Computer Science 4411: Computer Graphics
Spring Semester 2018 Final Examination
Instructor: CK Tang
Tuesday, May 29, 2018
4:30PM - 7:30PM at LG4 Student Common Rooms (Lift 3)

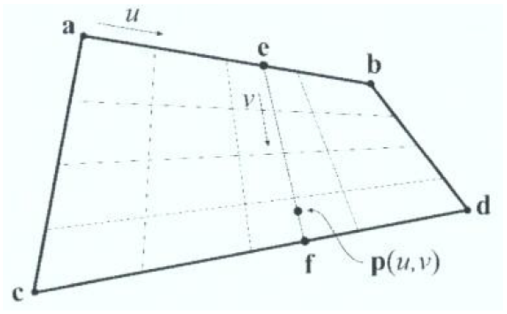
This is a **CLOSED-BOOK-CLOSED-NOTES** exam consisting of eighty (80) multiple choice questions and eight (8) long questions. **NO** computers or cellphones except approved scientific calculators are allowed. Follow the instructions carefully. Do not spend too much time on a single problem unless you have attempted all other problems. Unless otherwise stated only the simplest and correct expression can score full credits in mathematical problems. Please write legibly and keep the exam booklet stapled, except you may tear off the last sheet for rough work.

KEY

Problem	Points	your score
1 MULTIPLE CHOICES	80	
2 PARAMETRIC SURFACES	5	
3 ILLUMINATION	10	
4 ALL ABOUT MATRICES	15	
5 SHADING	10	
6 THREE DIMENSIONAL GEOMETRY	15	
7 GEOMETRIC TRANSFORM	15	
8 RAY INTERSECTIONS	15	
9 PERSPECTIVE PROJECTION	15	
Total	180	

2 Parametric Surfaces

The standard surface parameterization for a bilinear patch is $\mathbf{p}(u,v) = (1-u)(1-v)\mathbf{a} + (u)(1-v)\mathbf{b} + (1-u)(v)\mathbf{c} + (u)(v)\mathbf{d}$, with $(u,v) \in [0,1]^2$. Plugging in 0 and 1 for (u,v) quickly shows that $\mathbf{p}(0,0) = \mathbf{a}$, $\mathbf{p}(1,0) = \mathbf{b}$, $\mathbf{p}(0,1) = \mathbf{c}$, and $\mathbf{p}(1,1) = \mathbf{d}$. Come up with another surface parameterization that rescales the domain from $[0,1]^2$ to $[-1,1]^2$. That is, write out an equation for $\mathbf{p}'(-1,-1) = \mathbf{a}$, $\mathbf{p}'(1,-1) = \mathbf{b}$, $\mathbf{p}'(-1,1) = \mathbf{c}$, and $\mathbf{p}'(1,1) = \mathbf{d}$. Your answer $\mathbf{p}'(u,v)$ should be in terms of $u, v, \mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ but yu don't need to expand it.



We want to remap $[-1,1]$ to $[0,1]$, then use those numbers as input to $\mathbf{p}(u,v)$. That is:

$$\mathbf{p}'(u,v) = \mathbf{p}(\frac{u+1}{2}, \frac{v+1}{2})$$

Expanding:

$$\mathbf{p}'(u,v) = (1-\frac{u+1}{2})(1-(\frac{v+1}{2}))\mathbf{a}+(\frac{u+1}{2})(1-(\frac{v+1}{2}))\mathbf{b}+(1-\frac{u+1}{2})(\frac{v+1}{2})\mathbf{c}+(\frac{u+1}{2})(\frac{v+1}{2})\mathbf{d}$$

3 Colors and Illumination

3.1 Sunlight (6 pts)

Suppose that a shiny ground plane, $y = -2$, is illuminated by sunlight. Let the sun be in direction $(1, 2, 3)$ and let the camera be at position $(x, y, z) = (-6, 4, -4)$. Determine the position on the ground plane at which the peak of the highlight occurs.

The reflection direction $\mathbf{r}$ is $(-1, 2, -3)$. This is easy to see if you understand how the reflection direction is defined.

We first consider the ray from the reflection point to the eye:

$$(x, -2, z) = (-6, 4, -4) + t(-1, 2, -3)$$

Use the y coordinate to solve for t gives $t = -3$. Substituting gives $(x, y, z) = (-3, -2, 5)$.

3.2 Procedural Shading (4 pts)

Describe the visual effect of using the procedural shading formula:

$$I_{RGB}(x) = \left(\frac{1 + \mathbf{n}(\mathbf{x}) \cdot \hat{\mathbf{l}}}{2}\right)(1, 0, 0) + \left(\frac{1 - \mathbf{n}(\mathbf{x}) \cdot \hat{\mathbf{l}}}{2}\right)(0, 1, 1)$$

where $\hat{\mathbf{l}}$ is the unit light source direction.

The surface would be pure red at points where the normal points in direction $\mathbf{l}$ and it would be cyan at places where the normal direction is opposite to direction $\mathbf{l}$.

For all other points, the color would be vary between pure red and pure cyan.

The color would be neutral $(0.5, 0.5, 0.5)$ at points where the normal is perpendicular to the light source.

4 All About Matrices

4.1 OpenGL (5 pts)

Consider a sequence of OpenGL instructions that does the following. (You do not need to write out the instructions.) It defines a unit square object centered at the origin in the $z = 0$ plane. The unit square is transformed as follows. First, it is scaled by 2 in the y direction. Then, it is rotated by 45 degrees around the x axis. Then, it is translated by -5 in the z direction. What is the `GL_MODELVIEW` matrix that performs this transformation? (It is fine if your answer is a sequence of matrices.)

Assume the world coordinates are the same as camera coordinates, so the world to camera mapping is the identity.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.2 Camera Transfer (5 pts)

Suppose you have two cameras. Let $\mathbf{p}_1$ and $\mathbf{p}_2$ be the camera positions and let $\{\hat{\mathbf{x}}_1, \hat{\mathbf{y}}_1, \hat{\mathbf{z}}_1\}$ and $\{\hat{\mathbf{x}}_2, \hat{\mathbf{y}}_2, \hat{\mathbf{z}}_2\}$ be the respective camera axes. All of these are expressed in a world coordinate system. Write out the sequence of matrices that takes any scene point that is expressed in camera 1's coordinate system and maps it to camera 2's coordinate system.

The main idea is to map from camera 1 coordinates to world coordinates and then map from world coordinates to camera 2 coordinates.

$$\begin{bmatrix} & \hat{\mathbf{x}}_2^T & & 0 \\ & \hat{\mathbf{y}}_2^T & & 0 \\ & \hat{\mathbf{z}}_2^T & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & -\mathbf{p}_2 \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \mathbf{p}_1 \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & & 0 \\ \hat{\mathbf{x}}_1 & \hat{\mathbf{y}}_1 & \hat{\mathbf{z}}_1 & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.3 Texture Coordinates (5 pts)

A 300×200 texture image is mapped onto a 3D quad which is a parallelogram defined by vertices $\mathbf{p}_{00}, \mathbf{p}_{10}, \mathbf{p}_{01}, \mathbf{p}_{11}$. Pixel $(0, 0)$ in the texture maps to $\mathbf{p}_{00}$. Pixel $(299, 0)$ maps to $\mathbf{p}_{10}$, etc. What is the mapping from pixel (s_p, t_p) in the texture image to 3D coordinates on the quad?

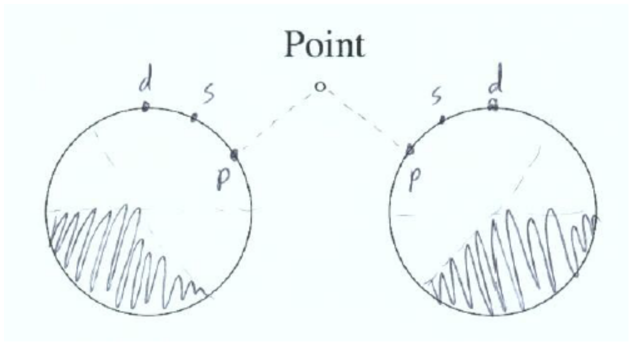
Write you answer as a product of matrices and be sure to specify the dimensions of the matrices.

$$\begin{bmatrix} \mathbf{p}_{10} - \mathbf{p}_{00} & \mathbf{p}_{01} - \mathbf{p}_{00} & \mathbf{p}_{00} \\ 0 & 0 & 1 \end{bmatrix}_{4\times 3} \begin{bmatrix} \frac{1}{299} & 0 & 0 \\ 0 & \frac{1}{199} & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

It was fine if you used 300 and 200 for the scaling matrix instead of 299 and 199.

5 Lighting

Consider the two-dimensional projection of a simple scene below, containing two spheres, a point light, and a vector that points toward either a directional light source at infinity or a far-away viewpoint (depending on the subproblem).



(a) (2 pts)If the two spheres have matte surfaces (i.e., perfectly diffuse surfaces), indicate the brightest spots on the spheres if only the directional light is turned on. Label the points with "d".

(b) (2 pts) If the two spheres have matte surfaces (i.e., perfectly diffuse surfaces), indicate the brightest spots on the spheres if only the point light is turned on. Label the points with "p".

(c) (3 pts) Shade the portion of the spheres, if any, that is completely unlit if both lights are on but there is no ambient light.

(d) (3 pts) Now assume that the point light is on, and the direction vector points to a far-away viewpoint or camera. If the spheres are specular, where will the center of the highlight be on each sphere? Label these points with "s", and also provide a brief explanation of why you think that is the right location.

s is the point where light coming from the point light reflects off the sphere straight to the viewer.

6 Three Dimensional Geometry

(a) (5 pts) Find the equation $\mathbf{n} \cdot \mathbf{p} = d$ of the plane which goes through the origin and is orthogonal to the planes

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \cdot \mathbf{p} = 1 \quad \text{and} \quad \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \cdot \mathbf{p} = 1$$

In your answer, $\mathbf{n}$ need not be unit vector.

The normal of the plane must be orthogonal to the normals of the given planes. Furthermore the distance of the plane to the origin is zero. Hence:

$$\mathbf{n} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -6 \end{bmatrix}, d = 0$$

(b) (10 pts) Given a point $\mathbf{q}_0$ and a vector $\mathbf{v}$ defining a line $\mathbf{q}(t) = \mathbf{q}_0 + t\mathbf{v}$ and a vector $\mathbf{n}$ and scalar value d defining a plane $\mathbf{n} \cdot \mathbf{p} = d$. Derive the parametric equation of the line obtained by orthographically projecting the line $\mathbf{q}(t)$ onto the given plane along $\mathbf{n}$. The spatial vectors $\mathbf{v}$ and $\mathbf{n}$ are unit vectors, and you can safely assume the line and the plane are not parallel. Your equation must be in the form $\mathbf{p}(t) = \mathbf{q}_{\text{onPlane}} + t\mathbf{v}_{\text{proj}}$, where $\mathbf{v}_{\text{proj}}$ is the direction of the projected line and $\mathbf{q}_{\text{onPlane}}$ is a point on this line. Express $\mathbf{q}_{\text{onPlane}}$ and $\mathbf{v}_{\text{proj}}$ in terms of $\mathbf{q}_0, \mathbf{v}, \mathbf{n}$ and/or d. Clearly explain the different steps of your derivation.

First we have to compute a point on the projected line – one such point is the intersection point of the line with the plane. The plane and line intersect if and only if

$\mathbf{n} \cdot (\mathbf{q}_0 + t_{\text{int}}\mathbf{v}) = d \leftrightarrow t_{\text{int}} = \frac{d - \mathbf{n} \cdot \mathbf{q}_0}{\mathbf{n} \cdot \mathbf{v}}$

Hence the intersection point is

$$\mathbf{q}_{\text{onPlane}} = \mathbf{q}_0 + t_{\text{int}}\mathbf{v} = \mathbf{q}_0 + \frac{d - \mathbf{n} \cdot \mathbf{q}_0}{\mathbf{n} \cdot \mathbf{v}}\mathbf{v}$$

The direction of the line is equal to the direction of v projected onto the plane. This is

$$\mathbf{v}_{\text{proj}} = \mathbf{v} - \frac{\mathbf{v} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}}\mathbf{n}$$

7 Geometric Transform

In class, we learned rotation by Euler angles, that is,

$$\begin{aligned}
R_x(\alpha) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
R_y(\beta) &= \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
R_z(\gamma) &= \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
R(\alpha,\beta,\gamma) &= R_x R_y R_z \\
&= \begin{bmatrix} \cos\gamma\cos\beta & \sin\gamma\cos\alpha + \cos\gamma\sin\beta\sin\alpha & \sin\gamma\sin\alpha - \cos\gamma\sin\beta\cos\alpha & 0 \\ -\sin\gamma\cos\beta & \cos\gamma\cos\alpha - \sin\gamma\sin\beta\sin\alpha & \cos\gamma\sin\alpha + \sin\gamma\sin\beta\cos\alpha & 0 \\ \sin\beta & -\cos\beta\sin\alpha & \cos\beta\cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

One problem associated with rotation using Euler angle is *Gimbal lock*.

(a) (5 pts) When $\beta = \frac{\pi}{2}$, What is $R(\alpha,\beta,\gamma)$? To ensure full credit, the *simplest expression* is required. The following trigonometric rules you learned in high school may be useful:

$$\begin{aligned}
\sin(a \pm b) &= \sin a\cos b \pm \cos a\sin b \\
\cos(a \pm b) &= \cos a\cos b \mp \sin a\sin b
\end{aligned}$$

How many degree of freedoms are lost? Show your work clearly on the next page, and write the answers in the space provided below:

Your simplest expression for $R(\alpha,\beta,\gamma) =$

$$\begin{bmatrix} 0 & \sin(\gamma+\alpha) & -\cos(\gamma+\alpha) & 0 \\ 0 & \cos(\gamma+\alpha) & \sin(\gamma+\alpha) & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The number of degrees of freedom lost =

The number of degrees of freedom lost = 1

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when $\beta = \frac{\pi}{2}$, $R(\alpha, \beta, \gamma)$ is

$$\begin{bmatrix} 0 & \sin\gamma\cos\alpha + \cos\gamma\sin\alpha & -(-\sin\gamma\sin\alpha + \cos\gamma\cos\alpha) & 0 \\ 0 & \cos\gamma\cos\alpha - \sin\gamma\sin\alpha & \cos\gamma\sin\alpha + \sin\gamma\cos\alpha & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \sin\theta & -\cos\theta & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\theta = \gamma + \alpha$.

To avoid Gimbal lock, we can represent rotations by *quaternions*. The representation is very similar to that of complex numbers. As you know, the system of complex numbers is defined in terms of $\mathbf{i}$, a square root of -1:

$$\mathbf{i} \cdot \mathbf{i} = -1$$

Quaternions are an extension of complex numbers. Instead of just $\mathbf{i}$, we have three different numbers that are all square roots of -1 labeled $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$

$$\mathbf{i} \cdot \mathbf{i} = -1, \mathbf{j} \cdot \mathbf{j} = -1, \mathbf{k} \cdot \mathbf{k} = -1,$$

The conjugate and magnitude of a quaternion are found in much the same way as complex conjugate and magnitude.

$$\begin{aligned} \mathbf{q} &= w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \\ &= [x\ y\ z\ w] \\ &= (s, \mathbf{v}) \text{ where } s = w, \text{ and } \mathbf{v} = [x\ y\ z] \\ \mathbf{q}' &= w - x\mathbf{i} - y\mathbf{j} - z\mathbf{k} \end{aligned}$$

The quaternion that computes a rotation about the unit vector $\mathbf{u}$ by an angle θ is

$$\begin{aligned} \mathbf{q} &= (s, \mathbf{v}) \\ s &= \cos\frac{\theta}{2} \\ \mathbf{v} &= \mathbf{u}\sin\frac{\theta}{2} \end{aligned}$$

We represent a point $\mathbf{p}$ in space by the quaternion $\mathbf{P} = (0, \mathbf{p})$. The desired rotation of that point is given by

$$\mathbf{P}_{\mathbf{rotated}} = \mathbf{q}'\mathbf{P}\mathbf{q} \tag{1}$$

(b) (6 pts) Given two quaternions $\mathbf{q_1} = (a, \mathbf{v})$ and $\mathbf{q_2} = (b, \mathbf{u})$, show:

$$\mathbf{q_1 q_2} = (ab - \mathbf{v} \cdot \mathbf{u}, a\mathbf{u} + b\mathbf{v} + \mathbf{v} \times \mathbf{u})$$

Show your work clearly.

$$\begin{aligned}
\mathbf{q}_1 \cdot \mathbf{q}_2 &= (a, \mathbf{v}) \cdot (b, \mathbf{u}) \\
&= (a + i\mathbf{v}_x + j\mathbf{v}_y + k\mathbf{v}_z) \cdot (b + i\mathbf{u}_x + j\mathbf{u}_y + k\mathbf{u}_z) \\
&= ab + ai\mathbf{u}_x + aj\mathbf{u}_y + ak\mathbf{u}_z + \\
&\quad i\mathbf{v}_x b + i\mathbf{v}_x i\mathbf{u}_x + i\mathbf{v}_x j\mathbf{u}_y + i\mathbf{v}_x k\mathbf{u}_z + \\
&\quad j\mathbf{v}_y b + j\mathbf{v}_y i\mathbf{u}_x + j\mathbf{v}_y j\mathbf{u}_y + j\mathbf{v}_y k\mathbf{u}_z + \\
&\quad k\mathbf{v}_z b + k\mathbf{v}_z i\mathbf{u}_x + k\mathbf{v}_z j\mathbf{u}_y + k\mathbf{v}_z k\mathbf{u}_z \\
&\quad = ab + ii\mathbf{v}_x\mathbf{u}_x + jj\mathbf{v}_y\mathbf{u}_y + kk\mathbf{v}_z\mathbf{u}_z + \\
&\quad ai\mathbf{u}_x + aj\mathbf{u}_y + ak\mathbf{u}_z + bi\mathbf{v}_x + bj\mathbf{v}_y + bk\mathbf{v}_z + \\
&\quad jk\mathbf{v}_y\mathbf{u}_z + kj\mathbf{v}_z\mathbf{v}_y + ik\mathbf{v}_x\mathbf{u}_z + ki\mathbf{v}_z\mathbf{u}_x + \\
&\quad ij\mathbf{v}_x\mathbf{u}_y + ji\mathbf{v}_y\mathbf{u}_x \\
&= ab - \mathbf{v} \cdot \mathbf{u} + a\mathbf{u} + b\mathbf{v} + \\
&\quad jk\mathbf{v}_y\mathbf{u}_z + kj\mathbf{v}_z\mathbf{v}_y + ik\mathbf{v}_x\mathbf{u}_z + ki\mathbf{v}_z\mathbf{u}_x + \\
&\quad ij\mathbf{v}_x\mathbf{u}_y + ji\mathbf{v}_y\mathbf{u}_x \\
&= (ab - \mathbf{v} \cdot \mathbf{u}, a\mathbf{u} + b\mathbf{v} + \mathbf{v} \times \mathbf{u})
\end{aligned}$$

(c) (4 pts) Use the result in (b) to show that Equation (1) represents the “right” rotation, that is, after the rotation, the coordinates of the rotation axis $\mathbf{u}$ does *not* change. Show your work clearly.

$$\begin{aligned}
\mathbf{q}' \cup \mathbf{q} &= \mathbf{q}'(0, \mathbf{u})\mathbf{q} \\
&= (\cos\frac{\theta}{2}, -\mathbf{u}\sin\frac{\theta}{2})(0, \mathbf{u})(\cos\frac{\theta}{2}, \mathbf{u}\sin\frac{\theta}{2}) \\
&= (\sin\frac{\theta}{2}, \mathbf{u}\cos\frac{\theta}{2})(\cos\frac{\theta}{2}, \mathbf{u}\sin\frac{\theta}{2}) \\
&= (\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}, \mathbf{u}\sin^2\frac{\theta}{2} + \mathbf{u}\cos^2\cos\frac{\theta}{2}) \\
&= (0, \mathbf{u})
\end{aligned}$$

Therefore, the rotation axis does not change.

8 Ray Object Intersections

Consider the following ray/surface intersection function.

```
function IntersectX(Point p, Vector d)
  t = (1 - (p.x + p.y + p.z)) / (d.x + d.y + d.z)
  if (t > 0) return t;
  else return PLUS_INFINITY;
```

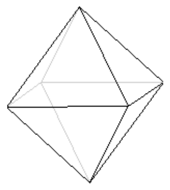
(a) (4 pts) Write down the equation of the surface this function returns for the intersection.

It intersects with the plane $x + y + z == 1$.

The following function returns the intersection of a ray with a cube:

```
function IntersectCube(Point p, Vector d)
  tx1 = (1 - p.x) / d.x;
  tx2 = (-1 - p.x) / d.x;
  ty1 = (1 - p.y) / d.y;
  ty2 = (-1 - p.y) / d.y;
  tz1 = (1 - p.z) / d.z;
  tz2 = (-1 - p.z) / d.z;
  t1 = max(min(tx1, tx2), min(ty1, ty2), min(tz1, tz2));
  t2 = min(max(tx1, tx2), max(ty1, ty2), max(tz1, tz2));
  if (t1 > 0) return t1;
  else if (t2 > 0) return t2;
  else return PLUS_INFINITY;
```

(b) (11 pts) Adapt this pseudocode to intersect a ray with an octahedron (see figure below) using a similar approach. Write your pseudocode on the next page if this page does not have enough space.



```
function IntersectOctahedron(Point p, Vector d)
   tpp1 = ( 1 - (p.x + p.y + p.z)) / (d.x + d.y + d.z);
   tpp2 = (-1 - (p.x + p.y + p.z)) / (d.x + d.y + d.z);
   tpm1 = ( 1 - (p.x + p.y - p.z)) / (d.x + d.y - d.z);
   tpm2 = (-1 - (p.x + p.y - p.z)) / (d.x + d.y - d.z);
   tmp1 = ( 1 - (p.x - p.y + p.z)) / (d.x - d.y + d.z);
   tmp2 = (-1 - (p.x - p.y + p.z)) / (d.x - d.y + d.z);
   tmm1 = ( 1 - (p.x - p.y - p.z)) / (d.x - d.y - d.z);
   tmm2 = (-1 - (p.x - p.y - p.z)) / (d.x - d.y - d.z);
   t1 = max(min(tpp1, tpp2), min(tpm1, tpm2), min(tmp1, tmp2), min(tmm1, tmm2));
   t2 = min(max(tpp1, tpp2), max(tpm1, tpm2), max(tmp1, tmp2), max(tmm1, tmm2));
   if (t1 > 0) return t1;
   else if (t2 > 0) return t2;
   else return PLUS_INFINITY;
```

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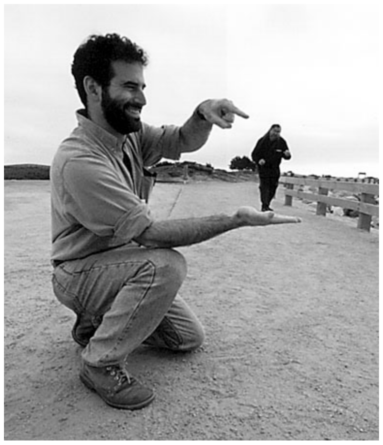
9 Perspective

Here is an unedited photograph of two normal-sized people:



The image above is 450 pixels high, and the two heads measure 90 and 15 pixels high. Assume all heads are 30cm high. The image on the camera's film plane is 24mm high.

(a) (8 pts) If I know that the person in the foreground is 2 meters from the camera, what is the camera's image plane distance (roughly, the focal length) and How far away is the other person from ?

At the distance of the close person, 90 pixels correspond to 30 cm. At the same distance, 450 pixels correspond to 150 cm, or 1.5 m. So the image is 1.5 m high at 2 m – a ratio of 3:4. When the image is 24mm high it is at a distance of (4:3) * 24 = 32mm. This is the image plane distance.
The second person is 6 times the distance of the first person, so 12 meters.

(b) (7 pts) If I know that the two people are standing 20 meters apart, what is the image plane distance and how far from the camera is the closer person?

Let the distance to the near person be d. The far person is at distance $6d$. The distance between the people is then $5d = 20$ meters, so $d = 4$ meters. This distance is twice that in the part 1, so the image plane distance is also doubled, and it is 64mm.

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