Lecture 9

Philosophy 109

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Participatory Recap

- Write the truth table definitions of our 5 connectives.

Negation

p	$\neg p$
T	F
F	T

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditionals

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Exercises

- Chapter 10 exercises.

Logical Truth and Falsity

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T	T	T	F	T
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p	p	$\wedge$	$\neg$	p
T	T	F	F	T
F	F	F	T	F

Logical Contingency

- A statement is **contingent** if it is neither tautological nor self-contradictory. In other words, on at least one row of its truth-table, it comes out true, and on at least one row, it comes out false. For example, the claim $A \to B$ is logically contingent.

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Equivalence

- Statements p and q are **equivalent** (written $p \Leftrightarrow q$) if they have the same truth-value on all interpretations (i.e., on each row of their truth-tables). For instance, $A \to B$ and $\neg A \vee B$ are logically equivalent.

A	B	A	$\to$	B	$\neg$	A	$\vee$	B
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F

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A	B	A	$\rightarrow$	B	$\neg$	A	$\vee$	B
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F

- Comprehension check: What's the difference between $p \leftrightarrow q$ and $p \Leftrightarrow q$?

Contradictoriness

- Statements p and q are **contradictory** if they have opposite truth-values on all interpretations. For instance, $A \rightarrow B$ and $A \wedge \neg B$ are contradictory.

A	B	A	$\rightarrow$	B	A	$\wedge$	$\neg$	B
T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	T	T	F
F	T	F	T	T	F	F	F	T
F	F	F	T	F	F	F	T	F

Inconsistency

- Statements p and q are **inconsistent** if there is no interpretation on which they are both true. For instance, $A \leftrightarrow B$ and $A \wedge \neg B$ are inconsistent. (Note: they are not contradictory!)

A	B	A	$\leftrightarrow$	B	A	$\wedge$	$\neg$	B
T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	T	T	F
F	T	F	F	T	F	F	F	T
F	F	F	T	F	F	F	T	F

Consistency

- Statements p and q are **consistent** if there's *at least one* interpretation on which they are both true. For example, $A \wedge B$ and $A \vee B$ are consistent.

A	B	A	$\wedge$	B	A	$\vee$	B
T	T	T	T	T	T	T	T
T	F	T	F	F	T	T	F
F	T	F	F	F	F	T	T
F	F	F	F	F	F	F	F

Relationships between Concepts

- Equivalence **doesn't** imply consistency. (Example?)
- Consistency **doesn't** imply equivalence. (Example?)
- Contradictoriness implies inconsistency. (Why?)
- Inconsistency **doesn't** imply contradictoriness. (Example?)

Semantic Equivalence Example 1

- Recall that we can translate "p unless q" either as $\neg q \rightarrow p$ or as $p \vee q$.
- That's because they're equivalent, which we demonstrate with the following truth-table.

p	q	$\neg$	q	$\rightarrow$	p	p	$\vee$	q
T	T	F	T	T	T		T	
T	F	T	F	T	T		T	
F	T	F	T	T	F		T	
F	F	T	F	F	F		F	

- Since their truth-tables are the same, we know $\neg q \rightarrow p \Leftrightarrow p \vee q$.

Semantic Equivalence Example 2

- $p \leftrightarrow q$ is equivalent to $(p \to q) \wedge (q \to p)$, as the following truth-table shows.

p	q	$(p \to q)$	$\wedge$	$(q \to p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	T	F	F	F
F	F	T	T	T	T

Example 3

- Intuitively, the truth-conditions for *exclusive or* ($\oplus$) are such that $p \oplus q$ is true just in case either p or q is true, but not both.
- The following truth-table shows how to represent exclusive or using our usual connectives.

p	q	$(p \vee q)$	$\wedge$	$\neg(p \wedge q)$	$p \oplus q$
T	T	T	F	F	F
T	F	T	T	T	T
F	T	T	T	T	T
F	F	F	F	T	F

The Exhaustive TT Method for Validity Testing

- Recall: an argument is **valid** if it is impossible for its premises to be true while its conclusion is false.

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- Recall: an argument is **valid** if it is impossible for its premises to be true while its conclusion is false.
- Let $p_1, \ldots, p_n$ be the premises of an LSL argument, and let q be the conclusion of the argument. We then have:

$$\begin{array}{c} p_1 \\ \vdots \\ p_n \\ \hline \therefore q \end{array}$$

is **valid** iff there is no row in the simultaneous truth-table of $p_1, \ldots, p_n$ and q such that all of $p_1, \ldots, p_n$ are true but q is false.

- In other words, the argument is invalid if we can find a row in the simultaneous truth-table that looks like this:

(Atomics)	p_1	$\dots$	p_n	q
$\dots$	T	T	T	F

- Consider the argument:

$$\begin{array}{l} A \\ A \rightarrow B \\ \hline \therefore B \end{array}$$

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$$\begin{array}{l} A \\ A \rightarrow B \\ \hline \therefore B \end{array}$$

- We can construct a simultaneous truth-table as follows:

A	B	A	A	$\rightarrow$	B	B
T	T	T		T		T
T	F	T		F		F
F	T	F		T		T
F	F	F		T		F

- Consider the argument:

$$\begin{array}{l} A \\ A \to B \\ \hline \therefore B \end{array}$$

- We can construct a simultaneous truth-table as follows:

A	B	A	$A \to B$	B
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- Verdict: VALID. There's no row where A and $A \to B$ are true, but B is false.

The Procedure

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2. Write out the symbolized argument.
3. Make a simultaneous truth-table for the symbolized argument
4. See if there's a row on which all the premises are true, but the conclusion's false. If not, it's valid. Otherwise, it's invalid.