Lecture 10

Philosophy 109

Caley Howland

March 4, 2020

Administrative Stuff

- Chapters 2 and 3 of Hardegree have lots of good practice problems for truth tables, and answers.
  - Just remember that he uses "$\sim$" for "$\neg$" and "&" for "$\wedge$"

The Procedure so far

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2. Write out the symbolized argument.
3. Make a simultaneous truth-table for the symbolized argument
4. See if there's a row on which all the premises are true, but the conclusion's false. If not, it's valid. Otherwise, it's invalid.

The Short-Cut Method for Validity Testing

$$\begin{array}{l} A \rightarrow (B \& E) \\ D \rightarrow (A \vee F) \\ \neg E \\ \hline \therefore D \rightarrow B \end{array}$$

- This argument has three premises with 5 atomic sentences.

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- A full truth-table would require 32 rows with 8 columns. Fully completing a truth-table would require 480 computations.
- So, we need something more practical.
- The idea is to work backwards from the assumption that we can find a row that invalidates the argument.

- Step 1: Assume there is an interpretation on which all three premises are true but the conclusion is false

A	B	D	E	F	$A \rightarrow (B \& E)$	$D \rightarrow (A \vee F)$	$\neg E$	$D \rightarrow B$
					T	T	T	F

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F			F		F	T	F		T		T	F

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F	F	T	F		F	T	F	T	T		T	F

- Step 4: **To make $D \to B$ false, we need D true but B false**

A	B	D	E	F	A	$\to$	$(B\&E)$	D	$\to$	$(A \vee F)$	$\neg E$	$D \to B$
F	F	T	F		F	T	F	T	T		T	F

- Step 5: Then, for $D \to (A \vee F)$ to be true, **we need $A \vee F$ to be true**, which gives:

- Step 4: **To make $D \to B$ false, we need D true but B false**

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- Step 6: Finally, since A is false, **the only way $A \vee F$ can be true is if F is true**, which completes the construction.

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Another Example

$$\begin{array}{l} \neg A \vee (B \rightarrow C) \\ E \rightarrow (B \& A) \\ C \rightarrow E \\ \hline \therefore C \leftrightarrow A \end{array}$$

- To test for validity, let's try the short method.
- Step 1: **Assume there's an interpretation in which all three premises are true, but the conclusion is false:**

Another Example

$$\begin{array}{l} \neg A \lor (B \to C) \\ E \to (B \& A) \\ C \to E \\ \hline \therefore C \leftrightarrow A \end{array}$$

- To test for validity, let's try the short method.
- Step 1: **Assume there's an interpretation in which all three premises are true, but the conclusion is false:**

A	B	C	E	$\neg A$	$\lor$	$(B \to C)$	E	$\to$	$(B\&A)$	$C \to E$	$C \leftrightarrow A$
					T			T		T	F

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  - Case 2: C is false and A is true
- Let's start with Case 1. If that doesn't work out, we'll try Case 2.

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- Step 3 (Case 1): Now, the only way **to make $E \rightarrow (B \& A)$ true is to make E false**. However, **since C is true, that means that $C \rightarrow E$ must be false**. But to show invalidity, **we need an interpretation where $C \rightarrow E$ comes out true**, since it's a premise!

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- So far, all we've shown is that there's no row in which (1) all the premises are true, (2) the conclusion is false **by way of** C **being true and** A **being false**. But we need to check the other case to determine if the argument is valid.

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- So, we've found a row in which all premises are true but the conclusion is false.
- Therefore, the argument is invalid.

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T	F	F	F	F	T	T	F	T	F	T	F

- So, we've found a row in which all premises are true but the conclusion is false.
- Therefore, the argument is invalid.
- Lesson: When there are multiple cases, you need to check each of them until you've either run out or you've arrived at a row that invalidates the argument.

Example 3

$$\frac{(A\&B) \vee (A\&C)}{\therefore A\&(B \vee C)}$$

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				T			F	

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F			F	T/F!!	F	F	F	F

- So we can't use Case 1 to invalidate the argument. We need to check the others.

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- If $B \vee C$ is false, **then B is false and C is false**. So, that means that $A \& B$ **and** $A \& C$ **are false**. So, Case 2 is a dead end.

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- Step 3 (Case 3): If A is false, then $A \& B$ **and** $A \& C$ **are also false**. Another dead end.

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F			F	T/F!!	F	F	F	T

- **Now we know the argument is** valid. There's no way to make the conclusion false without making the premise false as well. We've shown this by looking at each possibility and demonstrating that it doesn't work.

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- No.
- Does $A \leftrightarrow (B \vee C), B \to D, D \leftrightarrow C \vDash A \leftrightarrow D$?

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- Yes

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  1. You find an interpretation (i.e., a row of the truth-table) on which all the premises $p_1, \ldots, p_n$ of the argument are true but the conclusion q is false. *All you need to do* is (i) write down the relevant row of the truth-table, (ii) say that it's an interpretation on which the premises are all true but q is false, and (iii) say that the argument is invalid.

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  1. You find an interpretation (i.e., a row of the truth-table) on which all the premises $p_1, \ldots, p_n$ of the argument are true but the conclusion q is false. *All you need to do* is (i) write down the relevant row of the truth-table, (ii) say that it's an interpretation on which the premises are all true but q is false, and (iii) say that the argument is invalid.
  2. You discover that there's *no possible way* of making $p_1, \ldots, p_n$ all true and q false. Here, you need to *explain your reasoning*. It must be made clear that you have *exhausted all possible case* before concluding that the argument is valid. This needs to be spelled out step-by-step with each relevant case examined.

Less Cleverly

- Remember, you can always test for validity by making an entire truth-table.
  - Here, you need to check that on each row the premises come out true, the conclusion is also true.
- We saw this method could get very time-consuming.
- So, only do it if you really need to double check your short method.