Computer Science 4411: Computer Graphics
Spring Semester 2019 Final Examination
Instructor: CK Tang
Thursday, May 23, 2019
8:30AM - 11:30AM at LTL, CYT Bldg

This is a **CLOSED-BOOK-CLOSED-NOTES** exam consisting of eighty (80) multiple choice questions and six (6) long questions. **NO** computers or cellphones except approved scientific calculators are allowed, and any cellphones on your desk will be confiscated. Follow the instructions carefully. For multiple choice questions, mark your answer clearly using **HB pencil** in the bubble answer sheet provided. Do not spend too much time on a single problem unless you have attempted all other problems. Unless otherwise stated only the simplest and correct expression can score full credits in mathematical problems. Please write legibly and keep the exam booklet stapled, except you may tear off the last sheet for rough work.

KEY

Problem	Points	your score
1 MULTIPLE CHOICES	80	
2 BASIC RAY TRACING	10	
3 ADVANCED RAY TRACING	25	
4 ROTATIONS	10	
5 PROJECTIONS	15	
6 LIGHTING AND SHADING	15	
7 PARAMETRIC CURVES	25	
Total	180	

2 Basic Ray Tracing

2.1 Ray-triangle intersections

(a) Write down a mathematically correct description of a ray $\mathbf{r}$ that starts at eye vector $\mathbf{e} = (\frac{1}{2}, \frac{3}{4}, \frac{1}{2})$ and goes through the screen at position $\mathbf{s} = (\frac{1}{2}, \frac{3}{4}, \frac{3}{2})$.

The ray can be specified via:

$$\mathbf{r}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e}) \quad \text{for } t \in [0, \infty)$$

For the concrete values of $\mathbf{e}$ and $\mathbf{s}$ we get:

$$\mathbf{r}(t) = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{4} \\ \frac{1}{2} \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ for } t \in [0, \infty)$$

(b) Assume we want to check if our ray intersects with the triangle defined by the three vertex vectors $\mathbf{a} = (1, 1, 1)$, $\mathbf{b} = (0, 1, 1)$, and $\mathbf{c} = (1, 0, 1)$. Write down the parametric equation of the plane defined by this triangle. Use $\mathbf{a}$ as support vector and $\mathbf{b} - \mathbf{a}$ and $\mathbf{c} - \mathbf{a}$ as direction vectors. (The calculations in the following subproblems should become very simple then.)

If we chose the support and direction vectors as specified above, the correct equation is

$$\mathbf{p}(\beta, \gamma) = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

For the concrete values of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, we get:

$$\mathbf{p}(\beta, \gamma) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

(c) In order to verify if our ray $\mathbf{r}$ intersects with the above triangle, we want to calculate the intersection point of the ray with the plane from the previous subproblem. We do this by creating a linear equation system using the line and plane equations that we specified above. Create this linear equation system, solve it, and calculate the intersection point.

Equalizing the ray and plane equations gives us:

$$\begin{pmatrix} 1/2 \\ 3/4 \\ 1/2 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

Bringing all the variables to the left and all constant values to the right gives us:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \beta + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \gamma + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1/2 \\ 3/4 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/4 \\ 1/2 \end{pmatrix}$$

Because of the way we resorted and wrote our equations, we can easily write down the linear equation system in matrix notation:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta \\ \gamma \\ t \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/4 \\ 1/2 \end{pmatrix}$$

Because the left matrix is already the identity matrix, we dont even have to do any calculations, but can just write down the solutions:

$$\beta = 1/2, \gamma = 1/4, t = 1/2$$

Putting the value $t = 1/2$ into our ray equation, gives us the intersection point, i.e

$$\mathbf{r}(1/2) = (1/2, 3/4, 1)$$

(d) Use the solution of the linear equation system constructed in the previous subproblem to verify if the ray $\mathbf{r}$ intersects with the triangle or not. (Hint: remember that the way we constructed our plane is the same way in which we would create a related barycentric coordinates system. If you didn't do the calculation above, but know how to do this test, you can get at least some credit if you write down the correct conditions.)

Above, we have already used β and γ to denote the parameters in our parametric plane equation, because these are the same as the parameters in barycentric coordinates. Hence, we can just verify if they fulfill the condition for line-triangle intersection, i.e.

$$\beta > 0, \gamma > 0 \text{ and } \beta + \gamma < 1$$

In this case, they do, i.e.,

$$1/2 > 0, 1/4 > 0 \text{ and } \beta + \gamma = 3/4 < 1$$

so our ray does indeed intersect with the triangle.

3 Advanced Ray Tracing

3.1 Instancing

Mark the correct answer. No explanation is required. There is only one correct answer. Assume the following:

- O^* is an object created by multiplying an object O with a transformation matrix M.
- $\mathbf{r}$ is a ray created by multiplying a ray $\mathbf{r}^*$ with the inverse of this transformation matrix, i.e. with M^{-1}.
- $\mathbf{p}_i$ are the intersection points of $\mathbf{r}$ and object O.

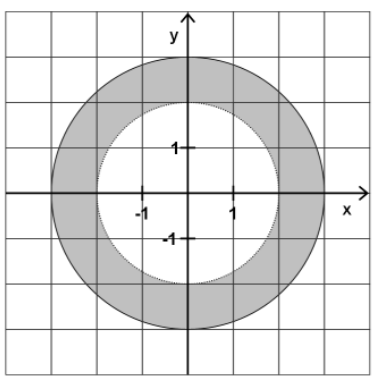
Given these assumptions, we can calculate the intersection points $\mathbf{p}_i$ of $\mathbf{r}^*$ and O^* with ...

(A) $M\mathbf{p}_i$
(B) $M^{-1}\mathbf{p}_i$
(C) $M^T\mathbf{p}_i$
(D) $(M^{-1})^T\mathbf{p}_i$
(E) $(M^T)^{-1}\mathbf{p}_i$
(F) none of the above

The correct answer is A.

3.2 Constructive solid geometry

We want to create an object in 2D that looks like the ring in the following image. We are using Constructive Solid Geometry and a solid circle C_1 with radius 2 and a solid circle C_2 with radius 3. Both circles are centered around the origin.



(a) Write down the concrete set operation that has been done to construct our ring.

$$C_2 - C_1$$

Note: if you wrote “difference” or “exclusion” instead of using the related mathematical symbol you got full credit as well.

(b) Now we want to use Constructive Solid Geometry to calculate the intersection points of our ring with the y-axis, i.e. with the line $x = 0$.

(i) Write down the intervals that you get when calculating the intersections with the original objects, i.e. the circles C_1 and C_2.

$$C_1 : [(0, -2), (0, 2)]$$
$$C_2 : [(0, -3), (0, 3)]$$

(ii) Write down the intervals that you get when applying Constructive Solid Geometry to the previously created intervals.

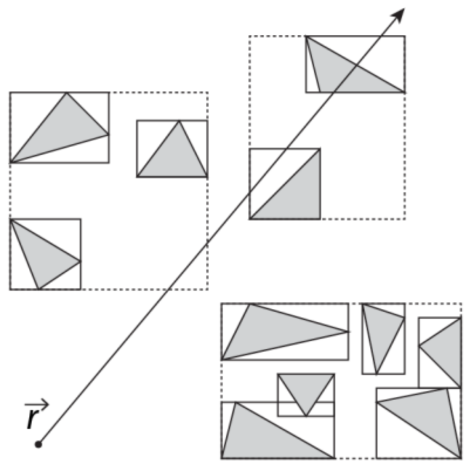
$$[(0, -3), (0, -2)], [(0, 2), (0, 3)]$$

(iii) How do you get the intersection points from the calculated intervals? (Note that it is not necessary to write them down but you should illustrate how to get them. The actual values are easy to see from the image and don't matter here. What we want to know is if you understood the procedure. One sentence can be enough to get full credit for this subproblem.)

The intersection points are just the borders of the intervals, i.e. $(0, -3)(0, -2)$, $(0, 2)$, and $(0, 3)$. Note that it was not necessary to write the actual numbers down in order to get full credit, if you gave the above explanation (or something similar).

3.3 Hierarchical bounding boxes for faster ray tracing

Assume we want to calculate the intersections of the ray **r** depicted in the diagram below with the gray triangles given in this scene. To speed things up, we put bounding boxes around our triangles (indicated by the solid rectangles) and hierarchically grouped them (indicated by the dotted rectangles).



In the following, just write down the correct number (no explanation required). Notice that the first question asks for the total number of intersection tests (i.e. the number of intersection tests with bounding *boxes* and the necessary tests with *triangles* therein).

When calculating the intersections using this structure of hierarchical bounding boxes . . .

How many intersection tests do we have to make? ____________________________

How many false positives do we get? ____________________________

How many false negatives do we get? ____________________________

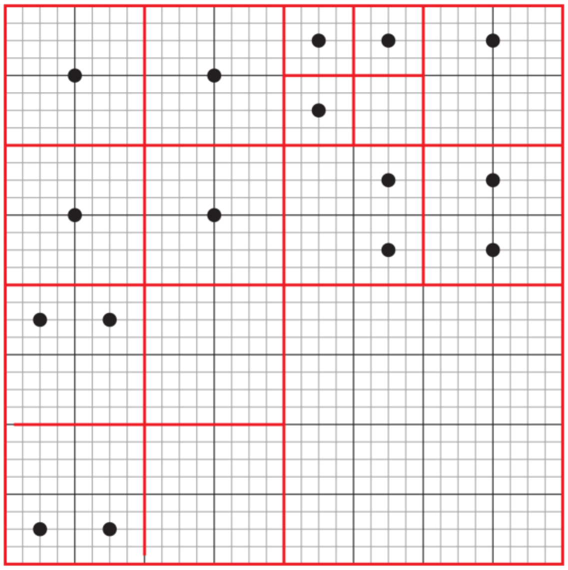
The correct solutions are 10, 2, and 0.

```
  3
 /
3   1
 \ /
  2
   \
    1
```

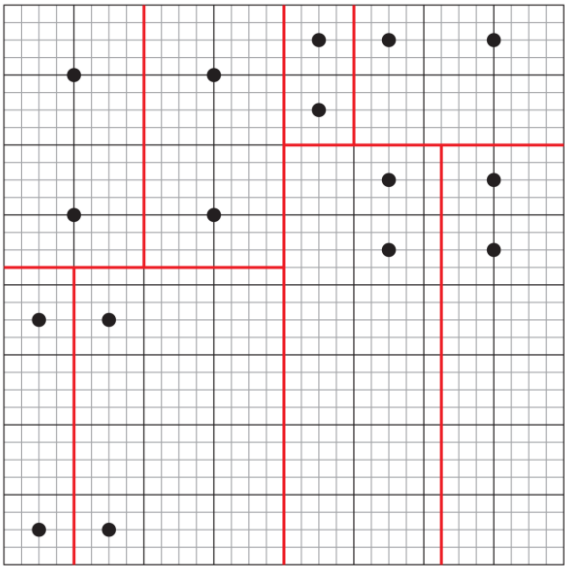
3.4 Space partitioning algorithms

Below you see the illustration of a 2D space containing objects (black dots). The grid structure has no real relevance but was just added to make drawing easier. Now we want to apply different space partitioning approaches to these scenes.

(a) Draw the cells into the image that we get when using the quadtree approach for space partitioning (which is the 2D version of the octree approach in 3D). Stop the space partitioning once each of the cells contains a maximum of two objects.



(b) Now we want to apply the BSP tree approach for space partitioning to the same scene. Draw the resulting cells into the image below. Start with a vertical split and then alternate between horizontal and vertical splits, where each split results in equal numbers in the two halves. Stop once each cell contains a maximum of two objects.



Note: use pencil for this question and if you made a mistake in your drawing that cannot be easily corrected, you can ask one of the assistants for another template (we have a few backups).

4 Rotations

In this problem, you will use quaternion multiplication to show that two rotations don't necessarily commute, i.e., the order in which they are performed matters. In particular, let the quaternions $\mathbf{q}_x$ and $\mathbf{q}_y$ represent rotations about the x and the y axes, by angles θ_x and θ_y, respectively. Use quaternion multiplication to show that $\mathbf{q}_x\mathbf{q}_y \neq \mathbf{q}_y\mathbf{q}_x$ by finding the respective simplest form of $\mathbf{q}_x\mathbf{q}_y$ and $\mathbf{q}_y\mathbf{q}_x$.

Hint: The unit quaternion corresponding to a rotation about axis $\mathbf{u}$ by angle θ is $\mathbf{q} = \langle \cos\frac{\theta}{2}; \sin\frac{\theta}{2}\mathbf{u}\rangle$. Therefore the two quaternions are

$$\mathbf{q}_x = \langle \cos\frac{\theta_x}{2}; \sin\frac{\theta_x}{2}, 0, 0\rangle$$

$$\mathbf{q}_y = \langle \cos\frac{\theta_y}{2}; 0, \sin\frac{\theta_y}{2}, 0\rangle$$

Recall the formula for quaternion multiplication,

$$< d; \mathbf{u} >< d'; \mathbf{u}' >=< dd' - \mathbf{u}\cdot\mathbf{u}'; d\mathbf{u}' + d'\mathbf{u} + \mathbf{u}\times\mathbf{u}' > .$$

Apply this formula (and using $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ and $\hat{\mathbf{y}}\times\hat{\mathbf{x}} = -\hat{\mathbf{z}}$) to derive your answers. Use the next blank page if you need more space for your derivations.

$$\mathbf{q}_x\mathbf{q}_y = \langle \cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}; \cos\frac{\theta_y}{2}\sin\frac{\theta_x}{2}, \cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}, \sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\rangle$$

$$\mathbf{q}_y\mathbf{q}_x = \langle \cos\frac{\theta_x}{2}\cos\frac{\theta_y}{2}; \cos\frac{\theta_y}{2}\sin\frac{\theta_x}{2}, \cos\frac{\theta_x}{2}\sin\frac{\theta_y}{2}, -\sin\frac{\theta_x}{2}\sin\frac{\theta_y}{2}\rangle$$

The resulting two quaternions are different due to the sign change on the last component, and thus the original x/y rotations do not commute.

5 Projections

Terminology of projections: **PP** is projection plane; **COP** is the center of projection; and **DOP** is direction of projection.

(a) What is the difference between parallel projection and perspective projection?
(in less than 20 words)

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Answer:
Parallel:      COP at an infinite distance from PP
Perspectively: COP at a  finite   distance from PP
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(b) What is the difference between orthographic projection and oblique projection?
(in less than 20 words)

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Answer:
Orthographic: DOP is     perpendicular to PP
Oblique:      DOP is not perpendicular to PP
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From what we learnt from the camera and projection lecture, we can transform an eye space coordinate $(x_{eye}, y_{eye}, z_{eye}, w_{eye})^T$ from eye space to clip space by right-multiplying it with the 4×4 projection matrix, $M_{projection}$.

$$\begin{pmatrix} x_{clip} \\ y_{clip} \\ z_{clip} \\ w_{clip} \end{pmatrix} = M_{projection} \begin{pmatrix} x_{eye} \\ y_{eye} \\ z_{eye} \\ w_{eye} \end{pmatrix}$$

Here, we have two projection matrices, M_A and M_B:

$$M_A = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -3 & -12 \\ 0 & 0 & -1 & 0 \end{pmatrix} \text{ and } M_B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Hint: both M_A and M_B are not skewed, i.e. they produce symmetric viewing frustums about the eye space z-axis.

(c) Is M_A a perspective projection matrix or a parallel projection matrix?
Justify your answer clearly with explanation.

`Answer:`

M_A is a perspective projection matrix because the W component of the resultant clip coordinate depends on eye_z. Therefore, we will have division by Z during the perspective division step.
Note: any reasonable explanation is acceptable.

(d) Is M_B a perspective projection matrix or a parallel projection matrix?
Justify your answer clearly with explanation.

`Answer:`

M_B is a parallel projection matrix because 1) the W component of the resultant clip coordinate is a constant and 2) the X and Y components of the resultant clip coordinate are just copies of eye_x and eye_y, i.e., no division by Z during the perspective division step.
Note: any reasonable explanation is acceptable.

(e) From what we learnt from the lecture, we can test whether a vertex is inside the viewing frustum by comparing its x_{clip}, y_{clip}, and z_{clip} against w_{clip}. Note that $(x_{clip}, y_{clip}, z_{clip}, w_{clip})$ is its clip space coordinate after projection transformation. Please state the method clearly.

Answer:

$$\begin{aligned} |w_{clip}| &\geq |x_{clip}| \\ |w_{clip}| &\geq |y_{clip}| \\ |w_{clip}| &\geq |z_{clip}| \,. \end{aligned}$$

(f) Given M_A as the current projection matrix, what is the clip coordinate of the eye point? What is the physical meaning of the result? (Hint: where is the eye point in eye space?)

Answer:

Right multiply $(0, 0, 0, 1)^T$ with M_A gives $(0, 0, -12, 0)$.
It is infinitely away at negative Z direction in the clip space.

(f) Find out the locations of the near and far clipping planes in eye space given M_A? That is, we have to compute n and f such that n and f are the eye-space distances from eye to near and far clipping planes, respectively.

$$\text{Hint}: M_A^{-1} \text{ is } \begin{pmatrix} 1/3 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1/12 & 1/4 \end{pmatrix} .$$

Answer:

Right multiply clip coordinates $(0, 0, -1, 1)$ and $(0, 0, 1, 1)$ with the inverse of M_A. Since we know that M_A represents a symmetric viewing frustums about the eye space z-axis. The resultant Z value will tell us the location of the clipping plane, i.e., we have $n = 3$ and $f = 6$.

(h) We would like to study the utilization of the depth buffer based on M_A. Now, we make up this equation $z_{eye}(t) = -n(1-t) - ft$ where $t \in [0,1]$. It is worth noting that $z_{eye}(t)$ denotes the z-coordinate from the eye point along the negative z-axis in the eye space. Thus, $z_{eye}(t)$ is linear to t in the eye space. If we transform $z_{eye}(t)$ to clip space using M_A, and then apply perspective division to go to the NDC space for depth buffering, is z_{NDC} still linear? Hint: you may (or may not) answer this by an example (consider three equidistant points along $z_{eye}(t)$).

Answer:

Consider eye coordinate $(0, 0, z_{eye}(t), 1)$, right multiply this coordinate with M_A gives clip coordinate $(0, 0, -3z_{eye} - 12, -z_{eye})$. Therefore, after perspective division, z_{NDC} is $(3z_{eye} + 12)/z_{eye}$. Therefore, z_{NDC} is non-linear with respect to changes in t.

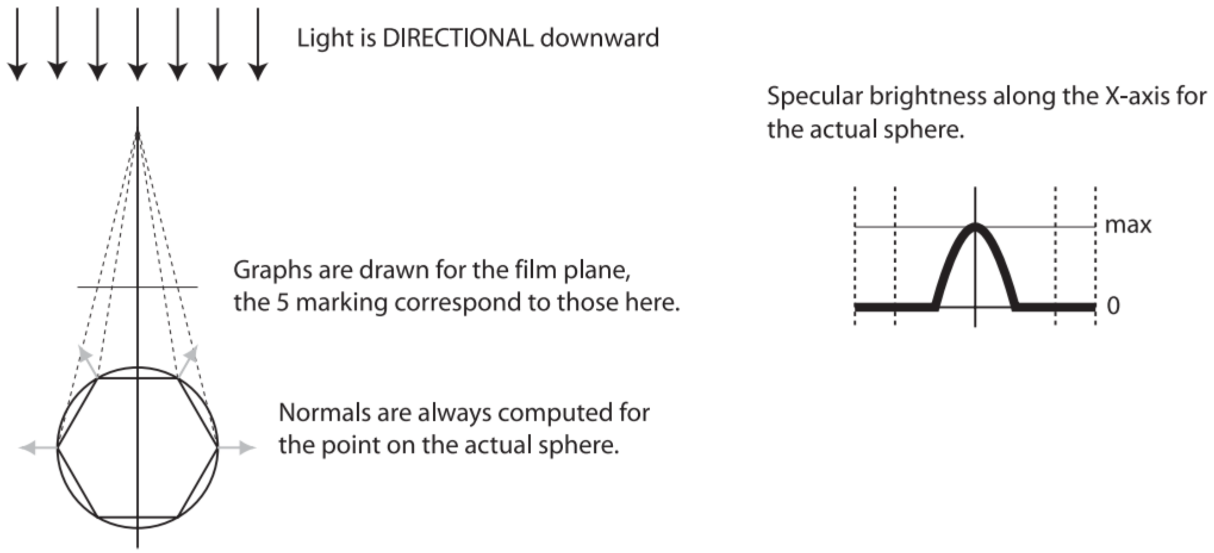
(i) What is the implication about depth buffer utilization based on your result in (h)?

Answer:

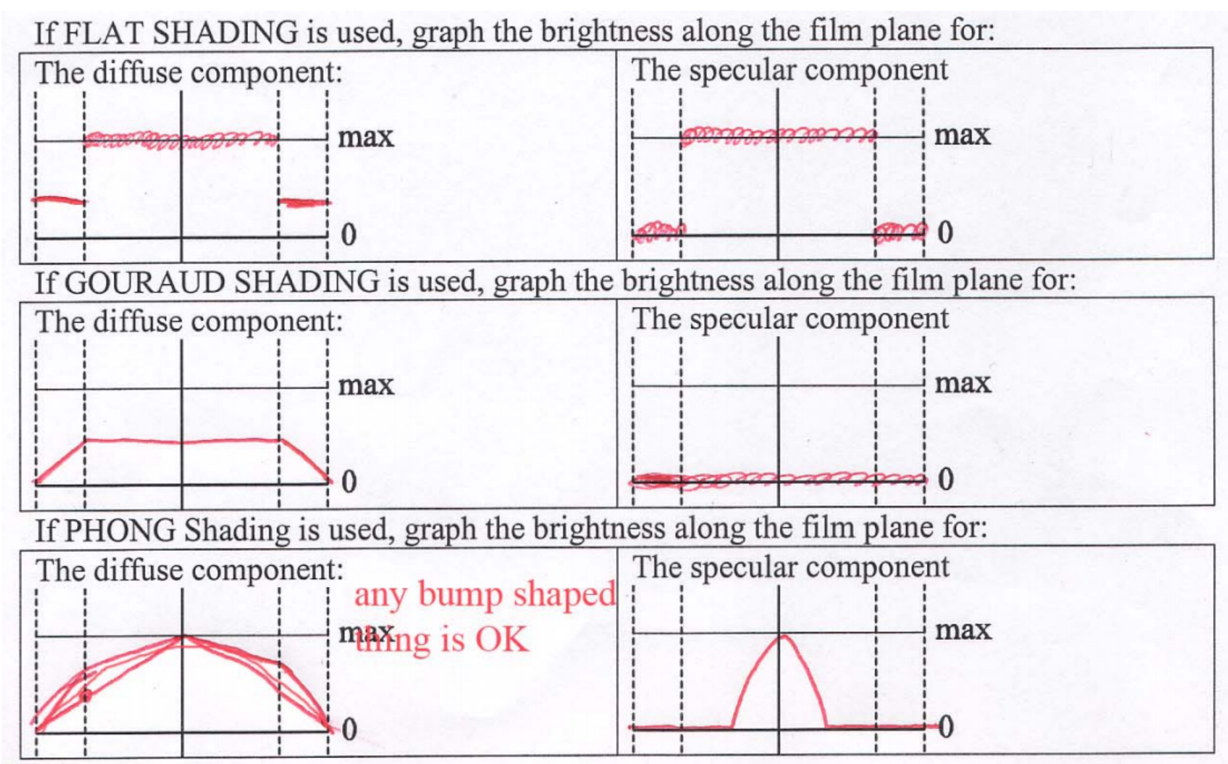
For perspective projection, the depth buffer precision is higher near the near plane than the far plane and decreases gradually.

6 Lighting and Shading

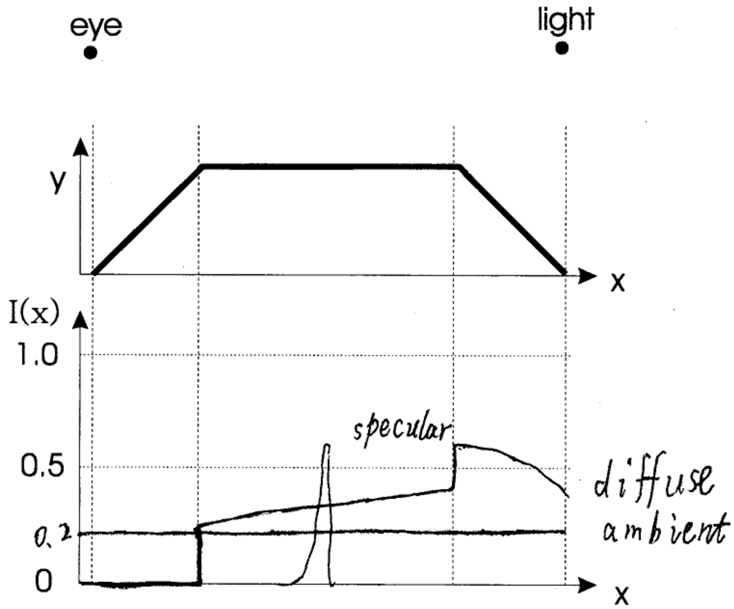
A sphere at the origin is approximated by a very small number of polygons. The camera is placed at $(0, 20, 0)$ and faces the sphere. A directional light source sends light downward from infinity along the y-axis, as shown here:



If the actual sphere (not the polygonal approximation) was lit perfectly, the specular component of the light along the film plane (in the x-direction for $z = 0$) is shown in the graph on the right. For the following questions, sketch a similar graph. (The important thing is the rough shape)



Sketch the ambient, diffuse and specular illumination for the following scene as a function of x. Assume the Phong illumination model, i.e., $I = k_a I_a + k_d I_d (N \cdot L) + k_s I_s (R \cdot V)^n$, where $k_a = 0.2, k_d = 0.6, k_s = 0.6, I_a = I_d = I_s = 1.0, n = 300$.



7 Parametric Curves

Suppose a Bézier curve $\mathbf{C}(u)$ is defined by the following four control points in the xy-plane: $\mathbf{P}_0 = (-2, 0), \mathbf{P}_1 = (-2, 4), \mathbf{P}_2 = (2, 4)$ and $\mathbf{P}_3 = (2, 0)$.

(a) What is the degree of $\mathbf{C}(u)$?

The degree is equal to the number of control points minus 1. Therefore, the degree of the given Bézier curve is 3.

(b) A Bézier curve $\mathbf{C}(u)$ defined by three control points $\mathbf{P}_0, \mathbf{P}_1$ and $\mathbf{P}_2$ can only be a parabola. Why? Elaborate your answer in some details.

Polynomials (in parametric form) can only represent parabolas, and circles, ellipses and hyperbolas require the use of rational form. Since Bézier curves are parametric polynomial curves, they can only be parabolas.

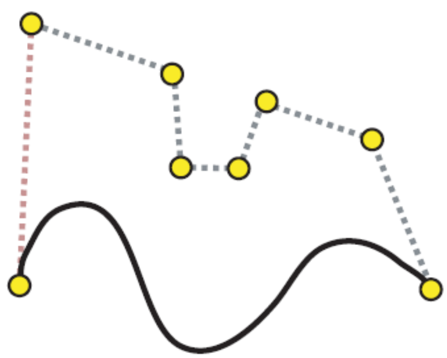
(c) Compute $\mathbf{C}(1/2)$ with de Casteljau's algorithm. You should show all computation steps in the form of a diagram showing the de Casteljau net. Complete the following diagram.

$$
\begin{tikzcd}
(-2,0) \arrow[dr] & & & \\
& (-2,2) \arrow[dr] & & \\
(-2,4) \arrow[ur] \arrow[dr] & & (-1,3) \arrow[dr] & \\
& (0,4) \arrow[ur] \arrow[dr] & & (0,3) \\
(2,4) \arrow[ur] \arrow[dr] & & (1,3) \arrow[ur] & \\
& (2,2) \arrow[ur] & & \\
(2,0) \arrow[ur] & & &
\end{tikzcd}
$$

(d) Divide the curve at $\mathbf{C}(1/2)$, and list respectively the control points on curve segment $[0, 0.5]$ and $[0.5, 1]$ in correct order.

From the above diagram, the "left" curve segment on $[0, 0.5]$ is a Bézier curve of degree 3 defined by control points $(-2, 0), (-2, 2), (-1, 3)$ and $(0, 3)$, and the "right" curve segment on $[0.5, 1]$ is a Bézier curve of degree 3 defined by control points $(0, 3), (1, 3), (2, 2)$ and $(2, 0)$.

(e) A programmer wrote a program for displaying Bézier curves. The following figure shows one of his results. He suspects his program has problems. Find two problems you can spot in this figure, and provide a brief explanation.



Note that the curve is not tangent to the first and last legs at the first and last control points. Moreover, since a portion of the curve is not in the convex hull of the control points, this violates the convex hull property.

(f) Suppose we have two Bézier curves, $\mathbf{C}_1(u)$ and $\mathbf{C}_2(u)$, where $\mathbf{C}_1(u)$ is defined by control points $\mathbf{P}_0 = (0, -2), \mathbf{P}_1 = (-2, -2), \mathbf{P}_2 = (-2, 0)$ and $\mathbf{P}_3 = (0, 0)$, and $\mathbf{C}_2(v)$ is defined by control points $\mathbf{Q}_0 = \mathbf{P}_3 = (0, 0)$, $\mathbf{Q}_1 = (2, 0), \mathbf{Q}_2 = (2, 3)$, and $\mathbf{Q}_3 = (0, 3)$, and $u, v \in [0, 1]$. Discuss the continuity at $\mathbf{P}_3 = \mathbf{Q}_0 = (0, 0)$. More precisely, are these two curves C^0, C^1, C^2, G^1, and G^2 and curvature continuous at $(0, 0)$? Circle the correct answer below:

C^0 continuous	yes	no
C^1 continuous	yes	no
C^2 continuous	yes	no
G^1 continuous	yes	no
G^2 continuous	yes	no
curvature continuous	yes	no

$\mathbf{C}_1(u)$ and $\mathbf{C}_2(u)$ are C^0, G^1 and C^1 continuous, but not G^2, C^2 and curvature continuous at (0, 0).

You must show your derivations. Without explicit calculation you will receive zero credit. You should find the following formulae for Bézeir curve useful:

$$\begin{aligned}
\mathbf{B}(t) &= (1-t)^3\mathbf{P}_0 + 3(1-t)^2 t\mathbf{P}_1 + 3(1-t)t^2\mathbf{P}_2 + t^3\mathbf{P}_3, \qquad t \in [0, 1] \\
\mathbf{B}'(t) &= 3(1-t)^2(\mathbf{P}_1 - \mathbf{P}_0) + 6(1-t)t(\mathbf{P}_2 - \mathbf{P}_1) + 3t^2(\mathbf{P}_3 - \mathbf{P}_2) \\
\mathbf{B}''(t) &= 6(1-t)(\mathbf{P}_2 - 2\mathbf{P}_1 + \mathbf{P}_0) + 6t(\mathbf{P}_3 - 2\mathbf{P}_2 + \mathbf{P}_1)
\end{aligned}$$

The curvature of $\mathbf{B}(t)$ is

$$\kappa(t) = \frac{\mathbf{B}'(t) \times \mathbf{B}''(t)}{|\mathbf{B}'(t)|^3}$$

where you may append 0 to make 3-vector for cross product calculation. If you forget the formula for computing cross product, simply write down the expression when applicable. The next page is blank if you need additional space for your derivations.

The curvature formula in the final exam version missed $'$ in the denominator but it does not change the conclusion on curvature continuity in the final answer.

Since the first and second derivatives of $\mathbf{C}_1(u)$ at $u = 1$ are $\mathbf{C}_1'(1)$ and $\mathbf{C}_1''(1)$, respectively, we have $\mathbf{C}_1(1) = \mathbf{P}_3 = (0, 0)$, $\mathbf{C}_1'(1) = (6, 0)$ and $\mathbf{C}_1''(1) = (12, -12)$. Similarly, the first and second derivatives of $\mathbf{C}_2(u)$ at $u = 0$ are $\mathbf{C}_2'(0)$ and $\mathbf{C}_2''(0)$, respectively, we have $\mathbf{C}_2(0) = \mathbf{P}_3 = (0, 0)$, $\mathbf{C}_2'(0) = (6, 0)$ and $\mathbf{C}_2''(0) = (-12, 18)$.

- Since $\mathbf{C}_1(1) = \mathbf{C}_2(0) = (0, 0)$, $\mathbf{C}_1(u)$ and $\mathbf{C}_2(u)$ are C^0-continuous at $(0, 0)$.
- Since $\mathbf{C}_1'(1) = \mathbf{C}_2'(0) = (6, 0)$, $\mathbf{C}_1(u)$ and $\mathbf{C}_2(u)$ are C^1-continuous at $(0, 0)$. They are also G^1-continuous at $(0, 0)$.
- Since $\mathbf{C}_1''(1) = (12, -12) \neq C_2''(0) = (-12, 18)$, $\mathbf{C}_1(u)$ and $\mathbf{C}_2(u)$ are *not* C^2-continuous at $(0, 0)$.
- Since $\mathbf{C}_1''(1) - C_2''(0) = (24, -30)$, which is not parallel to the tangent vector $(6, 0)$ at $(0, 0)$, $\mathbf{C}_1(u)$ and $\mathbf{C}_2(u)$ are *not* G^2-continuous at $(0, 0)$.
- The curvature of $\mathbf{C}_1(u)$ at $(0, 0)$ is computed as

  $$\kappa_1(1) = \frac{|\mathbf{C}_1'(1) \times \mathbf{C}_1''(1)|}{|\mathbf{C}_1'(1)|} = \frac{|(6, 0, 0) \times (12, -12, 0)|}{|(6, 0, 0)|^3} = \frac{1}{3}$$

  and the curvature of $\mathbf{C}_2(u)$ at $u = 0$ is computed as follows:

  $$\kappa_2(0) = \frac{|\mathbf{C}_2'(0) \times \mathbf{C}_2''(0)|}{|\mathbf{C}_2'(0)|} = \frac{|(6, 0, 0) \times (12, -18, 0)|}{|(6, 0, 0)|^3} = \frac{1}{2}$$

  Since $\kappa_1(1) \neq \kappa_2(0)$, $\mathbf{C}_1(u)$ and $\mathbf{C}_2(u)$ are not curvature continuous at $(0, 0)$.