# 2023 OSSM Integration Bee

## Team Match 1:

**Problem 1:** $\int \sin(x)\sin(\cos(x))\,\mathrm{d}x$.

**Problem 2:** $\int \sqrt[3]{(2x+7)^5}\,\mathrm{d}x$.

**Problem 3:** $\int \left(1 + x + x^2 + x^3 + x^4 + \cdots\right)\,\mathrm{d}x$. (Closed form answer required.)

## Team Match 2:

**Problem 4:** $\int \frac{e^x}{1+e^x}\,\mathrm{d}x$.

**Problem 5:** $\int (100x + 100)^{100}\,\mathrm{d}x$.

**Problem 6:** $\int_{-1}^{0}(x^4 + 4x^3 + 6x^2 + 4x + 1)\,\mathrm{d}x$.

## Team Match 3:

**Problem 7:** $\int (x^4 + 4)^2\,\mathrm{d}x$.

**Problem 8:** $\int_{-4}^{2}\left|\frac{1}{2}x + 1\right|\,\mathrm{d}x$.

**Problem 9:** $\int_0^\pi x\sin x\,\mathrm{d}x$.

## Team Match 4:

**Problem 10:** $\int_1^5 x^2\,\mathrm{d}x$.

**Problem 11:** $\int_{\pi/6}^{\pi/3}\left(\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x}\right)\,\mathrm{d}x$.

**Problem 12:** $\int x\sec x\tan x\,\mathrm{d}x$.

## Team Match 5:

**Problem 13:** $\int \frac{1}{(4x+5)^2}\,\mathrm{d}x$.

**Problem 14:** $\int \left(\cos^2 x - \sin^2 x\right)\,\mathrm{d}x$.

**Problem 15:** $\int x^2 e^x\,\mathrm{d}x$.

## Team Match 6:

**Problem 16:** $\int x\sqrt[3]{x^2 + 100}\,\mathrm{d}x$.

**Problem 17:** $\int \frac{x+2}{5x^2+20x+40}\,\mathrm{d}x$.

**Problem 18:** $\int \frac{x^{1+\sec^2 x}}{x^{\tan^2 x}}\,\mathrm{d}x$.

# Team Match 7 - Semifinals #1:

**Problem 19:** $\int \frac{4}{1+2x^2}\,\mathrm{d}x$.

**Problem 20:** $\int \frac{x}{x+37}\,\mathrm{d}x$.

**Problem 21:** $\int (x+1)\sec[(x+1)^2]\tan[(x+1)^2]\,\mathrm{d}x$.

# Team Match 8 - Semifinals #2:

**Problem 22:** $\int_{-2}^{2} \sqrt{4-x^2}\,\mathrm{d}x$.

**Problem 23:** $\int \sin^2 x \cos^3 x\,\mathrm{d}x$.

**Problem 24:** $\int \frac{1}{x^2-15x+50}\,\mathrm{d}x$.

# Team Match 9 - Finals:

**Problem 25:** $\int e^{x-e^x}\,\mathrm{d}x$.

**Problem 26:** $\int \arctan x\,\mathrm{d}x$.

**Problem 27:** $\int \frac{1}{1-\sqrt{x}}\,\mathrm{d}x$.

# Individual Match 1:

**Problem 28:** $\int \frac{\cos(\ln x)}{x}\,\mathrm{d}x$.

**Problem 29:** $f(x) = \begin{cases} 3 & x < -2 \\ -2 & -2 \le x \le 3 \\ 1 & x > 3 \end{cases}$

$\int_{-4}^{4} f(x)\,\mathrm{d}x$.

**Problem 30:** $\int 2^{x-2}\,\mathrm{d}x$.

# Individual Match 2:

**Problem 31:** $\int \frac{x}{1+x^2}\,\mathrm{d}x$.

**Problem 32:** $\int_{-\pi}^{\pi} |x|\,\mathrm{d}x$.

**Problem 33:** $\int \tan^2 x \, \mathrm{d}x$.

# Individual Match 3:

**Problem 34:** $\int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} \, \mathrm{d}x$.

**Problem 35:** $\int \left( e^x + e^{-x} \right)^2 \, \mathrm{d}x$.

**Problem 36:** $\int \frac{x^2 + 9}{3x^2 \sqrt{x}} \, \mathrm{d}x$.

# Individual Match 4:

**Problem 37:** $\int x^2 e^{x^3} \, \mathrm{d}x$.

**Problem 38:** $\int \left( \sqrt{2x} - \sqrt{3x} \right) \, \mathrm{d}x$.

**Problem 39:** $\int \left| e^{-x} + \frac{1}{x^2 + 1} \right| \, \mathrm{d}x$.

# Individual Match 5 - Semifinals #1:

**Problem 40:** $\int_{-1}^{3} x \left| x \right| \, \mathrm{d}x$.

**Problem 41:** $\int \left( 1 - x^2 + x^4 - x^6 + \cdots \right) \, \mathrm{d}x$. (Closed form answer required.)

**Problem 42:** $\int \frac{4 \tan x}{1 - \tan^2 x} \, \mathrm{d}x$.

# Individual Match 6 - Semifinals #2:

**Problem 43:** $\int_{-\infty}^{0} e^{2x} \, \mathrm{d}x$.

**Problem 44:** $\int x \ln x \, \mathrm{d}x$.

**Problem 45:** $\int \frac{1}{x \sqrt{x^4 - 1}} \, \mathrm{d}x$.

# Individual Match 7 - Finals:

**Problem 46:** $\int_{-\pi}^{\pi} x^4 \sin(x^3) \, \mathrm{d}x$.

**Problem 47:** $\int \frac{x^2 + 1}{x + 1} \, \mathrm{d}x$.

**Problem 48:** $\int e^x \sin x \, \mathrm{d}x$.

**Problem 49:** $\int \frac{1}{x^2 (x - 1)} \, \mathrm{d}x$.

**Problem 50:** $\int \frac{1}{\sqrt{4x - x^2}} \, \mathrm{d}x$.

# Tiebreakers:

**Problem 1:** $\int_{\pi/4}^{3\pi/4} \sin x \, \mathrm{d}x$.

**Problem 2:** $\int \dfrac{\ln x}{x} \, \mathrm{d}x$.

**Problem 3:** $\int e^{\sqrt{6}x} \, \mathrm{d}x$.

**Problem 4:** $\int_{1-\sqrt{2}}^{1+\sqrt{2}} \sqrt{2} \, \mathrm{d}x$.

**Problem 5:** $\int (x+1)(x-1) \, \mathrm{d}x$.

**Problem 6:** $\int_{25}^{100} \frac{1}{\sqrt{x}} \, \mathrm{d}x$.

**Problem 7:** $\int_0^{2\pi} (\cos x - \sin x) \, \mathrm{d}x$.

**Problem 8:** $\int_{\ln 2}^{\ln 3} e^x \, \mathrm{d}x$.

**Problem 9:** $\int \frac{x^2+1}{x} \, \mathrm{d}x$.

**Problem 10:** $\int \frac{1}{\sqrt{9-x^2}} \, \mathrm{d}x$.

**Problem 11:** $\int 5x\sqrt{x} \, \mathrm{d}x$.

**Problem 12:** $\int 2x \sin(x^2) \, \mathrm{d}x$.

**Problem 13:** $\int \frac{1}{\cos^2 x} \, \mathrm{d}x$.

**Problem 14:** $\int_0^1 \sqrt[20]{x} \, \mathrm{d}x$.

**Problem 15:** $\int_0^4 \left[ (x+4)^2 - 16x - (4-x)^2 \right] \, \mathrm{d}x$.

**Problem 16:** $\int \tan(x) \sec(x) \, \mathrm{d}x$.

**Problem 17:** $\int \sqrt{1-x} \, \mathrm{d}x$.

**Problem 18:** $\int e^2 \, \mathrm{d}x$.

**Problem 19:** $\int \frac{\sec^2 x}{1+\tan x} \, \mathrm{d}x$.

**Problem 20:** $\int_0^3 x \, \mathrm{d}x$.