Divide and Conquer Recursion + Merge Sort

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CSC 212

Based on slides from Prof. Carolina Ruiz's CS2223 at WPI

Announcements

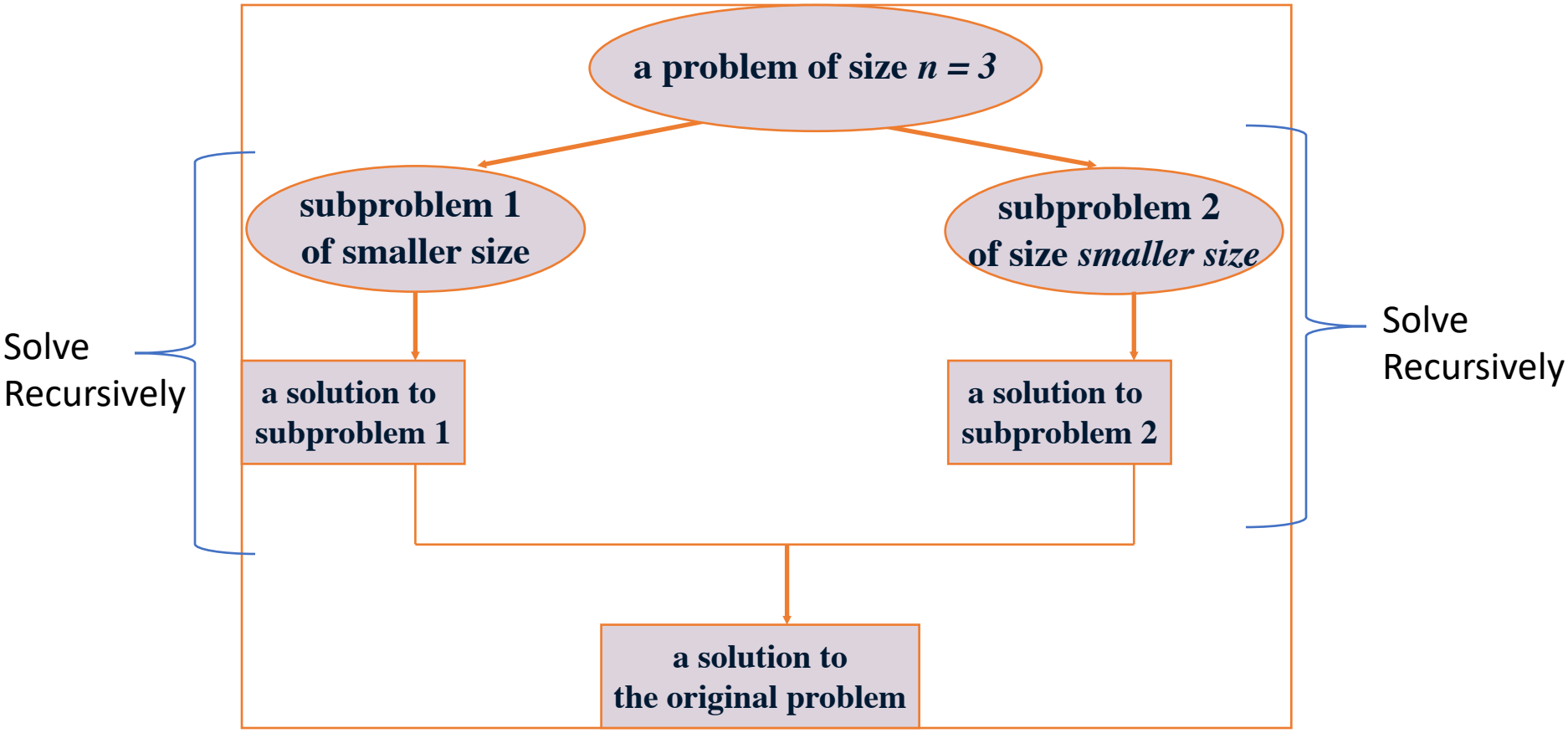
- Assignment 1 will be out on Thursday
- Quiz 2 will be next Tuesday (Oct 8)
- Slight change to office hours based on poll results and TA availability

Divide and Conquer Algorithms

- Before going to the details of today' lecture it is useful to under the concept of **Divide and Conquer Algorithms** and **Recursion**

- Divide and Conquer Algorithms
  - **Divide:** Break the larger problem into sub-problems that are smaller instances of the same problem
  - **Conquer:** the sub-problems are solved ***recursively***
    - If the sub-problem is really small, then solve in a straight-forward manner
  - **Combine:** combine the solutions of the sub-problems to find the solution of the original problem!

Visually Speaking!



Recursive Problem Solving

- What is Recursion?
  - A method of solving a problem where the solution depends on **solutions to smaller instances of the same problem**
- In effect
  - It is **basically a function calling itself** with a slightly smaller set of parameters
  - Can you see why Divide and Conquer Algorithms use it?
- **Recursive Algorithms MAY NOT be intuitive and need a lot of practice to work with!**

Example: Factorial

- Factorial of a positive number p (written as n!) is
  - p! = p*p-1*p-2*…*1
  - p! = 1, if p = 0

- Non-recursive Algorithm

```
def fact(n):
     for i in range(1,n+1):
         fact = fact * i
     return fact
```

- Recursive Algorithm

```
def fact(n):
     if n == 1:
        return 1
     else:
        return n*fact(n-1)
```

Example: Power of a Number

- Power of a number b^n (where n >=0) is defined as
  - b^n = b*b*b*b…*b (multiplied n times with itself)
  - b^n = 1, if n = 0

What will this be?

- Non-recursive Algorithm

```
def power(b,n):
      power = ___
      for i in range(n):
          power = power * b
      return power
```

- Recursive Algorithm

```
def power(b,n):
      if n == 0:
         return 1
      else:
         return b*power(b,n-1)
```

The Selection Sort Algorithm

```
Selection_Sort( A ):

   for i in range(len(A)-1):

    min_id = i
    for j in range(i+1, len(A)):
        if A[min_id] > A[j]:
            min_id = j

    swap(A[i], A[min_id])
```

Non-recursive Algorithm

i is the index of the array at which we want to "correctly" populate. Initially ZERO

Recursive Algorithm

```
Selection_Sort_Rec( A, n, i ):
   if i == n:
     return
   else:
     k = find_min_index(A, n, i)
     if k != i:
        A[i], A[k] = A[k], A[i]
     Selection_Sort_Rec(A,n,i+1)
```

find_min_index is a function to find the min element in an list A, starting at i and ending at n. It needs to be written separately

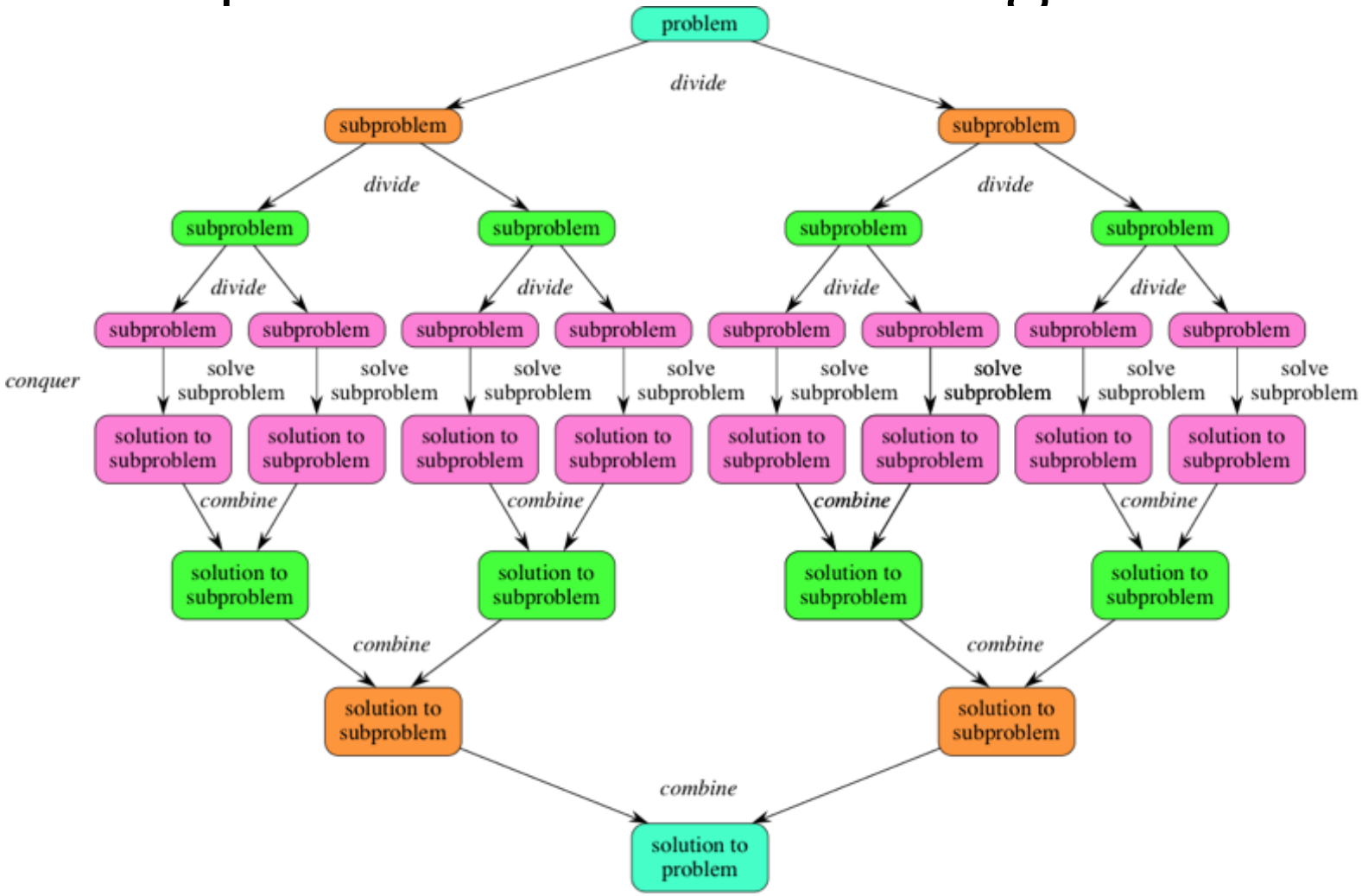
Practice: Convert a number to a String

- **Goal:** Write a function to take a number and converts it into a **string representation in any base** (i.e., binary, decimal, octal, hexadecimal
- So, the input is an integer
  - Example: 3, 17, 8, 19
- Produces an output is a **string representation of the number in a particular base**
  - Input – 3, 17, 8, 19
  - Output (Base 2) - 11, 10001, 1000, 10011 [in string format]
  - Output (Base 16) – 3, 11, 8, 13 [in string format]
- Of course do it with Recursion

Starter Code

```
def ToStr(n, base):
    str = "0123456789ABCDEF"
    if (n < base):
        return str[n]
    …
```

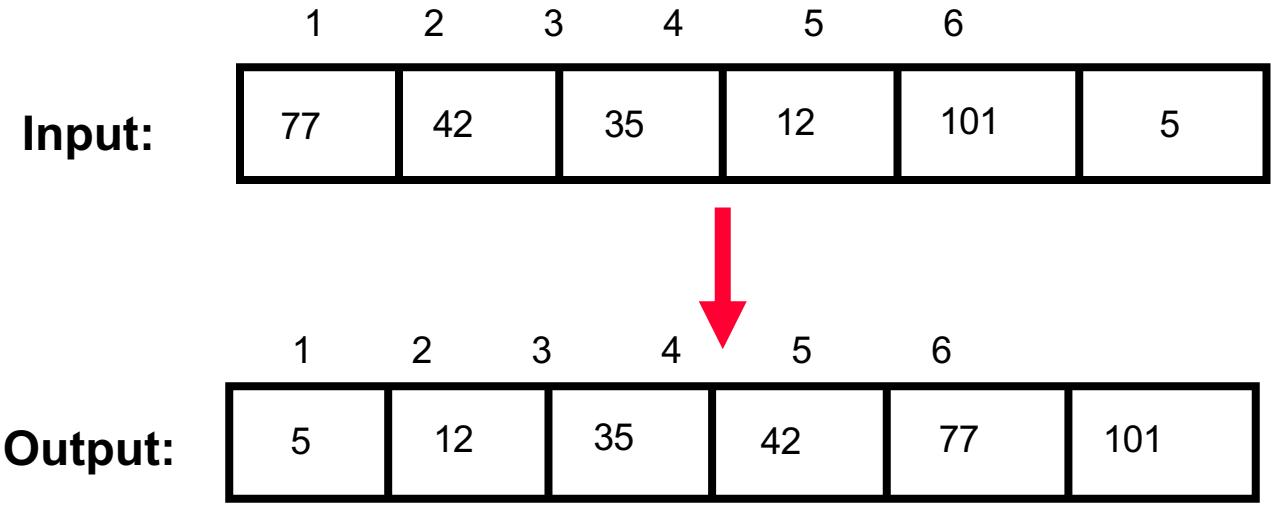
Coming back to Divide and Conquer Problem Solving



https://www.khanacademy.org/computing/computer-science/algorithms/merge-sort/a/divide-and-conquer-algorithms

Sorting: Problem Definition

- **Sorting takes an unordered collection and makes it an ordered one.**



How can we sort an array using divide and conquer approach?

Sorting Algorithms

- *Insertion Sort --- covered already*
- *Bubble Sort --- covered already*
- *Selection Sort --- covered already*
- **Merge Sort**
- Quick Sort
- Heap Sort
- …

Merge Sort (Pseudocode)

```
MergeSort(A)
   if A's size > 1
     Divide array A in halves
     Call MergeSort on first half.
     Call MergeSort on second half.
     Merge two results (combine).
```

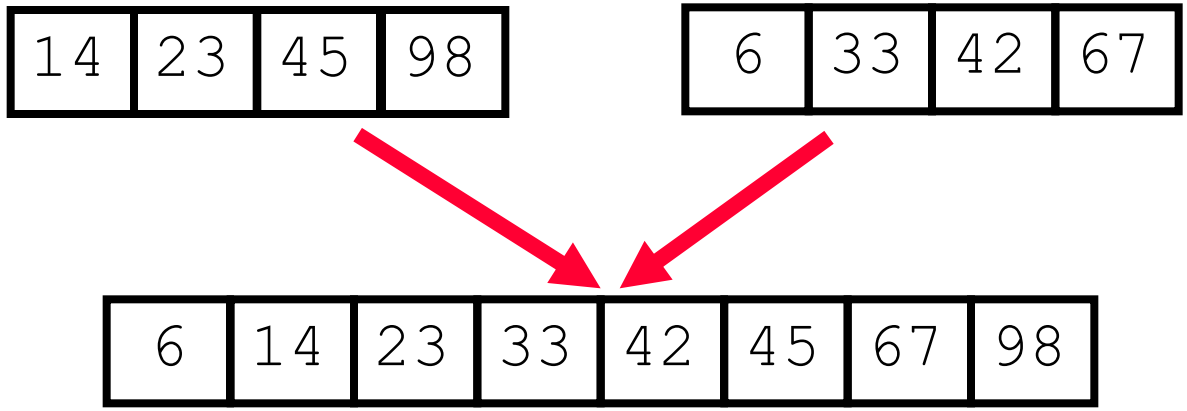
Recursion

Merge Sort Algorithm (Pseudocode)

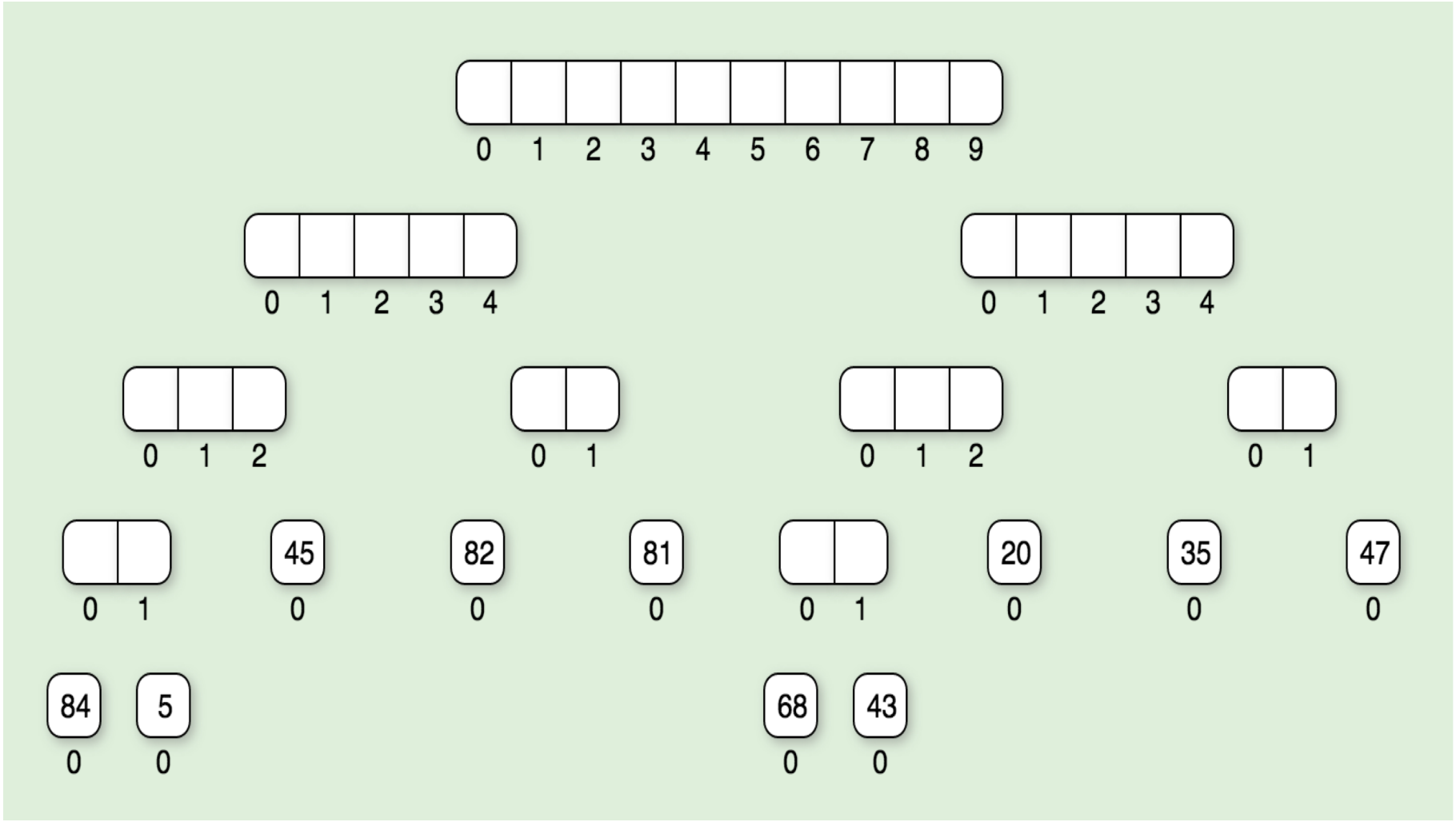
```
MergeSort(A, l, r)
   if l < r:
    m = (l + r)/2 //Divide
      MergeSort(A,l,m) //Conquer
      MergeSort(A,m+1,r)//Conquer
      Merge(A,l,m,r)// Combine
```

How to Merge?

- **Merge the sub-problem solutions together:**
  - **Compare the sub-array's first elements**
  - **Remove the smallest element and put it into the result array**
  - **Continue the process until all elements have been put into the result array**



Practice -- Merging



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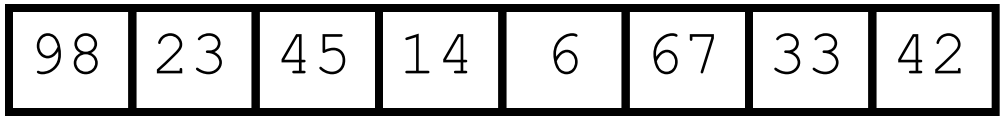

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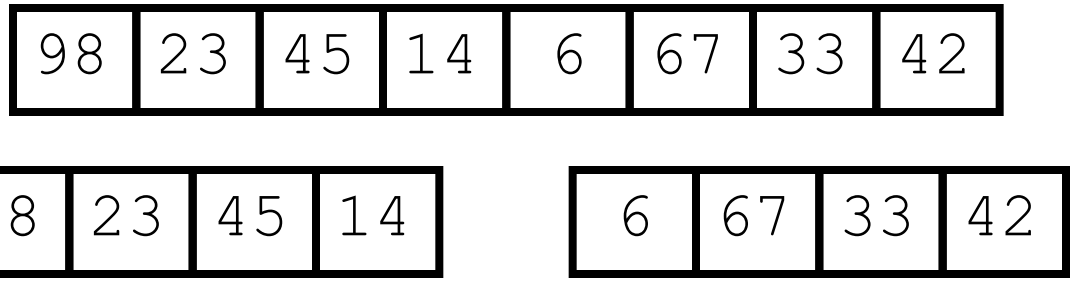

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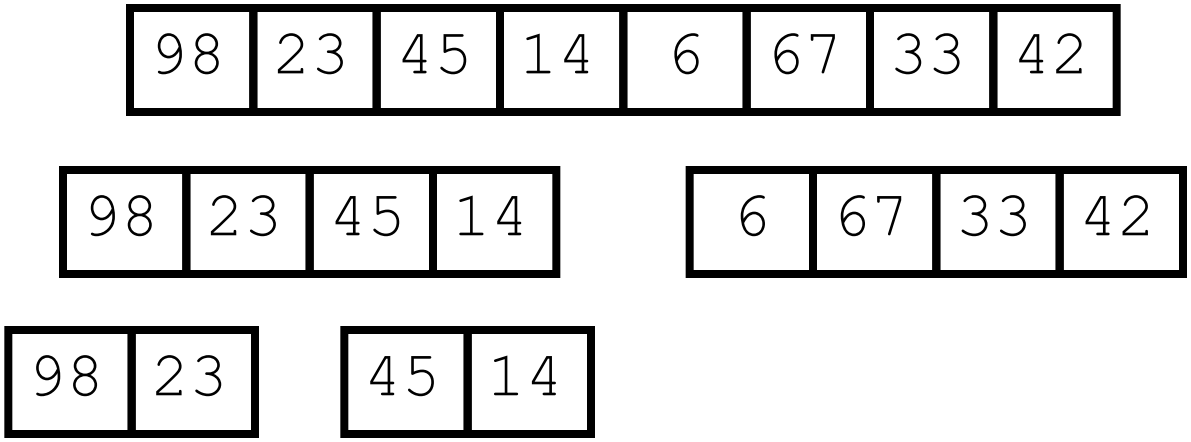

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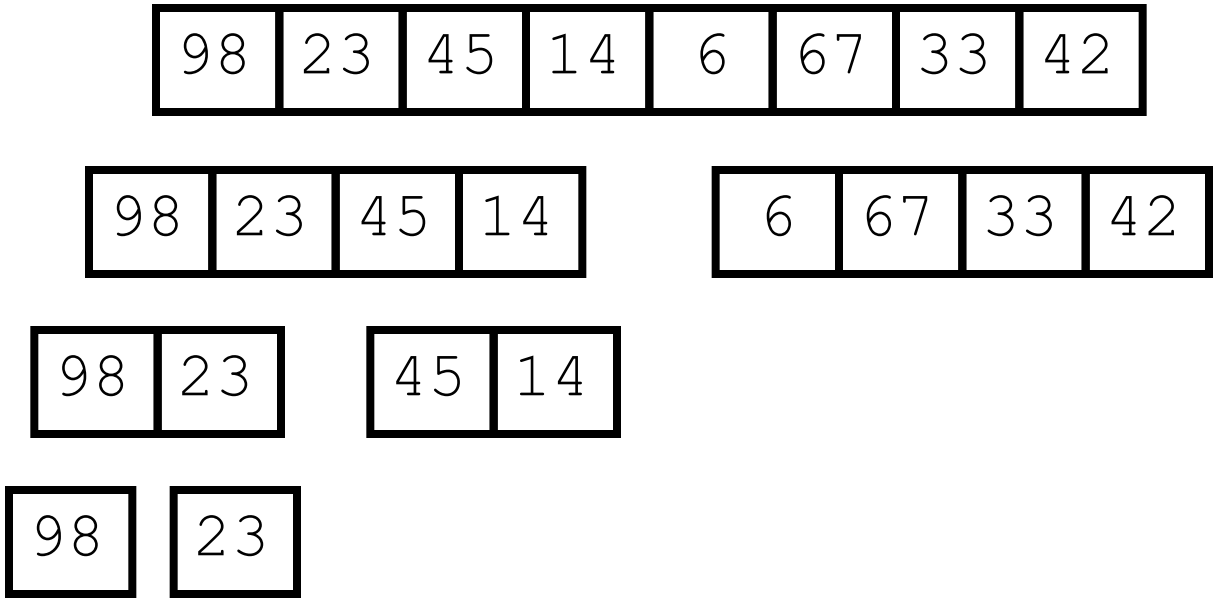

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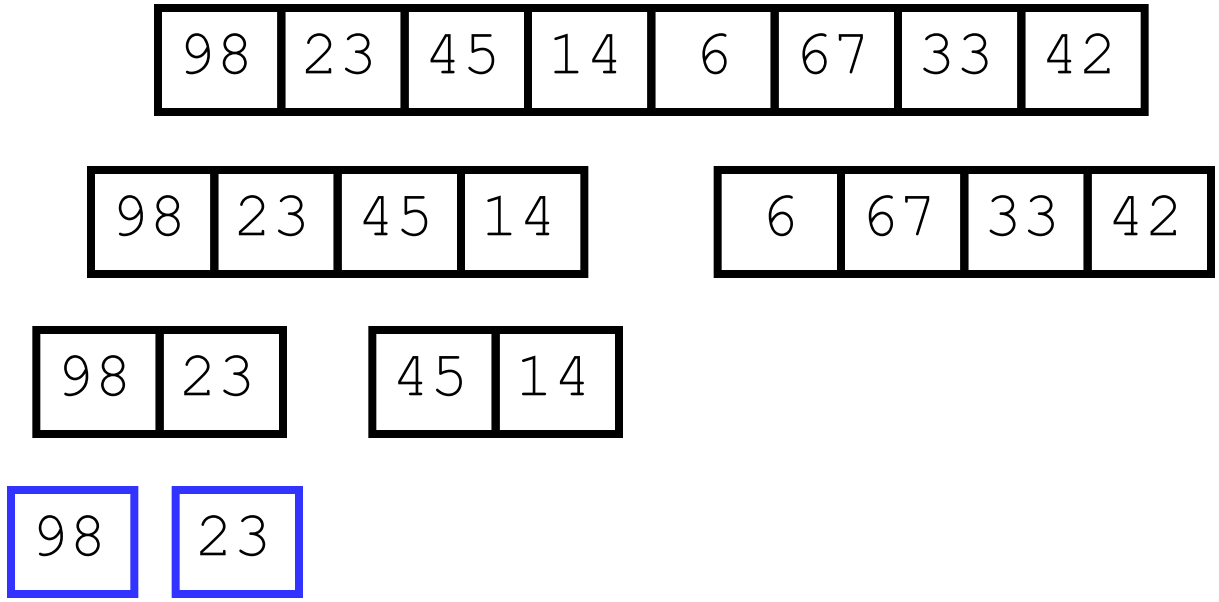

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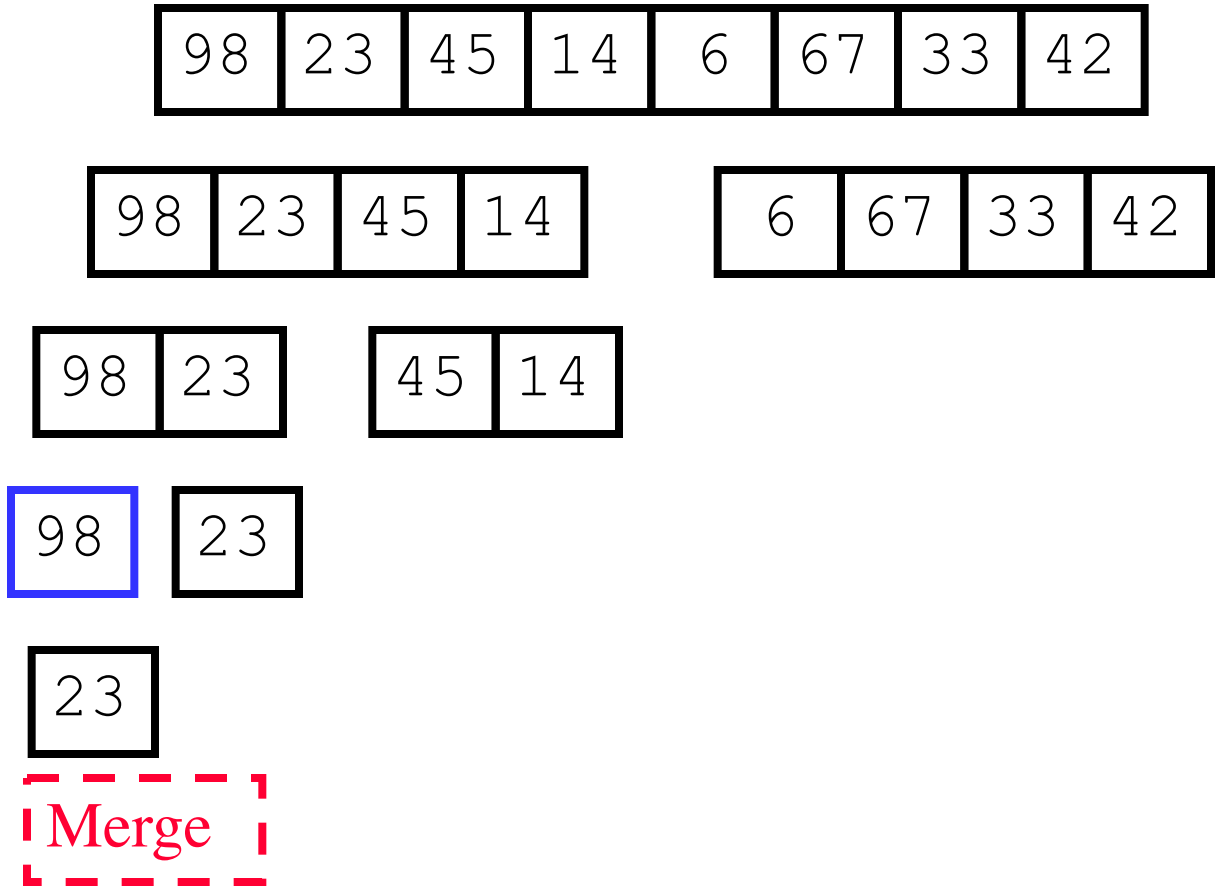

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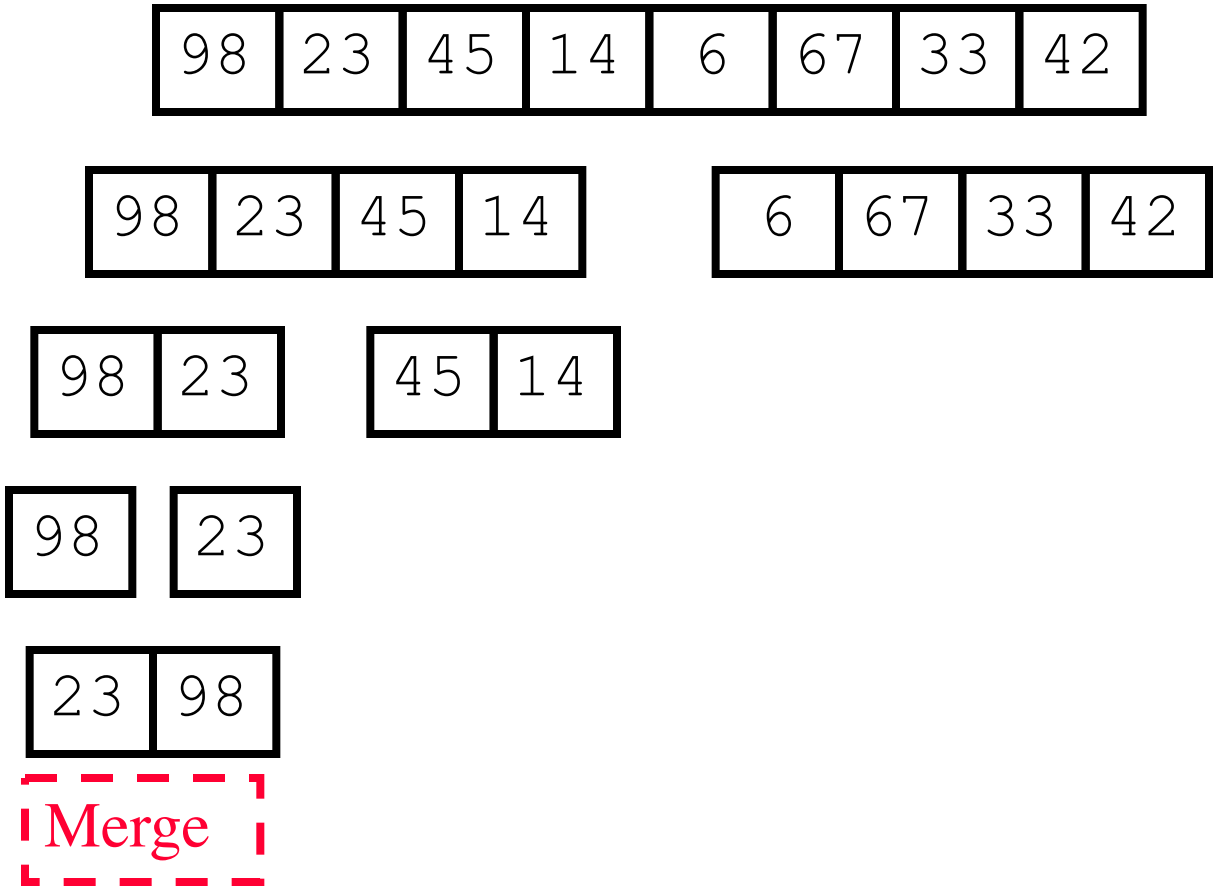

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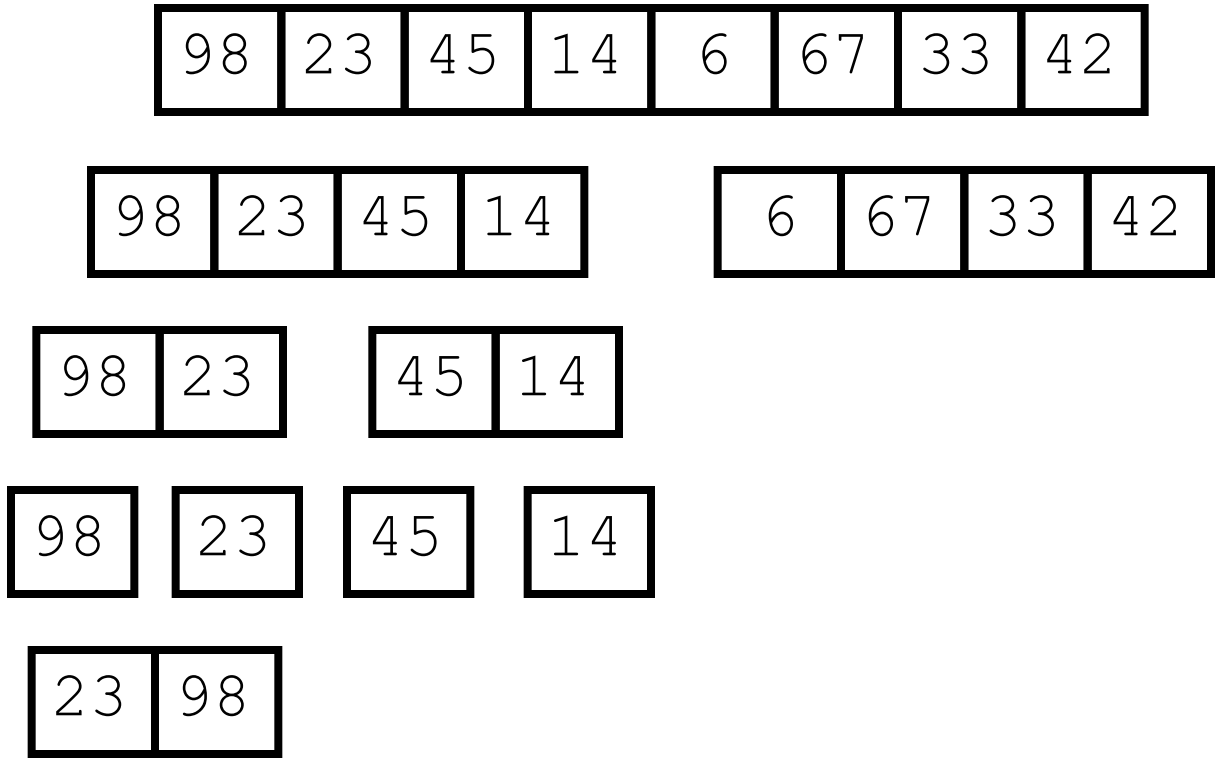

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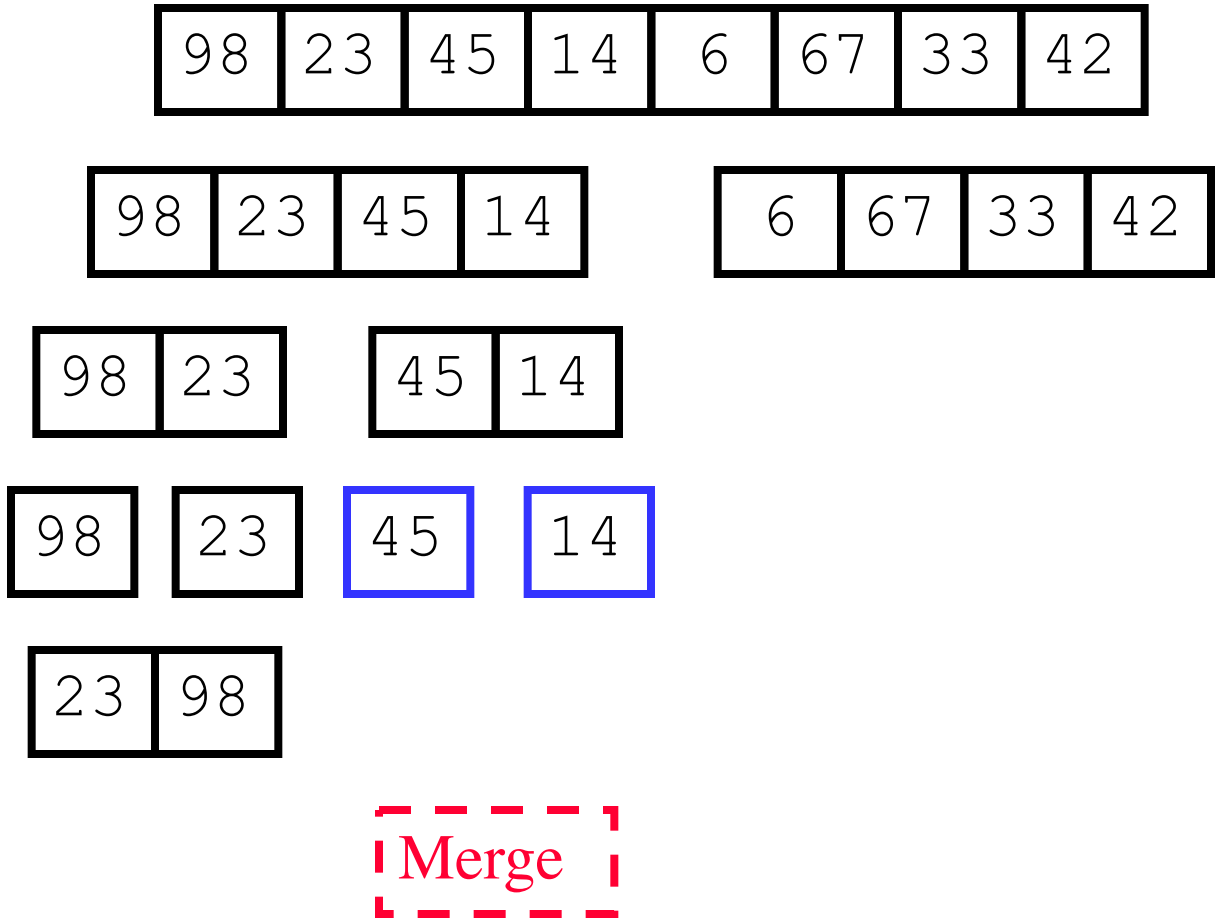

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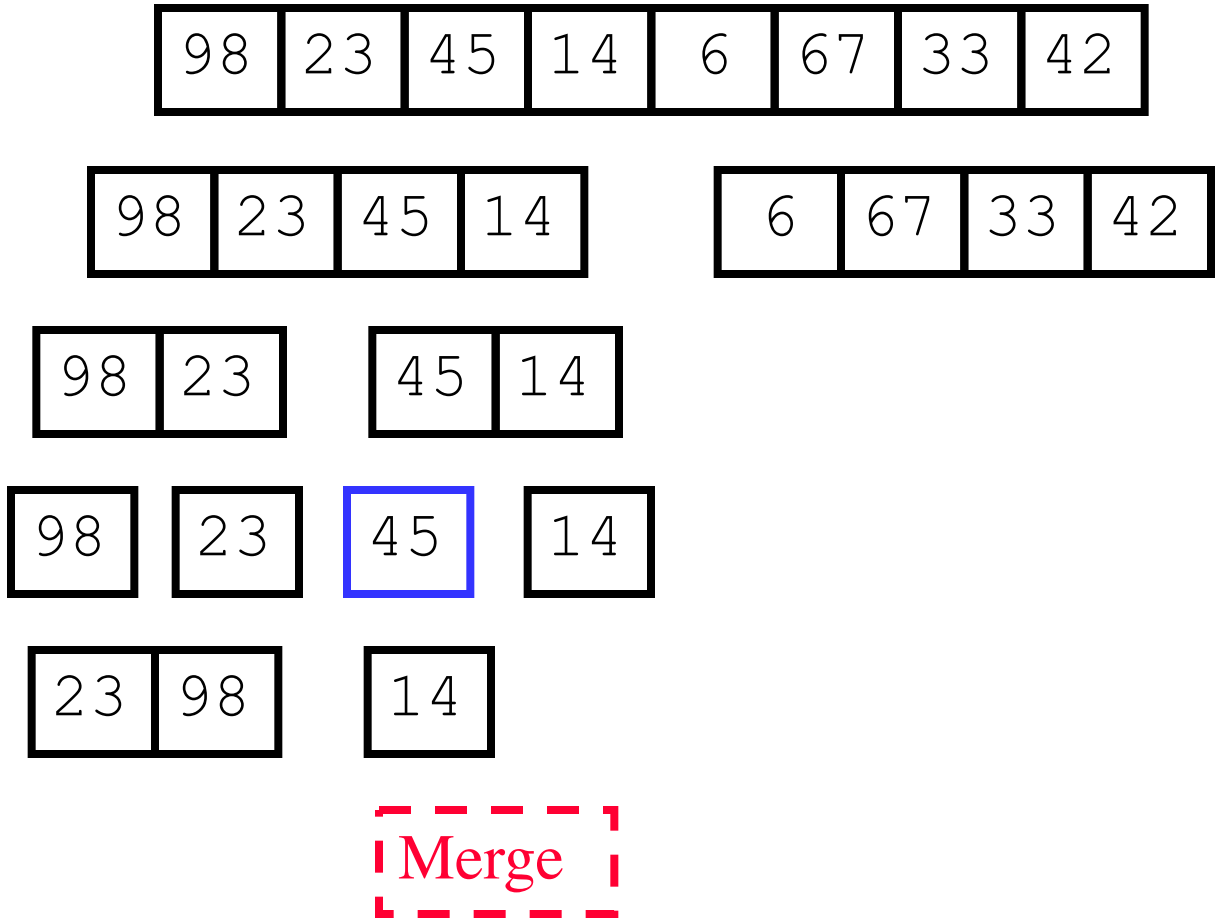

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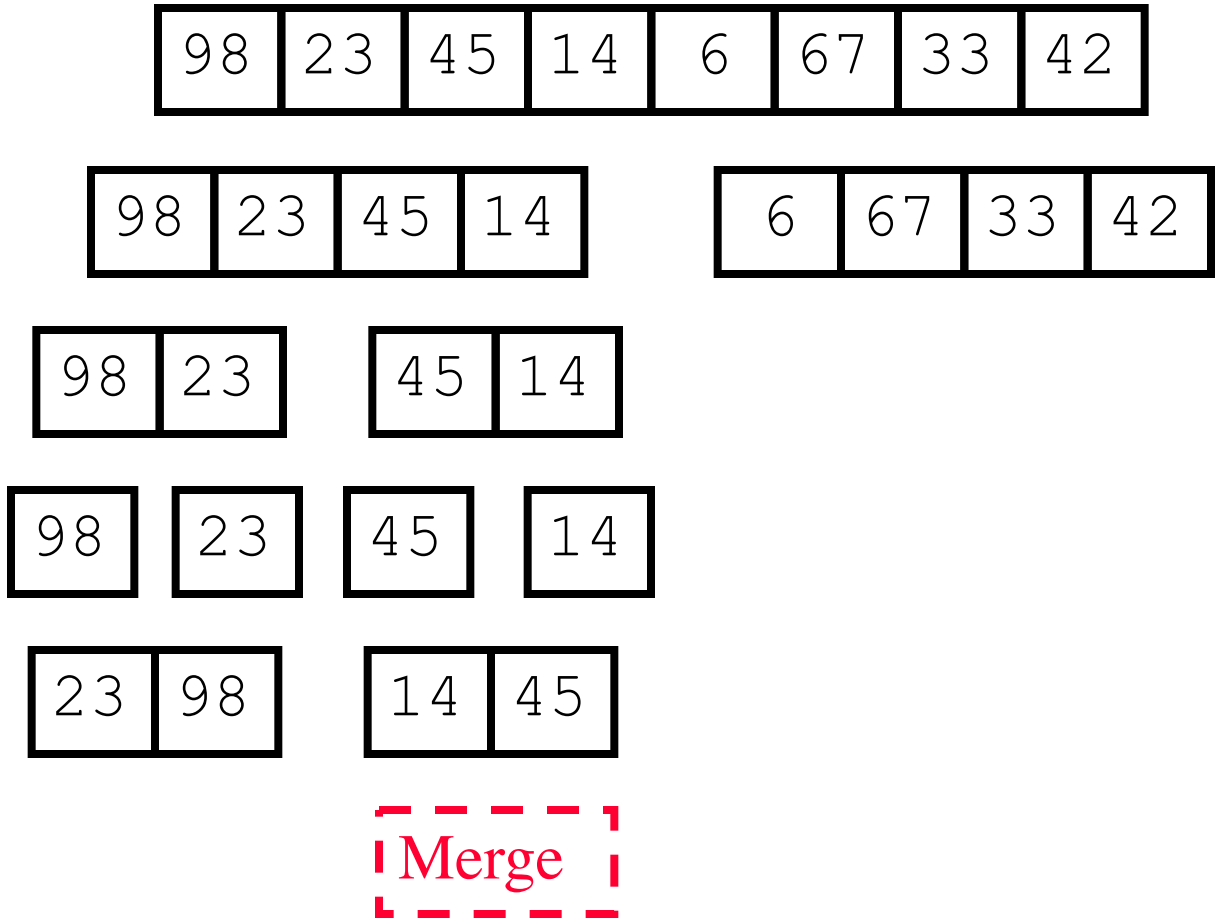

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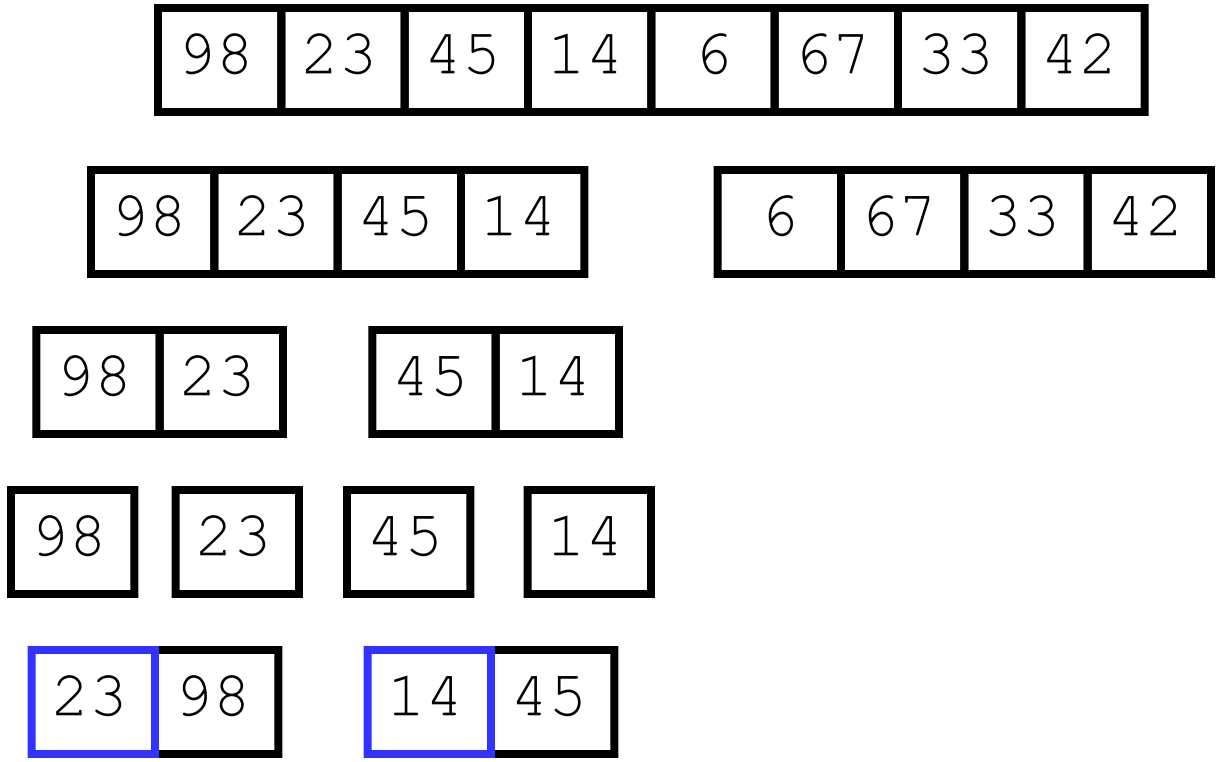

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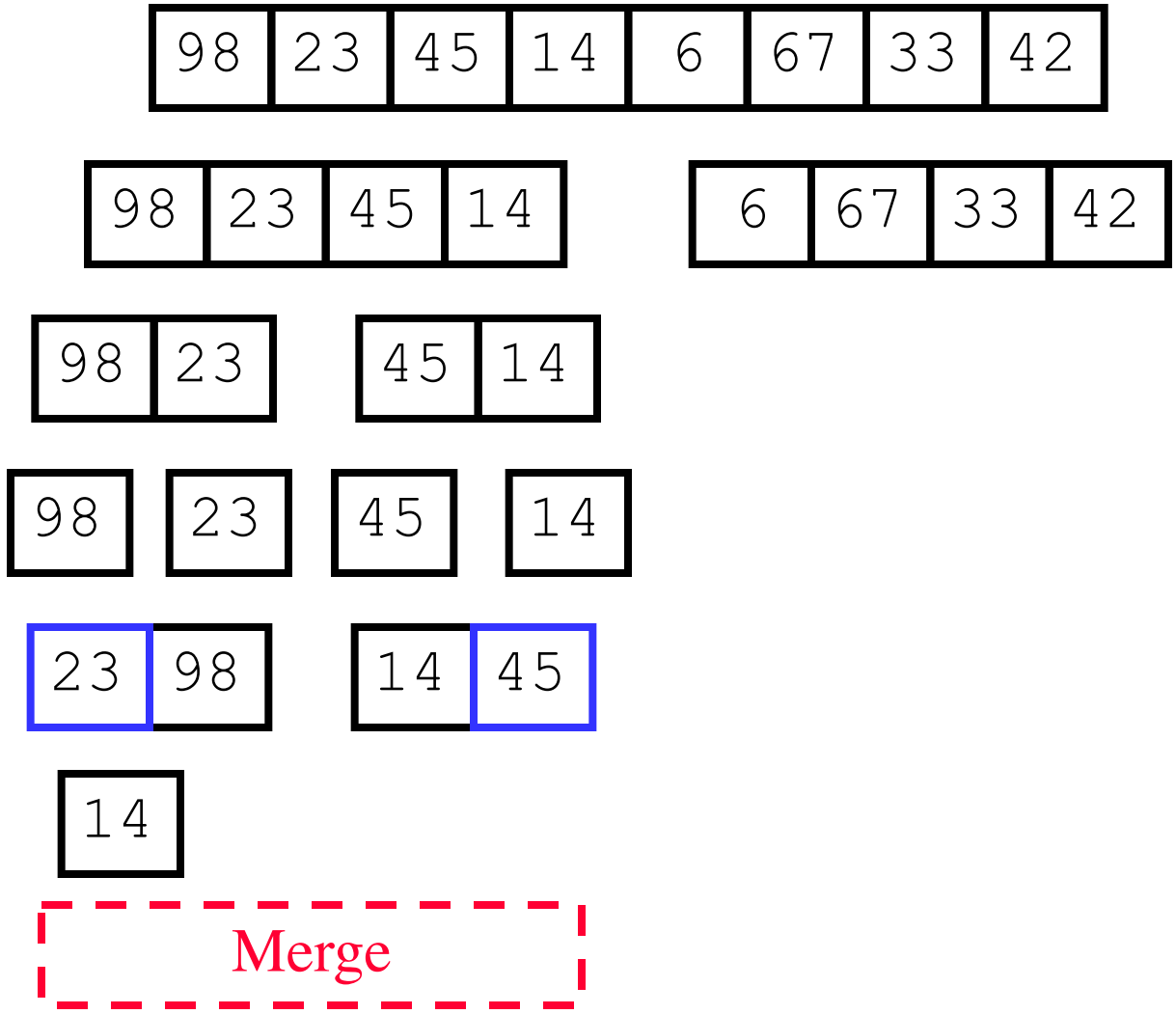

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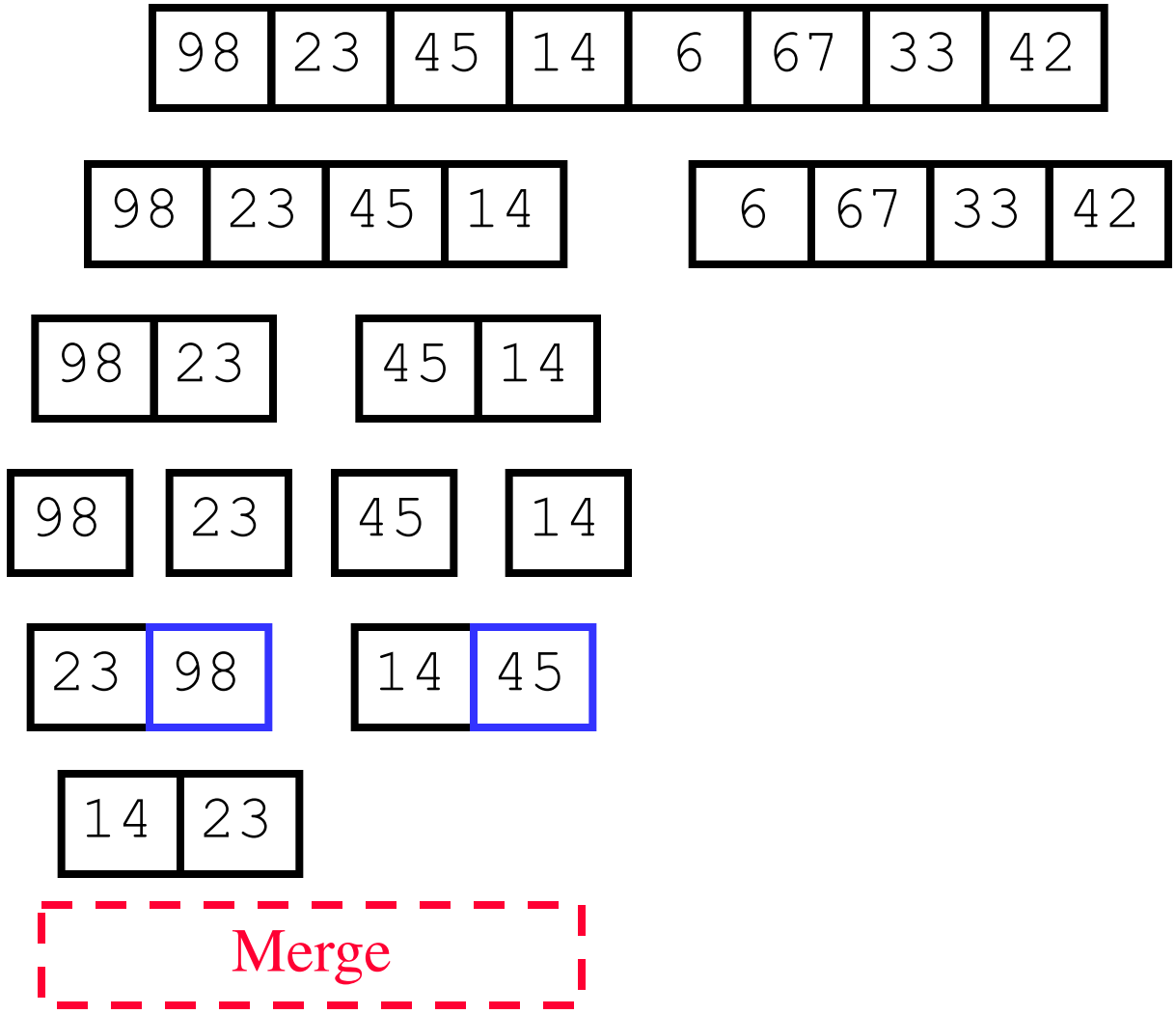

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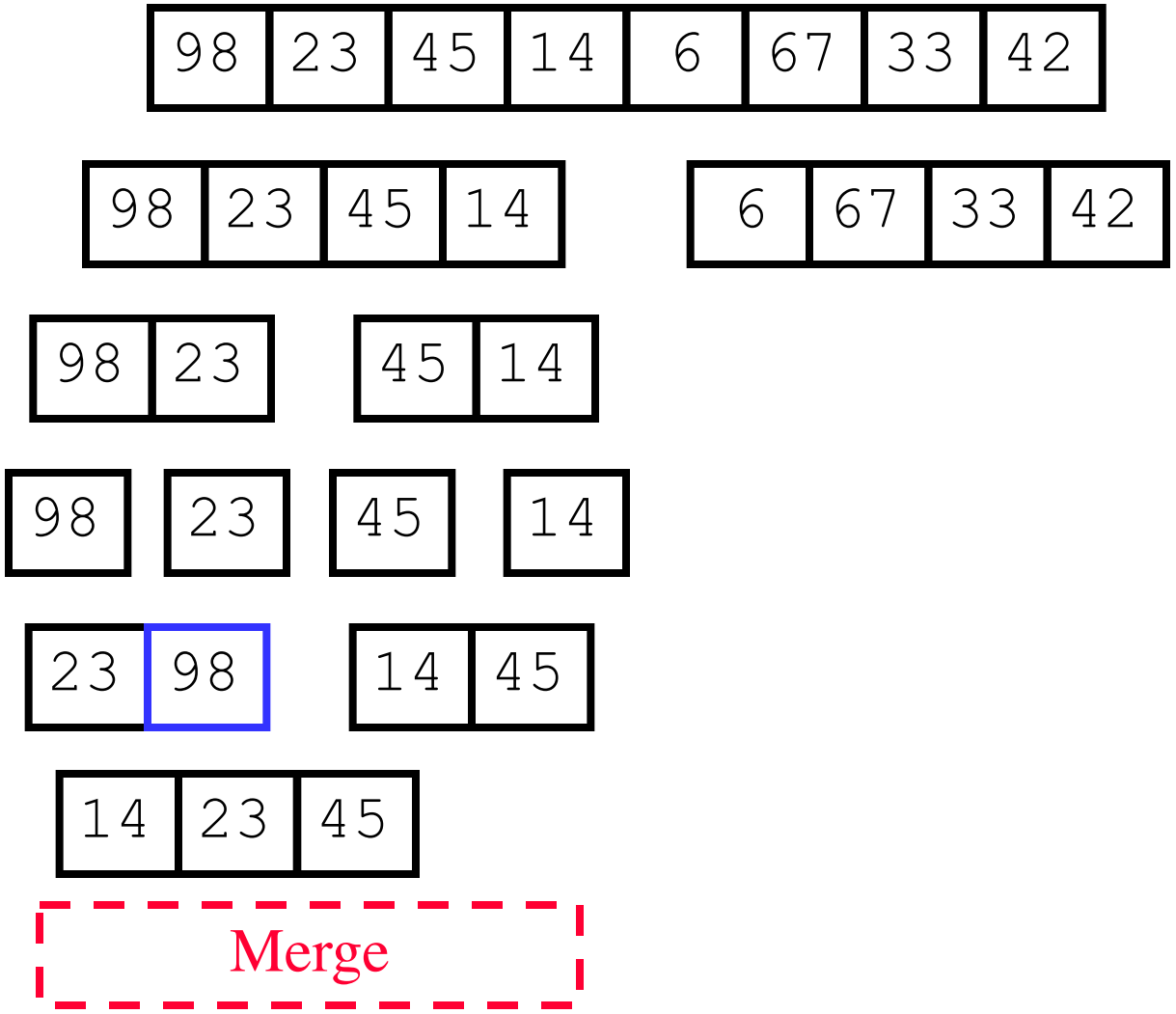

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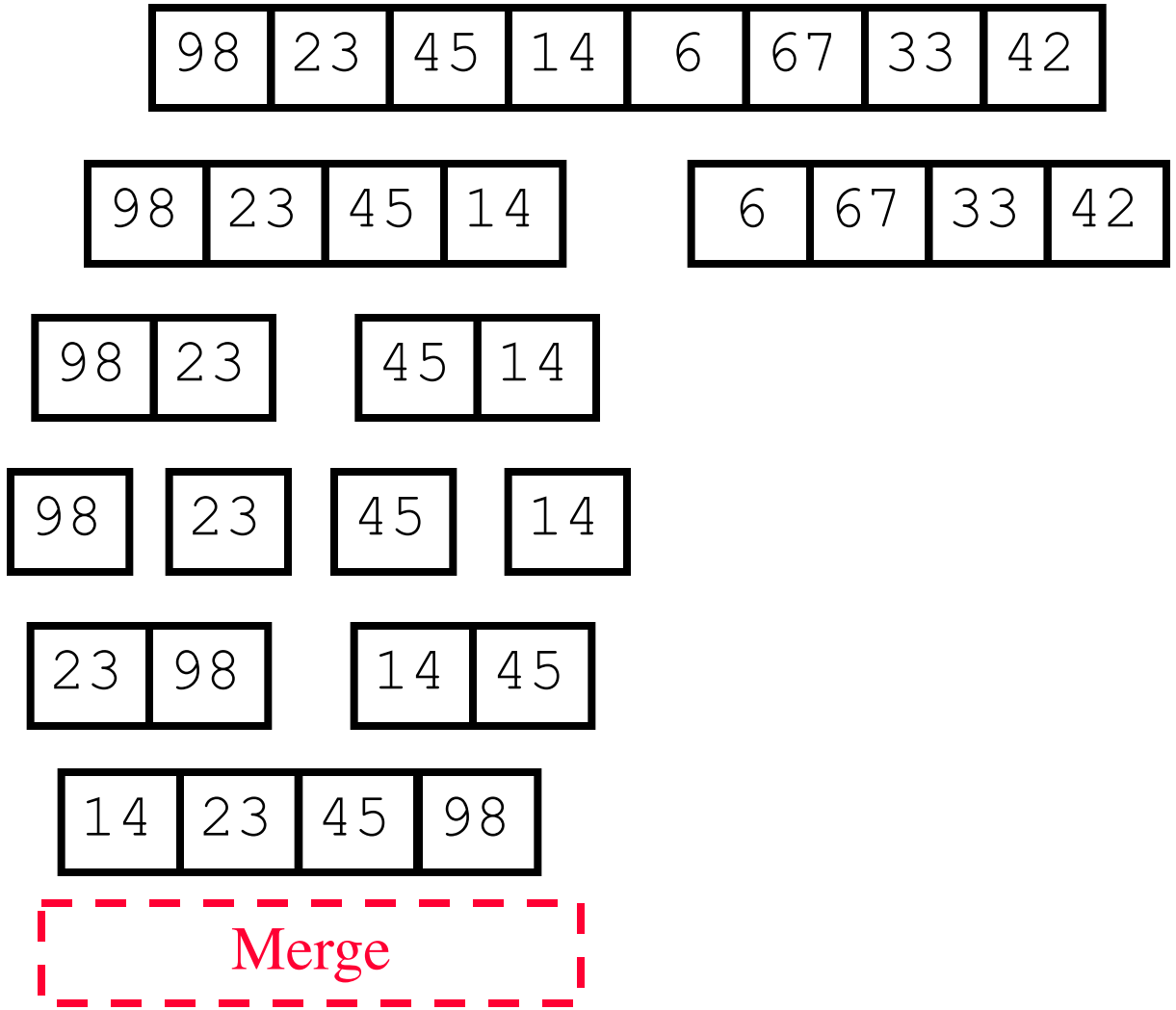

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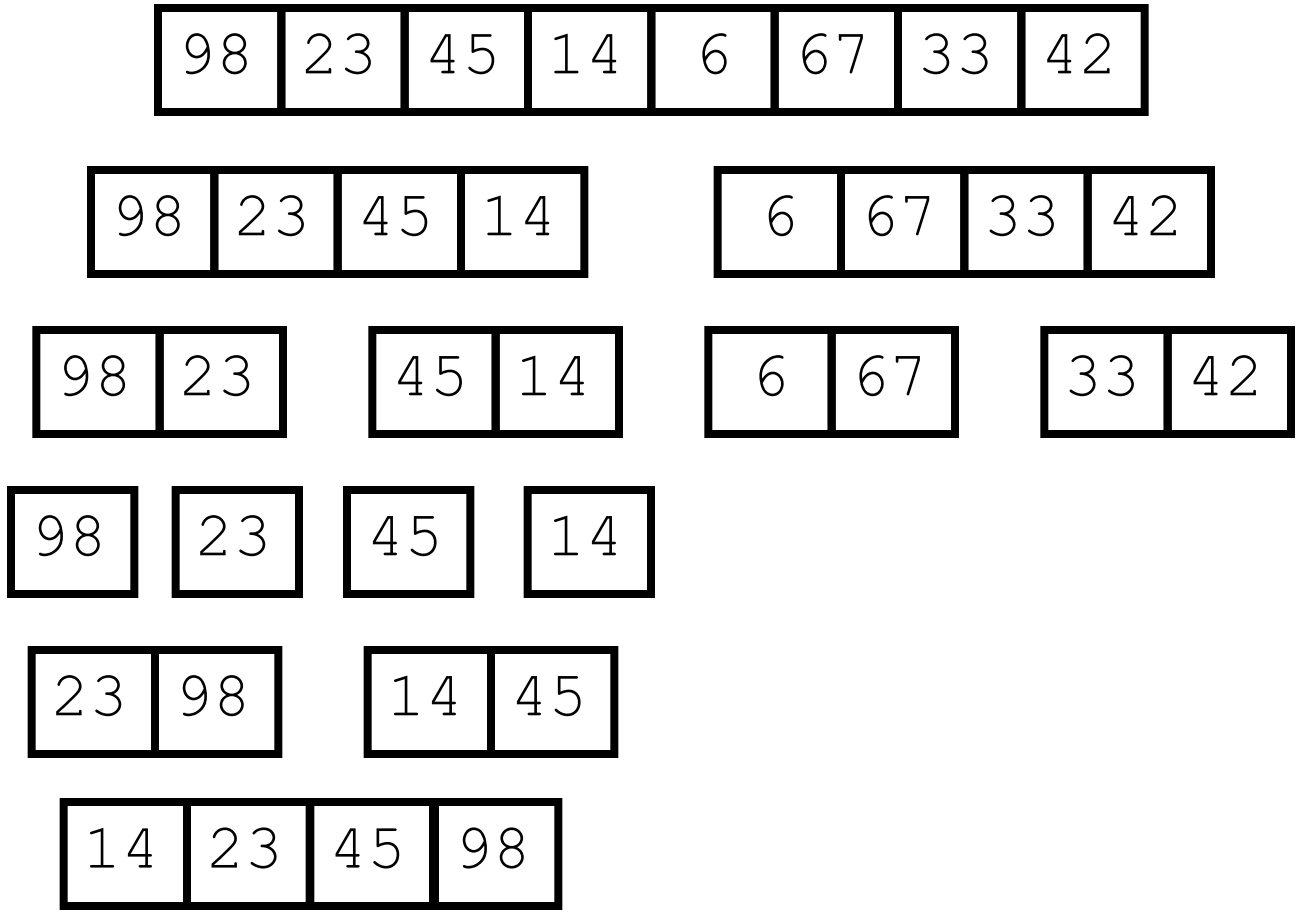

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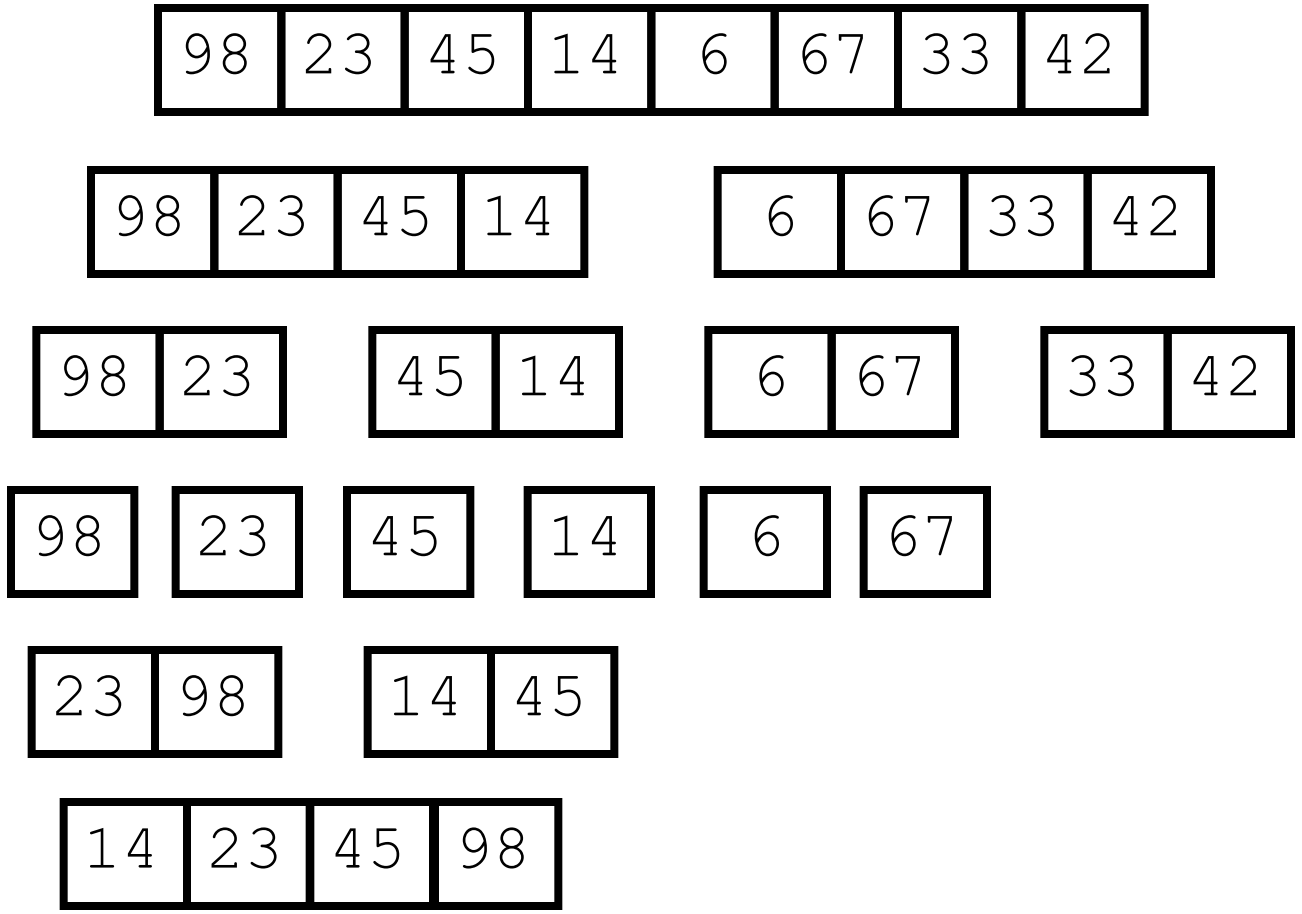

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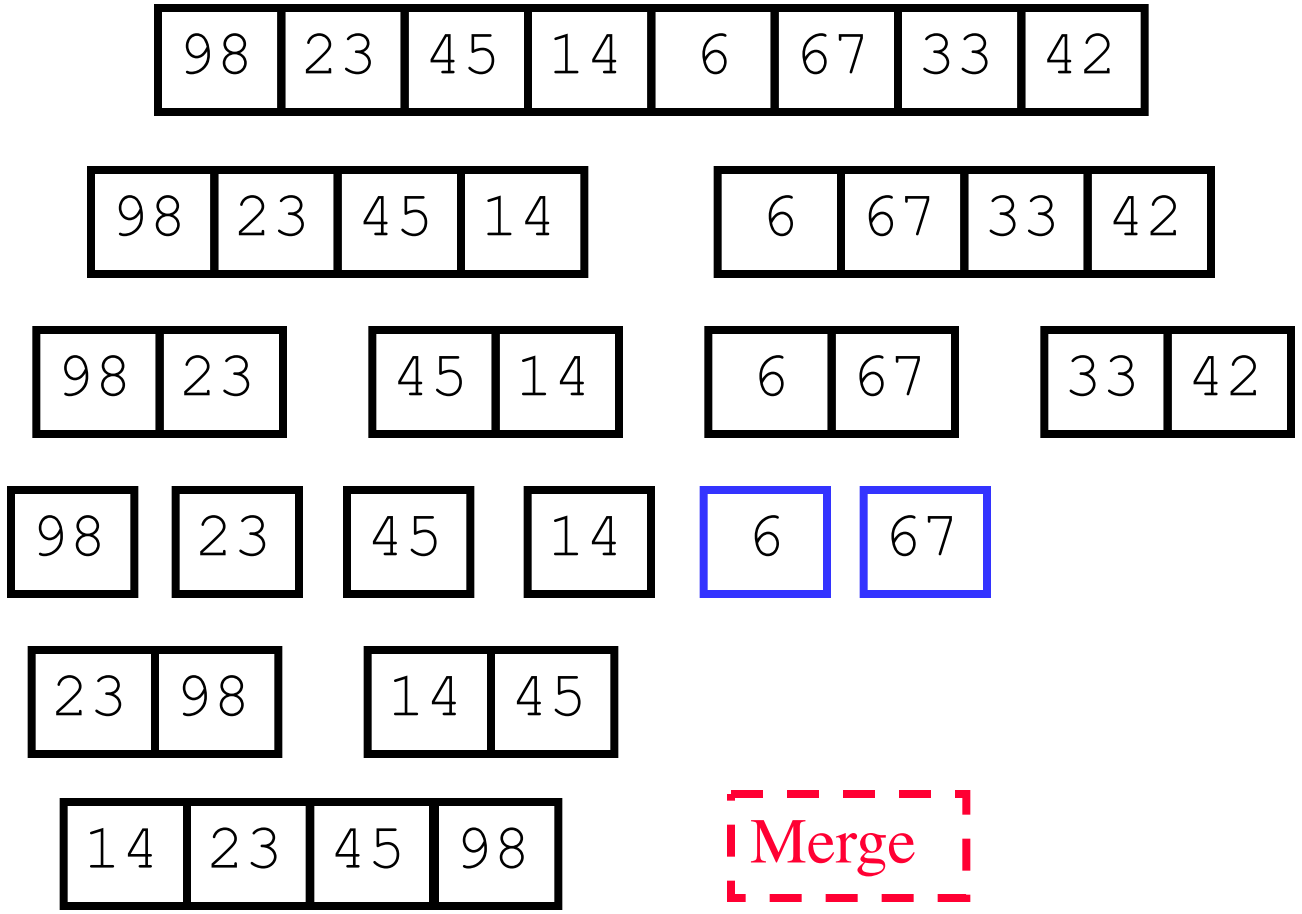

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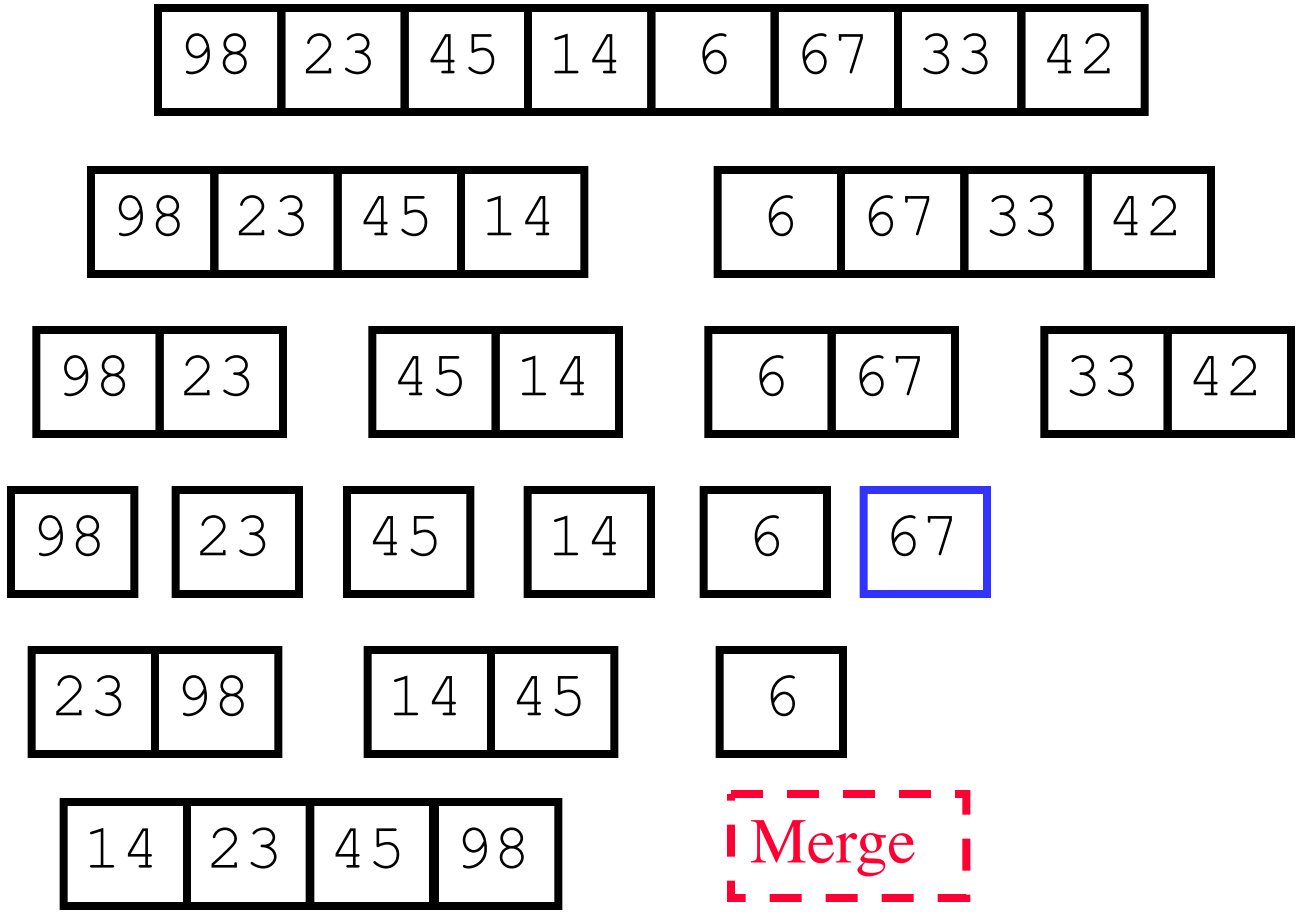

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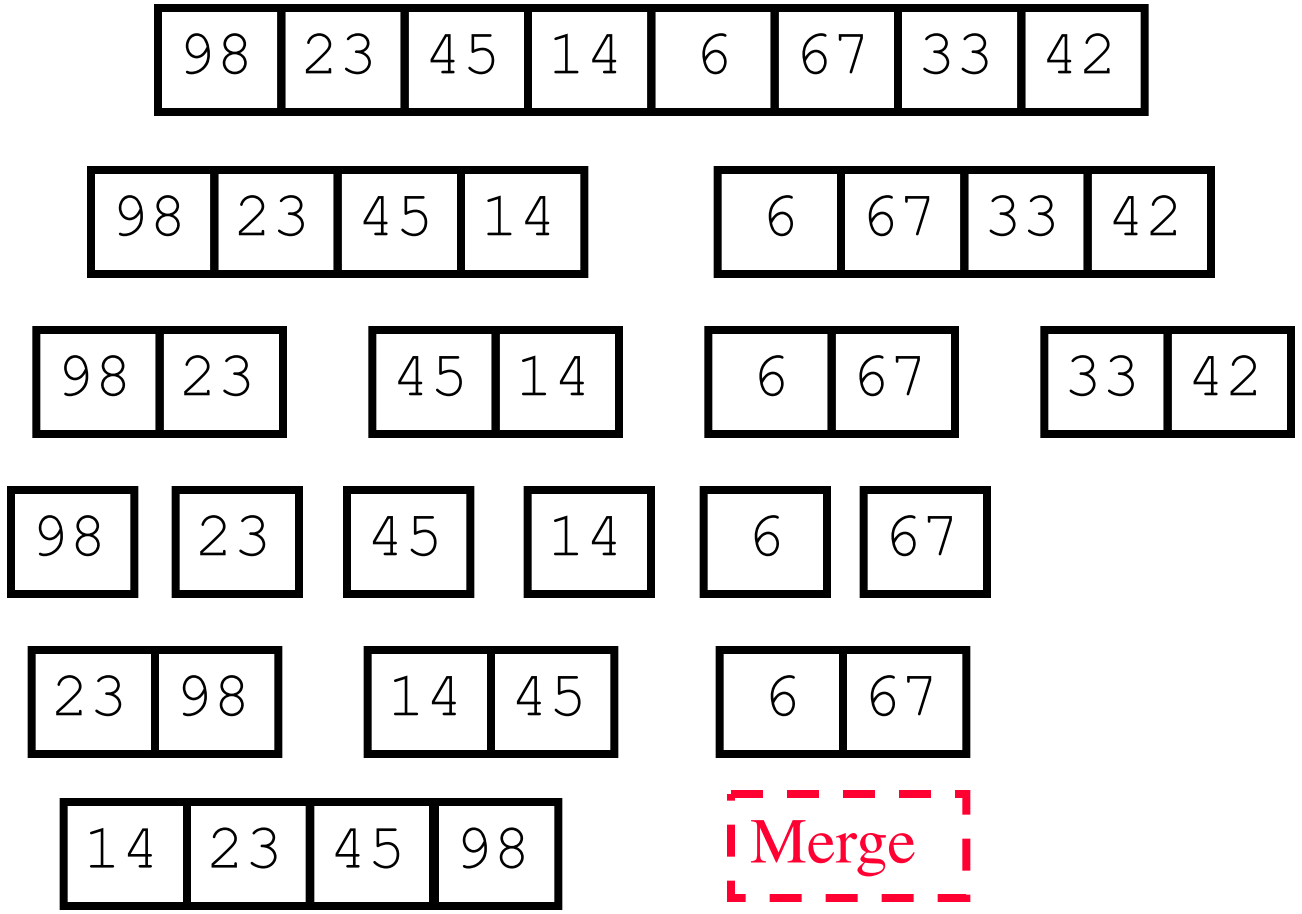

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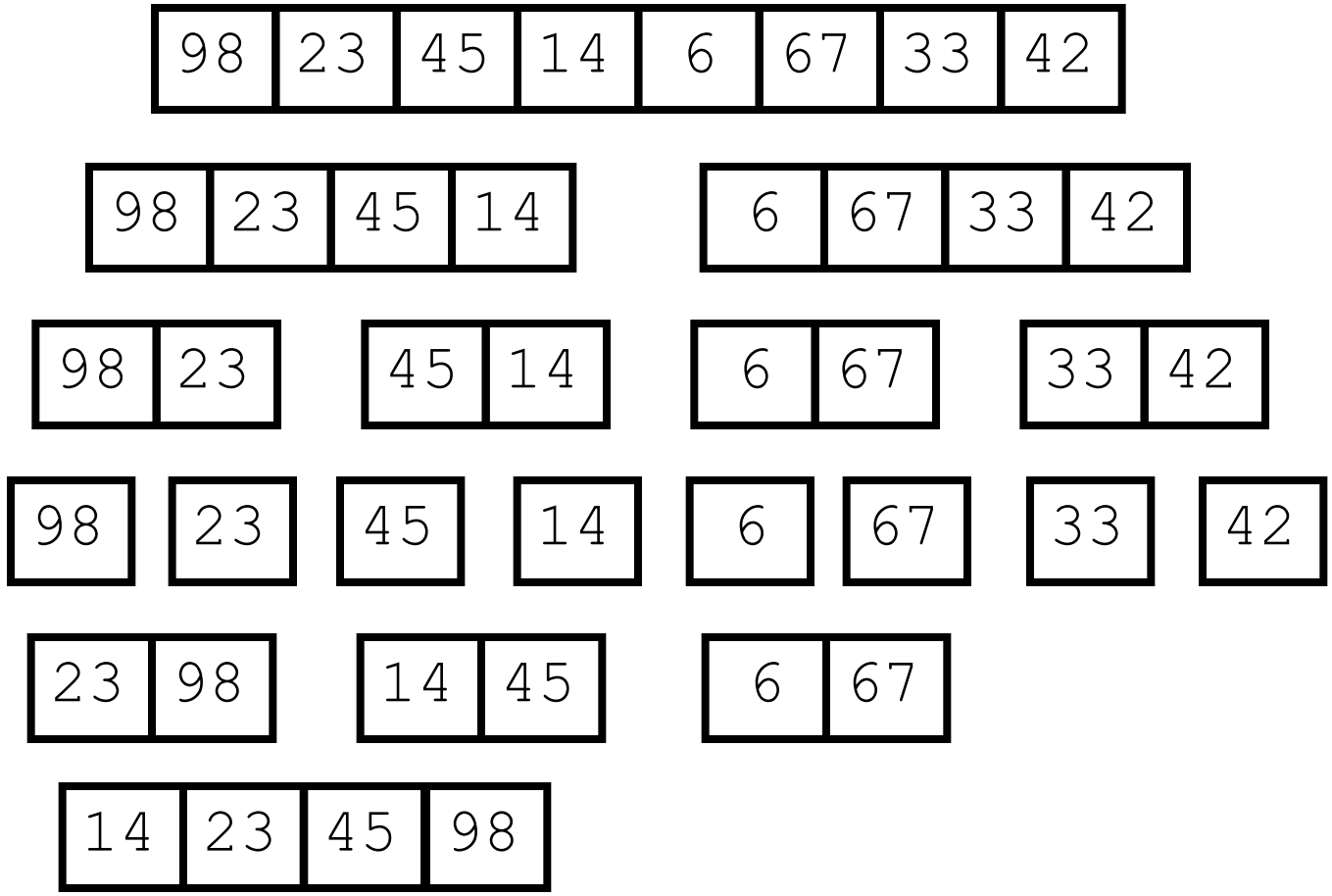

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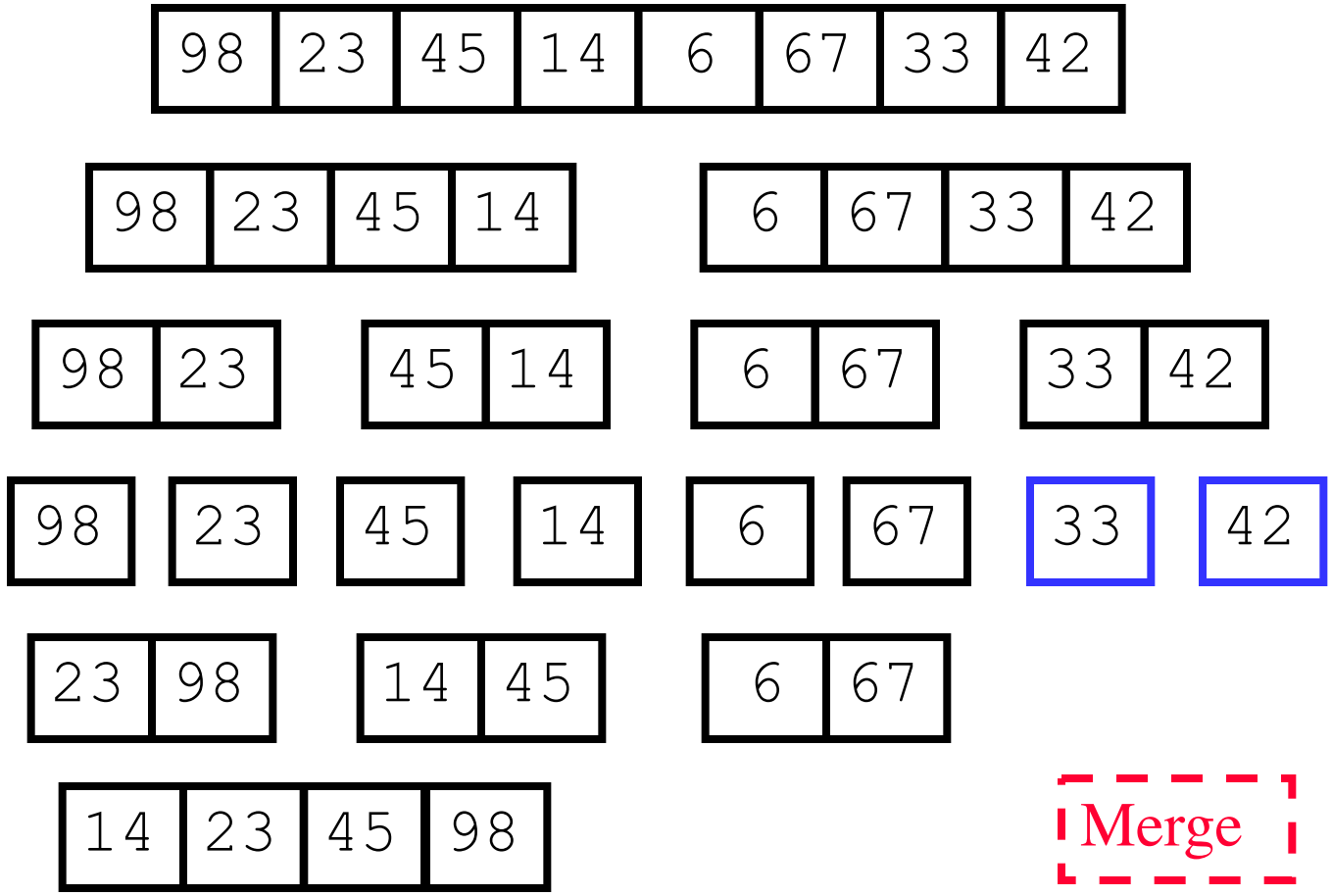

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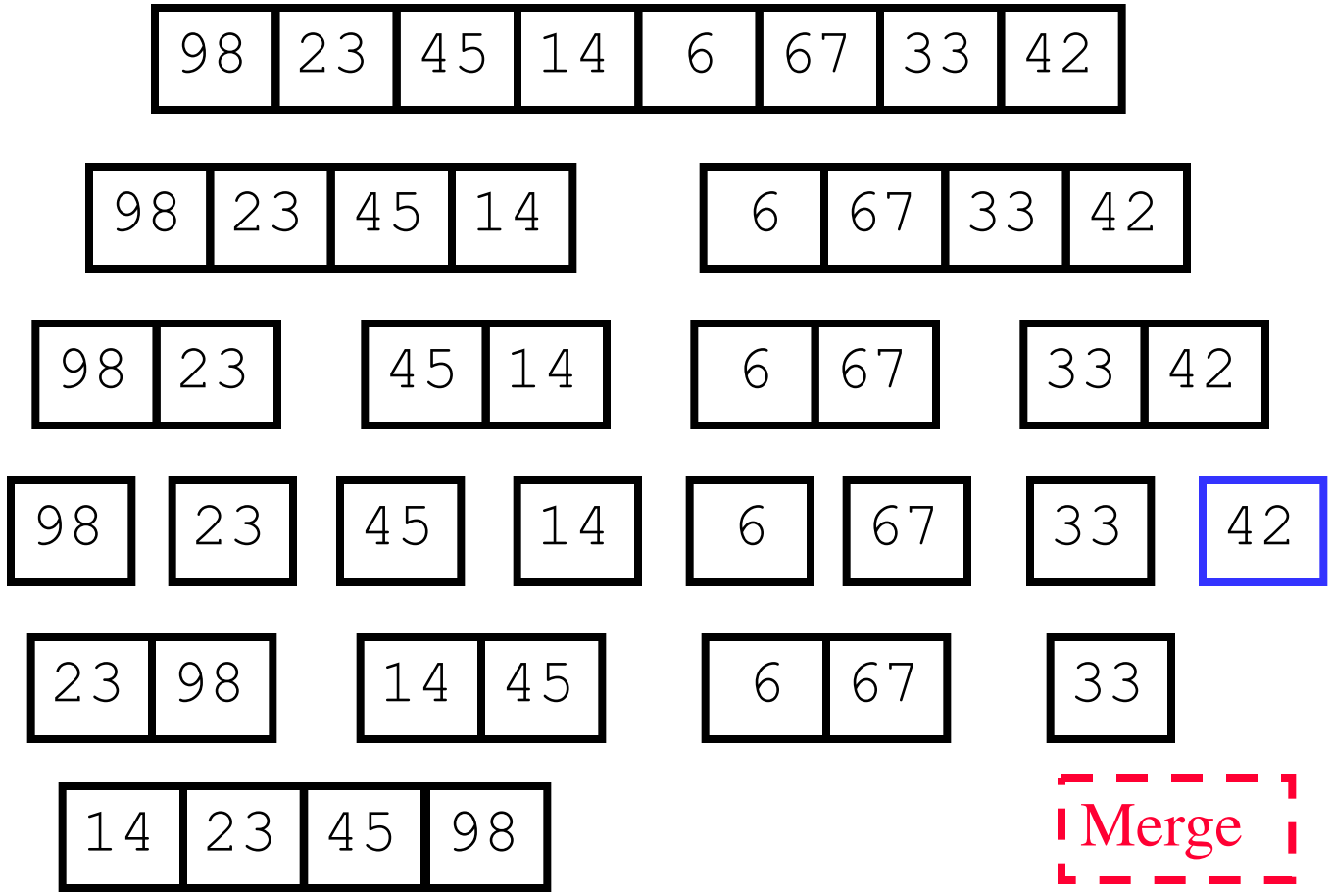

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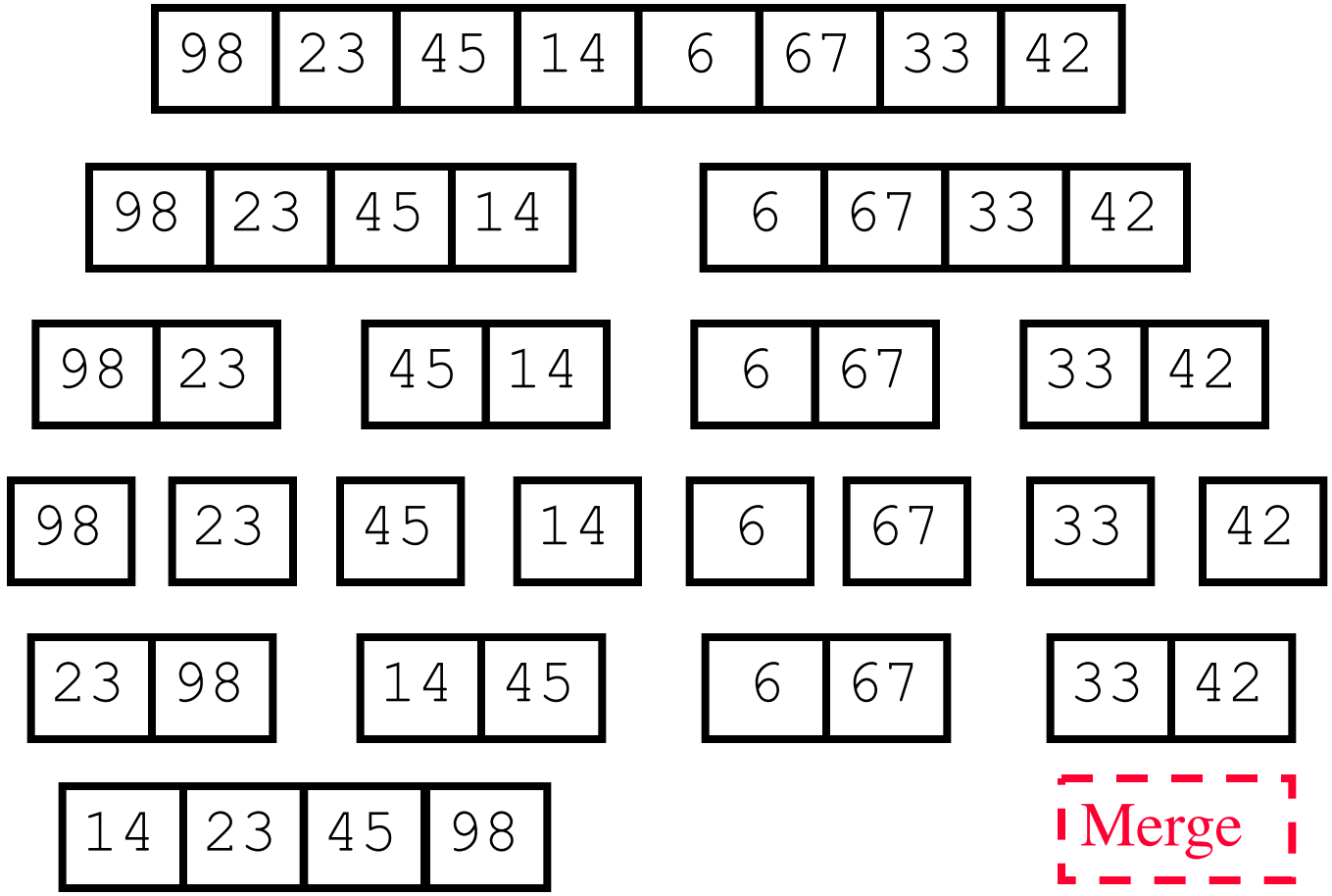

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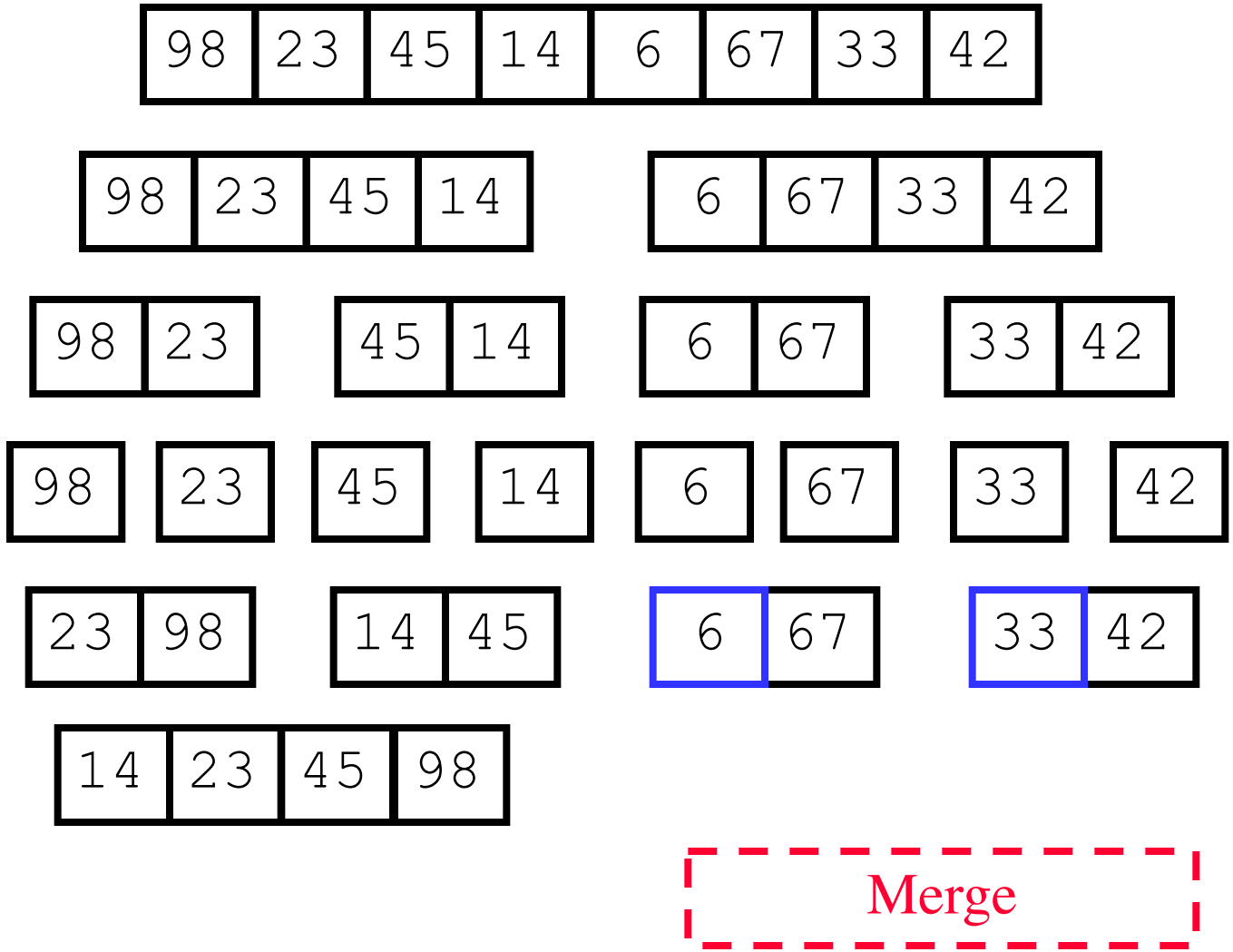

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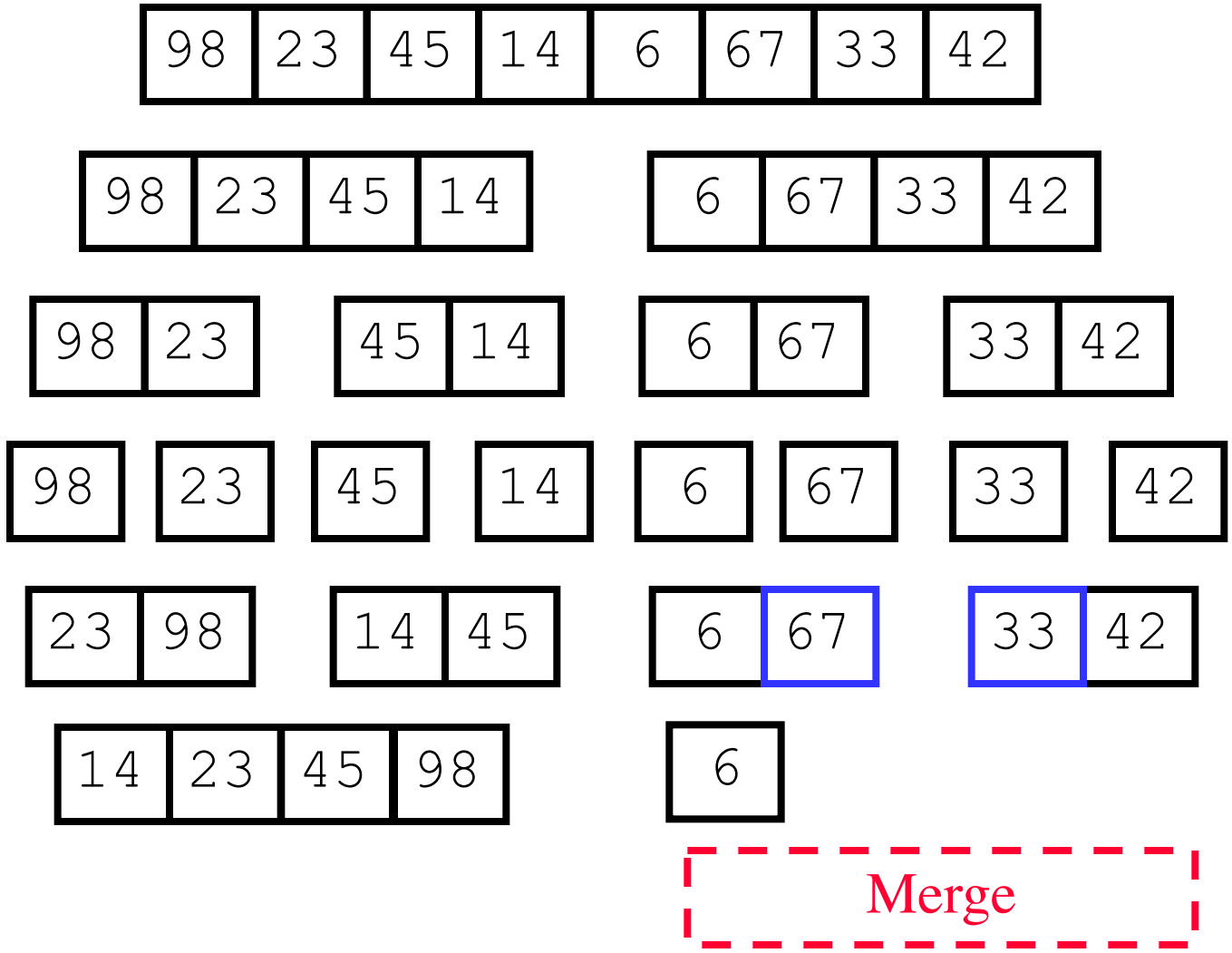

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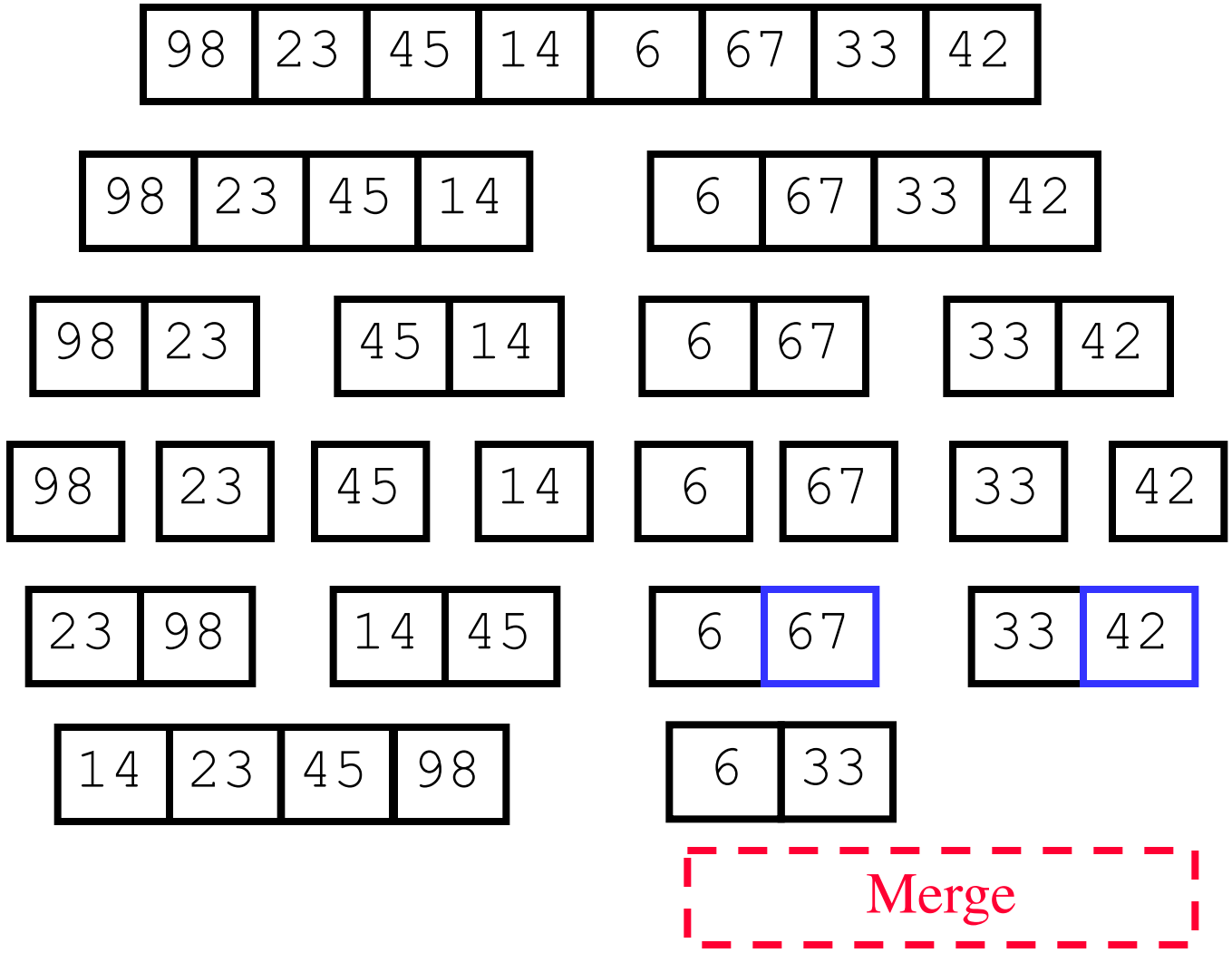

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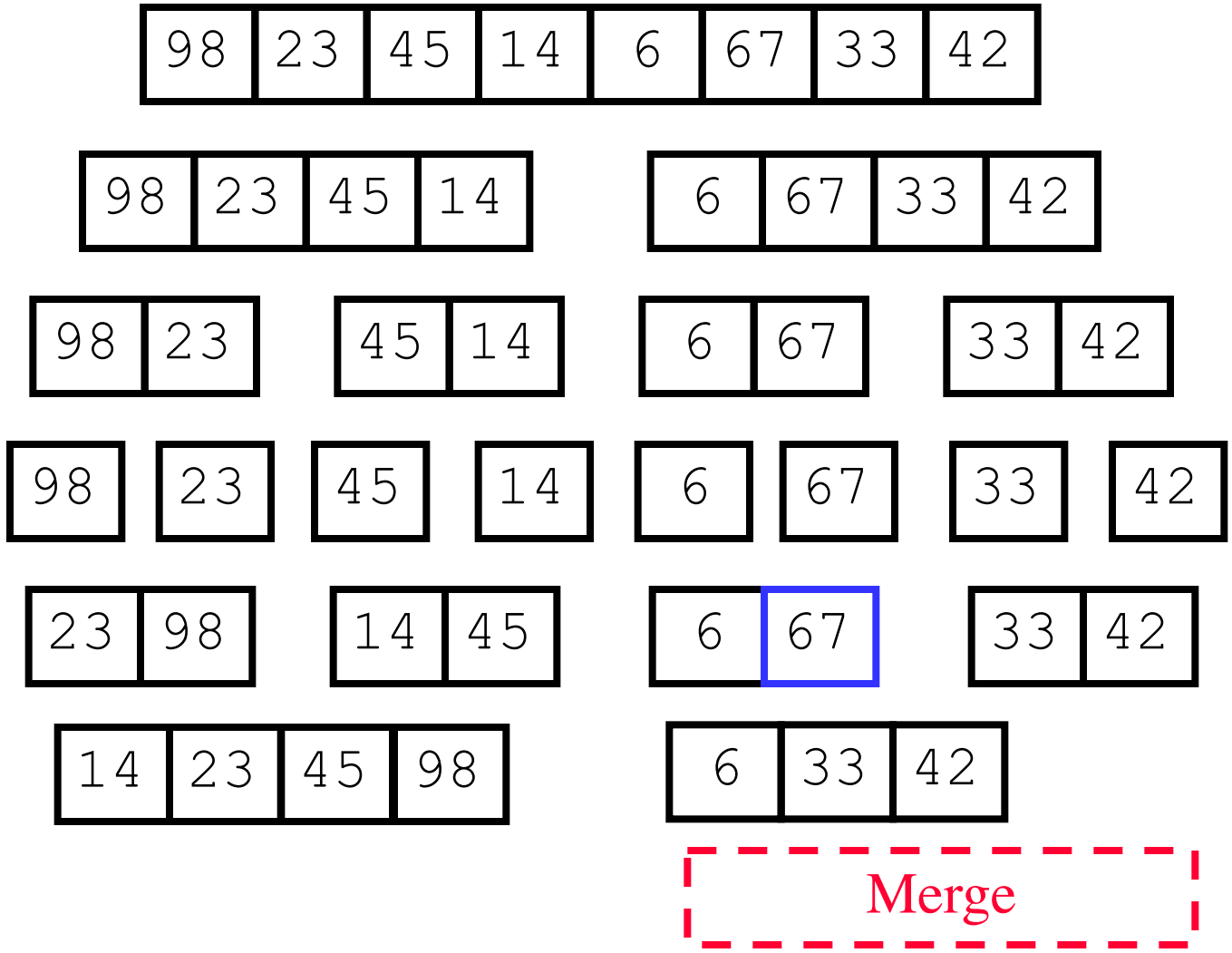

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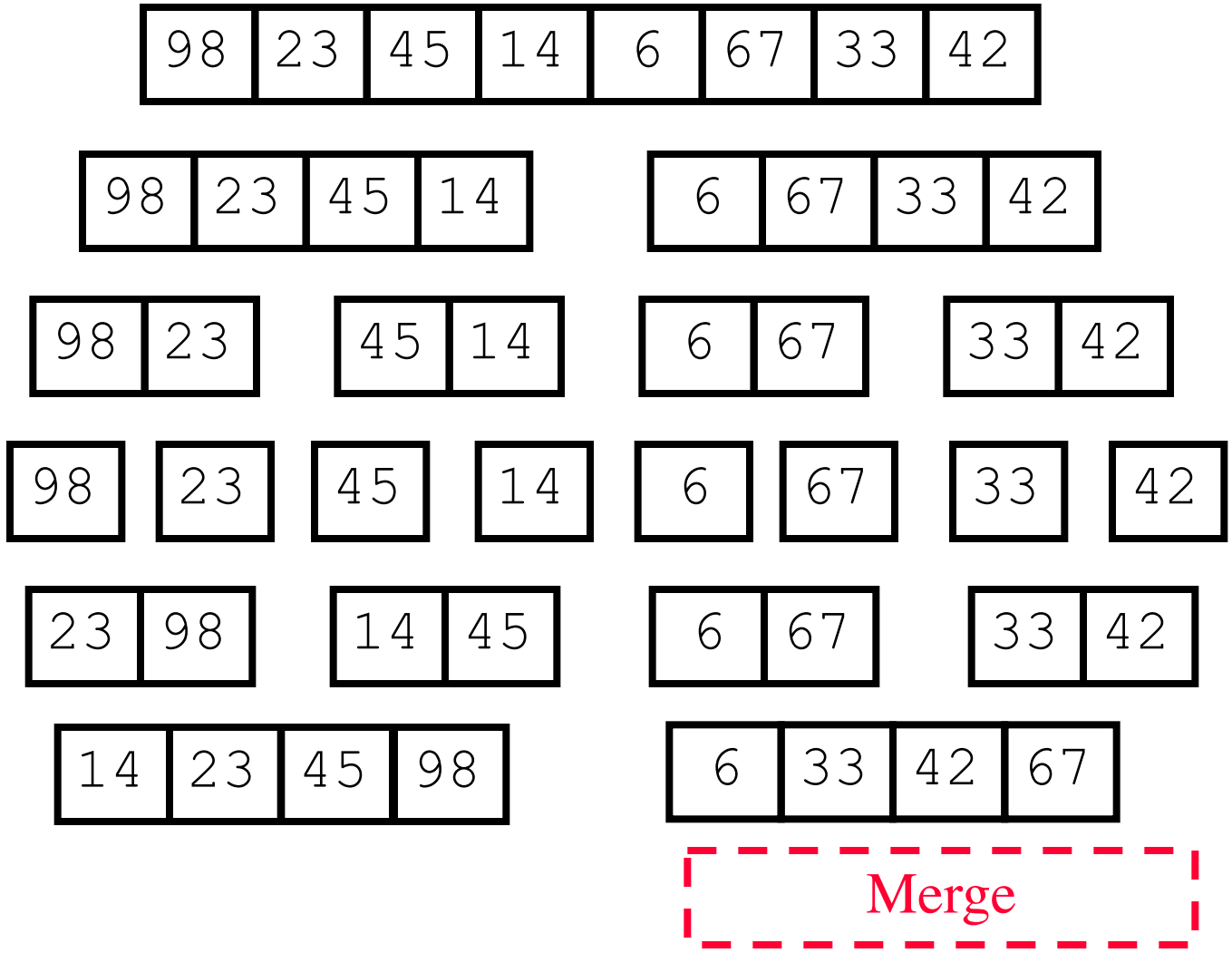

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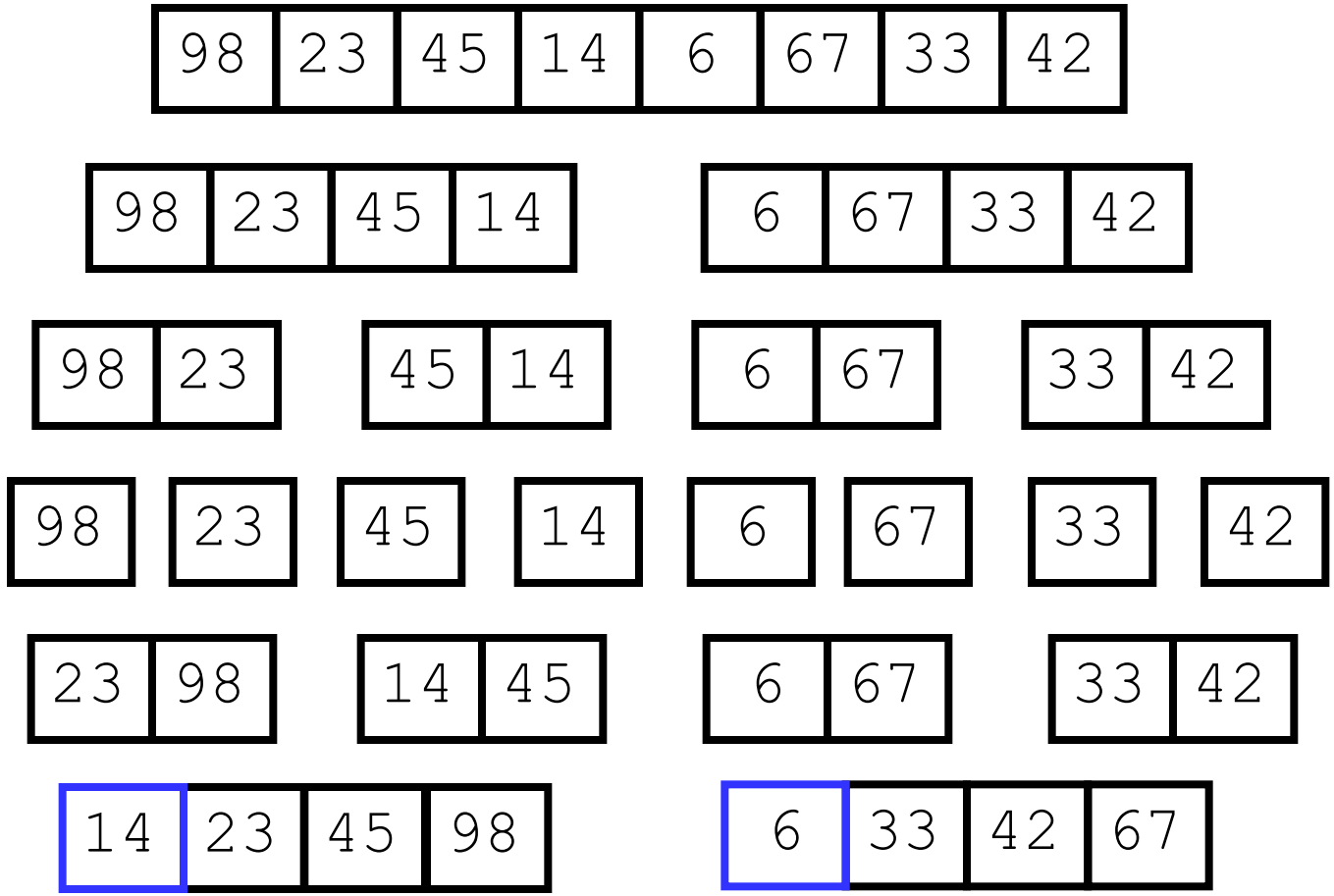

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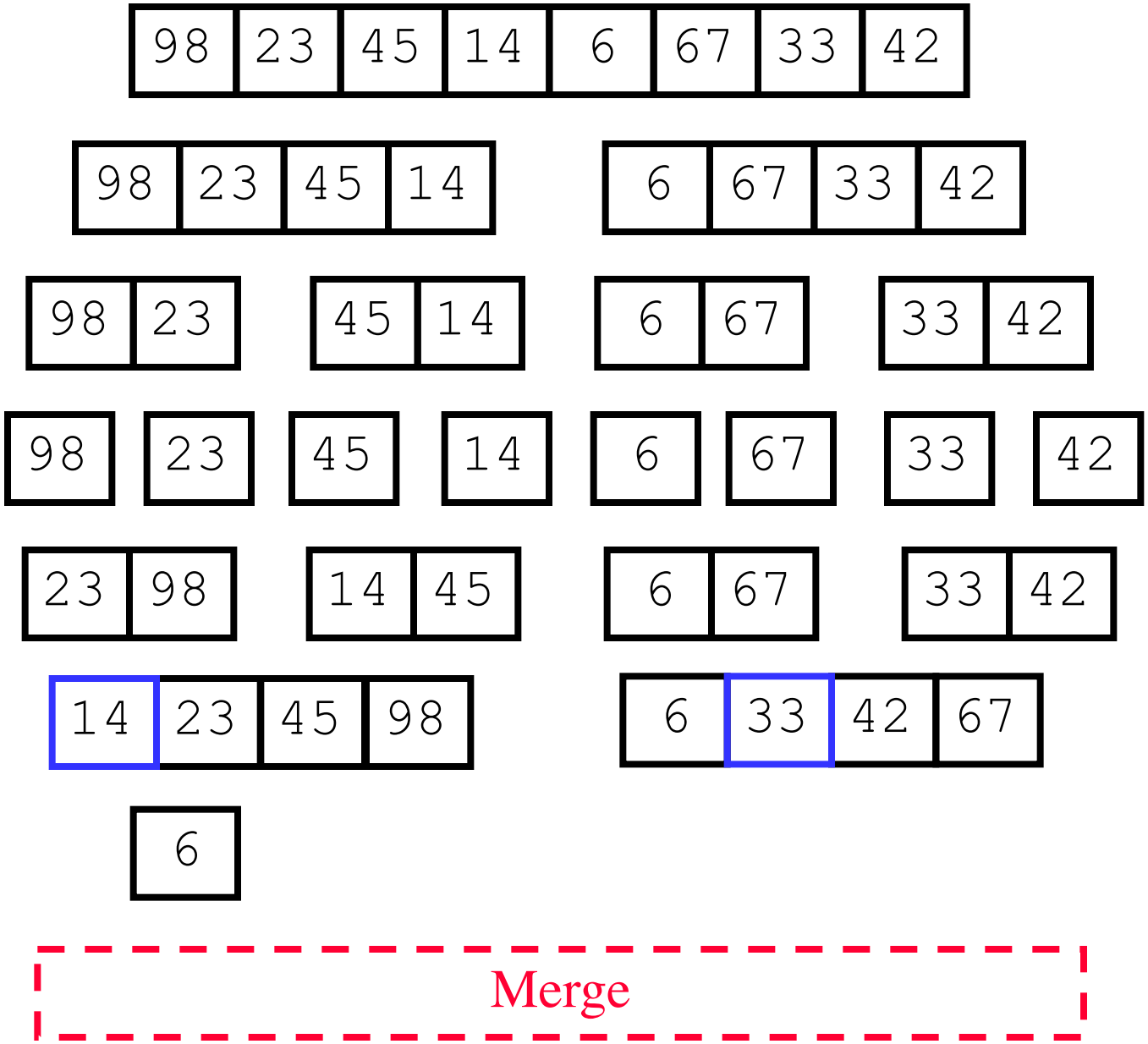

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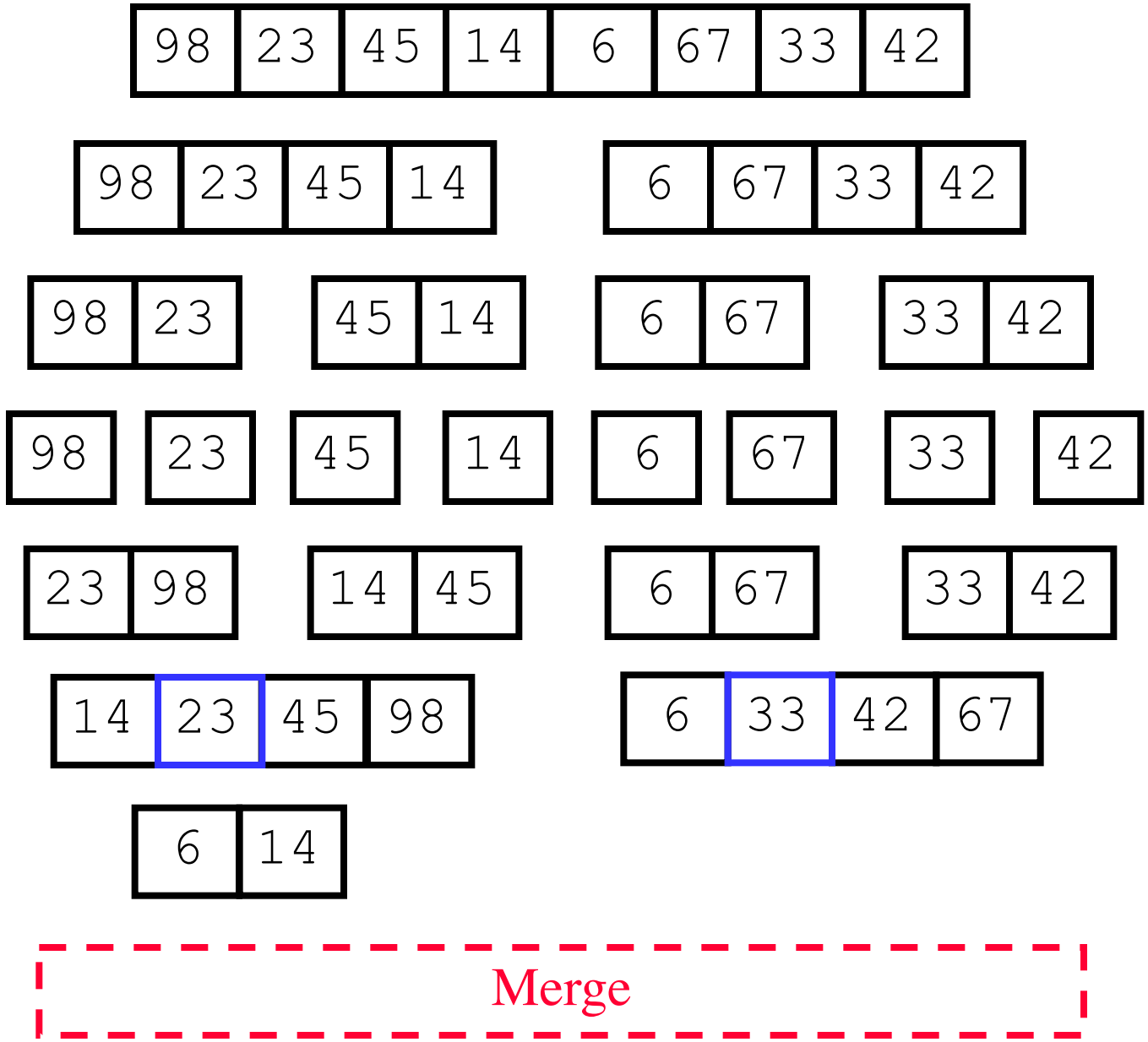

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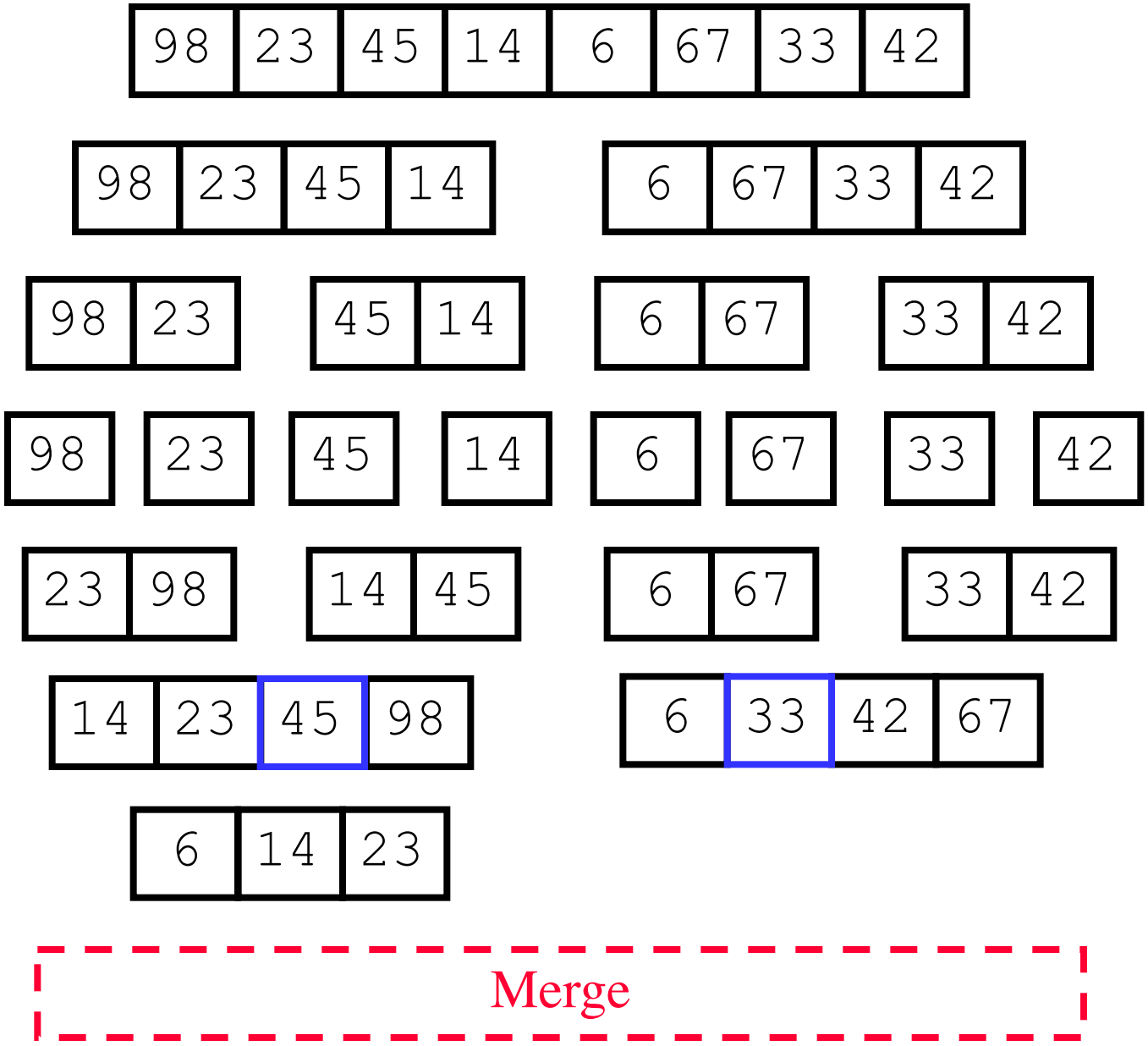

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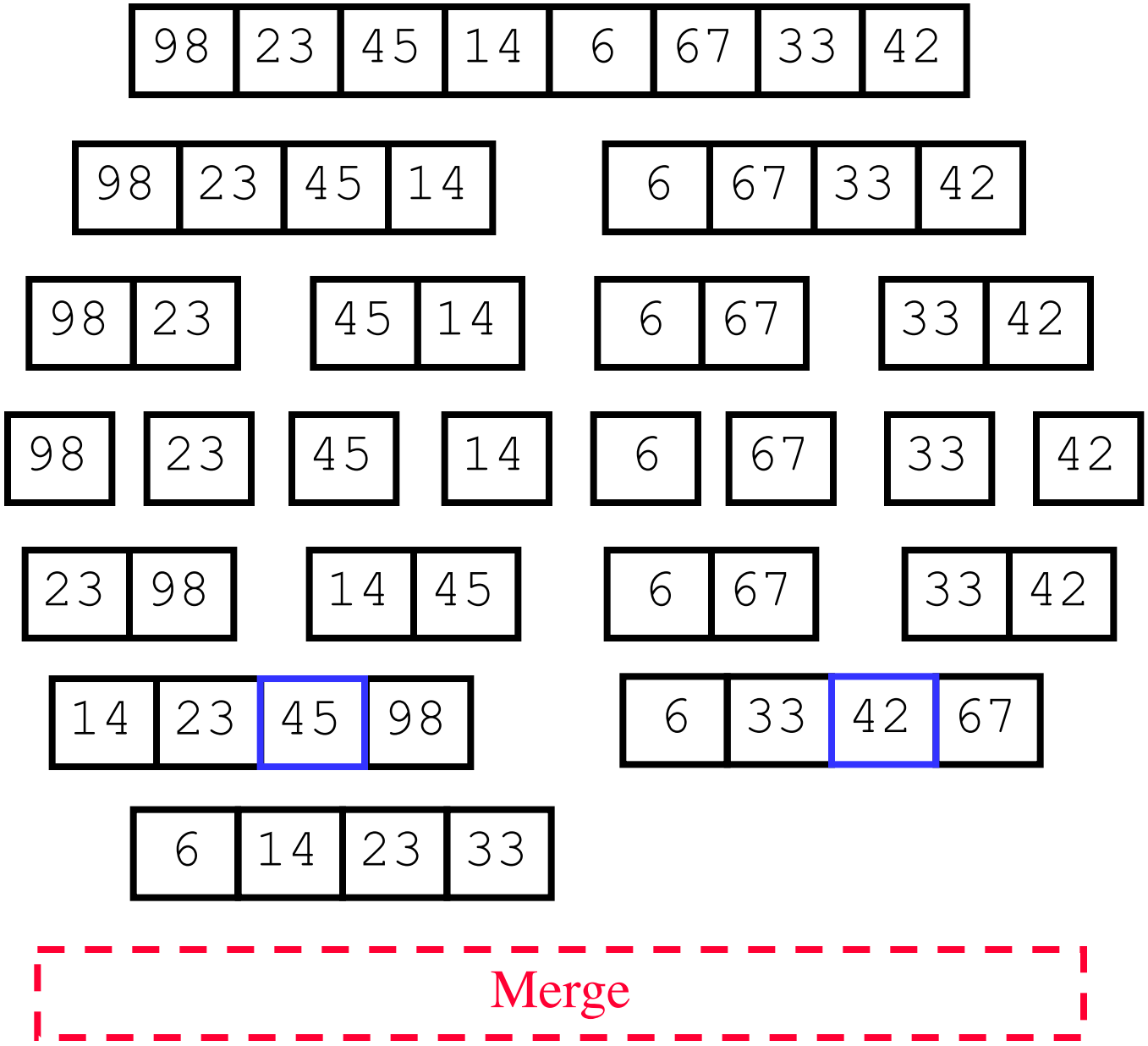

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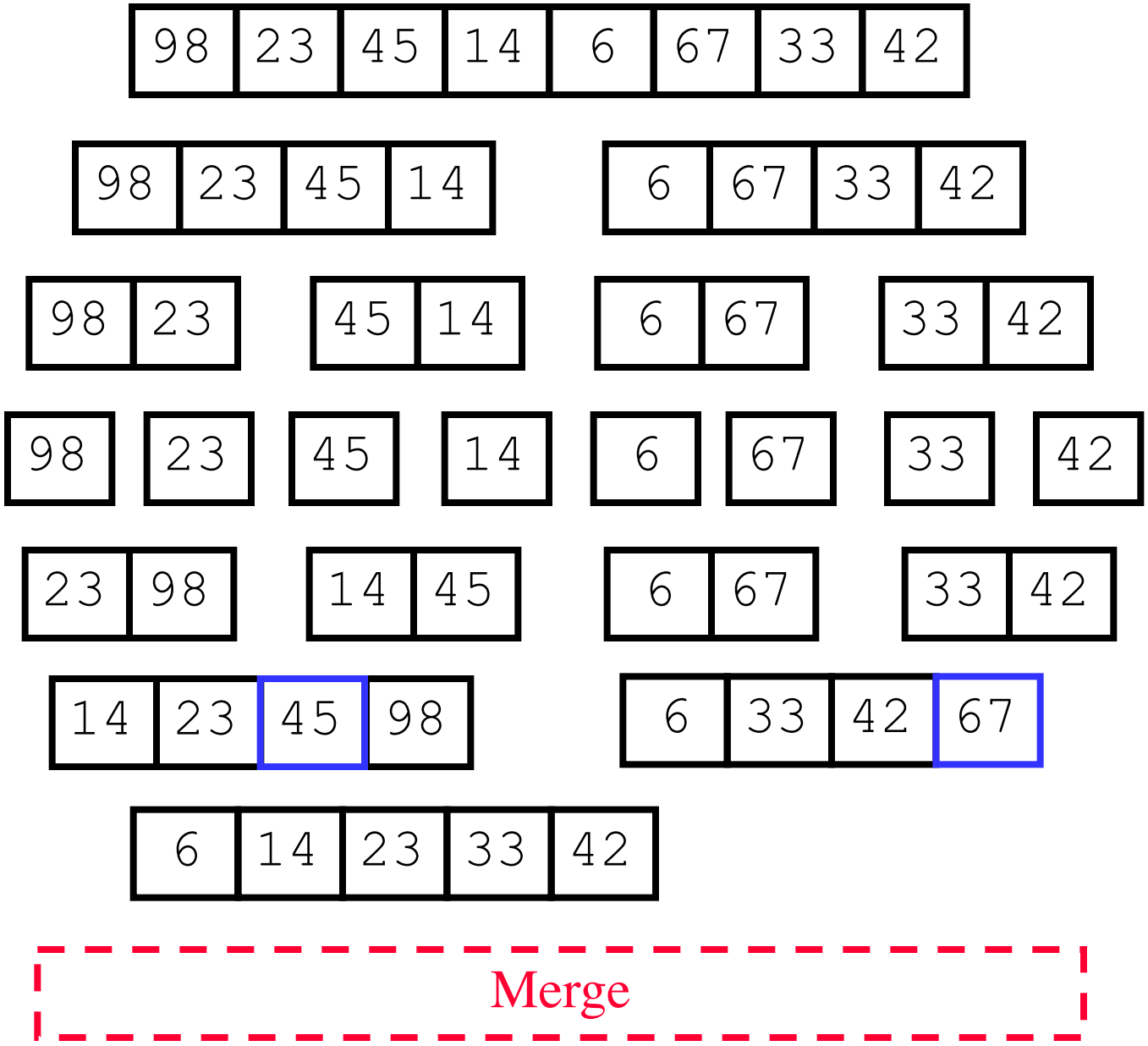

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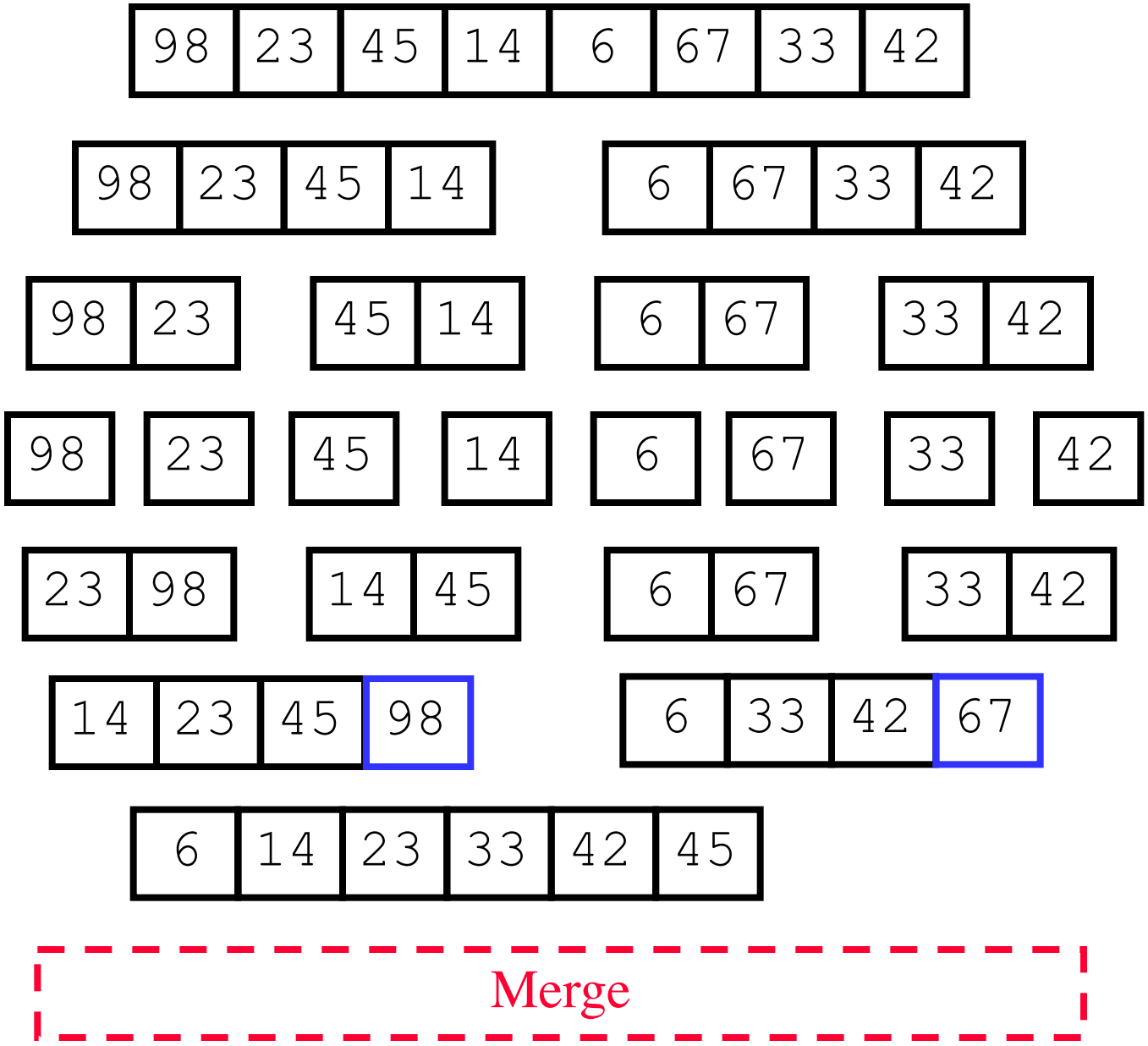
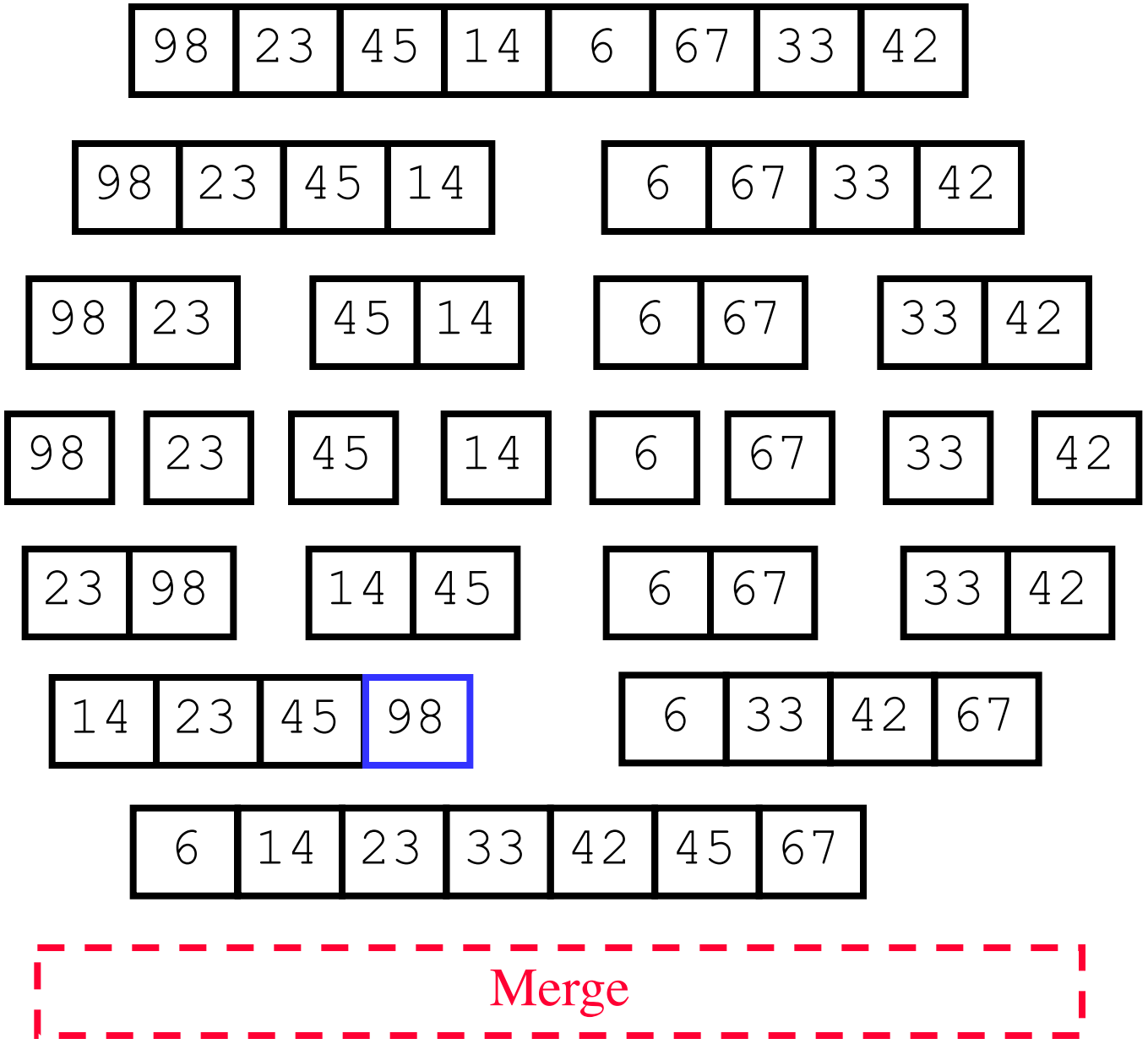
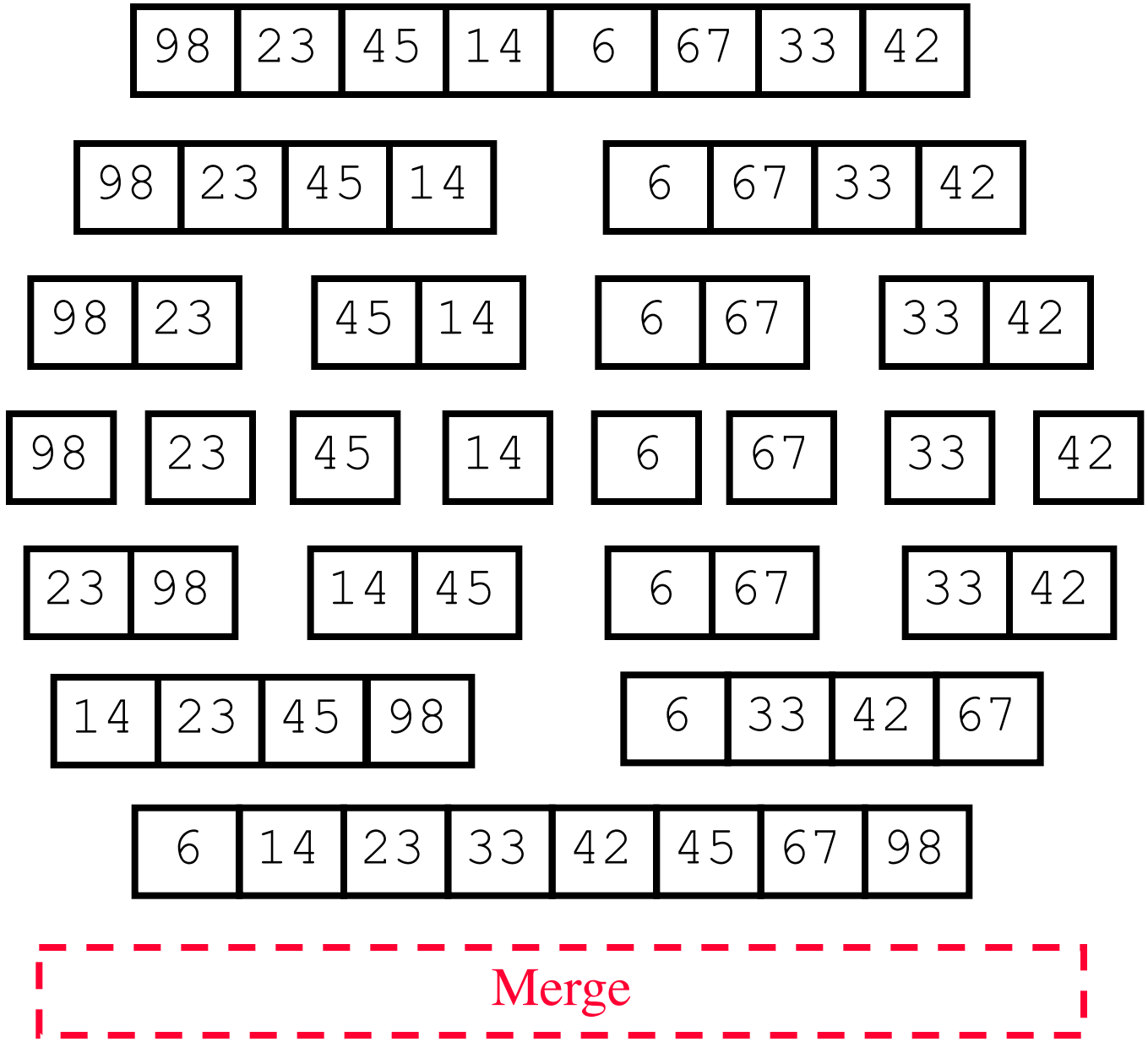


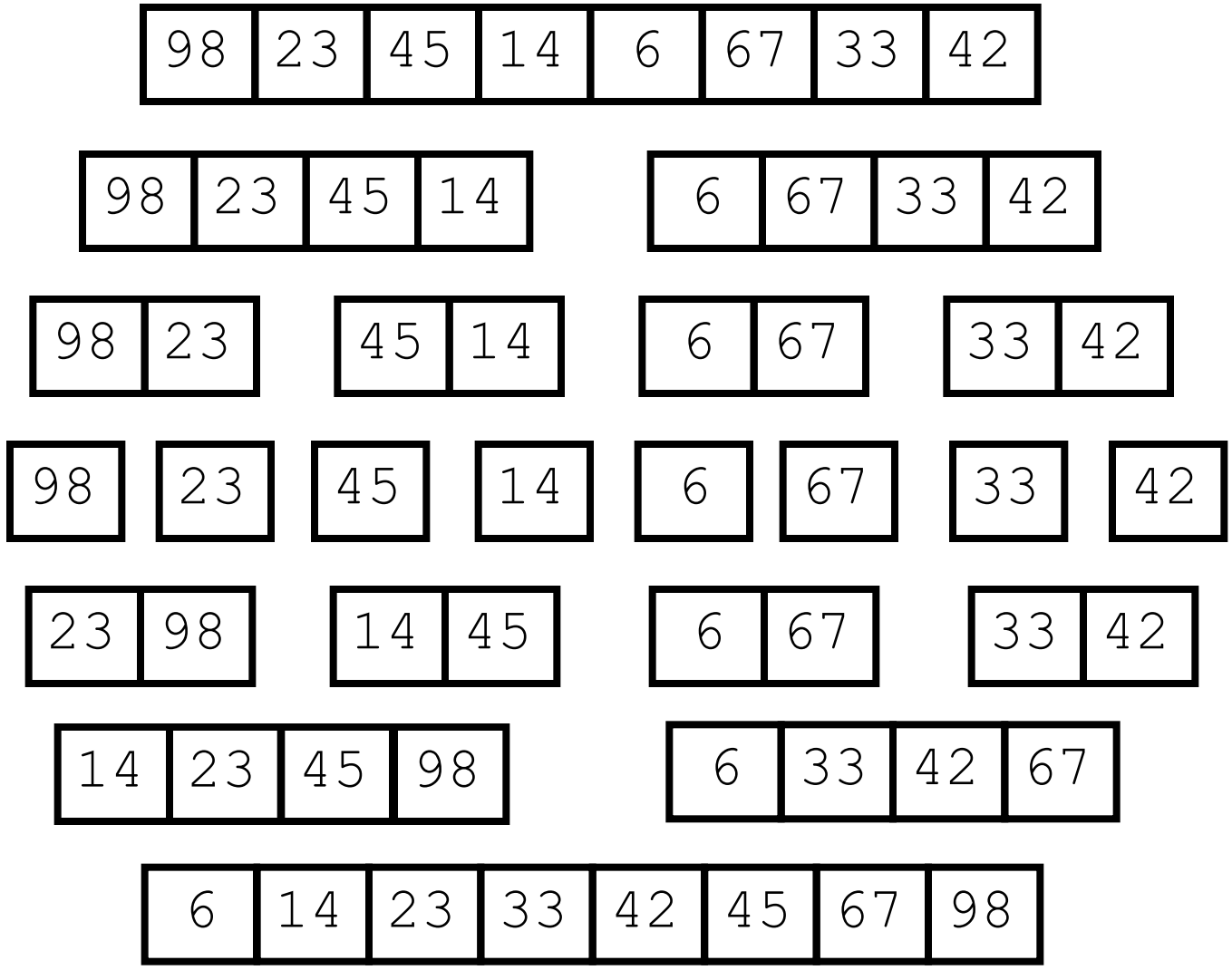
MERGE SORT EXAMPLE



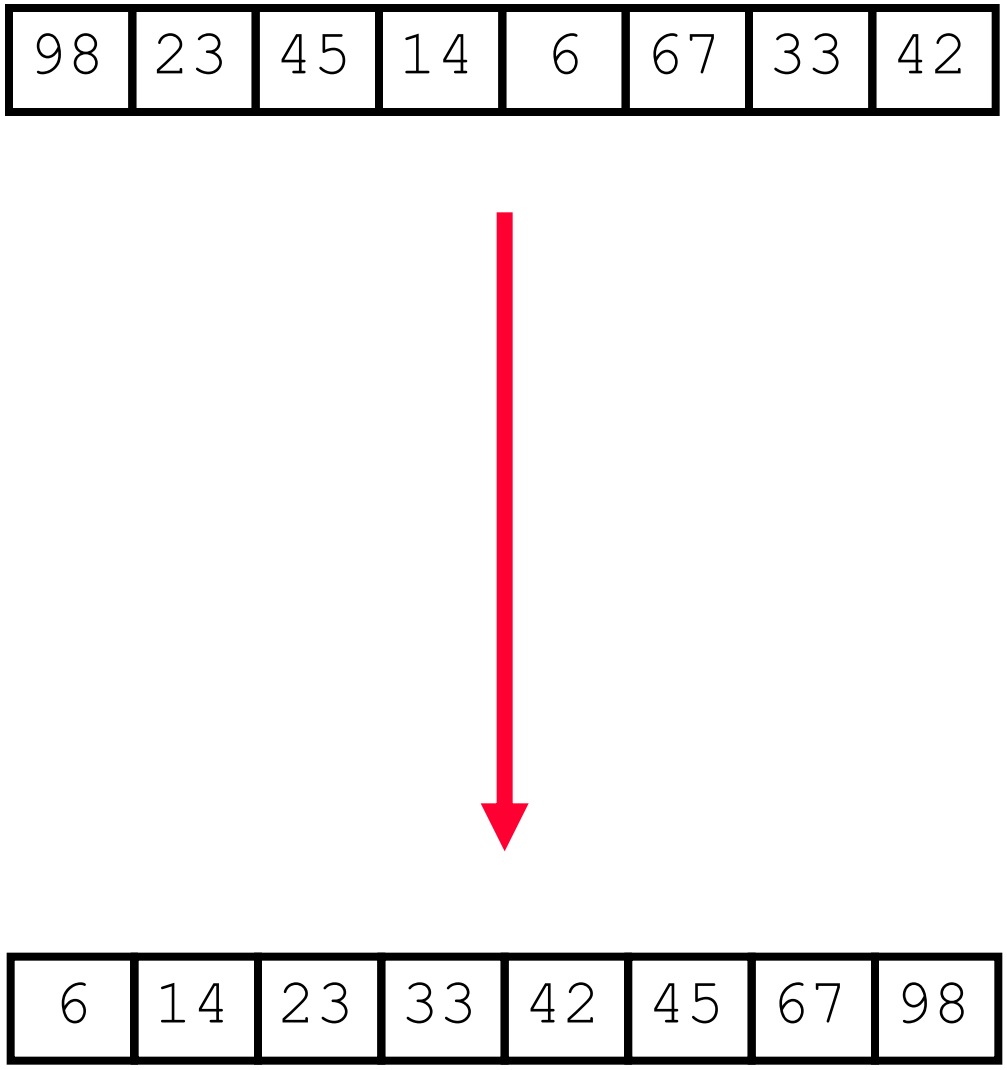
MERGE SORT EXAMPLE



MERGE SORT EXAMPLE

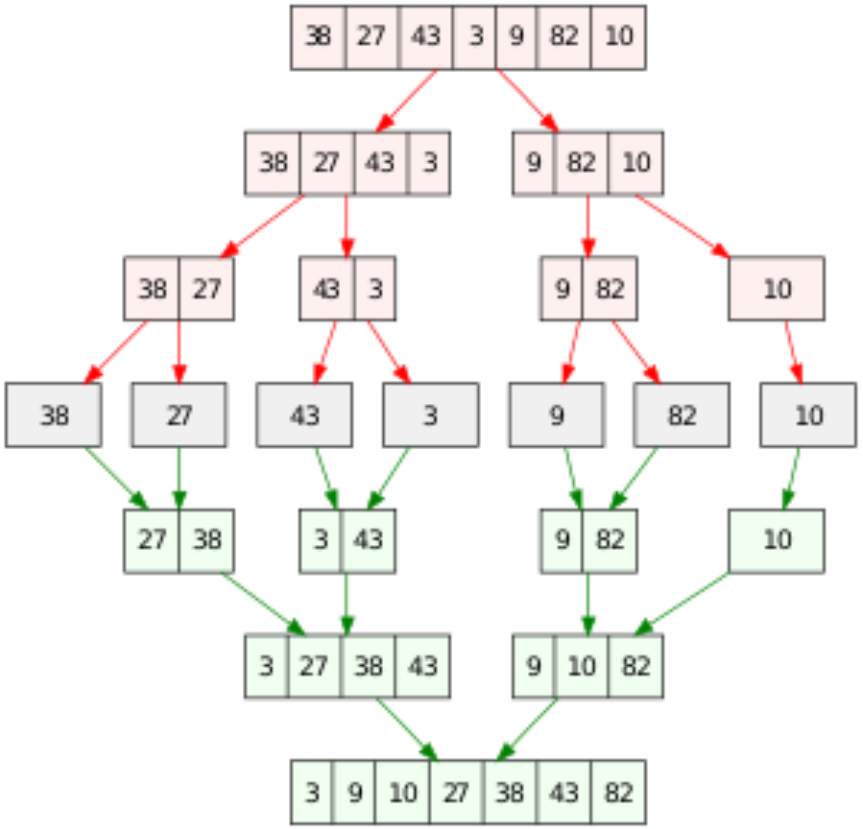


MERGE SORT EXAMPLE



Merge-Sort Memory Requirements

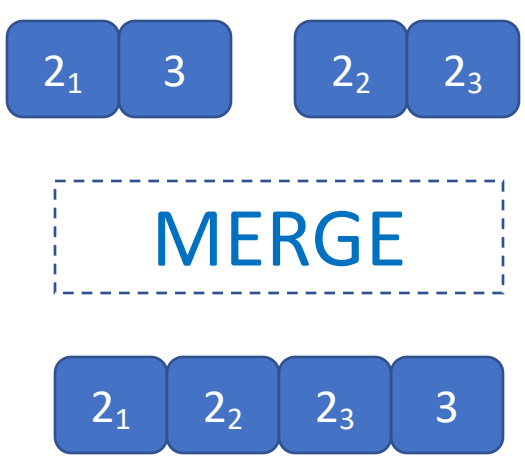
- The memory size of the input must be allocated for the sorted output to be stored.
  - **O(n)** space complexity
- In-place merge sort algorithm exist (quite cumbersome)



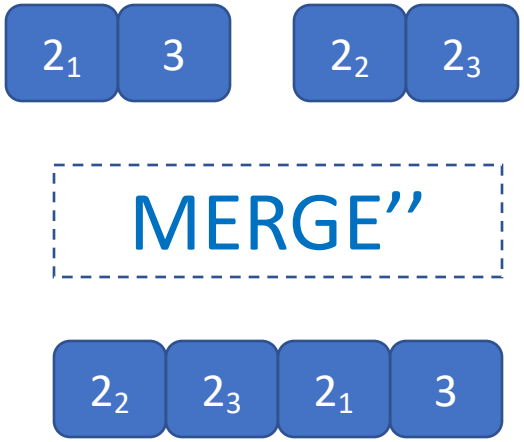
https://en.wikipedia.org/wiki/Merge_sort

Is Merge Sort Stable?

Example



STABILITY OF MERGE SORT?
It depends on the
Merge algorithm
used



Consider:
left array elements
before
right-array elements
if values equal
(the usual)

Consider:
right-array elements
before
left-array elements
if values equal
(alternate version)

Summary

- **Merge Sort** example of **Divide and Conquer Algorithms**
  - **Divide** the unsorted array **into two**
  - Until the sub-arrays only **contain one element**
  - Then **merge the sub-problem solutions** together
- It taken **O(n)** extra memory to sort
- Stability **depends** on the **merge** algorithm


That's all Folks!
Any Question ?