Hidden Surfaces

Reading

Required

- Foley *et al*, Chapter 15

Optional

- Hearn and Baker, 13.1-13.3, 13.6-13.7

The Quest for 3D

Construct a 3D hierarchical geometric model

- Define a virtual camera
- Map points in 3D space to points in an image
- Produce a wireframe drawing in 2D from a 3D object
- Of course, there's more work to be done…

Introduction

- Not every part of every 3D object is visible to a particular viewer. We need an algorithm to determine what parts of each object should get drawn.
- Known as “hidden surface elimination” or “visible surface determination”.
- Hidden surface elimination algorithms can be in three major ways:
  - Object space vs. image space
  - Object order vs. image order
  - Sort first vs. sort last
  - Still a very active research area
- Where would we use a hidden surface algorithm?

Object Space Algorithms

- Operate on geometric primitives
  - For each object in the scene, compute the part of it which isn't occluded by any other object, then draw.
  - Must perform tests at high precision
  - Resulting information is resolution-independent
- Complexity
  - Must compare every pair of objects, so $O(n^2)$ for n objects
  - Optimizations can reduce this cost, but…
  - Best for scenes with few polygons or resolution-independent output
- Implementation
  - Difficult to implement!
  - Must carefully control numerical error

Image Space Algorithms

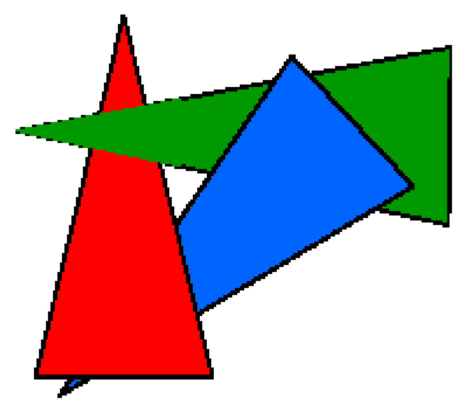
- Operate on pixels
  - For each pixel in the scene, find the object closest to the COP which intersects the projector through that pixel, then draw.
  - Perform tests at device resolution, result works only for that resolution
- Complexity
  - Must do something for every pixel in the scene, so at least O(R).
  - Easiest solution is to test projector against every object, giving O(nR).
  - More reasonable version only does work for pixels belonging to objects: O(nr), r is number of pixels per object
  - Often, with more objects, each is smaller, we estimate nr = O(R) in practice
- Implementation
  - Usually very simple!

Object Order vs. Image Order

- Object order
  - Consider each object only once - draw its pixels and move on to the next object
  - Might draw the same pixel multiple times
- Image order
  - Consider each pixel only once - draw part of an object and move on to the next pixel
  - Might compute relationships between objects multiple times

Sort First vs. Sort Last

- Sort first
  - Find some depth-based ordering of the objects relative to the camera, then draw from back to front
  - Build an ordered data structure to avoid duplicating work
- Sort last
  - Sort implicitly as more information becomes available

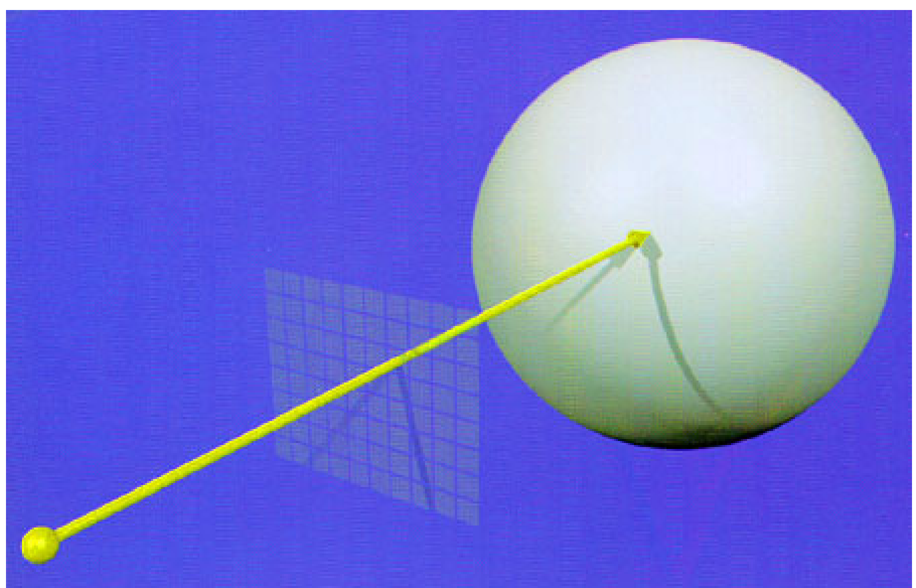


Important Algorithms

- Ray casting
- Z-buffer
- Binary space partitioning
- Back face culling

Ray Casting

- Partition the projection plane (PP) into pixels to match screen resolution
  - For each pixel p_i, construct ray (projector) from COP through PP at that pixel and into scene
  - Intersect the ray with every object in the scene, color the pixel according to the object with the closest intersection

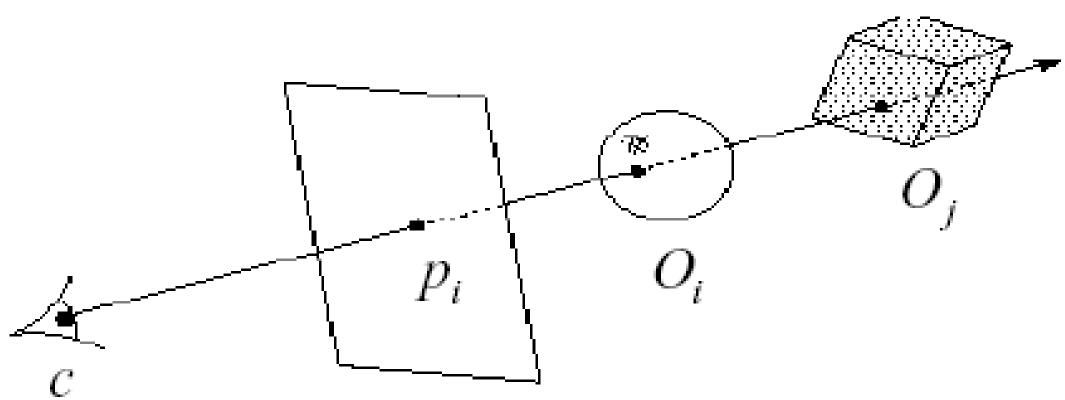


Ray Casting Implementation

- Parameterize the ray:

$$R(t) = (1-t)c + tp_i$$

  - If a ray intersects some object O_i, get parameter t such that first intersection with O_i occurs at $R(t_i)$
  - Which object owns the pixel?
- We will study ray-object intersections in more detail later (essential in ray tracing).



Aside: Definitions

- An algorithm exhibits *coherence* if it uses knowledge about the continuity of the objects on which it operates
- An *online* algorithm is one that doesn't need all the data to be present when it starts running
  - Example: insertion sort

Ray Casting: Analysis

Categorization:

- Easy to implement?
- Hardware implementation?
- Coherence?
- Memory intensive?
- Pre-processing required?
- Online?
- Handles transparency?
- Handles refraction?
- Polygon-based?
- Extra work for moving objects?
- Extra work for moving viewer?
- Efficient shading?
- Handles cycles and self-intersections?

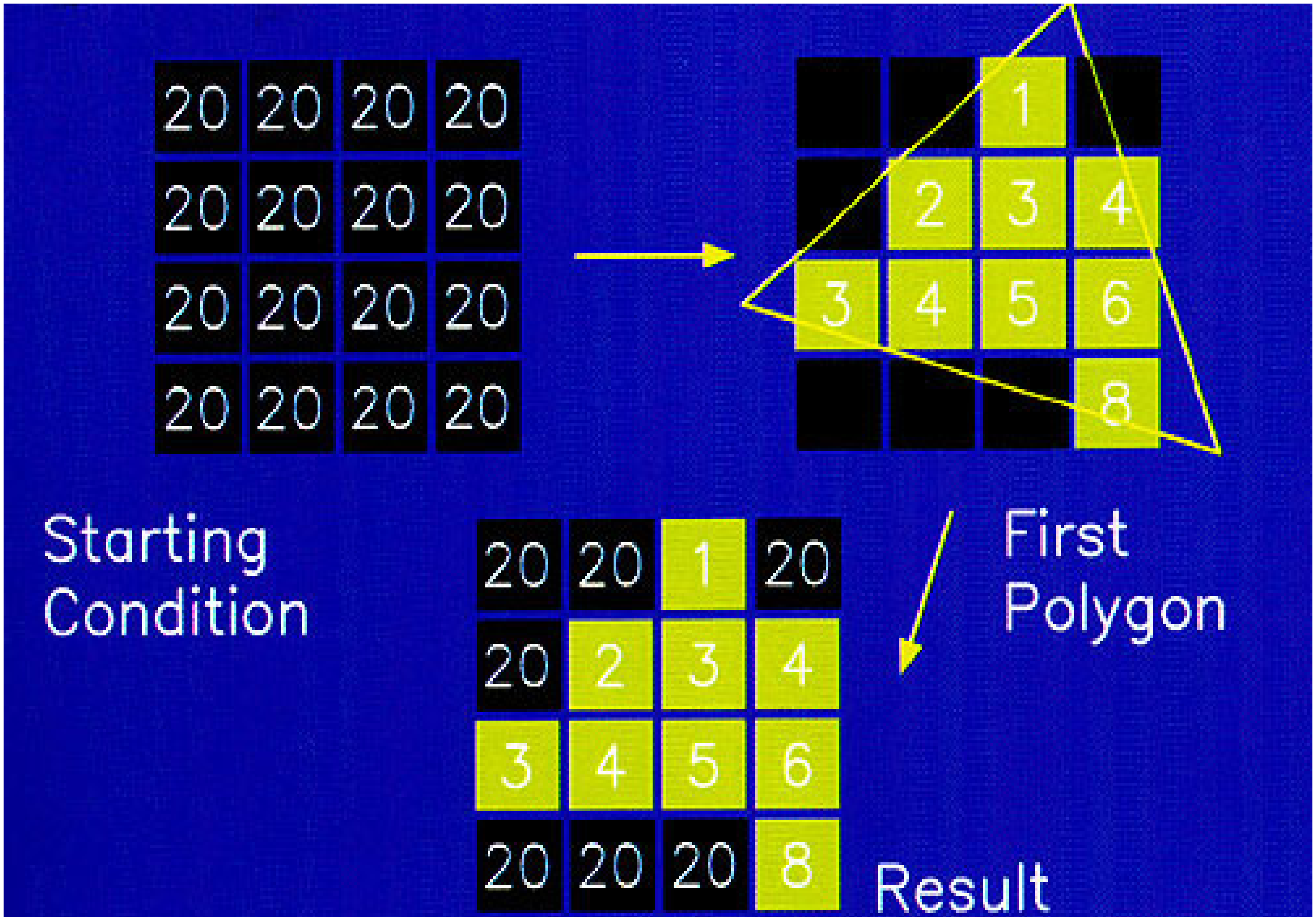
Z-buffer

- Idea: along with a pixel's red, green and blue values, maintain some notion of its *depth*
  - An additional channel in memory, like alpha
  - Called the depth buffer or Z-buffer
- When the time comes to draw a pixel, compare its depth with the depth of what's already in the framebuffer. Replace only if it's closer
  - Very widely used (in hardware, e.g. GeForce3/4)
- History
  - Originally described as "brute-force image space algorithm"
  - Written off as impractical algorithm for huge memories
  - Today, done easily in hardware

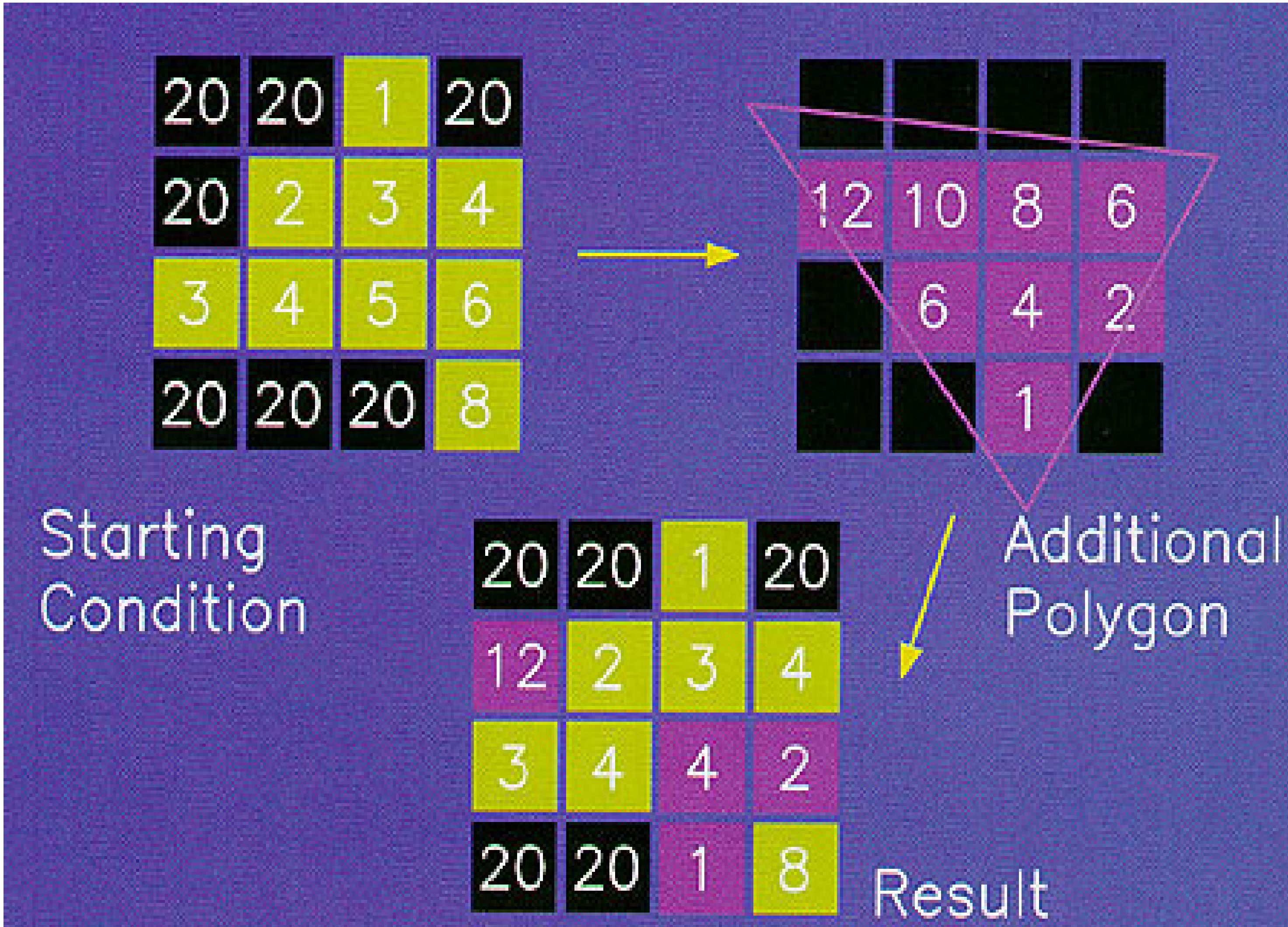
Z-buffer Implementation

```
for each pixel pi {
   Z-buffer[ pi ] = FAR
   Fb[ pi ] = BACKGROUND_COLOR
}
for each polygon P {
   for each pixel pi in the projection of P {
        Compute depth z and shade s of P at pi
        if z < Z-buffer[ pi ] {
                Z-buffer[ pi ] = z
                Fb[ pi ] = s
        }
   }
}
```

Z-Buffer Algorithm

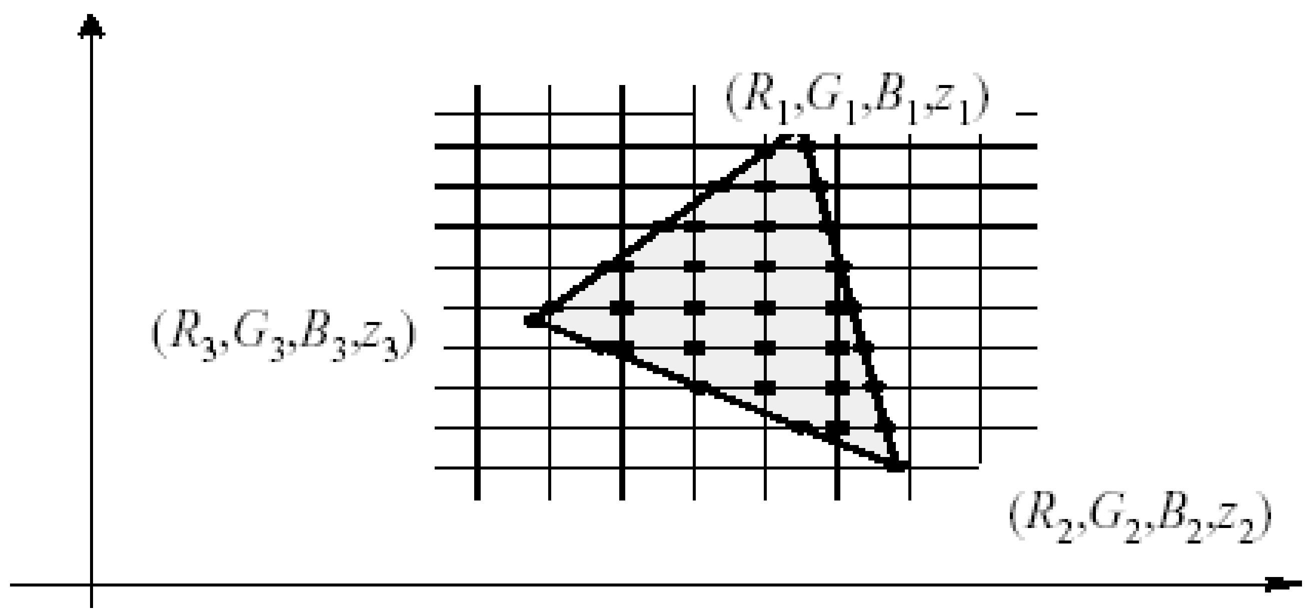


Z-Buffer Algorithm

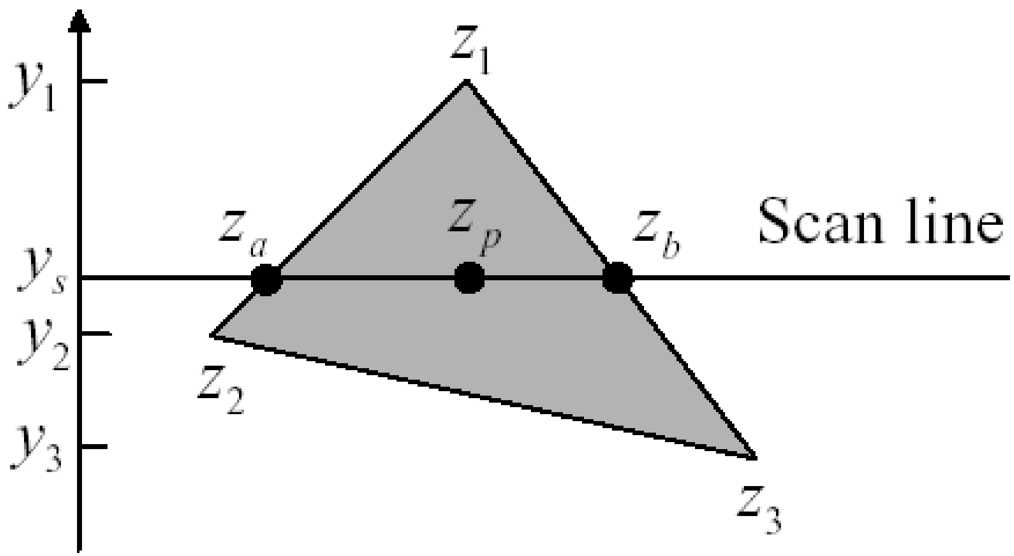


Z-buffer Tricks

- The shade of a triangle can be computed incrementally from the shades of its vertices (taking advantage of coherence)
- Can do the same with depth



Z Value Interpretation

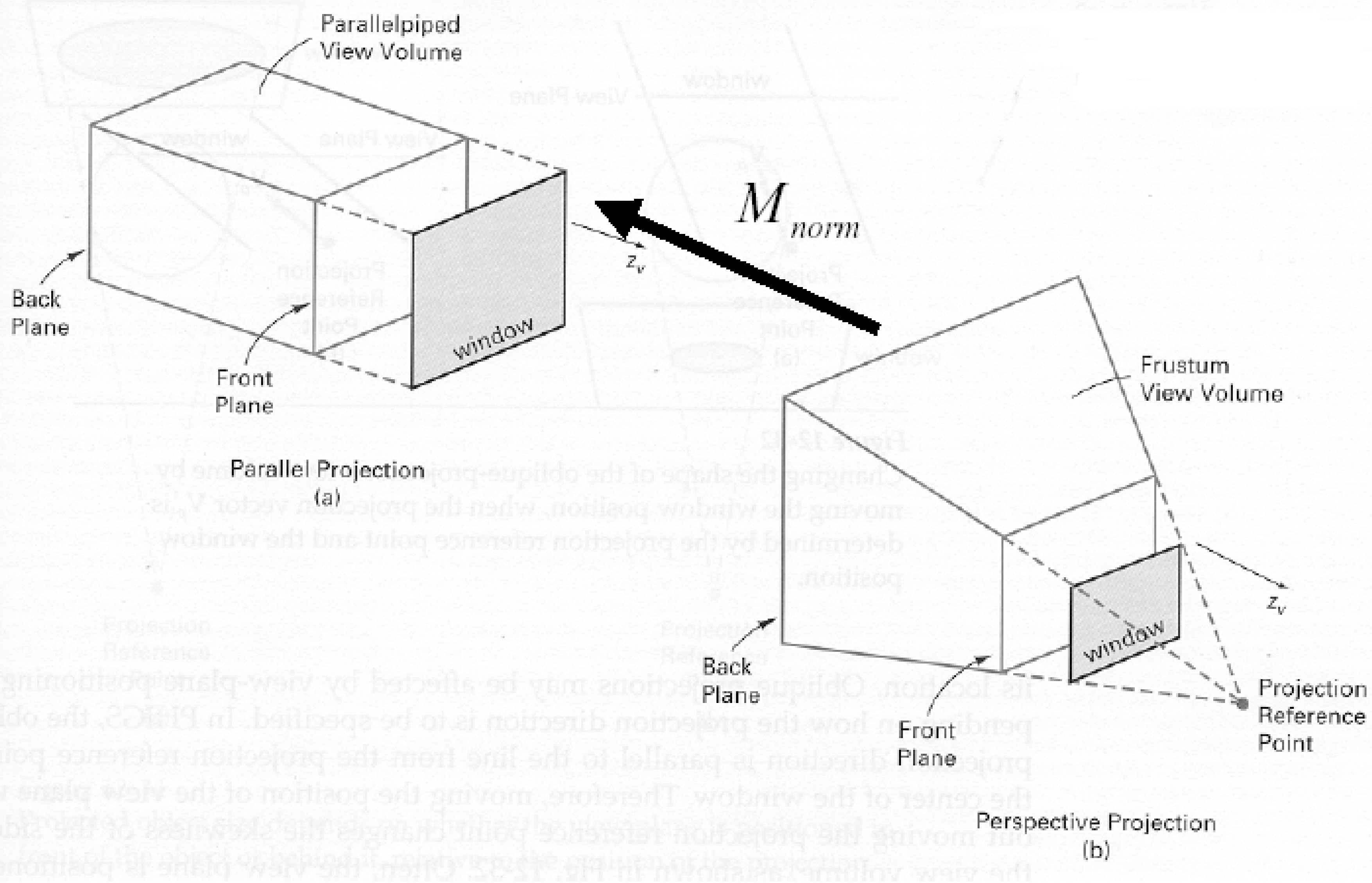


$$z_a = z_1 - (z_1 - z_2)\frac{y_1 - y_s}{y_1 - y_2}$$

$$z_b = z_1 - (z_1 - z_3)\frac{y_1 - y_s}{y_1 - y_3}$$

$$z_p = z_b - (z_b - z_a)\frac{x_b - x_p}{x_b - x_a}$$

Depth Preserving Conversion to Parallel Projection



Parallel Projection
(a)

Perspective Projection
(b)

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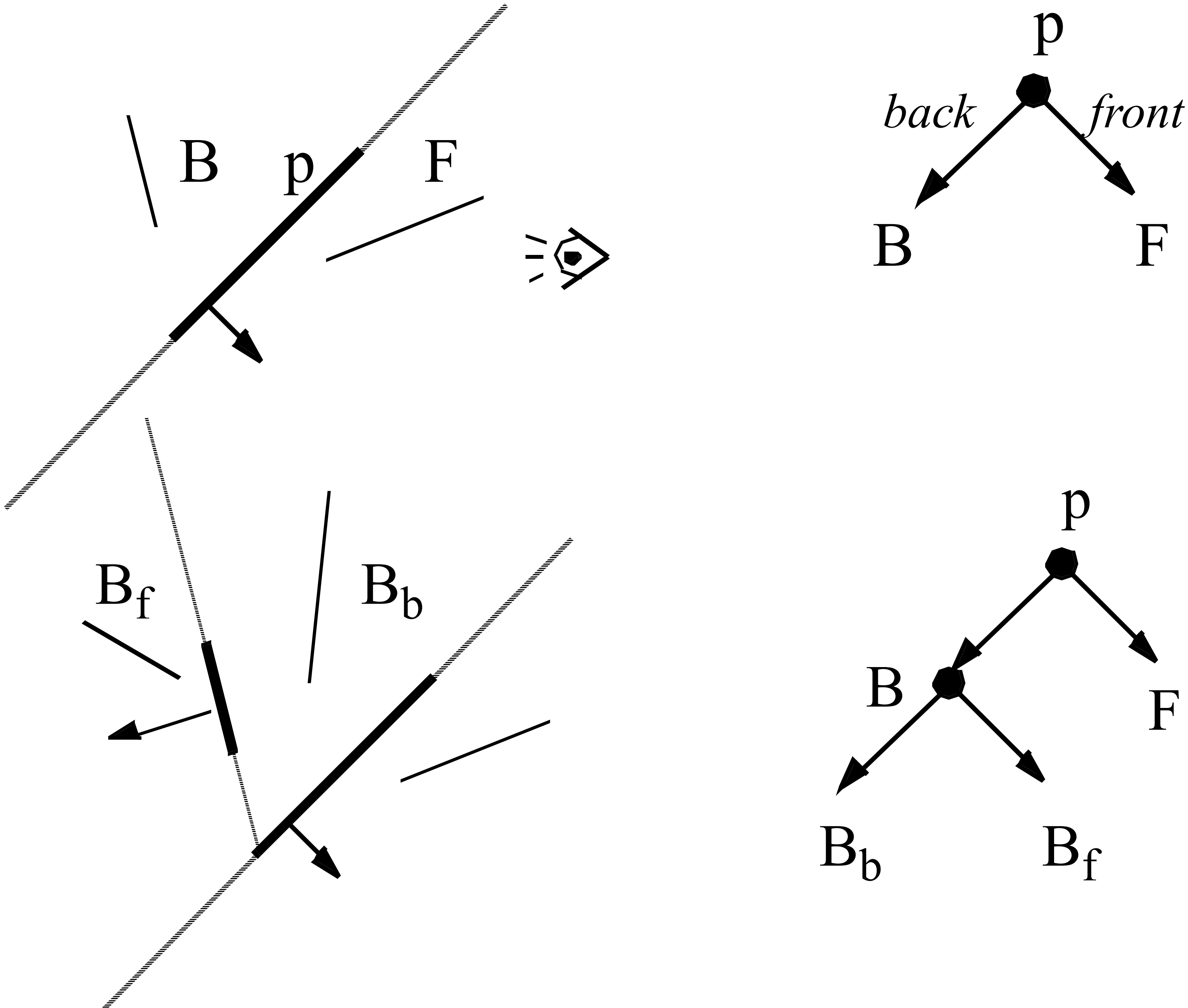
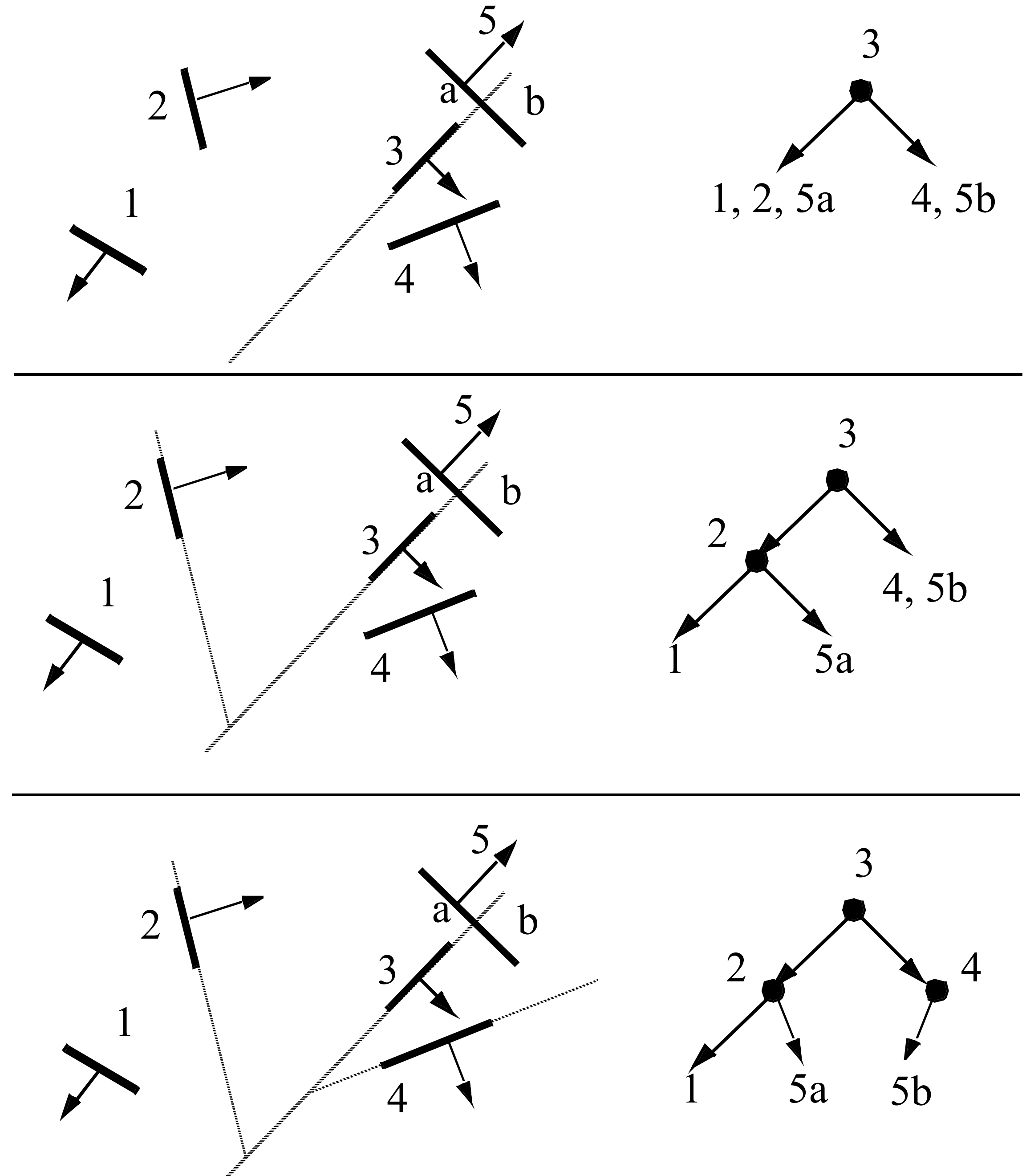
Binary Space Partitioning

- Goal: build a tree that captures some relative depth information between objects. Use it to draw objects in the right order.
  - Tree doesn't depend on camera position, so we can change viewpoint and redraw quickly
  - Called the binary space partitioning tree, or BSP tree
- Key observation: The polygons in the scene are painted in the correct order if for each polygon P,
  - Polygons on the far side of P are painted first
  - P is painted next
  - Polygons in front of P are painted last

Binary Space-Partitioning Tree

- Given a polygon p
- Two lists of polygons:
    - those that are behind(p) :B
    - those that are infront(p) :F
- If eye is in-front(p), right display order is B, p, F
- Otherwise it is F, p, B


B
p
F
B_f
B_b
p
back
front
B
F
p
B
F
B_b
B_f


5
a
b
2
3
1
4
3
1, 2, 5a
4, 5b
5
a
b
2
3
1
4
3
2
4, 5b
1
5a
5
a
b
2
3
1
4
3
2
4
1
5a
5b

BSP Tree Construction

```
BSPtree makeBSP( L: list of polygons ) {
    if L is empty {
          return the empty tree
    }
    Choose a polygon P from L to serve as root
    Split all polygons in L according to P
    return new TreeNode(
          P,
          makeBSP( polygons on negative side of P ),
          makeBSP( polygons on positive side of P ))
}
```

- Splitting polygons is expensive! It helps to choose P wisely at each step.
- Example: choose five candidates, keep the one that splits the fewest polygons

BSP Tree Display

```
showBSP( v: Viewer, T: BSPtree ) {
   if T is empty then return
   P := root of T
   if viewer is in front of P {
        showBSP( back subtree of T )
        draw P
        showBSP( front subtree of T )
   } else {
        showBSP( front subtree of T )
        draw P
        showBSP( back subtree of T )
   }
}
```

BSP Tree: Analysis

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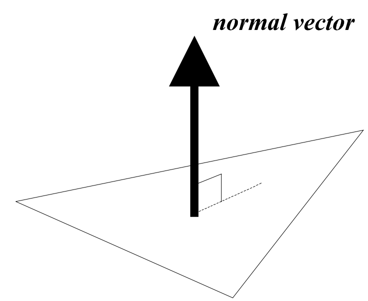
Back Face Culling

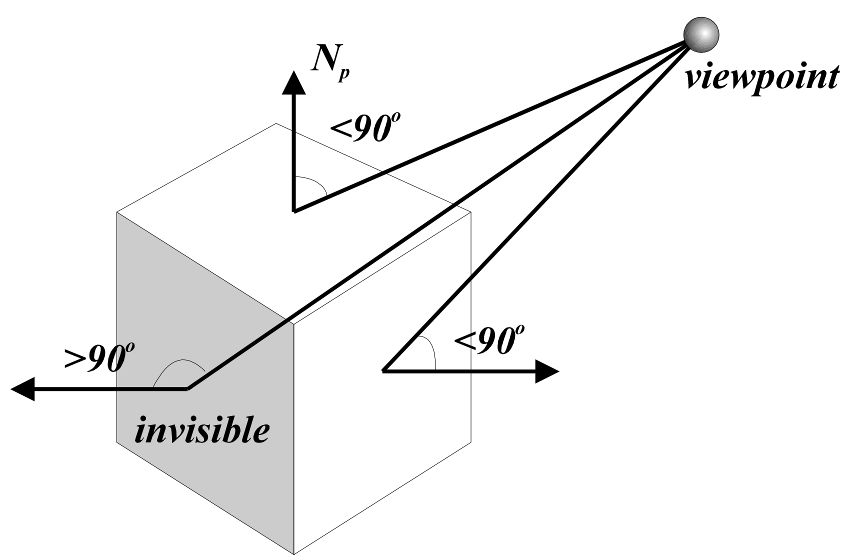
- Can be used in conjunction with polygon-based algorithms
  - Often, we don’t want to draw polygons that face away from the viewer. So test for this and eliminate (cull) backfacing polygons before drawing
- How can we test for this?

Back Face Culling

- What is a normal vector?
- It is a vector pointing in an orthogonal direction of a plane.

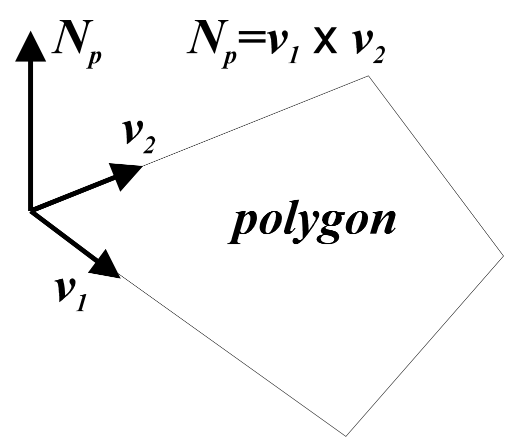


- A polygon is front facing when θ, the angle between the viewer and the normal to the polygon, lies between -90^o to $+ 90^o$ or $\cos \theta \geq 0$



Back Face Culling

- Use dot product to test for visibility.
- Visibility test: $N_p \cdot V > 0$
  where N_p is the polygon normal
  V is the line of sight (viewing) vector
- How to calculate polygon normal?



- A polygon is front facing when θ, the angle between the viewer and the normal to the polygon, lies between -90^o to $+90^o$ or $\cos\theta \geq 0$
- All vertices of polygons must be listed in the same order (either clockwise or anticlockwise)

Back Face Culling

- Eye space is the most convenient space in which to ‘cull’ polygon
- Remove all polygons that face away from the viewer
- For a scene that consists of only a single close object, it solves the hidden surface problem *completely*.
- Not work for scene with multiple objects
- Cannot remove all unnecessary polygons of a concave object
- In most cases, it is a preprocess to reduce invisible polygons.
- But it cannot removes all unnecessary triangles.

Summary

- Classification of hidden surface algorithms
- Understanding ray casting algorithms
- Understanding of Z-buffer
- Familiarity with BSP trees and back face culling