Multiple Torsional Viscous Damper Design

by

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Abstract

This project aims to analyze a disk-torsional spring-chain. The mode shapes of the system are found using modal analysis. The transmissibility from the base excitation to the last disk is found by using the corresponding element of the receptance matrix. The transmissibility is reduced by adding Houdaille dampers, which were optimzied using the built-in fminimax function of MATLAB. The damped frequencies were found by searching a small range before the the natural frequencies using built-in fminbnd function of MATLAB. The results showed that the minimum transmissibility is achived by putting a single absorber at the last disk and the limit on the total viscosity of the absorbers is the important parameter, while the number of disks has negligible effects.

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1 Introduction

In Figure 1, an (n+2)-degrees-of-freedom system that is going to be designed is shown, where $I_i = 100/n$, $k_i = 25n$. Four problems will be solved to design the system:

1. What are the natural frequencies and mode shapes of the n-degrees-of-freedom disk-torsional spring chain without the viscous torsional dampers for a given n?

2. Let the system without the viscous torsional dampers to be base excited. Find transmissibility for the last disk (Θ_n/Φ), assuming $\overrightarrow{\theta}(t) = \overrightarrow{\Theta}e^{i\omega t}$, $\overrightarrow{\varphi}(t) = \Phi e^{i\omega t}$, and plot the absolute value of the transmissibility in log-log axes for $0 \leq \omega \leq 1.5\omega_{n,\max}$.

3. For the (n+2)-degrees-of-freedom system shown in Figure 1 (viscous dampers are attached to the first and last disks) find the optimum c_{a1} and c_{a2} values that minimize the peak transmissibility for the last disk. I_{a1} and I_{a2} are given and between 0.1 and 0.3.

4. For a given $I_{a1} + I_{a2} = \mu$ value, find the optimum I_{a1}, I_{a2}, c_{a1}, c_{a2} values and location of the torsional dampers that minimize the peak transmissiblity for the last disk.

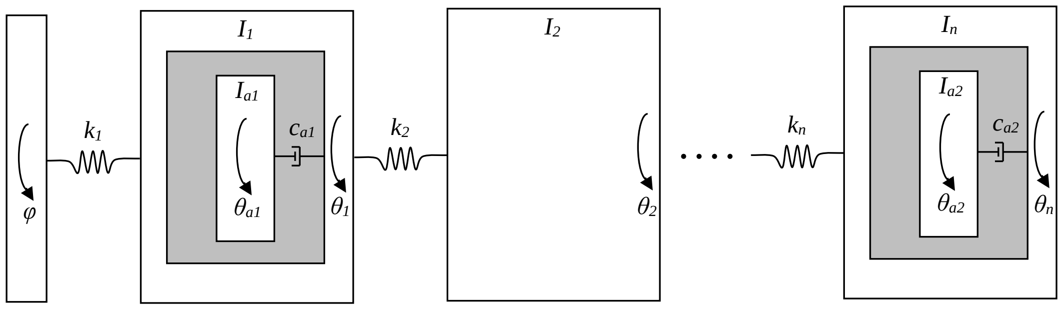

Figure 1: The system to be designed.

2 Theory

The equation of motion is given in Eq. (1) below in the matrix form.

$$\boldsymbol{M}\ddot{\vec{\theta}}(t) + \boldsymbol{C}\dot{\vec{\theta}}(t) + \boldsymbol{K}\vec{\theta}(t) = \vec{f}(t) \tag{1}$$

The force vector's first component can be written in terms of the excitation angular displacement by multiplying the latter by the stiffness between the base and the first inertia. The rest of its components are zero. The excitation and the resulting angular displacements are expressed as follows.

$$\vec{f}(t) = \begin{bmatrix} k & 0 & \cdots & 0 \end{bmatrix}^T \phi(t) = \vec{F}\exp(i\omega t) \tag{2}$$

$$\phi(t) = \Phi\exp(i\omega t) \tag{3}$$

$$\vec{\theta}(t) = \vec{\Theta}\exp(i\omega t) \tag{4}$$

The equation of motion can be reduced to Eq. (5) using Eq. (2), Eq. (3), and Eq. (4) by cancelling the exponential terms as the exponential terms which cannot be zero. These exponential terms cannot be zero as there will be no vibration in that case.

$$\boldsymbol{M}\ddot{\vec{\Theta}} + \boldsymbol{C}\dot{\vec{\Theta}} + \boldsymbol{K}\vec{\Theta} = \vec{F} \tag{5}$$

The damping matrix and the force vector are zero in the first part. Then, modal analysis was used to get the mode shapes and natural frequencies.

$$\boldsymbol{M}\ddot{\vec{\Theta}} + \boldsymbol{K}\vec{\Theta} = \vec{0}$$

$$\boldsymbol{M}\boldsymbol{M}^{-1/2}\ddot{\vec{Q}} + \boldsymbol{K}\boldsymbol{M}^{-1/2}\vec{Q} = \vec{0}$$

$$\boldsymbol{M}^{-1/2}\boldsymbol{M}\boldsymbol{M}^{-1/2}\ddot{\vec{Q}} + \boldsymbol{M}^{-1/2}\boldsymbol{K}\boldsymbol{M}^{-1/2}\vec{Q} = \vec{0}$$

$$\boldsymbol{I}\ddot{\vec{Q}} + \tilde{\boldsymbol{K}}\vec{Q} = \vec{0}$$

As you can see, with the above transformation, finding the natural frequencies becomes an eigenvalue problem. The eigenvectors of the mass normalized stiffness matrix ($\tilde{\boldsymbol{K}}$) are the mode shapes, and the square root of its eigenvalues are the natural frequencies.

Using Eq. (5), the receptance matrix ($\boldsymbol{\alpha}$) can be driven to find the transmissibility that is wanted.

$$\boldsymbol{M}\ddot{\vec{\Theta}} + \boldsymbol{C}\dot{\vec{\Theta}} + \boldsymbol{K}\vec{\Theta} = \vec{F}$$

$$-\omega^2\boldsymbol{M}\vec{\Theta} + i\omega\boldsymbol{C}\vec{\Theta} + \boldsymbol{K}\vec{\Theta} = \vec{F}$$

$$\left(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K}\right)\vec{\Theta} = \vec{F}$$

$$\vec{\Theta} = \left(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K}\right)^{-1}\vec{F}$$

$$\boldsymbol{\alpha} = \left(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K}\right)^{-1}$$

The transmissibility (T) is defined as shown in Eq. (6). Then, only a single entry of the receptance matrix can be used to improve the computation speed.

$$T = \left| \frac{\Theta_n}{\Phi} \right| \tag{6}$$

$$\Theta_n = \boldsymbol{\alpha}_{1,n} F_1$$

$$\Theta_n = \boldsymbol{\alpha}_{1,n} k \Phi$$

$$\frac{\Theta_n}{\Phi} = \boldsymbol{\alpha}_{1,n} k$$

$$T = |\boldsymbol{\alpha}_{1,n} k| \tag{7}$$

Transmissibility shown in Eq. (7) is plotted for a range of excitation frequencies and optimized using MATLAB. The natural frequencies found in the first part were used to find the peaks of transmissibility. After the damping is added, the damped frequencies are expected to be slightly smaller than the natural frequencies. Thus, the maximum transmissibility values are only searched n times in a range from 95% of the natural frequencies to themselves for the last two parts.

3 Results

The MATLAB code seen in Appendix A has been used to mainly calculate the vibration characteristics of systems with degrees of freedom 3 and 5. This code can be used to calculate different systems' vibration characteristics by changing the n and μ variables inside the code.

For the first two parts, where there are no absorbers, the mode shapes for the case with 3 and 5 degrees of freedom can be seen in Figure 2 and Figure 3, respectively. Here it is visible that the first three mode shapes are similar. This is the expected result as the systems' underlying physics are the same.

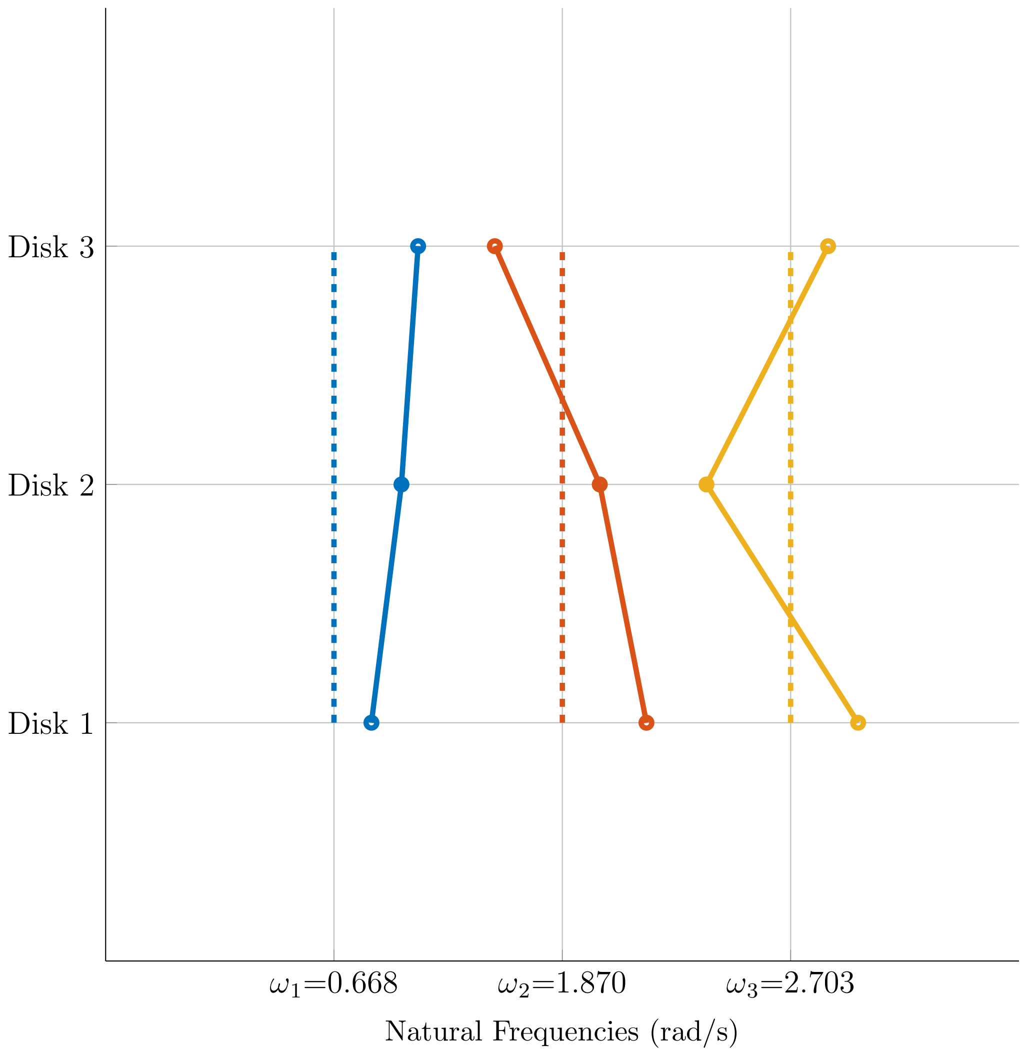

Figure 2: Mode Shapes n=3

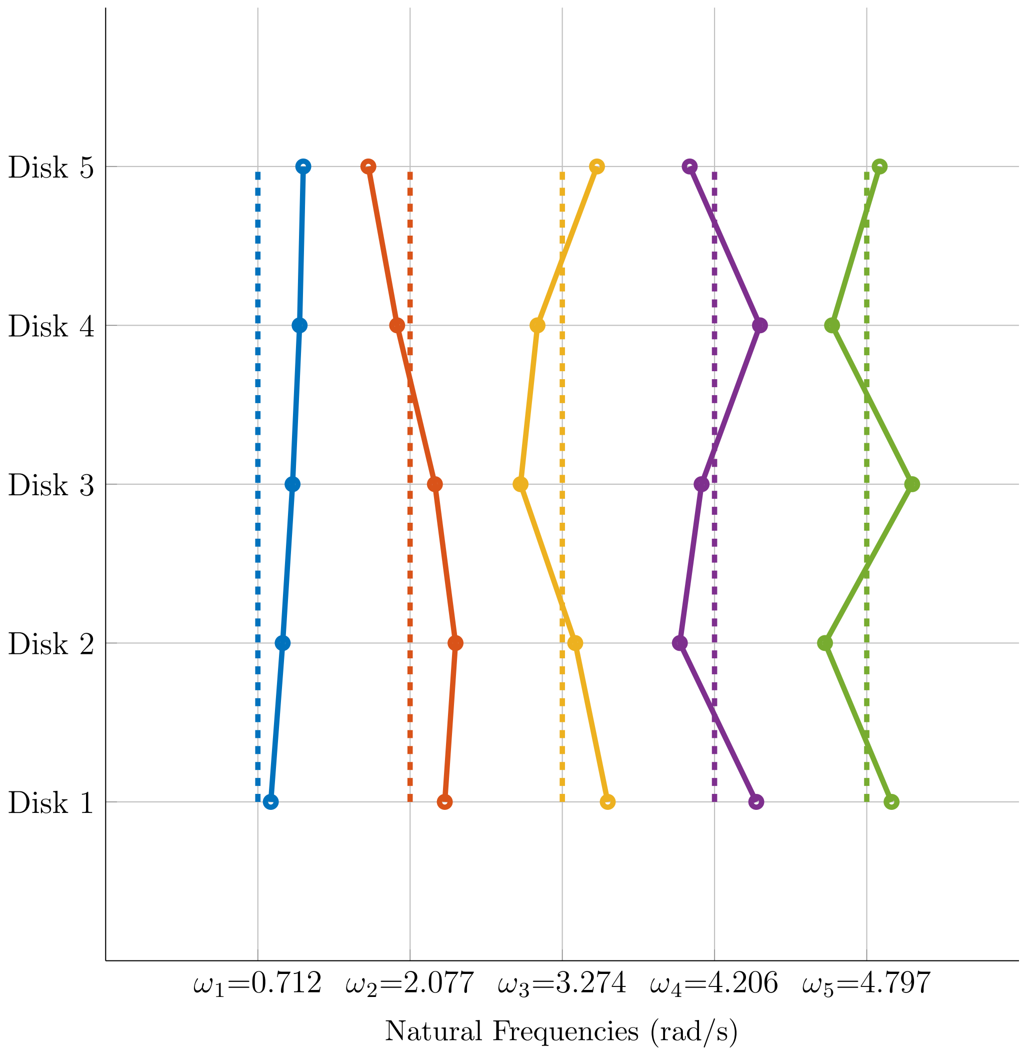

Figure 3: Mode Shapes n=5

The transmissibilities for these two cases for the undamped system are given below in Figure 4 and Figure 5. The resonance can be observed for the last disk for both cases where the transmissibility goes to infinity theoretically. Additionally, there are equal amounts of peaks and degrees of freedom.

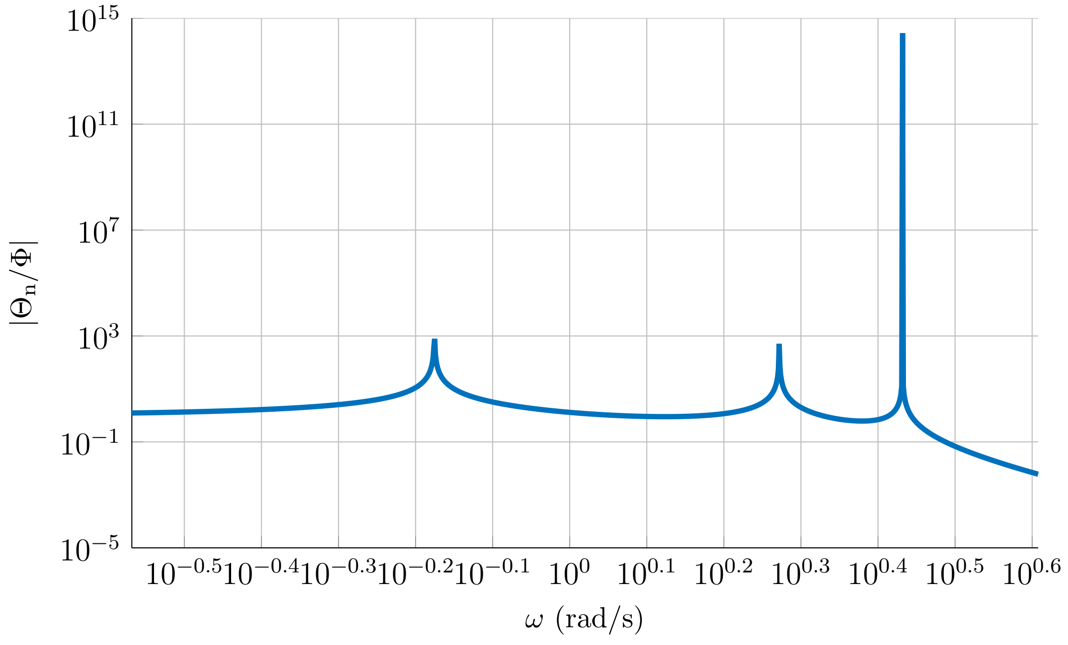

Figure 4: Part 2 Transmissibility n=3

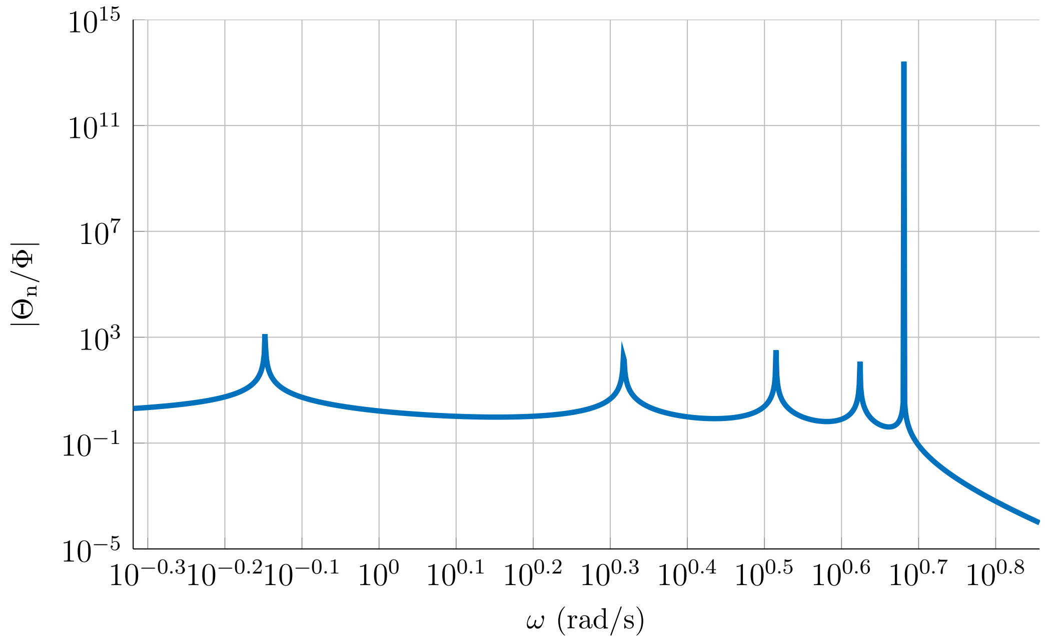

Figure 5: Part 2 Transmissibility n=5

As the absorbers are placed in the first and last disk, the peaks in Figure 4 and Figure 5 shift to the left slightly. This fact was used to efficiently find the maximum transmissibility of a design, as discussed in the previous section. The transmissibility of the solutions for the third part are visible in Figure 6 and Figure 7. The resonance is gone and the transmissibility is greatly reduced in this step.

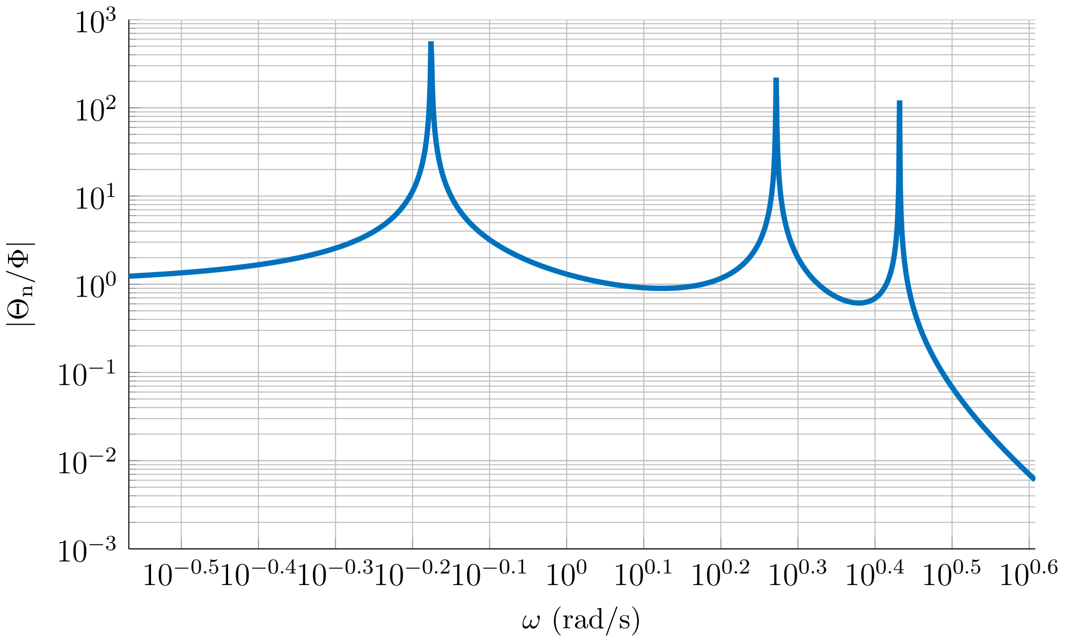

Figure 6: Part 3 Transmissibility n=3 u=0.3

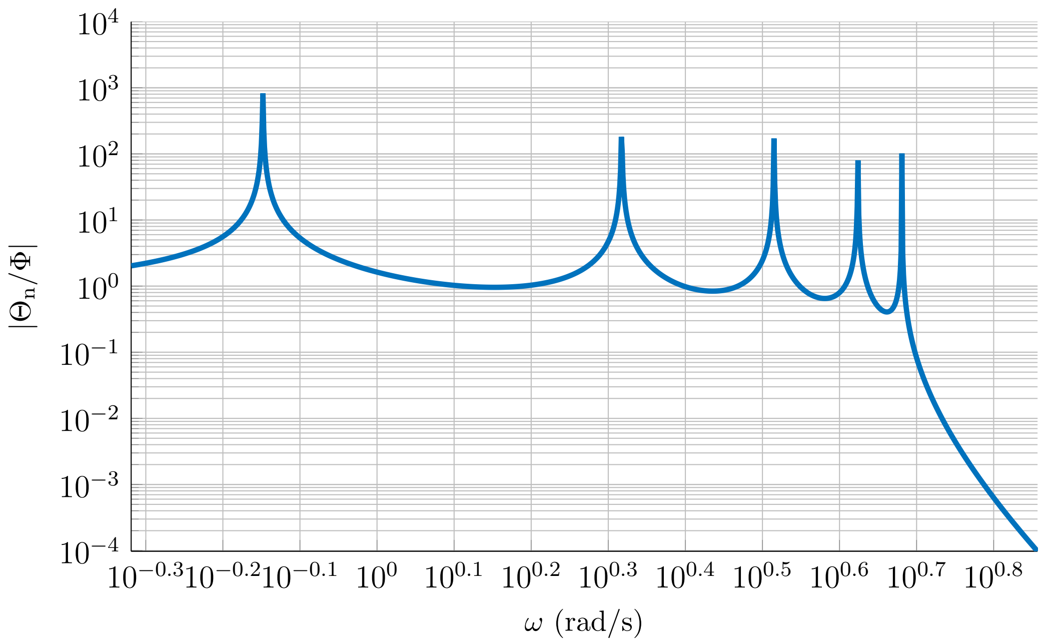

Figure 7: Part 3 Transmissibility n=5 u=0.3

In Figure 6 and Figure 7, all the peaks are at similar levels as expected. The peaks are close to each other in magnitude but not precisely the same. Hence, there is room for improvement if the constraint on the positions of the absorbers is removed. Figure 8 and Figure 9 show the solution of the last part, where the magnitude of the transmissibility peaks are very close to each other and the maximum transmissibility of the final design is even lower as can be seen in Table 1.

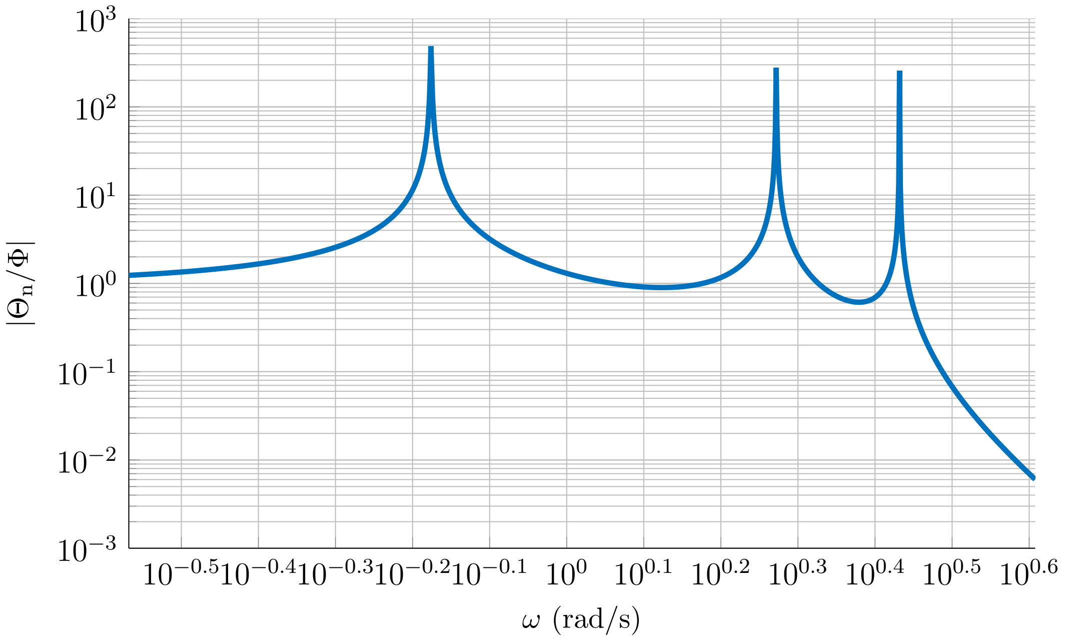

Figure 8: Part 4 Transmissibility n=3 u=0.3

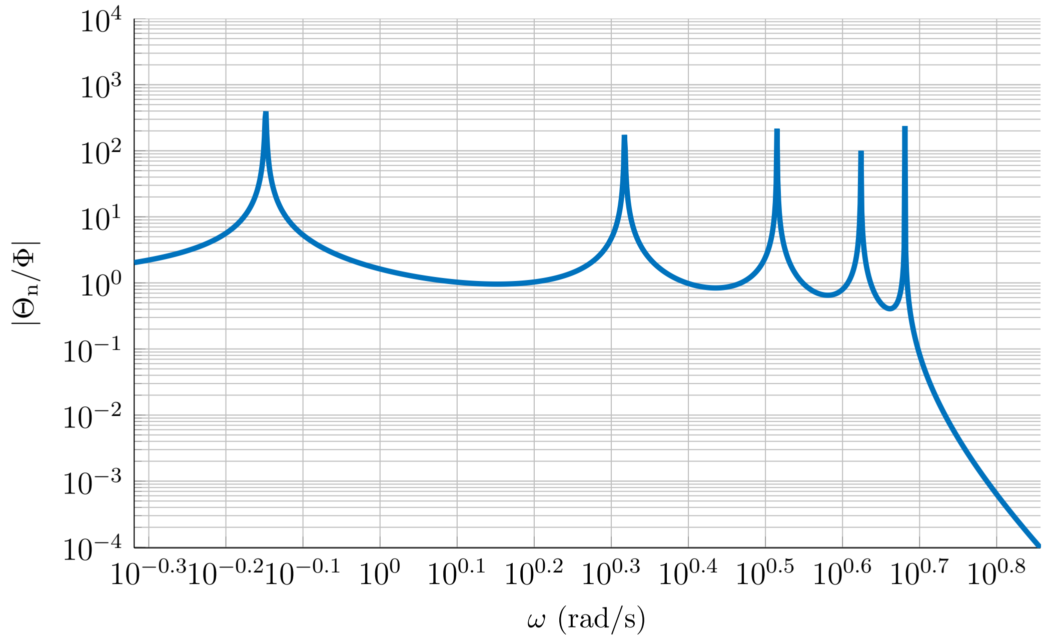

Figure 9: Part 4 Transmissibility n=5 u=0.3

In the last part, the optimizer also freely decides the inertia distribution between the absorbers. The result shows that all the inertia concentrated on the last disk. Basically, the result says that all the vibration absorption efforts should be spent on the disk whose transmissibility needs to be reduced. This is the expected answer and is true regardless of the total viscosity. Comparing Figure 9 and Figure 10 shows this. All the absorber inertia is concentrated on the last disk in both cases. As the one with a bigger μ has more absorber inertia, the maximum transmissibility is smaller for that case, as can be seen in Table 1.

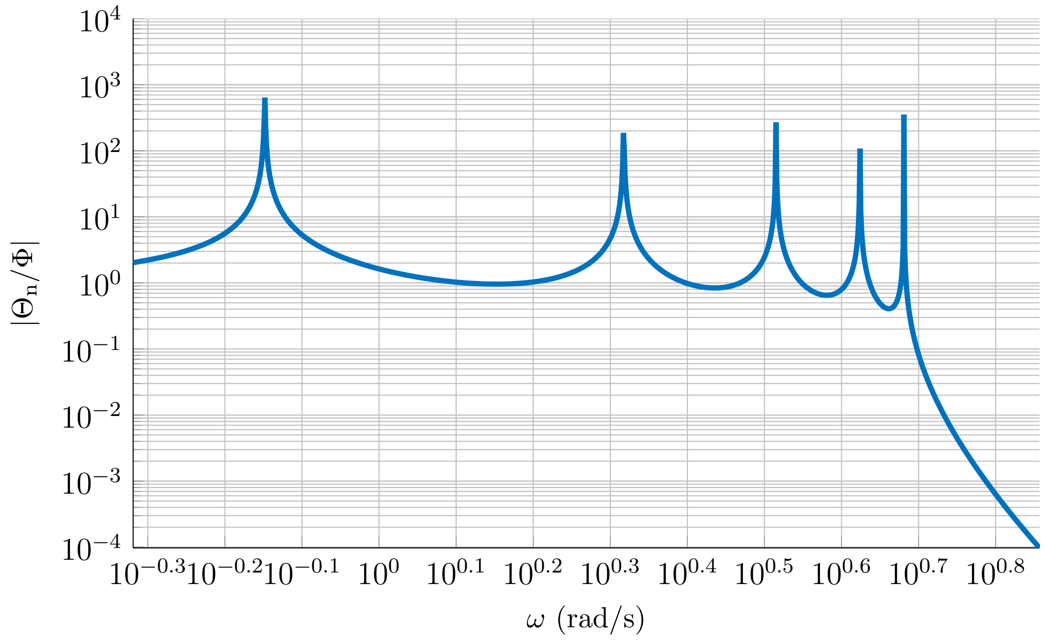

Figure 10: Part 4 Transmissibility n=5 u=0.2

In conclusion; for the first two parts where there are no dampers installed, the amount of disks govern the amount of transmissability the system shows. However in the last two parts where we have installed the dampers, the amount of disks between the input and output has less effect on the transmissibility than the absorber inertia as can be seen in Table 2 and Table 3. The disks in the middle act as a channel for the energy to travel. The important design decision is using an absorber on the last disk with a tuned damping coefficient. These can be read from Table 1.

Table 1: Maximum Transmissibilities

n	**u**	**Part 3**	**Part 4**
5	0.3	867.723	469.213
3	0.3	834.714	500.153
5	0.2	1299.786	703.329
4	0.2	1284.739	720.641
5	0.1	12642.484	1405.491

Table 2: Optimum Part 3 Absorber Designs

n	**u**	**ca1**	**ca2**
5	0.3	0.106	0.107
3	0.3	0.100	0.100
5	0.2	0.071	0.071
4	0.2	0.069	0.069
5	0.1	0.000	0.001

Table 3: Optimum Part 4 Absorber Designs

n	**u**	**na1**	**na2**	**Ia1**	**Ia2**	**ca1**	**ca2**
5	0.3	4	5	0.000	0.300	0.000	0.213
3	0.3	2	3	0.000	0.300	0.000	0.200
5	0.2	4	5	0.000	0.200	0.000	0.143
4	0.2	3	4	0.000	0.200	0.000	0.139
5	0.1	4	5	0.000	0.100	0.023	0.070

Appendix A: MATLAB Code

```
% me425 spring2022 project
% atalay gecgel sipahioglu

close('all');
clear();
clc();

print("me425 spring2022 project");
print("atalay gecgel sipahioglu");

% -----------------------------------------------------
% INPUT -----------------------------------------------
% -----------------------------------------------------

% Get the n and u from the user.

% Number of Disks (Min: 2, Max: 5)
n = 4;

% Total Houdaille Damper Viscosity (Min: 0.1, Max:
   0.3)
u = 0.2;

% Print
print("");
print("Input:");
print("~~~~~~");
print("[-] n = %4.0f", n);
print("[-] u = %4.2f", u);

% -----------------------------------------------------
% INITIAL CALCULATIONS --------------------------------
% -----------------------------------------------------

% Find I and k.
% Note: using `n`, `u` from the previous part.

% Rotational Inertia of a Disk
I = 100 / n;

% Torsional Stiffness Between Disks
k = 25 * n;
```

```
% Print
print("");
print("Initial Calculations:");
print("~~~~~~~~~~~~~~~~~~~~~~~");
print("[-] I = %5.1f", I);
print("[-] k = %5.1f", k);

% ----------------------------------------------------
% PART A ---------------------------------------------
% ----------------------------------------------------

% Find M and K.
% Use modal analysis to find the natural frequencies
   and mode shapes.
% Note: using `n`, `I`, `k` from the previous part.
% Note: using functions `f_M`, `f_K` that are defined
    at the end.

% Timer Start
c_start = tic();

% Inertia Matrix
M = f_M(n, 0, I, []);

% Stiffness Matrix
K = f_K(n, 0, k);

% First Transformation
M_ = M^(-1/2);
K_ = M_ * K * M_;

% Eigenvector and Eigenvalue Matrix
% Eigenvectors are the mode shapes.
% Eigenvalues are the squares of the natural
   frequencies.
[P, L] = eig(K_);

% Natural Frequencies in rad/s
w = zeros(n, 1);
for j = 1:n
    % Natural frequency is found by taking the square
        root of the eigenvalues.
    w(j) = L(j, j)^(1/2);
```

```
    % Mode shapes are found by making the first
       element of the eigenvectors positive.
    P(:, j) = P(1, j) / abs(P(1, j)) * P(:, j);
end

% Plot
figure();
hold('on');
grid('on');
colors = {'#0072BD', '#D95319', '#EDB120', '#7E2F8E',
    '#77AC30'};
xlim([0, n + 1]);
ylim([0, n + 1]);
xticks(1:n);
yticks(1:n);
xlabels = cell(1, n);
ylabels = cell(1, n);
for j = 1:n
    xlabels{j} = sprintf("\\omega_%.0f=%.3f", j, w(j)
       );
    ylabels{j} = sprintf("Disk %.0f", j);
end
xticklabels(xlabels);
yticklabels(ylabels);
xlabel("Natural Frequencies (rad/s)");
for j = 1:n
    plot(j + zeros(1, n), 1:n, '--', 'Color', colors{
       j}, 'LineWidth', 2);
    plot(j + P(:, j) / 2, 1:n, '-o', 'Color', colors{
       j}, 'LineWidth', 2);
end

% Elapsed Time
c_elapsed = toc(c_start);

% Print
print("");
print("Part A:");
print("~~~~~~~~");
print("[-] Elapsed Time: %5.2f s", c_elapsed);
prmat("[-] M", M, "%9.3f");
prmat("[-] K", K, "%9.3f");
prmat("[-] M_", M_, "%9.3f");
prmat("[-] K_", K_, "%9.3f");
```

```
for j = 1:n
    print("[*] w_%1.0f = %5.3f rad/s", j, w(j));
    prvec(sprintf("[*] v_%1.0f", j), P(:, j), "%5.1f
       ");
end

% ----------------------------------------------------
% PART B ---------------------------------------------
% ----------------------------------------------------

% Construct the range of excitation frequencies and
   find the transmissibilities
% using the repectance matrix.
% Note: using `w`, `n`, `M`, `K`, `k` from the
   previous part.
% Note: using function `f_C` that is defined at the
   end.

% Timer Start
c_start = tic();

% Excitation Frequency Range
w_e_range = max(w) * (10.^(-1:0.001:log10(1.5)));

% Damping Matrix
% This is just a zero matrix in this part.
C = f_C(n, 0, [], []);

% Plot Transmissibility Range
plot_T_range(n, w_e_range, M, C, K, k, "Part B
   Transmissibility");

% Elapsed Time
c_elapsed = toc(c_start);

% Print
print("");
print("Part B:");
print("~~~~~~~~");
print("[-] Elapsed Time: %5.2f s", c_elapsed);

% ----------------------------------------------------
% PART C ---------------------------------------------
% ----------------------------------------------------
```

```
% Optimize the maximum of the peaks in the
   transmissibility for the damping
% coefficients of the absorbers which are connected
   to 1 and 5.
% Note: using `u`, `n`, `I`, `k`, `w`, `w_e_range`
   from the previous part.
% Note: using functions `f_M`, `f_K`, `f_T_peaks`, `
   f_C` that are defined at the
% end.

% Timer Start
c_start = tic();

% Number of Absorbers
m = 2;

% Absorber Inertias
Ia = zeros(m, 1);
Ia(1) = u / 2;
Ia(2) = u / 2;

% Absorber Positions (Assumed to be unique for each
   absorber.)
na = zeros(m, 1);
na(1) = 1;
na(2) = n;

% Inertia Matrix
M = f_M(n, m, I, Ia);

% Stiffness Matrix
K = f_K(n, m, k);

% Optimization Parameter Vector
% [ca1, ca2]
f_ca = @(x) [x(1); x(2)];

% Initial Value
x_0 = [0.5, 0.5];

% Lower Bound
% 10^-10 instead of 0, such that receptance matrices
   are not singular.
```

```
x_lb = [1e-10, 1e-10];

% Optimized Function
x_f = @(x) f_T_peaks(n, w, M, f_C(n, m, na, f_ca(x)),
    K, k);

% Optimization Options
x_options = optimoptions('fminimax');
x_options.MaxIterations = 100;
x_options.MaxFunctionEvaluations = 1000;
x_options.Display = 'off';

% Optimization Results
[x, ~, x_maxfval] = fminimax(x_f, x_0, [], [], [],
   [], x_lb, [], [], x_options);

% Absorber Dampings
ca = f_ca(x);

% Minimized Maximum Transmissibility
T_min = x_maxfval;

% Damping Matrix
C = f_C(n, m, na, ca);

% Plot Transmissibility Range
plot_T_range(n, w_e_range, M, C, K, k, "Part C
   Transmissibility");

% Elapsed Time
c_elapsed = toc(c_start);

% Print
print("");
print("Part C:");
print("~~~~~~~~");
print("[-] Elapsed Time: %5.2f s", c_elapsed);
prvec("[-] na", na, "%7.0f");
prvec("[-] Ia", Ia, "%7.3f");
prvec("[-] ca", ca, "%7.3f");
prmat("[-] M", M, "%9.3f");
prmat("[-] C", C, "%9.3f");
prmat("[-] K", K, "%9.3f");
print("[-] T_max = %.3f", T_min);
```

```
% ----------------------------------------------------
% PART D ---------------------------------------------
% ----------------------------------------------------

% Optimize all the possible combinations of the
   absorber positions over the
% damping coefficients and absorber inertias.
% Select the one with the smallest transmissibility.
% Note: using `m`, `n`, `w`, `u`, `I`, `k`, `
   w_e_range` from the previous part.
% Note: using functions `f_T_min`, `f_M`, `f_C`, `f_K
   ` that are defined at the
% end.

% Timer Start
c_start = tic();

% Absorber Positions (Assumed to be unique for each
   absorber.)
na = zeros(m, 1);

% Absorber Inertias
Ia = zeros(m, 1);

% Absorber Dampings
ca = zeros(m, 1);

% Minimum Maximum Transmissibility
T_min = Inf;

% For all possible combinations...
na_combinations = nchoosek(1:n, m);
for j = 1:size(na_combinations, 1)
    na_j = na_combinations(j, :);

    % Optimize
    [Ia_j, ca_j, T_min_j] = f_T_min(n, m, w, na_j, u,
        I, k);

    % ... replace if better.
    if T_min > T_min_j
        na = na_j;
        Ia = Ia_j;
        ca = ca_j;
        T_min = T_min_j;
    end
end

% Inertia Matrix
M = f_M(n, m, I, Ia);

% Damping Matrix
C = f_C(n, m, na, ca);

% Stiffness Matrix
K = f_K(n, m, k);

% Plot Transmissibility Range
plot_T_range(n, w_e_range, M, C, K, k, "Part D
   Transmissibility");

% Elapsed Time
c_elapsed = toc(c_start);

% Print
print("");
print("Part D:");
print("~~~~~~~~");
print("[-] Elapsed Time: %5.2f s", c_elapsed);
prvec("[-] na", na, "%7.0f");
prvec("[-] Ia", Ia, "%7.3f");
prvec("[-] ca", ca, "%7.3f");
prmat("[-] M", M, "%9.3f");
prmat("[-] C", C, "%9.3f");
prmat("[-] K", K, "%9.3f");
print("[-] T_max = %.3f", T_min);

% ---------------------------------------------------
% CONSTRUCTION FUNCTIONS ----------------------------
% ---------------------------------------------------

% For creating the inertia matrix. In the case with
   no absorbers give `m` as
% `0`, `Ia` as `[]`.
function M = f_M(n, m, I, Ia)
    % Inertia Matrix
    M = zeros(n + m);
```

```
    for j = 1:n
        M(j, j) = I;
    end
    for j = 1:m
        M(n + j, n + j) = Ia(j);
    end
end

% For creating the damping matrix. In the case with
   no absorbers give `m` as
% `0`, `na` and `ca` as `[]`. The resulting matrix
   will be just zeros.
function C = f_C(n, m, na, ca)
    % Damping Matrix
    C = zeros(n + m);
    for j = 1:m
        C(n + j, n + j) = ca(j);
        C(n + j, na(j)) = -ca(j);
        C(na(j), n + j) = -ca(j);
        C(na(j), na(j)) = ca(j);
    end
end

% For creating the stiffness matrix. In the case with
    no absorbers give `m` as
% `0`.
function K = f_K(n, m, k)
    % Stiffness Matrix
    K = zeros(n + m);
    for j = 1:n
        if j > 1
            K(j, j - 1) = -k;
        end
        if j < n
            K(j, j + 1) = -k;
            K(j, j) = 2 * k;
        else
            K(j, j) = k;
        end
    end
end

% -----------------------------------------------------
% TRANSMISSIBILITY FUNCTIONS ------------------------
```

```matlab
% ------------------------------------------------------

% For designing the absorbers in part D. Minimizes
   the maximum of the peaks
% using `fminimax` and returns the optimum parameters
   . Only needs to know the
% positions of the absorbers. The combination of all
   possible absorber positions
% can be iterated over to get the best desing. The
   total absorber inertia is
% equated to viscosity by giving a linear equality
   constraint to `fminimax`.
function [Ia, ca, T_min] = f_T_min(n, m, w, na, u, I,
    k)
    % Stiffness Matrix
    K = f_K(n, m, k);

    % Optimization Parameter Vector
    % [Ia1, Ia2, ca1, ca2]
    f_Ia = @(x) [x(1); x(2)];
    f_ca = @(x) [x(3); x(4)];

    % Initial Value
    x_0 = [u / 2, u / 2, 0.5, 0.5];

    % Lower Bound
    % 10^-10 instead of 0, such that receptance
       matrices are not singular.
    x_lb = [1e-10, 1e-10, 1e-10, 1e-10];

    % Optimized Function
    x_f = @(x) f_T_peaks(n, w, f_M(n, m, I, f_Ia(x)),
        f_C(n, m, na, f_ca(x)), K, k);

    % Linear Equality Constraint
    x_Aeq = [1, 1, 0, 0];
    x_beq = u;

    % Optimization Options
    x_options = optimoptions('fminimax');
    x_options.MaxIterations = 100;
    x_options.MaxFunctionEvaluations = 1000;
    x_options.Display = 'off';
    % Optimization Results
    [x, ~, x_maxfval] = fminimax(x_f, x_0, [], [],
       x_Aeq, x_beq, x_lb, [], [], x_options);

    % Absorber Inertias
    Ia = f_Ia(x);

    % Absorber Dampings
    ca = f_ca(x);

    % Minimized Maximum Transmissibility
    T_min = x_maxfval;
end

% For finding the peaks in the transmissibility plot
   very fast and secure. It
% always returns the peaks because it uses the
   natural frequencies. With the
% damping the peaks shift to left slightly. Looks for
    the peaks via iterating
% over the natural frequencies and using `fminbnd`
   starting from the 95% of the
% natural frequency to 100% of it. Fast because it
   looks for a very small
% window. Guaranteed to find the peaks because it
   does not do grid search.
function T_peaks = f_T_peaks(n, w, M, C, K, k)
    % Transmissibility Peaks
    T_peaks = zeros(1, n);
    for j = 1:n
        % Optimization Parameter Vector
        % [w_e]
        % Lower Bound
        % The damped frequencies should be just a
           little smaller.
        x_lb = w(j) * 0.95;

        % Upper Bound
        x_ub = w(j);

        % Optimized Function
        x_f = @(x) -f_T(n, x, M, C, K, k);

        % Optimization Options
        x_options = optimset('fminbnd');
        x_options.MaxIterations = 100;
        x_options.MaxFunctionEvaluations = 1000;
        x_options.Display = 'off';

        % Optimization Results
        [~, x_fval] = fminbnd(x_f, x_lb, x_ub,
           x_options);

        % Maximum Transmissibility
        T_peaks(j) = -x_fval;
    end
end

% For calculating a range of transmissibilities for
   the given range of
% excitation frequencies.
function T_range = f_T_range(n, w_e_range, M, C, K, k
   )
    % Transmissibility Range
    T_range = zeros(size(w_e_range));
    for j = 1:length(w_e_range)
        T_range(j) = f_T(n, w_e_range(j), M, C, K, k)
            ;
    end
end

% For calculating the transmissibility of the last
   disk for the given excitation
% frequency. Very fast because it only uses the
   necessary element of the
% receptance matrix.
function T = f_T(n, w_e, M, C, K, k)
    % Receptance Matrix
    a = (-w_e^2 * M + 1i * w_e * C + K)^ - 1;

    % Transmissibility
    T = abs(a(1, n) * k);
end

% -----------------------------------------------------
% OUTPUT FUNCTIONS ------------------------------------
% -----------------------------------------------------
% For plotting the transmissibility over a range of
   excitation frequencies.
function plot_T_range(n, w_e_range, M, C, K, k, name)
    % Plot
    figure();
    set(gca, 'YScale', 'log');
    set(gca, 'XScale', 'log');
    hold('on');
    grid('on');
    xlabel('\omega (rad/s)');
    ylabel('|\Theta_n/\Phi|');
    plot(w_e_range, f_T_range(n, w_e_range, M, C, K,
       k), 'LineWidth', 2);
    title(name);
end

% For easier general printing. Puts new line at the
   start.
function print(varargin)
    fprintf('%s\n', sprintf(varargin{:}));
end

% For printing matrices.
function prmat(name, matrix, element)
    print("%s [%.0f, %.0f]: ", name, size(matrix, 1),
        size(matrix, 2));
    for k = 1:size(matrix, 1)
        for j = 1:size(matrix, 2)
            fprintf(element, matrix(k, j));
        end
        fprintf("\n");
    end
end

% For printing vectors.
function prvec(name, vector, element)
    fprintf("%s [%.0f]: ", name, length(vector));
    for k = 1:length(vector)
        fprintf(element, vector(k));
    end
    fprintf("\n");
end
```