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Choice of Functional Form for Hedonic Price Equations[1]

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The appropriate functional form for a hedonic price equation cannot in general be specified on theoretical grounds. In this paper, a statistical procedure for the choice of functional form is proposed. A highly general functional form is specified that yields all other functional forms of interest as special cases. Likelihood ratio tests are used to test the appropriateness of alternative forms. The procedure is illustrated using cross section microdata for housing. For the case considered, the functional forms most commonly used in previous studies are strongly rejected.

I. INTRODUCTION

A hedonic price equation is a reduced-form equation reflecting both supply and demand influences. Therefore, the appropriate functional form for the hedonic equation cannot in general be specified on theoretical grounds.[2] The lack of a firm theoretical basis for the choice of functional form is unfortunate since the results obtained using the hedonic approach often depend critically on the functional form used.

In practice, the choice of functional form has usually been based mainly on considerations of convenience in dealing with the problem at hand. While reference has sometimes been made to experimentation with respect

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[2]See Rosen [33].

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to the goodness of fit of alternative forms, the comparison of goodness of fit has generally not been embedded in a relevant statistical framework. Also, the range of forms considered has been limited to forms that place severe restrictions on the underlying demand and supply functions, including nonjointness in both consumption and production.[3]

The failure to use a relevant statistical framework in choosing among functional forms is surprising, since Griliches [17] early drew attention to an article by Box and Cox [6] that provided an appropriate methodology. The Box–Cox methodology has received considerable use in other applications,[4] but has been largely ignored in hedonic studies. Goodman [14] and Linneman [29] provide important recent exceptions but continue the tradition of considering only forms that impose highly restrictive assumptions on the underlying demand and supply functions.

One way to avoid the imposition of theoretically unwarranted restrictions is to use a flexible functional form.[5] This approach has been used extensively in other applications,[6] but has been ignored in studies employing hedonic price equations. While the use of flexible functional forms would eliminate the problem of using a form that is overly restrictive, the question of which flexible form would remain, as would the question of which, if any, restrictions were warranted.

In this paper, we propose a procedure for choice of functional form for hedonic price equations that combines the best features of the Box–Cox and flexible functional form approaches.[7] A general functional form is specified that yields all other functional forms of interest as special cases. Likelihood ratio tests are then used to test the appropriateness of alternative functional forms for the hedonic equation. The application of this procedure

[3] The principal functional forms used have been the linear, Kain and Quigley [21], King [26]; log-linear, Kravis and Lipsey [27]; and semilog, Griliches [16], Kain and Quigley [21], Ohta and Griliches [30], and Triplett [34]. In some cases, the functional form used has been modified by the limited use of interaction terms among the explanatory variables. See, for example, Grether and Mieszkowski [15].

[4] See Heckman and Polachek [19], White [35], and Zarembka [36, 37]. Kau and Lee [22, 23] have examined the functional form of urban density gradients, and Kau and Sirmans [24] have examined land value functions.

[5] A flexible functional form provides a second order approximation to an arbitrary twice differentiable functional form; see Diewert [10] and Lau [28]. Examples of flexible forms are the translog, quadratic, generalized square root quadratic, square root quadratic, and generalized Leontief.

[6] See, for example, Atkinson and Halvorsen [2], Berndt and Wood [5], Christensen, Jorgenson, and Lau [7, 8], Diewert [9, 10, 11], Glandon and Pollakowski [12, 13], Halvorsen [18], and Lau [28].

[7] A similar, but somewhat more restrictive, approach has been used in different contexts by Appelbaum [1], Berndt and Khaled [4], and Kiefer [25]. See Footnote 10.

is illustrated by the estimation of a hedonic price equation for housing with cross section microdata.

II. THE MODEL

The general functional form that incorporates all other functional forms of interest as special cases is:

$$P^{(\theta)} = \alpha_0 + \sum_{i=1}^{m} \alpha_i Z_i^{(\lambda)} + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_{ij} Z_i^{(\lambda)} Z_j^{(\lambda)}, \tag{1}$$

where P is price, the Z_i are attributes, $\gamma_{ij} = \gamma_{ji}$,[8] and $P^{(\theta)}$ and $Z_i^{(\lambda)}$ are Box–Cox transformations,[9]

$$\begin{aligned} P^{(\theta)} &= (P^{\theta} - 1)/\theta, && \theta \neq 0, \\ &= \ln P, && \theta = 0, \\ Z_i^{(\lambda)} &= (Z_i^{\lambda} - 1)/\lambda, && \lambda \neq 0, \\ &= \ln Z_i, && \lambda = 0. \end{aligned}$$

The transformations are continuous around $\theta = 0$ and $\lambda = 0$ since the limit for the $\theta \neq 0$ case as $\theta \to 0$ is $\ln P$ and the limit for the $\lambda \neq 0$ case as $\lambda \to 0$ is $\ln Z_i$. Equation (1) will be referred to as the quadratic Box–Cox functional form.[10]

The translog form, Christensen, Jorgenson, and Lau [7, 8],

$$\ln P = \alpha_0 + \sum_i \alpha_i \ln Z_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln Z_i \ln Z_j,$$

is obtained from the quadratic Box–Cox form by imposing the restrictions

[8] The restrictions $\gamma_{ij} = \gamma_{ji}$ are imposed for purposes of identification only and place no effective constraints on the generality of the form. If the restrictions were not imposed, each coefficient γ_{ij} would be replaced by $1/2(\gamma_{ij} + \gamma_{ji})$ and the γ_{ij} and γ_{ji} could not be separately identified.

[9] A still more general functional form could be obtained by allowing $\lambda_i \neq \lambda_j$. However, since all of the usual flexible functional forms impose the restriction $\lambda_i = \lambda_j = \lambda$, this generalization does not appear to be worth the considerable increase in computational cost that it would involve.

[10] We know of no studies using a form similar to the quadratic Box–Cox to analyze the choice of functional form for hedonic price equations. Kiefer [25] uses a similar form to analyze the choice of functional form for indirect utility functions, but imposes the restriction $\theta = \lambda$. Berndt and Khaled [4] use a similar form for cost functions, but impose the restriction $\theta = 2\lambda$. Appelbaum [1] uses Box–Cox transformations of the explanatory variables in choosing between quadratic forms for production functions, but does not transform the dependent variable.

$\theta = 0$ and $\lambda = 0$. The log-linear form is obtained as a special case of the translog form by imposing the further restrictions $\gamma_{ij} = 0$, all i, j.

To facilitate the derivation of other functional forms, (1) is rewritten:

$$P = \left\{1 + \theta\left[\alpha_0 + \sum_i \alpha_i\left(\frac{Z_i^\lambda - 1}{\lambda}\right)\right.\right.$$

$$\left.\left. + \frac{1}{2}\sum_i \sum_j \gamma_{ij}\left(\frac{Z_i^\lambda - 1}{\lambda}\right)\left(\frac{Z_j^\lambda - 1}{\lambda}\right)\right]\right\}^{1/\theta}. \tag{3}$$

Imposing the restrictions $\theta = \lambda = 1$ in (3) yields the quadratic form, Lau [28],

$$P = c_0 + \sum_i c_i Z_i + \sum_i \sum_j \gamma_{ij} Z_i Z_j, \tag{4}$$

where

$$c_0 = 1 + \alpha_0 - \sum_i \left(\alpha_i - \frac{1}{2}\sum_j \gamma_{ij}\right), c_i = \alpha_i - \sum_j \gamma_{ij}.$$

The linear form can be obtained from the quadratic form by imposing the restrictions $\gamma_{ij} = 0$, all i, j, in (4).

The generalized square root quadratic form [11] is obtained by imposing the restrictions $\theta = 2$, $\lambda = 1$ in (3),

$$P = \left(b_0 + \sum_i b_i Z_i + \sum_i \sum_j \gamma_{ij} Z_i Z_j\right)^{1/2}, \tag{5}$$

where

$$b_0 = 1 + 2\alpha_0 - \sum_i \left(2\alpha_i - \sum_j \gamma_{ij}\right), b_i = 2\left(\alpha_i - \sum_j \gamma_{ij}\right).$$

[11] The generalized square root quadratic form is a generalization of the square root quadratic form introduced by Diewert [11].

The square root quadratic form, Diewert [11],

$$P = \left(\sum_i \sum_j \gamma_{ij} Z_i Z_j \right)^{1/2} \tag{6}$$

is obtained by imposing the restrictions $b_0 = b_i = 0$ in (5).[12]

A generalized nonhomogeneous version of the generalized Leontief form, Diewert [9], is obtained by imposing the restrictions $\theta = 1$, $\lambda = \frac{1}{2}$ in (3),

$$P = a_0 + \sum_i a_i Z_i^{1/2} + 2 \sum_i \sum_j \gamma_{ij} Z_i^{1/2} Z_j^{1/2}, \tag{7}$$

where

$$a_0 = 1 + \alpha_0 - 2 \sum_i \left(\alpha_i - \sum_j \gamma_{ij} \right), a_i = 2 \left(\alpha_i - 2 \sum_j \gamma_{ij} \right).$$

A linear homogeneous version of the generalized Leontief form,

$$P = 2 \sum_i \sum_j \gamma_{ij} Z_i^{1/2} Z_j^{1/2} \tag{8}$$

is obtained by imposing the restrictions $a_0 = a_i = 0$ in (7).[13] The linear form can be obtained from (7) by imposing the restrictions $a_i = 0$ and $\gamma_{ij} = 0$, $i \neq j$.

A functional form frequently used in previous studies of hedonic price equations is the semilog form,

$$\ln P = d_0 + \sum_i d_i Z_i .$$

The semilog form is obtained as a special case of the quadratic Box–Cox functional form by imposing the restrictions $\theta = 0$, $\lambda = 1$, and $\gamma_{ij} = 0$, all i, j.

In order to test whether a particular functional form is appropriate, the restrictions corresponding to that functional form are tested using a likelihood ratio test. Estimation and testing procedures are discussed in the next section.

[12] The restrictions $b_0 = b_i = 0$ imply that $\alpha_i = \Sigma_j \gamma_{ij}$ and $\Sigma_i \alpha_i = 1 + 2\alpha_0$. The square root quadratic form is linear homogeneous.

[13] The restrictions $a_0 = a_i = 0$ imply that $\alpha_i = 2\Sigma_j \gamma_{ij}$ and $\Sigma_i \alpha_i = 1 + \alpha_0$.

III. ESTIMATION PROCEDURES[14]

Including a stochastic disturbance term in (1), the equation to be estimated is

$$P_k^{(\theta)} = \alpha_0 + \sum_{i=1}^{m} \alpha_i Z_{ki}^{(\lambda)} + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_{ij} Z_{ki}^{(\lambda)} Z_{kj}^{(\lambda)} + \epsilon_k, \tag{9}$$

where $P_k^{(\theta)}$ is the kth observation on the transformed price variable; $Z_{ki}^{(\lambda)}$ is the kth observation on the transformed attribute i; θ, λ, α_0, α_i, and γ_{ij} are unknown parameters; and ϵ_k is the disturbance term. It is assumed that for the true functional form (i.e., the true θ and λ) the disturbance term is normally and independently distributed with zero mean and constant variance.

Under the assumption of normality, the probability density function for the *transformed* dependent variable may be written as,

$$f\left(P_k^{(\theta)}\right) = \left(2\pi\sigma^2\right)^{1/2} \exp\left[-\frac{\left(\alpha_0 + \sum_i \alpha_i Z_{ki}^{(\lambda)} + \frac{1}{2}\sum_i \sum_j \gamma_{ij} Z_i^{(\lambda)} Z_j^{(\lambda)}\right)^2}{2\sigma^2}\right]. \tag{10}$$

The probability density function for the *untransformed* dependent variable is,

$$f(P_k) = f\left(P_k^{(\theta)}\right) J, \tag{11}$$

where J is the Jacobian of the inverse transformation from the transformed dependent variable to the actually observed dependent variable,

$$J = \left| \frac{dP_k^{(\theta)}}{dP_k} \right| = P_k^{\theta} - 1. \tag{12}$$

The likelihood function for a sample of n observations on the untransformed dependent variable is the product of the density for each observation. Maximizing this function or its logarithm yields estimates of θ, λ, α_0, α_i, and γ_{ij}.

[14]This section is based on Box and Cox [6].

Ordinary least-squares regression programs can be used to obtain the maximum likelihood estimates. Note that, for a given θ and λ, (9) is linear in the parameters α_0, α_i, and γ_{ij}. By a suitable redefinition of the variables, the equation can be rewritten in matrix form as

$$P^{(\theta)} = X\beta + \epsilon, \tag{13}$$

where X is the matrix

$$\begin{bmatrix} 1 & Z_{11}^{(\lambda)} & \dots & Z_{1m}^{(\lambda)} & Z_{11}^{(\lambda)} \cdot Z_{11}^{(\lambda)} & \dots & Z_{1m}^{(\lambda)} \cdot Z_{1m}^{(\lambda)} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 1 & Z_{n1}^{(\lambda)} & \dots & Z_{nm}^{(\lambda)} & Z_{n1}^{(\lambda)} \cdot Z_{n1}^{(\lambda)} & \dots & Z_{nm}^{(\lambda)} \cdot Z_{nm}^{(\lambda)} \end{bmatrix}$$

and β is the column vector $[\alpha_0\ \alpha_1 \dots \alpha_m\ \gamma_{11} \dots \gamma_{mm}]$.

The likelihood function for a sample of n observations on the untransformed dependent variable may then be written as

$$(2\pi\sigma^2)^{-n/2} \exp\left[\frac{(P^{(\theta)} - X\beta)'(P^{(\theta)} - X\beta)}{2\sigma^2}\right] \prod_{k=1}^{n} P_k^{\theta-1}. \tag{14}$$

For a given θ and λ, (14) is, except for a constant factor, the likelihood for a standard least-squares problem. Therefore, the maximum likelihood estimates of the β's are the ordinary least-squares estimates for the dependent variable $P^{(\theta)}$, and the estimate of σ^2 for a given θ and λ is

$$\hat{\sigma}^2(\theta,\lambda) = \frac{SSR}{n},$$

where SSR is the residual sum of squares.

Thus, for a fixed θ and λ, the maximized log likelihood is, except for a constant,

$$L_{\max}(\theta,\lambda) = -\frac{1}{2} n \ln \sigma^2(\theta,\lambda) + (\theta - 1) \sum_{k=1}^{n} \ln P_k, \tag{15}$$

where the second term is obtained from the Jacobian. To maximize over the entire parameter space it is necessary only to estimate (13) for alternative values for θ and λ and find the values of θ and λ for which (15) is maximized.

A test of whether a particular functional form is acceptable is performed by testing the null hypothesis that the parameters of the hedonic equation satisfy the relevant restrictions. The hypothesis tests are based on the large sample theory result that, under the null hypothesis, twice the difference in the logarithmic likelihood between a null and an alternative hypothesis is distributed as χ^2 with the number of degrees of freedom equal to the difference in the number of unrestricted parameters.

This result can also be used to form confidence regions around the estimates $(\hat{\theta}, \hat{\lambda})$ obtained using the unrestricted Box–Cox quadratic form. A 100 $(1 - \alpha)$ percent confidence region consists of all points (θ^*, λ^*) which satisfy the inequality

$$L_{\max}(\hat{\theta},\hat{\lambda}) - L_{\max}(\theta^*,\lambda^*) < \tfrac{1}{2}\chi_2^2(\alpha).$$

For $\alpha = 0.01$, $\frac{1}{2}\chi_2^2 = 4.605$.

The translog, quadratic, generalized square root quadratic, and nonhomogenous version of the generalized Leontief functional forms involve restrictions on θ and λ only, and thus tests of these functional forms consist simply of determining whether the corresponding values (θ^*, λ^*) fall within the appropriate confidence region.

The semilog form is also tested directly on the quadratic Box–Cox form. The log-linear form is tested conditional on acceptance of the translog form. The square root quadratic form is tested conditional on acceptance of the generalized square root quadratic form. The linear form is tested conditional on the quadratic form and is also tested conditional on the generalized Leontief form. The linear homogeneous version of the generalized Leontief form is tested conditional on acceptance of the nonhomogeneous version.

Because the testing procedure is only partially nested, it will not in general indicate that there is one and only one acceptable functional form for a particular application. If more than one of the nonnested alternatives were accepted, the choice among them could be based on several criteria. First, it might be possible to apply a nonnested testing procedure. For example, Berndt *et al.* [3] provide a Bayesian rationale for choosing among alternative flexible functional forms on the basis of the size of the log likelihood. Second, if the choice is narrowed to two or more of the standard hedonic forms (the linear, semilog, and log-linear), there may be theoretical grounds for preferring one to another. Finally, in the absence of firm statistical or theoretical grounds for choosing among the acceptable functional forms, the choice can be based on convenience in dealing with the problem at hand.

IV. APPLICATION

The use of the quadratic Box–Cox procedure for choice of the functional form for a hedonic price equation is illustrated by an application to housing. Given the nature of housing, it is useful to characterize it as consisting of attributes representing not only the actual structure, but also the locational amenities which purchase of housing provides households. We thus assume that the housing market is most appropriately viewed as consisting of implicit markets for each of the attributes of housing.

The data set employed consists of a sample of 5727 single-family owner-occupied dwelling units in the San Francisco Bay Area. The primary data source is the 1965 Bay Area Transportation Study Commission (B.A.T.S.C.) survey of about 29,000 households. The housing data from this survey are supplemented by data describing neighborhood characteristics and employment accessibility. The specific variables employed in this illustration are number of rooms, age, lot size, median income in census tract, median number of rooms in census tract, percent dwelling units owner-occupied in census tract, and an employment accessibility index.[15]

The alternative functional forms of the hedonic price equation are estimated using ordinary least squares. In estimating the unrestricted quadratic Box–Cox form a grid search was preformed over values of θ and λ between -1.0 and 2.0 in order to find the values of θ and λ which maximize the log likelihood. The optimum optimorum for the quadratic Box–Cox form was found to be $\theta = 0.06$, $\lambda = 0.28$. The parameter estimates are generally quite precise. Of the 36 parameters estimated, 23 are significant at the 0.01 level using two-tailed tests, and an additional three are significant at the 0.05 level.[16]

The 99% confidence region around $\theta = 0.06$, $\lambda = 0.28$ is shown in Fig. 1. As indicated by the shape of the confidence region, the value of the log likelihood was substantially more sensitive to the value of the transformation parameter for the dependent variable than to the value of the transformation parameter for the independent variables.

[15] The structural data are drawn from the B.A.T.S.C. Household Survey, 1965. Census tract data are taken from the 1960 Census of Housing and the 1960 Census of Population. The employment accessibility index is the gravity index $\Sigma_j E_j/d_j^2$, where E_j = total employment in the jth census tract, d_j = freeflow driving time from the census tract in question to the jth one, and the summation is done over all census tracts except the one in question. The employment data are obtained from the B.A.T.S.C. Employment Inventory, 1964; driving times are obtained from the Bay Area Simulation Study, 1968.

See Pollakowski [31, 32] for a more detailed discussion of the B.A.T.S.C. data and the construction of specific variables.

[16] The parameter estimates and their standard errors are available from the authors upon request.

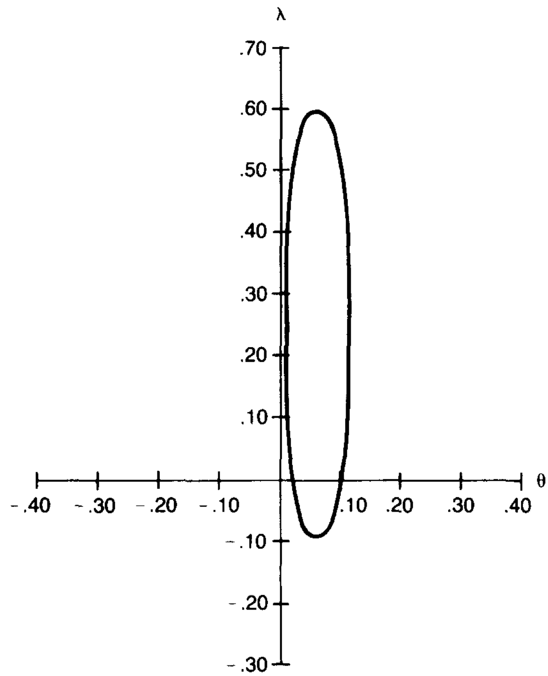


FIG. 1. 99% Confidence region.

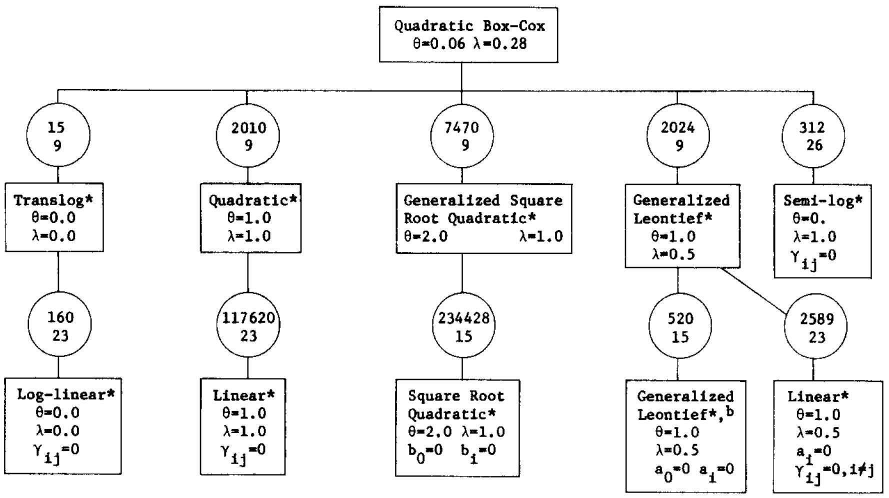
The translog, quadratic, generalized square root quadratic, and nonhomogeneous version of the generalized Leontief forms involve only the imposition of restrictions on the values of θ and λ. The values of (θ, λ) for each form are: translog (0.0, 0.0), quadratic (1.0, 1.0), generalized square root quadratic (2.0, 1.0), and nonhomogeneous version of the generalized Leontief form (1.0, 0.5). As shown in Fig. 1, the values of θ and λ corresponding to each of these forms lie outside the 99% confidence interval, and all of these forms are accordingly rejected at the 0.01 level.

The test statistics and critical χ^2 values for each of the tests of hypotheses are shown in Table 1. Since all other forms except the semilog are conditional on the rejected translog, quadratic, generalized square root quadratic, or nonhomogeneous version of the generalized Leontief forms, they need not be tested further. Nevertheless, it is interesting to note from Table 1 that these forms are rejected even conditional on acceptance of the more general forms of which they are special cases.

The semilog form is obtained from the quadratic Box–Cox form by imposing the restrictions $\theta = 0$, $\lambda = 1$, and $\gamma_{ij} = 0$. As shown in Table 1,

TABLE 1

Tests of Hypotheses[a]



[a] The top number in each circle is the value of the test statistic and the bottom number is the critical value at the 0.01 level.

[b] Linear homogeneity imposed.

*The form is rejected at the 0.01 level.

the semilog form is also strongly rejected at the 0.01 level. Thus the three functional forms most commonly used in previous research, the linear, log-linear, and semilog, are all strongly rejected.

V. CONCLUDING COMMENTS

Previous studies employing hedonic price equations have generally used highly restrictive functional forms chosen in a largely arbitrary manner. Since the choice of functional form can have a major effect on the conclusions reached, it is clearly preferable to base the choice on relevant statistical procedures.

In this paper, a highly general functional form is specified which yields all other functional forms of interest as special cases. Therefore, it is possible to use likelihood ratio tests to test the appropriateness of the alternative functional forms.

The application of this procedure is illustrated by the estimation of hedonic price equations for housing using cross section microdata. For this case, all specific functional forms, including those most commonly used in previous studies, are rejected.[17]

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[17] If a specific functional form were accepted, the resulting estimators would be "preliminary test" estimators with unknown statistical properties, and the usual hypothesis tests would not be valid. See Judge and Bock [20].

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