Trees + Binary Search Tree + Priority Queues

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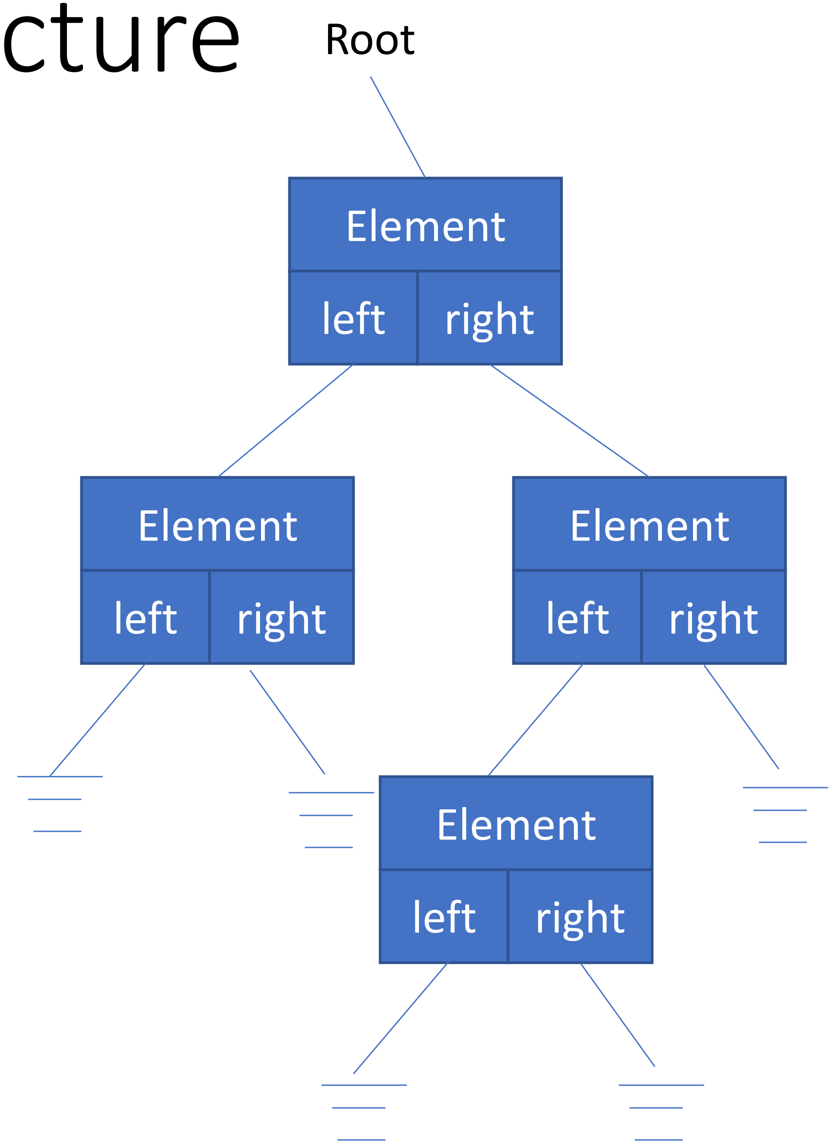
CSC 212

Based on slides from Prof. Carolina Ruiz at WPI

The Tree Data Structure

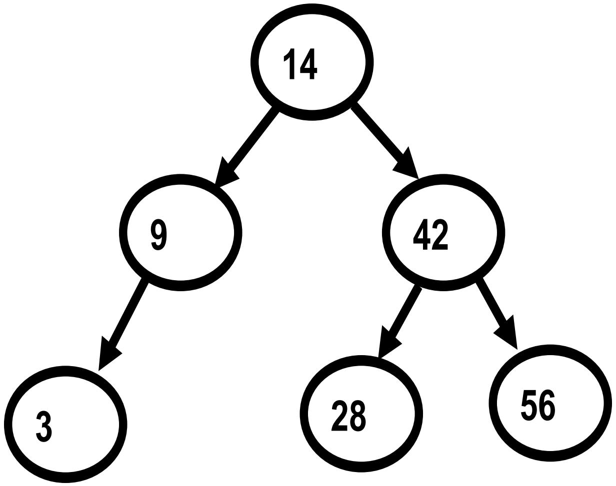
- A tree data structure is like a doubly linked list except now
  - we have a **left-child and right-child**
  - A **root instead of head**
- The leaf nodes have Null (empty object like None in Python) for it's children
- In the HeapSort case, we imagined an array as a binary tree
- **Here, we actually store information in memory in a tree format.**



Binary Search Tree Definitions

- **Binary tree**
  - **Each node has at most two children**

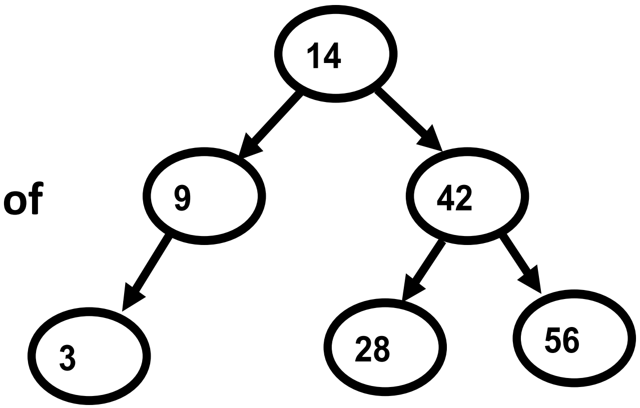
- **Binary Search Tree (BST): Is a binary tree where:**
  - **Left subtree is always less than the node**
  - **Right subtree is always greater than or equal the node**



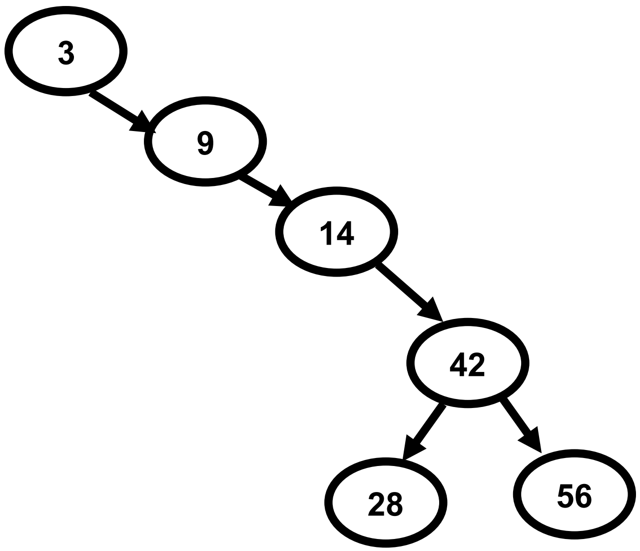
As is usual in Tree diagrams. The Null (None) Links for leaf nodes are not shown, unless needed

Height of BST

- **Can be balanced**
  - **Height of left subtree of the root ≈ Height of right subtree of the root**
  - **In this case the height O(lg n)**

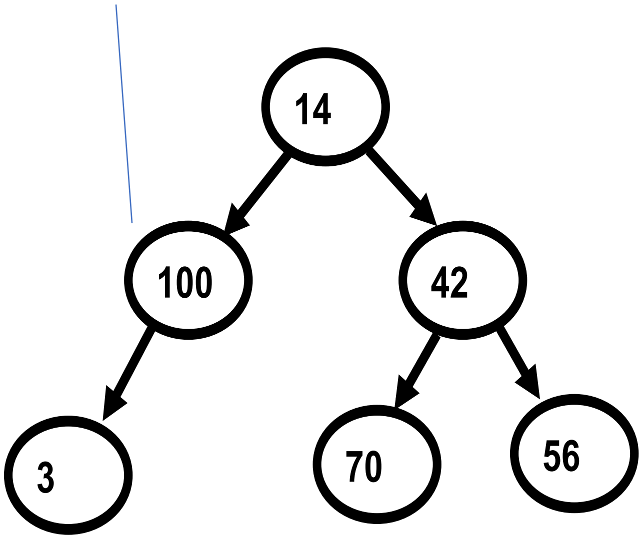


- **Can be un-balanced**
  - **In this case the height is worst case O(n)**

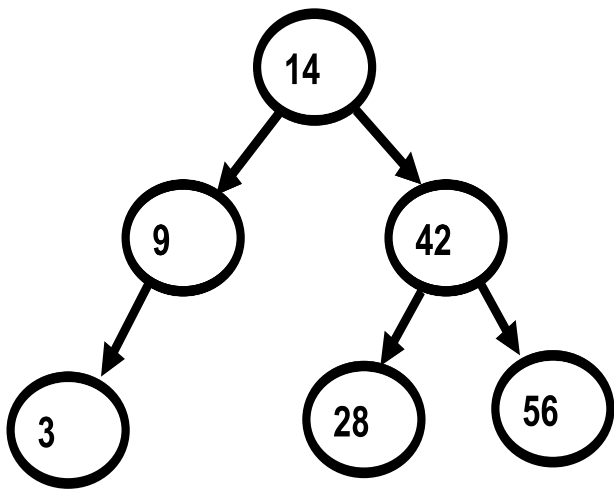


Binary Search Trees (BST)

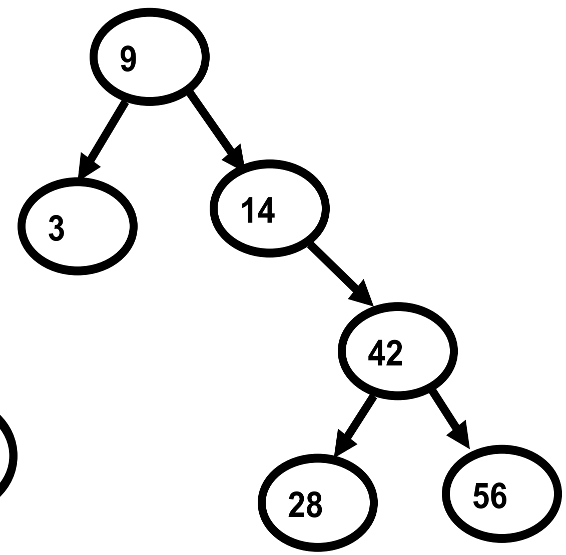
100 as the left child of 14



Binary tree (but not BST)



Binary Search Tree (Balanced)



Binary Search Tree (Un-balanced)

BST Traversal

- With Linked List we talked about traversing through it.
- We can similarly traverse through BST (or any Tree) as well
  - Not as simple as going through a chain that is the Linked List
  - Also there are more than one way to traverse a tree top-bottom
- **Three main types**
  - **Pre-Order Traversal**
  - **Post-Order Traversal**
  - **In-Order Traversal**

Traversing a Binary Search Tree (BST)

Pre-Order

Outline of Pre-Order Traversal

- **For each node**
  - **Do the work first (Current)**
  - **Traverse Left**
  - **Traverse Right**

- **Work can be anything (E.g., print)**

Pre-Order Traversal Procedure (Pseudocode)

```
def Pre_Order(Link):
    if Link != None:
        Do_Whatever( Link.data )
        Pre_Order( Link.left_child )
        Pre_Order( Link.right_child )
```

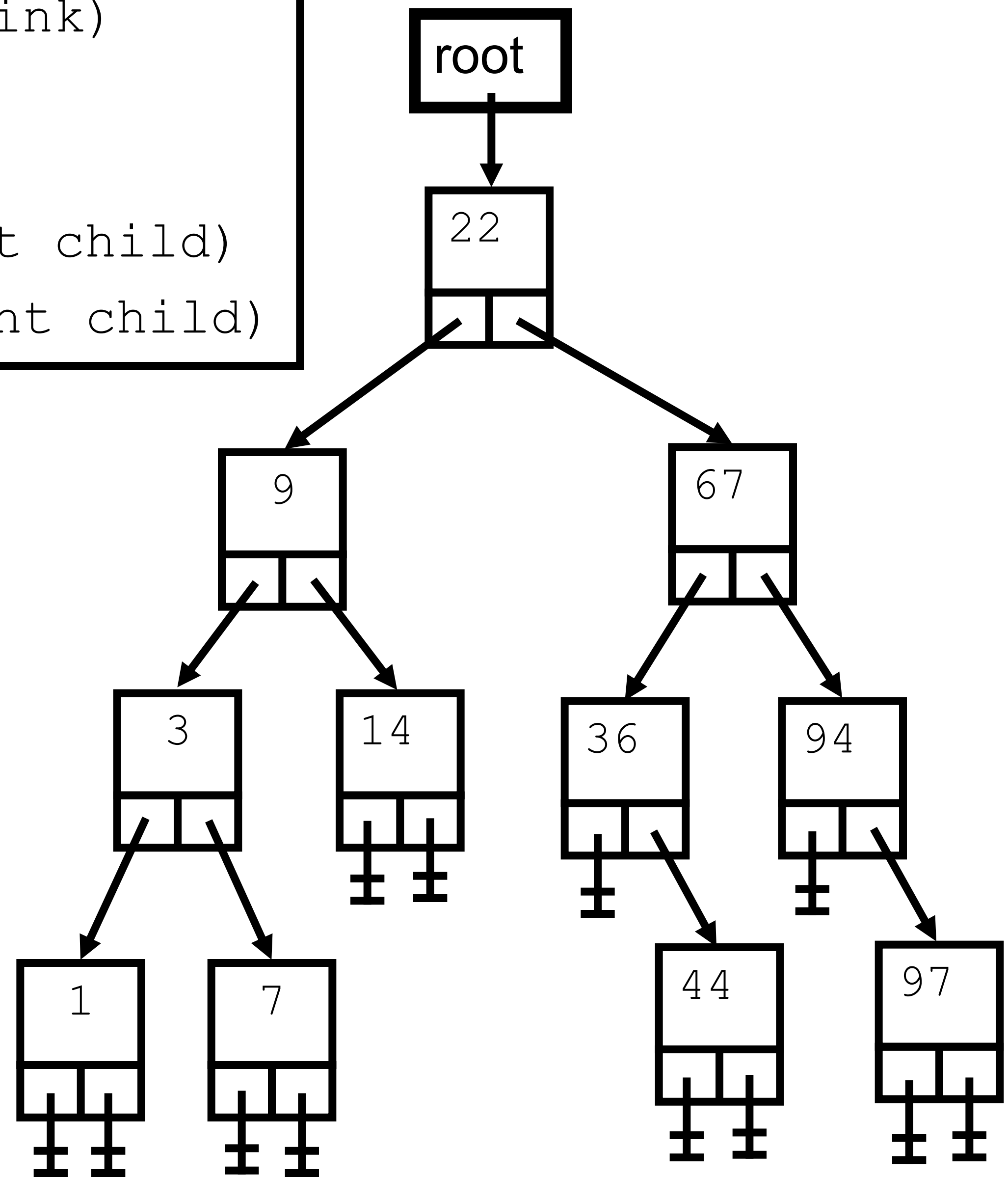
Start at the “Root” for a full tree traversal

E.g., print

Two recursive calls

```
def PreOrderPrint(Link)
 Link NOT None?
L print(data)
R PreOrderPrint(left child)
P PreOrderPrint(right child)
```

Output



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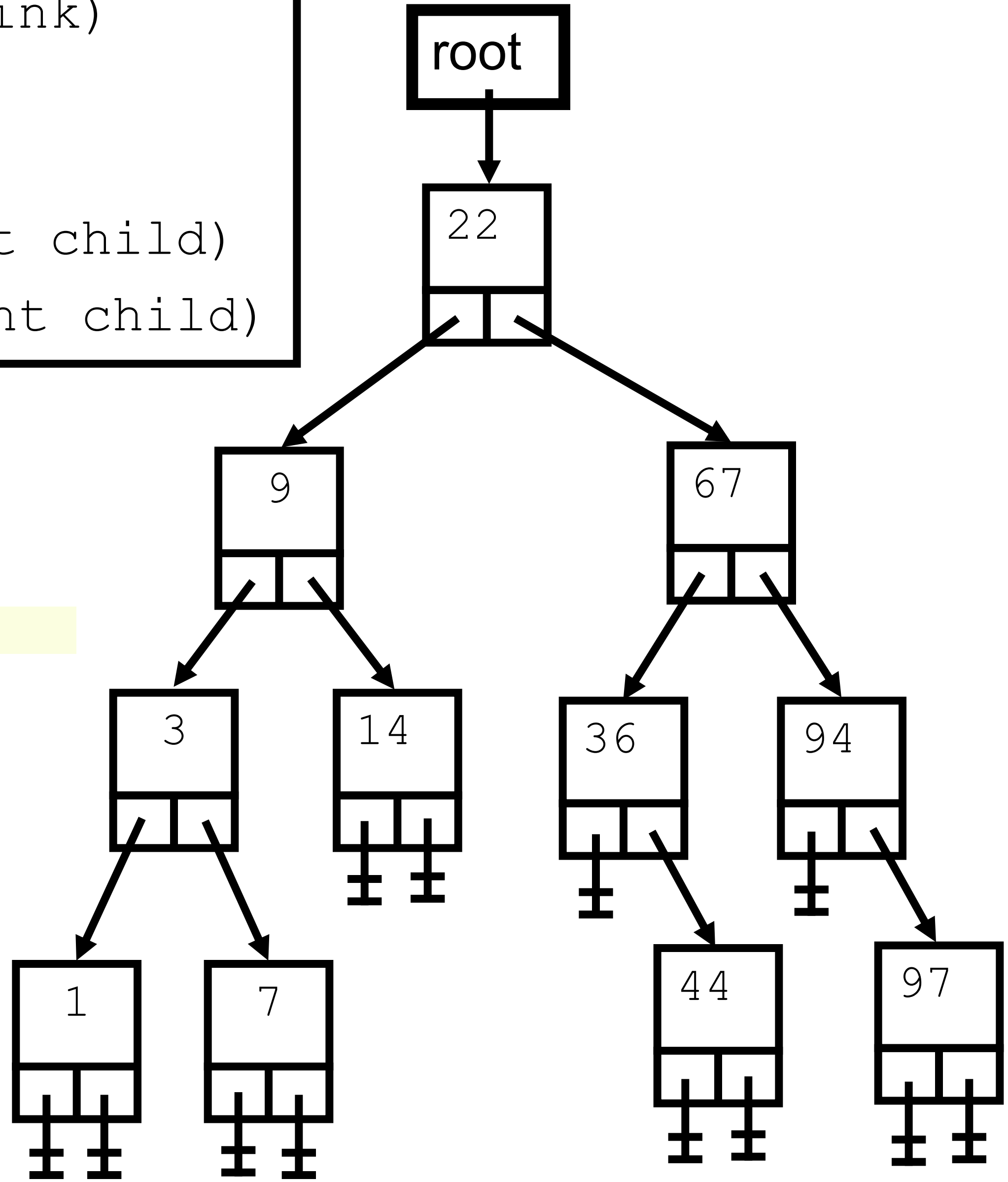
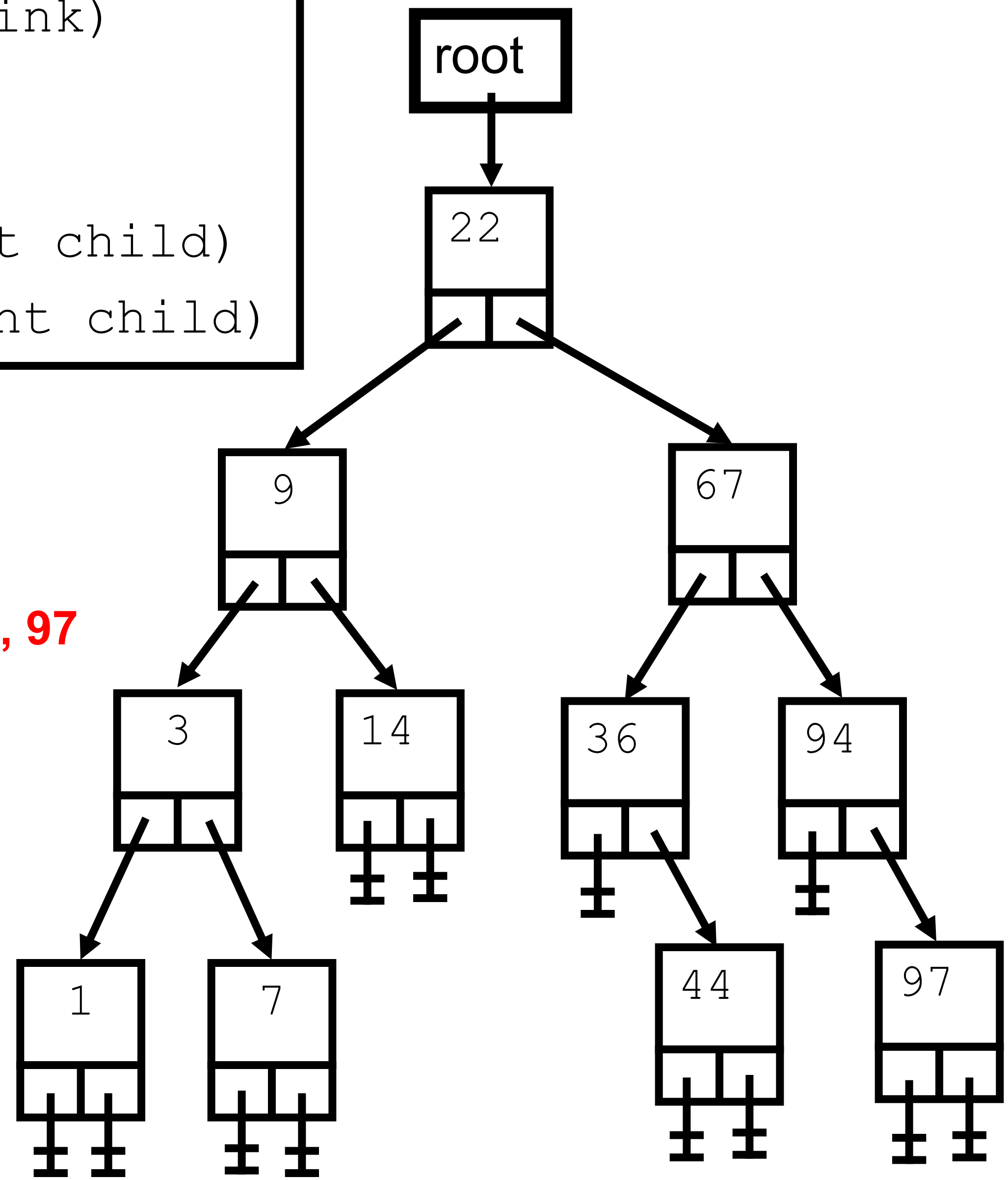


Output

22

```
def PreOrderPrint(Link)
 Link NOT None?
P print(data)
L PreOrderPrint(left child)
R PreOrderPrint(right child)
```



Output

22, 9, 3, 1, 7, 14, 67, 36, 44, 94, 97

Traversing a Binary Search Tree (BST)

Post-Order

Outline of Post-Order Traversal

- **For each node**
  - **Traverse Left**
  - **Traverse Right**
  - **Do the work (Current)**

Post-Order Traversal Procedure (Pseudocode)

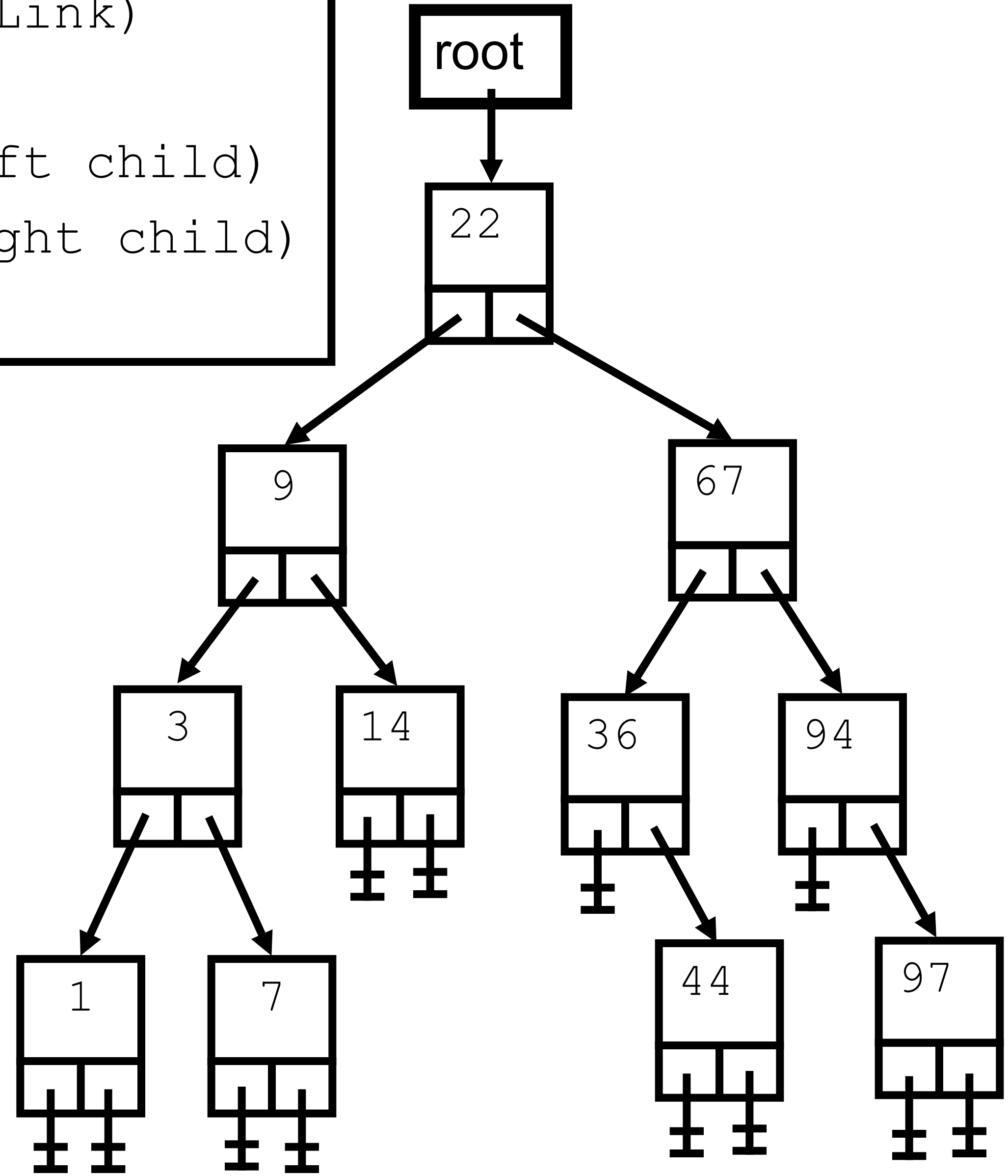
```
def Post_Order(Link):
    if Ptr != NIL:
        Post_Order( Link.left_child )
        Post_Order( Link.right_child )
        Do_Whatever( Link.data )
```

Start at the “Root” for a full tree traversal

Two recursive calls

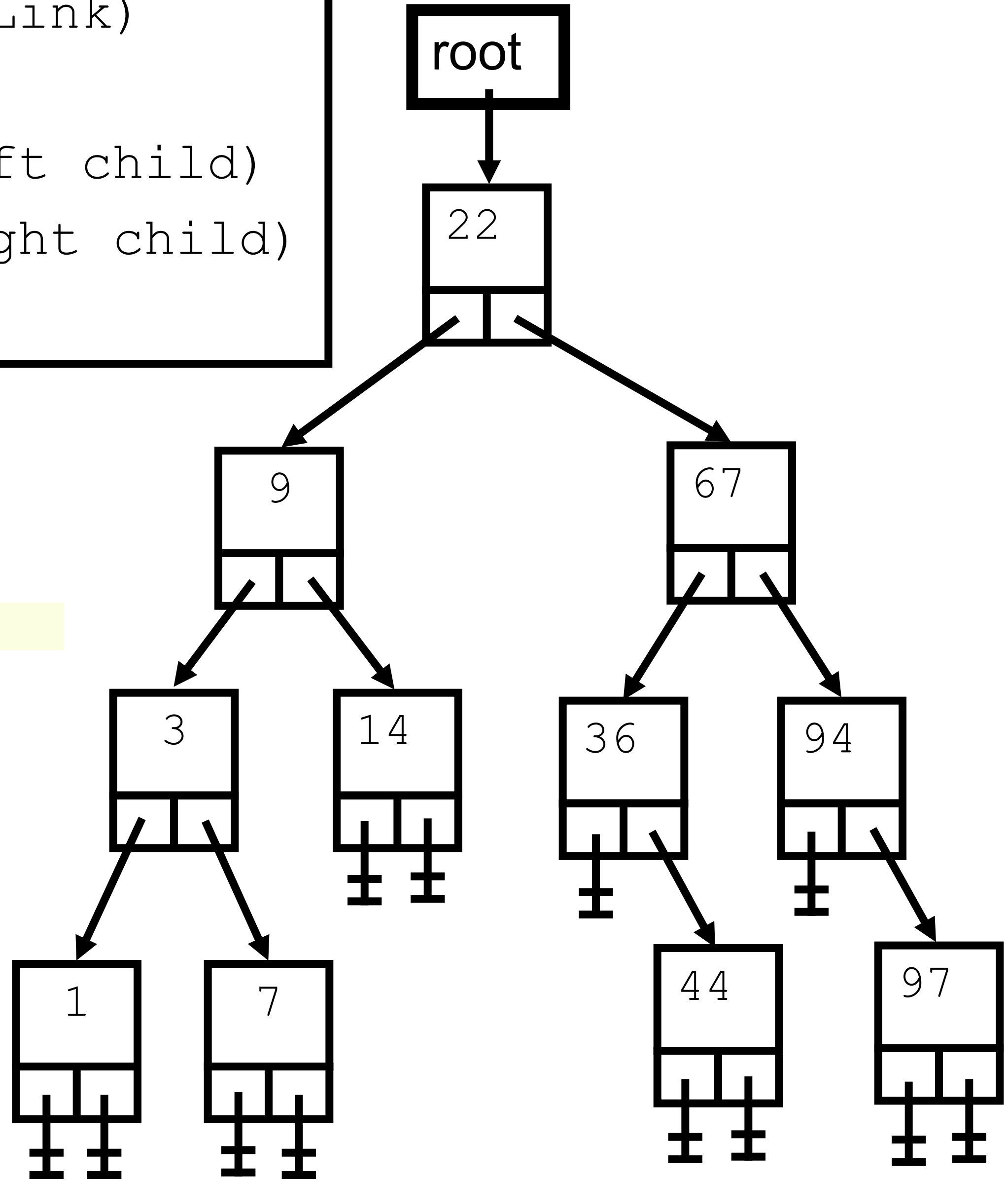
E.g., print

```
def PostOrderPrint(Link)
 Link NOT None?
L PostOrderPrint(left child)
R PostOrderPrint(right child)
P print(data)
```



Output

```
def PostOrderPrint(Link)
 Link NOT None?
L PostOrderPrint(left child)
R PostOrderPrint(right child)
P print(data)
```



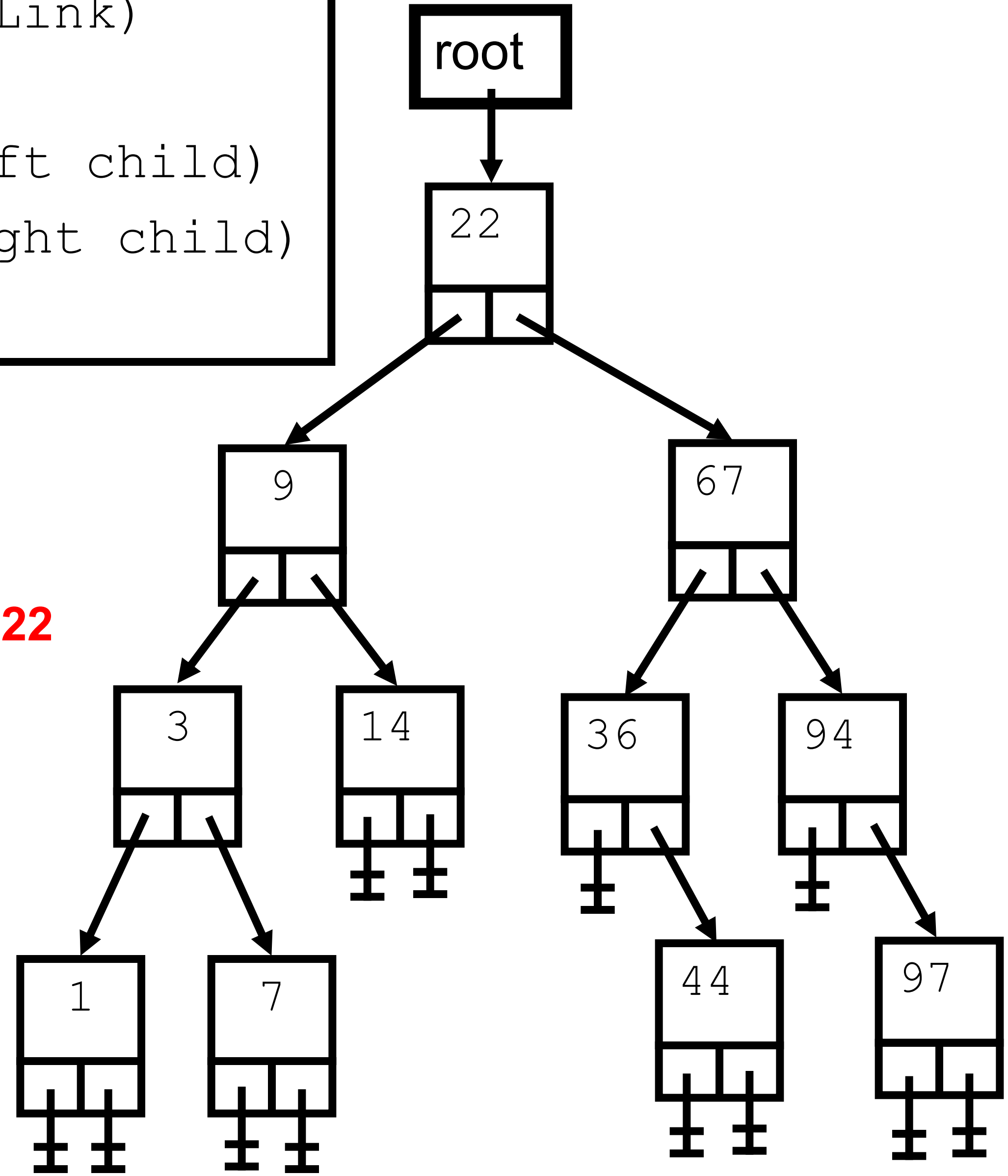
Output

1

```
def PostOrderPrint(Link)
 Link NOT None?
L PostOrderPrint(left child)
R PostOrderPrint(right child)
P print(data)
```

Output

1, 7, 3, 14, 9, 44, 36, 97, 94, 67, 22



Traversing a Binary Search Tree (BST)

In-Order

Outline of In-Order Traversal

- **For each node**
  - **Traverse Left**
  - **Do the work (Current)**
  - **Traverse Right**

In-Order Traversal Procedure (Pseudocode)

```
def In_Order(Link):
    if Link != None:
        In_Order( Link.left_child )
        Do_Whatever( Link.data )
        In_Order( Link.right_child )
```

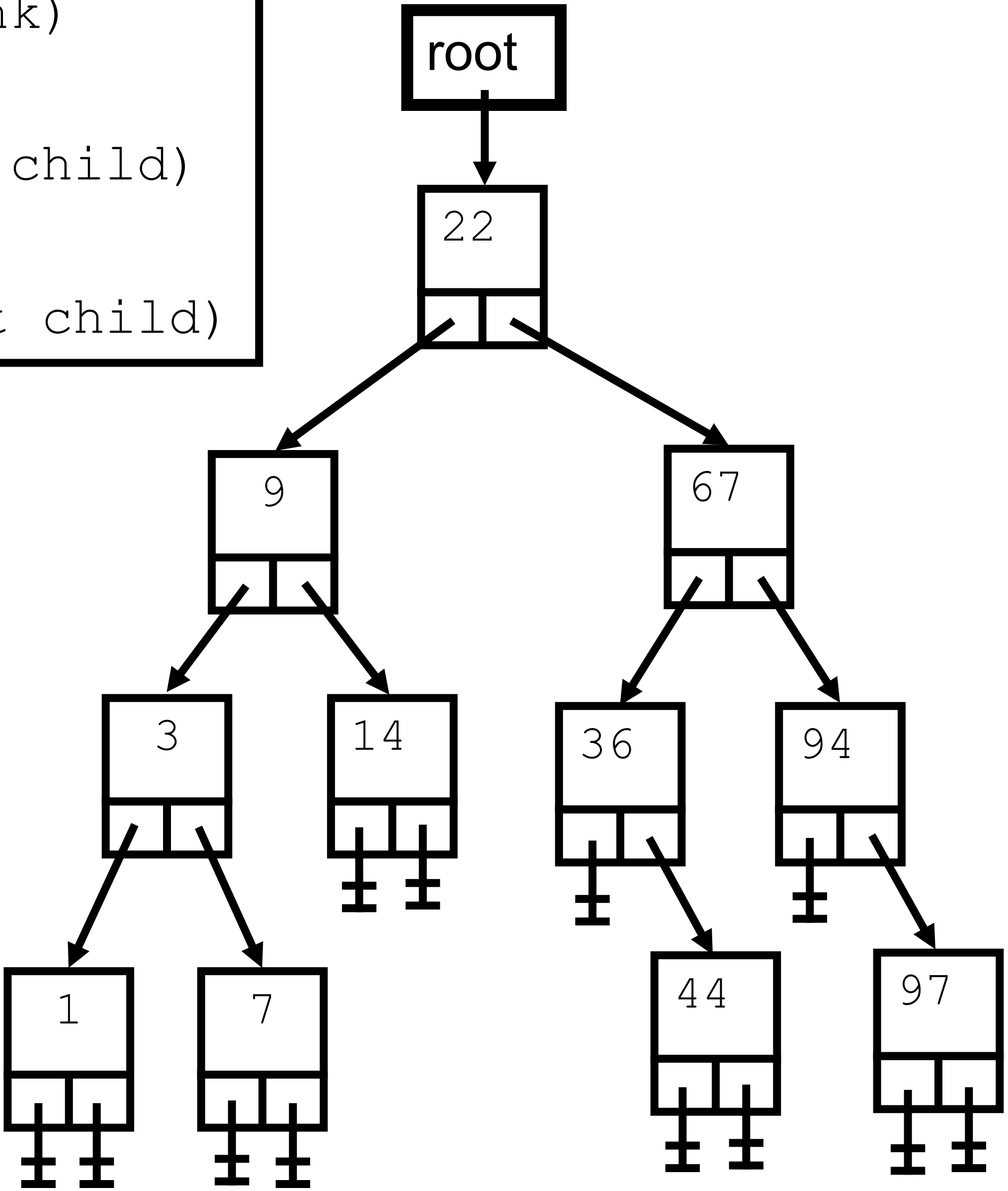
Start at the “Root” for a full tree traversal

Two recursive calls

E.g., print

```
def InOrderPrint(Link)
 Link NOT None?
L InOrderPrint(left child)
P print(data)
R InOrderPrint(right child)
```

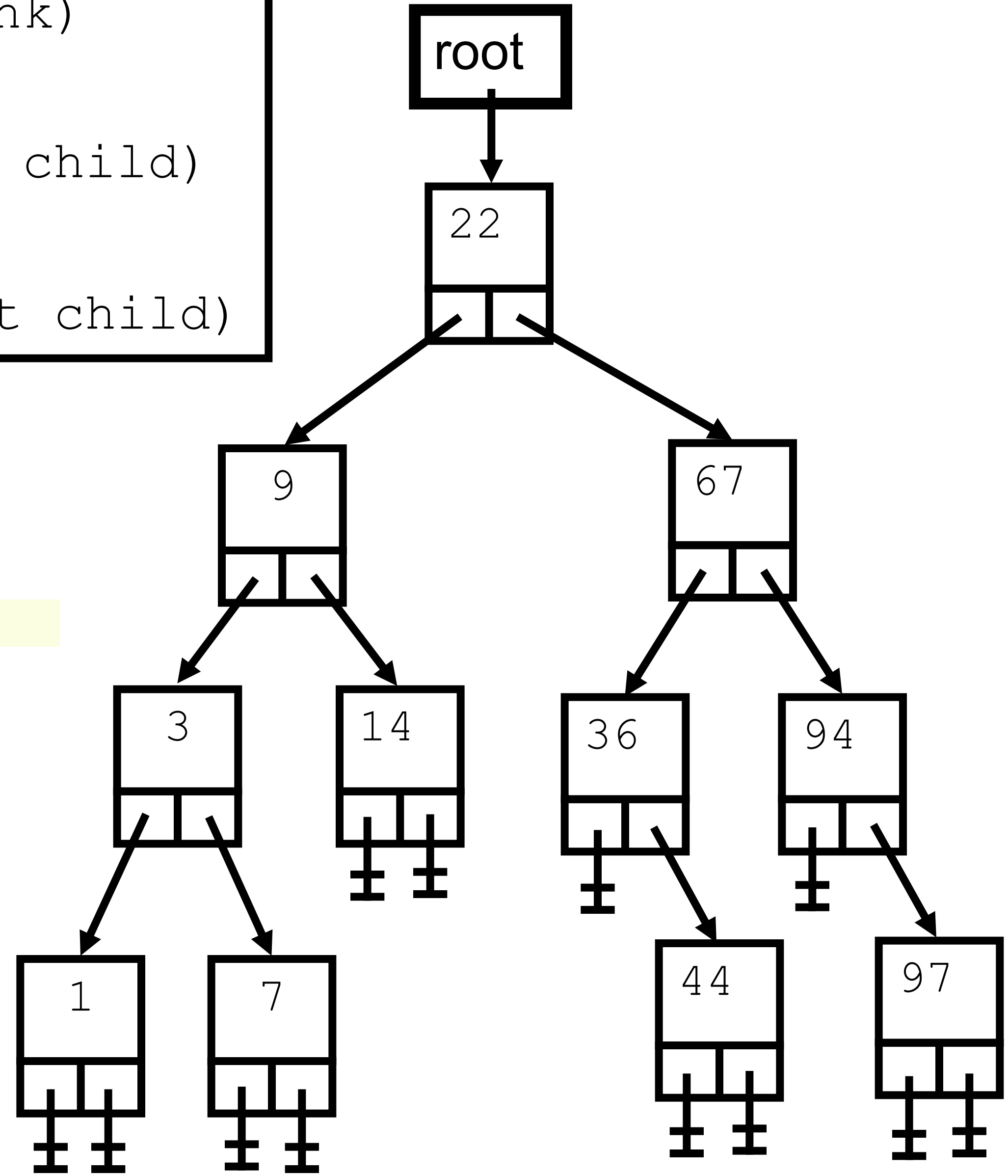
Output



```
def InOrderPrint(Link)
 Link NOT None?
L InOrderPrint(left child)
P print(data)
R InOrderPrint(right child)
```

Output

1



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 Link NOT None?
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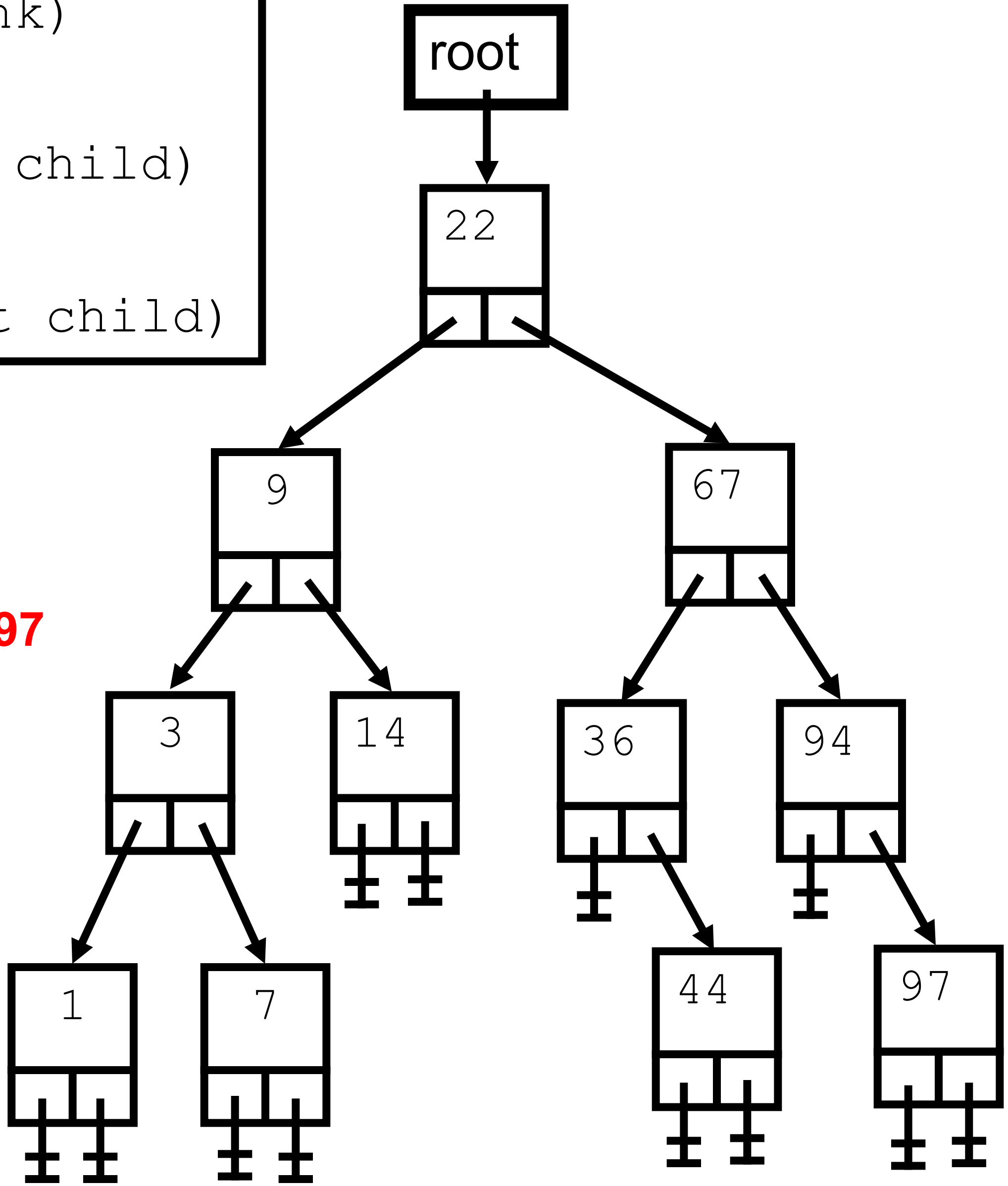
```

Output

1, 3, 7, 9, 14, 22, 36, 44, 67, 94, 97

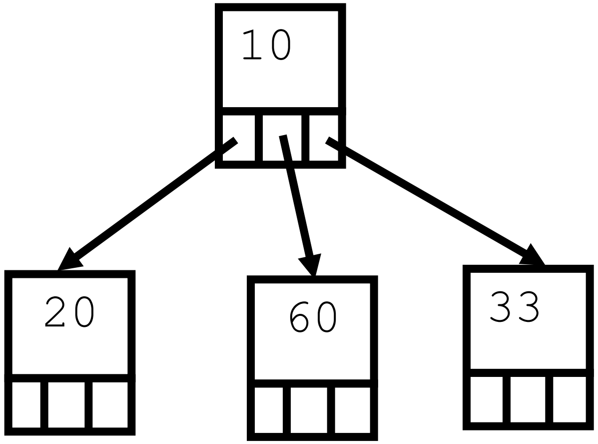
In-order traversal produces a sorted list



Notes

- **The three traversal types (Pre-, In-, Post-)Order apply to all types of binary trees**
  - **Whether binary search tree (BST) or not**

- **In-Order traversal is applicable only for binary trees**
  - **Not for N-way trees, where N > 2**



- **3-way tree**
- **Pre-Order and Post-Order traversal are applicable**
- **In-Order traversal is not applicable**

Analysis of Tree Traversal

- **Assume the tree has n nodes**

- **Any the three types of traversal is linear O(n)**

- **Each node is visited only once (E.g., printing is done once)**

Searching in a Tree: A few Observations

- Accessing a item from a linked list takes O(N) time for an arbitrary element

- Binary Search Trees can improve upon this:
  - reduce access to O(lg N) time for the average case
  - expands on the binary search technique (from Oct 24) and allows insertions and deletions

Searching in a Tree: Problem Definition

- **Input**
  - Binary tree
  - Value k to search for

Algorithm depends on whether it is binary search tree (BST) or not

Searching Binary Trees (Not BST)

```
Tree-Search(Link to root, value k)
        if Link == NULL:
            return 0
        else if Link.Data == k:
            return 1;
        else:
         Tree-Search(Link.left, k)
         Tree-Search(Link.right, k)
```

Pseudocode

If the current node contains k, return true

Call the left and right subtrees recursively (Will check the LEFT subtree first)

Example: Search For 6

Tree-Search(Link to root, value k)

if Link == NULL:

return 0

else if Link.Data == k:

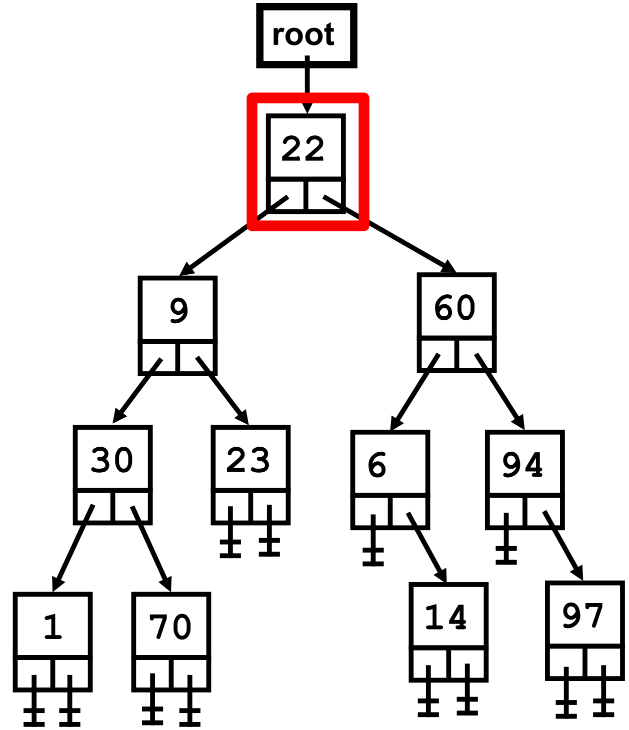
return 1;

else:

Tree-Search(Link.left, k)

Tree-Search(Link.right, k)

Pseudocode



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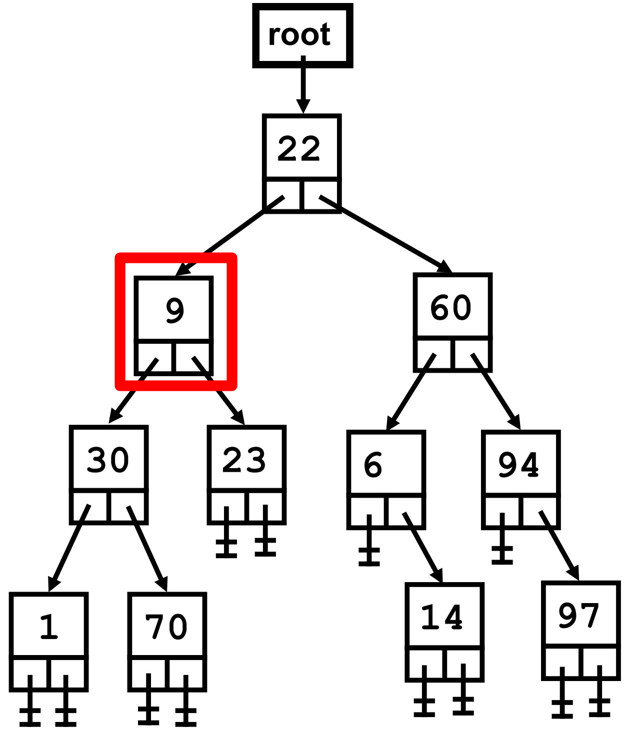
Pseudocode



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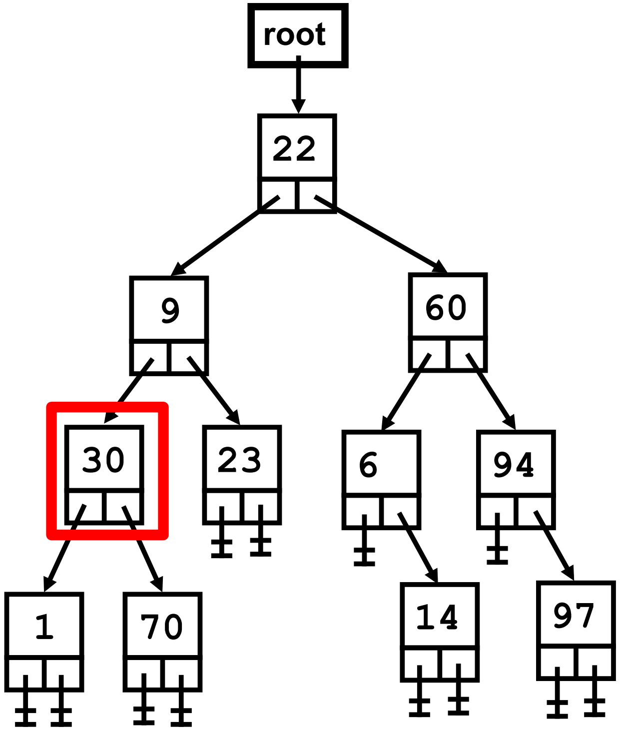

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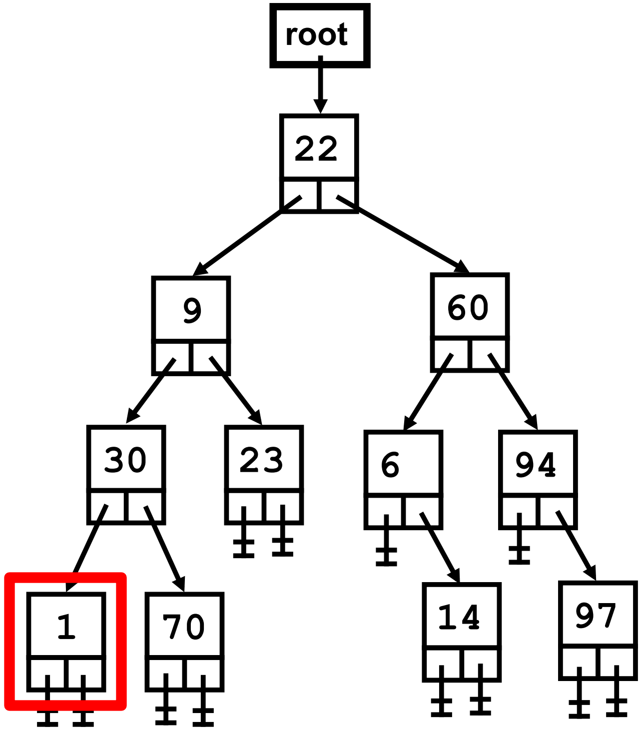

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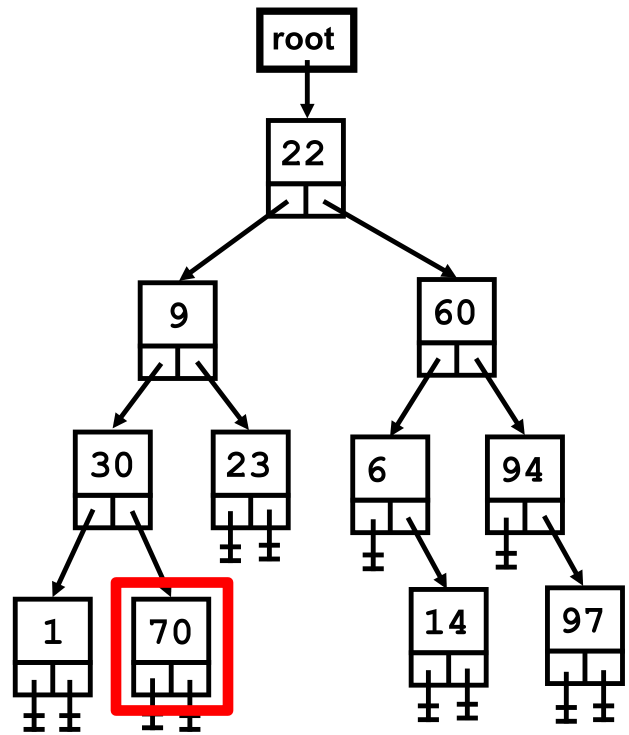
Pseudocode



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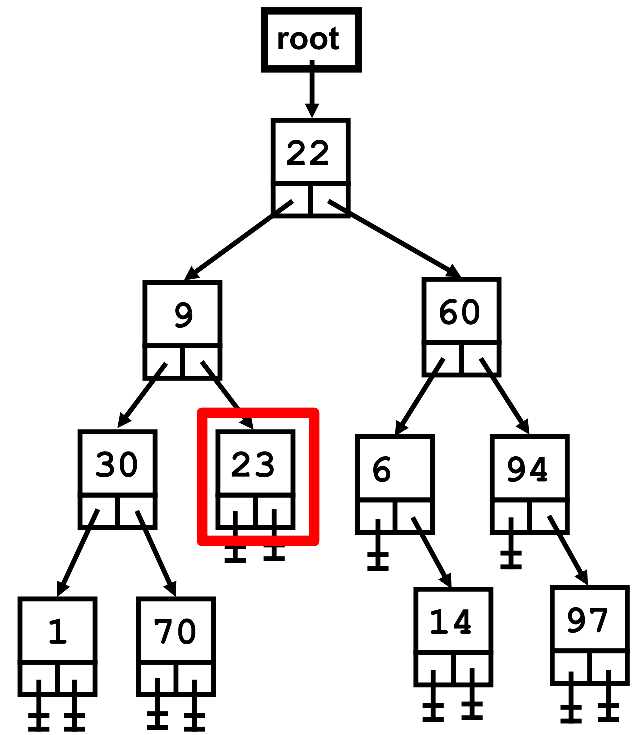

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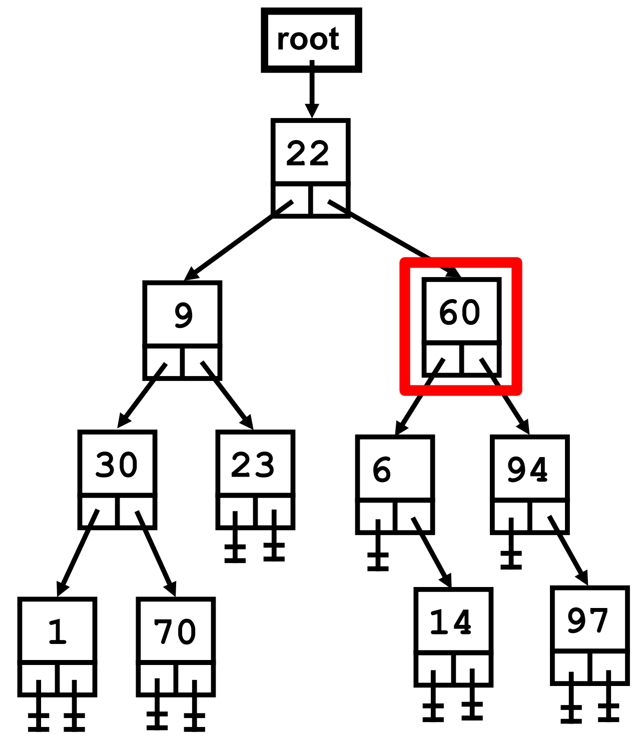

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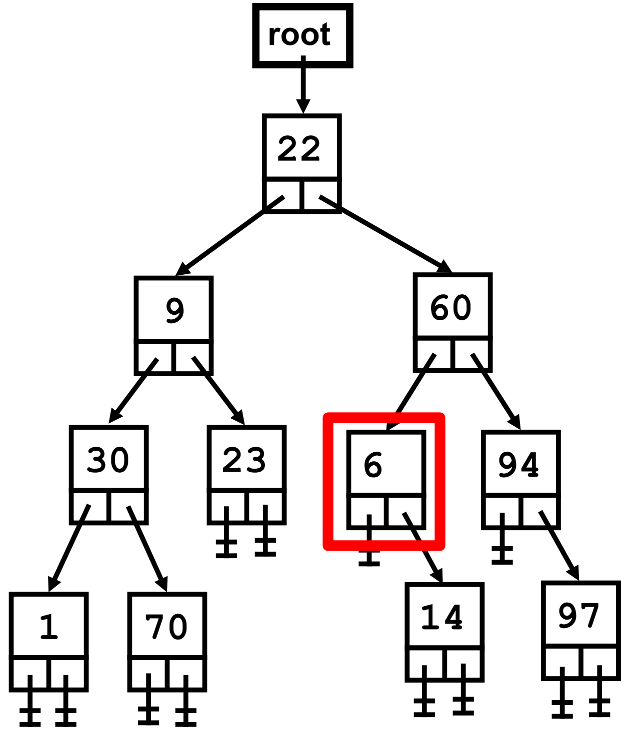
```

Pseudocode

Similar to Pre-Order traversal
- Check the node first
- Check the Left subtree
- Check the Right subtree



Time Complexity For Binary Tree (Not BST) Searching

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Tree-Search(Link to root, value k)
        if Link == NULL:
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        else:
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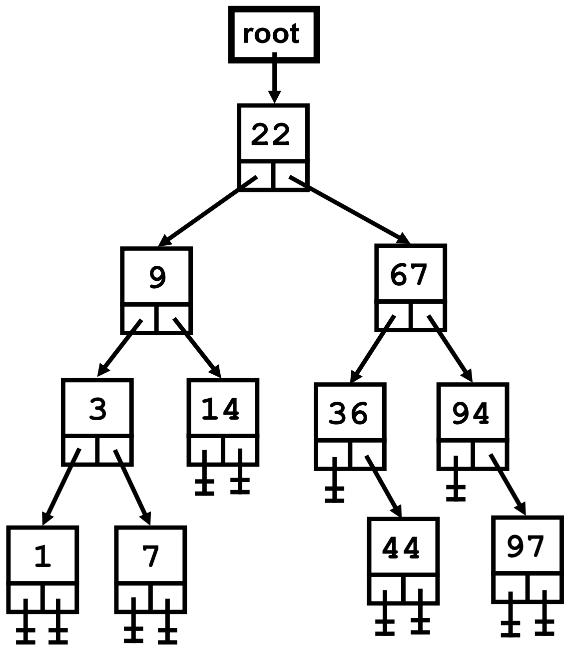
```

- **How many times each node is checked?**
  - **At most once**
- **Searching a binary tree is linear → O(n)**

Find k in Binary Search Trees (BST)

- **Should make use of the BST property to search in an efficient way**

```
Tree-Search(Link to root, value k)
        if Link == NULL:
            return 0
        else if Link.Data == k:
            return 1;
        else if k < Link.Data
            return Tree-Search(Link.left, k)
        else:
            return Tree-Search(Link.right, k)
```



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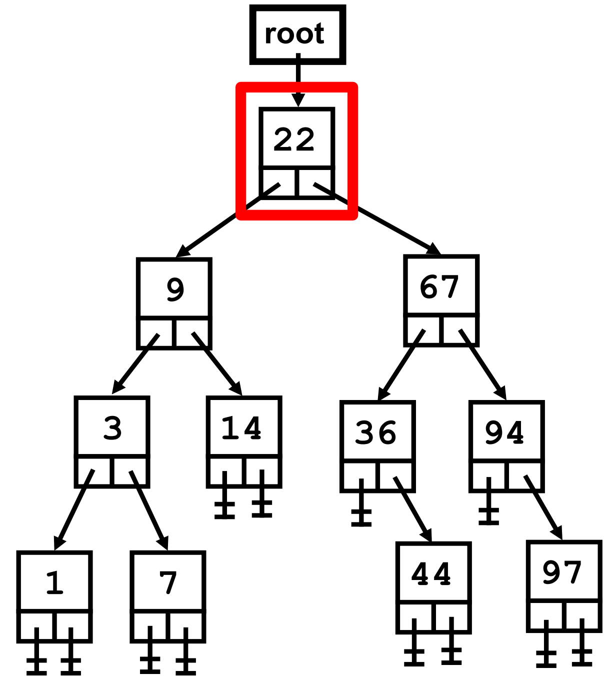

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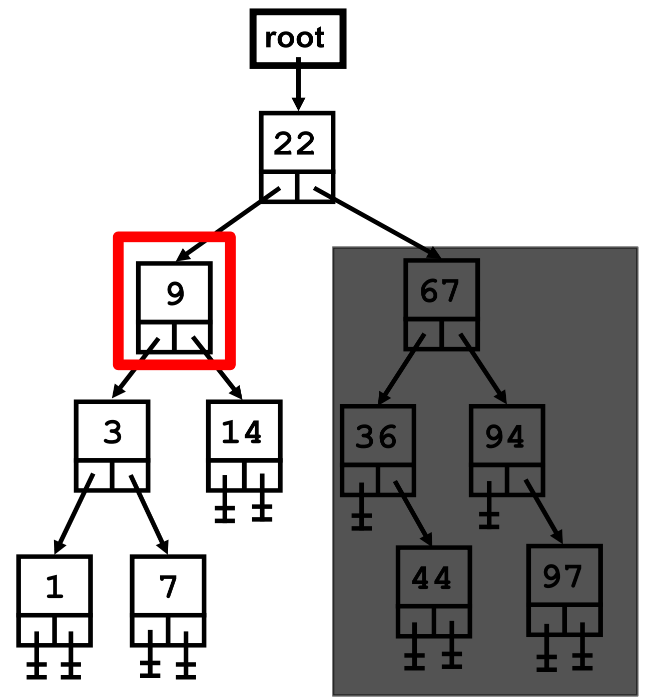

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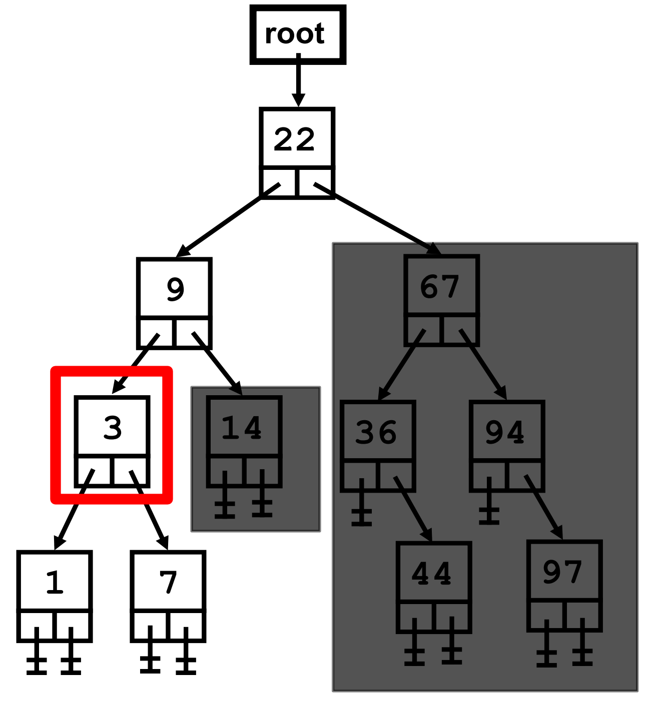

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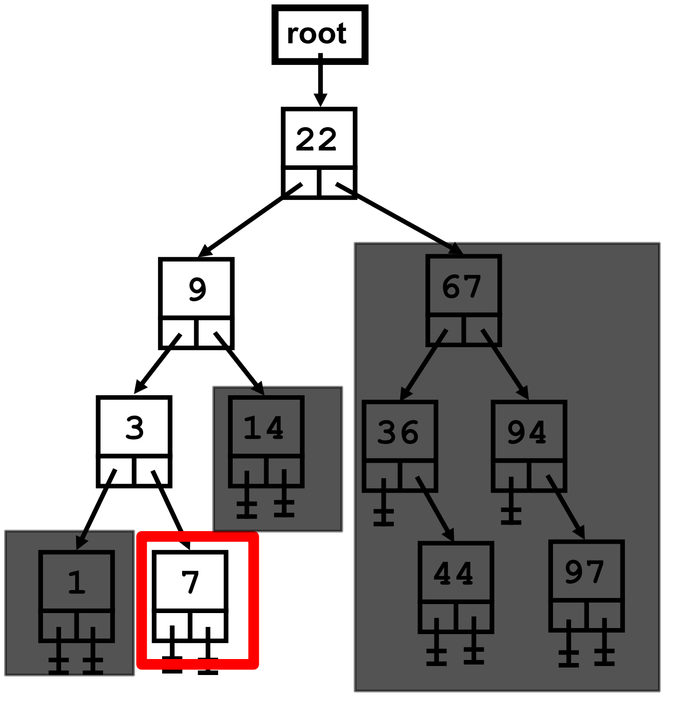

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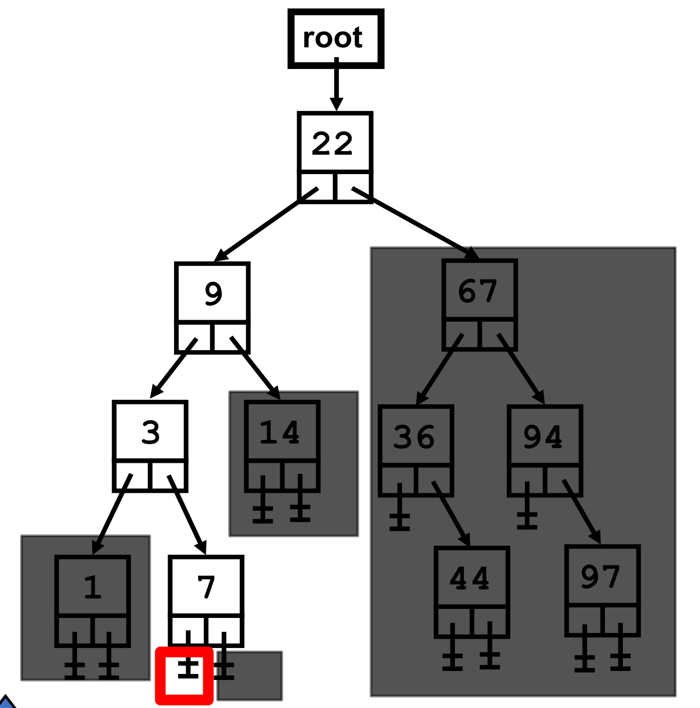
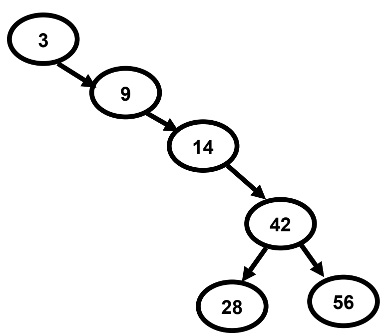


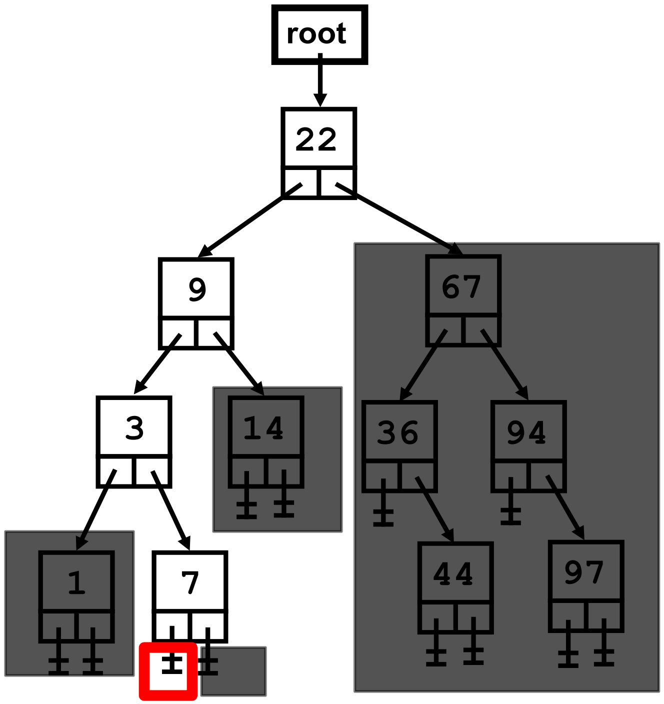
Not Found

Time Complexity For BST Searching

- **In BST search we follow ONLY on path from root to a leaf**

- **What is the *Worst Case Complexity?***
  - **O(h), where *h* is the tree height = lg n**
  - **If BST is balanced ➔ O (lg n)**
  - **If BST is not balanced ➔ O (n)**





Insertion and Deletion in BST

- In Binary Search (from Oct 24), we needed to sort the an array of elements first before we could search in O(lg n) time.
  - Otherwise, one has to search every element in the array and that has O(n) time-complexity

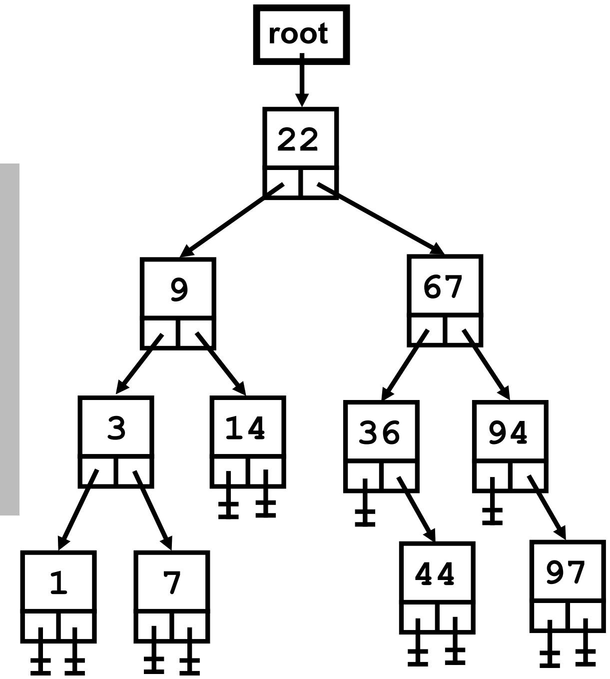
- With BST we can store elements in such a way that we can always get O(lgn) time-complexity for search.

- It depends on how we insert and delete elements in the BST such that **the BST property is always maintained**

Insertion of *V* in BST

- **Search for the correct empty place to put the new node**
  - **Correct: if V >= current node → move right**
    if V < current node → move left
  - **Empty: Find link with Null value**

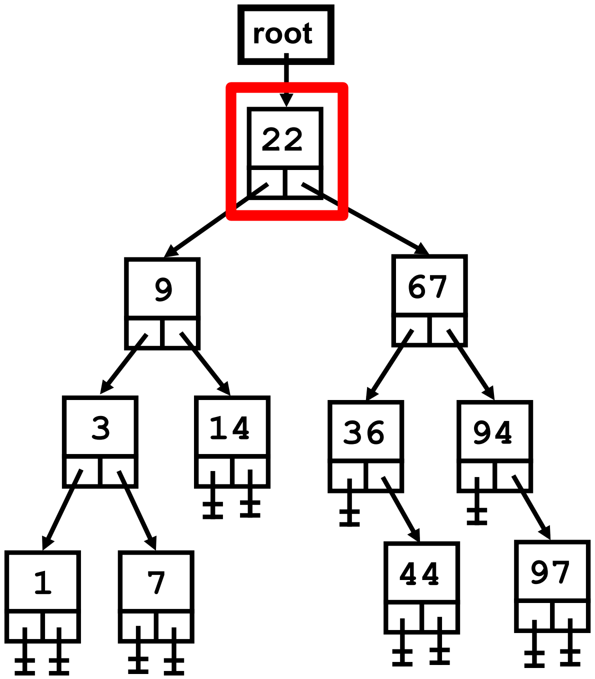
Pseudocode



Example: Insert value 6

- **Search for the correct empty place to put the new node**
  - **Correct: if V >= current node → move right**
    if V < current node → move left
  - **Empty: Find link with Null value**

Pseudocode



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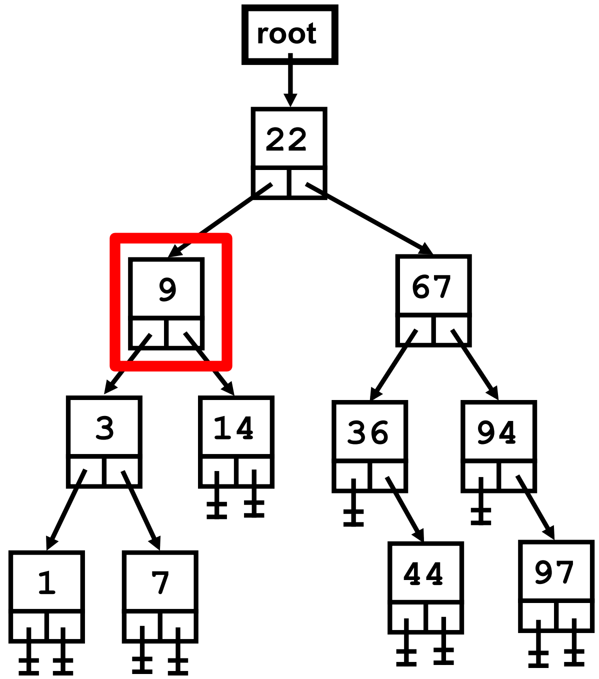

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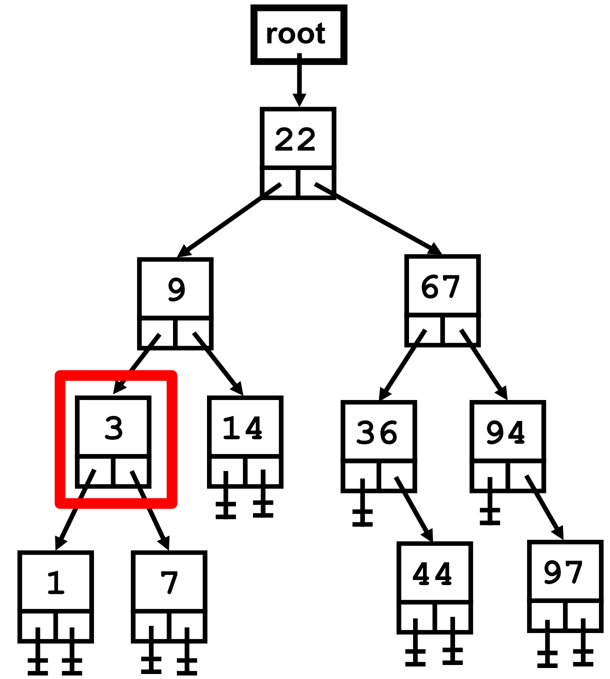

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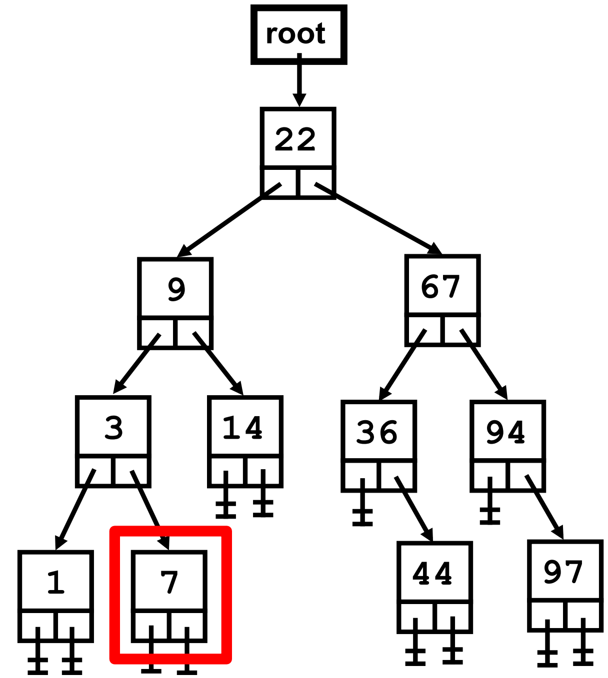

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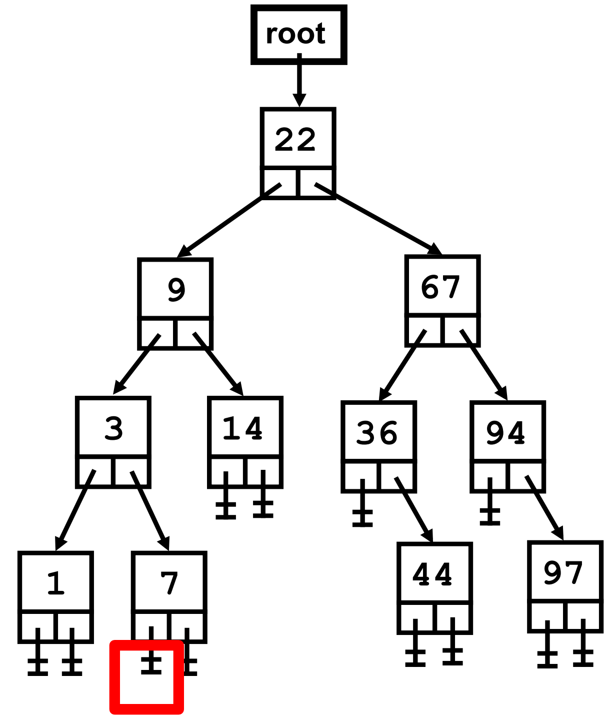
  - **Empty: Find link with Null value**

Pseudocode

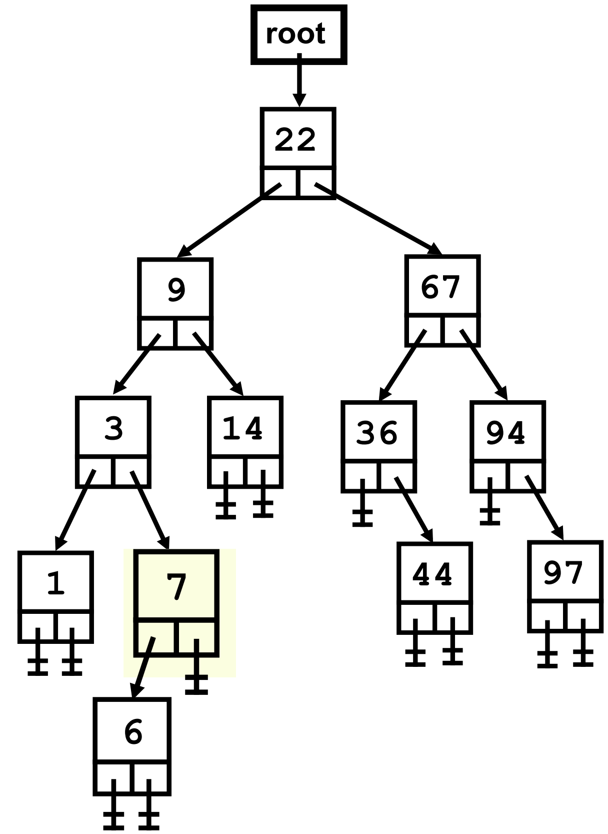


Example: Insert value 6

- **Search for the correct empty place to put the new node**
  - **Correct: if V >= current node → move right**
    if V < current node → move left
  - **Empty: Find link with Null value**

Pseudocode

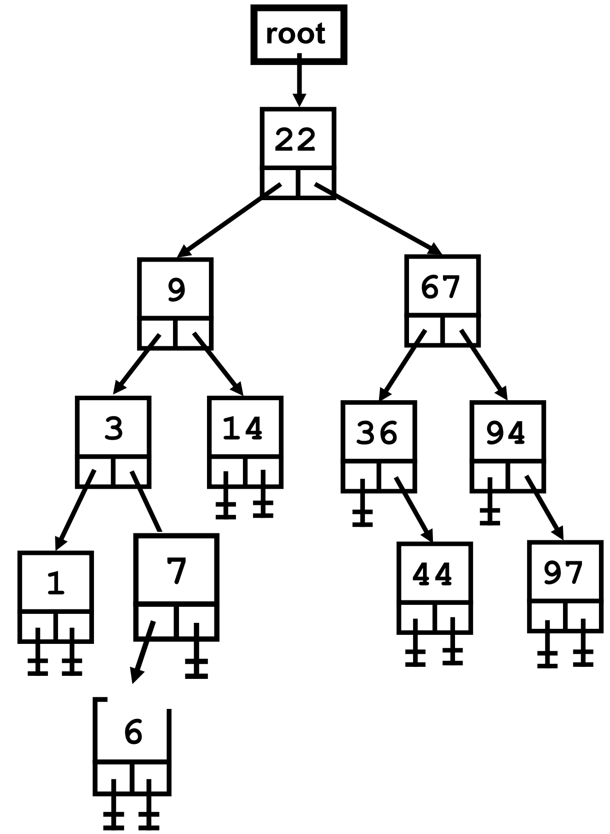
Time complexity for insertion is O(h),
h: is the tree height



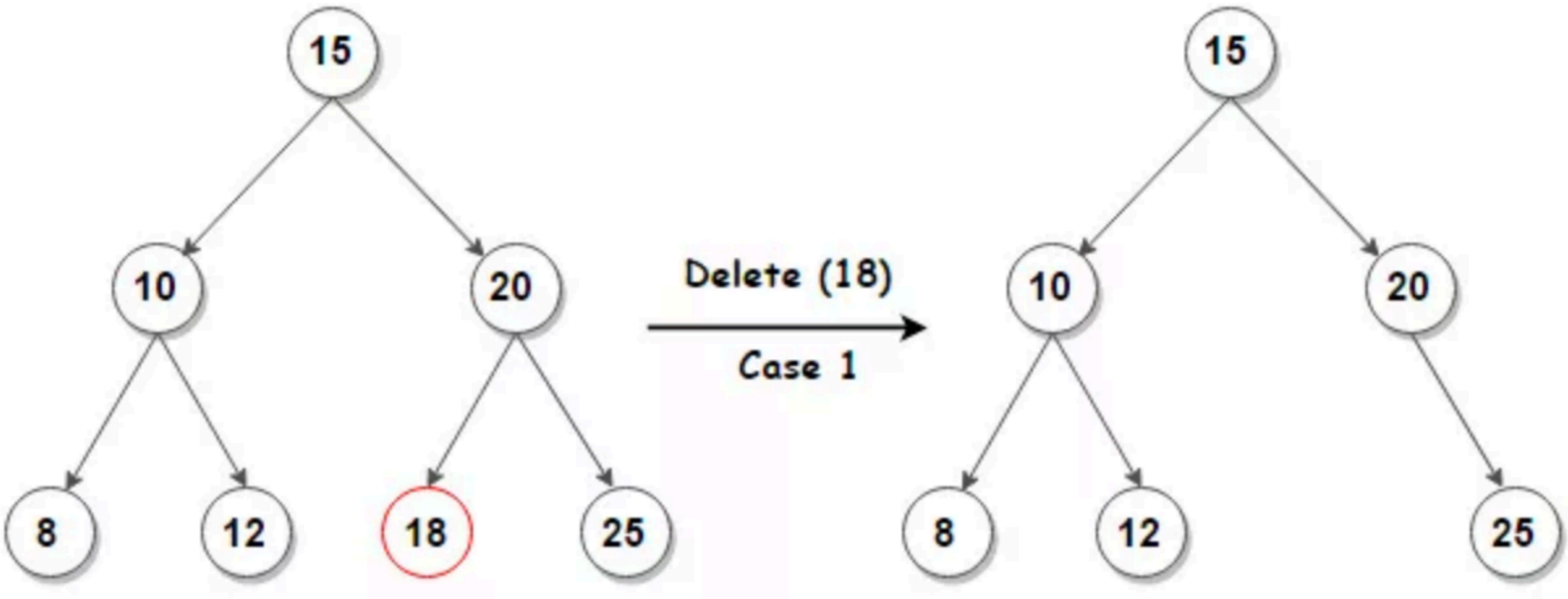
Deletion of *V* from BST

- **Search for V in the tree**
  - **Not Found** ➔ **We are done**
  - **Found**
    - **Case 1: Node V has no children**
      - **Delete V**
    - **Case 2: Node V has one child**
      - **The child takes the place of V**
    - **Case 3: Node V has two children**
      - **Find the predecessor node of V → Say X**
      - **Put X in the place of V**

How to find this value (Predecessor of V)??

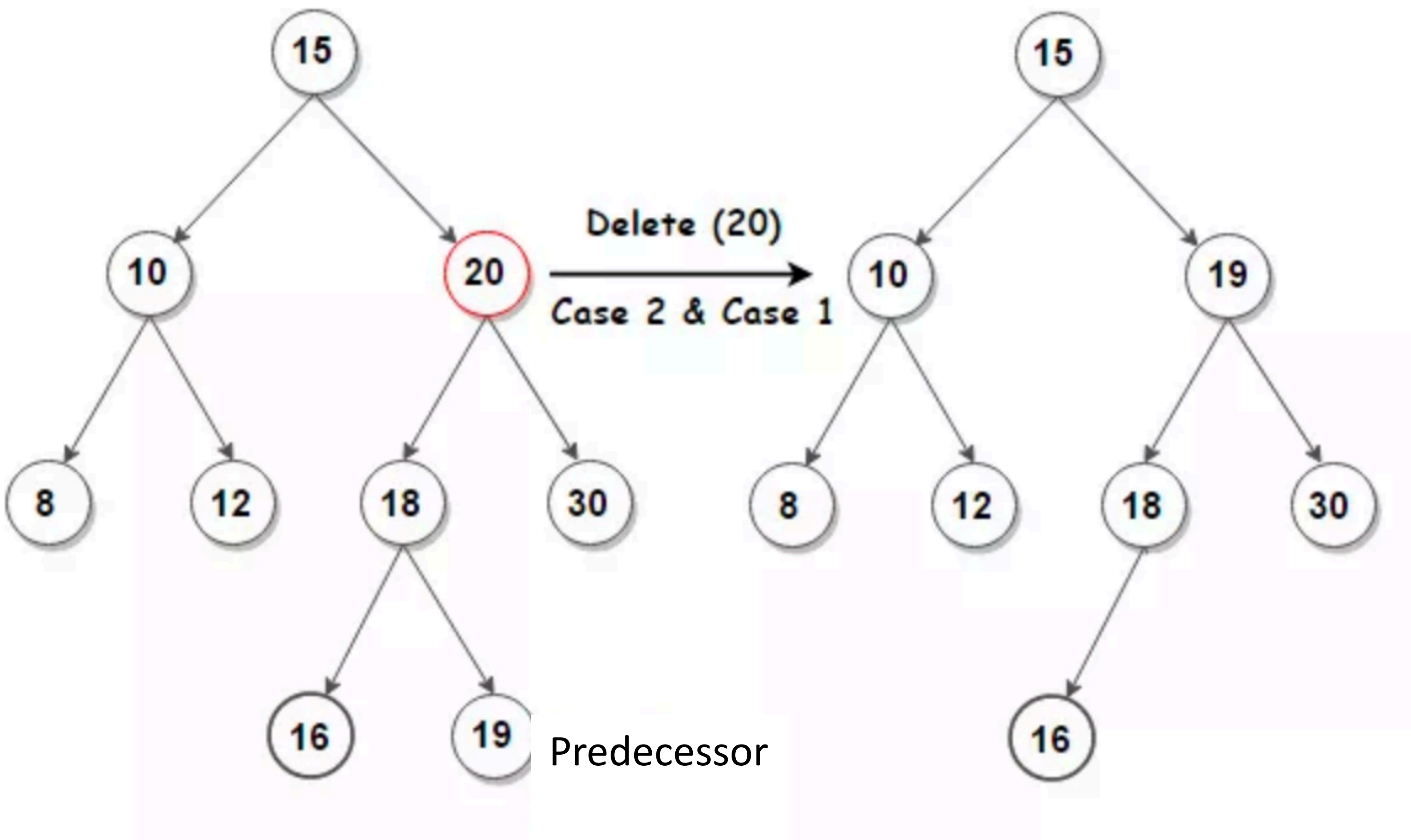


Example of BST Deletion:



Courtesy: https://www.techiedelight.com/deletion-from-bst/

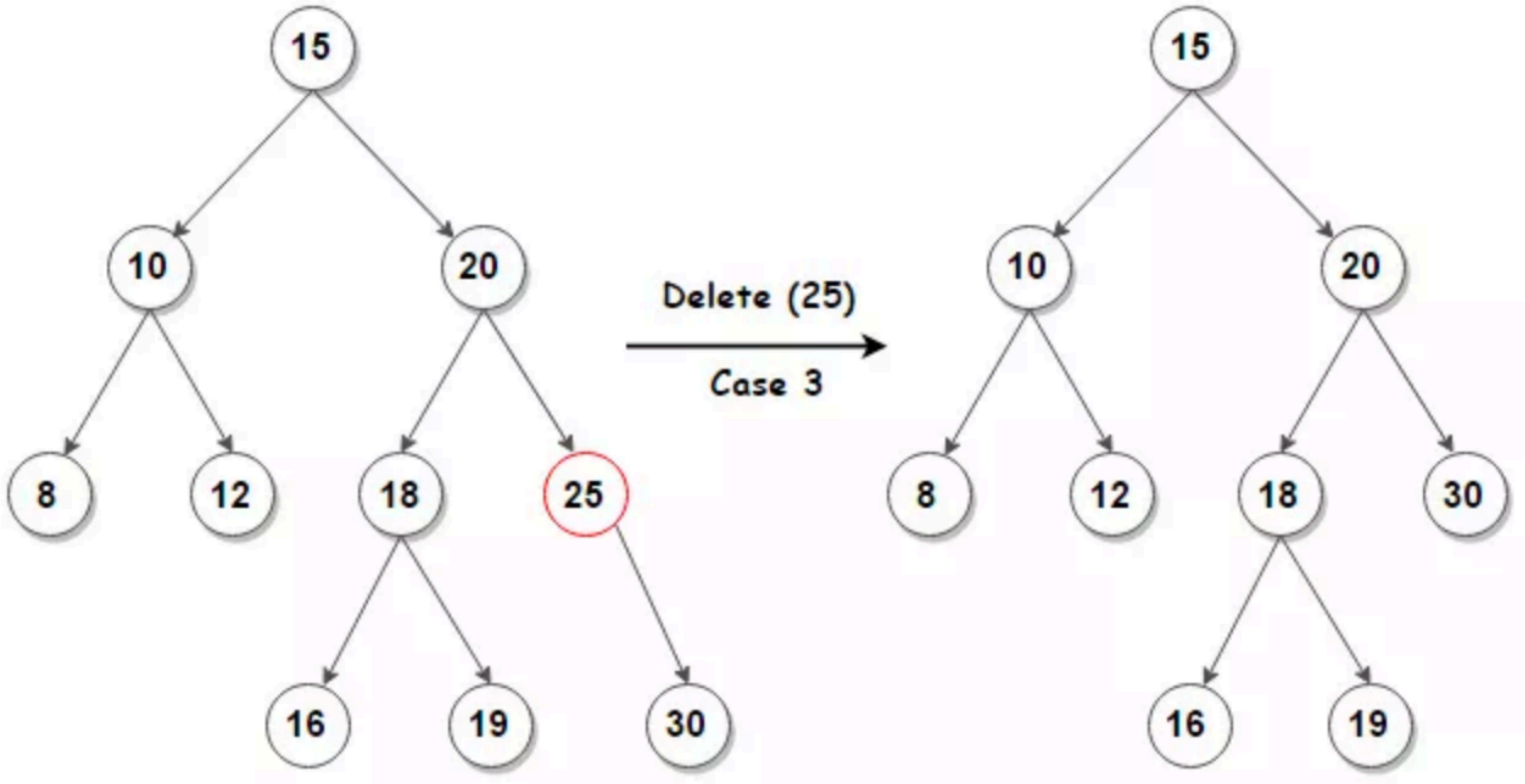
Example of BST Deletion:



Predecessor

Courtesy: https://www.techiedelight.com/deletion-from-bst/

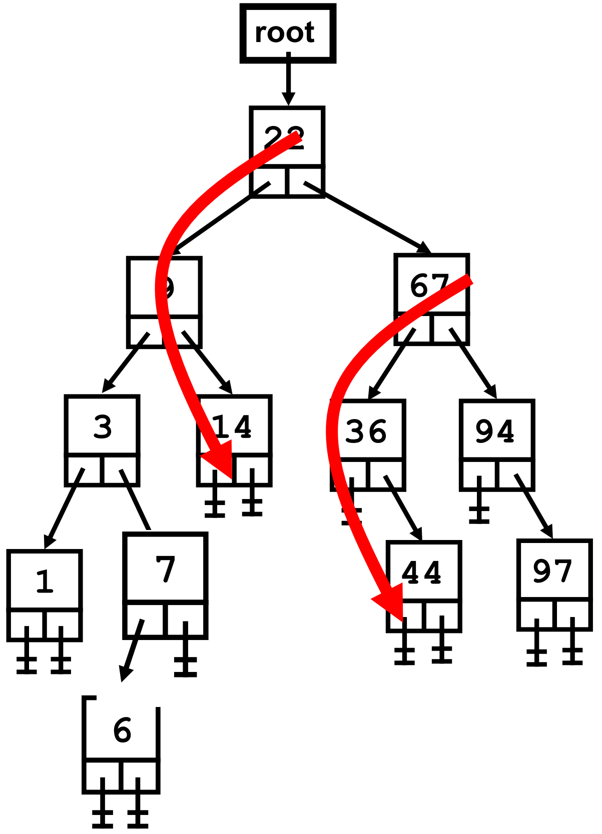
Example of BST Deletion:



Courtesy: https://www.techiedelight.com/deletion-from-bst/

Find the Predecessor Of Given Node

- **Predecessor of V: is the smallest value larger than V**
- **How to find it**
  - **Go to the LEFT child node**
  - **Follow the RIGHT link all the way (until None)**
- **Predecessor of 22**
  - **14**
- **Predecessor of 67**
  - **44**



Time complexity for finding the predecessor is O(h), h is the tree height

Priority Queues

Heaps (RECAP)

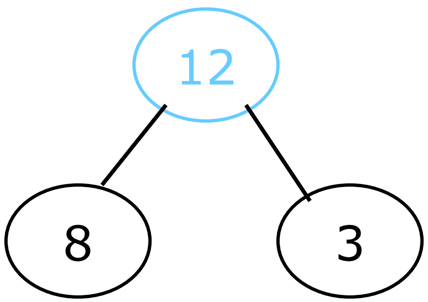
- A Binary Tree with a specific property like BST – but different, of course.

- Heaps (MAX-HEAPS or MIN-HEAPS) were used as mental model for sorting in HeapSort

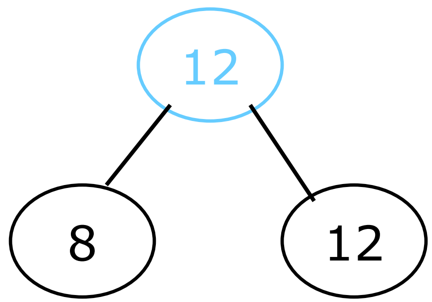
- The MAX-HEAPS/MIN-HEAPS can also be created in memory!

The MAX-Heap property (RECAP)

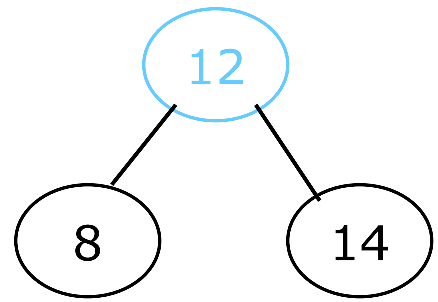
- A node has the **MAX-Heap property** if the **value in the node is >= the values in its children**



Blue node has MAX-heap property



Blue node has MAX-heap property



Blue node does not have MAX-heap property

- All **leaf nodes automatically have the MAX-Heap property**
- A **binary tree is a MAX-heap if *all* nodes in it have the MAX-heap property**
- You can similarly have MIN-heap(with the smallest element as parent)

MAX-Heap and Min-Heaps are generally called HEAPS

Priority Queues

- A queue where we add objects, each with a value ("priority").
- Priority queues are very common for *job scheduling*
- Two Types:
  - **Max-Priority Queue** ← we use MAX-HEAPS
  - **Min-Priority Queue** ← we use MIN-HEAPS

Operations on Priority Queues (Assume Max Queue)

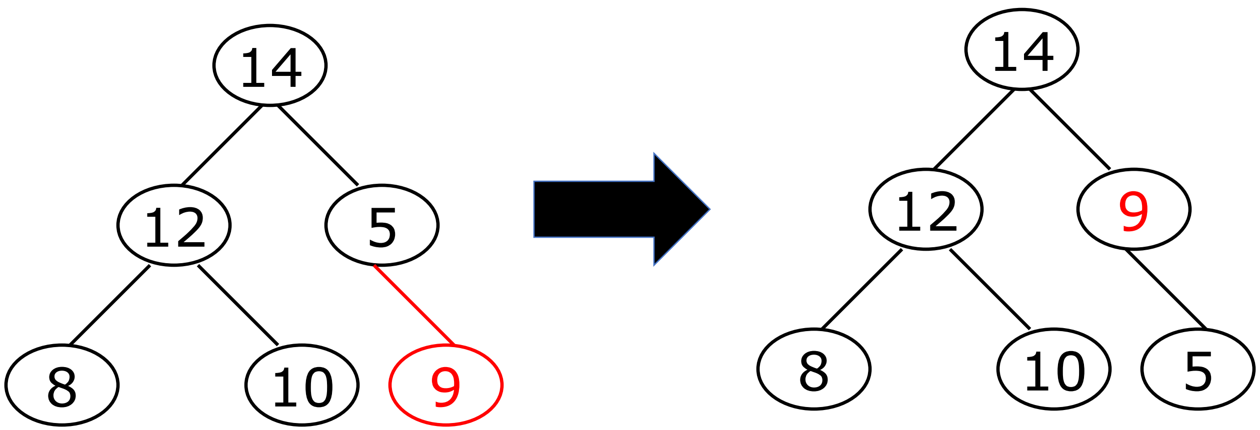
- 1 → **Add** a **new** object with priority K
- 2→ **Return** the object with the **highest** priority
- 3 → **Remove** the object with the **highest** priority
- 4 → **Increase** the **priority** of object O

Heap data structure can implement all these operations efficiently

1- Add New Object With Priority 9

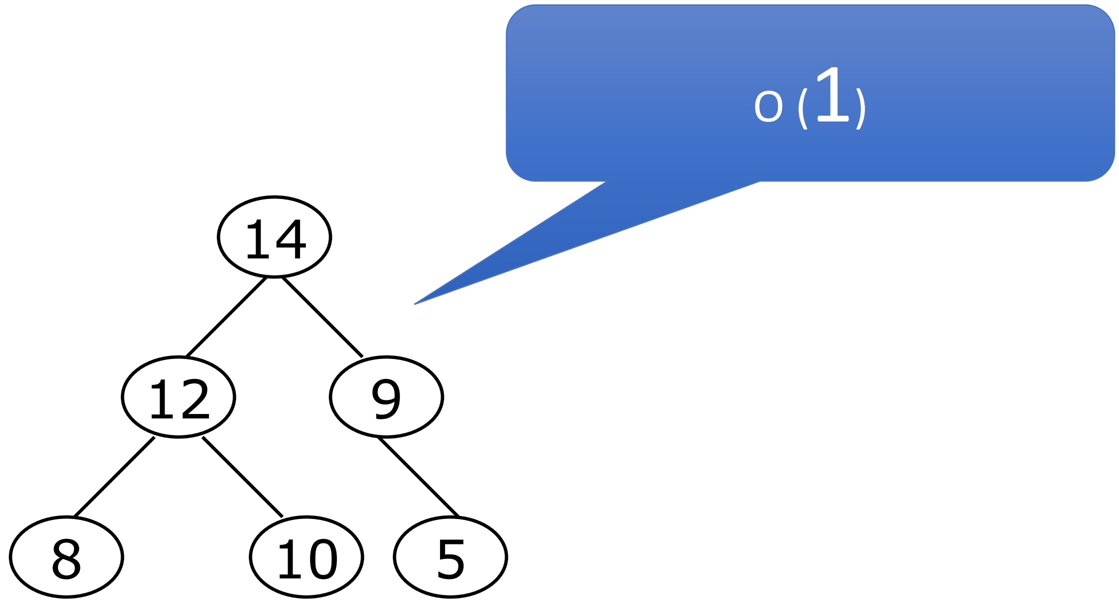
- Add the object to the heap
- Check parent and move node **upward iteratively**

O (Log n)



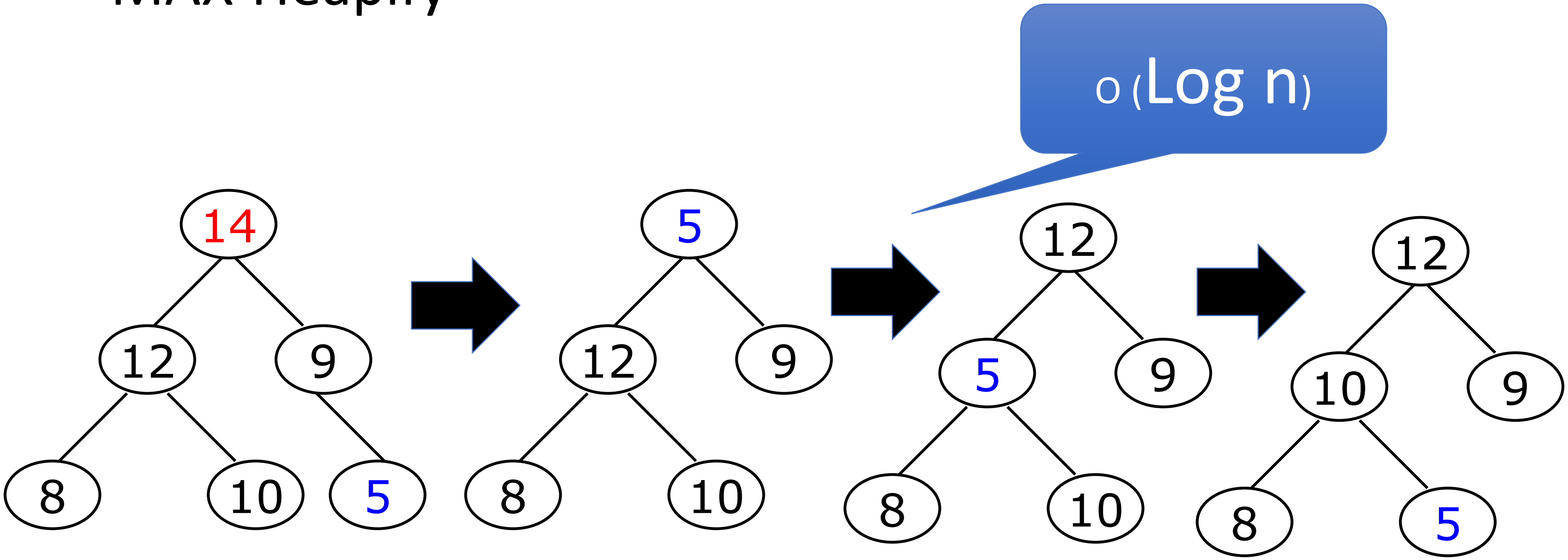
2- Return the Highest-Priority Object

- Return the root of the tree
- Same as: Return the first element in the Heap array
- In our example, return 14



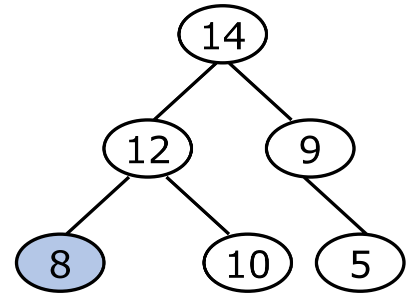
3- Remove the Highest-Priority Object

- Remove the root of the tree
- Replace it with the lowest right-most object
- MAX-Heapify

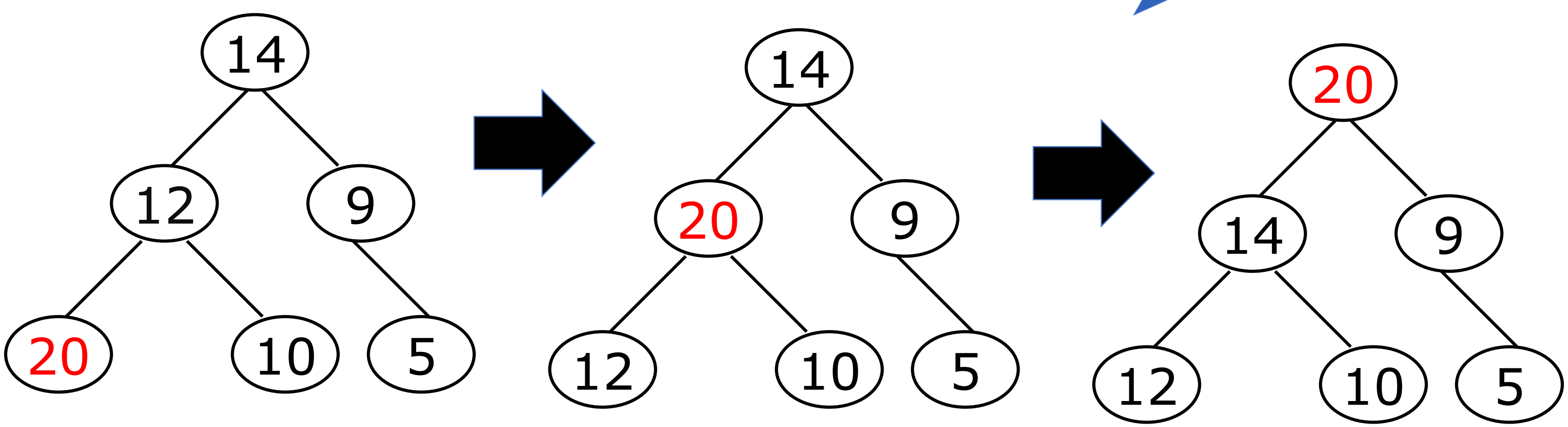


4- Increase the Priority of Object O

- Change 8 to 20
- Check parent and move node **upward iteratively**



O (Log n)



Next Class…

- Quiz 4 will be on Tuesday (Nov 12)

- Quiz 4 will cover all the materials from Nov 5 and Nov 7 (inclusive)


That's all Folks!
Any Question ?