Lecture 8

Philosophy 109

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Ch. 3: Semantics of TFL

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- The semantics of TFL is relatively simple.
  - All that matters is the **conditions under which a sentence is true**.
  - The truth-value of a complicated expression is true or false based on the truth-values of the atomic sentences in it.
  - So, you look at each atomic sentence and decide whether it's true or false, and semantics tells you whether the whole sentence is true or false.

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  - Tables show how the truth value of each type of complex sentence depends on the constituent parts.
  - Truth-tables provide a precise way of thinking about logical possibility.
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  - Truth-tables provide a precise way of thinking about logical possibility.
  - Each row—aka, **interpretation**—gives a unique way of how the world could be. The actual world is just one of these rows.
- Later, we'll also use truth-tables to determine whether an argument in TFL is valid.

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- The different connectives name different functions.
- Truth tables are a way of representing truth functions.
- Later, we will see they can represent probability functions, too.

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- But this is equivalent to other ways.
- T is equivalent to $\top$ and 1.
- F is equivalent to $\bot$ and 0.

Negation

- We'll start with negation because that's the simplest.

p	$\neg p$
T	F
F	T

- This says that if p is true, then $\neg p$ is false, and if p is false, then $\neg p$ is true.
- Example:
  - It is not the case that Wagner wrote operas. ($\neg W$)
  - $\neg W$ is true just in case W is false, and $\neg W$ is false just in case W is true.

Conjunction

p	q	$p \wedge q$
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- Remember that $\wedge$ corresponds well to the English *and*.
    - *John and Bill are painters* is true just in case it's true that John's a painter, and it's also true that Bill's a painter.
    - If it's false that John's a painter or it's false that Bill's a painter or both then the whole sentence is false that John and Bill are painters.

Disjunction

p	q	$p \vee q$
T	T	T
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    - In TFL, for $p \vee q$ to be true, we need p to be true, or q to be true, or both p and q to be true. This is the *inclusive or*.

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  - In TFL, for $p \vee q$ to be true, we need p to be true, or q to be true, or both p and q to be true. This is the *inclusive or*.
  - Often in English, we have an *exclusive or*.
  - "You can either join the NBA or become a lawyer" seems to imply that you can't do both.
  - Watch out for this when filling out truth-tables.

Conditionals

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- The TFL $\to$ isn't great at matching the English "if...then" and "only if".
  - We have to make some sacrifices here because we want all connectives to be truth-functional.
  - Plus it turns out that there isn't agreement in philosophy of language and linguistics about what the English indicative conditional means.

English v. TFL conditional

Consider:

M: The moon is made of green cheese.

O: Life exists on other planets.

E: Life exists on Earth.

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- If the moon is made of green cheese, then life exists on other planets.
- If life exists on other planets, then life exists on earth.

In TFL, both of these sentences are true. $M \rightarrow O$ is true because M is false. $O \rightarrow E$ is true because E is true, so it doesn't matter whether O is true. What do you think about the English conditionals?

The Biconditional

p	q	$p \leftrightarrow q$
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- The TFL $p \leftrightarrow q$ comes out true just when p and q have the same truth-value.
- Note that there doesn't need to be any real connection between p and q.

TFL v. English Biconditional

Consider:

M: The moon is made of green cheese.

U: There are unicorns.

E: Life exists on earth.

B: Beyoncé sang at the Superbowl in 2013.

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In TFL, the translations of both of these sentences ($M \leftrightarrow U$ and $E \leftrightarrow B$) are true. M's truth-value is F, and so is U's. So, $M \leftrightarrow U$ is true. E's truth-value is T, and so is B's. So, $E \leftrightarrow B$ is true.

- Remember, we want our semantics to be truth-functional.
  - The truth-value (T/F) of the whole sentence depends on the truth-value of the parts.
  - We have to make some sacrifices. Not every connective can correspond exactly to English: English often has meanings which go beyond the truth functional.
  - Once we require truth-functionality, the tables we gave for $\vee, \rightarrow,$ and $\leftrightarrow$ are the best we can do.

All the Truth-Functional Connectives

- Since the semantics is truth-functional, when we have two sentence p and q, there are $2^4 = 16$ possible connectives.

p	q	$\top$	$\mid$	$\rightarrow$	$\neg p$	$\leftarrow$	$\neg q$	$\leftrightarrow$	NOR	$\vee$	$\nleftrightarrow$	q	$\nleftarrow$	p	$\nrightarrow$	$\wedge$	$\bot$
T	T	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F
T	F	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	T	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	F	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F

Constructing Truth-Tables for TFL Sentences

- Now that we have the truth-table definitions of the five TFL connectives ($\vee, \wedge, \neg, \leftrightarrow, \rightarrow$), we can construct truth-tables for any TFL sentence.
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    4. Place the atomic sentence letters in the left-most columns, in alphabetical order from left to right.
    5. Place p in the right-most column.

6. Start in the nth column: the one furthest down in the alphabet. Use the pattern TFTFTF... until the column is filled. Then move one column left and use the pattern TTFFTTFF... until the column is filled. Then move one column left and use the pattern TTTTFFFFTTTTFFFF.... until the column is filled. In other words, always double the number of consecutive Ts and Fs each time you move left. Keep going until everything is filled in.

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7. Finally, compute the truth-values of p in each row of the table. First, copy the truth-values of the atoms, then compute the negations, conjunctions, etc. which compose p and write the truth-values under that connective. Finally, compute the value of the main connective of p, at which point you're done.

More on Step 7

- The intuitive idea is that we start with the atomic sentences and use their truth-values to determine the truth-values of the next-simplest kinds of sentences. We then use those to determine the truth-values of the simplest after that and so on until we've determined p's truth-value.
- We need to break p down into its subsentential parts until we get to the bottom.

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- B's done, but $C \wedge \neg D$ needs to be broken down into C and $\neg D$, since $\wedge$ is its main connective.
- Finally, $\neg D$ has only one subsentence: D.

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  - We determine the truth-values of $B \rightarrow (C \wedge \neg D)$ based on the truth-values of B and $C \wedge \neg D$.
  - Finally, we determine the truth-value of $A \leftrightarrow (B \rightarrow (C \wedge \neg D))$ based on the truth-values of A and $B \rightarrow (C \wedge \neg D)$

Step-by-Step Example

Let p be $A \leftrightarrow (B \wedge A)$

A	B	A	$\leftrightarrow$	$(B$	$\wedge$	$A)$

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F	T	F	T	T	F	F
F	F	F	T	F	F	F