Theory of Covert Search in Noise Backgrounds Correctly Predicts Asymmetrical Spatial Distributions of Misses and False Alarms

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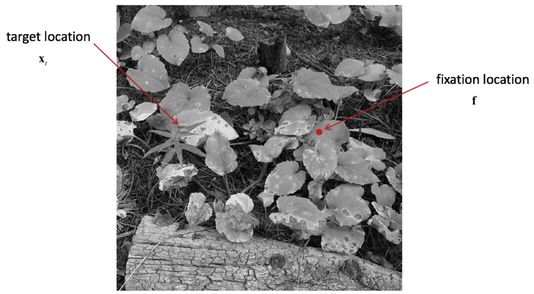
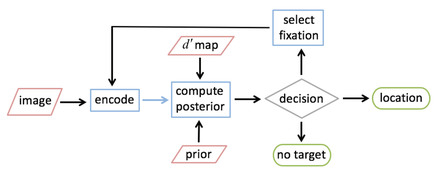
TEXAS

Background and Motivation



Generic Framework



Key Components of Theory

- **Target and background properties** $L(\mathbf{x}), C(\mathbf{x}), S(\mathbf{x}), \|T_t(\mathbf{x})\|$
- **Variation in retinal processing across the visual field** $F(\mathbf{x})$
- **Intrinsic location uncertainty** $U(\mathbf{x})$
- **Covert selection and attenuation** $prior(\mathbf{x}), A(\mathbf{x})$
- **Fixation selection**

Covert Search Theory

$$R(\mathbf{x}) \sim \begin{cases} \mathcal{N}\left[d'(\mathbf{x})\rho_{TT}(\mathbf{x}-\mathbf{x}_T),1\right] & present \\ \mathcal{N}[0,1] & absent \end{cases}$$ normalized template response map

$$d'(\mathbf{x}) \propto A(\mathbf{x})\left[a\frac{\|T_t(\mathbf{x})\|}{L(\mathbf{x})C(\mathbf{x})S(\mathbf{x})}F(\mathbf{x})\right]^{\beta}$$ d' map with no location uncertainty

$$l(\mathbf{x}) = d'(\mathbf{x})R(\mathbf{x}) - 0.5d'^2(\mathbf{x})$$ log likelihood ratio

$$\max_{\mathbf{x}}\left[l(\mathbf{x}) + \ln prior\left(\mathbf{x}|U(\mathbf{x})\right)\right] > \gamma$$ present/absent decision rule

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}}\left[l(\mathbf{x}) + \ln prior\left(\mathbf{x}|U(\mathbf{x})\right)\right]$$ location if present

$U(\mathbf{x})$ intrinsic location uncertainty map

Current Goal: Test of Covert Theory in Noise Backgrounds

$$d'(\mathbf{x}) \propto \left[\frac{a}{\sigma}F(\mathbf{x})\right]^{\beta}$$ white noise backgrounds and no covert attenuation: $A(\mathbf{x}) = 1$

$prior\left(\mathbf{x}|U(\mathbf{x})\right)$ prior probability over location

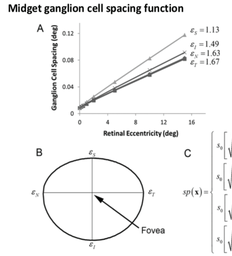
$U(\mathbf{x})$ intrinsic location uncertainty map

Visual Field Detectability

Ganglion Cells: Critical Visual Information Bottleneck

Midget ganglion cell spacing function

Wässle et al. *(1989) Nature*



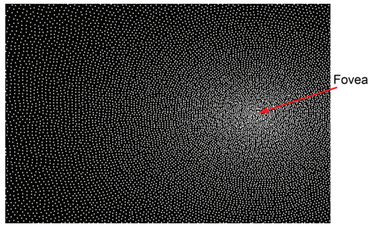
Average from 6 human eyes

Drasdo et al. (2007) *Vis. Res.*

$$sp(\mathbf{x}) = \begin{cases} s_0\left[\sqrt{\frac{x^2}{\varepsilon_x^2}+\frac{y^2}{\varepsilon_y^2}}+1\right] & x \ge 0, y \ge 0 \\ s_0\left[\sqrt{\frac{x^2}{\varepsilon_x^2}+\frac{y^2}{\varepsilon_y^2}}+1\right] & x < 0, y \ge 0 \\ s_0\left[\sqrt{\frac{x^2}{\varepsilon_x^2}+\frac{y^2}{\varepsilon_y^2}}+1\right] & x \ge 0, y < 0 \\ s_0\left[\sqrt{\frac{x^2}{\varepsilon_x^2}+\frac{y^2}{\varepsilon_y^2}}+1\right] & x < 0, y < 0 \end{cases}$$

Bradley et al. (2014) *JOV*

Detectability Falls Inversely with GC Spacing



Foveation factor

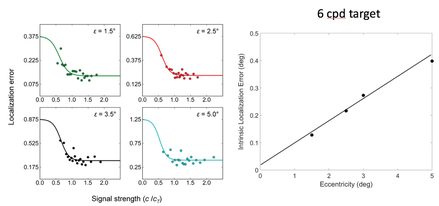
$$F(\mathbf{x}) = k_0\frac{1+k_{sp}sp(0)}{1+k_{sp}sp(\mathbf{x})}$$

k_{sp} = target dependent constant

Rovamo et al. (1978) *Nature*
Bradley et al. (2014) *JOV*

Uncertainty

Intrinsic Position Uncertainty Increases in Proportion GC Spacing

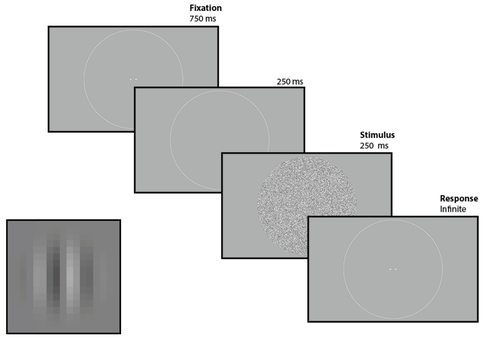


$U(\mathbf{x}) = k_U sp(\mathbf{x})$ k_u = target dependent constant $k_u = 26.9$

Michel & Geisler (2011) *JOV*

$$d'_U(\mathbf{x}) = \ln\left(\frac{\exp\left[d'(\mathbf{x})\right]+U(\mathbf{x})}{1+U(\mathbf{x})}\right)$$ closed-form approximation

Detection/Search Experiments



Target: 6 cpd raised cosine sine wave stimulus.

Background: White Noise (20% RMS). Pixels subtend 2 arcmin.

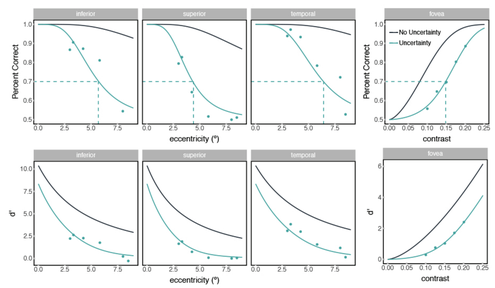
Foveal Task: 5 levels of contrast (blocked). Yes/No Task. Fixed noise (20% RMS) .

Peripheral Task: 5 levels of eccentricity (blocked). Yes/No. Fixed noise (20% RMS), fixed signal (amplitude 35%).

Search Task: Polar sampling. Fixed signal, fixed noise.

Detection Results

Psychometric Functions
Detection. White Noise. 6-cpd Target



Identification Performance With Intrinsic Uncertainty

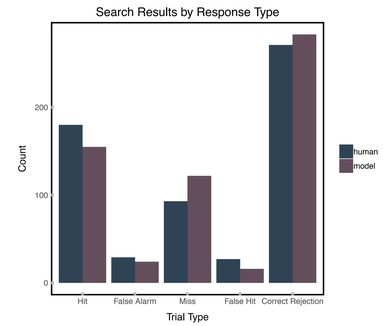
$$d'_U(\mathbf{x}) = \ln\left(\frac{\exp\left[d'(\mathbf{x})\right]+U(\mathbf{x})}{1+U(\mathbf{x})}\right)$$ Closed-form approximation Geisler (2018) JoV

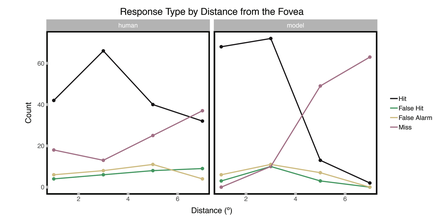
$$d'(\mathbf{x}) = \left(\frac{aF(x)}{\sigma_b}\right)^{\beta} \qquad F(\mathbf{x}) = k_0\frac{1+k_{sp}sp(0)}{1+k_{sp}sp(\mathbf{x})}$$

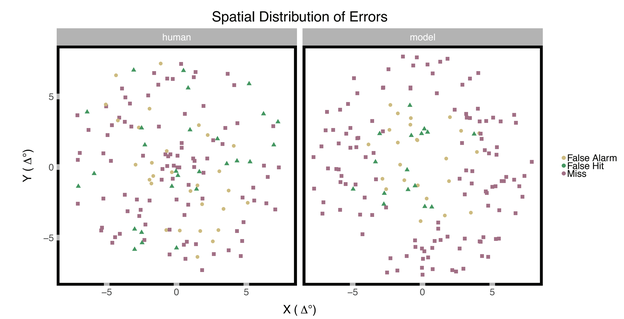
Unknowns to estimate from preliminary measurements:

k_0, k_{sp}, β, k_u $k_0 = 3.2$ $k_{sp} = 1.07$ $\beta = 1.56$ $k_u = 26.9$

Covert Search Results

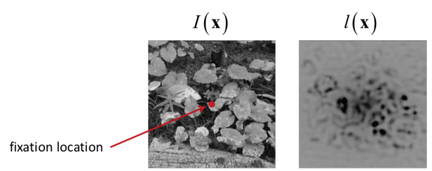






Future Directions and Conclusions

Covert Search in Natural Backgrounds



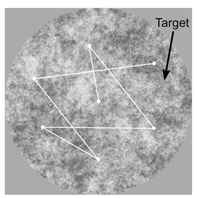
target amplitude foveation factor

$$d'(\mathbf{x}) \propto A(\mathbf{x})\left[a\frac{\|T_t(\mathbf{x})\|}{L(\mathbf{x})C(\mathbf{x})S(\mathbf{x})}F(\mathbf{x})\right]^{\beta}$$

masking factors

$\mathbf{f}$ = fixation location

Multiple Fixation Search



Covert Search Model + Fixation Selection

- **Covert search in noise backgrounds is approximated by an optimal search strategy.**
- **Absence of errors in the fovea differs from human spatial distribution of errors.**
- **Retinal factors and uncertainty alone may not explain human covert search.**