Subdivision Surfaces

Reading

- Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 10.2 (available in library).

Subdivision curves

Idea:

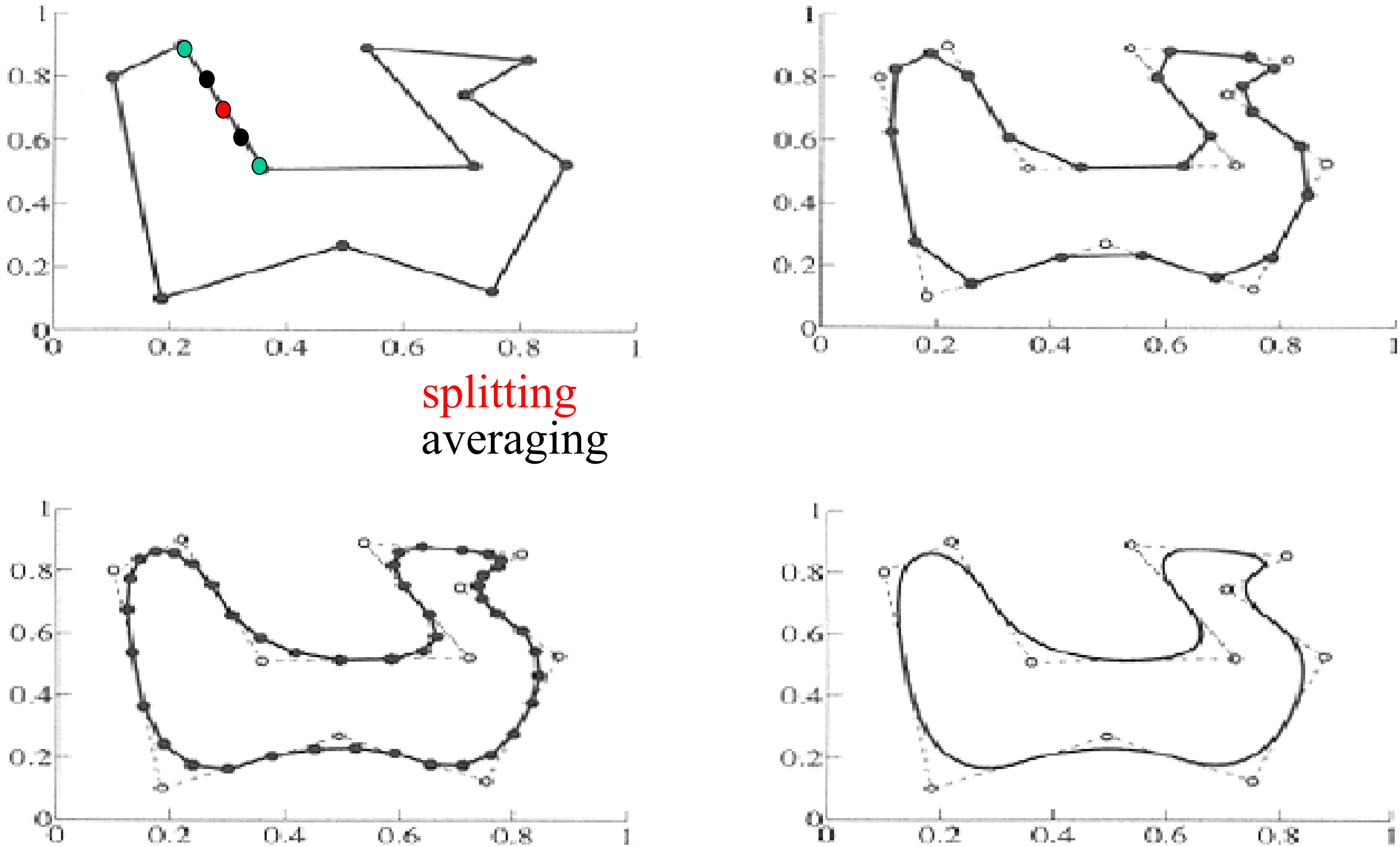
- repeatedly refine the control polygon

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow \cdots$$

$$C = \lim_{i \to \infty} P_i$$

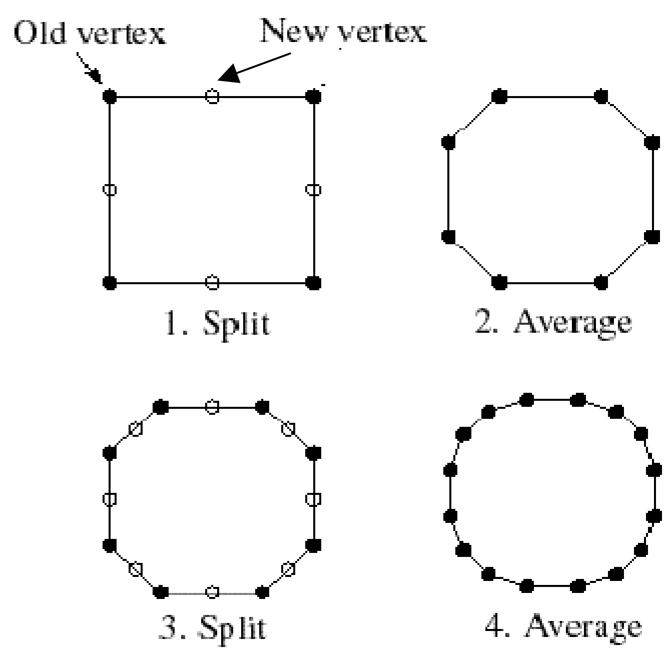
- curve is the limit of an infinite process:

Subdivision curves



Chaikin's algorithm

- Chaikin introduced the following "corner-cutting" scheme in 1974:
  - Start with a piecewise linear curve
  - Insert new vertices at the midpoints (the **splitting step**)
  - Average each vertex with the "next" neighbor (the **average step**)
  - Go to the splitting step



Averaging masks

- The limit curve is a quadratic B-spline!
- Instead of averaging with the nearest neighbor, we can generalize by applying an averaging mask during the averaging step:

$$r = (\ldots, r_{-1}, r_0, r_1, \ldots)$$

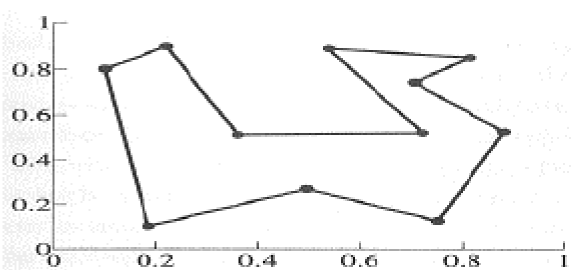
- In case of Chaikin’s algorithm:

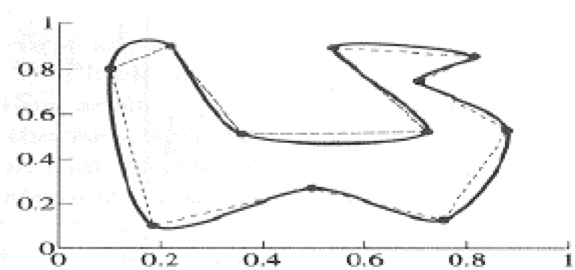
$$r = (0.5, 0.5)$$

DLG interpolating scheme (1987)

- Slight modification to algorithm:
  - splitting step introduces midpoints
  - averaging step *only changes midpoints*
- For DLG (Dyn-Levin-Gregory) use:

$$r = \frac{1}{16}(-2,6,10,6,-2)$$

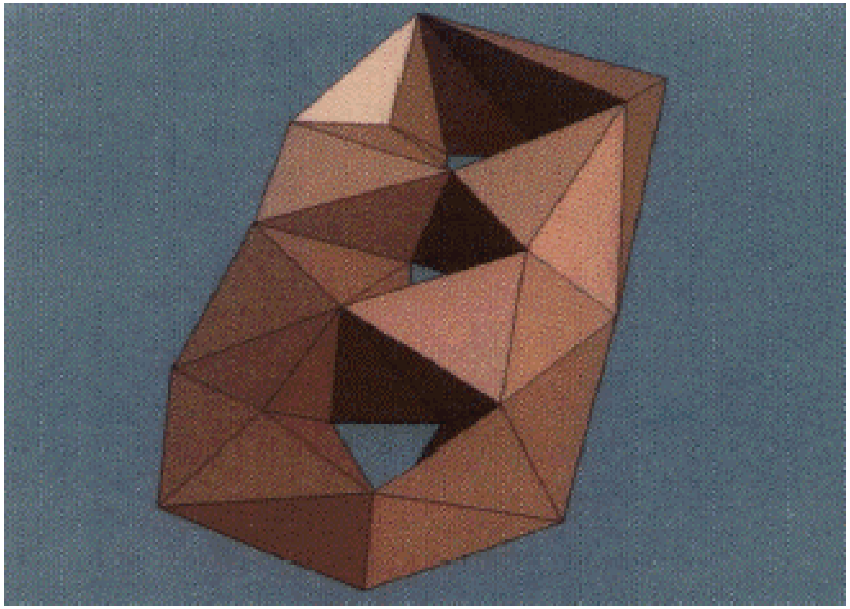


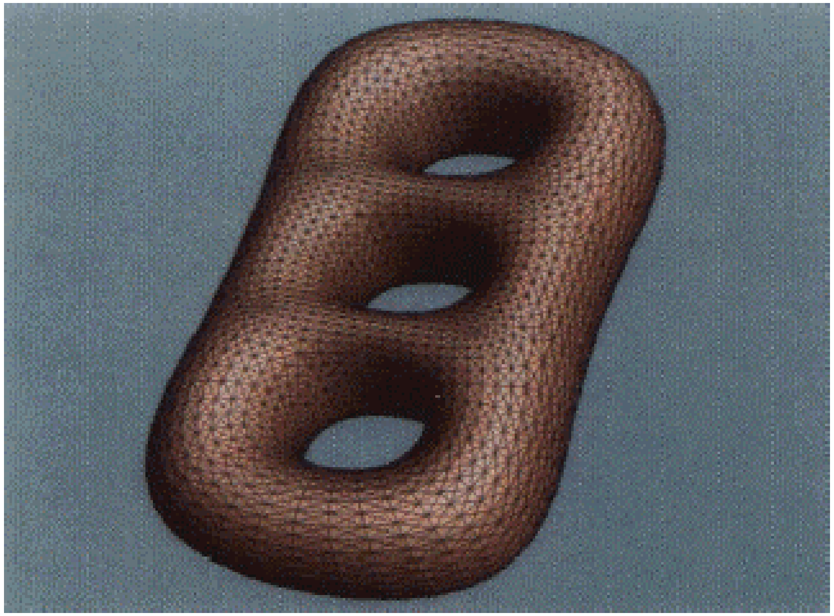


- Since we are only changing the midpoints, the points are the averaging step do not move.

Building Complex Models

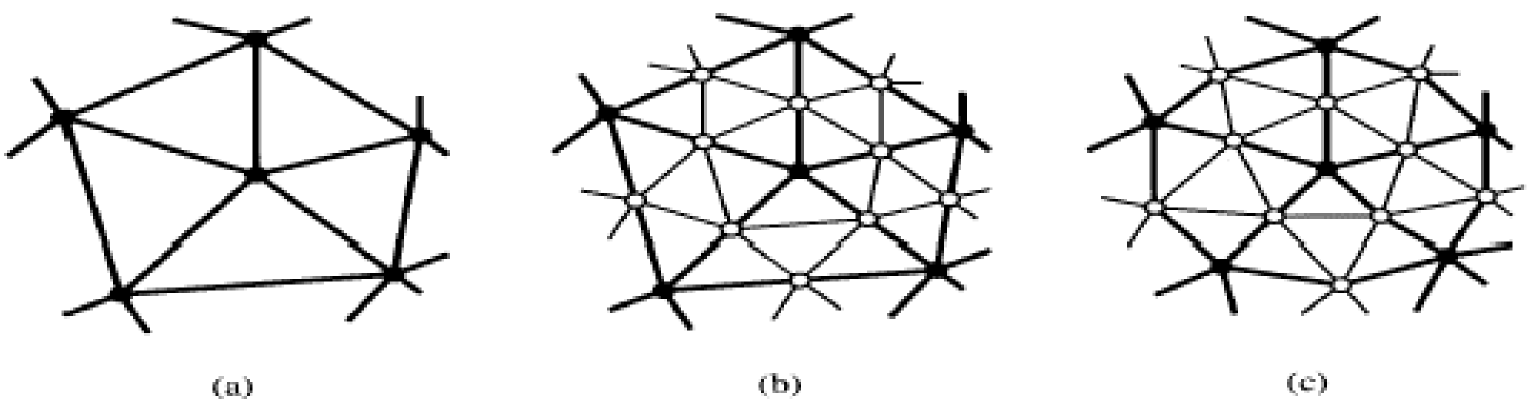
- This simple idea can be extended to build subdivision surfaces.





Subdivision surfaces

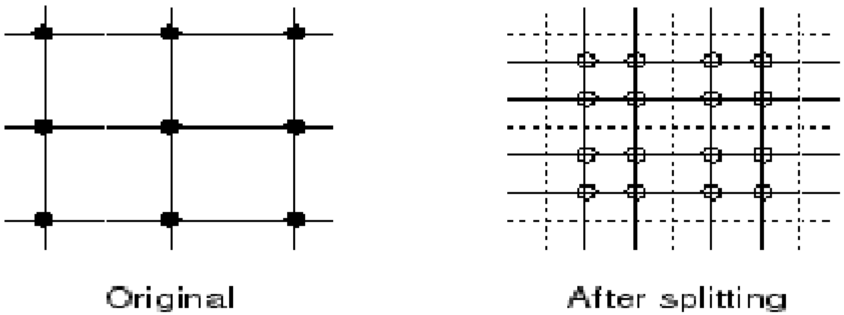
- Chaikin’s use of subdivision for curves inspired similar techniques for subdivision.
- Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

  using splitting and averaging $\sigma = \lim_{j \to \infty} M^j$
-
  

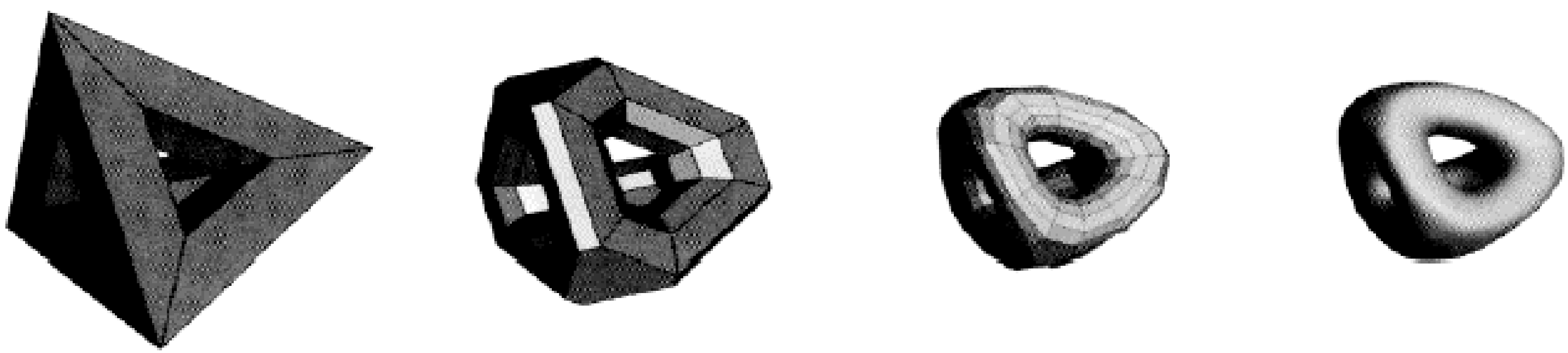
  – face schemes

Vertex schemes

- A vertex surrounded by n faces is split into *n* subvertices, one for each face:

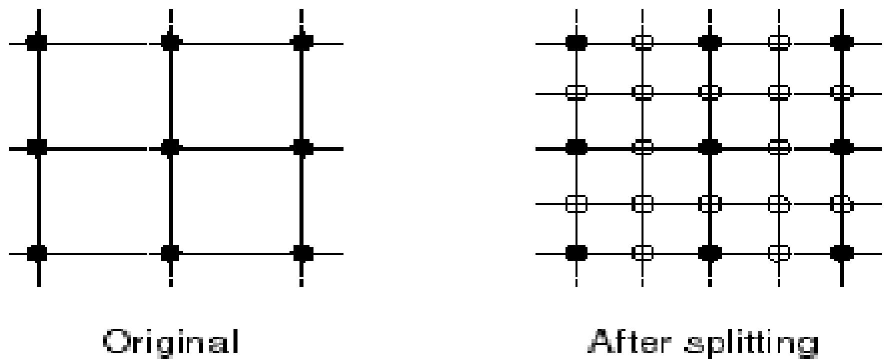


- Doo-Sabin subdivision:

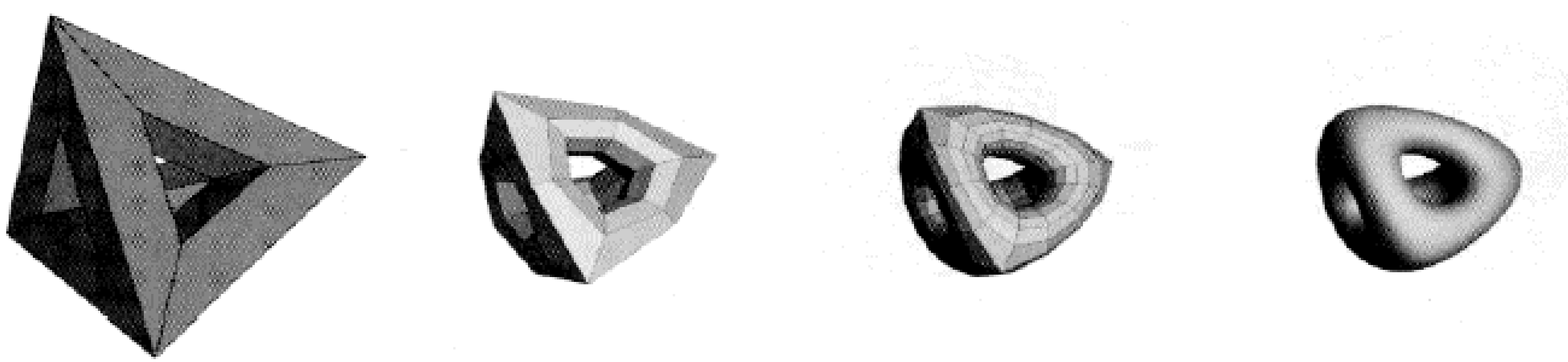


Face schemes

- Each quadrilateral face is split into four subfaces:

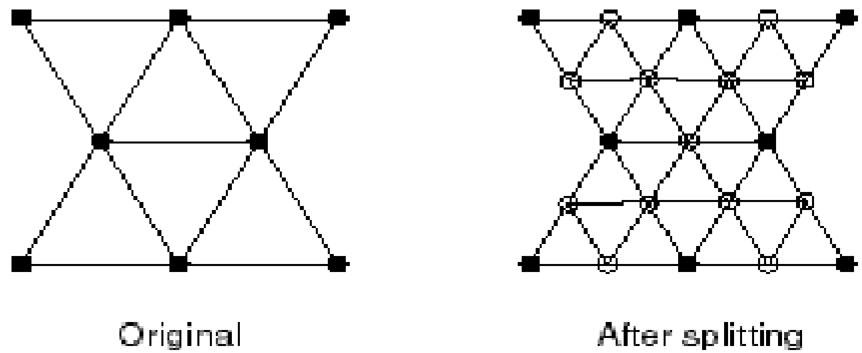


- Catmull-Clark subdivision:

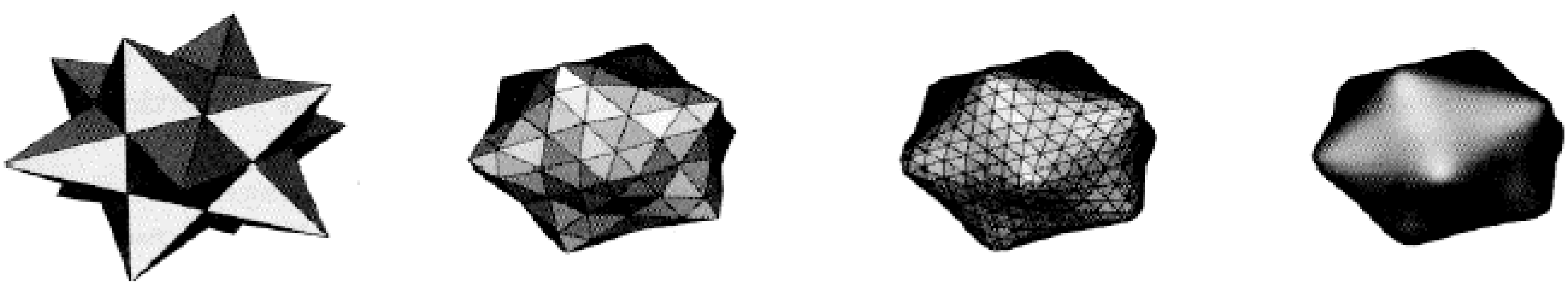


Face scheme, cont.

- Each triangular face is split into four subfaces:

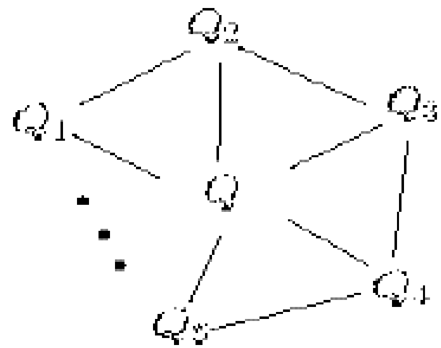


- Loop subdivision:

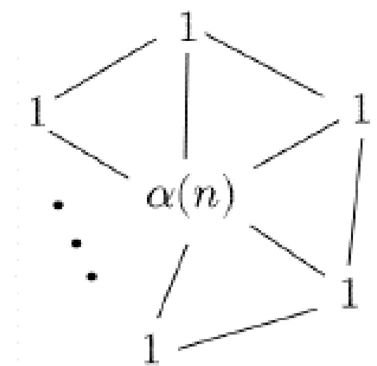


Averaging step

- Once again we can use averaging **masks** for the averaging step:



Vertex labeling



Averaging mask

$$Q \leftarrow \frac{\alpha(n) + Q_1 + \cdots + Q_n}{\alpha(n) + n}$$

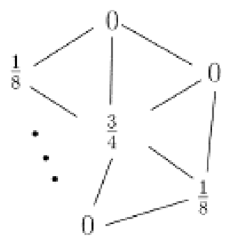
where

$$\alpha(n) = \frac{n(1 - \beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi / n))^2}{32}$$

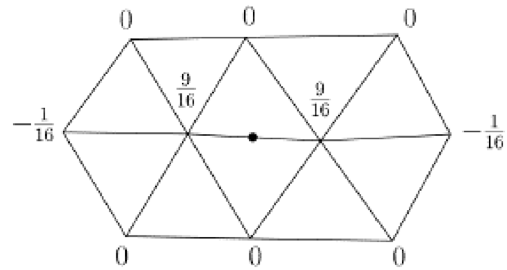
(carefully chosen to ensure smoothness.)

Adding creases without trim curves

- Sometimes, particular feature such as a crease should be preserved. With B-spline surfaces, this required the use of trim curves.
- For subdivision surfaces, we just modify the subdivision mask:

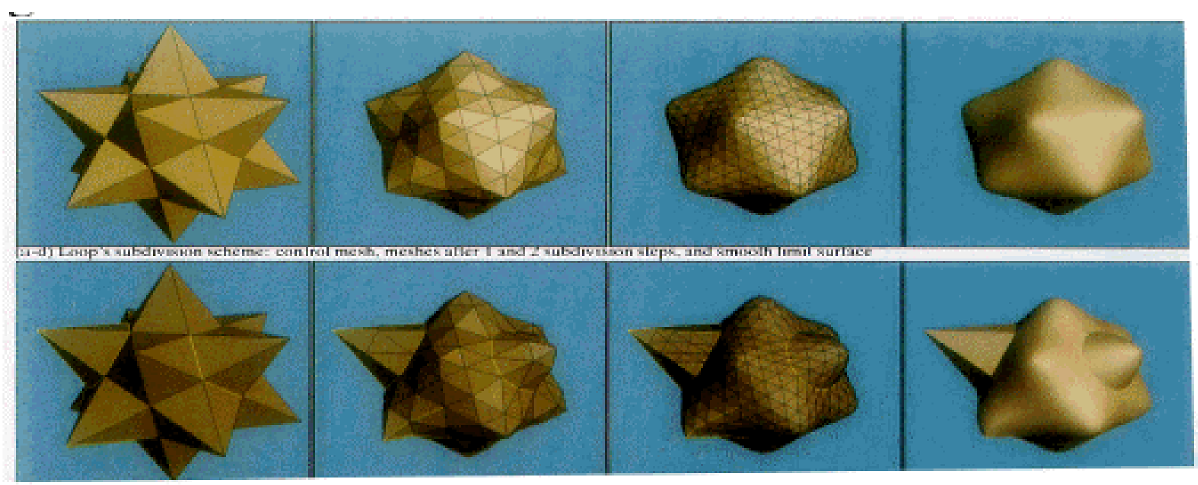


Loop crease/boundary edge



Buttery crease/boundary edge

- This gives rise to G0 continuous surfaces.



Creases without trim curves

- Here's an example using Catmull-Clark surfaces: