# Market Value of Rarity: A Theory of Fair Value and Evidence from Rare Baseball Cards*

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**Abstract**

We investigate the market value of rarity theoretically and empirically. Prior studies find that the market value of rarity follows a power law, but this finding lacks a theoretical foundation. We provide a micro-foundation for this finding, demonstrating that the observed power law emerges in a competitive market where agents have rank-dependent utility preferences. The model leads to two new theoretical insights: (i) the rank of an item within a set of close substitutes and the quantity of that item known to exist are both natural measures of rarity, but rank is predicted to perform better; (ii) there is a systematic relationship between the estimated slope and intercept from a regression of log price on log rank. When we test the model on data from over 4000 auction records of rare baseball cards, we find that a regression with only log rank and grading company explains 60% of the variation in log price.

JEL Classification: D00

Keywords: Rarity, Rank Dependent Utility, Baseball Cards, Power Laws, Zipf's Law

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# 1. Introduction

Emerging evidence reveals that the market value of rarity follows a power law. That is, the log of price for a rare item is proportional to that item's log of rarity, measured by quantity in existence.[1] Such a relationship has been documented for rare coins (Dickie, Delorme Jr, and Humphreys, 1994; Koford and Tschoegl, 1998), for rare vinyl music records (Cameron and Sonnabend, 2020), and for digital collectibles known as non-fungible tokens that are linked to block-chain technology (Lee, 2022; Mekacher, Bracci, Nadini, Martino, Alessandretti, Aiello, and Baronchelli, 2022). Given the robustness of the relationship and the different contexts in which it has been discovered, it is natural to ask if there is an underlying theory that can account for how the market values rarity. Thus far, the literature lacks a theoretical explanation for this basic finding. Our paper addresses this gap, which furthers our understanding of rarity by providing a plausible explanation for the current empirical findings, and by delivering new testable hypotheses.

We use the notion of fair price to model the relative value of two rare items when they are close substitutes. In particular, we model the perceived relative value of rare items through their perceived relative marginal search costs, as if there exists a perfectly competitive market. In a classic survey study, Kahneman, Knetsch, and Thaler (1986) find that consumers consider price increases to be fair only if they coincide with increases in costs. Rotemberg (2005) assumes a fairness constraint that prices cannot be set too high over marginal cost without being considered unfair.[2] The literature continues to conceptualize a fair price as a price that is close to the marginal cost.[3] In general, the fair price can be thought of as a behavioral mechanism that allows market participants to coordinate their beliefs about the value of rarity.

We first provide a theoretical benchmark in which market participants are risk-neutral expected utility agents. This setup reveals that although an intrinsic value for rarity has been argued to be a behavioral bias that is inconsistent with rational behavior (John, Melis, Read, Rossano, and Tomasello, 2018), a *relative* market value of rarity can emerge from rational behavior. We show

[1] Equivalently, $price \propto rarity^{-\gamma}$, where $\propto$ denotes proportionality and $-\gamma$ is the *power coefficient*. The literature also refers to $\gamma$ as the *elasticity of price* with respect to rarity.

[2] Rotemberg (2005) also assumes that for a given elasticity of demand, consumers expect prices to rise by the same percent change as marginal cost. He adds that "Increases in price that do not correspond closely to increases in costs are likely to trigger consumer anger."

[3] For example, Eyster, Madarász, and Michaillat (2021) comment "The first element of our theory is that customers dislike paying prices marked up heavily over marginal costs because they find these prices unfair."

that in a perfectly competitive market, the price of a rare item is proportional to the inverse of its rarity (measured by the quantity of the item known to exist). That is, the price follows a power law with a power coefficient of negative one, or equivalently, the elasticity of price with respect to rarity equals one. The reason is that in a perfectly competitive market, the price ratio of two items equals the ratio of their marginal search costs. As an example, consider the case where agents open sealed packs of baseball cards to find rare prizes (similar to scratching off lottery tickets). Intuitively, if an item is twice as rare, the marginal search cost of finding that item is twice as large, and hence, its price (value) in a perfectly competitive market must be twice as high.

Prior empirical studies have consistently found that the elasticity of price with respect to rarity (measured as quantity) is less than one. We show that the empirically observed power law emerges from a generalization of our baseline model with rank-dependent utility (RDU) preferences (Quiggin, 1982; Yaari, 1987; Tversky and Kahneman, 1992; Diecidue and Wakker, 2001).[4] The difference between expected utility (EU) and RDU agents is that a standard RDU agent is systematically biased and overweighs small probabilities of extreme outcomes. In the competitive market setting, this bias leads an RDU agent (who searches for rare prizes by scratching off lottery tickets) to systematically *underestimate* the relative marginal cost of finding a prize that is twice as rare. Thus, the price of an item twice as rare *less* than doubles. In other words, in a competitive market with RDU agents, the elasticity of price with respect to rarity is less than one. The version of RDU that we apply is the Yaari (1987) dual theory of choice under risk with a specific empirically validated probability weighting function (Diecidue, Schmidt, and Zank, 2009; Abdellaoui, l'Haridon, and Zank, 2010).

In addition to capturing the empirically observed power law, the model with RDU agents leads to three new theoretical insights. First, with RDU agents, the notion of rank naturally emerges as a measure of rarity. More specifically, the model with RDU agents suggests that an item's rank within an ordered set of close substitutes is a natural measure of rarity that might better capture the qualitative concept of rarity. In an ordered set of items (consisting of goods that are close substitutes), the rank of item $i$ is the number of items known to exist that are at least as good

[4]The RDU framework is one of the promising generalizations of expected utility theory (EU) as it can rationalize empirical violations of the EU independence axiom and a preference for purchasing both lottery tickets and insurance while preserving basic properties such as transitivity and first-order stochastic dominance. A standard axiomatic characterization of RDU that differs only from EU by weakening the independence axiom is provided by Abdellaoui (2002).

as $i$. The item's rank enters the pricing formula when the setting is extended from EU agents to RDU agents. In contrast, each prior study used a measure of the quantity (in existence) of a rare item as the measure of that item's rarity. For example, Koford and Tschoegl (1998) who study the value of rare coins used a coin's mintage as their measure of quantity. Cameron and Sonnabend (2020) who study the value of rare vinyl music records used a large database of music records owned by collectors to determine the quantity of each item that has survived. Lee (2022) who studied the value of non-fungible tokens of professional basketball players used a non-fungible token's circulation as the measure of quantity.

Using rank as the measure of rarity, the model further predicts that if the regressions of log price on log rank are estimated for each year of data, then small fluctuations in the estimated slope of regressions are accompanied by large changes of the estimated intercept in the opposite direction. In particular, the second new insight is that the model predicts a negative linear relationship between the slope and intercept. That is, when the slope of the log price versus log rank plot is shallower, prices of the rarest items are lower as well. This is because when market participants are more biased, they assign lower values to rare objects compared to rational agents, and the effect grows larger as the object becomes rarer.

The third new insight provides a theoretical link between the degree to which the market deviates from efficiency (with greater deviations reflected by greater probability distortions) and a regularity known as Zipf's law which describes an inverse relationship between size and rank.[5] In our setting, Zipf's law emerges in the market (e.g., the tenth rarest item has approximately one tenth of the value of the rarest item among a set of close substitutes) in the limit as the degree of probability distortions approach zero (i.e., as the agents come closer to the expected utility benchmark).

We empirically investigate the theoretical predictions using data on rare baseball card sales spanning 22 of the most valuable baseball cards in the post-WWII era. In particular, our data set consists of 4,264 sales across 21 different types of professionally graded vintage (pre-1970) baseball cards, and 51 sales of serially numbered variants of the most valuable post-1970 baseball card (issued in 2009).

[5] Zipf's law has been identified in a variety of contexts in the social sciences, perhaps most notably in characterizing the distribution of city sizes and firm sizes (Gabaix, 2009).

Our data set has several unique features that collectively distinguish it from prior studies. First, the prices of the items span roughly five orders of magnitude (from \$50 to nearly \$4 million). In particular, we observe that the prices for 21 of the 22 different cards in our data set span at least three orders of magnitude across the sales of the lowest ranked to the highest ranked instances of each card. In contrast, the data on rare coins in Koford and Tschoegl (1998) spans roughly two orders of magnitude (from approximately \$200 to \$15,000).[6] Similarly, the data on rare vinyl music records in Cameron and Sonnabend (2020) spans roughly two orders of magnitude (from \$300 to \$27,500).

Second, the auctions in our data placed no bounds on the highest market price that can be realized. In contrast, Lee (2022) notes that the marketplace for non-fungible tokens that he uses imposes an upper bound on a seller's asking price of \$250,000. This restriction effectively truncates the right tail of the distribution for the rarest items and potentially limits the inferences that can be made about the market value of rarity. In our data, 17 of the 22 different cards have their highest sale price above \$250,000, including four different cards that have a sale price above \$1 million. Importantly, as our data set has no imposed limit on the sale price and as there are high graded examples of all cards in our data set, we can observe the market value of rarity for the rarest items.

Third, the baseball players in 21 of the 22 cards have been retired for decades, and these 21 cards were all issued more than half a century ago, such that the value of these cards is not affected by changes in a player's statistics or changes in expectations about a player's future performance. In contrast, the Lee (2022) data set generally consists of current basketball players and fluctuations in their performance have additional effects on the market price.

Fourth, each card is professionally graded which standardizes two instances of the same card with the same grade from the same grading company, and which makes card quality objective, observable, and commonly known.

This makes our setting particularly clean for empirical analysis since we can control for a card's condition and other observable characteristics. In contrast, in markets for collectibles and original artwork, controlling for observable characteristics is often difficult, if not impossible. As is usually the case in collectibles markets, the data in Cameron and Sonnabend (2020) does not control for

[6]In addition, their price data is based on a price guide estimate of seller ask prices and does not use the prices buyers paid for the items.

the condition of the items sold.

Fifth, our data span a longer period of time, from 2005 through 2021, which enables us to investigate time series properties of our data such as whether stock market sentiment affects the market value of rarity. In contrast, the data in Lee (2022) spans two months, and the data in Koford and Tschoegl (1998) comes from dealer ask prices in two years. The data in Cameron and Sonnabend (2020) spans nine years, but they do not study such time series properties of their data.

To empirically investigate the theoretical prediction regarding rank as a measure of rarity, we use our data on professionally graded vintage cards and identify a measure of rank for each card. The rank of a card with grade G is the number of instances of that card with a grade at least as high as G. The distribution of cards receiving each grade by the prominent grading companies is publicly available online. In contrast, consistent with the prior literature, one might measure rarity as the quantity of a card with grade G known to exist. While the measure of quantity does have an intuition as a measure for rarity (a "mint" condition card for which 100 others have been graded mint condition is rarer than if 1000 others have been graded mint condition), the measure of rank incorporates this information in addition to relevant information about higher quality items, making it a more informative measure. Supporting the theoretical prediction that an item's rank within an ordered set of close substitutes is a more natural measure of rarity than the conventional measure of quantity, we find empirically that rank provides a better fit to the data. In particular, we find that rank provides a higher $\mathrm{R}^2$ than quantity, even after controlling for grade, card fixed effects and year fixed effects. Across ten regression specifications, rank consistently generates a higher adjusted $\mathrm{R}^2$, which, in some cases, is more than 10 percentage points higher than the adjusted $\mathrm{R}^2$ from using quantity. In our baseline specification without card or year fixed effects, quantity generates an adjusted $\mathrm{R}^2$ of 44%, compared to an adjusted $\mathrm{R}^2$ of 60% generated by using rank as the measure of rarity. In addition to rank providing a better fit, we find that the estimated coefficients on log rank are more stable than the estimated coefficients on log quantity across our ten regression specifications with different sets of control variables.

To test the theoretical prediction of a negative linear relationship between the slope and intercept estimated from a regression of log price on log rank, we estimate the slope and intercept for each year. We find empirical support for the predicted negative linear relationship between the estimated slopes and intercepts.

Regarding the theoretical link between deviations from market efficiency (via probability distortions) and Zipf's law, we find empirically that the estimated degree of probability distortion is smaller for the vintage (pre-1970) cards in our data set and closer to Zipf's law than it is for the only modern card in our data set which depicts a currently active baseball player. This is consistent with the intuition that the vintage card market where no new information about players' performance or career enters the market is closer to an efficient market compared to the market for currently active athletes where there is room for speculation on a player's future performance.[7]

We also use the time variation in the market value of rarity to test for a relationship between stock market sentiment and the market value of rarity. As a novel empirical finding, we document find that the market value of rarity is higher in years following higher stock market sentiment.

The literature on the market value of rarity has developed more rapidly in recent years. The earliest paper investigating the relationship between price and rarity appears to be Dickie et al. (1994). As indicated, our paper is most closely related to Koford and Tschoegl (1998), Lee (2022), and Cameron and Sonnabend (2020). A recent study by Etro and Stepanova (2018) takes a complementary approach by identifying a power law for artist talent. Other studies have examined art market fluctuations (Lovo and Spaenjers, 2018; Penasse and Renneboog, 2022). Our results regarding the annual fluctuations of rare items' prices contribute to this literature as well.

The remainder of the paper is organized as follows: Section 2 introduces the model and studies its implications. Section 3 summarizes our data set. Section 4 presents our empirical results. Section 5 provides further discussion of our results and their broader implications. Section 6 concludes.

# 2. Model

This section proposes a cost-based approach to determine the market value of rarity, which can be treated as a fair price when the relative value of rarity is considered in general.

## *2.1. The Relative Value of Rarity*

We define the relative value of rarity as the relative market price of two objects that are (almost) perfect substitutes but differ in a measure of rarity. A near-perfect example of such objects is two

[7]Consistent with this intuition, in a stock market context, Liu, Moskowitz, and Stambaugh (2021) argue that decade-old information is efficiently priced.

baseball cards that are almost identical but differ in their print run. We consider a perfectly competitive environment in which the price of an object equals its marginal search cost, i.e., the expected cost of finding a rare item through buying lottery tickets. For vintage cards that are now found at auctions or baseball card conventions rather than from packs of cards, the marginal search cost can be viewed as the cost of finding another version of the particular card one is looking for at auction. High graded versions of some vintage cards have appeared only once or not at all in our data set.[8] Thus, in a perfectly competitive market, the relative price of two objects equals the ratio of their marginal (search) costs.

Consider a special lottery whose prizes are baseball cards. For instance, imagine a company that prints baseball cards and sells them individually, each in a sealed package. The card inside the sealed package is revealed once the package is bought and opened. A few cards are rare and valuable, but most are not valued. In short, buying one sealed card is the same as buying a lottery ticket, and that is how we refer to it (this is indeed close to how baseball cards are sold in reality). We denote this lottery by $(\pi_1, p_1; \pi_2, p_2; \cdots; \pi_K, p_K; 1 - \sum_{i=1}^{K} \pi_i, 0)$, where $\pi_i$ is the probability of finding the $i$'th card (i.e., the $i$'th lottery prize), and $p_i$ represents both the $i$'th lottery prize and its market value. Without loss of generality, we rank the prizes based on their rarity so that $\pi_1 < \pi_2 < \cdots < \pi_K$, i.e., the first card is the rarest and the $K$'th card is the least rare. Note that finding any prize at all is a rare event and has a low probability. Let $n_i$ be the print run of card $i$, $N$ the total number of lottery tickets, and $c$ the cost of buying one baseball card lottery. Then, $\pi_i = \frac{n_i}{N}$, and the (expected) marginal cost of finding an $i$'th card is $\frac{c}{K\pi_i} = \frac{cN}{Kn_i}$.[9] Thus, in a competitive market with expected utility (EU) agents, the price ratio satisfies $\frac{p_i}{p_1} = \frac{\pi_1}{\pi_i} = \frac{n_1}{n_i}$, showing how the price of a rare item depends on its relative rareness.

The rarity literature uses print run ($n_i$) as the measure of rarity.[10] Our discussion above shows that with risk-neutral EU agents, price is proportional to the inverse of rarity, implying a price elasticity of rarity that equals one (i.e., -$\partial \ln p_i / \partial \ln n_i = 1$). In particular, $p_i = p_1 n_1 / n_i$, where the proportionality factor is the total value of the rarest card ($p_1 n_1$).

[8] For instance, the PSA 10 Reggie Jackson rookie card for which only one such example is known, appeared at auction only once in the seventeen years covered by our data set. Perhaps due in part to the difficulty of finding another, the card sold for over $1 million.

[9] Note that with the total cost of $cN$, one can buy all the cards. Ex ante, it is as if the $cN/K$ portion of the cost is spent on each of the $K$ types (with different rarity), and $cN/(Kn_i)$ is spent on each instance of card $i$.

[10] Non-Fungible Tokens (NFTs) have a moment circulation number that is a digital counterpart of $n_i$ (Lee, 2022). For rare coins, mintage is the counterpart of $n_i$ (Koford and Tschoegl, 1998).

We summarize this in the following proposition.

PROPOSITION 1: *In a competitive market with risk-neutral expected utility agents, an otherwise equivalent object that is $n$ times rarer is $n$ times as valuable. In other words, $p_i = (p_1 n_1)\frac{1}{n_i}$, where $n_i$ is the print run of the object $i$.*

In contrast to the above result, the rarity literature finds that the empirical relation $p_i \propto n_i^{-\theta}$ with $\theta \in (0.3, 0.8)$ is a good empirical model for the relative value of rare items (Lee, 2022). Here, drawing on the behavioral framework of rank dependent utility (RDU), we shed some light on why the empirical elasticity parameter $\theta$ is consistently estimated to be less than one. Then, we show that the concept of rank could potentially explain the relative values better, and finally, we make a new prediction based on the RDU model for which we offer empirical support in Section 4.

It is well known in behavioral economics that decision makers (DMs) facing small probabilities toward either positive or negative extreme outcomes do not comport with the expected utility (EU) framework's prediction in that DMs seemingly use distorted probabilities. Particularly, it is a well-known empirical finding that a DM overweights small incremental probabilities for extreme good (or bad) outcomes. The RDU framework captures this behavior via an inverse-S shape probability weighting function. That is, an RDU agent distorts probabilities in a systematic way and uses "decision weights" instead of physical probabilities when calculating an expected value, which we briefly explain below. Note that throughout, the utility is assumed linear, and agents deviate from the risk-neutral EU framework only through the probability weighting function, as in Yaari (1987).

Consider the lottery $(\pi_1, p_1; \pi_2, p_2; \cdots; \pi_K, p_K; 1 - \sum_{i=1}^{K} \pi_i, 0)$ as before in which the prizes (baseball cards) are ordered by rarity, i.e., the rarest prize is ordered first, and so on. Importantly, we assume baseball card lottery buyers behave according to RDU and *not* EU. Thus, a lottery buyer inflates the (small) probability of the event of finding an $i$'th ordered item or a better prize, which is called rank, $\rho_i \coloneqq \sum_{j=1}^{i} \pi_j$.[11] Specifically, the DM replaces $\rho_i$ with $W(\rho_i)$, where the probability weighting function $W(\cdot)$ is concave for small probabilities, hence, inflating them. A commonly used probability weighting function in the literature is the constant relative sensitivity (CRS) weighting function that is axiomatized in Diecidue et al. (2009) and later investigated in

[11] The reader might recall that in the RDU literature, rank is typically defined as the probability $1 - \rho_i + \pi_i$, however, either definition can be used. See Diecidue and Wakker (2001) for an excellent exposition. For empirical work, defining the rank as a (probability) measure of distance from the top prize has a clear practical advantage, since the worst prize might be hard to identify or might be missed by the researcher.

Abdellaoui et al. (2010). For small probabilities, the CRS function is $W(\rho_i) = \delta^{1-\gamma}\rho_i^\gamma$ for $\rho_i \leq \delta$, with $\delta \in (0,1)$, but the coefficient $\delta^{1-\gamma}$ is not currently of our interest. As the reader might be interested, a value of $\gamma \in (0.4, 0.8)$ is typically suggested by empirical studies.[12] Many other probability weighting functions, including the specification by Tversky and Kahneman (1992), converge to a power function for small probabilities as well. The next proposition shows how the value of a rare object is related to the object's rank and quantity. For convenience, we use an equivalent measure of rank, $r_i := \sum_{j=1}^{i} n_j$, i.e., the number of cards at least as good as the $i$'th prize. Clearly, we have $r_i = N\rho_i$.

PROPOSITION 2: *With linear utility and a probability weighting function that is a power law in small probabilities (such as CRS), an RDU agent's value of an i'th ordered rare object is approximately the following:*

$$p_i \approx \left(\frac{p_1 n_1^\gamma}{\gamma}\right)\frac{r_i^{1-\gamma}}{n_i}, \quad \text{for } i > 1, \tag{1}$$

*where $r_i$ is the object's rank, $n_i$ is its print run, and the approximation is accurate when the probabilities of finding rare objects are small (or when $\gamma$ is close to one).*

*Proof.* In a competitive environment, the relative price of rare cards equals the ratio of (expected) marginal costs of finding the cards in the lottery. But for RDU agents, the (expected) marginal cost is proportional to the inverse of decision weights instead of probabilities, i.e., $p_i \propto w_i^{-1}$ (instead of $p_i \propto \pi_i^{-1}$). Let $w_i$ denote the decision weight associated with rank $\rho_i$ and probability $\pi_i$. Then, we have $w_i = W(\rho_i) - W(\rho_{i-1})$ for $i > 1$.[13] Assuming a CRS probability weighting function, we have $W(\rho_i) = \delta^{1-\gamma}\rho_i^\gamma$ for small $\rho_i$ (given that we have small winning probabilities) and the first order approximation with respect to $\rho_i$ yields $\rho_i^\gamma - \rho_{i-1}^\gamma \approx \gamma\rho_i^{\gamma-1}\pi_i$ (this is the Taylor expansion of $\rho_{i-1}^\gamma$ around $\rho_i$ which holds for $i > 1$). Put together, we have

$$\frac{p_i}{p_1} = \frac{w_1}{w_i} = \frac{W(\rho_1)}{W(\rho_i) - W(\rho_{i-1})} = \frac{\rho_1^\gamma}{\rho_i^\gamma - \rho_{i-1}^\gamma} \approx \frac{\rho_1^\gamma}{\gamma\rho_i^{\gamma-1}\pi_i} = \frac{(N\rho_1)^\gamma}{\gamma}\frac{r_i^{1-\gamma}}{n_i} = \frac{n_1^\gamma}{\gamma}\frac{r_i^{1-\gamma}}{n_i}, \text{ for } i > 1.$$

The second to last equality follows since $\pi_i = \frac{n_i}{N}$, and $\rho_i = \sum_{j=1}^{i}\frac{n_j}{N}$, and the last one since $\rho_1 = \pi_1$. In practice, the winning probabilities ($\pi_i$) are tiny and the approximation is accurate.[14] Note that the proof can be extended to any probability weighting function that approaches a power function

[12] See Blavatskyy (2005), Table 1, for a summary of estimates for $\gamma$ from nine empirical studies. Figure 4 in the appendix depicts a CRS probability weighting function with $\gamma = 0.80$ and $\delta = 0.30$ for readers' reference.

[13] See, for example, Diecidue and Wakker (2001).

[14] Figure 5 in the appendix shows how accurate the approximation is for a conservative set of parameters.

for small probabilities. $\square$

Importantly, since $\gamma \in (0,1)$ and $r_i$ increases in $n_i$, Proposition 2 implies that as $n_i$ increases, the price $p_i$ does not drop by $1/n_i$, but it drops at a slower rate. In other words, an econometrician that regresses log price on log quantity finds a slope coefficient shallower than negative one. Thus, the theory is in line with the empirical findings. The intuition behind Proposition 2 is that a more biased DM has a more concave probability weighting function (smaller $\gamma$) and assigns a relatively larger decision weight to the event of finding a rarer object. The RDU agent behaves as though it is "easier" to find a rarer prize, and underestimates its marginal search cost. Thus, as opposed to the case with EU agents, when an object becomes rarer, its price increases by less than $n^{-1}$.

Another significance of Proposition 2 is that it suggests that log price regressions that use rank could perform better than log price regressions that use quantity as the measure of rarity. The intuition is that as $n_i$ increases, $r_i$ increases, but not necessarily the other way around. Hence, rank can possibly contain more relevant information than quantity. Later in the paper we show empirically that rank performs significantly better than the usual rarity measure of quantity (equivalent to print run). Further support for the use of rank as a more relevant measure of rarity comes from textual data in the auction listings, which suggest that decision makers pay more attention to the rank than to the print run. In particular, in online auctions of vintage baseball cards, sellers can put a note in the description of the card. We observe that many sellers choose to point out how many instances of that card with a *higher* grade are known to exist rather than pointing out how many instances of that card with the same grade are in existence.

The next corollary shows that under a certain condition, however, the rank and quantity can be used interchangeably in the regression of log price on log rank. Moreover, it gives us exactly the typical equation of the rarity literature, i.e., a log-linear relationship between price and quantity.

COROLLARY 1: *If $n_i$ is approximately exponentially increasing in $i$, that is, $n_i \approx b\,\kappa^i$ with $b > 0$ and $\kappa > 1$, then we approximately have $r_i \approx \frac{\kappa}{\kappa-1} n_i$, and for the RDU agent in Proposition 2, we have*

$$p_i \approx \Big(\frac{p_1 n_1^{\gamma}}{\gamma}\frac{\kappa}{\kappa-1}\Big) r_i^{-\gamma}, \text{ for } i > 1. \tag{2}$$

*Proof.* If $n_i = b\,\kappa^i$, then

$$r_i = \sum_{j=1}^{i} n_j = b \sum_{j=1}^{i} \kappa^j = b\,\kappa \frac{\kappa^i - 1}{\kappa - 1} \approx b\,\kappa \frac{\kappa^i}{\kappa - 1} = \frac{\kappa}{\kappa - 1} n_i.$$

Finally, using Proposition 2 and replacing $n_i$ with $\frac{\kappa-1}{\kappa} r_i$ gives us the result. □

The intuition behind Corollary 1 is that $r_i$ can be thought of as the integral of $n_i$ over $i$, and $n_i$ is the derivative of $r_i$ with respect to $i$.[15] But a function and its derivative are proportional if and only if the function is exponential. Thus, if $n_i \propto \kappa^i$, we approximately have $r_i \propto n_i$, and the rest follows from Proposition 2.

One can also use data to motivate the condition $n_i \propto \kappa^i$. We find that as a statistical model, $log(n_i) = b_0 + b_1 i + \epsilon_i$ offers a very good fit in a subset of our data, exactly where the rank and print run measures both give us an excellent empirical fit for the relative prices. As a side note, another model with a good empirical fit is the power function ($n_i \propto i^\kappa$ for some $\kappa > 1$), which gives a similar result to Equation (2), but with different constants.

### *2.2. Systematic Bias and the Market Value of Rarity*

The behavioral economics literature finds that different individuals vary in their attitude towards small winning probabilities. That is, although all individuals may inflate a small probability of winning a large prize, some individuals have a larger bias. The RDU framework captures a larger bias with a smaller $\gamma$ parameter. Moreover, one might expect to observe small fluctuations in market participants' bias from year to year.[16] As the number of market participants change from year to year, small fluctuations in $\gamma_t$ might have a sizeable impact on the price of rare items.

In this section, we investigate the model's prediction about a change in the level of the market's bias through a comparative static analysis of $\gamma_t$. Recall that our marginal cost argument together with the RDU framework implies that the competitive market price of the prize $i$ is $p_{i,t} = \frac{c}{K w_{i,t}}$. Similar to Proposition 2, we can write $w_{i,t}$ in terms of $\rho_i$, $\pi_i$ and $\gamma_t$. This, gives us the price of the $i$'th ordered card, using which we can easily do a comparative static analysis in $\gamma_t$.

[15] If $i$ varies continuously, then $r_i$ is exactly the integral of $n_i$ over $i$.

[16] For instance, one might expect to see that a more liquid market behaves closer to the rational expectations benchmark (i.e., a $\gamma_t$ closer to one, where the subscript $t$ shows that $\gamma$ is subject to small temporal changes). In particular, with more market participants, it is more likely that there are EU agents who determine the prices since the RDU agents undervalue the items relative to EU agents for values of $\gamma$ less than unity.

COROLLARY 2: *For the RDU agent in Proposition 2, the price of the i'th ordered card is*

$$p_{i,t} \approx \frac{c}{K}\frac{1}{\gamma_t}\left(\frac{\rho_i}{\delta}\right)^{1-\gamma_t}\frac{1}{\pi_i}, \quad \text{for } i > 1, \tag{3}$$

*which is strictly increasing in $\gamma_t$ if $\rho_i \ll \delta$. In other words, a higher level of bias (lower $\gamma_t$) suppresses the price level.*

*Proof.* Take the partial derivative of the log of price with respect to $\gamma_t$. Note that $\log p_{i,t} = \log(\frac{c}{K\pi_i}) - \log\gamma_t + (\gamma_t - 1)\log(\frac{\delta}{\rho_i})$. Thus,

$$\frac{\partial \log p_{i,t}}{\partial \gamma_t} = -\frac{1}{\gamma_t} + \log\left(\frac{\delta}{\rho_i}\right),$$

which is larger than zero when $\rho_i$ is small enough compared to $\delta$. This is a soft condition as the estimates of $\delta$ are around 0.3 (Abdellaoui et al., 2010), and typically, $\rho_i$ is two or more orders of magnitude smaller than $\delta$. □

Similar to Proposition 2, the intuition behind Corollary 2 is that since the price of a rare item in the competitive market is determined by its marginal search cost, the price is proportional to the inverse of the "perceived" probability of finding a rare item in a lottery. But a more biased individual estimates a lower marginal cost, and hence, sets a lower price.

The significance of Corollary 2 is that it provides us with a new relationship regarding rare items' prices that we can empirically test in our data.

Using the approximation in Corollary 1, we have $p_{i,t} \approx p_{1,t}(\frac{r_i}{r_1})^{-\gamma_t}$, and thus, $\log p_{i,t} \approx \log(p_{1,t}r_1^{\gamma_t}) - \gamma_t \log r_i$. That is, the slope of log price plotted against log rank is $-\gamma_t$, and its intercept is $\log(p_{1,t}r_1^{\gamma_t})$. In the model, $p_{1,t} = \frac{c}{K}\frac{1}{w_{1,t}}$, and with the CRS probability weighting function, $w_{1,t} = \delta^{1-\gamma}\rho_1^{\gamma_t}$, and thus, $p_{1,t} = \frac{c}{K\delta}(\frac{N\delta}{r_1})^{\gamma_t}$, and $\log(p_{1,t}r_1^{\gamma_t}) = \log(\frac{c}{K\delta}) + \gamma_t \log(N\delta)$.[17] Hence, if there is a change in the slope ($-\gamma_t$), the model predicts a change in the intercept in the opposite direction due to $\gamma_t \log(N\delta)$, assuming parameters $\delta$, and $c$ do not vary too much over time. The following corollary summarizes the above analysis.

COROLLARY 3: *Consider the RDU agent in Proposition 2. Then, in the plot of log price versus log rank, a small change in the slope (e.g., from year to year) is accompanied by a larger negative change in the intercept. The proportionality factor is approximately $-\log(N\delta)$, a negative number since $N$ is large.*

[17] Note that for $p_{1,t}$, the expression is exact and not an approximation.

Based on Corollary 3, we predict that when the slope of the log price versus log rank plot is shallower, its intercept drops significantly. To have a gauge on the size of the effect, if the agents roughly go through about ten thousand cards to find the rarest instance of a specific card, and with $\delta$ being 0.3, the annually estimated intercept should be roughly about eight times as volatile as the annually estimated slope.[18] That is, the model predicts a large change in the level of prices (measured by the intercept) as the estimated $\gamma_t$ varies, and this is precisely what we find in our data, which we present in Section 4. We want to emphasize two points here. In general, there is no need to have a shallower slope accompanied by a smaller intercept. Without a theory, one might as well imagine that a shallower slope means a constant intercept, but a larger value for less rare items (in fact, this seems to be what the previous literature suggests). Second, when comparing two different cards, it is possible to have a larger intercept and a shallower slope. For instance, Lee (2022) finds a shallower slope for more popular (and hence, more expensive) NFTs, which we discuss in Section 5 when comparing two subsets of our data.

### *2.3. On the Sources of Demand for Rarity*

For the sake of completeness, we want to briefly mention that in our cost-based analysis, we assumed that each *individual* baseball card seller faces a perfectly elastic demand, which requires the existence of an aggregate demand for rare items. In a general equilibrium, this aggregate demand determines the price of a lottery ticket, which we treated as the constant parameter ($c$). However, a change in $c$ does not affect the elasticity of price with respect to relative rarity ($\gamma$), which is our main focus. In appendix A, we offer a discussion on the sources of aggregate demand for rare items, and in particular, we offer a compelling example of a case that shows, in part, this demand is due to some individuals striving to complete their collections of rare items.[19]

### *2.4. Taking the Model to Data*

The model in this section can be viewed as a mental framework that provides a fair relative price for buyers and sellers of rare items. When comparing two items that are similar (closely comparable) but have different levels of rarity, the model provides a benchmark as to how market

[18] Note that $-\log(N\delta) \approx -8$, for $\delta = 0.3$ and $N = 10,000$.

[19] We thank a referee that brought this to our attention.

participants interpret relative rarity as a gauge for the relative cost of acquiring rare items. That said, like any asset pricing model, the success and usefulness of this “fair price model for relative rarity” should be judged empirically and through testing the model’s predictions. The following sections provide empirical evidence regarding the model’s predictions in this section. We show that the results are in line with the model’s predictions.

# 3. Data

We gathered data on the highest selling baseball cards of the modern era (post-WWII). We identified 22 different cards that have ever sold for over $100,000. These cards include the most recognizable and sought after cards issued since WWII. Each card features a baseball player (e.g., Mickey Mantle) from a set (e.g., Topps, Bowman, or Leaf) with a limited print run at the time of issue. Importantly, each card in the dataset is professionally graded by a grading company (e.g., PSA, SGC, or BGS).

The sample of cards was determined from a list of the most valuable baseball cards.[20] We included all cards on the list from the post-WWII era (the cards issued in 1948 or later). This era is identified as the modern era of baseball cards in an authoritative reference book (Beckett, 2010).

Only graded card sales were included in the data set since grading standardizes the cards and makes different examples of the same card comparable in terms of their condition, rarity, and desirability. As a result, we can treat two Mickey Mantle 1952 Topps cards with a grade of PSA 9 as essentially identical items. The major grading companies provide freely accessible population reports on their websites which record the number of cards receiving each grade for each card they have graded.[21] Since half a century has passed after the vintage cards were issued and they have been widely collected for decades, with graded cards selling for a premium, the population reports of the grading companies is fairly representative of the true population.

The data set of all graded versions of these 22 cards sold by the two major auction houses specializing in these items (Goldin and Heritage) consists of 4,327 auction records from 2003 to

[20] The list of cards was obtained from the link: https://www.stadiumtalk.com/s/most-expensive-baseball-cards-985687df1bbe45c5 (accessed September 2021).

[21] The entire population of cards graded by PSA (and the number receiving each grade) is available to be searched at the PSA population report database (https://www.psacard.com/pop/baseball-cards/20003). Similarly, the entire population of cards graded by SGC (and the number receiving each grade) is available to be searched at the SGC population report database (https://gosgc.com/pop-report).

**Table 1.** Summary of Cards in Data Set.

**Notes:** The table lists all 22 cards in our sample. The 'max price' and 'min price' columns display the highest and lowest sales of each card in our sample. The 'max grade' and 'min grade' columns display the highest and lowest card grade (on a scale where 0 and 10 are respectively the lowest and highest grades possible) of each card in our sample. The $\overline{r}_i$ and $\underline{r}_i$ columns display the highest and lowest ranks of each card that are observed in our sample. The 'trading volume' columns displays the total number of sales of each card in our sample. Unlike the vintage cards which are not serially numbered, the Mike Trout Bowman Chrome refractor card was issued in seven serially numbered varieties with print runs ranging from 500 produced to only 1 produced. For brevity, the set Bowman Chrome refractors is abbreviated as 'Chrome' in the table. Prices are in US dollars, rounded to the nearest dollar.

| Player | Year | Set | Max price | Min price | Max grade | Min grade | $\overline{r}_i$ | $\underline{r}_i$ | Trading Volume |
|---|---|---|---|---|---|---|---|---|---|
| Hank Aaron | 1954 | Topps | 645,500 | 239 | 9 | 0 | 5 | 4955 | 347 |
| Ernie Banks | 1954 | Topps | 144,000 | 120 | 9 | 0 | 10 | 3112 | 185 |
| Yogi Berra | 1948 | Bowman | 93,000 | 144 | 10 | 0 | 1 | 1456 | 61 |
| Roberto Clemente | 1955 | Topps | 1,107,000 | 143 | 9 | 0 | 12 | 4777 | 358 |
| Reggie Jackson | 1969 | Topps | 1,005,600 | 58 | 10 | 0 | 1 | 7130 | 182 |
| Sandy Koufax | 1955 | Topps | 384,000 | 159 | 9 | 0 | 9 | 7888 | 446 |
| Mickey Mantle | 1951 | Bowman | 750,000 | 508 | 9 | 0 | 6 | 1906 | 255 |
| Mickey Mantle | 1952 | Topps | 2,880,000 | 2868 | 9 | 0 | 9 | 1614 | 315 |
| Mickey Mantle | 1954 | Bowman | 168,000 | 102 | 9 | 0 | 3 | 3817 | 230 |
| Mickey Mantle | 1956 | Topps | 382,400 | 90 | 10 | 0 | 5 | 7478 | 341 |
| Willie Mays | 1951 | Bowman | 338,400 | 191 | 9 | 0 | 8 | 1635 | 196 |
| Willie Mays | 1952 | Topps | 478,000 | 213 | 9 | 0 | 2 | 2155 | 195 |
| Stan Musial | 1948 | Bowman | 43,200 | 102 | 10 | 0 | 1 | 1634 | 83 |
| Stan Musial | 1948 | Leaf | 312,000 | 191 | 9 | 0 | 3 | 830 | 57 |
| Andy Pafko | 1952 | Topps | 57,600 | 50 | 8 | 0 | 3 | 1924 | 60 |
| Satchel Paige | 1948 | Leaf | 432,000 | 1080 | 8 | 0 | 5 | 156 | 23 |
| Jackie Robinson | 1948 | Leaf | 444,000 | 454 | 9 | 0 | 8 | 1283 | 169 |
| Pete Rose | 1963 | Topps | 717,000 | 239 | 10 | 0 | 1 | 4596 | 243 |
| Nolan Ryan | 1968 | Topps | 612,360 | 84 | 10 | 0 | 1 | 10419 | 427 |
| Duke Snider | 1949 | Bowman | 264,000 | 191 | 10 | 1.5 | 1 | 584 | 51 |
| Warren Spahn | 1948 | Leaf | 252,000 | 91 | 10 | 0 | 1 | 733 | 40 |
| Mike Trout | 2009 | Chrome | 3,800,000 | 13,530 | 10 | 9 | 1 | 956 | 51 |

2021. The data for years 2003 and 2004 is sparse with just 2 data points for 2003 and 10 data points for 2004. This is small given that our data covers 22 different cards. Our final data set consists of all observations from 2005 through 2021 (4,315 auction records) as these were the years with more than 20 observations per year.[22]

We collected data of all auction sales of professionally graded versions of these 22 cards from the two most prominent auction houses that sell rare baseball cards: Heritage Auctions (www.HA.com) and Goldin Auctions (www.GoldinAuctions.com).[23] Both auction houses have sold at least one card on the above list for over $1 million. Heritage Auctions sold a mint condition (PSA 9) version of

[22]Our results are robust to whether or not the data from 2003 and 2004 are included.

[23]We collected data on all past auction sales available from both websites which are freely accessible to the general public (Heritage requires free user registration) from the earliest sales in the database (beginning in 2003) through December, 2021.

the 1952 Topps Mickey Mantle for $2.88 Million in April, 2018.[24] Goldin Auctions sold a mint condition (BGS 9) 2009 Bowman Chrome Mike Trout superfractor parallel (the only one issued) for $3.84 million in August, 2020.[25]

The final data set consists of (i) all 4,264 sales from January 2005, through December, 2021, of the 21 vintage cards in either the Heritage or Goldin auctions database that were graded by PSA or SGC, (ii) the PSA and SGC population report data for all 21 vintage cards, permitting us to calculate a card's rank, and (iii) all 51 sales through December, 2021, of graded 2009 Bowman Chrome Mike Trout refractor parallel cards in the Heritage or Goldin auctions database.

The complete list of cards in our data set consists of the 22 cards in Table 1 issued by three set manufacturers: Leaf, Bowman, and Topps. The cards in the table are reasonably representative of the most valuable baseball cards of the post-WWII era.

The data provides a reasonable coverage of each card: It contains at least 100 sales for each of 14 different cards and at least 40 sales of each card except for the 1948 Leaf Satchel Paige card for which 23 auction records are available.

A notable feature of our data set, as indicated in Table 1, is the large variation in prices between the highest and lowest ranked instances of the same card with many cards varying in price by four or five orders of magnitude over a short time period. For instance, in 2021, a PSA 0 1969 Topps Reggie Jackson card (with a rank of 7130) sold for $144. In the same year, the only known PSA 10 instance of that card sold for over $1,000,000. Another notable feature of the data set is the extreme degree of rarity that is observed in our sample. In particular, 20 of the 22 cards in our data set have a version of the card with rank less than 10 that sold in our sample, enabling us to observe the market value of rarity among the rarest cards.

Of the 22 different cards in our analysis, 21 were issued between 1948 and 1969, and the last card was issued in 2009. We separate our analyses into the 21 vintage cards (4,264 auction records) and the 2009 card, a refractor rookie card of Mike Trout produced by Bowman Chrome (51 auction records). This separation was done for multiple reasons: The players depicted on the pre-1970 cards are all retired, whereas Mike Trout is an active player. Most importantly, the Trout card differs from the others in that it was serially numbered. The dominant grading card companies

[24] Information about the sale is available at https://sports.ha.com/heritage-auctions-press-releases-and-news/mint-mickey-mantle-1952-topps-rookie-sets-world-record-in-9.83-million-sports-card-auction-at-heritage.s?releaseId=3416

[25] Information about the sale is available at https://goldinauctions.com/LotDetail.aspx?inventoryid=63942

also differ for pre-1970 cards and the Trout card.[26]

### 3.1. *The Trout Card*

It is difficult to measure the market value of rarity since rare items typically have higher qualities. The 2009 Mike Trout card provides an unusual opportunity to measure the value of rarity since it is a case with a large variation in prices over a short time period for *identical* products with different degrees of rarity. In particular, there are seven different refractor parallel versions of the Trout card, each of which has sold in graded form via Goldin Auctions in 2020. The seven different versions are: (i) regular refractor (print run of 500); (ii) X-fractor (print run of 225); (iii) blue refractor (print run of 150); (iv) gold refractor (print run of 50); (v) orange refractor (print run of 25); (vi) red refractor (print run of 5); (vii) superfractor (print run of 1). The seven versions have exactly the same image of Trout and differ only in: (i) the color of the border of the card (*e.g.*, white, blue, gold, orange, red) and the way in which the card refracts light, as shown in Figure 1; and (ii) the serial number on the back of the card. It thus seems reasonable to assume that the Trout cards of the same grade (by the same grading company) are identical in every dimension except for rarity.

Since the print run is known for each type of refractor, we construct our measure of rarity as the rank of the Trout card using its print run information. The rank of the Trout refractor card of print run X is specified to be the total print run of Trout 2009 Bowman Chrome refractor parallels of print runs at least as small as X. (*i.e.*, the number of Trout refractors at least as rare as X). Specifically, as only one superfractor was issued, that card has a rank of 1 among 2009 Bowman Chrome Trout refractors. A red refractor then has a rank of 6. An orange refractor has rank 31, and gold and blue refractors have ranks 81 and 231, respectively. The X-fractor has rank 456 and the regular refractor has rank 956. All the Trout cards in our sample are graded 9, 9.5, or 10, so virtually all the heterogeneity across the Trout cards is due to the variation in their rarity.

[26] It is well-known that the three major card grading companies are PSA, SGC, and BGS (https://sportscollectorsdigest.com/collecting-101/history-of-grading-cards-psa-bgs-sgc-dominating). PSA has substantial market share in grading both modern and vintage cards, whereas SGC is PSA's primary competitor for grading vintage (pre-1980) cards and BGS is PSA's primary competitor for grading modern (post-1980) cards.

### *3.2. Vintage cards*

Vintage cards are *not* serially numbered. However, the quantity is known for each grade level. Thus, we can construct our measure of rank for each of the 21 vintage cards, such that the rank of a card is the number of cards at least as good as that card. Specifically, the rank for a card of grade G evaluated by grading service S is specified to be the number of cards which have received a grade of at least G (from S).

We searched the PSA and SGC population report databases to obtain the rank corresponding to each possible grade for each of the 21 vintage cards. For example, from the PSA population report (accessed September 2020) for the 1952 Topps Mantle card, 1614 cards had been graded by PSA. All cards graded 0 therefore have rank 1614.[27] In contrast, 3 cards have been graded 10, which then share a rank of 3, while the next 6 cards have been graded 9, and hence share a rank of 9 (*i.e.*, 3+6), and so on.

## 4. Empirical Results

We first present the results for the Trout card and then for the vintage cards. As noted in Section 1, Koford and Tschoegl (1998), Cameron and Sonnabend (2020), and Lee (2022) run regressions of log price on log quantity (that is equivalent to print run in our data), and find evidence that the market value of rarity follows a power law. In the present setting, our regression of log price on log rank is justified by the theoretical model in Section 2.

### *4.1. The Trout Card*

For the Trout 2009 Bowman Chrome refractor card, we run regressions of log price, $p_i$, against log rank, $r_i$. The variable $Year_i$ is the year in which the auction took place, and the grade of card $i$ is $Grade_i$ (ranging from 0 to 10). The regression equation is:

$$log(p_i) = \beta_0 + \beta_1 log(r_i) + \beta_2 Grade_i + \beta_3 Year_i + \epsilon_i, \quad i = 1, ..., 51. \tag{4}$$

[27] Cards that PSA certifies are not counterfeit, but which are not in good enough condition to grade even a 1 are graded "PSA Authentic." This is what we refer to as a grade of 0, and many sellers indeed refer to such cards as "PSA 0 Authentic." Thus, cards graded PSA authentic are ranked below cards graded PSA 1 or higher.

**Table 2.** Regressions of log price, $p_i$, against log rank, $r_i$, and control variables for the Trout card. The table displays regressions for all graded card sales of the different versions of the Mike Trout 2009 Bowman Chrome refractor card in the Heritage and Goldin auctions databases. The data ranges from February, 2017, through December, 2021. The log rank is determined by the serial number on the back of the card. The control variables include the grade of the card (Grade), which ranges between 0 and 10, and the year in which the auction took place (Year). The robust standard errors are in parentheses. All coefficients are significant at any conventional significance level. The regression equation is

$$log(p_i) = \beta_0 + \beta_1 log(r_i) + \beta_2 Grade_i + \beta_3 Year_i + \epsilon_i, \quad i = 1, ..., 51.$$

| | (1) Log $p_i$ | (2) Log $p_i$ | (3) Log $p_i$ | (4) Log $p_i$ |
|---|---|---|---|---|
| Log Rank | -0.60*** | -0.63*** | -0.63*** | -0.67*** |
| | (0.05) | (0.05) | (0.04) | (0.04) |
| Grade | | 0.75*** | | 0.85*** |
| | | (0.21) | | (0.17) |
| Year | | | 0.30*** | 0.33*** |
| | | | (0.06) | (0.07) |
| $N$ | 51 | 51 | 51 | 51 |
| adj. $R^2$ | 0.78 | 0.82 | 0.85 | 0.91 |

Robust standard errors in parentheses; *** $p < 0.01$

The results for different specifications of regression Equation (4) are presented in Table 2. Specification (1) in Table 2 shows that log rank explains 78% of the log price variation without any controls (such as for the grade of the card or the year of the transaction). Specifications (2) and (3) show that Grade and Year make moderate increases in the adjusted $R^2$. The adjusted $R^2$ increases to 91% in the final specification that includes both Grade and Year. Across all four specifications, we see that the slope coefficient on $log(r_i)$ is quite stable, and in each case, it is negative and highly significant.

The log-log relationship for the Trout card is shown graphically in the top panel of Figure 1. The figure plots log price against log rank for all Trout cards sold in 2020 with the same grade (BGS 9.5). The $R^2$ is 0.97. The bottom panel of Figure 1 displays images of six different types of Trout cards with a grade of BGS 9.5. Five of these types correspond to the five clusters of points in the top panel. As the figure suggests, the Trout cards can be viewed as perfect substitutes that differ only in their rarity.


**Figure 1.** The top panel graphs the log auction sale price against the log rank of all thirteen Mike Trout 2009 Bowman Chrome rookie card refractors with a grade of BGS 9.5 (Gem Mint) that sold at Goldin or Heritage Auctions in 2020, with prices ranging from \$22,800 to \$922,500. The R-squared is 0.97. The bottom panel displays six versions of the card graded 9.5 with manufacturer print runs of 5 (red), 25 (orange), 50 (gold), 150 (blue), 225 (X-fractor), and 500 (regular refractor). Five of these versions correspond to the five clusters of points in the top panel.

### *4.2. Vintage Cards*

The regression for the vintage card data is given by Equation (5). Subscript $i$ runs over the auctioned items, each a vintage card in Table 1. $p_i$ is the price at which the card was sold, and $r_i$ is the rank of the auctioned card. $PSA_i$ is an indicator variable that equals 1 if card $i$ is graded by PSA, and zero if it is graded by SGC. $Grade_i$ is the grade that card $i$ received from its grading company (on a scale from 0 to 10). $Year_i$ is the auction year of item $i$, an integer between 2005 and 2021, which captures a trend in prices.

$$log(p_i) = \beta_0 + \beta_1 log(r_i) + \beta_2 PSA_i + \beta_3 Grade_i + \beta_4 Year_i + \epsilon_i, \quad i = 1, \cdots, 4264. \tag{5}$$

A feature of our data is that we can treat two cards as identical if they have the same grade from the same grading company, from which we can then identify the market value of rarity. For instance, two PSA 8's of the same card are identical items, but if one card is graded by PSA and the other by SGC they are no longer identical. The importance of controlling for the grading company is clearly visible in Figure 2, where one can see two separate lines, one for each grading company, and Figure 6 in the appendix, that shows the pattern holds across all cards in the sample.

Table 3 reports the results from ten specifications of regression Equation (5) for all 4,264 sales of the vintage cards. For parsimony, in our baseline specification (Specification (1) in Table 3), we keep only the two variables (rank and grading company) that are needed to identify identical cards from the perspective of a buyer. Specification (1) in Table 3 shows that regressing log price against log rank and controlling only for the grading company, but without controls for the type of card, the grade of the card, or the year of the transaction explains 60% of the price variation across 4,264 auction records. Controlling for Grade has no effect on the adjusted $R^2$. Specifications (5)-(10) control for the card fixed effects. This is done by augmenting the regression with dummy variables for each type of vintage card in Table 1 (i.e., 20 dummy variables for 21 vintage cards). Specifications (9)-(10) replace the year trend with year fixed effects (again, by including a dummy variable for each year except 2005). Including card and year fixed effects increases the adjusted $R^2$, but importantly, the rank and PSA coefficients are very significant and stable throughout. With all controls included, Specification (10) explains 95% of the log price variation in the sample.

Recall from Section 2 that a more efficient market (closer to the theoretical benchmark with risk-neutral EU agents), should have a slope on log rank closer to -1. The vintage card market is generally more liquid and has a longer history than the market for the Trout card. Many traders in the vintage card market have experience collecting and trading vintage cards for decades. In addition, in a stock market context, Liu et al. (2021) argue that decade-old information is priced correctly. Specifically, their premise is that any current mispricing of a stock gets corrected in less than ten years. For a less liquid market such as baseball cards, it may take longer for mispricing to be corrected. In our setting, all information about players' performance for the vintage cards is at least half a century old, which one might expect to be priced correctly. In contrast, for the Trout card, the information is being updated in real time since Mike Trout is an active player. The streams of new information about Trout and his performance create an avenue for mispricing in the short-run that is absent for the vintage cards. For these reasons, we predict that the vintage card market more closely approximates an efficient market than the market for the Trout card. Supporting this prediction, we find in comparing the coefficients on log rank in Table 2 for the modern Trout card to the coefficients on log rank in Table 3 for the vintage cards, that the slope coefficient for the Trout card ranges between -0.60 and -0.67, whereas the slope coefficient for the vintage cards ranges between -0.82 to -0.98. Thus, with reference to the theoretical model with RDU agents, the market for the Trout card seems to have a larger systematic bias (leading to larger probability distortions) than the market for vintage cards, which more closely approximates an efficient market.

The coefficient for the year captures the trend in prices, which can be used to estimate the average annual return for the cards. For vintage cards, this net return is 15% (i.e., $e^{0.14} - 1$), whereas the net return on the market portfolio over the same period (January, 2005, through December, 2021) was approximately 12.1%.[28] The Trout card had an estimated annual net return of 39.1% (i.e., $e^{0.33} - 1$).[29] Since Mike Trout is an active player who had a meteoric rise to fame, buying a Trout card in our sample is similar to investing in Microsoft in 1990, fundamentally different from investing in vintage cards, whose players have long been retired.

[28]This value was obtained by first computing the market return (adding the stock market excess return and risk-free rate) from Kenneth French's data library, computing the average monthly market return over our sample period and then computing the corresponding annualized return.

[29]Annual net return for year $t$ is defined as $(P_t - P_{t-1})/P_{t-1}$, where $P_t$ is the price at year $t$.

**Table 3.** The table displays regressions of log price, $p_i$, against log rank, $r_i$, and control variables. The log rank of each card is constructed from the PSA and SGC population reports. The data includes all sales of the 21 vintage cards in the sample at Heritage or Goldin auctions between 2005 and 2021. Year denotes the year in which the auction took place, PSA is a variable for the grading company (1 if graded by PSA), and Grade denotes the grade of the card (ranging from 0 to 10). The regressions summarized in Columns (5) through (10) have card fixed effects. The regressions summarized in Columns (9) and (10) also have year fixed effects. Robust standard errors (clustered at the card level or the year and card level where applicable) are shown in parentheses. The regression equation is:

$$log(p_i) = \beta_0 + \beta_1 log(r_i) + \beta_2 PSA_i + \beta_3 Grade_i + \beta_4 Year_i + \epsilon_i, \quad i = 1, ..., 4264.$$

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| | Log $p_i$ | Log $p_i$ | Log $p_i$ | Log $p_i$ | Log $p_i$ | Log $p_i$ | Log $p_i$ | Log $p_i$ | Log $p_i$ | Log $p_i$ |
| Log Rank | -0.87*** | -0.98*** | -0.90*** | -0.98*** | -0.89*** | -0.86*** | -0.92*** | -0.82*** | -0.93*** | -0.83*** |
| | (0.01) | (0.02) | (0.01) | (0.02) | (0.02) | (0.04) | (0.02) | (0.03) | (0.01) | (0.02) |
| PSA | 1.83*** | 2.04*** | 1.72*** | 1.89*** | 1.79*** | 1.73*** | 1.68*** | 1.51*** | 1.67*** | 1.48*** |
| | (0.04) | (0.05) | (0.03) | (0.04) | (0.05) | (0.07) | (0.04) | (0.06) | (0.02) | (0.04) |
| Grade | | -0.10*** | | -0.08*** | | 0.02 | | 0.07*** | | 0.08*** |
| | | (0.01) | | (0.01) | | (0.02) | | (0.01) | | (0.01) |
| Year | | | 0.15*** | 0.15*** | | | 0.14*** | 0.14*** | | |
| | | | (0.00) | (0.00) | | | (0.01) | (0.01) | | |
| Year FE | | | | | | | | | Y | Y |
| Card FE | | | | | Y | Y | Y | Y | Y | Y |
| $N$ | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 |
| adj. $R^2$ | 0.60 | 0.60 | 0.68 | 0.68 | 0.86 | 0.86 | 0.93 | 0.94 | 0.95 | 0.95 |

Robust Standard errors in parentheses; $^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

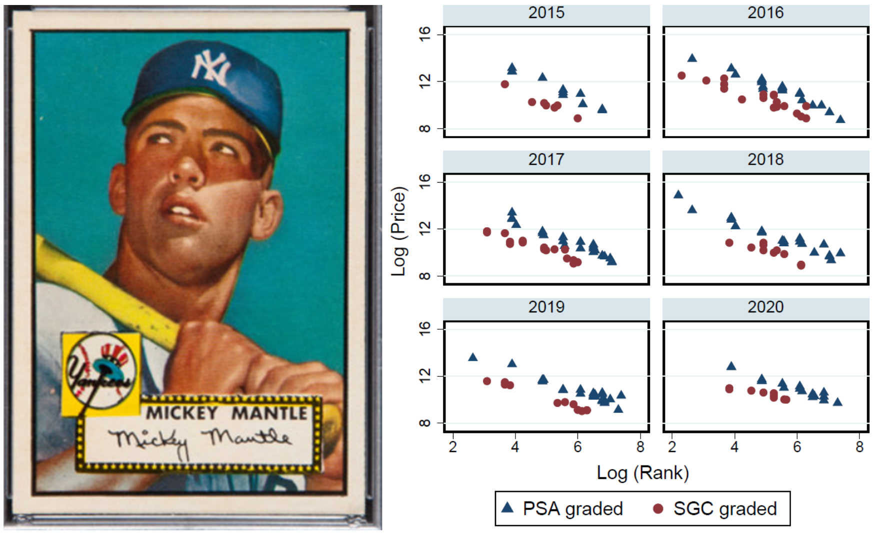



**Figure 2.** The right panel graphs the log auction sale price against the log rank of the Mickey Mantle 1952 Topps Cards that sold at Goldin or Heritage Auctions in years 2015 through 2020. The blue triangles denote PSA graded cards. The red circles denote SGC graded cards. The left panel depicts the mint (PSA 9) example of the iconic Mickey Mantle card that sold at Heritage auctions in 2018 for $2.88 million.

### *4.3. Quantity versus Rank*

An implication that follows naturally from the rank-dependent utility framework in Section 2 is that an item's rank (within an ordered set of close substitutes) may be a more accurate measure of rarity than the print run (the quantity of the rare item known to exist). As noted, this possibility is supported by auction listings which highlight the number of cards with higher grades rather than the number of cards with the same grade that are known to exist. In this section, we empirically investigate if rank performs better than quantity.

To further motivate the difference between quantity and rank as measures of rarity, consider two scenarios for different versions of a rare card. Suppose the versions are identical in every respect except for the print run, as printed on the card. In the first scenario, there are two versions with print runs of 100 and 1000, respectively. In the second scenario there are ten versions, with print runs of 100, 200, 300, ...., 1000, respectively. While prior approaches would treat the card with a print run of 1000 as equally rare in both scenarios, in the first scenario there are 1100 cards that are at least as rare as the card with a 1000 print run, while in the second scenario, there are 5500 cards that are at least as rare. As the example suggests, the rank of an item within an ordered set of close substitutes might better reflect its perceived rarity than the absolute quantity of the item.

Table 4 reports the results from specifications of regression Equation (5) that use the quantity of card $i$ known to exist (with the same grade as card $i$ from the same grading company that graded card $i$), $n_i$, instead of the rank of card $i$, $r_i$. We predict that using rank as the measure of rarity will provide a better fit to the data. Comparing the adjusted $R^2$ values in Tables 3 and 4, reveals that across all ten regression specifications, rank delivers a noticeably better fit than quantity. In the baseline Specification (1) in Tables 3 and 4, replacing $log(n_i)$ with $log(r_i)$ increases the adjusted $R^2$ by 16 percentage points. The difference in adjusted $R^2$ is also 10 percentage points or more in five other specifications and is 4 percentage points in the full specification with all control variables. In this latter case, replacing log $n_i$ with log $r_i$ increases the adjusted $R^2$ from 91% to 95%.

In addition to providing a better fit, we see from comparing Tables 3 and 4 that the coefficients for $log(r_i)$ are more stable across the different regression specifications than the coefficients for $log(n_i)$. Whereas the coefficients on $log(r_i)$ differ by less than 0.20 across the ten regression specifications in Table 3, the coefficients on $log(n_i)$ differ by nearly 0.50 across the ten regression specifications in Table 4.

Table 5 performs a similar analysis for the Trout card. Comparing Tables 2 and 5, we see that rank performs only slightly better than print run for the Trout card with an adjusted $R^2$ of 91% (compared to 90%) with all control variables included. Corollary 1 provides the explanation as to why the use of rank instead of quantity can substantially improve the empirical fit for the vintage cards but only slightly improve the fit for the Trout card. According to Corollary 1, the reason is that for the Trout card, quantity can be well approximated by an exponential function (i.e., $n_i \approx b\,\kappa^i$ for some $b > 0$ and $\kappa > 1$).

To test this hypothesis, Figure 7 in the appendix plots the log print run for the seven different versions of the Trout refractor versus the card order (with cards ordered from lowest print run to highest print run). The exponential approximation is indeed very good as it explains 96% of the variation in the print run of the Trout card in the regression $log(n_i) = a_0 + a_1 i + \epsilon_i$, $i = 1, ..., 7$. In the appendix, we show that the exponential approximation for quantity is not as excellent for the vintage cards as it is for the Trout card. However, the approximation is still reasonable with an adjusted $R^2$ of 33% for the PSA graded cards and 43% for the SGC graded cards. The details of the regression are presented in Table 7 in the appendix.

**Table 4.** The table displays regressions of log price, $p_i$, against log $n_i$, where $n_i$ corresponds to the quantity of card $i$ known to exist (at the grade level of card $i$ from the same grading company that graded card $i$). Log $n_i$ of each card is constructed from the PSA and SGC population reports. The data includes all sales of the 21 vintage cards in the sample at Heritage or Goldin auctions between 2004 and 2021. Year denotes the year in which the auction took place, PSA is a variable for the grading company (1 if graded by PSA), and Grade denotes the grade of the card (ranging from 0 to 10). The regressions summarized in Columns (5) through (10) have card fixed effects. The regressions summarized in Columns (9) and (10) also have year fixed effects. Robust standard errors (clustered at the card level or the year and card level where applicable) are shown in parentheses. The regression equation is

$$log(p_i) = \beta_0 + \beta_1 log(n_i) + \beta_2 PSA_i + \beta_3 Grade_i + \beta_4 Year_i + \epsilon_i, \quad i = 1, ..., 4264.$$

| | (1) Log $p_i$ | (2) Log $p_i$ | (3) Log $p_i$ | (4) Log $p_i$ | (5) Log $p_i$ | (6) Log $p_i$ | (7) Log $p_i$ | (8) Log $p_i$ | (9) Log $p_i$ | (10) Log $p_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Log $n_i$ | -0.87*** | -0.73*** | -0.87*** | -0.72*** | -0.80*** | -0.43*** | -0.79*** | -0.38*** | -0.81*** | -0.39*** |
| | (0.02) | (0.02) | (0.02) | (0.02) | (0.02) | (0.03) | (0.02) | (0.03) | (0.02) | (0.02) |
| PSA | 2.25*** | 1.74*** | 2.14*** | 1.57*** | 2.10*** | 1.08*** | 1.99*** | 0.85*** | 1.99*** | 0.83*** |
| | (0.05) | (0.05) | (0.05) | (0.05) | (0.08) | (0.07) | (0.08) | (0.07) | (0.06) | (0.04) |
| Grade | | 0.25*** | | 0.27*** | | 0.40*** | | 0.44*** | | 0.45*** |
| | | (0.01) | | (0.01) | | (0.01) | | (0.01) | | (0.01) |
| Year | | | 0.12*** | 0.13*** | | | 0.11*** | 0.14*** | | |
| | | | (0.01) | (0.00) | | | (0.01) | (0.01) | | |
| Year FE | | | | | | | | | Y | Y |
| Card FE | | | | | Y | Y | Y | Y | Y | Y |
| $N$ | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 | 4264 |
| adj. $R^2$ | 0.44 | 0.52 | 0.49 | 0.58 | 0.65 | 0.82 | 0.70 | 0.89 | 0.72 | 0.91 |

Robust Standard errors in parentheses; $^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table 5.** Regressions of log price against log print run with control variables for the Trout card. The table displays regressions for all graded card sales of the different versions of the Mike Trout 2009 Bowman Chrome refractor card in the Heritage or Goldin auctions database. The data ranges from February, 2017, through December, 2021. The print run is determined by the serial number on the back of the card. The control variables include the grade of the card (Grade), which ranges between 0 and 10, and the year in which the auction took place (Year). The robust standard errors are in parentheses. The regression equation is:

$$log(p_i) = \beta_0 + \beta_1 log(n_i) + \beta_2 Grade_i + \beta_3 Year_i + \epsilon_i, \quad i = 1, ..., 51.$$

| | (1)<br>Log $p_i$ | (2)<br>Log $p_i$ | (3)<br>Log $p_i$ | (4)<br>Log $p_i$ |
|---|---|---|---|---|
| Log $n_i$ | -0.67*** | -0.71*** | -0.71*** | -0.75*** |
| | (0.06) | (0.06) | (0.05) | (0.05) |
| Grade | | 0.76*** | | 0.86*** |
| | | (0.21) | | (0.18) |
| Year | | | 0.30*** | 0.33*** |
| | | | (0.07) | (0.08) |
| $N$ | 51 | 51 | 51 | 51 |
| adj. $R^2$ | 0.77 | 0.81 | 0.84 | 0.90 |

Robust standard errors in parentheses; *** $p < 0.01$

### 4.4. *Intercept versus Slope*

Corollaries 2 and 3 of the RDU model in Section 2 predict a negative relationship between the estimated time series of slope and intercept from the regression of log price on log rank. This subsection presents the results from testing this prediction. We estimate the slope and intercept each year by regressing log price on log rank while controlling for the grading company. In particular, for each $t$ from 2005 to 2021, we run the following regression:

$$log(p_{i,t}) = \beta_{0,t} + \beta_{1,t} log(r_{i,t}) + \beta_{2,t} PSA_{i,t} + \epsilon_{i,t}, \ i = 1, ..., N_t. \tag{6}$$

$N_t$ denotes the number of auctions in year $t$. From the regressions, we obtain an estimate of the intercept, $\hat{\beta}_{0,t}$, and an estimate of the slope, $\hat{\beta}_{1,t}$ for each year. We then run the regression:

$$\hat{\beta}_{0,t} = \alpha_0 + \alpha_1 \hat{\beta}_{1,t} + \epsilon_t, \quad t = 2005, ..., 2021. \tag{7}$$

The $R^2$ from regressing the estimated intercept on the estimated slope in Equation (7) is 91%, and the estimated $\alpha_1$ is about -8. Figure 3 shows the intercept and slope data points for each year used in regression Equation (7).[30] Allowing for small fluctuations in the slope from year-to-year, Figure 3 reveals a clear negative relationship between the estimated intercept and estimated slope as predicted by the theory. Since regression Equation (6) contains no card fixed effects, the intercept $\hat{\beta}_{0,t}$ can be interpreted as the average price (across all cards) of the rarest vintage cards for each year, which we later refer to as the price of rarity.

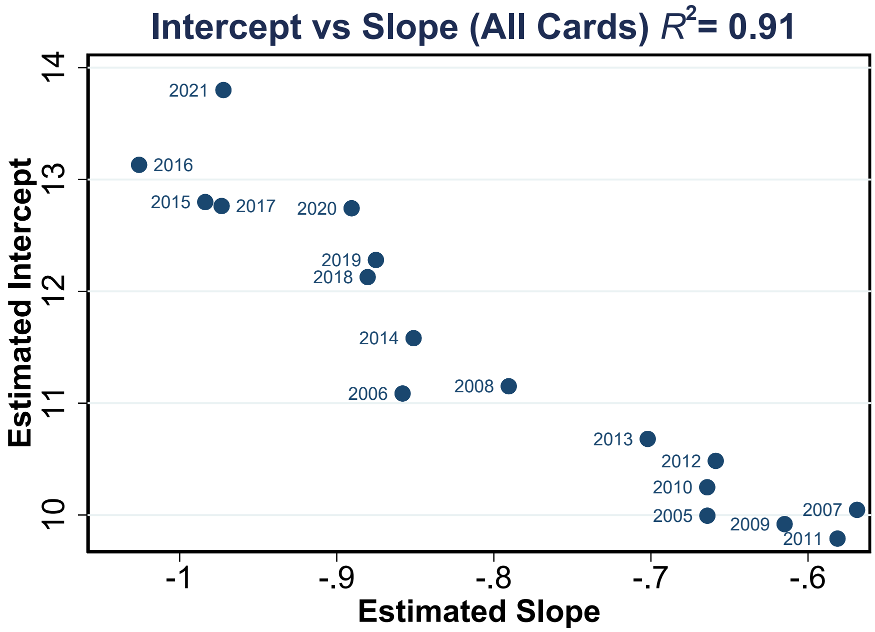
As a robustness check, we also estimated (6) using regressions of log price on log rank, PSA, and card fixed effects, using the Mickey Mantle 1956 Topps card as the reference card (i.e., adding dummy control variables in (6) for all other 20 vintage cards). We recorded the intercept and slope for each year and plotted the result in the bottom panel of Figure 3. We estimated $\alpha_1$ similar as before, using Equation (7). The results are shown graphically in Figure 3 and are tabulated in Table 8, for Equation (6) and in Table 9, for Equation (7) in the appendix. From the figure and tables it is clear that the results are very robust.[31]

Unfortunately, the 51 observations of the Trout card only span 2017-2021, and 2017-2018 contain only two observations each, so a similar time series analysis with our Trout card data is not feasible.

Before we conclude this section, we want to point out that the regression Equation (7) contained independent variables that were previously estimated, i.e., annually estimated slopes and intercepts in (6). In general, when one uses estimates instead of data in a regression, the standard errors of those estimates have be accounted for. For example, one can bootstrap year $t$ regression Equation (6) to create 10,000 slope and intercept coefficients for each year, and then use each set of annual coefficients in the regression Equation (7) to create 10,000 estimates for $\alpha_1$, using which one finds a significance level. This can be particularly important when one has barely significant coefficients. However, in our case, the coefficients are so significant and the standard errors are so small that it makes no difference whether we treat the annual slope and intercept as data or as estimates.

[30]In unreported regressions we also controlled for the year to account for a deterministic time trend, the risk-free rate to control for monetary policy, the ratio of government expenditures to GDP to control for fiscal policy, the NBER recession indicator, consumer price index inflation, and GDP growth to control for the macroeconomic environment, and the aggregate stock market return to control for the market condition. The only control variable that is significant is the year. The estimated coefficient $\alpha_1$ remains significant and is similar in magnitude when the control variables are included in the regression.

[31]The relationship in Figure 3 is robust to using any other vintage card as the reference card when adding the card fixed effect dummy controls.



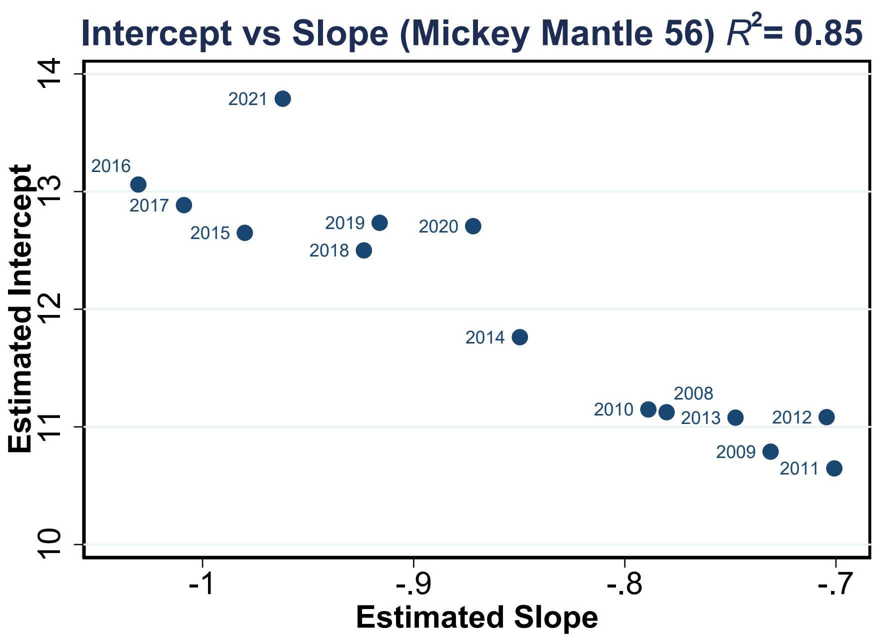




**Figure 3.** The top panel of the figure plots the estimated intercept against the estimated slope for each year (2005–2021), with the estimates from regressions of log price on log rank, while controlling for the grading company (PSA). The bottom panel plots the estimated intercept versus estimated slope across time (2008–2021) when the fixed effects are also included in the annual regressions and the reference card is the Mickey Mantle 1956 Topps card. There are not enough data for the bottom panel prior to 2008 for an annual estimate.

### 4.5. *Stock Market Sentiment and the Price of Rarity*

The intercept, $\beta_{0,t}$, in Equation (6) is the (log) price of rarity, in that it is the average (log) price of the rarest vintage cards traded in year $t$. Having estimated $\beta_{0,t}$, we briefly explore the connection between excessive stock market activity and the price of rarity. There are periods in the stock market when excessive demand causes the stock prices to soar, such as when the price-earnings ratio gets too high or when market sentiment indexes indicate an over-priced market. As the stock prices get too high, other investment opportunities, such as art and memorabilia, will seem more attractive to typical investors.[32] Thus, we expect to see a rise in the price of rarity immediately following periods of excessive stock market activity. To test this hypothesis, we use three well-established sentiment indicators: (i) the Shiller cyclically adjusted price-earnings ratio (CAPE); (ii) the Baker and Wurgler (2006) index of market sentiment (BW); (iii) the Michigan index of consumer sentiment (ICS).[33]

The sentiment index for a year is the average of monthly values over that year. As a fourth variable, we consider an index that directly measures the abnormal trading volume in the stock market (AVOL).[34] We standardize each index to have a mean of zero and a standard deviation of one for our sample period.

We run the following regression for each index, $X$, in the set {CAPE, BW, ICS, AVOL} to forecast the estimated intercept:

$$\hat{\beta}_{0,t} = \alpha_0 + \alpha_1 X_{t-1} + \alpha_2 Year_t + \epsilon_t, \quad t = 2005, ..., 2021. \tag{8}$$

Controlling for the year accounts for a possible trend in the intercept over time. As shown in Table 6, each of the sentiment indexes positively and significantly predicts the market value of rarity.

[32] For instance, amid high stock prices in 2021, CNN Business interviewed Ken Goldin of Goldin Auctions about the baseball card market. CNN reported "Goldin said for the first time in his career, he's fielding calls from hedge funds interested in gaining exposure." (`https://www.cnn.com/2021/02/12/investing/baseball-cards-markets/index.html`).

[33] The data for CAPE is available from Robert Shiller's website (`http://www.econ.yale.edu/~shiller/data.htm`). The Baker-Wurgler index is available from Jeffrey Wurgler's website (`https://pages.stern.nyu.edu/~jwurgler/`). The Michigan index is available at `https://data.sca.isr.umich.edu/data-archive/mine.php`. The BW index is constructed as the first principal component of five stock market sentiment proxies and it is intended to co-move with market anomalies related to investor sentiment. The proxies are (i) the value-weighted dividend premium; (ii) the first-day returns on IPO's; (iii) IPO volume; (iv) the closed-end fund discount; (v) equity share in new issues. For a detailed description of these variables, see Baker and Wurgler (2006).

[34] Abnormal trading volume, from Chen, Tang, Yao, and Zhou (2022), is constructed from the ratio of the end of month trading volume divided by the average monthly trading volume over the past year. The ratio is constructed for each stock and then averaged for the whole market to create the index.

For instance, Column (1) of the table indicates that a one-standard deviation increase in the Shiller price-earnings ratio predicts an immediate increase in the price of the rarest items by a factor of $e^{0.57} = 1.77$.

The positive relation between sentiment indexes and the intercept supports our prediction that the market value of rarity is higher following periods of higher market sentiment. Given the negative relation between the slope and intercept predicted by the theory in Section 2, we predict that analogous regressions with $\beta_{1,t}$ as the dependent variable will yield coefficients of the opposite sign, and about 8 times smaller in size.[35] We find this to be the case as shown in Columns (5)–(8) of Table 6. The regression summarized in Column (5) indicates that a one standard deviation increase in the CAPE predicts an increase in the elasticity by 0.07 (which makes the slope of the log-log plot of price versus rank steeper and closer to -1).

[35]The slope of the plot in Figure 3 is about -8, as one can see from Table 9 in the appendix.

**Table 6.** The table displays predictive regressions of the estimated intercept ($\hat{\beta}_{0,t}$) and slope ($\hat{\beta}_{1,t}$) from Equation (6). The predictors are three prominent sentiment indicators and one direct measure of abnormal stock market activity. Namely, the Shiller cyclically adjusted price-earnings ratio (CAPE), the Baker and Wurgler (2006) stock market sentiment index (BW), the Michigan index of consumer sentiment (ICS), and abnormal trading volume (AVOL) from Chen et al. (2022). Each predictor, lagged one year, is used to forecast the intercept in Columns (1)–(4) and is used to forecast the slope in Columns (5)–(8). All regressions in the table also control for the time trend. The data is annual, spanning the years 2005 through 2021, except for AVOL, which is available until 2017. The monthly indexes are converted to annual series by taking the average. For convenience in interpreting the coefficients, the independent variables are standardized to have a mean of zero and a standard deviation of one.

| | (1)<br>$\hat{\beta}_{0,t}$ | (2)<br>$\hat{\beta}_{0,t}$ | (3)<br>$\hat{\beta}_{0,t}$ | (4)<br>$\hat{\beta}_{0,t}$ | (5)<br>$\hat{\beta}_{1,t}$ | (6)<br>$\hat{\beta}_{1,t}$ | (7)<br>$\hat{\beta}_{1,t}$ | (8)<br>$\hat{\beta}_{1,t}$ |
|---|---|---|---|---|---|---|---|---|
| $\text{CAPE}_{t-1}$ | 0.57** | | | | -0.07* | | | |
| | (0.22) | | | | (0.04) | | | |
| $\text{BW}_{t-1}$ | | 0.87*** | | | | -0.10** | | |
| | | (0.27) | | | | (0.04) | | |
| $\text{ICS}_{t-1}$ | | | 0.44** | | | | -0.07** | |
| | | | (0.18) | | | | (0.03) | |
| $\text{AVOL}_{t-1}$ | | | | 0.47** | | | | -0.07** |
| | | | | (0.14) | | | | (0.02) |
| Trend (Year) | Y | Y | Y | Y | Y | Y | Y | Y |
| $N$ | 16 | 16 | 16 | 13 | 16 | 16 | 16 | 13 |
| adj. $R^2$ | 0.72 | 0.75 | 0.72 | 0.62 | 0.48 | 0.48 | 0.54 | 0.47 |

Robust Standard errors in parentheses; $^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$.

## 5. Discussion

This section contains further discussion of the results, as well as the limitations of our analysis.

### *5.1. Application to Zipf's Law*

The inverse relationship between size and rank is often referred to as Zipf's law (see Gabaix, 2009). For instance, Zipf's law for city sizes is the empirical finding that within a large geographical region, when cities are ranked by population (the largest ranked first) a city's population is approximately proportional to the inverse of its rank (e.g., the tenth largest city has approximately one tenth of the population of the largest city).

Our results show that for vintage cards, for which no new information about the players' performance enters the market, the relation between the price of a rare item and its rank is close to Zipf's law (with an estimated elasticity parameter between 0.82 and 0.98 across our ten regression specifications in Table 3). There are two points that we want to mention in this regard. First, from Corollary 1, Zipf's law emerges in a market when the limit of $\gamma$ goes to one. In other words, in a more efficient market with less biased agents, the price of rare items roughly follows Zipf's law.

Second, we see deviations from Zipf's law when the card belongs to an active player. The elasticity parameter for the card of the active player Mike Trout is between 0.6 and 0.67 from the regressions in Table 2. This seems to be related to how new information about the status of a player is incorporated into the price, as we do not see this in any of the players in our vintage cards (a quick glance at Figure 6 shows that all vintage cards have an elasticity parameter close to one for 2016 through 2019 consistently). Consistent with this finding, Lee (2022) reports that more popular NFT moments have a slightly lower elasticity parameter (0.72 versus 0.78). Of course, in his setting, the difference is small since all moments belong to active players. A possible explanation is that when the status of a player increases, the market size goes up, but disproportionately with biased agents. This has two effects, it lowers the elasticity parameter, but at the same time, it increases the price. Thus, for active players, we possibly lose the clear intercept versus slope result of Section 4 since that result hinges on the other parameters in the intercept remaining relatively constant, but the above-mentioned change in demand violates that condition.

### 5.2. *Application to Investment in Rare Items*

A potentially important insight from the model and the empirical analyses of determinants of slope and intercept in Section 4 pertains to investment in rare items. We established that for the vintage cards, as the theory predicted, there is a negative relation between the slope and the intercept of the log-log plot of price versus rank. Moreover, high sentiment periods (e.g., periods with a high CAPE or BW index) are immediately followed by high rarity prices (i.e., high intercepts). Thus, for an investor in rarity with a long horizon, it is best to buy when sentiment measures are low (and hence the prices are low) and wait until the sentiment measures are high. But importantly, the change in sentiment changes both the intercept and the slope of the log-log curve of price versus rank. As the intercept increases, the slope becomes steeper too. This immediately implies that the highest percent change in prices (i.e., the highest returns) belongs to the rarest items. In contrast, the clockwise rotation of the curve means that an investor who times the market (in terms of buying when sentiment is low and selling when sentiment is high) might even lose money in practice if the investor buys an item that is not very rare. Of course, this analysis might not hold for recent cards where many parameters of the model can change simultaneously.

### *5.3. Application to Product Design*

An implication of our analysis is that the rank of different versions of a product may itself be a source of value to consumers. One natural application of the market value for rarity and the methodology proposed here is in marketing and product design. Bowman Chrome randomly inserted the Trout cards into sealed packs of baseball cards. As such, they profited only from the expectation of what a Trout card could be worth, given the low probability of finding it in a pack.

In principle, a firm can benefit directly by designing a product line analogous to the Trout cards such that the products are otherwise close substitutes that differ in their rarity (e.g., limited editions). Similarly, products paired with a brand name or a celebrity could potentially capitalize on the market value for rarity by creating parallel versions of a product that have extreme rarity.

### *5.4. Limitations*

The model developed in Section 2 applies most directly to settings in which goods that are close substitutes differ primarily in their rarity. One limitation is that in many markets, items differ on multiple dimensions in addition to their rarity. For such cases, a more general framework in which rarity is only one determinant of a good's value could be developed.

A second limitation is that the model operates in a perfectly competitive market in which the distribution of rare items is taken as exogenous. While this seems plausible in many contexts, a complementary approach that tries to determine the optimal distribution of rarity could also analyze strategic factors that might affect the endogenous level of rarity along the lines of Moldovanu, Sela, and Shi (2008) on strategic supply.

In addition to strategic factors that might affect the supply of rare items, a third limitation is that the model in Section 2 does not account for strategic bidding in which bidders in auctions systematically deviate from bidding their true valuation for a rare item.[36] However, this limitation does not pose an issue for our results so long as there is only a positive correlation between RDU agents' valuations of rare items and their bids, which is a soft condition to assume. As an example, if the agents bid a constant fraction of their valuations, our power law relationships and analyses remain intact (up to changes in proportionality constants).

# 6. Conclusion

We showed theoretically that (i) a power law relationship for the market value of rarity can emerge in equilibrium in a market with risk-neutral expected utility agents; and (ii) to explain the observed power coefficient, a market with rank-dependent utility agents provides a better approximation. The latter case with RDU agents yields a theoretical insight that an item's rank (among close substitutes) serves as a natural measure of rarity.

We showed empirically that rank provides a better fit to the data than the measure of quantity used in previous studies. Our data reveals that the power law relationship holds across roughly five orders of magnitude, and that it explains 60% of the log price variation in our dataset of 4,264 auction records of vintage baseball cards even without card or year fixed effects. Our results provide evidence that the market value of rarity is remarkably robust as it follows a similar pattern across the different cards in our sample and across time.

From a broader perspective, our results provide a micro-foundation for a basic market-level phenomenon. Given the robustness of the market value for rarity, future research might investigate potential foundations for this finding from biology or evolutionary psychology. It is already known that collecting behavior dates back at least to Mesopotamia in the third millennium BCE (Thomason, 2017). Research has also found a neural basis for collecting behavior, suggesting it emerged from the hunter-gatherer lifestyle in which it increased the probability of survival (Anderson, Damasio, and Damasio, 2005). Further research along these lines might help to bridge the biological, psychological, and economic determinants of the market value of rarity.

[36] See Karni and Safra (1987) and Segal (1988), which study the relationship between non-expected utility preferences and incentive compatibility.

# Appendix A. Sources of Demand for Rarity

An important assumption in deriving the relative value of rarity is that there exists an aggregate demand for rare items, that is, a fraction of aggregate income is spent on acquiring rare items.

**Preference for Complete Sets:** One possible reason for why there is a demand for rare items at all is that there are many individuals that as a hobby engage in collecting rare items such as baseball cards, and these individuals strive to complete their collection of some "full set," which in turn creates aggregate demand for individual cards in those sets.[37] Since it is common knowledge among collectors that other collectors also demand complete sets, the value of the rarest cards increases exponentially as a large number of collectors need those cards to complete their sets.

The demand for completing a set is evident in the following account of the demand for the 1952 Topps Andy Pafko card, who was not considered a star player unlike other players with such expensive cards, but happened to be a rare card in a "perfect set."

***The Andy Pafko card and the 1952 Topps Set*:** In addition to valuing star players, some collectors also value cards that help them complete a set. A classic example is the Andy Pafko 1952 Topps card in our dataset. Pafko is the only player in our vintage dataset who is not in the baseball hall of fame.

The Augusta Chronicle notes in a 2002 article (Chronicle, 2002), "According to Topps, the first card in the 1952 set, Andy Pafko of the Brooklyn Dodgers, is one of the hardest to find because collectors sorted their cards by number and wrapped rubber bands around their stacks. The result was a glut of damaged Andy Pafko cards."

Newsday (Cohen, 1998) attempts to give an economics lesson in their 1998 article on the Pafko card. The article states: "Ok, Class. An Andy Pafko baseball card sells for more than a house and you want to know why?" The article notes, "1952 Topps is considered by many collectors to be the ultimate modern-era baseball card set. Pafko is No. 1 in that set," and the article adds that most surviving 1952 Topps Pafko cards are in poor condition. Reporting that a PSA 10 Pafko card sold for \$83,870, the article comments, "For the buyer, it was the perfect beginning to a perfect set."

[37] One can easily find references to "ultimate set," "perfect set," and other such phrases in the baseball card literature.

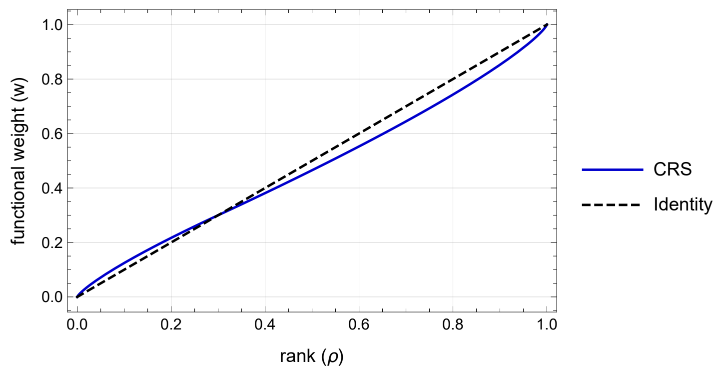
***Sets of hall of fame players*:** A preference for complete sets may be relevant for cards beyond the Andy Pafko card. Collectors may have their own target sets they collect (such as the set of Mickey Mantle cards or the set of the first Topps cards of hall of fame players). Cards that are rarer or in more collectors' target sets will then command higher prices as more collectors require these cards to obtain their maximum utility from completing their target sets.

***Sets of rare coins and music records*:** Other studies also identify completing a set as an important motivation for collecting. A study of rare U.S. coins by Dickie et al. (1994) identifies collecting a "complete set of vintages and mints" of a given type of coin as an important motive for a broad class of coin collectors. In the study of rare vinyl music records, Cameron and Sonnabend (2020) also note that "set completion can lead to escalating willingness to pay by an individual collector."

**Preference for Status:** Demand for rare items might also emerge from a more basic preference for status. A preference for status has previously been incorporated into economic theory by Bernheim (1994). In the context of collecting, a preference for status might create a desire to collect items of high-status individuals (such as star baseball players), as a means of increasing one's own status as perceived by others. Rarer items (of high-status individuals) might provide a means of signaling higher status for one's self, to the extent that status provides a means of ranking one's self above others and rarer items are naturally ranked higher than less rare items (among items that are otherwise close substitutes).

## Appendix B. Supplementary Figures

**Figure 4.** An example of a CRS probability weighting function.

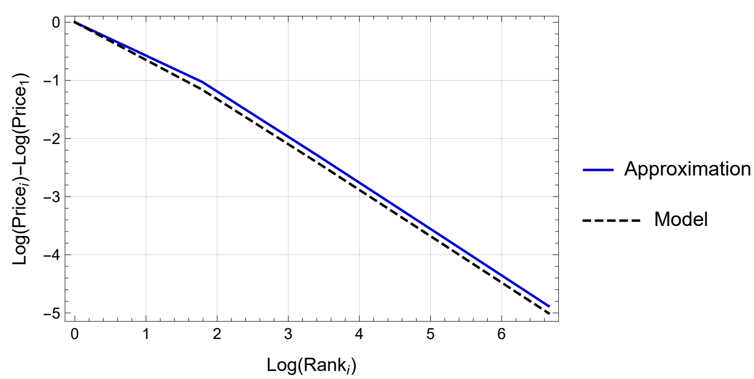



Note: The plot depicts a constant relative sensitivity (CRS) probability weighting function:

$$W(\rho) = \begin{cases} \delta^{1-\gamma}\rho^{\gamma}, & \text{if } 0 \leq \rho \leq \delta \\ 1-(1-\delta)^{1-\gamma}(1-\rho)^{\gamma}, & \text{if } \delta < \rho \leq 1, \end{cases}$$

with parameters $\gamma = 0.80$, and $\delta = 0.30$. The higher slope (compared to the identity) where $\rho$ is close to zero shows that small incremental probabilities are exaggerated by the rank-dependent utility (RDU) agent.

**Figure 5.** RDU model fair relative price of rare objects and its approximation.




Note: The plot depicts log of relative prices implied by the RDU model $log(p_i) - log(p_1)$, and its approximation:

$$\frac{p_i}{p_1} = \frac{w_1}{w_i} = \frac{W(\rho_1)}{W(\rho_i) - W(\rho_{i-1})} = \frac{\rho_1^{\gamma}}{\rho_i^{\gamma} - \rho_{i-1}^{\gamma}} \approx \frac{\rho_1^{\gamma}}{\gamma \rho_i^{\gamma-1}\pi_i} = \frac{(N\rho_1)^{\gamma}}{\gamma}\frac{r_i^{1-\gamma}}{n_i} = \frac{n_1^{\gamma}}{\gamma}\frac{r_i^{1-\gamma}}{n_i}, \text{ for } i > 1.$$

with parameters $\gamma = 0.80$, $N = 10,000$, and $n_i = 5^i$ for $i = 0, ..., 4$. Even with these conservative parameters, the approximation seems to work well, and an econometrician who uses the approximation should recover the slope and intercept almost identical to the true values.

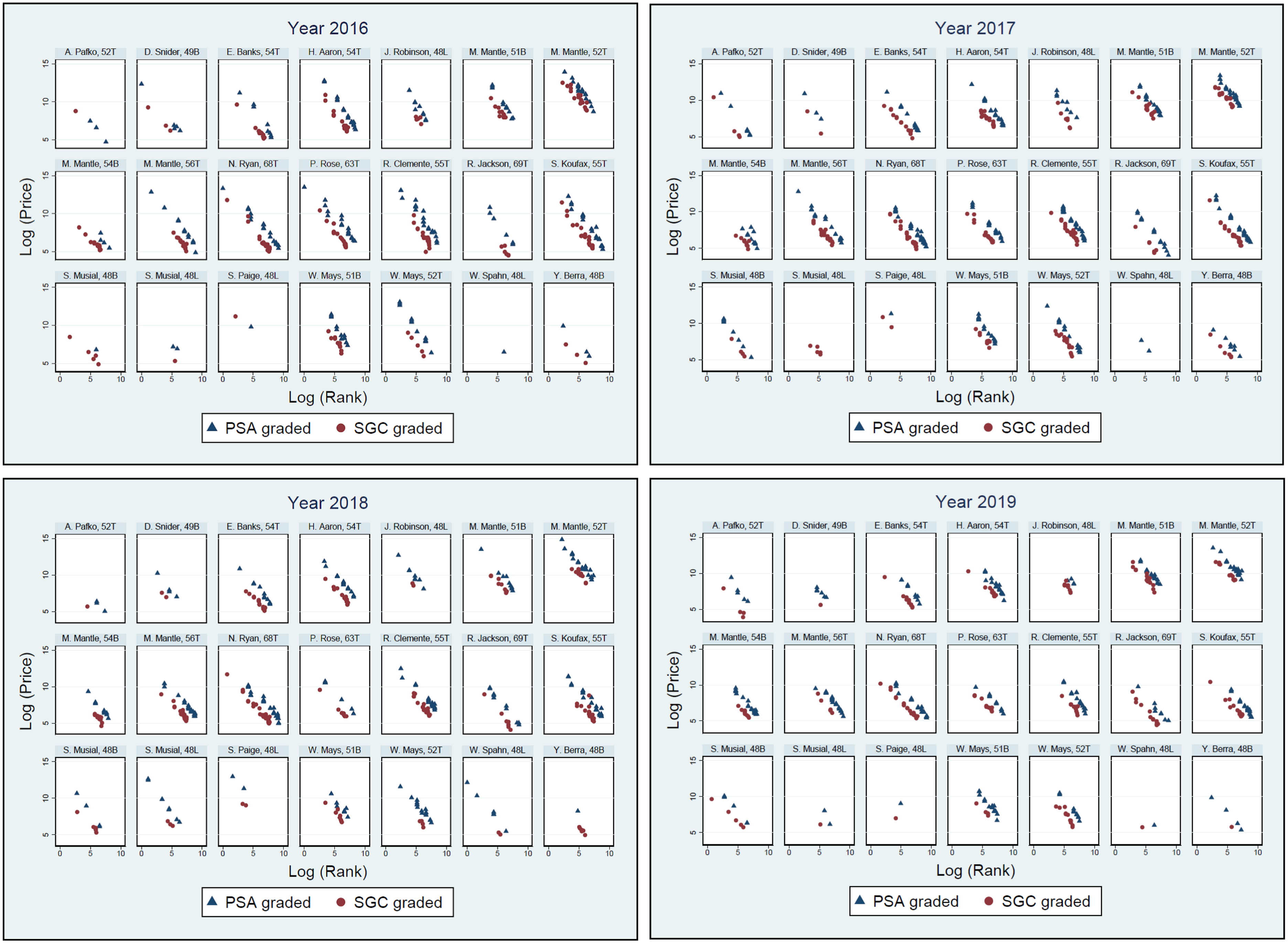



**Figure 6.** Plot of log price versus log rank for each of the 21 vintage cards in the sample sold in 2016, 2017, 2018, and 2019. T = Topps; B = Bowman; L = Leaf. Number denotes year issued (1948-1969).

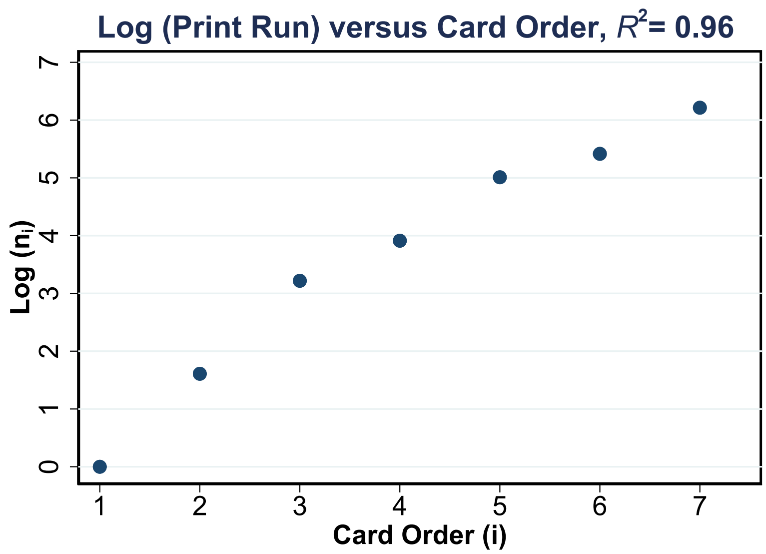



**Figure 7.** The figure plots the log print run of the seven versions of the Mike Trout 2009 Bowman Chrome Refractor card against the order of the cards (with the different cards ordered from lowest print run to highest print run). The reported $R^2$ in the title belongs to the regression $log(n_i) = a_0 + a_1 i + \epsilon_i$, for $i = 1, ..., 7$.

# Appendix C. Supplementary Tables

**Table 7.** The table displays the adjusted $R^2$ from three regressions of log quantity against card order. The regression in row (1) is for the Trout card. Quantity, $n_i$ is the print run of Trout card with order $i$ (where cards are ordered 1 through 7 from the card with the lowest print run to the card with the highest print run). Unlike the Trout card, $n_i$ for the vintage cards are created using the grading companies' population reports. The regression in row (2) is for the vintage cards graded by PSA. Quantity, $n^{psa}_{j,g}$, is the quantity of card $j$ receiving grade $g$ by PSA. The regression in row (3) is for the vintage cards graded by SGC. Quantity, $n^{sgc}_{j,g}$, is the quantity of card $j$ receiving grade $g$ by SGC.

| | Data | Regression Equation | adj. $R^2$ |
|---|---|---|---|
| (1) | Trout Card | $log(n_i) = a_0 + a_1 i + \epsilon_i, \quad i = 1, ..., 7.$ | 0.95 |
| (2) | Vintage (PSA) | $log(n^{psa}_{j,g}) = a_{0,j} + a_{1,j}\, g + \epsilon_{g,j}, \quad g = 0, 1, ..., 10, \quad j = 1, ..., 21.$ | 0.30 |
| (3) | Vintage (SGC) | $log(n^{sgc}_{j,g}) = a_{0,j} + a_{1,j}\, g + \epsilon_{g,j}, \quad g = 0, 1, ..., 10, \quad j = 1, ..., 21.$ | 0.39 |

**Table 8.** The table displays the intercept and slope coefficients from regression Equation (6) for all vintage cards in the data set (the three left columns) and the estimates for the Mickey Mantle 1956 Topps card (the three right columns). For all vintage cards, the coefficients are estimated each year from 2005 through 2021. For the Mickey Mantle 1956 Topps card, there are only a small number of transactions in our data for the years before 2008. Consequently, the coefficients are estimated for this card beginning in 2008. The regression equation from which the coefficients are estimated is reproduced below. In the regression, $log(p_{i,t})$ is the log of the price of card $i$ in period $t$, $log(r_{i,t})$ is the lof of the rank of card $i$ in period $t$, and $PSA_{i,t}$ is an indicator variable that equals one if the card was graded by PSA and that equals zero otherwise. The coefficients for the regression with the Mantle 1956 Topps card are denoted in the table with an $m$ superscript to distinguish them from the coefficients for the regression with all vintage cards.

$$log(p_{i,t}) = \beta_{0,t} + \beta_{1,t}log(r_{i,t}) + \beta_{2,t}PSA_{i,t} + \epsilon_{i,t},\ i = 1, ..., N_t;\ t = 2005, ..., 2021.$$

| | All Vintage Cards | | | Mantle 1956 Topps | | |
|---|---|---|---|---|---|---|
| Year | $\hat{\beta}_{0,t}$ | $\hat{\beta}_{1,t}$ | $\hat{\beta}_{2,t}$ | $\hat{\beta}^m_{0,t}$ | $\hat{\beta}^m_{1,t}$ | $\hat{\beta}^m_{2,t}$ |
| 2005 | 9.99 | -0.66 | 1.14 | | | |
| 2006 | 11.09 | -0.86 | 1.73 | | | |
| 2007 | 10.05 | -0.57 | -0.04 | | | |
| 2008 | 11.15 | -0.79 | 1.12 | 11.12 | -0.78 | 1.11 |
| 2009 | 9.92 | -0.61 | 1.66 | 10.79 | -0.73 | 1.18 |
| 2010 | 10.25 | -0.66 | 1.39 | 11.15 | -0.79 | 1.27 |
| 2011 | 9.79 | -0.58 | 0.94 | 10.65 | -0.70 | 1.17 |
| 2012 | 10.48 | -0.66 | 1.03 | 11.08 | -0.70 | 1.13 |
| 2013 | 10.68 | -0.70 | 1.28 | 11.08 | -0.75 | 1.34 |
| 2014 | 11.58 | -0.85 | 1.48 | 11.76 | -0.85 | 1.58 |
| 2015 | 12.80 | -0.98 | 1.87 | 12.65 | -0.98 | 1.76 |
| 2016 | 13.13 | -1.03 | 1.91 | 13.06 | -1.03 | 1.85 |
| 2017 | 12.76 | -0.97 | 1.75 | 12.88 | -1.01 | 1.74 |
| 2018 | 12.13 | -0.88 | 1.75 | 12.50 | -0.92 | 1.71 |
| 2019 | 12.28 | -0.88 | 1.72 | 12.73 | -0.92 | 1.64 |
| 2020 | 12.74 | -0.89 | 1.57 | 12.71 | -0.87 | 1.57 |
| 2021 | 13.80 | -0.97 | 1.82 | 13.79 | -0.96 | 1.70 |

**Table 9.** The table displays the intercept and slope coefficients from regression Equation (7) for all vintage cards in the data set (left column) and the estimates for the Mickey Mantle 1956 Topps card (right columns). The regression equation is reproduced below:

$$\hat{\beta}_{0,t} = \alpha_0 + \alpha_1 \hat{\beta}_{1,t} + \epsilon_t, \quad t = 2005, ..., 2021.$$

| | All Cards $\hat{\beta}_{0,t}$ | Mantle 1956T $\hat{\beta}_{0,t}$ |
|---|---|---|
| $\alpha_1$ | -8.11*** | -8.09*** |
| | (0.64) | (0.98) |
| $\alpha_0$ | 4.98*** | 5.06*** |
| | (0.47) | (0.84) |
| $N$ | 17 | 14 |
| $R^2$ | 0.91 | 0.85 |

Robust standard errors in parentheses.
$^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$.

# Appendix D. Data Sources

This Appendix contains the sources of data used in the paper.

1. **List of Most Valuable Baseball Cards:** The link below (accessed September 2021) provides a list of the most valuable baseball cards. Our data set includes all cards on the list in the modern era of baseball cards (post-WWII): `https://www.stadiumtalk.com/s/most-expensive-baseball-cards-985687df1bbe45c5`.

2. **Population of PSA graded cards:** Below is the link to the database of the entire population of cards graded by PSA: `https://www.psacard.com/pop/baseball-cards/20003`.

3. **Population of SGC graded cards:** Below is the link to the database of the entire population of cards graded by SGC: `https://gosgc.com/pop-report`.

4. **Link to Baseball Card Data Set:** We will provide a link to our data set of the vintage baseball cards and the Mike Trout card to facilitate further research in this area. The data was collected and is freely available from the websites of the two major auction houses for graded baseball cards: Heritage Auctions (`https://www.HA.com`), and Goldin auctions (`https://www.GoldinAuctions.com`). (Heritage requires free user registration).

5. **CAPE:** The data for the cyclically adjusted price-earnings ratio (CAPE) is available from Robert Shiller's website at `http://www.econ.yale.edu/~shiller/data.htm`. The index is monthly. We converted the monthly index to an annual series by taking the average over months. The data spans our full sample period from 2005-2021.

6. **BW:** The Baker and Wurgler (2006) index of market sentiment is available from Jeffrey Wurgler's website at `https://pages.stern.nyu.edu/~jwurgler/`. The index is monthly. We converted the monthly index to an annual series by taking the average over months. The data spans our full sample period from 2005-2021.

7. **Michigan ICS:** The Michigan Index of Consumer Sentiment is available at `https://data.sca.isr.umich.edu/data-archive/mine.php`. The index is monthly. We converted the monthly index to an annual series by taking the average over months. The data spans our full sample period from 2005-2021.

8. **Abnormal Stock Market Trading Volume:** This data was used in the paper from Chen et al. (2022). It was provided to us directly by the authors of that paper. The index is monthly. We converted the monthly index to an annual series by taking the average over months. The data overlaps with our sample period for the years 2005-2017.

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