Lecture 12

Philosophy 109

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  - **Proposal #3** "P $\therefore$ C" is strong just in case (1) the probability of $\sim C$ given that P is low, and (2) P is *positively relevant* to C — i.e., the probability of $\sim C$ given that P is lower than the probability of $\sim C$.

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Proposal 3 and Relevance

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- If we return to our card example, we can see how relevance works.
- The probability that c is a not a spade, given that c is black is 1/2. That is, the probability that $\sim C$, given that P is 1/2.
- This is not low (i.e., it is not less than 1/2). But, it is *lower* than the probability that c is not a spade (i.e., the probability of $\sim C$), which is 3/4.

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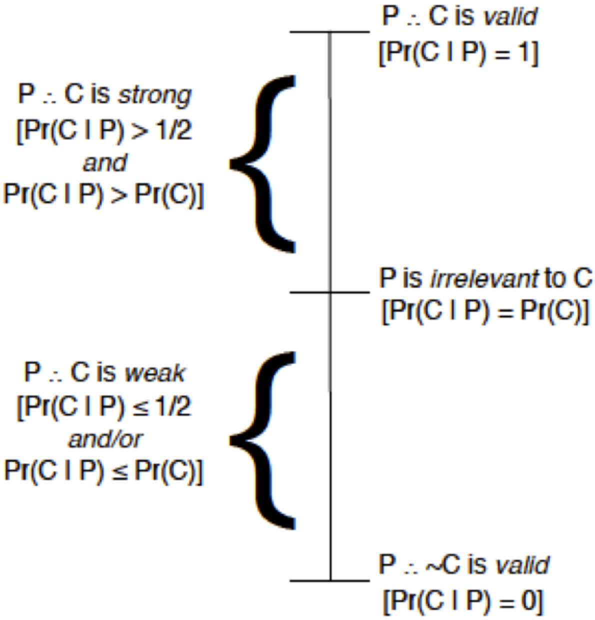
  - The probability ofC given that P = 1/2, which is greater than the probability of C = 1/4).
- Thus, the argument "P ∴ C" does not come out strong on proposal #3 (because it fails the low conditional probability requirement). But this argument does satisfy the positive relevance requirement.
- With proposal #3 in hand, we can now explain how our "scale" of argument strength works.
  - It depends on $Pr(C|P)$ and $Pr(C|P) > Pr(C)$.


P ∴ C is *valid*
[Pr(C | P) = 1]
P ∴ C is *strong*
[Pr(C | P) > 1/2
and
Pr(C | P) > Pr(C)]
P is *irrelevant* to C
[Pr(C | P) = Pr(C)]
P ∴ C is *weak*
[Pr(C | P) ≤ 1/2
and/or
Pr(C | P) ≤ Pr(C)]
P ∴ ~C is *valid*
[Pr(C | P) = 0]

Monotonicity

- A crucial difference between validity vs strength is that deductive validity is monotonic, but inductive strength is non-monotonic.
- If an argument is valid, then it cannot be rendered invalid merely by adding additional premises to it. Let's think about why this is true.

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- $\mathcal{A}'$ is weak. So adding a new premise—Premise (Q)—made it weak.

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  - (R) John has yellow stains on his fingers and smells like tobacco.
  - (S) John rolls cigars for a living.
- And so on. . . This non-monotonicity of inductive strength is a key feature. Diagramming can help explain why this can happen.

Numerical Probability

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  - Sometimes, we've discussed games of chance for which there are clear, *numerical* (inductive) probabilities.
- But, we haven't seen a precise, general way to think about (or calculate) numerical probabilities of LSL sentences. This is our current topic.

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- I will call these Probabilistic Truth Tables (or sometimes "Stochastic Truth Tables").
- A probabilistic truth-table is just a truth-table with an additional column. That column gives the inductive **probabilities of each of the possible worlds** (i.e., the inductive probabilities of each of the rows).
- It helps to start with a simple example (involving a game of chance).

- Let's start with our example of drawing a single card (c) at random from a standard deck of playing cards. There are various properties that our card might have. Here are two examples (as atomic LSL sentences).
  - $H = c$ is a heart. $R = c$ is a red card.

Probabilistic Truth Table

- A **probabilistic truth-table** (involving H and R) will look like this.

World (w_i)	H	R	$Pr(w_i)$	
w_1	T	T	1/4	$[= Pr(H \& R)]$
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- The new right hand column assigns probabilities to these worlds.

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- Real numbers include Naturals, Rationals, and Irrationals.
- Real numbers are uncountably infinite.
- We will generally round things so as not to deal with irrationals.

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  - Finite Additivity: for any mutually exclusive propositions p or q, $Pr(p \vee q) = Pr(p) + Pr(q)$.

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- This last idea, that probability functions describe beliefs, is called "personalism", and is part of what's called Bayesianism

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- Plus, that is how to read Titelbaum: Titelbaum uses $cr(\cdot)$ to make it explicit that is the kind of probability he is interested in.

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- Once we have world probabilities assigned (involving some atomic sentences), we can then calculate the probability of *any LSL statement* (involving those atomic sentences). In fact, the calculation is easy.
- The probability of an LSL sentence s is just **the sum of the world probabilities of the possible worlds in which s is true**.

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- Once these world probabilities are given, all other probabilities can be calculated.

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- Therefore, $Pr(H \rightarrow R) = 1/4 + 1/4 + 1/2 = 1$.
  - This makes sense, since all hearts are red cards. Thus, we can be certain that $H \rightarrow R$ is true.

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- So, $Pr(R \rightarrow H) = 1/4 + 0 + 1/2 = 3/4$.
- According to Proposal 1 for inductive strength, the argument $R \therefore H$ is going to come out strong, since $Pr(R \rightarrow H) = 3/4$, which is high.