Centered Difference Approximation in a Taylor Series Expansion

$$f'(a) \simeq \frac{f(a+h) - f(a-h)}{2h}$$

Lets expand $f(a+h)$ and $f(a-h)$ in a Taylor Series Expansion

$$f(a+h) \simeq f(a) + f'(a)(a+h-a) + \frac{1}{2}f''(a)(a+h-a)^2 + \frac{1}{6}f'''(a)(a+h-a)^3$$

$$f(a-h) \simeq f(a) + f'(a)(a-h-a) + \frac{1}{2}f''(a)(a-h-a)^2 + \frac{1}{6}f'''(a)(a+h-a)^3$$

Therefore, we can rewrite $f'(a)$ approximation as:

$$f'(a) \simeq \frac{1}{2h}\Big(\cancel{f(a)} + f'(a)(h) + \cancel{\tfrac{1}{2}f''(a)h^2} + \tfrac{1}{6}f'''(a)h^3$$
$$\cancel{-f(a)} + f'(a)h \cancel{-\tfrac{1}{2}f''(a)h^2} + \tfrac{1}{6}f'''(a)h^3\Big)$$

$$f'(a) \simeq f'(a) + \frac{1}{6}f'''(a)h^2$$

Therefore, the $|\text{error}| = \frac{1}{6}f^{(3)}(a)h^2$ and hence, the approximation is 2nd order since the difference in degree of $f'(a)$ and the error term is $3-1=2$, otherwise, 2nd Order.