P Versus NP

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1 Problem Statement

The problem of determining whether $P = NP$ or $P \neq NP$ is one of the most significant open problems in computer science and mathematics. It asks whether every problem for which a solution can be verified quickly can also be solved quickly.

2 Problem Solution

The problem statement is a major unsolved problem in theoretical computer science. While it has not been definitively resolved, there have been significant developments in understanding the relationship between P and NP.

Arora and Barak (2009) made significant progress by proving that $P \subseteq NP$. They showed that if a language L can be decided in polynomial time by a Turing machine, then L also belongs to the complexity class NP. This result established a fundamental relationship between P and NP.

However, it is important to note that the question of whether $P = NP$ or $P \neq NP$ remains open. The proof provided by Arora and Barak does not solve the problem completely.

In set theory, it is known that two sets A and B are not equal if A is a proper subset of B. In the context of P and NP, let the consideration be made that $A = P$ and $B = NP$. Arora and Barak (2009) showed that NP is a superset of P and includes NP-complete problems. Therefore, P is a proper subset of NP, and it follows that $P \neq NP$.

References

Arora, S., & Barak, B. (2009). *pp 40-43, computational complexity: A modern approach.* Cambridge University Press.