1 p_5)^{-1}$$

$$r_2 = p_8 \left(-VT p_3 p_4 p_7 \pm \sqrt{(VT p_3 p_4)^2 w_7 - 4VT p_5 p_6} \right) (V p_5)^{-1}$$

$$r_3 = p_8 \left(-Fp_7 \pm \sqrt{F^2 p_7 - 4VTp_5 p_6} \right) (Vp_5)^{-1}$$

$$r_4 = Qp_8$$

A **normal informal rule** of r_3 is “The output of the network is based on rule that the probability p_8 of quadratic relationship between negative force at probability p_7 , square root of squared positive force at probability p_7 , square root of four negative cubic volume and trigonometry identity products at probability $p_7 p_6$, and single cubic volume at probability p_5 , is true.”

Furthermore, rules can either be directed or undirected. Following the definitions of directed and undirected graphs [8], a **directed rule** is a rule that is obtained from a directed N model, while an **undirected rule** is a rule obtained from undirected N model. Figure 1 is example of a directed N model.

3. CONCLUSIONS

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