# Modality and Translation

Philosophy 109

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September 13, 2019

# Administrative Stuff

- Reading and exercises for next time:
  - ForAllx Chapter 5.
  - Exercises A, B, and C: Even numbers only.
- Homework 1 is due **Sept. 23rd**
  - Posted to Sakai.
  - Upload the homework to sakai in .pdf or Word format.
  - Upload under the Assignments section.
  - Late Policy: no late work will be accepted without significant excuse; all extensions must be requested ahead of time.

# Non-formal Validity?

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- This is tricky: the validity here relies on knowing the meaning of bachelor.
- But logic is formal.

# Non-formal Validity?

- Consider:
  - P1 John is a bachelor
  - P2 All bachelors are unmarried.
  - C John is unmarried.
- This time, even if you don't know the meaning of the terms, you can tell the argument is valid.
- We can check this by replacing the terms with nonsense ones:
  - P1 John is a bleep.
  - P2 All bleeps are bloops.
  - C John is bloop.
- Formal logic is concerned with this kind of validity.

# Logical Form

- We want to determine which arguments are valid based merely on their form.
- Much of the course will focus on *sentential* logic (aka *propositional* logic).
- This allows us to represent the most basic formal features of arguments.

# Logical Form

- Consider the following valid argument, argument 9:
  - P1 Lisa is a Californian
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- Argument 9 has a valid form: any argument with the same form is valid.
- Let's represent this form abstractly, as argument 9f:
  - P1 P
  - P2 If P, then Q
  - C $\therefore$ Q

# Logical Form

- Argument 9f:

  P1 P
  P2 If P, then Q
  C ∴ Q
- In 9f, "P" and "Q" stand for any declarative statements whatsoever.
  - They are like variables in algebra class, except instead of numbers, they represent statements.
- No matter which statements you plug in for P and Q in 9f, you always get a valid argument out.

# Sentential Form

## Sentential Form

The **sentential form** of an argument is obtained by replacing each basic (aka atomic) sentence in the argument with a single letter.

- Basic or atomic sentences are sentences that don't have any other sentence as a part.
  (a) Andy is a philosopher and Andy is a Packers fan.
  (b) It is not the case that Amy is six feet tall.
  (c) Grass is green.
  (d) Either it will rain today or it will be sunny.
- (a), (b), and (d) are not basic/atomic. They are *complex* or *compound*.
- Only (c) is atomic.

# Comprehension Check

- Consider argument 10:

  *If Tom is in his Freemont home, then he's in California. Tom is in California. Therefore, Tom is in his Freemont home.*

- What is the form of 10?
- is 10 valid?

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- 11:

  P1 $p$

  C $\therefore q$ or not $q$

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- 12 is valid because because $p$ and not $p$ is a logical falsehood.
  - so it's impossible for the premises to be true and the conclusion false, because its impossible for the premise to be true.

# A Few Valid and Invalid Sentential Forms

| Sentential Form | Name | Valid/Invalid |
|---|---|---|
| p<br>If $p$ then $q$<br>$\therefore q$ | Modus Ponens | Valid |
| $q$<br>If $p$ then $q$<br>$\therefore p$ | Affirming the Consequent | Invalid |
| not $q$<br>if $p$ then $q$<br>$\therefore$ not $p$ | Modus Tollens | Valid |

# A Few Valid and Invalid Sentential Forms

| Sentential Form | Name | Valid/Invalid |
|---|---|---|
| Not $p$<br>If $p$ then $q$<br>$\therefore$ not $q$ | Denying the Antecedent | Invalid |
| If $p$ then $q$<br>If $q$ then $r$<br>$\therefore$ if $p$ then $r$ | Hypothetical Syllogism | Valid |
| Not $p$<br>Either $p$ or $q$<br>$\therefore q$ | Disjunctive Syllogism | Valid |

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|---|---|---|
| 13f | P1 | $p$ |

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- In this course we will stick with sentential logic, which we will more formally call **truth-functional logic (TFL)**. You can take Philosophy 201 to learn about more advanced kinds of logic.

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- The formal language of truth-functional logic we will call **TFL** for short.

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  - Basically, what is sententially valid is what we can prove to be valid using our formal system of truth-functional logic.
- The formal language of truth-functional logic we will call **TFL** for short.
- Note: Even if an argument fails to be sententially valid, it could be valid according to a richer logical theory.

# TFL

- We will develop a precise theory of sentential validity, and a few techniques for deciding whether a sentential form is valid or not.
- We will do this using the formal language of TFL.

# TFL

- TFL is a formal language that comes with its own special symbols.
- We will use upper-case letters to stand for atomic (or simple) sentences.
    - $A, B, C, \dots$
- Special symbols, called *connectives* (or sometimes *operators*) put sentences together.
- And we will use parentheses to group sentences together.

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# Connectives of TFL

| Symbol | Name/Function | Translation |
|---|---|---|
| $\neg$ or $\sim$ | negation | not |
| & or $\wedge$ | conjunction | and |
| $\vee$ | disjunction | or |
| $\rightarrow$ | conditional | If..., then... |
| $\leftrightarrow$ | biconditional | if and only if |

# The Five Kinds of Non-Basic TFL Sentences

Conjunctions: $p \wedge q$. Constituents $p$ and $q$ are called conjuncts.

Disjunctions: $p \vee q$. Constituents $p$ and $q$ are called disjuncts

Conditionals: $p \rightarrow q$. $p$ is the antecedent. $q$ is the consequent

Biconditionals: $p \leftrightarrow q$. $p$ is the left-hand side, $q$ is the right-hand side

Negations: $\neg p$. The sentence $p$ is called the negated sentence

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- Note: $p$ and $q$ need not be atomic sentences.
  - e.g., $(A \wedge B) \vee \neg C$ is a disjunction.

# Translation to TFL

- The first step in evaluating arguments is translating them into TFL.
- Later, we will get a clearer understanding of what these connectives mean.
- Each of these connectives are what we call "truth functional": their meaning is exhausted by their truth functions.
- All of the logical structure in sentential logic comes from these connectives.

# The Meaning of the Connectives

- The connectives are truth functional.
  - This means their meaning is exhausted by their effect on the truth values.
- They are functions in the mathematical sense:
  - Input the truth values of the constituent sentences, and the connective outputs a truth function for the whole sentence.
- So we can give the meanings with something called a "truth table".

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|---|---|
| T | F |
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| p | q | $p \wedge q$ |
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| $p$ | $q$ | $p \leftrightarrow q$ |
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| T | F | F |
| F | T | F |
| F | F | T |

# Meanings

- We will talk much more about meanings or "semantics" next week.
- Truth tables turn out to be essential for sentential logic.
- For now, I just want you to have a basic understanding so that you can do translations.