Homework #1

Assigned: May 5, 2003
Due: March 20, 2003
(collection box outside 3561)

Directions

Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to talk over the problems with classmates but please answer the questions on your own.

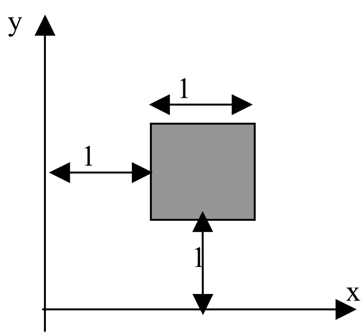
Name:______________Key______________

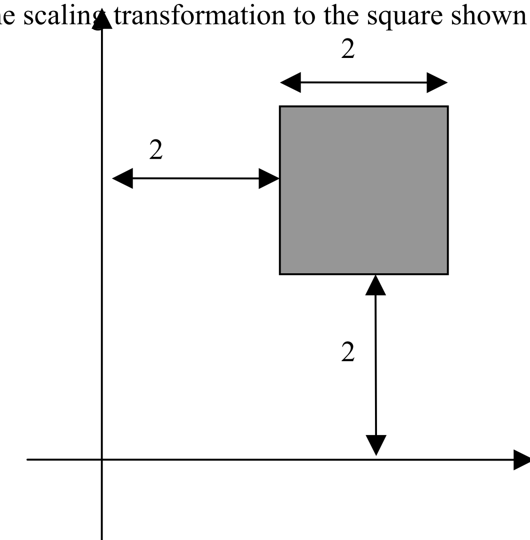
1) *Uniform scalings*. Consider uniform scalings in 2D Euclidean space, i.e., on the plane.

a. Write the 3x3 homogeneous-coordinate matrix that corresponds to a uniform scaling by a factor *k*.

$$\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

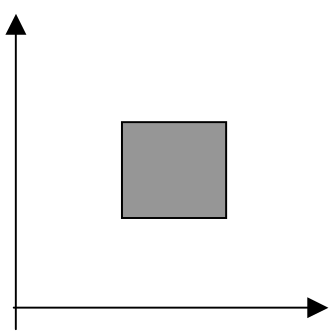
b. For $k = 2$ sketch the result of applying the scaling transformation to the square shown below.





c. The standard scaling transformation you used above is relative to the origin. Suppose we wanted the scaling to be relative to the center of the square, with the effect shown below. Or, more generally, we wanted to scale relative to some arbitrary point with coordinates (a,b). What would be the corresponding matrix in the homogeneous coordinates?

$$\begin{bmatrix} 1 & 0 & fa \\ 0 & 1 & fb \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$



2) *World to screen transformation*. The world to screen transformation is also called *viewing transformation*. It is the composition of a projection onto the 2D *viewplane* with a 2D transformation between the viewplane and the *viewport*, which is the region of the screen where the image is to appear. There are several ways of specifying a set of *viewing parameters* to define the viewing transformation. The specification should be easy to understand by users, and therefore should refer to entities whose geometrical meaning is clear. Computation of the various matrices involved should be transparent to users. In one particular system, the viewing transformation is specified by the following parameters:

- The viewpoint **p**.
- A sphere of radius R and centered at a reference point **r**.

In this simple system, the user should ensure that the sphere he or she specifies encloses the object to be displayed, and that the sphere does not enclose the viewpoint. In addition, the following assumptions are made:

- The reference point is the origin of the (x_v, y_v) coordinate system in the viewplane.
- The viewpoint and the reference point define the line of sight. The line of sight is perpendicular to the viewplane.
- The orientation of the viewplane coordinate system is as defined in Fig. 1. We assume that the entire configuration (viewpoint, viewplane, sphere, and reference point) has been translated so that the reference point is at the origin. The coordinate system (X_t, Y_t, Z_t) is constructed as follows: The *z* axis coincides with the line of sight **rv**. The *x* axis is tangent to the parallel to the sphere at the point where the line of sight intersects the sphere. And the *y* axis is tangent to the meridian at the same point. The coordinate system (X_v, Y_v, Z_v) is (X_t, Y_t, Z_t) translated to the origin, and therefore the two coordinate systems have the same orientation.
- The viewpoint coincides with the entire window, whose width *W* and height *H* are known to the system through interaction with the window manager. (The window need not cover the whole screen).

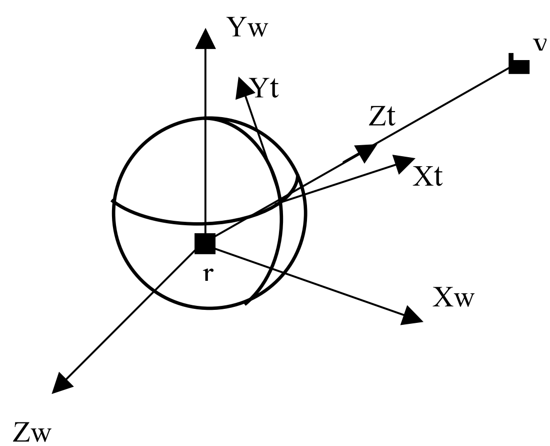


Figure 1

The world to screen transformation consists of the following steps:

a. Translate the center of the sphere to the origin. Let M_1 be the homogeneous transform that corresponds to a translation by the vector –**r**. Let r = (rx,ry,rz). Write M_1.

$$\begin{bmatrix} 1 & 0 & 0 & -r_x \\ 0 & 1 & 0 & -r_y \\ 0 & 0 & 1 & -r_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. When the viewpoint is on the positive y_w axis (top view) by convention we set

$$X_v = X_w,\ Y_v = -Z_w,\ Z_v = Y_w.$$

And for a bottom view, when the viewpoint is on the negative y_w axis we set

$$X_v = X_w,\ Y_v = Z_w,\ Z_v = -Y_w$$

When the viewpoint is not on the y axis of the world coordinate system, what are X_v, Y_v, Z_v?

Z_v	=	v-r /	‖v-r‖
X_v	=	Y_w x Z_v /	‖Y_wxZ_v‖
Y_v	=	Z_v x X_v	

c. Rotate the axes so that the viewplane coincides with the world coordinate system. To do this, first compute the viewplane coordinate system (X_v,Y_v,Z_v). Then, construct the matrix that corresponds to the world to viewplane coordinate system transformation and then invert it. Let this rotation matrix be M_2. Write M_2.

$$\begin{bmatrix} X_v^t & 0 \\ Y_v^t & 0 \\ Z_v^t & 0 \\ 0 & 1 \end{bmatrix}$$

d. Apply a perspective transform M_3 with viewpoint on the z axis at a distance d = ||v – r ||. Write M_3.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix}$$

e. Project orthographically on the xy plane. Let M_4 be this projection. Write M_4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

f. Scale to fit in the viewport. Suppose k_1 and k_2 are the scaling factors in x and y direction, respectively. Construct this transform M_5.

$$\begin{bmatrix} k & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

g. Let W and H be the width and height of the viewport, and M_6 be the corresponding translation matrix. Write M_6.

$$\begin{bmatrix} 1 & 0 & 0 & W/2 \\ 0 & 1 & 0 & H/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

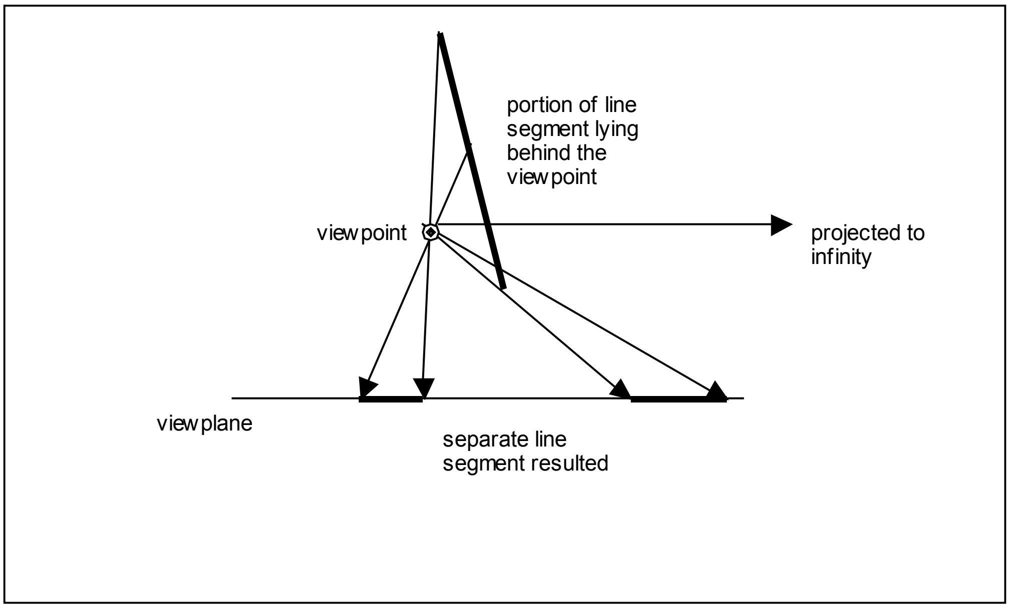
h. Compute the final viewing matrix M_{ws} by composing all the previous transformations:

$$M_6 M_5 M_4 M_3 M_2 M_1$$

3) Wireframe displays. Suppose you are given a wireframe model, i.e., a set of straight-line segments, and you want to display it in perspective. Should you clip before or after the projection transformation? Does it make a difference? Why?

A given set of straight-line segments should be clipped before computing the perspective projection.

There is a difference if there exists some point lying on some line segment of the given set, which is behind the viewpoint. Such points maybe projected to the line of infinity, or making the projected line segment disconnected into separate pieces.



4) *Image Filters*. Each of the matrices below is actually a convolution filter for image processing. In addition, we have included a median filter. Next to each filter, identify the image (next page) that would result from applying the filter. In addition, write all the characteristics from the following list that apply:

Mean blurring	Blurring in x	Edge enhancing
Gaussian blurring	Blurring in y	Rotating
Edge-preserving blurring	Gradient in x	Translating
	Gradient in y	Identity (no effect)

a. $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

Image:___3________

_Gaussian blurring___

b. $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

Image:____2_______

_Edge enhancing___

c. $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Image:__6_________

_Gradient in x___

d. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Image:___1______

_Identity________

e. $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Image:____8______

_Gradient in y_____

f. $\frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

Image:__5________

_Mean blurring____

g. $\frac{1}{5}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Image:____4______

_Blurring in x_____

h. Median filter over 4x4 region.

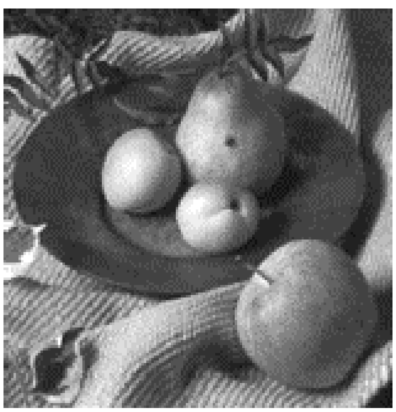
Image:___7________

Edge-preserving blurring

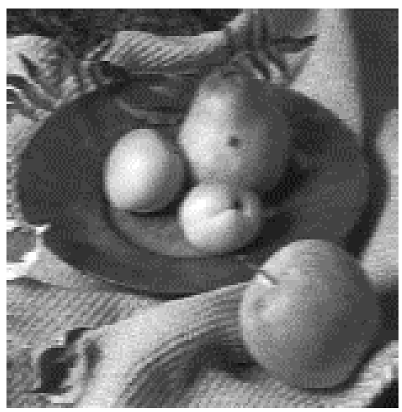
1)

1. Original image

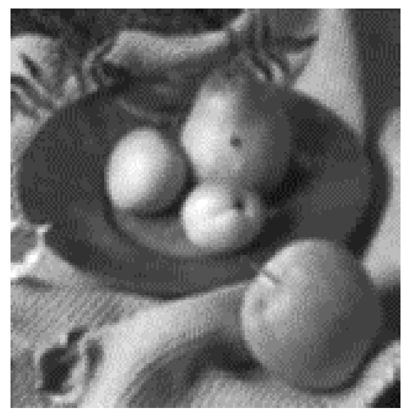
2



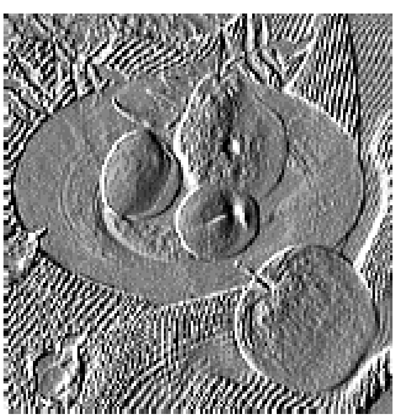
3



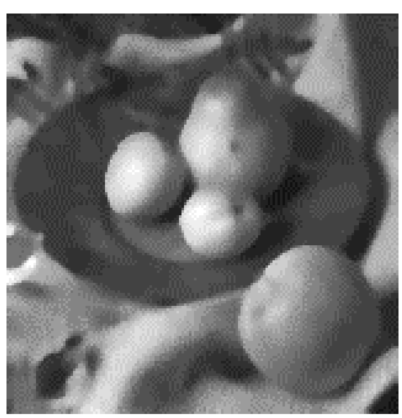
4



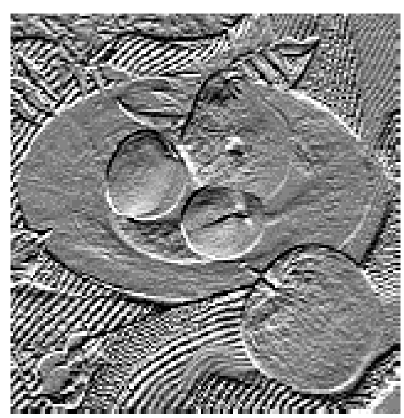
5



6 [after adding 128 to each pixel]

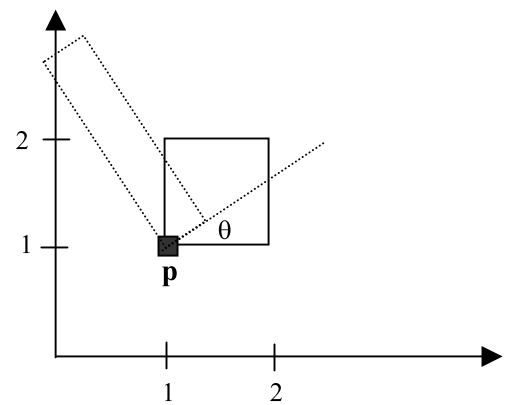


7



8 [after adding 128 to each pixel]

5) *Transformations*. Let's assume you want to perform a scale **S**(1/2,2), and then perform a rotation **R**(θ), with respect to a point **p** = (1,1). Fig. 2 shows how a unit cube (solid) sitting at p would be transformed into a rotated (dotted) by this process.



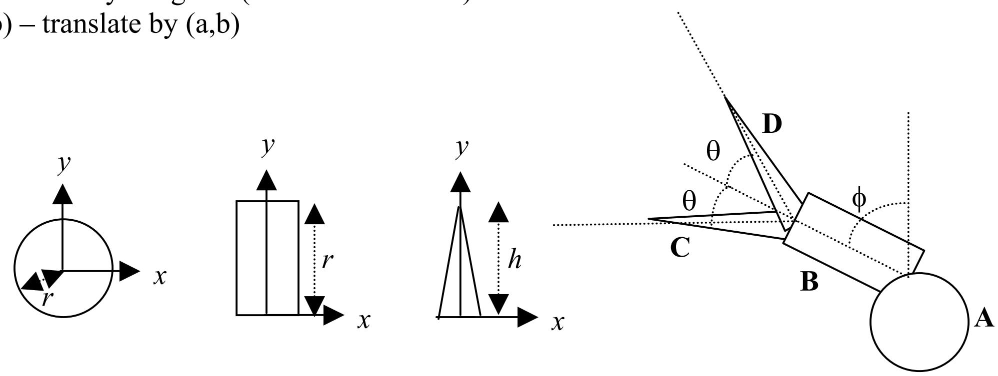
Write out the sequence of matrics, in terms of **S**(1/2,2), **R**(θ), and translation, **T**(c,d), that would be needed to perform this transformation. You may leave all matrices in symbolic form – do **not** expand them into 3x3 matrices. θ is the only variable, you must fill in all other arguments as numbers.

T(1,1)*R(θ)*S(1/2,2)*T(-1,-1)

6) *Hierarchies*. Suppose you want to model the pinching figure shown in Fig. 3. The pincher is made of four parts, labeled A, B, C, and D and each part is drawn as one of the three primitives as given below.

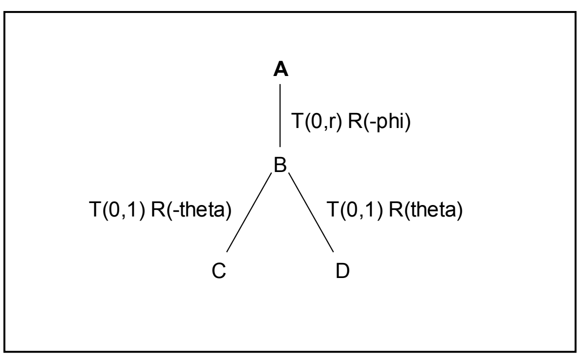
The following transformations are also available to you:

R(t) – rotate by t degrees (counter clockwise)
T(a,b) – translate by (a,b)



Primitives Pincher diagram

a. Construct a tree to specify the pincher that is rooted at **A**. Along each of the edges of the tree, write expressions for the transformations that are applied along that edge, using the notation given above (you do not need to write out the matrices). Remember that order is important, since matrix multiplication is not commutative in general!



b. Write out the full transformation expression for the part labeled **C**.
T(0,r)R(-phi)T(0,1)R(-theta)

7) *Projections*. Complete the following table, summarizing properties of these two types of projections.

Property	Perspective	Parallel
Parallel lines remain parallel [Y/N]	____N_____	_____Y_____
Angles are preserved [Y/N]	____N____	_____Y____
Lengths vary with distance to eye [Y/N]	____Y_____	_____N_____

Under perspective projections, any set of parallel lines that are not parallel to the projection plane will converge to a "vanishing point." Vanishing points of lines parallel to a principal axis *x*, *y*, or *z* are called "principal vanishing points".

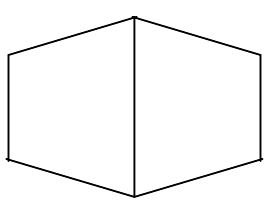
a. How many different vanishing points can a perspective drawing have?

infinite

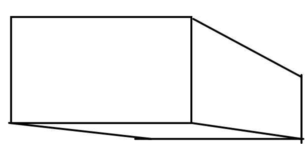
b. How many different principal vanishing points can a perspective drawing have?

three

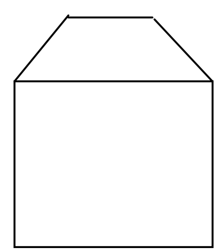
c. How many vanishing points are in each of the following drawings?



2



1



1

8) Short answer. Please provide a short answer (typically one or two sentences) to each of the following questions. In each case, you must justify your answer.

a. How are the electrons coming out of the red, green, blue electron guns of a color monitor different?

They aren't. In fact, all the electrons that are cooked off are the same, but they strike different types of phosphors that emit light with distinctly different spectra

b. At 24 bits per pixel (true color), can a monitor produce every color perceptible to the human eye?

No; an RGB monitor only can express combinations of three colors and we cannot express the entire gamut using these.

c. Is the median filter a convolution filter?

It is not, since we cannot express it as a function of position.

d. Why do we use 4x4 matrix for transformation in 3D space?

Because we are using homogenous coordinates; by adding the 4^{th} coordinate to our position vector and using the 4^{th} column in the transformation matrix we can express translation just as easily as rotation, scaling, etc.

-end-