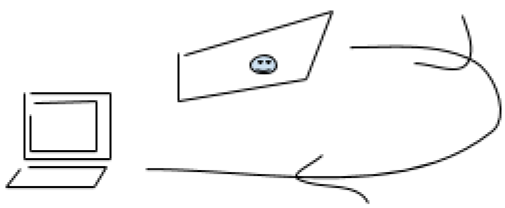
Lecture 2: Thurs Jan 19



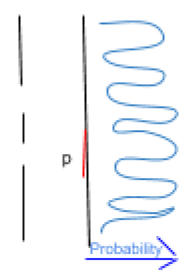
Using time dilation, you could travel billions of years in the future and get results to hard problems. Fun! But you'd need a *LOT* of energy, and if you have that much energy in one place you basically become a black hole. Not so fun!

Computational Universality says that there aren't any computers that could exist which could solve a problem that ours can't already.

The Extended Church-Turing thesis says that if you can't solve a problem in polynomial time on today's computers then no one will ever be able to. Quantum mechanics challenges this. With quantum computers you could solve some problems faster that with a classical computer. With that said, however, there could still be a quantum equivalent to the ECTT.

Feynman said that everything about quantum mechanics could be encapsulated in **The Double Slit Experiment.**

In the double slit experiment, you shoot photons through a wall with two narrow slits. Where the photon lands is probabilistic. If we plot where photons appear on the back wall, some places are very likely, some not.



Note that this itself isn't the weird part, we could totally justify this happening. What's weird is as follows. For some interval:

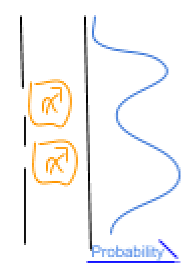
Let P be the probability that the photon lands on the interval.

Let P_1 be the probability that the photon lands on the interval if only slit 1 is open.

Let P_2 be the probability that the photon lands on the interval if only slit 2 is open.

You'd think that $P = P_1 + P_2$, but that's not the case. Dark fringes that exist with two slits end up being hit by photons if only one slit is open.

The weirdness isn't that "God plays dice," but rather that "these aren't normal dice"



You may think to measure which slit the photon went through, but doing so *changes* the measurements into something that makes more sense. Note that this isn't really a matter of having a conscious observer: if the information about which slit the photon went through leaks out in any way, the results go back to looking like they obey classical probability.

As if nature says "What? Me? I didn't do anything!"

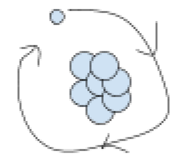
This is called **Decoherence**.

Decoherence is why the usual laws of probability look like they work in everyday life. A cat isn't in superposition because it interacts with normal stuff every day. These interactions essentially leak information about the 'cat system' out.

It's important to note that this relates to particles in isolation. Needing particles to be in isolation is why it's so hard to build a quantum computer.

The story of physics between 1900 and 1926 is that scientists kept finding things that didn't fit with the usual laws of mechanics or probability. They usually came up with hacky solutions that explained a thing without connecting it to much else. That is, until Schrodinger, etc. came up with quantum mechanics.

A normal quantum physics class would go through this process of experimental proof to arrive at quantum mechanics, but we're just going to accept the rules as given and see what we can do from there.



For example take the usual high school model of the electron, rotating around a nucleus in a fixed orbit. Scientes realized that this model would mean that the electron would need to be constantly losing energy until it hit the nucleus. To explain this (and many other phenomenon) scientists modified the laws of probability.

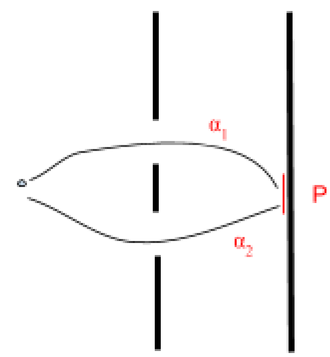
Instead of using probabilities $p \in [0,1]$ they started using **Amplitudes** $\alpha \in \mathbb{C}$. Amplitudes can be positive or negative and can have an imaginary part.

The central claim of quantum mechanics is that to explain a system you'd need to give one amplitude for each particle for each possible configuration of the particles.

The Born Rule says that the probability you see a particular outcome is the absolute value of the amplitude squared.

$$P = |\alpha|^2$$
$$= |\text{The real amplitude}|^2 + |\text{The imaginary amplitude}|^2$$

So let's see how amplitudes being complex leads them to act differently from probabilities. Lets revisit the Double Slit Experiment considering **Interference**. We'll say that:



the total amplitude of a photon landing in a spot	α
is the amplitude of it going through the first slit	α_1
plus the amplitude of it going through the second slit.	α_2

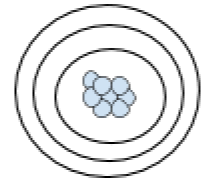
$$P = |\alpha|^2 = |\alpha_1 + \alpha_2|^2$$
$$= |\alpha_1|^2 + |\alpha_2|^2 + 2\,|\alpha_1\alpha_2|$$

If $\alpha_1 = ½$ and $\alpha_2 = -½$, then interference means that if both slits are open $P = 0$, but if only one of them is open, $P = ¼$.

So then to justify the electron not spiraling into the nucleus:

We say that, yes, there are many paths where the electron does do that, but their amplitudes are all positive and negative and they end up canceling each other out.

With some physics we won't cover in this class, you discover that all possibilities where amplitudes don't cancel each out leads to discrete shells where electrons can sit in.



We use **Linear Algebra** to model states of systems as vectors and the evolution of systems in isolation as transformations of vectors.

$$M \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \alpha_1' \\ \alpha_2' \end{pmatrix}$$

For now, we'll consider classical probability. Let's <u>look at flipping a coin</u>:

$$\begin{array}{c l} \begin{pmatrix} p \\ q \end{pmatrix} & \begin{array}{l} \text{tails} \\ \text{heads} \end{array} \end{array} \qquad p, q > 0 \qquad p + q = 1$$

We model this with a vector listing both possibilities and assigning a variable to each.

We can apply a transformation, like <u>turning the coin over</u>.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} q \\ p \end{pmatrix}$$

Turning the coin over means the prob that the coin *was* heads is now the probability that the coin *is* tails. If it helps, you can think of the transformation matrix as:

$$\begin{pmatrix} P(\text{tails}|p) & P(\text{tails}|q) \\ P(\text{heads}|p) & P(\text{heads}|q) \end{pmatrix}$$

We could also <u>flip the coin fairly</u>.

$$\begin{pmatrix} ½ & ½ \\ ½ & ½ \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} ½ \\ ½ \end{pmatrix}$$

Which means regardless of previous position, both possibilities are equally likely.

Let's say we <u>flip the coin, and if(we get heads){we flip again}, but if(we get tails){we turn it to heads}</u>.

$$\begin{pmatrix} 0 & ½ \\ 1 & ½ \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} q/2 \\ p + q/2 \end{pmatrix}$$

Does that make sense?

If we say that p, q are P(tails) and P(heads) after the first flip:

Then the probability the coin will land on tails in the end is:

0 if (it lands on tails on the first flip) and

½ if (it lands on heads and we flip again).

So we sum those values.

The probability that the coin will land on heads in the end is:

1 if(it lands on tails on the first flip) and

½ if (it lands on heads and we flip again).

So we sum those values.

So what matrices CAN be used as transformations?

Firstly, we know that <u>all entries have to be non-negative</u> (because classical probabilities can't be negative).

We can also say that <u>all columns must add to 1</u>, since we need the sum of initial probabilities to equal the sum of the transformed probabilities (both should equal 1).

$$A \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = (i^{th} \text{ column of } A)$$

We can see this clearly by using basis vectors.

A matrix of this form is called a **Stochastic Matrix**.

Now let's say we want to flip two coins, or rather, two bits. For the first coin P(a) = P(getting 0), P(b) = P(getting 1). For the second coin we'll use P(c) and P(d).

$$\begin{pmatrix} a \\ b \end{pmatrix}\begin{matrix} 0 \\ 1 \end{matrix} \quad \begin{pmatrix} c \\ d \end{pmatrix}\begin{matrix} 0 \\ 1 \end{matrix}$$

To combine the two probabilities we'll use the **Tensor Product**.

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} P_{00} \\ P_{01} \\ P_{10} \\ P_{11} \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$$

It's worth noting that not all combinations are possible.

For example:

$$\begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$$

Would mean that

$(\frac{1}{2})(\frac{1}{2}) = abcd = (0)(0)$

Therefore it can't be a tensor product.

Let's say that if(the first bit is 1){we want to flip the second bit}

$$\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \\ \end{array} \begin{pmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \end{pmatrix} \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array}$$

$$\quad 00 \;\; 01 \;\; 10 \;\; 11$$

We'd do:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$$

This is called the **Controlled NOT** it comes up in quantum mechanics.

Quantum mechanics basically follows this process to model states in quantum systems except that it uses amplitudes instead of probabilities.

$$\begin{pmatrix} & & \\ & U & \\ & & \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix}$$

Where $\sum_{i=1}^{n} |A_i|^2 = 1 = \sum_{i=1}^{n} |B_i|^2$ and you're measuring with probability $|\alpha_i|^2$