Centered Difference of Approximation of the 2nd Derivative in a Taylor Series Expansion

$$f''(x) \simeq \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Lets expand the terms via Taylor Series Expansion

$$f(x+h) \approx f(x) + f'(x)(h) + \tfrac{1}{2} f''(x)h^2 + \tfrac{1}{6} f'''(x)h^3 + \tfrac{1}{24} f^{(4)}(x)h^4$$

$$f(x) \approx f(x)$$

$$f(x-h) \approx f(x) - f'(x) + \tfrac{1}{2} f''(x)h^2 - \tfrac{1}{6} f'''(x)h^3 + \tfrac{1}{24} f^{(4)}(x)h^4$$

$$f''(x) \approx \frac{1}{h^2}\Big(\cancel{f(x)} + \cancel{f'(x)h} + \tfrac{1}{2} f''(x)h^2 + \cancel{\tfrac{1}{6} f'''(x)h^3} + \tfrac{1}{24} f^{(4)}(x)h^4$$

$$- \cancel{2f(x)}$$

$$+ \cancel{f(x)} - \cancel{f'(x)h} + \tfrac{1}{2} f''(x)h^2 - \cancel{\tfrac{1}{6} f'''(x)h^3} + \tfrac{1}{24} f^{(4)}(x)h^4 \Big)$$

$$f''(x) \approx f''(x) + \tfrac{1}{24} f^{(4)}(x)h^2 \rightarrow \text{Therefore, } |\text{error}| = \tfrac{1}{24} f^{(4)}(x)h^2$$