

Texture Mapping

Reading

Required

- Hearn & Baker 14.8-14.9
- Angel, pages 373-386

Optional

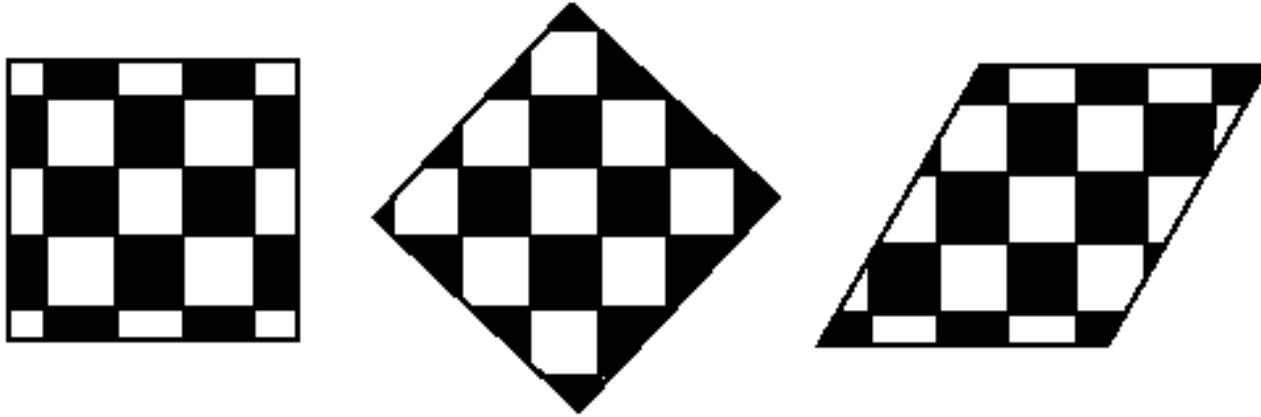
- Paul S. Heckbert. Survey of texture mapping. Computer Graphics and Applications 6(11): 56-67, November 1986
<http://www.cs.cmu.edu/afs/cs/user/ph/www/texsurv.ps.gz>

Texture Mapping

- Texture mapping allows you to take a simple polygon and give it the appearance of something much more complex
 - Due to Ed Catmull, PhD thesis, 1974
 - ensures that “all the right things” happen as a texture polygon is transformed and rendered

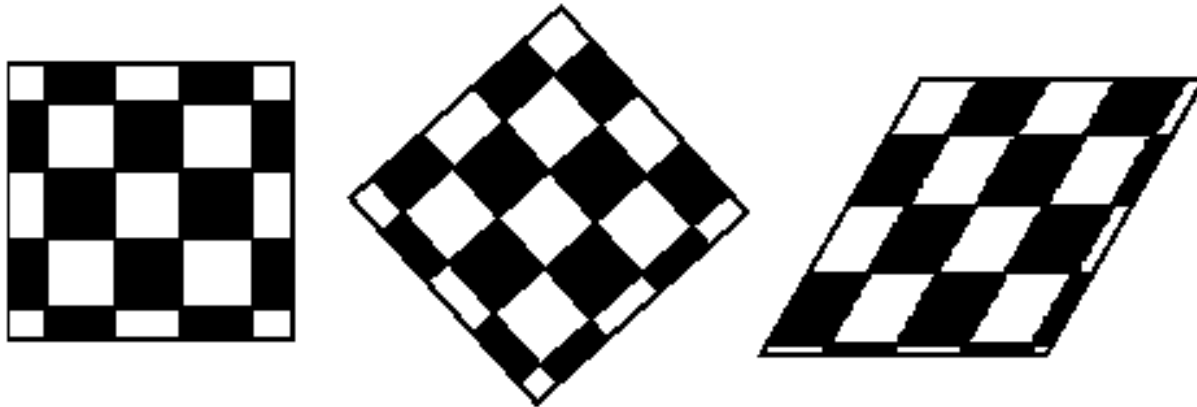


Non-Parametric Texture Mapping



- With non parametric texture mapping:
 - Texture size and orientation are fixed
 - Unrelated to size and orientation of polygon
 - Gives a cookie-cutter effect

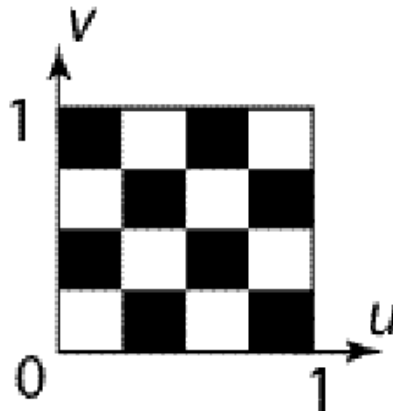
Parametric Texture Mapping



- With parametric texture mapping, texture size and orientation are tied to the polygon:
 - Separate texture space and screen space
 - Texture the polygon as before but in texture space
 - Deform (render) the textured polygon into screen space

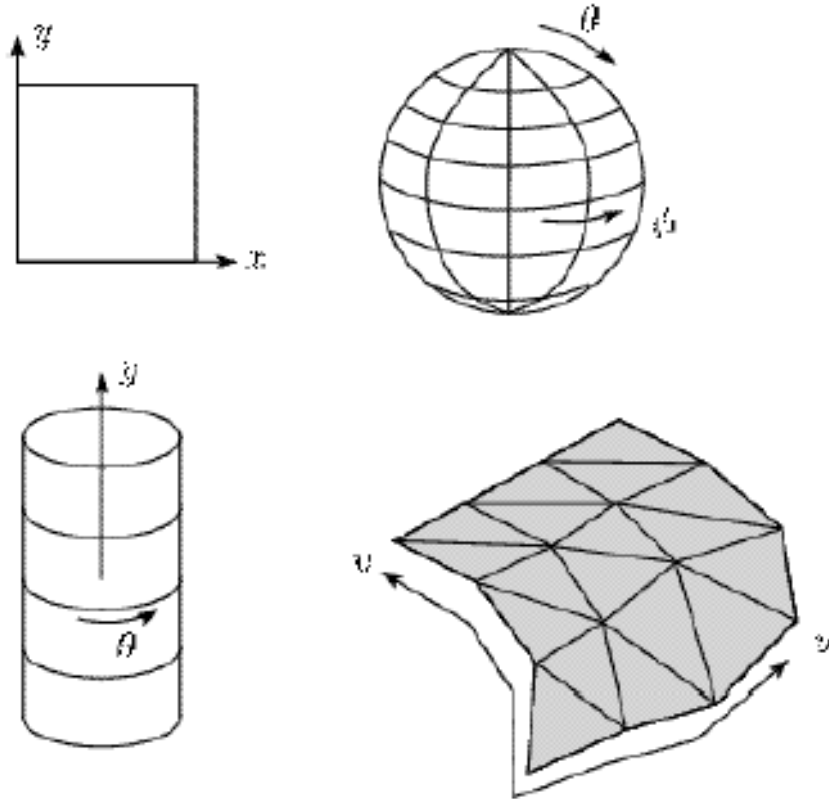
Implementing Texture Mapping

- A texture lives in its own image coordinates parameterized by (u,v) :



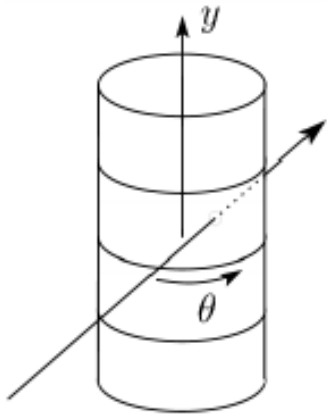
Implementing Texture Mapping

- It can be wrapped around many different surfaces:

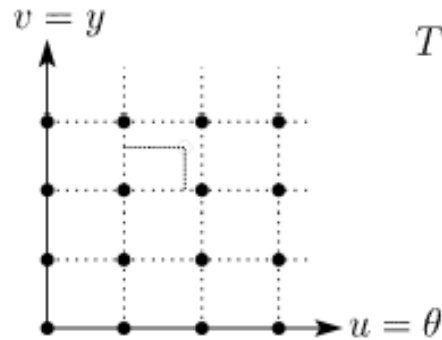


Texture Resampling

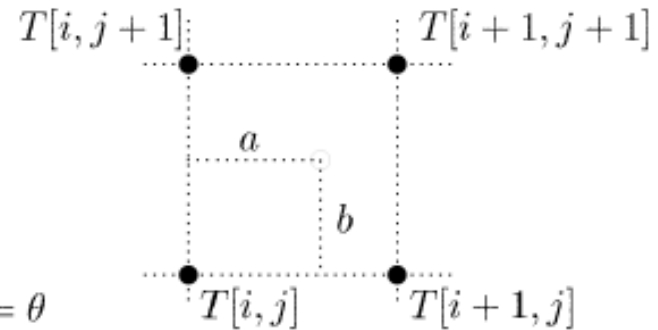
- What do we do when the texture sample lands between texture pixels?



Ray intersection



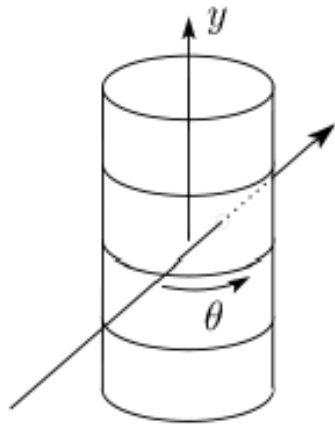
Mapping to texture pixels



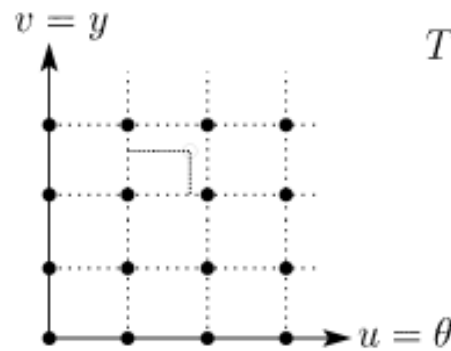
Close-up

- We resample. Common choice is **bilinear resampling**.

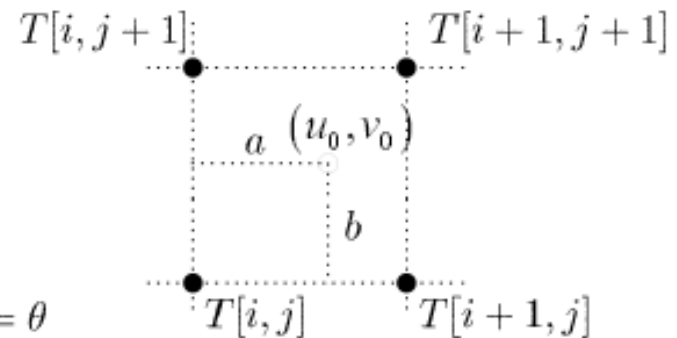
Bilinear Resampling



Ray intersection



Mapping to texture pixels



Close-up

$$T(u_0, v_0) =$$

$$T(i\Delta + a, j\Delta + b) = \underline{\hspace{2cm}} T[i, j] +$$

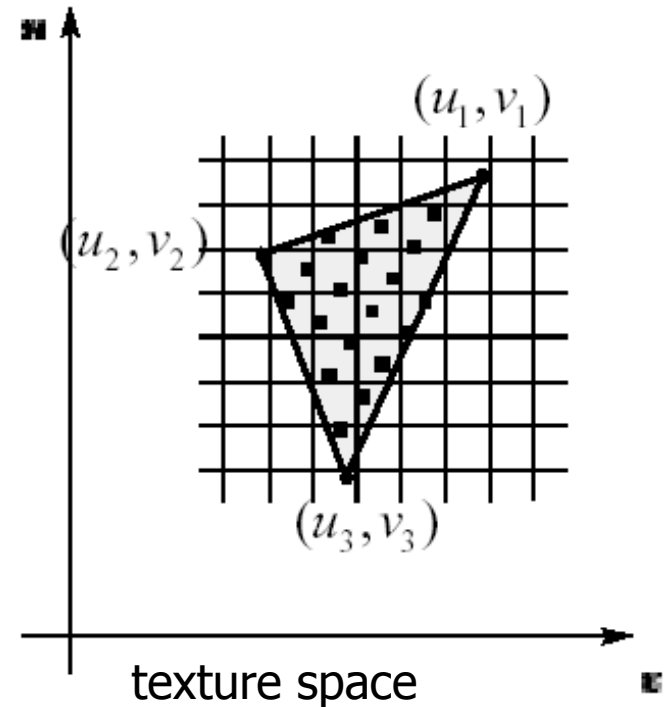
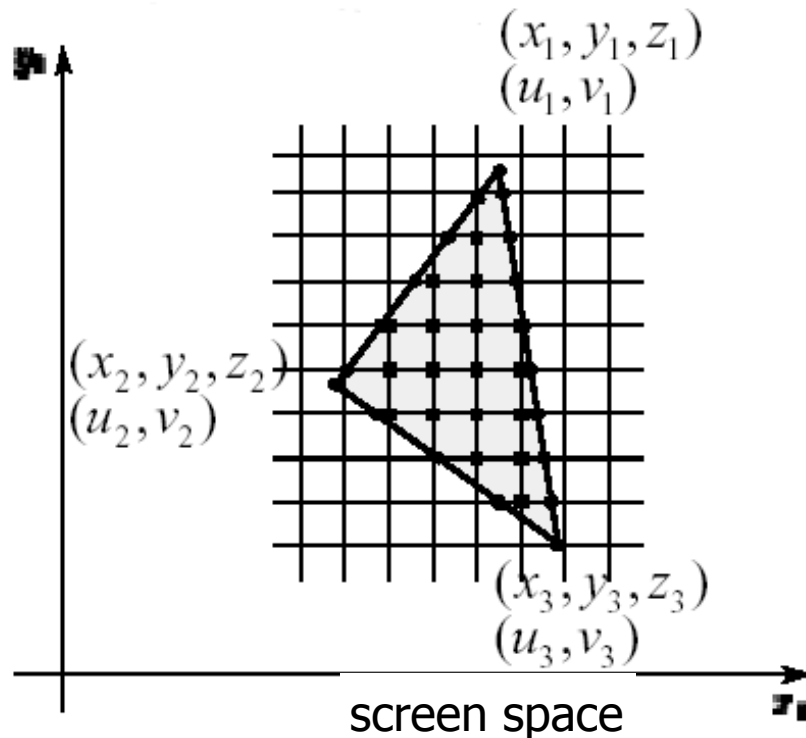
$$\underline{\hspace{2cm}} T[i+1, j] +$$

$$\underline{\hspace{2cm}} T[i, j+1] +$$

$$\underline{\hspace{2cm}} T[i+1, j+1]$$

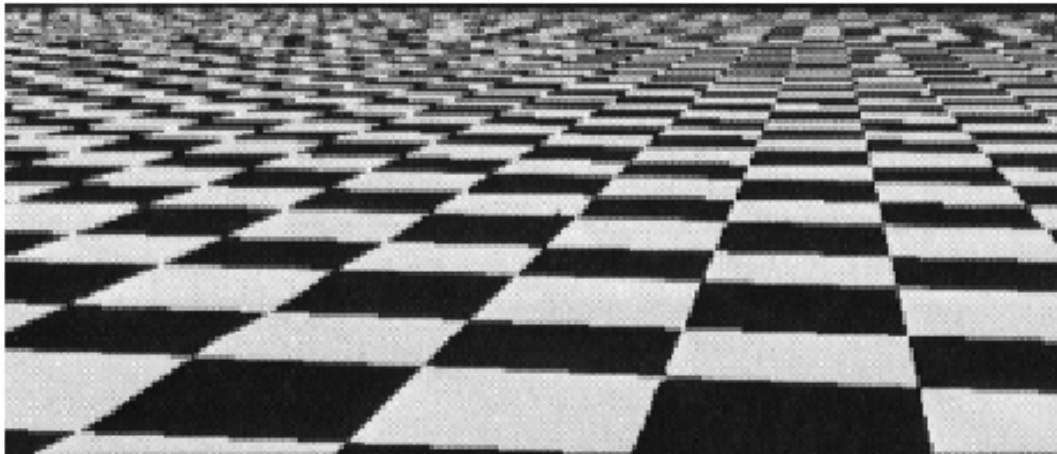
Implementing, cont'd

- Texture mapping can also be handled in z-buffer algorithms
 - Scan conversion is done in screen space, as usual
 - Each pixel is colored according to the texture
 - Texture coordinates are found by Gouraud-style interpolation



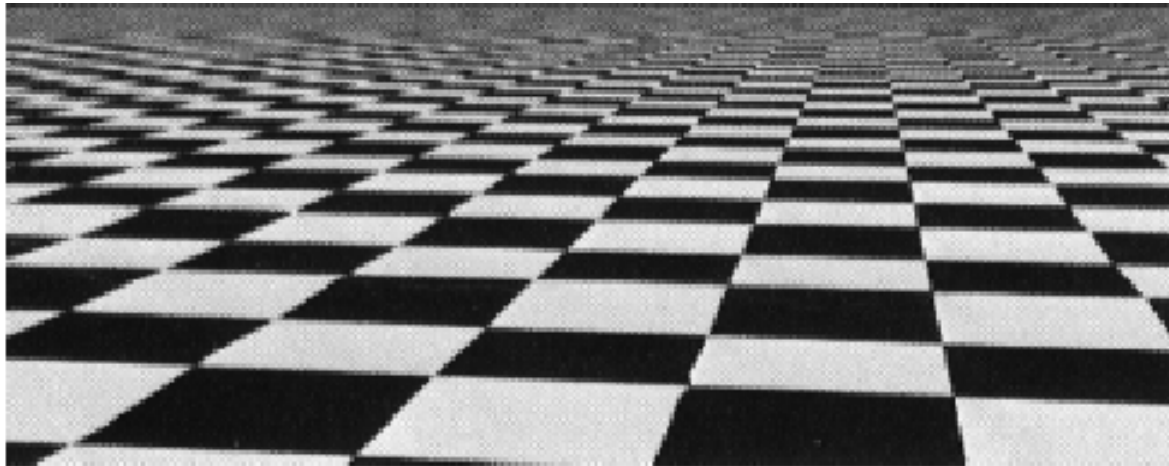
Antialiasing

- If you point-sample the texture map, you get aliasing (stair-casing effect):



Antialiasing

- Proper antialiasing requires area averaging in the texture:

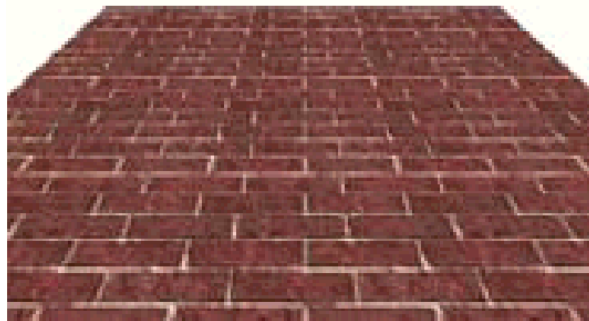


What is Aliasing?

- This is the phenomenon that occurs when we undersample a signal. It causes certain high frequency feature to appear as low frequencies.
- To eliminate aliasing we have to either band limit (low pass filter) our input signal or sample it at a higher rate.
- The first approach (i.e. prefiltering) is more efficient.

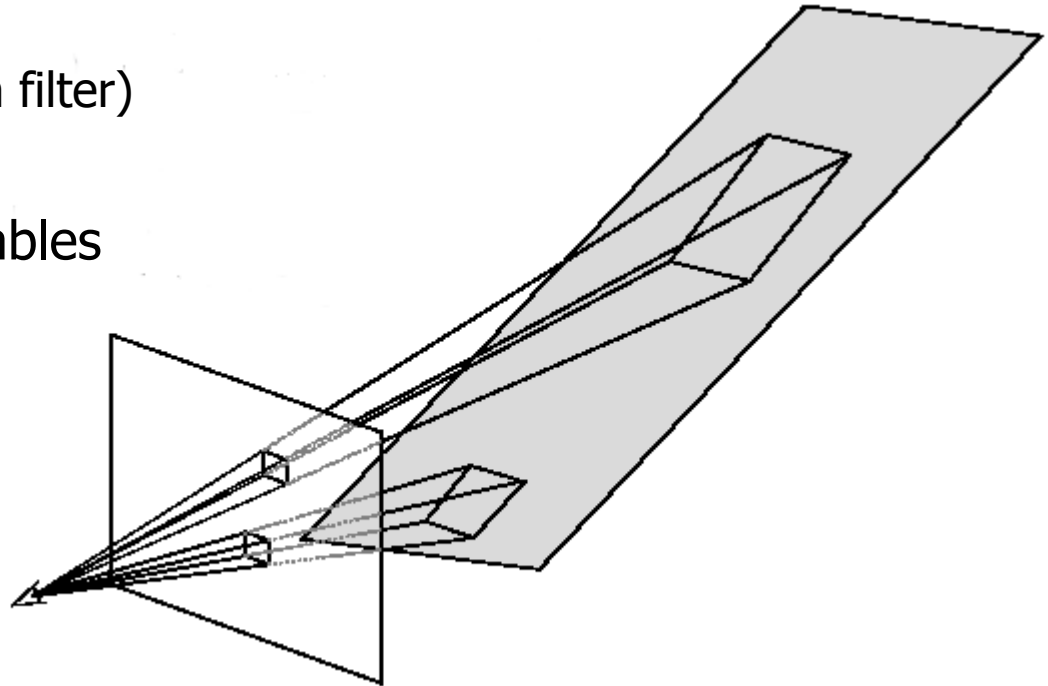
Spatial filtering

- We prefilter the texture to remove the high frequencies that show up as artifacts in the final rendered image.
- The prefiltering required in the undersampling case is basically a spatial integration (summing) over the extent of the sample.



Computing Average Colors

- Computationally difficult part is prefiltering (summing) over the covered pixels:
- Several methods have been used:
 1. Brute force
 - Just sum (mean filter)
 2. Mip maps
 3. Summed Area Tables



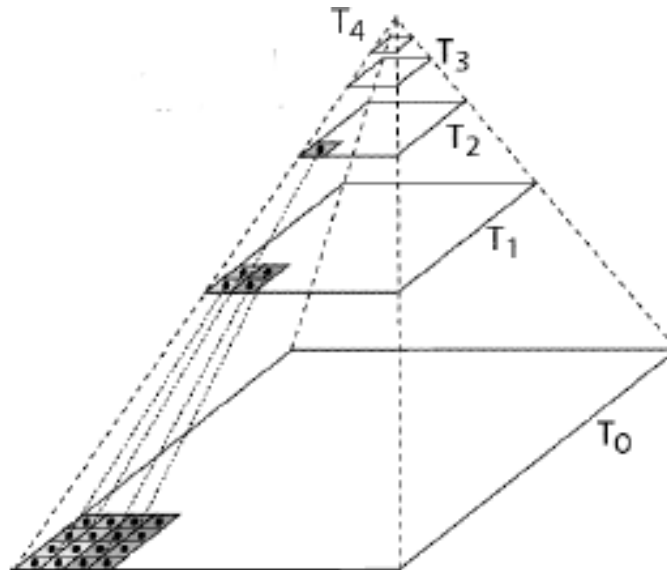
Mip Maps



- Lance Williams, 1983
 - “multum in parvo” – many things in a small place”
 - Keep textures prefiltered at multiple resolutions
 - Figure out two closest levels
 - Linear interpolate between the two

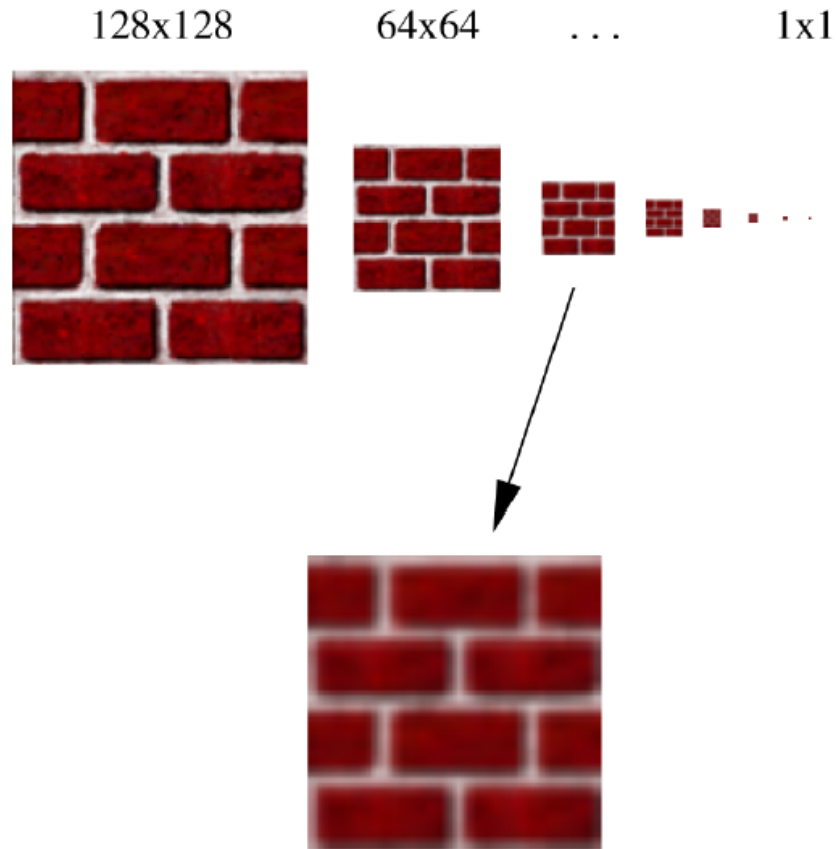
Mip Map Pyramid

- The mip map hierarchy can be thought of as an image pyramid:
 - Level 0 ($T_0[i,j]$) is the original image.
 - Level 1 ($T_1[i,j]$) averages over 2×2 neighborhoods of original.
 - Level 2 ($T_2[i,j]$) averages over 4×4 neighborhoods of original
 - Level 3 ($T_3[i,j]$) averages over 8×8 neighborhoods of original



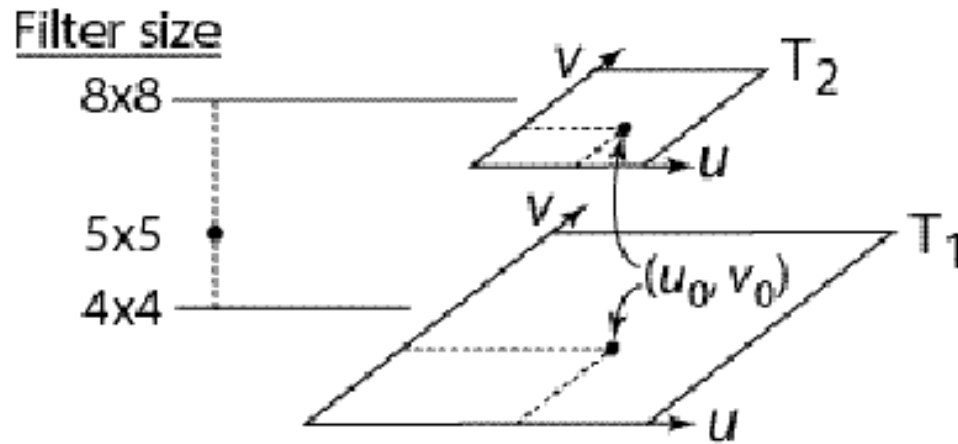
Mip Maps

1. Figure out two closest levels
2. Linear interpolate between the two



Mip Map Resampling

- What would the mip-map return for an average over a 5x5 neighborhood at location (u_0, v_0) ?



- How do we measure the fractional distance between levels?
- What if you need to average over a non-square region?

Summed Area Table (SAT)



- Recall in Calculus (in continuous or discrete form):

$$\int_a^b f(x)dx = \int_{-\infty}^b f(x)dx - \int_{-\infty}^a f(x)dx$$

$$\sum_{i=k}^m f[i] = \sum_{i=0}^m f[i] - \sum_{i=0}^k f[i]$$

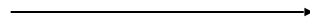
What is a SAT?

5 0 9 9

4 7 8 8

0 0 3 7

1 6 8 3



10 23 51 78

5 18 37 55

1 7 18 28

1 7 15 18

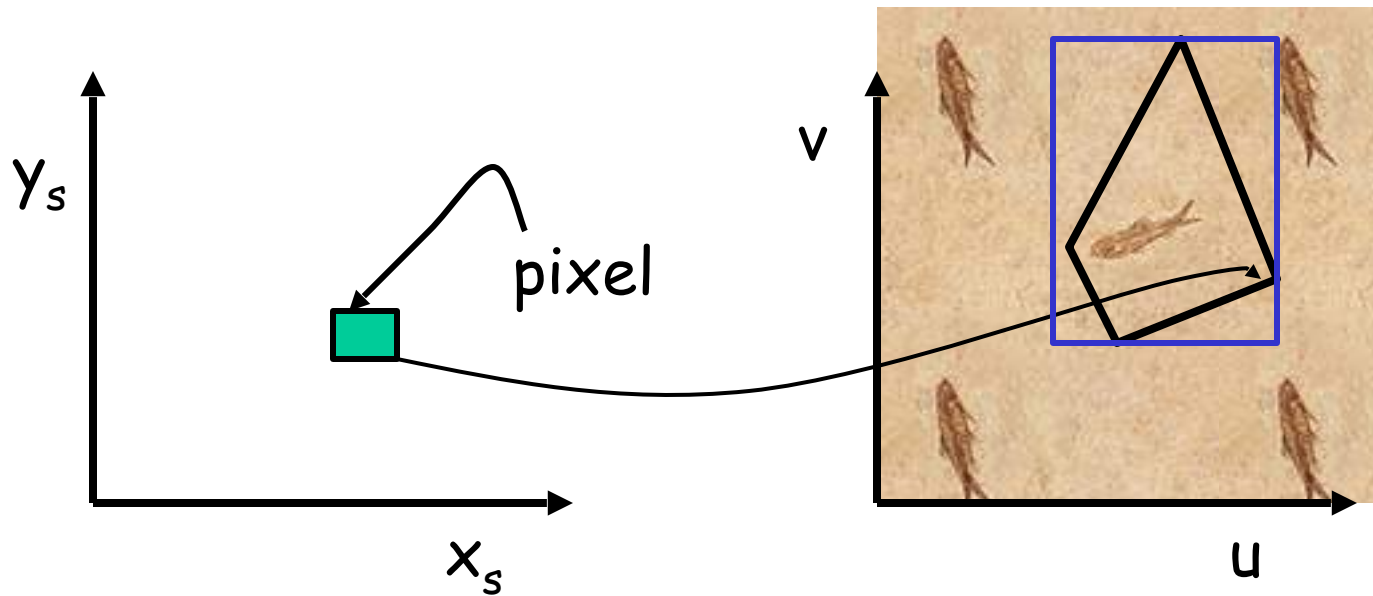
Summed Area Table



- Due to Frank Crow (1984).
- Keep sum of everything below and to the left.
- Use four table lookup.
- Requires more memory (2-4 times the original)
- Gives less blurry results.

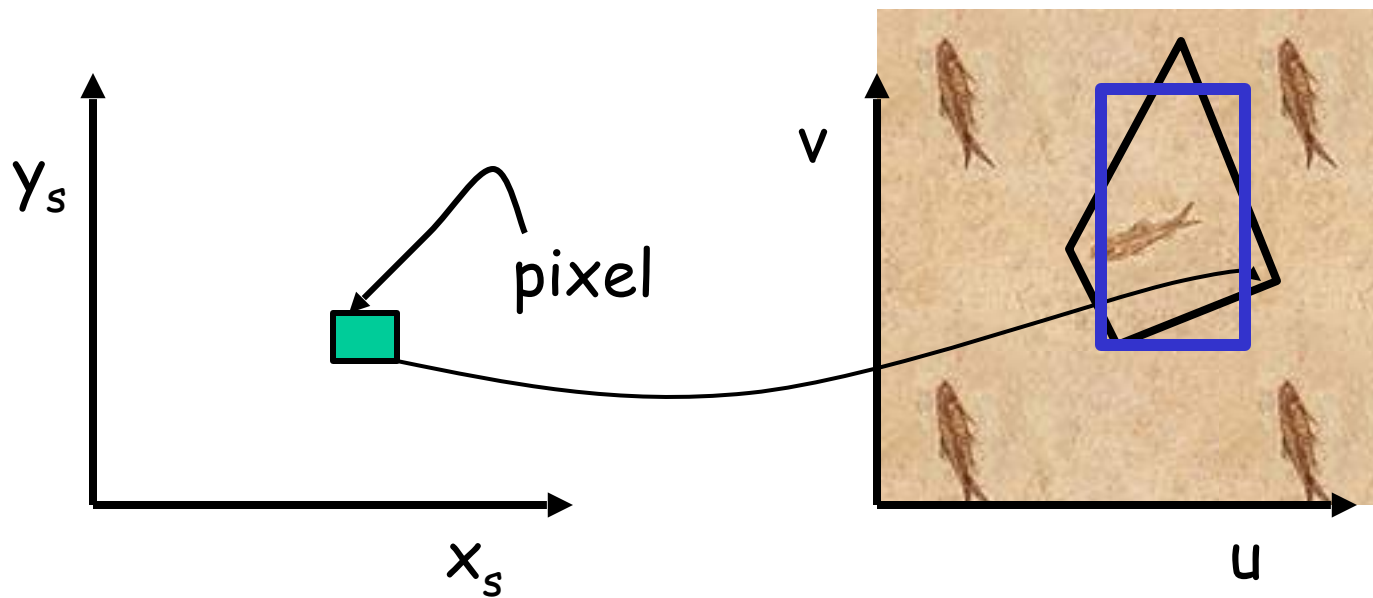
Summed Area Table (SAT)

- Determining the rectangle:
 - Find bounding box and calculate its aspect ratio



Summed Area Table (SAT)

- Determine the rectangle with the same aspect ratio as the bounding box and the same area as the pixel mapping.

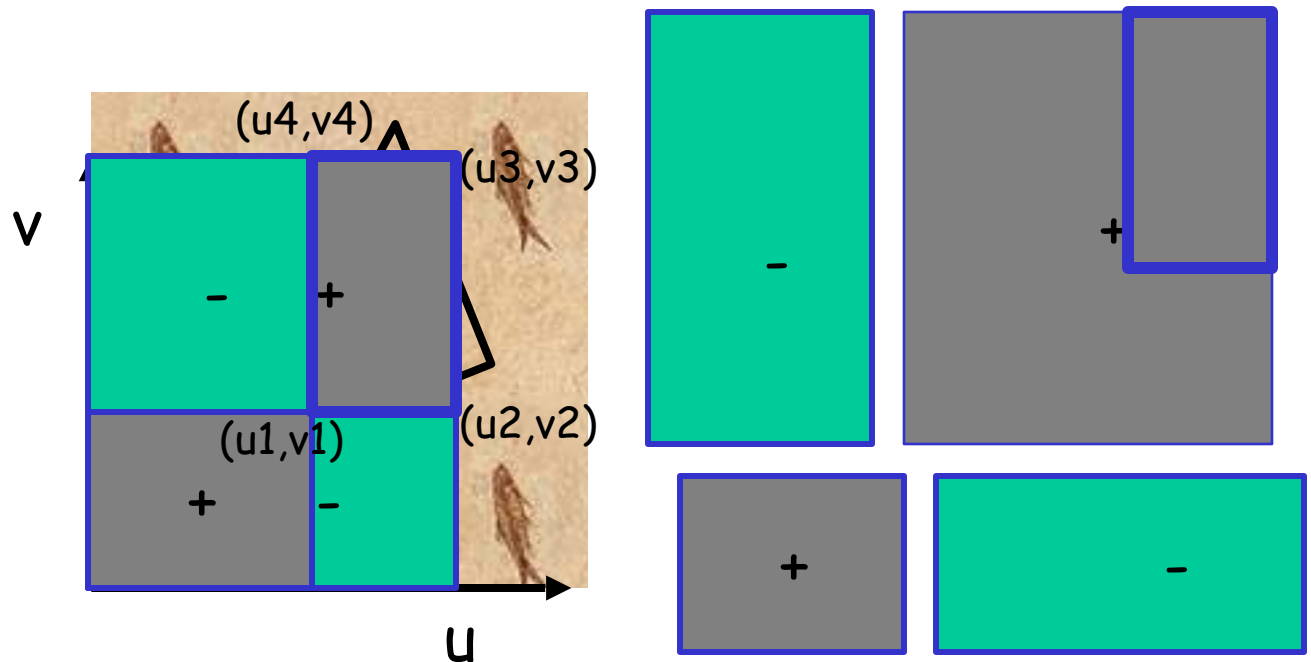


Summed Area Table (SAT)

- Center this rectangle around the bounding box center.
- Formula:
 - $\text{Area} = \text{aspect_ratio} * x * x$
 - Solve for x – the width of the rectangle
- Other derivations are also possible using the aspects of the diagonals, ...

Summed Area Table (SAT)

- Calculating the color
 - We want the average of the texture element colors within this rectangle

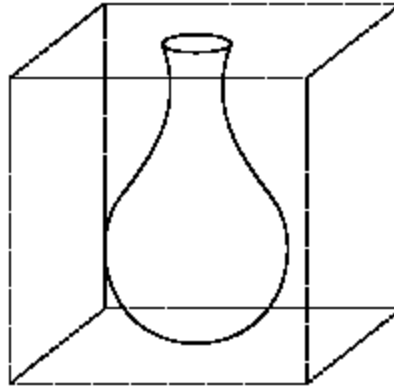


Summed Area Table (SAT)

- To get the average, we need to divide by the number of texels falling in the rectangle.
 - $\text{Color} = \text{SAT}(u_3, v_3) - \text{SAT}(u_4, v_4) - \text{SAT}(u_2, v_2) + \text{SAT}(u_1, v_1)$
 - $\text{Color} = \text{Color} / ((u_3 - u_1) * (v_3 - v_1))$

Solid Textures

- Q: What kinds of artifacts might you see from using a marble wallpaper instead of a real marble?



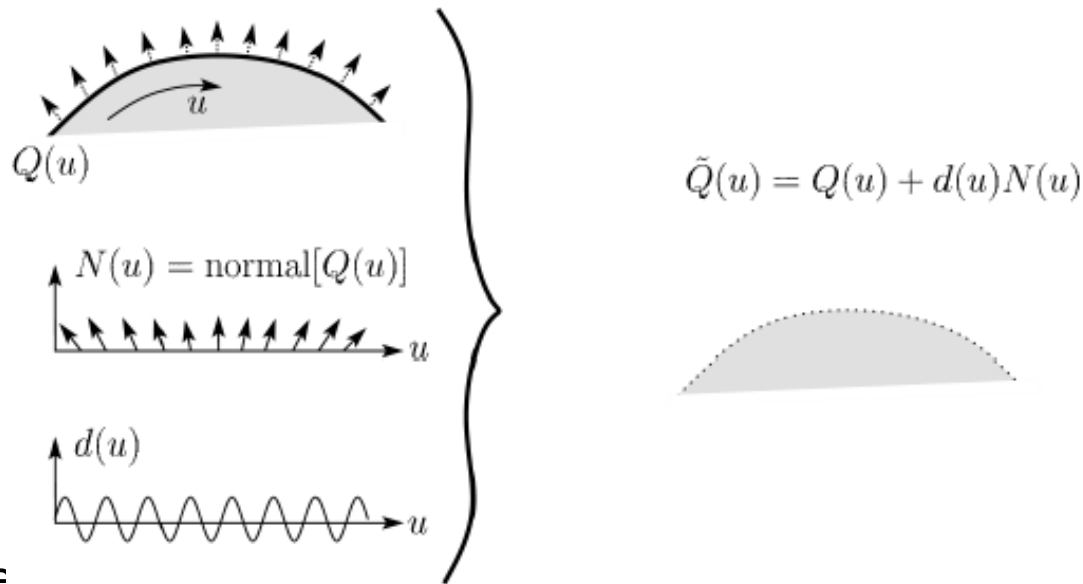
- One solution is to use model-space coordinates to index into a 3D texture.
 - Like “carving” the object from the material
- The mathematics is easy; but coming up with the solid texture is difficult...

Solid Textures



Displacement Mapping

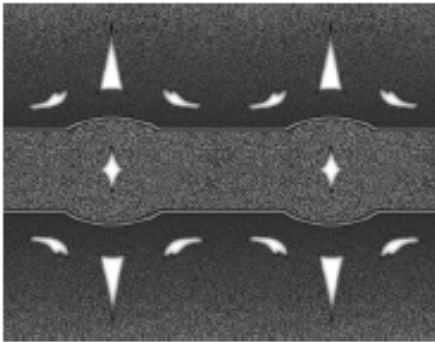
- In displacement mapping, a texture is used to perturb the surface geometry itself:



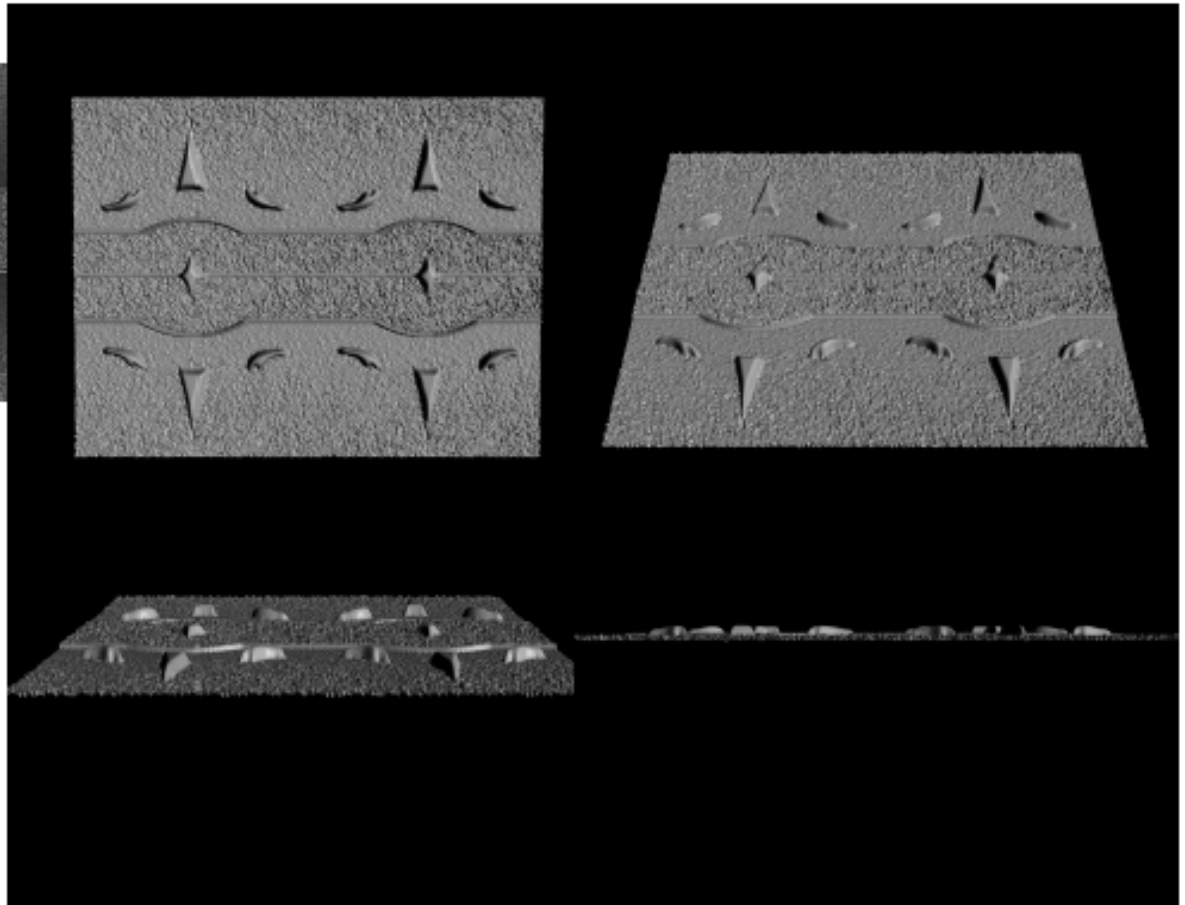
- Silhouettes are correct.
- Requires doing additional hidden surface calculations.

Displacement Mapping

Input texture:



Displacement map over rectangular surface:



Bump Mapping

- Textures can be used for more than just color

$$I = k_a I_a + \sum_i f(d_i) I_{li} \left(k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R})_+^{n_s} \right)$$

- In bump mapping, a texture is used to perturb the normal:
 - The normal is perturbed according to the partial derivatives of the texture.



- These bumps 'animate'

Bump Mapping Example



Original rendering

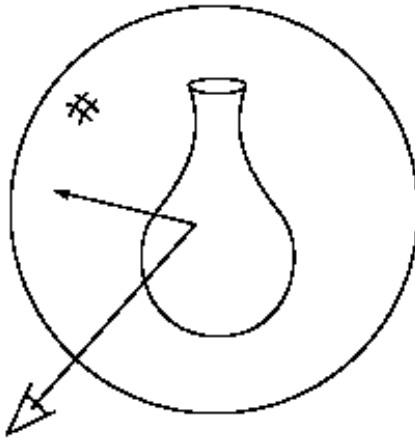


Rendering with bump map
wrapped around a cylinder

Environment Mapping



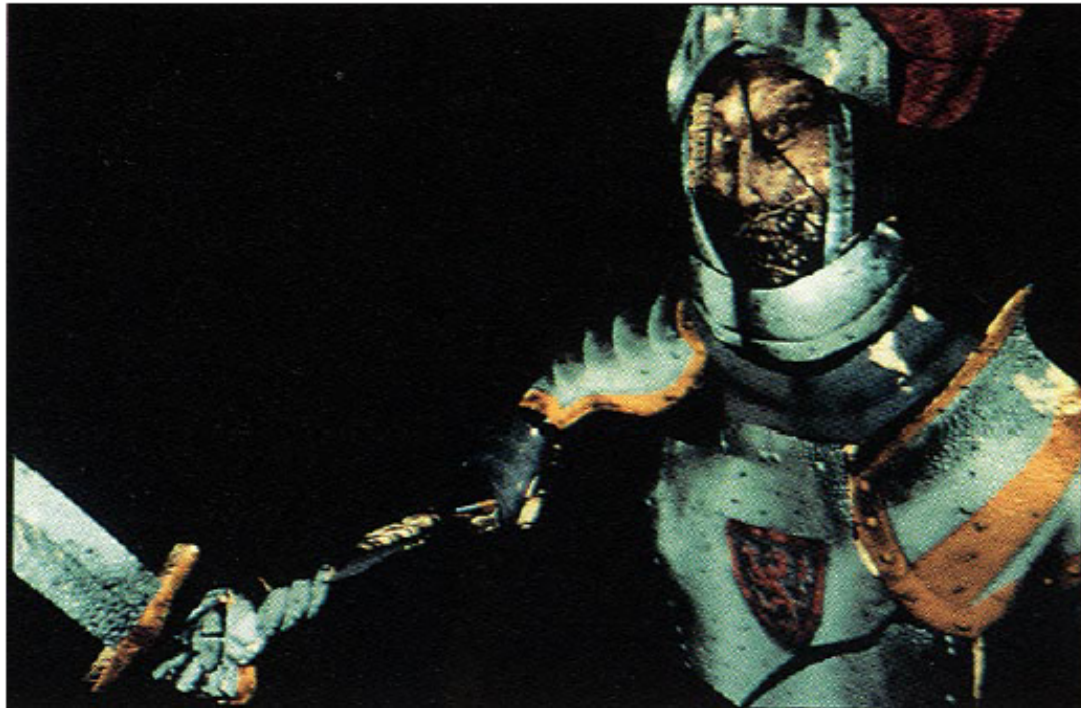
Environment Mapping



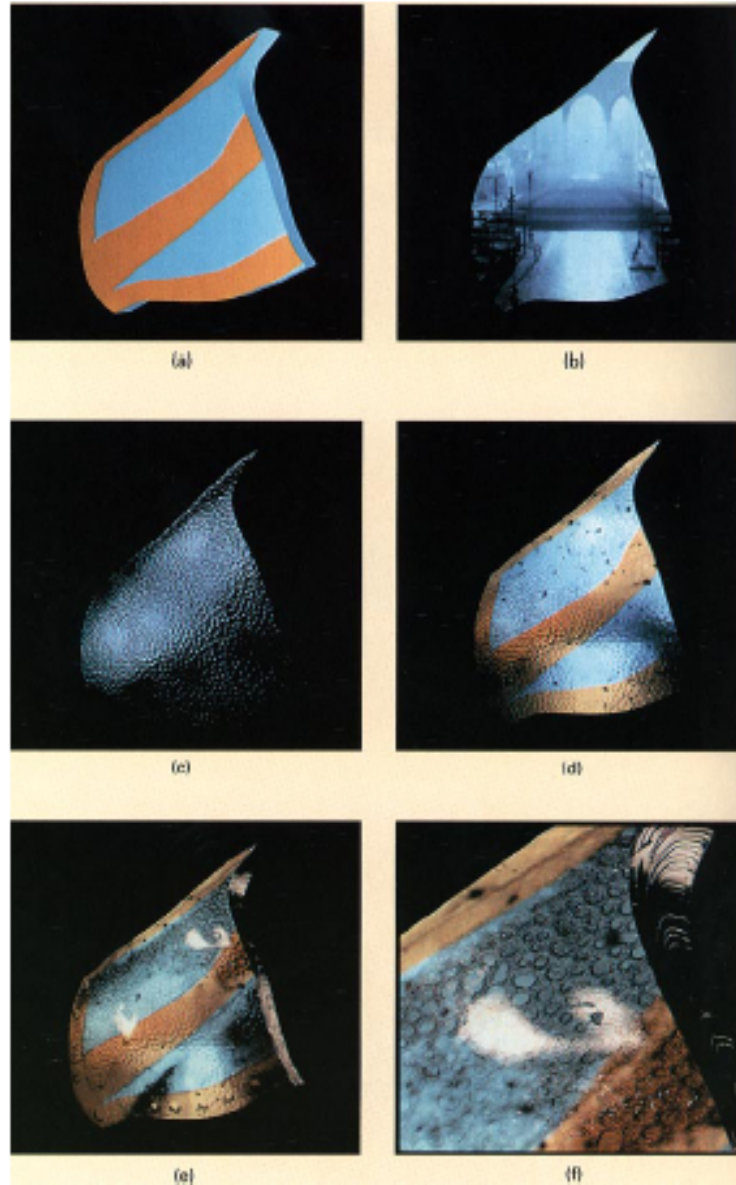
- A.k.a. reflection mapping
- Use texture to model object's environment
- Rays are bounced off objects into environment to determine color of illumination
- Works well when there is just a single object
- With some simplifications can be implemented in hardware
- Ray tracer can be extended to handle refractions as well

Combining Texture Maps

- Using texture maps in combination to achieve better effects.



Combining Texture Maps



Summary

What to take from this lecture:

- What texture mapping is and what is it good for
- Understanding the various approaches to antialiased textured mapping
 - Brute force
 - Mipmaps
 - Summed area tables
- Additional effect with texture mapping techniques
 - Bump mapping
 - Displacement mapping
 - Environment mapping