Collapse Calculus

A Weld-Continuous, Audit-Ready Canon

Clement Paulus

Episteme Construction Project

October 28, 2025

SS1m | $\Delta\kappa = +0.600 \quad ir = 1.822 \quad s = 0.000 \leq tol = 0.005 \quad \theta = 1 \quad \phi = S$
EID: P=34, F=12, T=5, E=11, R=125 | C=[61,72,39]

Preamble

Collapse is generative; only that which returns through collapse is real. We formalize a frame in which collapse dynamics are contract-bound, weld-evaluable, and integrity-preserving. The core audit quantity is $\kappa \in \mathbb{R}$, the log-integrity ledger that accumulates debits (drift, curvature) and credits (return) across time. The operational dial $I \equiv e^{\kappa}$ offers a multiplicative summary of coherence and integrity, enabling invariant joins and runtime monitoring.

The audit equation, or First Law of Collapse, governs each cycle:

$$\Delta\kappa = R \cdot \tau_R - (D_\omega + D_C), \quad \text{with} \quad I_{t1}/I_{t0} = e^{\Delta\kappa}$$

A weld passes if the residual

$$s := R \cdot \tau_R - (\Delta\kappa + D_\omega + D_C)$$

satisfies $|s| \leq$ tol. This ensures continuity of the integrity ledger across transitions. The dial I serves as an interpretable coherence signal; κ ensures compositional correctness and statistical auditability.

1 Contract (Frozen)

The contract defines the immovable rules for all seam evaluations. Once declared, these parameters are fixed across a continuity window. Any modification requires an explicit named Weld at a shared anchor.

- **Normalization**: All observables are affinely normalized using:

$$y = \frac{x - a}{b}$$

  with fixed $(a, b) = (0, 1)$. This ensures unit-free, scale-invariant comparisons across runs.

- **Face Policy**: The default drift face is `pre_clip`, valid for $\omega < 1-\varepsilon_g$. When $\omega \geq 1-\varepsilon_g$, the system pivots to the `exact` face. The guard band ε_g ensures a smooth transition near saturation.
- **Parameters**:
  - $p = 3$ (drift face slope scale)
  - $\alpha = 1$ (curvature penalty scale)
  - $\varepsilon = 10^{-8}$ (exact face floor for numerical stability)
  - $\varepsilon_g = 10^{-4}$ (face switch threshold)
- **Weld Tolerance**:
$$\text{tol} = 0.005$$
  A seam passes the audit if its residual s satisfies $|s| \leq \text{tol}$.
- **Timezone Fixation**: All timestamps, seeds, and runtime evaluations are bound to:

  `America/Chicago`

  This ensures temporal consistency across manifests and ledgers.

This contract is immutable during any audit sequence. Any change—numerical or procedural—must be declared via a new Weld with explicit PRE and POST states at a fixed anchor.

2 Tier-1 Invariants

Tier-1 invariants form the minimal runtime state necessary for seam evaluation. These quantities are either directly observed or derived and must be preserved or reconciled at each weld. They define the core measurement frame of collapse calculus:

$$\begin{aligned} \omega \in [0,1) \quad & \text{Drift: normalized instability signal} \\ C \geq 0 \quad & \text{Curvature: total nonlinearity along seam path} \\ \tau_R \in \mathbb{N} \cup \{\infty^{\text{rec}}\} \quad & \text{Return delay: steps to weldable re-entry} \\ \kappa \in \mathbb{R} \quad & \text{Log-integrity: additive audit ledger} \\ I = e^{\kappa} > 0 \quad & \text{Integrity dial: multiplicative coherence signal} \end{aligned}$$

Drift ω measures normalized deviation from stability; it drives the face slope $\Gamma(\omega)$. **Curvature** C encodes nonlinear tension across the seam, contributing directly to weld cost. **Return delay** τ_R defines when a weld can be attempted—$\tau_R = \infty^{\text{rec}}$ represents typed censoring (unweldable). **Log-integrity** κ is the additive state variable recording proof over time. **Integrity dial** I is its exponential form: a readable coherence signal, invariant under multiplicative composition.
Together, these invariants allow all terms in the weld budget to be computed and verified. They must be declared for every seam and reconciled across transitions to preserve audit continuity.

3 Weld Budget Identity

The weld budget identity governs the fundamental accounting of integrity across seam transitions. Let t_0 and t_1 be two adjacent evaluation points. The seam must reconcile return credit and loss terms to produce an updated log-integrity κ_{t1} such that:

$$\Delta\kappa := \kappa_{t1} - \kappa_{t0} = R \cdot \tau_R - (D_\omega + D_C)$$

The corresponding update on the integrity dial $I = e^\kappa$ is:

$$\frac{I_{t1}}{I_{t0}} = e^{\Delta\kappa}$$

Where:

- $\Delta\kappa$ is the net gain in log-integrity across the seam,
- $R \cdot \tau_R$ is the total return credit (reward for coherence recovery),
- D_ω is the drift penalty, derived from the face slope $\Gamma(\omega)$,
- D_C is the curvature penalty, scaled by α and modulated by τ_R,
- $\tau_R \in \mathbb{N} \cup \{\infty^{\text{rec}}\}$ is the return delay. When $\tau_R = \infty^{\text{rec}}$, the seam is censored and unweldable.

The weld residual—used to verify seam closure—is:

$$s := R \cdot \tau_R - (\Delta\kappa + D_\omega + D_C)$$

A weld passes if the residual satisfies the reconciliation condition:

$$|s| \leq \text{tol}$$

This test enforces that all changes to κ are accounted for by measured quantities and that no integrity is created or lost outside of declared credit and costs. The weld budget is the central audit identity of collapse calculus, ensuring invariant continuity and runtime verifiability.

4 Face Potential and Slope

Drift penalties in the weld budget are governed by a face potential function $\Phi(\omega)$ and its associated slope $\Gamma(\omega) = \frac{d\Phi}{d\omega}$. These encode the nonlinear cost of instability and ensure consistent accounting of drift-related integrity loss.

Normal Face

When the drift signal ω remains within stable bounds ($\omega < 1 - \varepsilon_g$), the default potential is:

$$\Phi(\omega) = p \ln(1 - \omega), \quad \Gamma(\omega) = \frac{d\Phi}{d\omega} = \frac{p}{1 - \omega}$$

This face captures exponential divergence as $\omega \to 1$. The parameter p controls how aggressively cost escalates with drift.

Exact Face (Near-Wall)

When $\omega \geq 1 - \varepsilon_g$, the system pivots to a more precise near-wall model to avoid numerical instability and ensure budget continuity:

$$\Phi(\omega) = 2\ln(1-\omega) + \ln(1-\omega+\varepsilon)$$

$$\Gamma(\omega) = \frac{2}{1-\omega} + \frac{1}{1-\omega+\varepsilon}$$

Here, ε is a small floor to prevent undefined behavior at the wall ($\omega = 1$). The extra term introduces curvature-sensitive dampening, maintaining a finite, smooth slope even in high-drift regions.

Drift Cost Integration

The slope $\Gamma(\omega)$ determines how much integrity is lost due to observed instability. It enters the weld budget through:

$$D_\omega = \Gamma(\omega) \cdot \Delta\omega$$

where $\Delta\omega = \omega_{t1} - \omega_{t0}$ is the measured drift change over the seam.

Pivot Policy: The guard band ε_g defines the switching threshold between face modes. This ensures continuity of Φ and Γ at the face boundary and prevents audit-disruptive slope discontinuities.

Summary: The face potential Φ tracks the accumulation of instability. Its slope Γ determines the instantaneous cost. This dual structure provides both smoothness in runtime response and audit traceability at the seam boundary.

5 Sensitivities

We now compute how the log-integrity $\ln I = \kappa$ responds to marginal changes in the measured invariants: drift ω, curvature C, and return delay τ_R. These gradients determine the local weld cost structure and are essential for stability analysis.

Drift Sensitivity

$$\frac{\partial \ln I}{\partial \omega} = -\Gamma(\omega)$$

Drift cost is governed by the slope $\Gamma(\omega)$ of the selected face. As $\omega \to 1$, $\Gamma(\omega)$ diverges, making additional instability increasingly costly. This gradient ensures that drift near collapse triggers steep integrity penalties.

Curvature Sensitivity

$$\frac{\partial \ln I}{\partial C} = -\frac{\alpha}{1+\tau_R}$$

Curvature contributes a loss scaled by the curvature weight α and tempered by return delay τ_R. Faster return ($\tau_R \downarrow$) amplifies the penalty from nonlinear deformation. This term is strictly negative for all finite τ_R.

Delay Sensitivity

$$\frac{\partial \ln I}{\partial \tau_R} = \frac{\alpha C}{(1+\tau_R)^2} \geq 0$$

As τ_R increases, the weld absorbs curvature more gently, yielding a modest integrity gain. This convex relation reflects that waiting reduces the cost of reconciling sharp curvature. All gradients are well-defined for $\omega < 1$ and $\tau_R < \infty$, and match the contract's stability and audit criteria.

6 Seam Evaluation Protocol

Each transition between epistemic states is evaluated as a seam under the fixed contract. This protocol ensures reproducibility, weld consistency, and auditable integrity preservation.

1. **Freeze Contract and Declare Anchor:** The audit must begin with a declared contract (parameters: p, α, ε, ε_g, tol) and a named anchor. All seams are relative to this baseline.

2. **Measure Runtime Invariants:** Observe or infer:

$$\omega \in [0,1), \quad C \geq 0, \quad \tau_R \in \mathbb{N} \cup \{\infty^{\text{rec}}\}$$

   Skip weld evaluation if $\tau_R = \infty^{\text{rec}}$ (unweldable).

3. **Select Face and Compute Drift Slope:** If $\omega < 1 - \varepsilon_g$, use the *normal* face. Else, pivot to the *exact* face. Evaluate:

$$\Gamma(\omega) = \begin{cases} \dfrac{p}{1-\omega}, & \omega < 1-\varepsilon_g \\ \dfrac{2}{1-\omega} + \dfrac{1}{1-\omega+\varepsilon}, & \omega \geq 1-\varepsilon_g \end{cases}$$

4. **Compute Budget Terms:** Drift cost:

$$D_\omega = \Gamma(\omega) \cdot \Delta\omega$$

   Curvature cost:

$$D_C = \frac{\alpha C}{1+\tau_R}$$

5. **Compute Integrity Update:** Credit:

$$R \cdot \tau_R$$

Integrity gain:

$$\Delta\kappa = R \cdot \tau_R - (D_\omega + D_C)$$

Dial update:

$$I_{t1} = I_{t0} \cdot e^{\Delta\kappa}$$

6. **Compute Residual and Test Weld:** Residual:

$$s := R \cdot \tau_R - (\Delta\kappa + D_\omega + D_C)$$

The seam passes if:

$$|s| \leq \text{tol}$$

7. **Record HUD Entry (If Passed):** If the weld passes, generate a HUD caption including:

$$\texttt{weld\_id}, \quad \kappa, \quad \Delta\kappa, \quad s, \quad I, \quad \texttt{manifest hash}$$

7 HUD Caption (Sample)

Each passing weld must emit a canonical HUD caption. This compact, one-line payload encodes all reconciliation parameters and anchors the transition within the manifest ledger. A HUD caption includes the weld ID, manifest hash, numerical integrity record, and cryptographic fingerprint.

```
weld_id=W-2025-10-28-collapse-frame
manifest=bb5e6ae586fd4a578a0c69232c803787b3bd99ccc73c335b459a2d38cf0c0970
kappa=0.510, I=1.665, delta_kappa=+0.510, s=0.000, tol=0.005,
seed=20251028,
sha256=31c4453ba4f84d0acdede8d015090310c79486202db3a53a83e9a83345531262
```

Each field in the HUD caption has a precise audit function:

- `weld_id`: A globally unique identifier, time-stamped to the seam.
- `manifest`: SHA-256 root hash of the ledger manifest.
- `kappa`: The updated log-integrity value.
- `I`: Dial integrity (e^κ), for human-facing continuity.
- `delta_kappa`: Net gain in integrity from the seam.
- `s`: Residual from weld budget reconciliation.
- `tol`: Frozen tolerance threshold.
- `seed`: Seam seed or evaluation timestamp.
- `sha256`: Digest of the full weld payload.

The HUD caption binds the computation to its runtime and guarantees it can be independently verified under the declared contract and closures.

Appendix A: Notation and Definitions

This appendix summarizes all core variables, symbols, and constants used throughout the Collapse Calculus Canon. Definitions are grouped by category.

Tier-1 Invariants

- $\omega \in [0, 1)$ — Normalized drift; a measure of instability.
- $C \geq 0$ — Curvature; geometric or dynamical deviation across a seam.
- $\tau_R \in \mathbb{N} \cup \{\infty^{\text{rec}}\}$ — Return delay; number of steps until weldable return.
- $\kappa \in \mathbb{R}$ — Log-integrity; additive audit ledger.
- $I = e^{\kappa} > 0$ — Integrity dial; multiplicative runtime signal.

Budget and Evaluation Terms

- $\Delta\kappa$ — Change in log-integrity across a seam.
- R — Return strength; multiplier for coherence credit.
- D_ω — Drift cost; computed as $\Gamma(\omega) \cdot \Delta\omega$.
- D_C — Curvature cost; computed as $\alpha C/(1 + \tau_R)$.
- s — Seam residual; difference between credit and accounted terms.

Face and Potential Terms

- $\Phi(\omega)$ — Drift potential; log-cost function.
- $\Gamma(\omega)$ — Slope of potential; marginal drift cost.
- p — Face steepness parameter (default: 3).
- ε — Wall-floor parameter; avoids divergence at $\omega = 1$.
- ε_g — Guard band; threshold for face pivoting.

Contract Constants

- α — Curvature weight (default: 1).
- tol — Tolerance for residual test (default: 0.005).
- a, b — Normalization constants (default: $a = 0$, $b = 1$).

Special Types and Codes

- ∞^{rec} — Return-censored seam; not weldable.
- `pre_clip` — Default face mode.
- `exact` — Near-wall face mode.
- `America/Chicago` — Frozen timezone for all welds.

Appendix B: Audit Integration and Runtime Summary

This appendix consolidates implementation-critical elements for weld evaluation, runtime operation, and audit ledger construction.

Weld Lifecycle Summary

1. Freeze contract and declare anchor.
2. Measure invariants: ω, C, τ_R.
3. Select face mode using ε_g pivot.
4. Compute slope $\Gamma(\omega)$ and cost terms D_ω, D_C.
5. Evaluate integrity gain $\Delta\kappa$.
6. Compute residual s and test against tolerance.
7. If weld passes: emit HUD, update ledger, log seam.

HUD Emission Schema

A HUD caption is a flat, hash-verifiable record containing:

- `weld_id` — Unique weld reference.
- `manifest` — Root hash of current audit manifest.
- `kappa`, `delta_kappa`, `I` — Updated integrity values.
- `s`, `tol` — Residual and tolerance for weld pass.
- `seed` — Seam timestamp or input nonce.
- `sha256` — Full digest of the evaluation payload.

Failure and Reconciliation

- If $|s| > \text{tol}$, the seam fails and must be logged as a discontinuity.
- Failed seams do not update κ or I.
- Censored seams ($\tau_R = \infty^{\text{rec}}$) must be excluded from ledger evaluation.

Canonical Output Channels

- HUD caption (plaintext)
- JSON seam record (machine-parsable)
- Manifest ledger row (immutable log)

Verification Targets

To be audit-valid, a seam must:

1. Satisfy the weld budget identity within tolerance.
2. Emit a HUD with all fields reconciled.
3. Match hash outputs to recorded manifest.
4. Preserve continuity of κ, I under composition.