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SCT221-0211/2021

1
b i) Binomial

$$b(x;n,p) = \frac{n!}{x!(n-x)!} p^x q^{n-x} \quad \text{where } q = 1-p$$

$$M_x(t) = \sum_{x=0}^{n} e^{xt} \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} (pe^t)^x \frac{n!}{x!(n-x)!} q^{n-x}$$

$$= (q+pe^t)^n$$

Mean;

$$\frac{d}{dt} M_x(t) = n(q+pe^t)^{n-1} pe^t$$

$$= npe^t(q+pe^t)^{n-1}$$

$$E(x) = np(q+p)^{n-1} = np \quad \text{, after evaluating } t=0$$

Variance;

$$\frac{d^2}{dt^2} M_x(t) = npe^t(q+pe^t)^{n-2}\{q+npe^t\}$$

$$E(x^2) = np(q+p)^{n-2}(q+np)$$

$$= np(q+np)$$

$$V(x) = E(x^2) - \{E(x)\}^2$$

$$= np(q+np) - n^2p^2$$

$$= npq$$

ii) Poisson

$$P_r(X=n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

MGF:

$$M_x(t) = \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} e^{tn} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{(\lambda e^t)^n}{n!}$$

$$= e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t-1)}$$

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Mean:

$$\frac{d}{dt} M_x(t) \Big|_{t=0} = \lambda e^{-\lambda} e^{t} e^{\lambda e^{t}}$$

$$= \lambda e^{-\lambda} \cdot e^{0} \cdot e^{\lambda e^{0}}$$

$$= \lambda e^{-\lambda} \cdot 1 \cdot e^{\lambda}$$

$$= \lambda$$

Variance:

$$\frac{d^2}{dt^2} M_x(t) = \lambda e^{-\lambda} \left[e^{t} e^{\lambda e^{t}} \cdot \lambda e^{t} + e^{\lambda e^{t}} \cdot e^{t} \right]$$

Evaluating $t=0$;

$$= \lambda e^{-\lambda} \left[e^{\lambda} \cdot \lambda + e^{\lambda} \right]$$

$$= \lambda^2 + \lambda$$

$$Var: = (\lambda^2 + \lambda) - (\lambda)^2$$

$$= \lambda$$

iii) Exponential

$$f_x(x) = \lambda e^{-\lambda x}$$

$$M_x(t) = \int_0^{\infty} e^{tx} \cdot \lambda e^{-\lambda x} \, dx$$

$$= \int_0^{\infty} \lambda e^{x(t-\lambda)} \, dx$$

$$= \frac{\lambda}{t-\lambda} e^{x(t-\lambda)} \Big|_{x=0}^{x=\infty}$$

$$= \lim_{x \to \infty} \left[\frac{\lambda}{t-\lambda} e^{x(t-\lambda)} - \frac{\lambda}{t-\lambda} \right]$$

$$= \frac{\lambda}{t-\lambda} \left[\lim_{x \to \infty} e^{x(t-\lambda)} - 1 \right]$$

$$= \frac{\lambda}{t-\lambda} [0-1] \quad = \frac{\lambda}{\lambda - t}$$

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Mean:

$$E(x) = \int_0^\infty x \lambda e^{-\lambda x}\,dx$$

$$= \lambda\left[\left|\frac{-xe^{-\lambda x}}{\lambda}\right|_0^\infty + \frac{1}{\lambda}\int_0^\infty e^{-\lambda x}\,dx\right]$$

$$= \lambda\left[0 + \frac{1}{\lambda}\,\frac{-e^{-\lambda x}}{\lambda}\right]_0^\infty$$

$$= \lambda\,\frac{1}{\lambda^2}$$

$$= \frac{1}{\lambda}$$

Variance:

$$E(x^2) = \int_0^\infty x^2 \lambda e^{-\lambda x}$$

$$= \frac{2}{\lambda^2}$$

$$Var(x) = E(x^2) - E(x)^2$$

$$= \left(\frac{2}{\lambda^2}\right) - \left(\frac{1}{\lambda}\right)^2$$

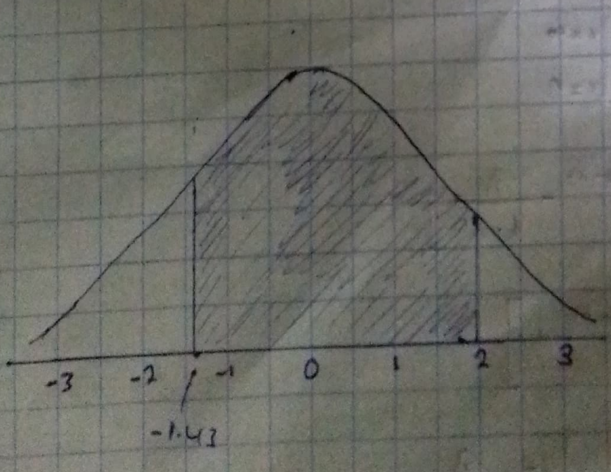
$$= \frac{1}{\lambda}$$

2. Stand normal table and graph sketche

(i) $p(-1.43 \le z \le 2)$

$$= p(z<2) - (1 - p(z<1.43))$$
$$= 0.9772 - (1 - 0.9236)$$
$$= 0.9008$$

90.08% of the area lies between -1.43 and 2



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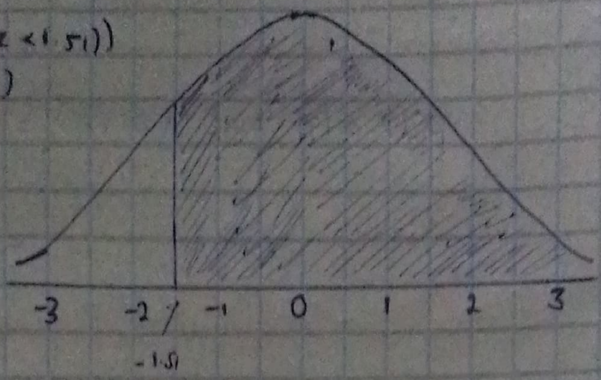
(ii) $P(z > -1.51)$

~~$= 1 - p(z < 1.51)$~~
~~$= 1 - 0.9345$~~
~~$= 0.0655$~~

$= 1 - (1 - p(z < 1.51))$
$= 1 - (1 - 0.9345)$
$= 0.9345$



93.45% of the area lies above -1.51

(iii) $\mu = 40$, $\sigma^2 = 100$, find $p(25 \le x \le 55)$

$SD = \sqrt{100} = 10$

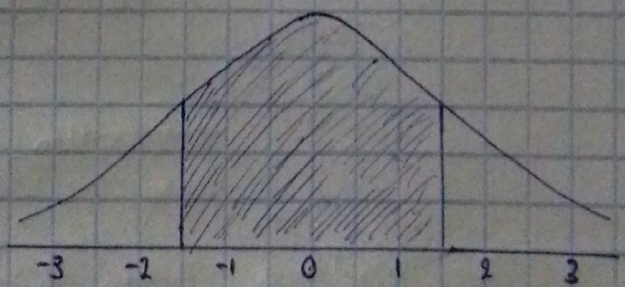
$z = \frac{x - \mu}{\sigma}$

when $x = 25$, $z = \frac{25 - 40}{10} = -1.5$

when $x = 55$, $z = \frac{55 - 40}{10} = 1.5$

$p(-1.5 \le z \le 1.5)$

$= p(z < 1.5) - (1 - p(z < 1.5))$
$= 0.9332 - (1 - 0.9332)$
$= 0.8664$



86.64% of the area lies between -1.5 and 1.5