Lecture 13

Philosophy 109

Caley Howland

Administrative Stuff

- **Required** for next time:
  - Instead of new reading, do the "Chances Worksheet" on sakai.
  - Exercises: Forallx Leeds, Chapter 10, Parts A and B.
- Schedule is updated on Sakai
- Answer key to probabilistic truth table exercises will be posted shortly.
- HW 4 will be posted soon.
- I have your blue books for the exam.

Conditional Probability

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Definition of Conditional Probability

$$Pr(p|q) = \frac{Pr(p \wedge q)}{Pr(q)}$$

- This definition makes sense, because we want to look at how probable p is within the q-worlds. And, that's just the ratio of $Pr(p \wedge q)$ to $Pr(q)$.

Inductive Strength: Precise

Precise Inductive Strength

An argument $P \therefore C$ is inductively strong iff:
(1) C is probable given P, i.e., $Pr(C|P) > 1/2$, and
(2) P is **positively relevant** to C, i.e., $Pr(C|P) > Pr(C)$

Rules for Calculating $Pr(p)$ and $Pr(p|q)$

- Once you're given a probabilistic truth-table, there are only two rules for calculating numerical probabilities.
- A probabilistic truth-table specifies world-probabilities for all possible worlds involving some fixed set of TFL atomic sentences.

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Note: If $Pr(q) = 0$, then $Pr(p|q)$ is undefined, since you can't divide by 0. So, we always assume $Pr(q) > 0$ when we use $Pr(p|q)$.

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- Disjunctive Normal form: any sentence can be expressed in this form by a disjunction of state descriptions:
  - $p \Leftrightarrow (p \wedge q) \vee (p \wedge \sim q)$

Some Derived Rules

- We can use generic probabilistic truth-tables to derive additional rules. Here are some examples.

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- $Pr(p|q) \times Pr(q) = Pr(p \wedge q)$
- Alternative: $Pr(p|q) = \frac{a}{a+c} \times a + c$

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- But, by (5), we know that $Pr(A|A \vee \neg A) = Pr(A)$. So, (ii)'s premise is always irrelevant to its conclusion.
- Therefore, by Proposal #3, argument (i) is stronger than argument (ii).

Monotonicity

- A crucial difference between validity vs strength is that deductive validity is monotonic, but inductive strength is non-monotonic.
- If an argument is valid, then it cannot be rendered invalid merely by adding additional premises to it. Let's think about why this is true.

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- $\mathcal{A}'$ is weak. So adding a new premise—Premise (Q)—made it weak.

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  - (S) John rolls cigars for a living.
- And so on. . . This non-monotonicity of inductive strength is a key feature. Diagramming can help explain why this can happen.