Quick Sort

Instructor: Krishna Venkatasubramanian

CSC 212

Partly based on slides from Prof. Carolina Ruiz's CS2223 at WPI

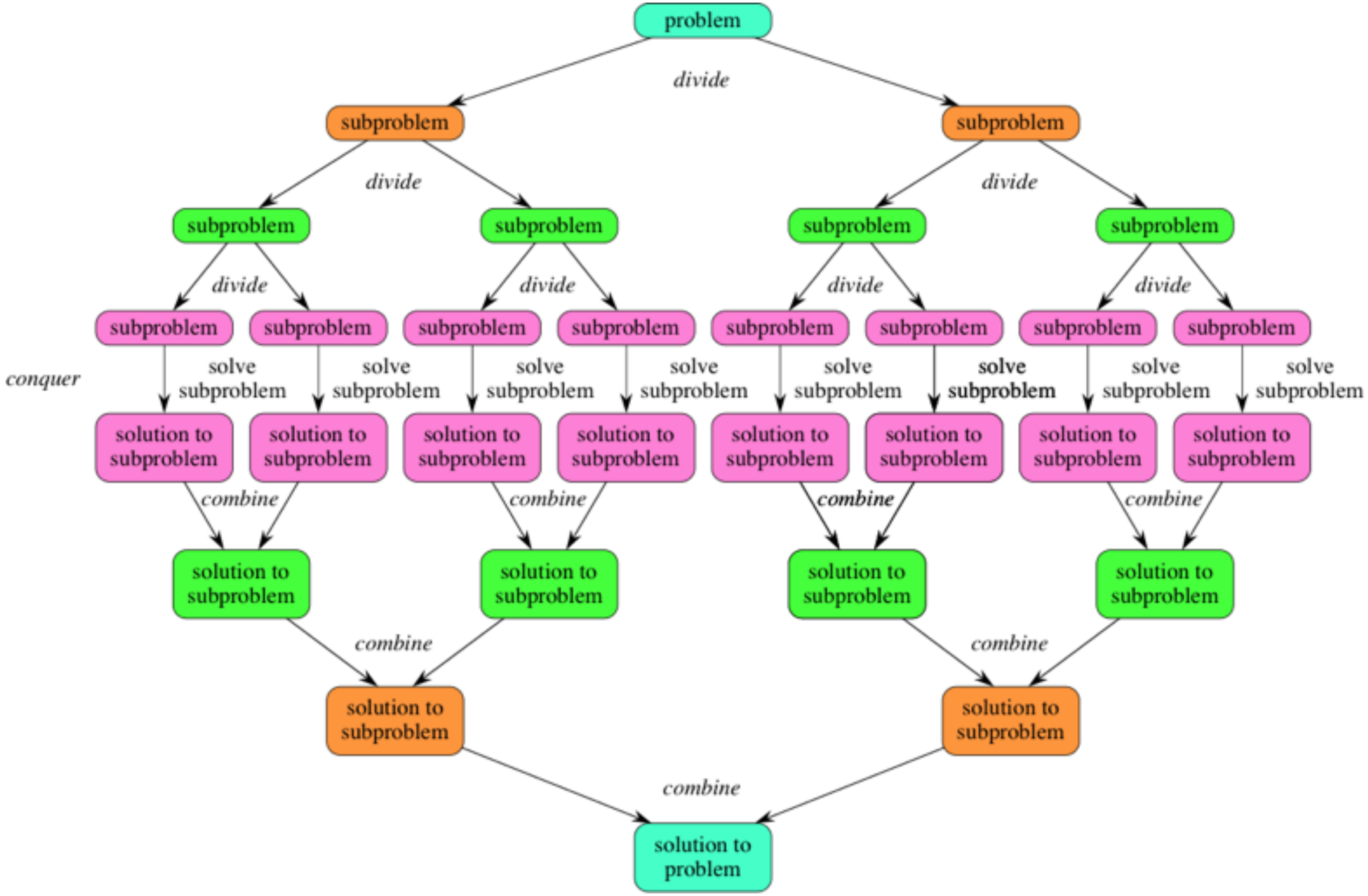
Divide and Conquer Algorithms

- **Divide:** Break the larger problem into sub-problems that are smaller instances of the same problem

- **Conquer:** the sub-problems are solved ***recursively***
  - If the sub-problem is really small, then solve in a straight-forward manner

- **Combine:** combine the solutions of the sub-problems to find the solution of the original problem!

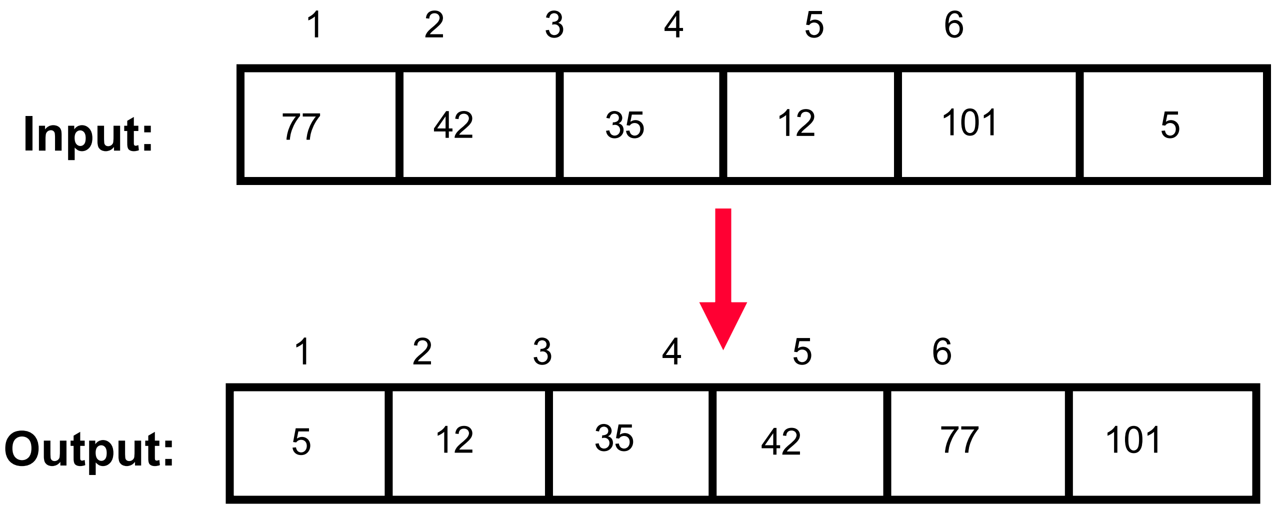
Visually Speaking



https://www.khanacademy.org/computing/computer-science/algorithms/merge-sort/a/divide-and-conquer-algorithms

Sorting: Problem Definition

- **Sorting takes an unordered collection and makes it an ordered one.**



Input:

1	2	3	4	5	6
77	42	35	12	101	5

Output:

1	2	3	4	5	6
5	12	35	42	77	101

How can we sort an array using divide and conquer approach?

Sorting Algorithms

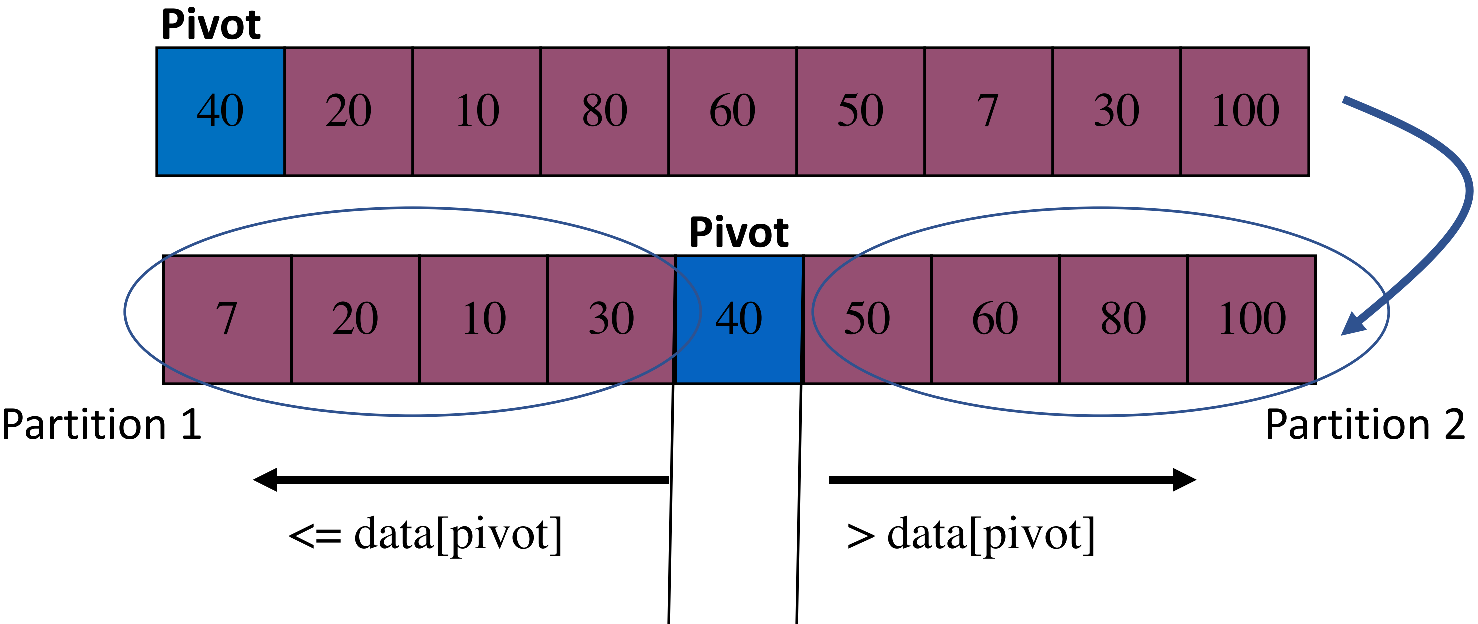
- *Insertion Sort --- covered already*
- *Bubble Sort --- covered already*
- *Selection Sort --- covered already*
- *Merge Sort --- covered already*
- **Quick Sort**
- Linear-Time Sort
- Heap Sort
- …

Quicksort Algorithm

```
QuickSort(A,left,right)
  if right-left +1 == 1
    return
  else
    pivot = Partition(A,left,right)
    QuickSort(A,left, pivot)
    QuickSort(A,pivot+1,right)
```

How to Partition

- Given an array A
  - Pick one element to use as *pivot.*
  - Partition elements into two sub-arrays:
    - Elements **less than or equal to pivot**
    - Elements **greater than pivot**



Pivot

40	20	10	80	60	50	7	30	100

Pivot

7	20	10	30	40	50	60	80	100

Partition 1

Partition 2

<= data[pivot]

> data[pivot]

Partition Example

We are given array of n integers to sort:

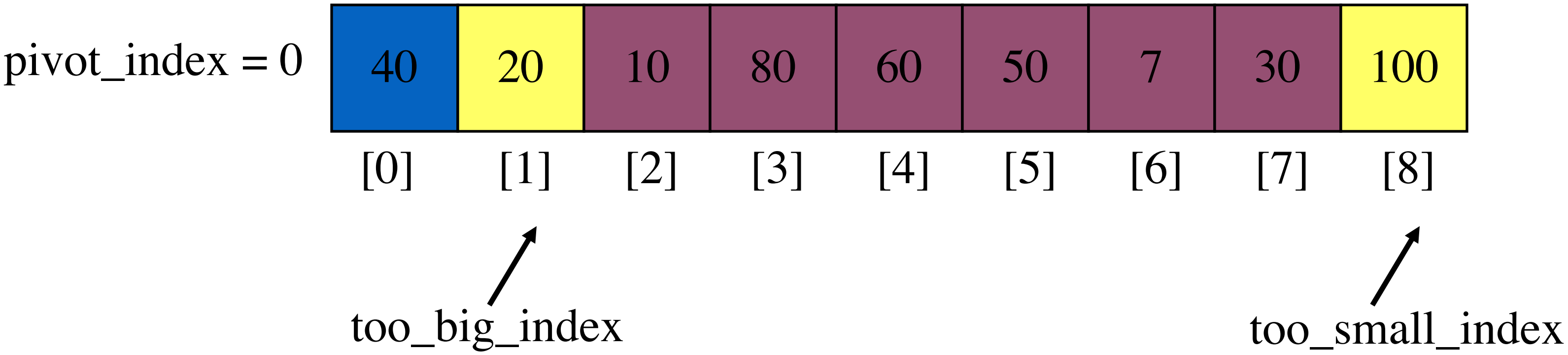
40	20	10	80	60	50	7	30	100

Partition Example

There are a **number of ways to pick the pivot element**. In this example, we **will use the first element** in the array:

40	20	10	80	60	50	7	30	100

Partition Example



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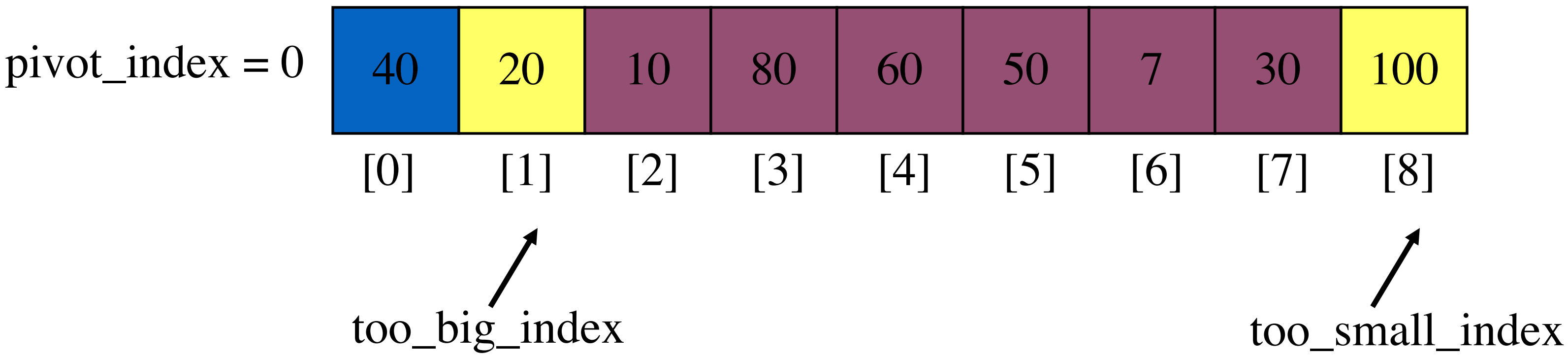

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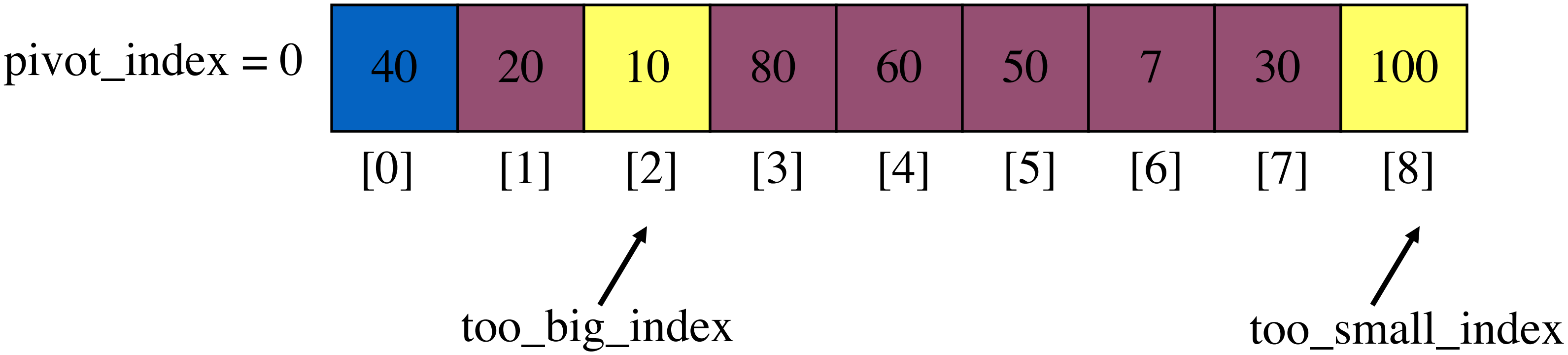

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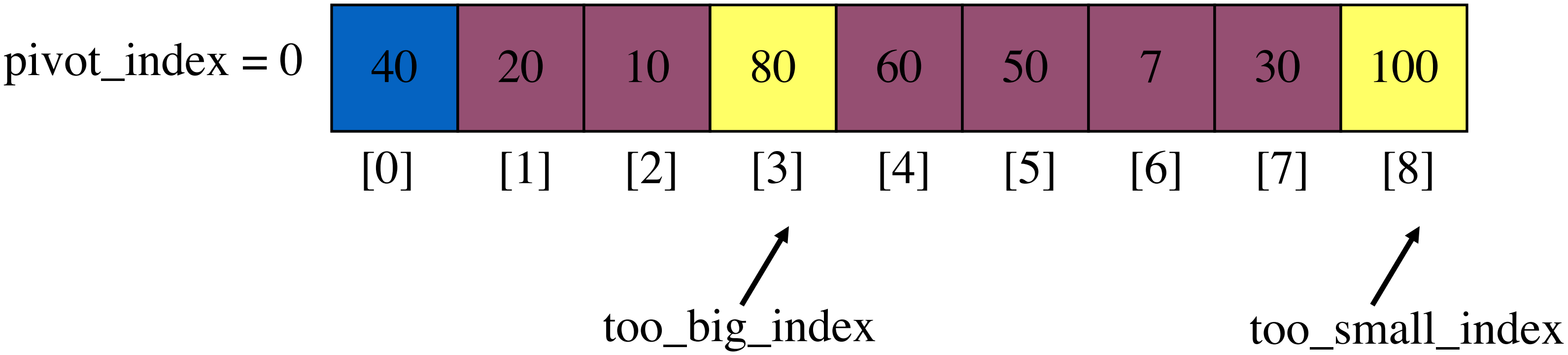

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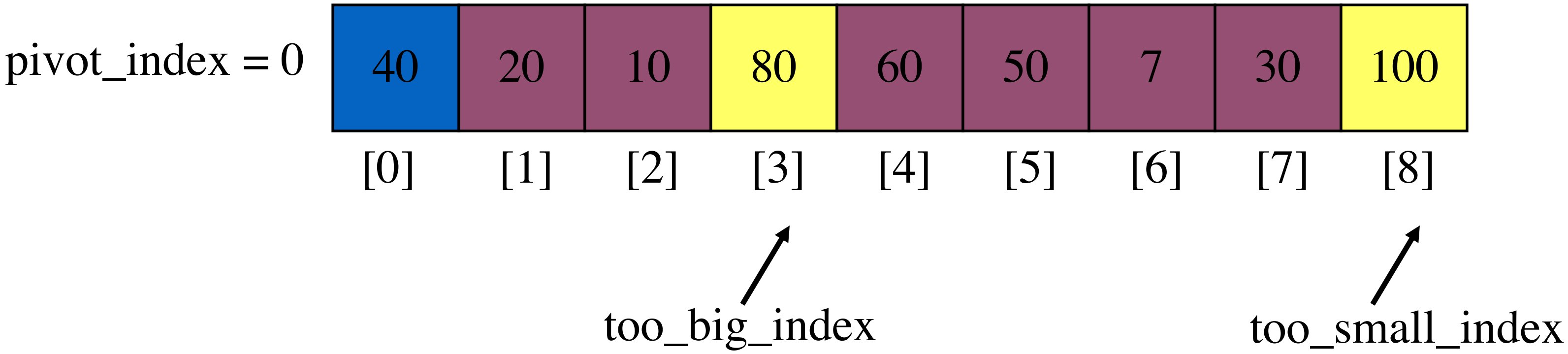

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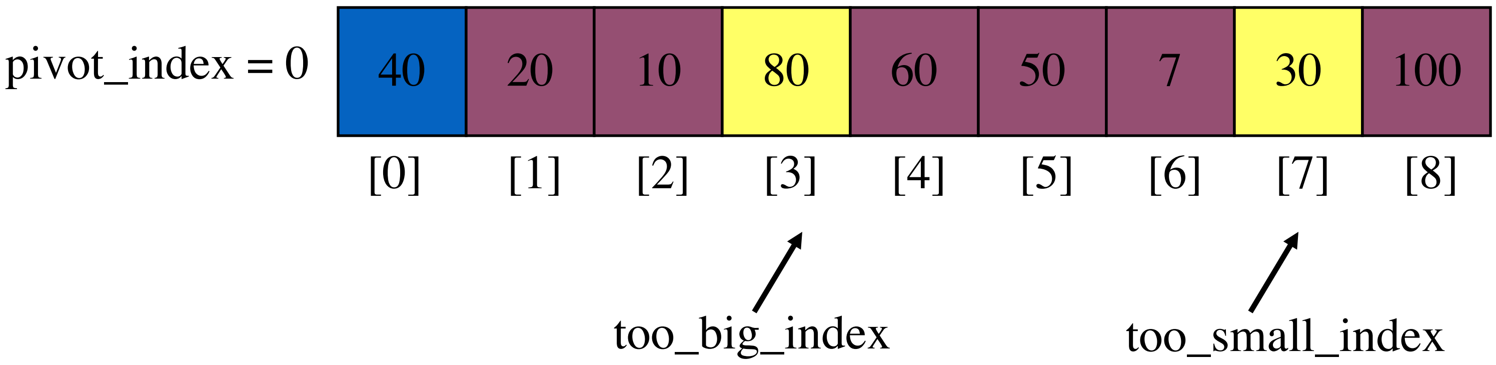

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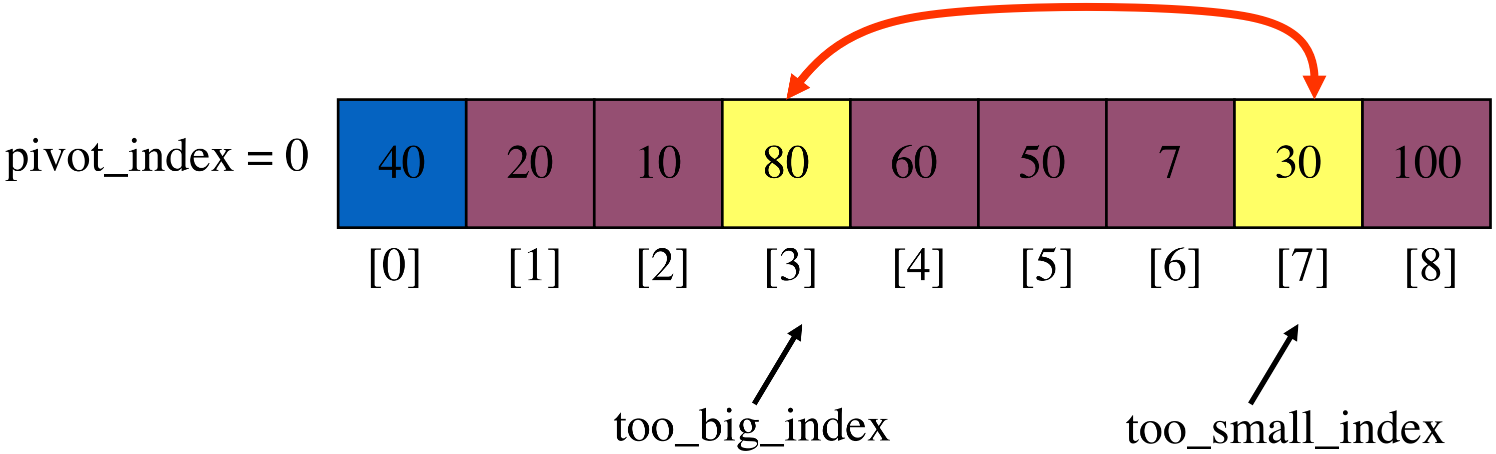

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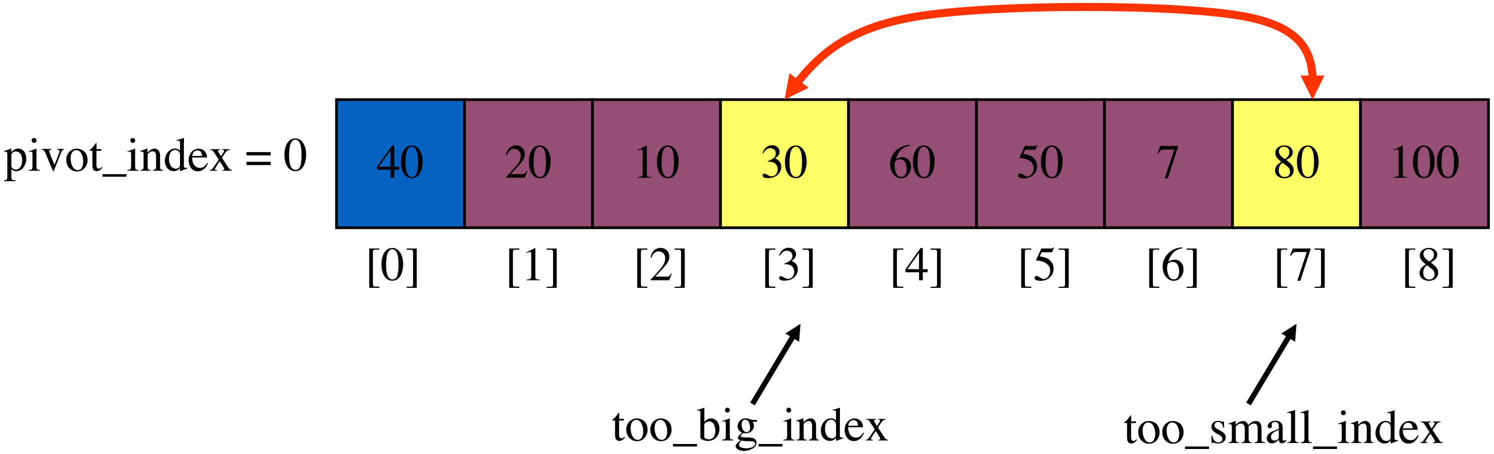

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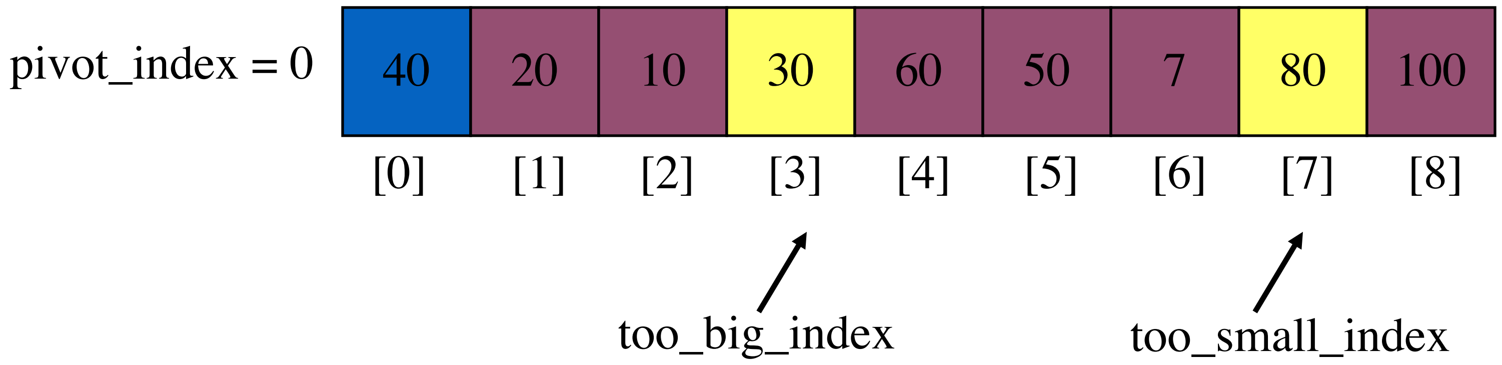

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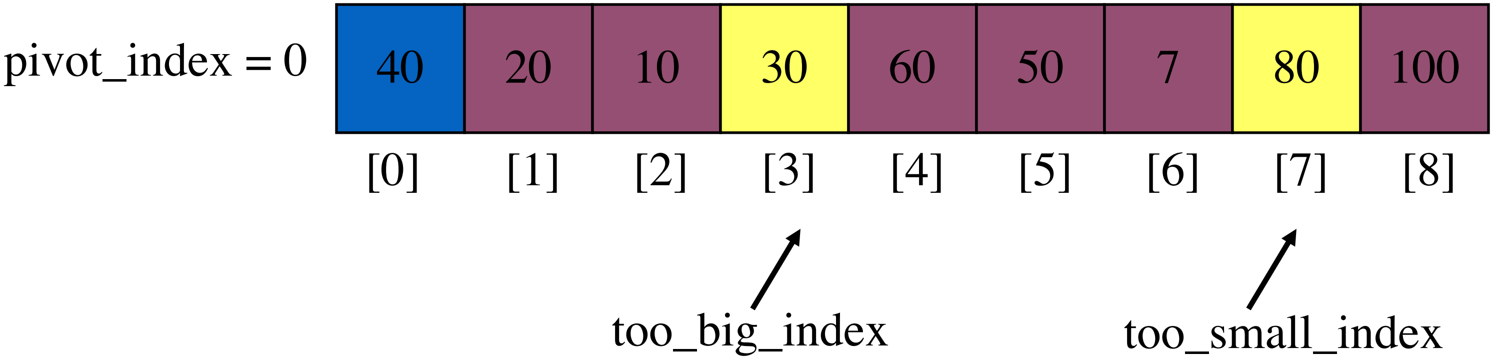

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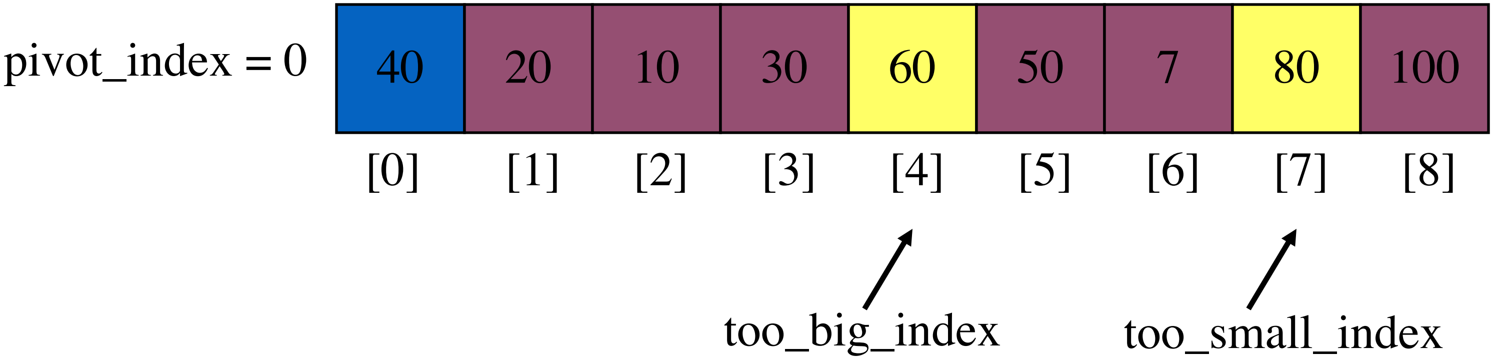

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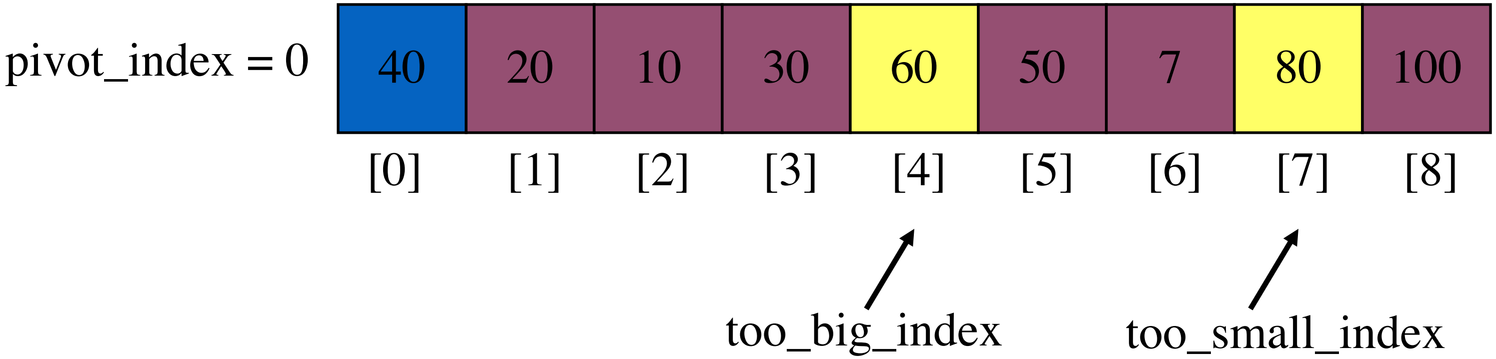

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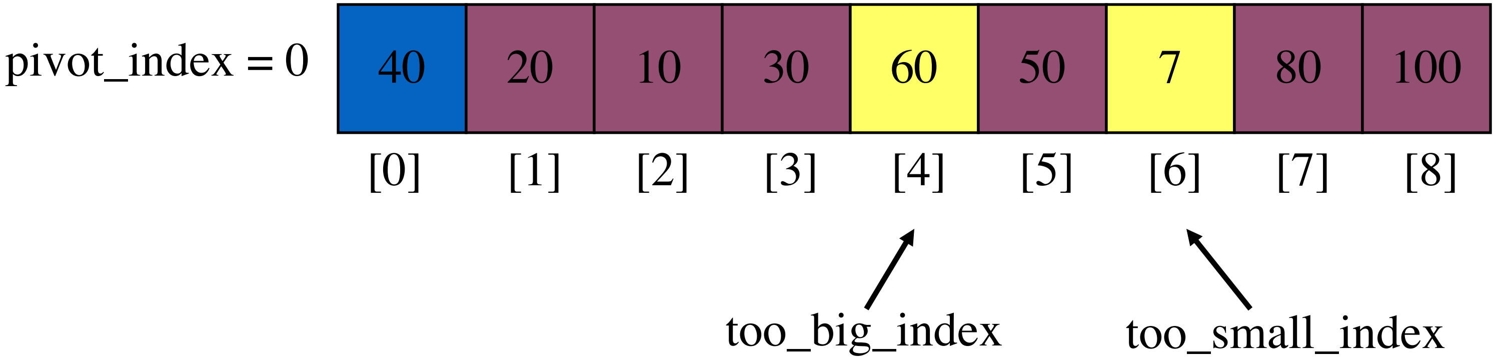

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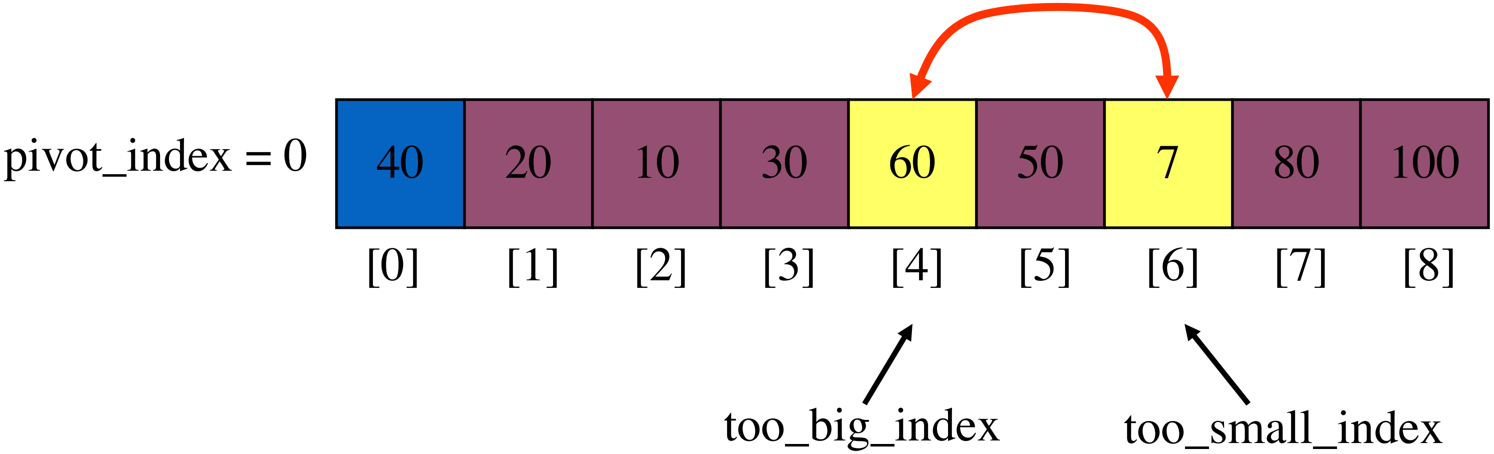

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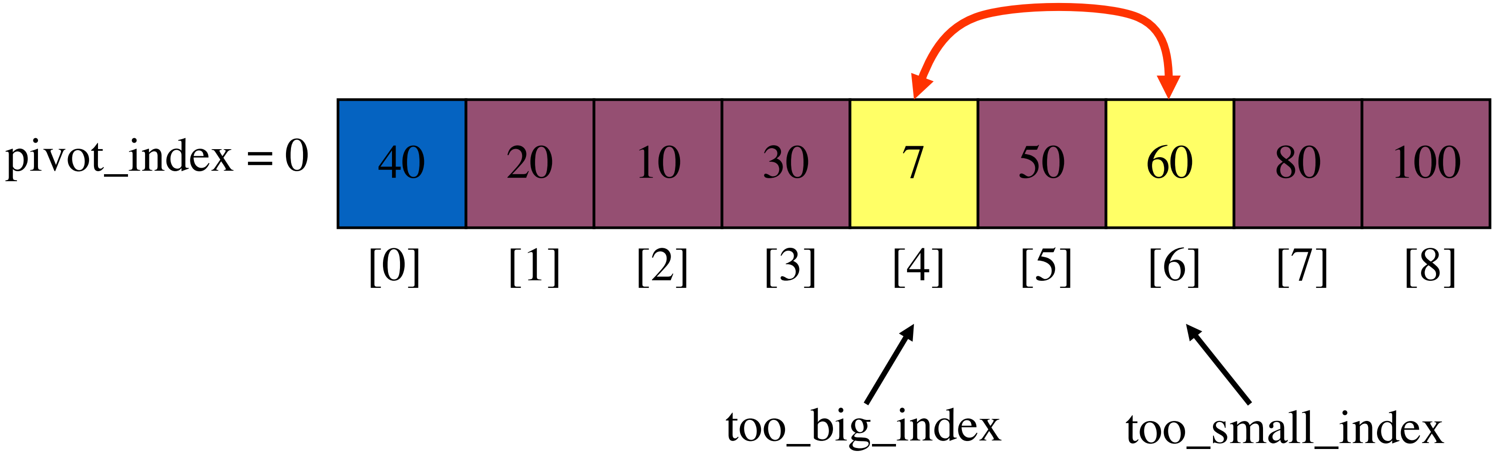

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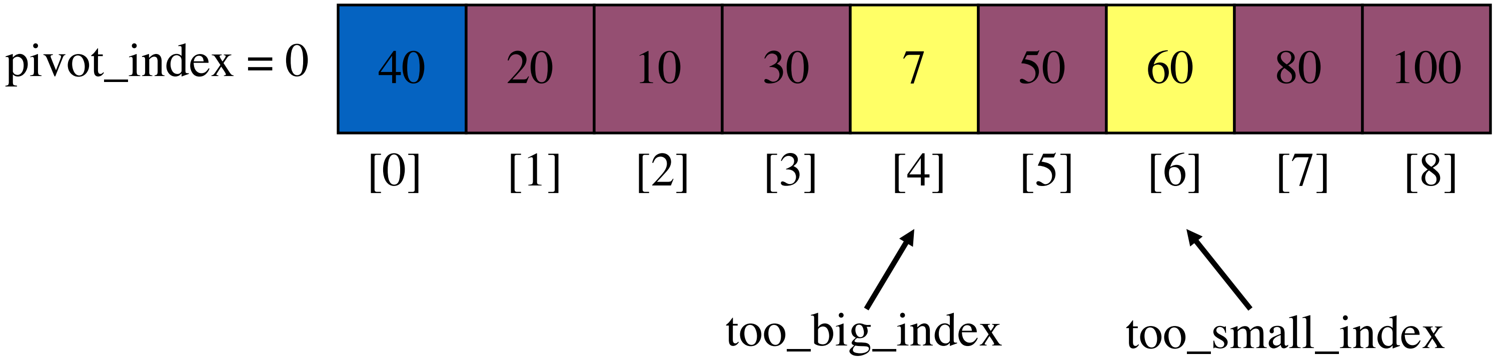

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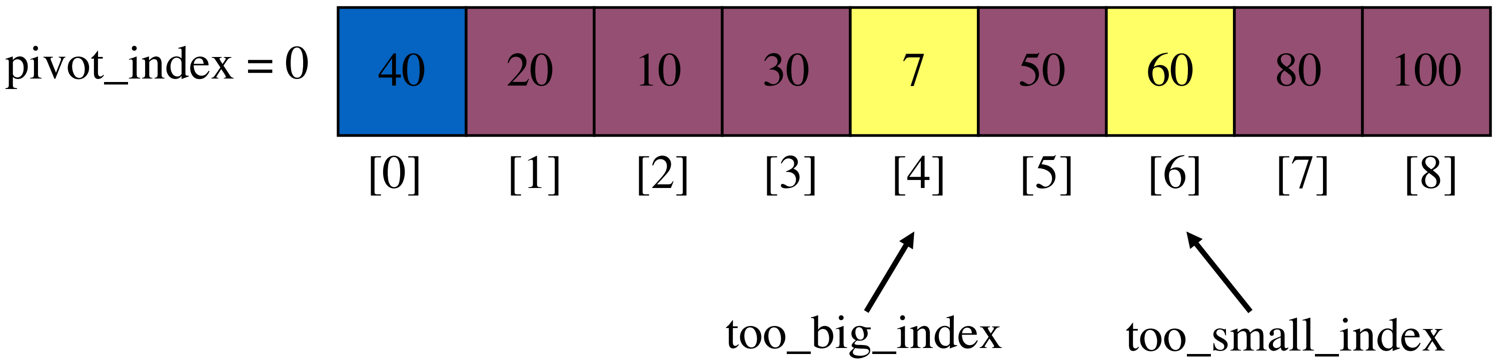

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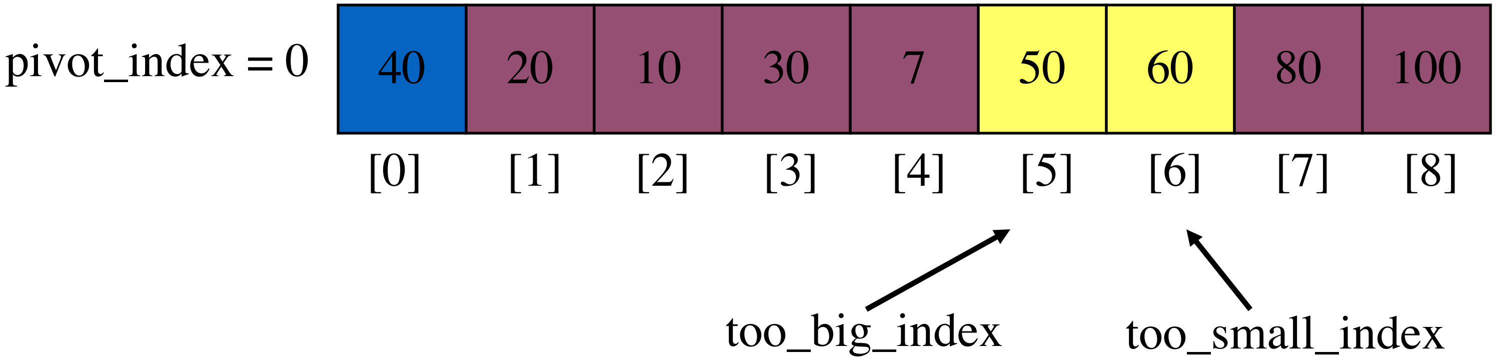

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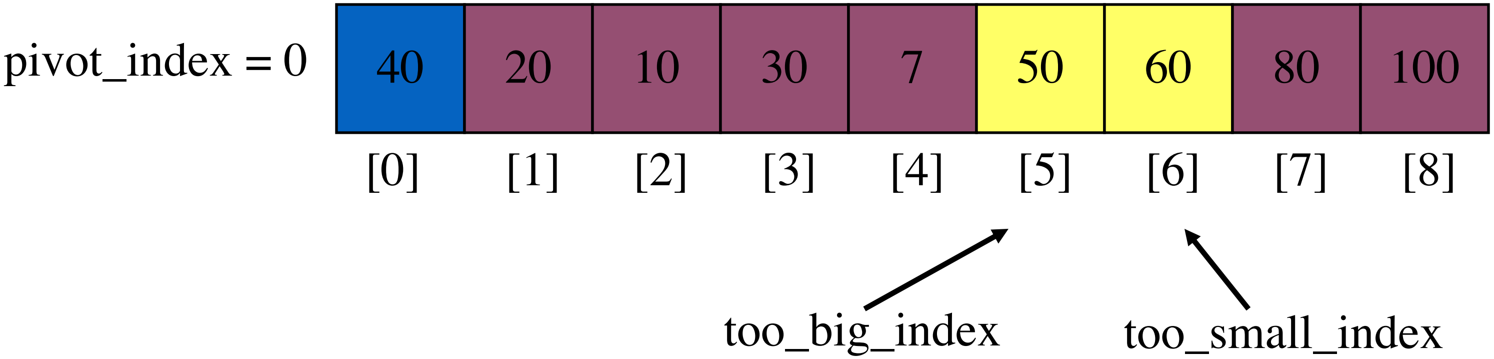

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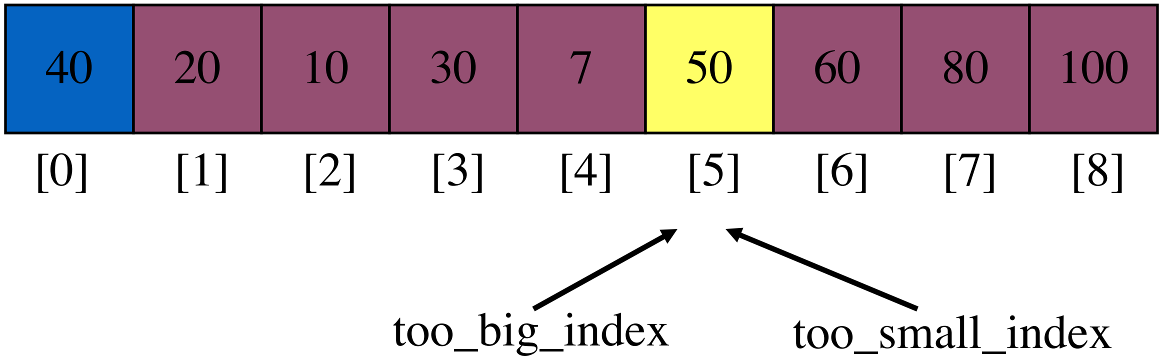

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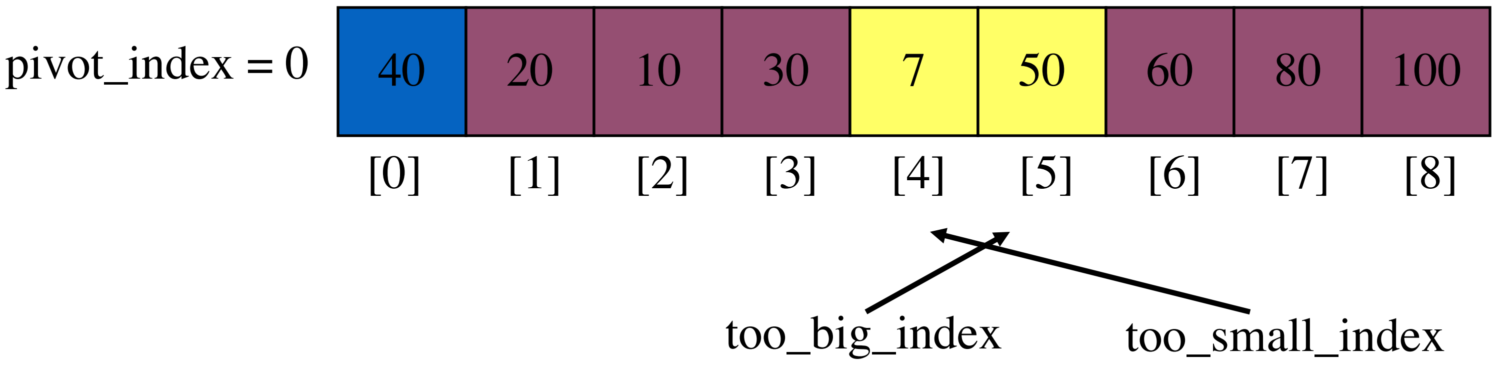

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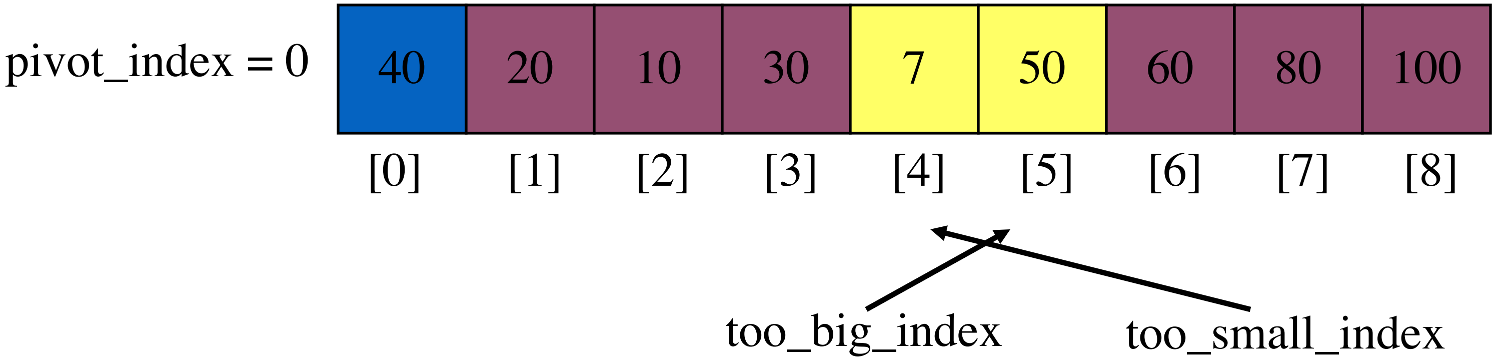

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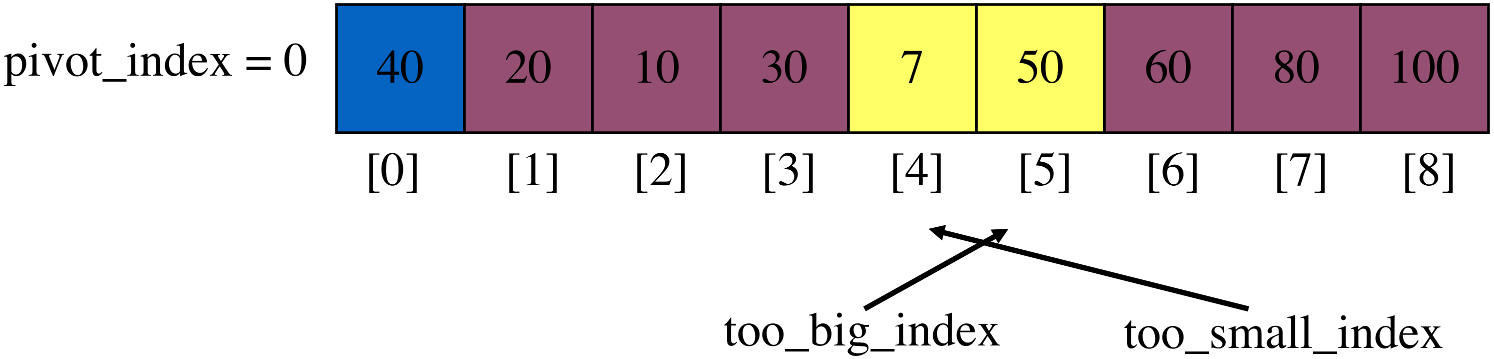

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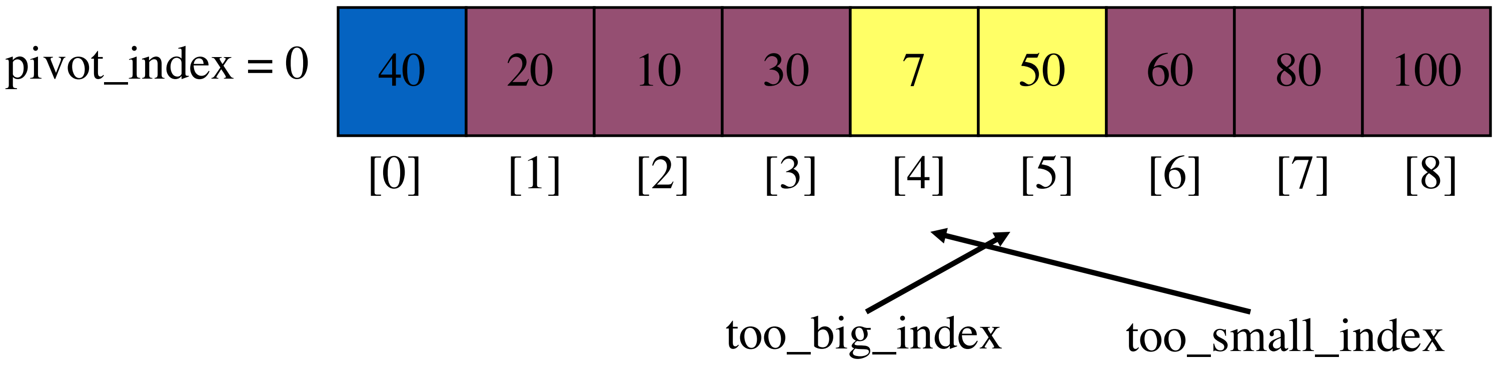

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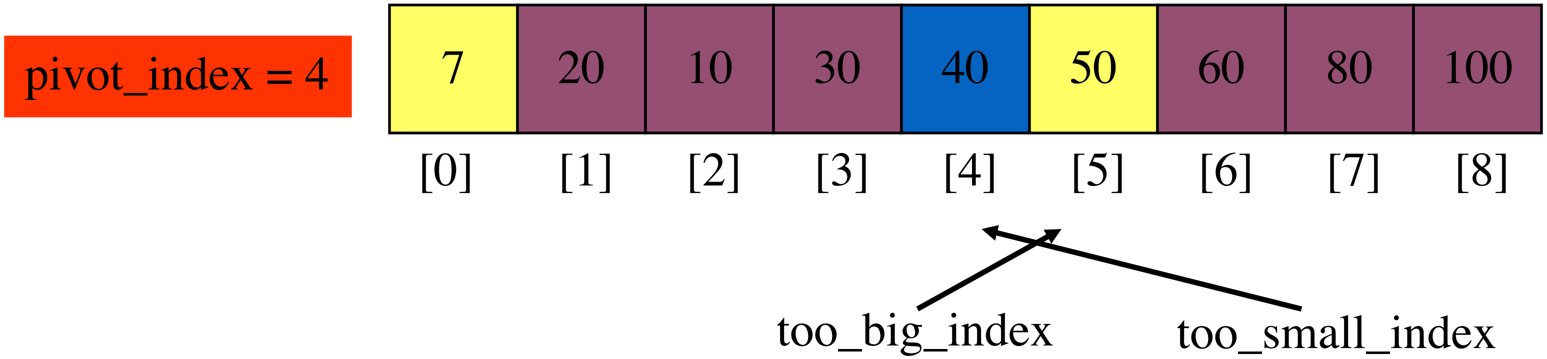


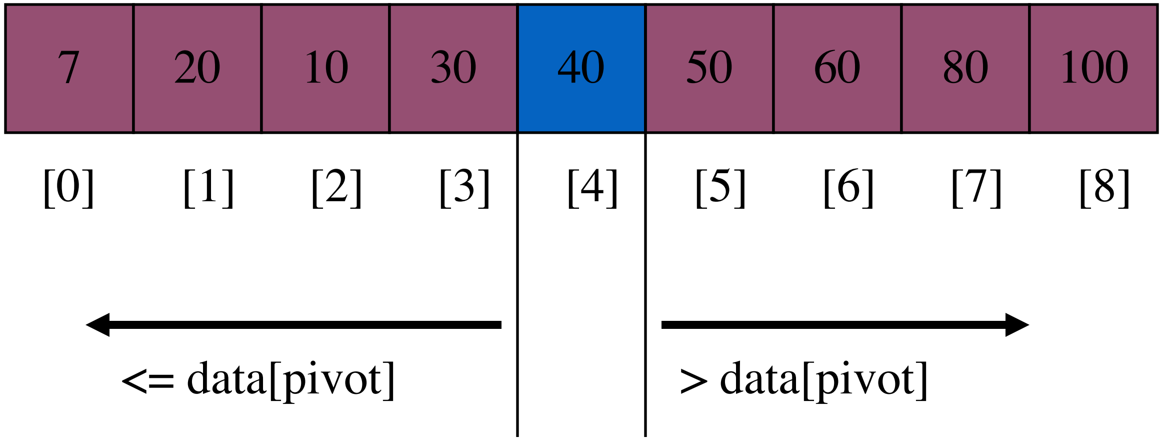
pivot_index = 4

7	20	10	30	40	50	60	80	100
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

too_big_index

too_small_index

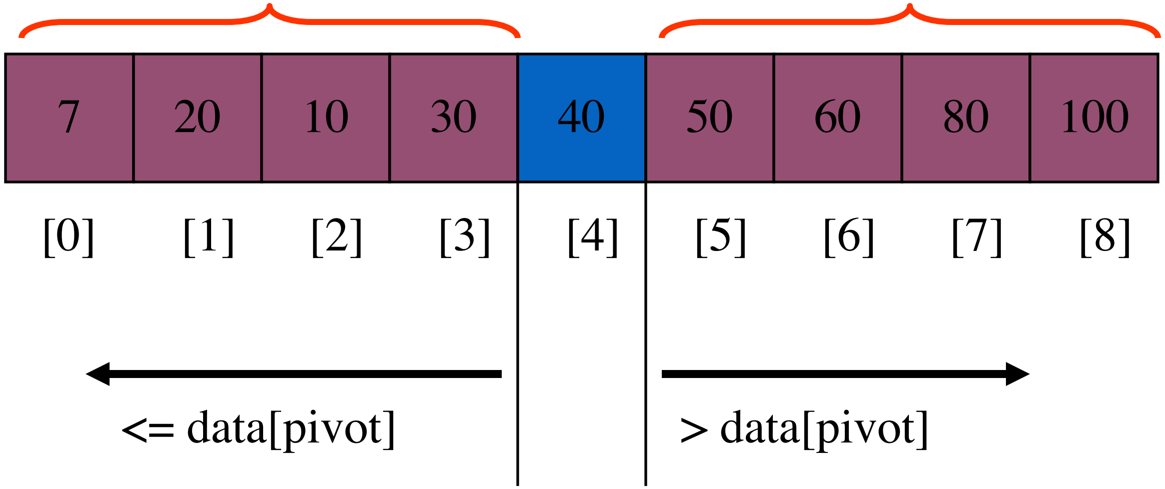
Partition Result



[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
7	20	10	30	40	50	60	80	100

All this is done **in-place**, and does not require extra memory

Recursion: Quicksort Sub-arrays

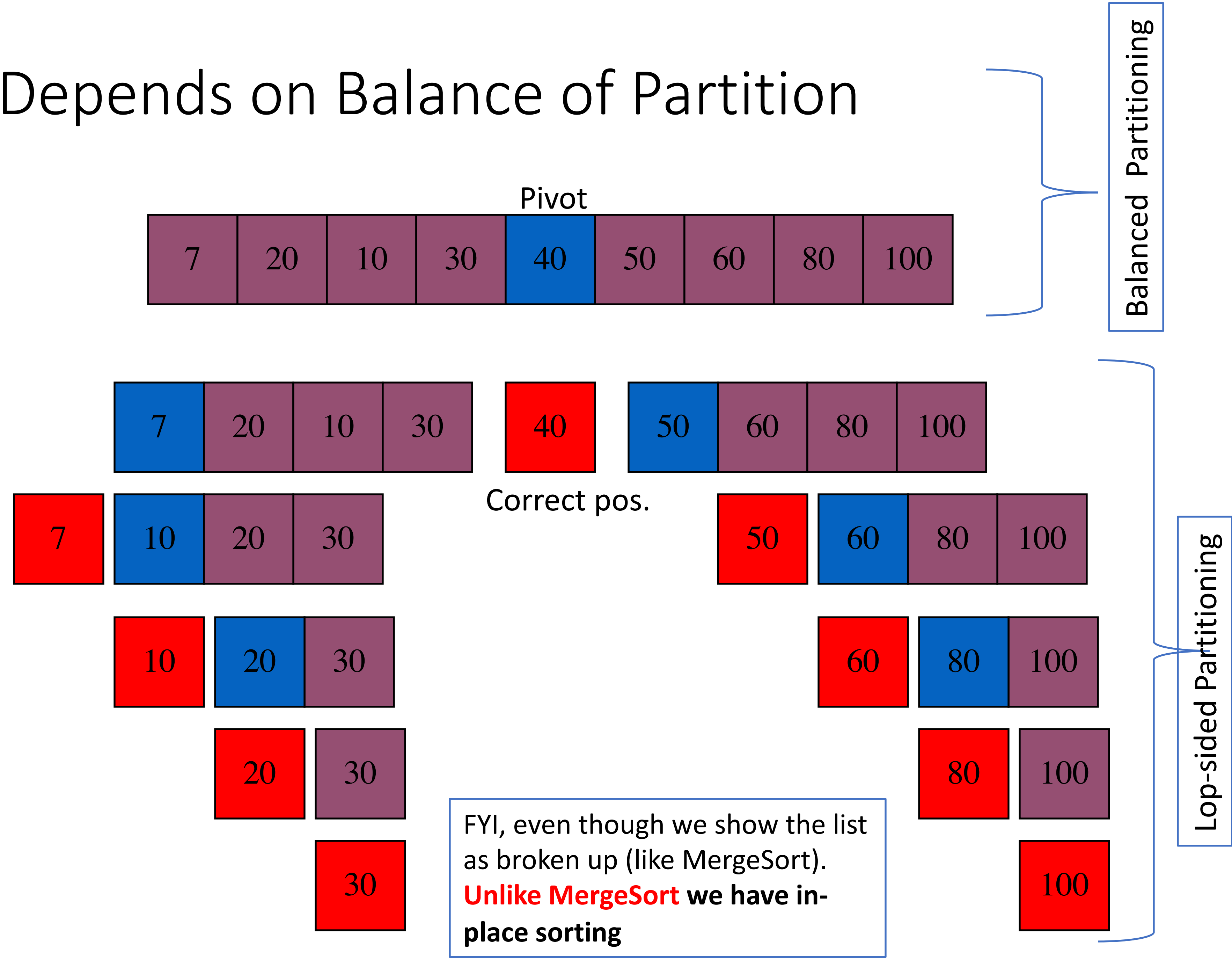


Quicksort Analysis

- Assume that keys are random, uniformly distributed.
- What is **best case running time?**
  - Recursion:
    - Partition splits array in two sub-arrays of size n/2
    - Quicksort each sub-array
- Depth of recursion tree?
  - **$O(\log_2 n)$**
- Number of accesses in partition?
  - **O(n)**
- Best case running time: **$O(n \log_2 n)$**

Worst case running time?

Depends on Balance of Partition



FYI, even though we show the list as broken up (like MergeSort). **Unlike MergeSort we have in-place sorting**

That's all Folks!
Any Question?