2023 OSSM Integration Bee

Team Match 1:

Problem 1: $\int \sin(x)\sin(\cos(x))\,\mathrm{d}x$.

Solution. $\cos(\cos(x)) + C$

Problem 2: $\int \sqrt[3]{(2x+7)^5}\,\mathrm{d}x$.

Solution. $\frac{3}{16}(2x+7)^{8/3} + C$

Problem 3: $\int \left(1 + x + x^2 + x^3 + x^4 + \cdots\right)\,\mathrm{d}x$. (Closed form answer required.)

Solution. $-\ln|1-x| + C$

Team Match 2:

Problem 4: $\int \frac{e^x}{1+e^x}\,\mathrm{d}x$.

Solution. $\ln(1+e^x) + C$

Problem 5: $\int (100x+100)^{100}\,\mathrm{d}x$.

Solution. $\frac{1}{10100}(100x+100)^{101} + C$

Problem 6: $\int_{-1}^{0}(x^4 + 4x^3 + 6x^2 + 4x + 1)\,\mathrm{d}x$.

Solution. $\frac{1}{5}$

Team Match 3:

Problem 7: $\int (x^4+4)^2\,\mathrm{d}x$.

Solution. $\frac{1}{9}x^9 + \frac{8}{5}x^5 + 16x + C$

Problem 8: $\int_{-4}^{2} \left|\frac{1}{2}x + 1\right|\,\mathrm{d}x$.

Solution. 5

Problem 9: $\int_0^\pi x \sin x \, \mathrm{d}x$.

Solution. π

Team Match 4:

Problem 10: $\int_1^5 x^2 \, \mathrm{d}x$.

Solution. $124/3$

Problem 11: $\int_{\pi/6}^{\pi/3} \left(\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} \right) \mathrm{d}x$.

Solution. $\pi/6$

Problem 12: $\int x \sec x \tan x \, \mathrm{d}x$.

Solution. $x \sec x - \ln|\sec x + \tan x| + C$

Team Match 5:

Problem 13: $\int \frac{1}{(4x+5)^2} \, \mathrm{d}x$.

Solution. $\frac{-1}{16x+20} + C$

Problem 14: $\int \left(\cos^2 x - \sin^2 x \right) \mathrm{d}x$.

Solution. $\frac{1}{2} \sin(2x) + C$

Problem 15: $\int x^2 e^x \, \mathrm{d}x$.

Solution. $(x^2 - 2x + 2)e^x + C$

Team Match 6:

Problem 16: $\int x \sqrt[3]{x^2 + 100} \, \mathrm{d}x$.

Solution. $\frac{3}{8}(x^2 + 100)^{4/3} + C$

Problem 17: $\int \frac{x+2}{5x^2+20x+40} \, \mathrm{d}x$.

Solution. $\frac{1}{10}\ln(5x^2 + 20x + 40) + C$

Problem 18: $\int \frac{x^{1+\sec^2 x}}{x^{\tan^2 x}}\,\mathrm{d}x.$

Solution. $\frac{1}{3}x^3 + C$

Team Match 7 - Semifinals #1:

Problem 19: $\int \dfrac{4}{1 + 2x^2}\,\mathrm{d}x.$

Solution. $2\sqrt{2}\arctan\left(\sqrt{2}x\right) + C$

Problem 20: $\int \frac{x}{x+37}\,\mathrm{d}x.$

Solution. $x - 37\ln|x + 37| + C$

Problem 21: $\int (x+1)\sec[(x+1)^2]\tan[(x+1)^2]\,\mathrm{d}x.$

Solution. $\frac{1}{2}\sec[(x+1)^2] + C$

Team Match 8 - Semifinals #2:

Problem 22: $\int_{-2}^{2}\sqrt{4 - x^2}\,\mathrm{d}x.$

Solution. 2π

Problem 23: $\int \sin^2 x \cos^3 x\,\mathrm{d}x.$

Solution. $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$

Problem 24: $\int \frac{1}{x^2 - 15x + 50}\,\mathrm{d}x.$

Solution. $\frac{1}{5}\ln|x - 10| - \frac{1}{5}\ln|x - 5| + C$

Team Match 9 - Finals:

Problem 25: $\int e^{x-e^x} \, \mathrm{d}x$.

Solution. $-e^{-e^x} + C$

Problem 26: $\int \arctan x \, \mathrm{d}x$.

Solution. $x \arctan x - \frac{1}{2} \ln(1+x^2) + C$

Problem 27: $\int \frac{1}{1-\sqrt{x}} \, \mathrm{d}x$.

Solution. $-2 \ln|1 - \sqrt{x}| - 2\sqrt{x} + C$

Individual Match 1:

Problem 28: $\int \frac{\cos(\ln x)}{x} \, \mathrm{d}x$.

Solution. $\sin(\ln x) + C$

Problem 29: $f(x) = \begin{cases} 3 & x < -2 \\ -2 & -2 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$

$\int_{-4}^{4} f(x) \, \mathrm{d}x$.

Solution. -3

Problem 30: $\int 2^{x-2} \, \mathrm{d}x$.

Solution. $\frac{1}{4\ln 2} 2^x + C$

Individual Match 2:

Problem 31: $\int \frac{x}{1+x^2} \, \mathrm{d}x$.

Solution. $\frac{1}{2} \ln(1+x^2) + C$

Problem 32: $\int \frac{x}{1+x^2} \, \mathrm{d}x$.

Solution. $\frac{1}{2}\ln(1+x^2) + C$

Problem 33: $\int_{-\pi}^{\pi} |x| \, \mathrm{d}x$.

Solution. π^2

Problem 34: $\int \tan^2 x \, \mathrm{d}x$.

Solution. $\tan x - x + C$

Individual Match 3:

Problem 35: $\int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} \, \mathrm{d}x$.

Solution. $\frac{-1}{\sin x + \cos x} + C$

Problem 36: $\int (e^x + e^{-x})^2 \, \mathrm{d}x$.

Solution. $\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x + C$

Problem 37: $\int \frac{x^2+9}{3x^2\sqrt{x}} \, \mathrm{d}x$.

Solution. $\frac{2}{3}\sqrt{x} - \frac{2}{x\sqrt{x}} + C$

Individual Match 4:

Problem 38: $\int x^2 e^{x^3} \, \mathrm{d}x$.

Solution. $\frac{1}{3}e^{x^3} + C$

Problem 39: $\int \left(\sqrt{2x} - \sqrt{3x}\right) \, \mathrm{d}x$.

Solution. $\frac{2}{3}(\sqrt{2} - \sqrt{3})x^{3/2} + C$

Problem 40: $\int \left| e^{-x} + \frac{1}{x^2+1} \right| \, \mathrm{d}x$.

Solution. $-e^{-x} + \arctan x + C$

Individual Match 5 - Semifinals #1:

Problem 41: $\int_{-1}^{3} x \, |x| \, \mathrm{d}x$.

Solution. $\frac{26}{3}$

Problem 42: $\int \left(1 - x^2 + x^4 - x^6 + \cdots \right) \mathrm{d}x$. (Closed form answer required.)

Solution. $\arctan x + C$

Problem 43: $\int \frac{4 \tan x}{1 - \tan^2 x} \, \mathrm{d}x$.

Solution. $- \ln |\cos(2x)| + C$

Individual Match 6 - Semifinals #2:

Problem 44: $\int_{-\infty}^{0} e^{2x} \, \mathrm{d}x$.

Solution. $1/2$

Problem 45: $\int x \ln x \, \mathrm{d}x$.

Solution. $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

Problem 46: $\int \frac{1}{x\sqrt{x^4 - 1}} \, \mathrm{d}x$.

Solution. $\frac{1}{2} \sec^{-1}(x^2) + C$

Individual Match 7 - Finals:

Problem 47: $\int_{-\pi}^{\pi} x^4 \sin(x^3) \, \mathrm{d}x$.

Solution. 0

Problem 48: $\int \frac{x^2 + 1}{x + 1} \, \mathrm{d}x$.

Solution. $\frac{1}{2}x^2 - x + 2 \ln |x + 1| + C$

Problem 49: $\int e^x \sin x \, \mathrm{d}x$.

Solution. $\frac{1}{2}e^x \left(\sin x - \cos x \right) + C$

Problem 50: $\int \frac{1}{x^2(x-1)} \, \mathrm{d}x.$

Solution. $-\ln|x| + \frac{1}{x} + \ln|x-1| + C$

Problem 51: $\int \frac{1}{\sqrt{4x-x^2}} \, \mathrm{d}x.$

Solution. $\arcsin \frac{x-2}{2} + C$

Tiebreakers:

Problem 1: $\int_{\pi/4}^{3\pi/4} \sin x \, \mathrm{d}x$.

Solution. $\sqrt{2}$

Problem 2: $\int \dfrac{\ln x}{x} \, \mathrm{d}x$.

Solution. $\frac{1}{2}(\ln x)^2 + C$

Problem 3: $\int e^{\sqrt{6}x} \, \mathrm{d}x$.

Solution. $\frac{\sqrt{6}}{6} e^{\sqrt{6}x} + C$

Problem 4: $\int_{1-\sqrt{2}}^{1+\sqrt{2}} \sqrt{2} \, \mathrm{d}x$.

Solution. 4

Problem 5: $\int (x+1)(x-1) \, \mathrm{d}x$.

Solution. $\frac{1}{3}x^3 - x + C$

Problem 6: $\int_{25}^{100} \frac{1}{\sqrt{x}} \, \mathrm{d}x$.

Solution. 10

Problem 7: $\int_0^{2\pi} (\cos x - \sin x) \, \mathrm{d}x$.

Solution. 0

Problem 8: $\int_{\ln 2}^{\ln 3} e^x \, \mathrm{d}x$.

Solution. 1

Problem 9: $\int \frac{x^2+1}{x} \, \mathrm{d}x$.

Solution. $\frac{1}{2}x^2 + \ln|x| + C$

Problem 10: $\int \frac{1}{\sqrt{9-x^2}} \, \mathrm{d}x$.

Solution. $\arcsin \frac{x}{3} + C$

Problem 11: $\int 5x\sqrt{x}\, \mathrm{d}x$.

Solution. $2x^{5/2} + C$

Problem 12: $\int 2x \sin(x^2)\, \mathrm{d}x$.

Solution. $-\cos(x^2) + C$

Problem 13: $\int \frac{1}{\cos^2 x}\, \mathrm{d}x$.

Solution. $\tan x + C$

Problem 14: $\int_0^1 \sqrt[20]{x}\, \mathrm{d}x$.

Solution. $\frac{20}{21}$

Problem 15: $\int_0^4 \left[(x+4)^2 - 16x - (4-x)^2 \right]\, \mathrm{d}x$.

Solution. 0

Problem 16: $\int \tan(x) \sec(x)\, \mathrm{d}x$.

Solution. $\sec x + C$

Problem 17: $\int \sqrt{1-x}\, \mathrm{d}x$.

Solution. $-\frac{2}{3}(1-x)^{3/2} + C$

Problem 18: $\int e^2\, \mathrm{d}x$.

Solution. $e^2 x + C$

Problem 19: $\int \frac{\sec^2 x}{1+\tan x}\, \mathrm{d}x$.

Solution. $\ln|1+\tan x| + C$

Problem 20: $\int_0^3 x\, \mathrm{d}x$.

Solution. $9/2$