Stacks, Queues, Linked-Lists

Instructor: Krishna Venkatasubramanian

CSC 212

Announcements

- Assignment 2 release Out.

- Next Quiz next Tuesday (Nov 12)

Dealing with data…

- How to use it ?
- How to store it ?
- How to process it ?
- How to gain “knowledge” from it ?
- How to keep it secret?

How should data be stored?

Depends on your requirement

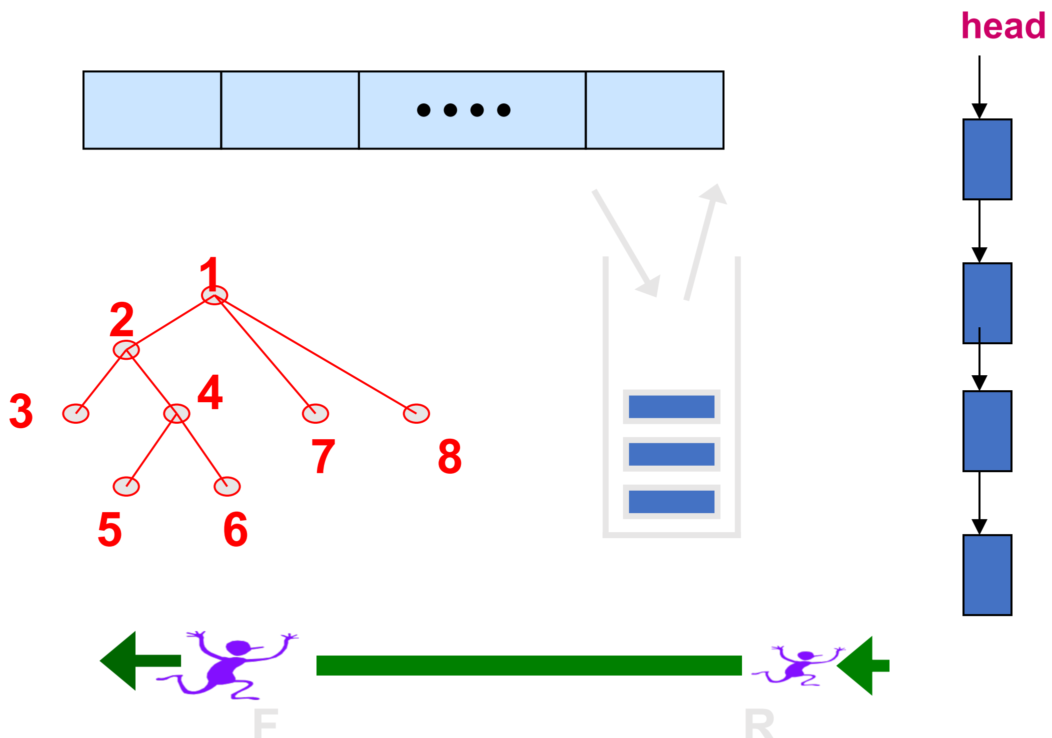
"We back up our data on sticky notes because sticky notes never crash."

Data is diverse ..

But we have some building blocks

Elementary Data “Structures”

- **`Arrays`**
- **`Lists`**
- **`Stacks`**
- **`Queues`**
- **`Trees`**



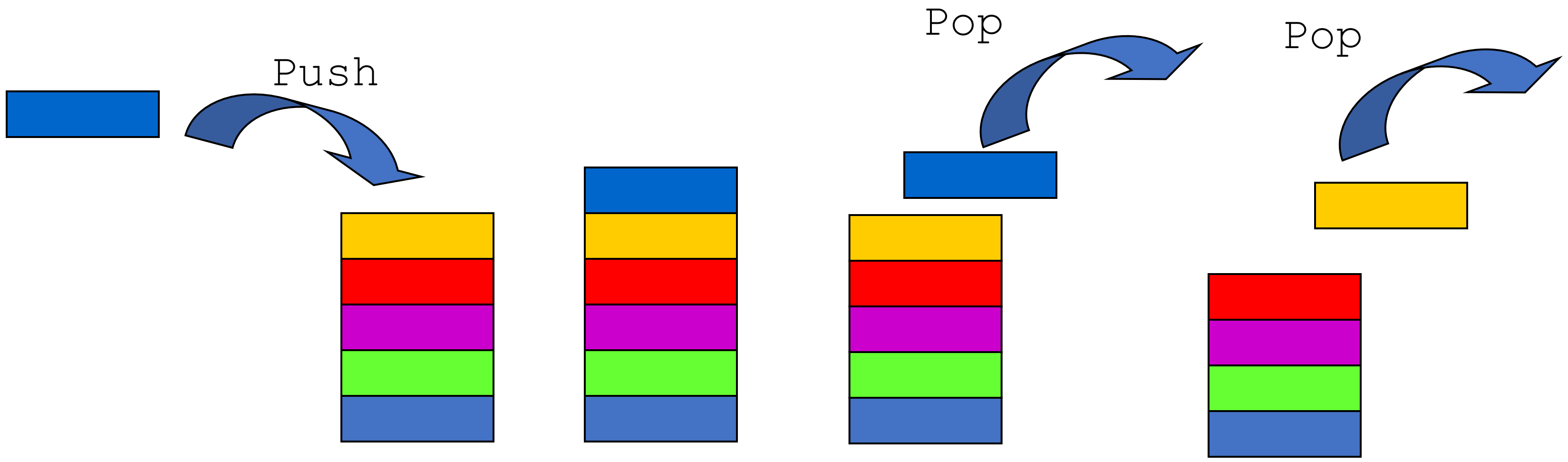
In some languages these are basic data types – in others they need to be implemented

Stacks

Stack

A list for which Insert and Delete are allowed only at one end of the list (the *top*)

- LIFO – Last in, First out



Stack Example: Postfix Calculation

- Postfix notation places operator after operands
  - Infix: (3 + 2)* 10
  - Postfix: 3 2 + 10 *
- **Postfix Advantages:**
  - Do not need expressions to be parenthesized
    - Actually: It does not need any parentheses as long as each operator has a **fixed number of operands**.
    - Example:
      - An operator like "+ (addition)" needs two operands (numbers) that it can add.
      - A+B. here A and B are operands and + is the operator
  - Fewer operations need to be entered to perform typical calculations
  - Make fewer errors

Stack Example: Postfix Calculation

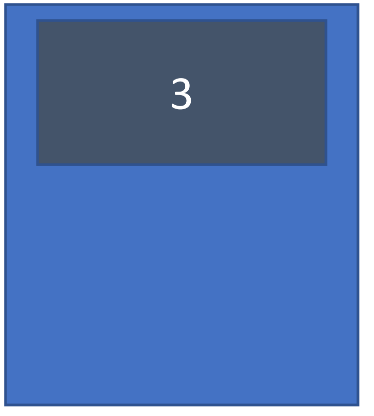
- **Postfix notation** places operator after operands
  - Infix: (3 + 2)* 10
  - Postfix: 3 2 + 10 *

- **A Stack can be used to calculate Postfix notations**
  - Push symbols as they appear
  - Whenever we read an operator, **pop two operands**
  - Evaluate operation, push result

Stack Example

A Stack Data Structure

3 2 + 10 *

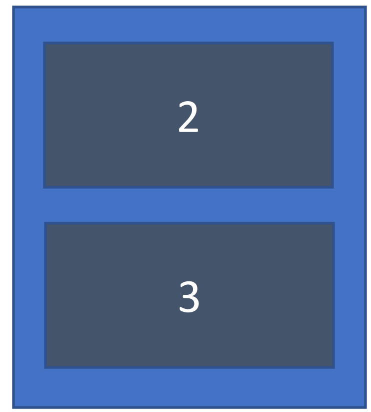


Push element

Stack Example

A Stack Data Structure

3 2 + 10 *



Push element

Stack Example

A Stack Data Structure

3 2 + 10 *

Pop two element

Stack Example

A Stack Data Structure

3

2

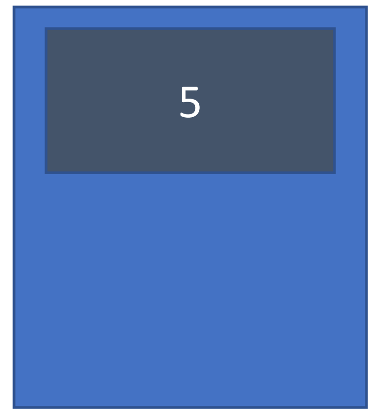
3 2 + 10 *

Pop two element

Stack Example

A Stack Data Structure

3 2 + 10 *

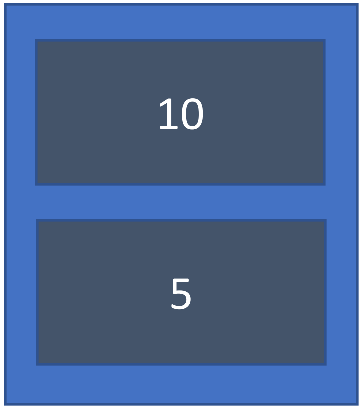


Perform operation and push result back

Stack Example

A Stack Data Structure

3 2 + 10 *

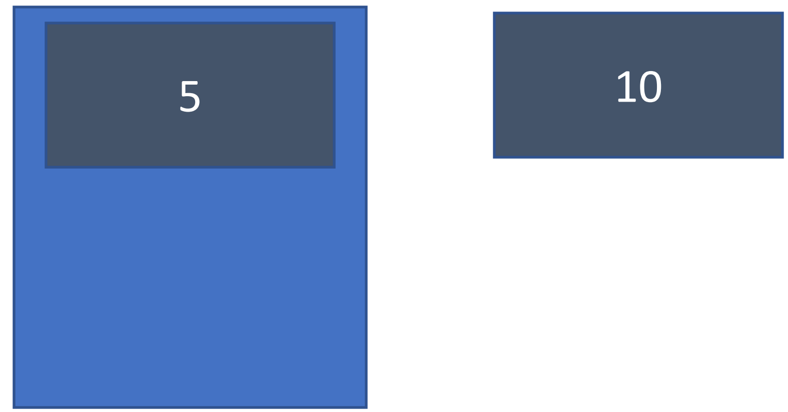


Push element

Stack Example

A Stack Data Structure

3 2 + 10 *

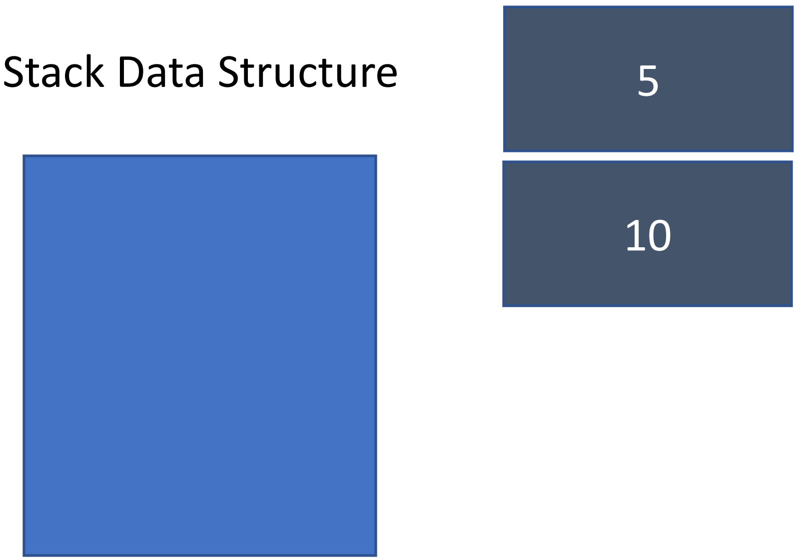


Pop two element

Stack Example

A Stack Data Structure

3 2 + 10 *

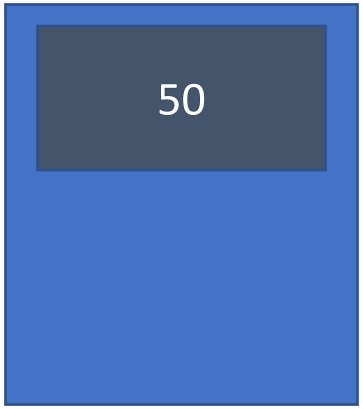


Pop two element

Stack Example

A Stack Data Structure

3 2 + 10 *



Perform operation and push result back

Exercise

- At home draw out the stack to solve the following Postfix expression

$$15\ 7\ 1\ 1 + - \div 3 \times 2\ 1\ 1 + + -$$

- The answer you should get is **5**

What else is this good for ?

- Page-visited history in a Web browser
- Undo sequence in a text editor
- Saving local variables when one function calls another, and this one calls another
- Matching parenthesis in text and code

Abstract data types and Data Structures

Data Structure
a way to store and organize data to facilitate access and modifications.

Ex. array, linked list, later in the course: hash table, heap, …

Abstract Data Type (ADT)
a set of data values and associated operations that are precisely specified independent of any particular implementation.

Ex. stack, queue, … later in the course: dictionary,…

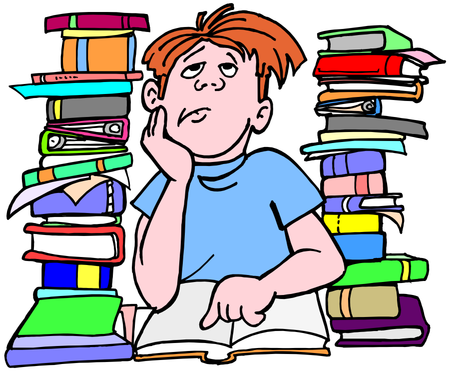
- ADT describe the functionality of data structures
- Data structures are implemented as ADT
  - how is the data stored?
  - which algorithms implement the operations?

Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure

- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

Stack ADT

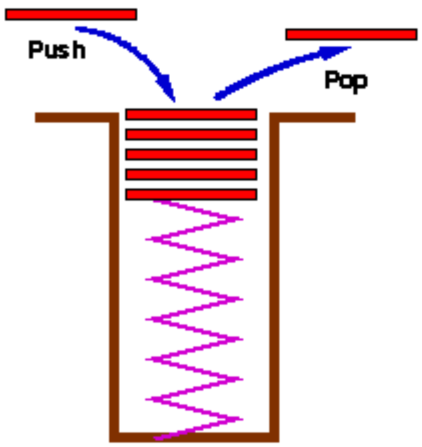


Objects:

A finite sequence of nodes

Operations:

- create
- push: Insert element at top
- top: Return top element
- pop: Remove and return top element
- isEmpty: test if the stack is empty



Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be “thrown” by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException

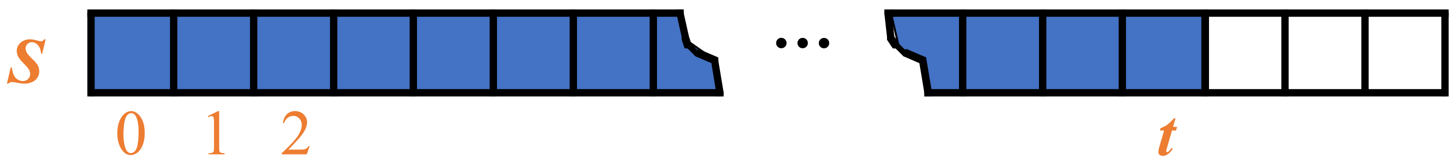
Exercise: Stacks

- Describe the output of the following series of stack operations
  - Push(8)
  - Push(3)
  - Pop()
  - Push(2)
  - Push(5)
  - Pop()
  - Pop()
  - Push(9)
  - Push(1)

Stack Visuzalization

https://www.cs.usfca.edu/~galles/visualization/StackArray.html

Array-Based Stack



- we add elements from left to right
- a variable keeps track of the index of the top element

```
class CapacitatedStack:
      def __init__(self):
            self.capacity = 20
            self.items = self.capacity*[None]
            self.count = 0
```

- capacity of stack is capped to size of array

Array-Based Stack

```
class CapacitatedStack:
    def __init__(self):
        self.capacity = 20
        self.items = self.capacity*[None]
        self.count = 0
```

- How can we implement the operations **size**, **isEmpty**, **push** and **pop**?

```
def size(self):
    return self.count
```

```
def isEmpty(self):
    return self.count == 0
```

```
def push(self, item):
    if self.count < self.capacity:
            self.items[self.count] = item
            self.count += 1
    else: raise Exception("CapacitatedStack overflow.")
```

```
def pop(self):
    if self.count > 0:
            self.count -= 1
            return self.items[self.count]
    else:
            raise Exception("Cannot pop from empty stack")
```

What are running times of these operations?

Growable Array-based Stack

- Fixed-capacity stack: fast but not very useful

- How can we make an array-based stack that has unlimited capacity?
  - **Incremental strategy**: increase the size of the array by a constant c when capacity is reached
  - **Doubling strategy**: double the size of the array when capacity is reached

- **Problem: arrays cannot be resized. You can only copy over elements to a new array**

Growable Array-based Stack

```
def push(self, item):
    if self.count >= self.capacity:
        # doubling, incremental would be += ...
        self.capacity *= 2
        copy_items = self.capacity*[None]
        for i in range(self.count): copy_items[i] = self.items[i]
        self.items = copy_items

    self.items[self.count] = item
    self.count += 1
```

What's the runtime of push?

- when the stack doesn't expand?

  O(1)

- when it does expand?

  Incremental: **O(n)**

  Doubling: **O(n)**

  WHY? : need to copy the current elements in the stack (array) to the new one

Stacks/Growable Arrays in Python

- The list data type in Python is based on a growable array with doubling strategy --- can be used to build Stack ADT

s.append(x)	PUSH	Can be used to implement appends *x* to the end of the sequence, here, end of the list is the top of the stack
s.pop()	POP	Retrieves and removes the item from the end of list
s == []	IS_EMPTY	Checks if stack is empty
s[-1]	TOP	Gets the item from the end of the list

- The Stack ADT operations :
  - push/ isEmpty/ top / pop can be performed in ➔ O(1)

Complexity of implementing Python data structures

https://www.ics.uci.edu/~brgallar/week8_2.html

Queues

Queues

Queue

Stores a set S of elements with insertions and deletions follow a FIFO (first-in, first-out) scheme

Operations for a Queue ADT

enqueue(S, x): inserts element x into S

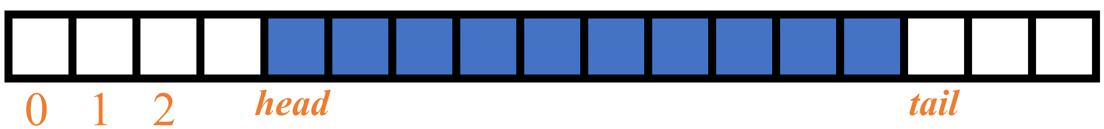
dequeue(S): removes and returns the element first inserted into S

size(S): returns the number of elements in S

isEmpty(S): indicates whether S is the empty set

Growable Array-based Queue

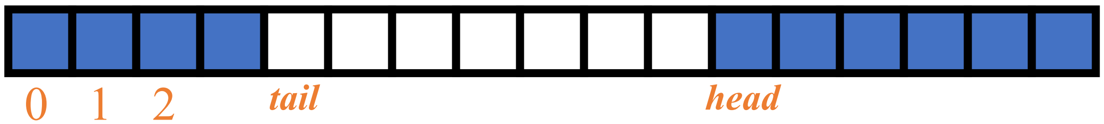
- We can also implement a queue using an growing array, but with a slight complication
- Unlike a stack, we need to **keep track of the head and the tail** of the queue



- What happens if the tail reaches the end of the array, but there's still room at the front? Is the queue full?

Growable Array-based Queue

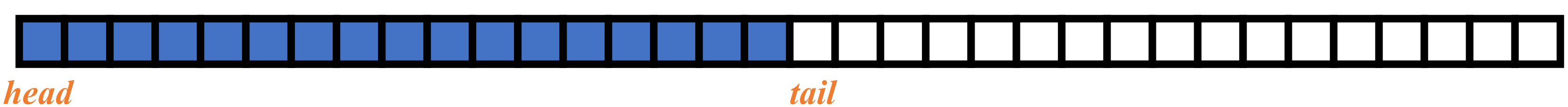
- **Wrap the queue!**



- **Expand the array when queue is completely full**
  - When copying, “unwind” the queue so the head starts back at 0

```
enqueue(x):
  if size == capacity:
    double array and copy contents
    reset head and tail pointers
  data[tail] = x
  tail = (tail + 1) % capacity
  size++
```

```
dequeue():
  if size == 0:
    error(“queue empty”)
  element = data[head]
  head = (head + 1) % capacity
  size--
  return element
```



Queues in Python

- Do Python lists provide an efficient implementation of Queues if used "directly"?

- No:
  - enqueue(x): s.append(x) in O(1) time, but
  - dequeue(): s.pop(0) in O(n) time **[Not the same as s.pop() used in a stack. Notice the index in the call to pop]**

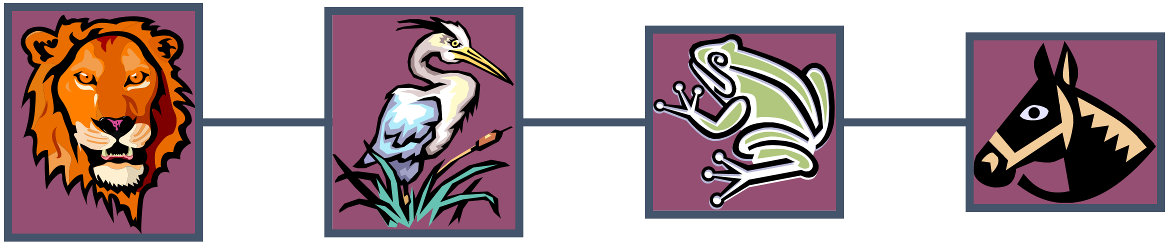
- Deques (double-ended Queues) are provided as collections.deque

```
from collections import deque
S = deque([2, 3, 5])
S.append(7)
S.popleft()
S
```

```
deque([3, 5, 7])
```

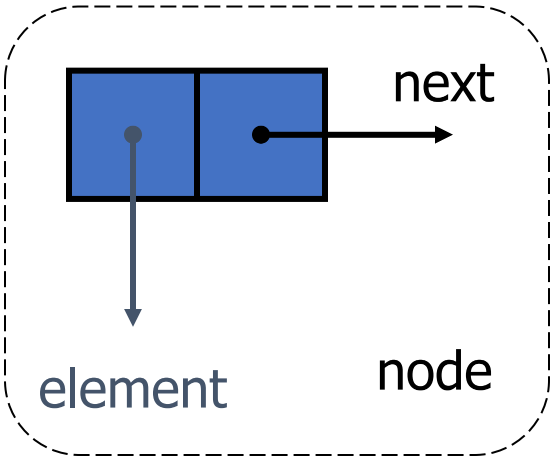
- deques are not implemented using arrays but doubly-linked lists

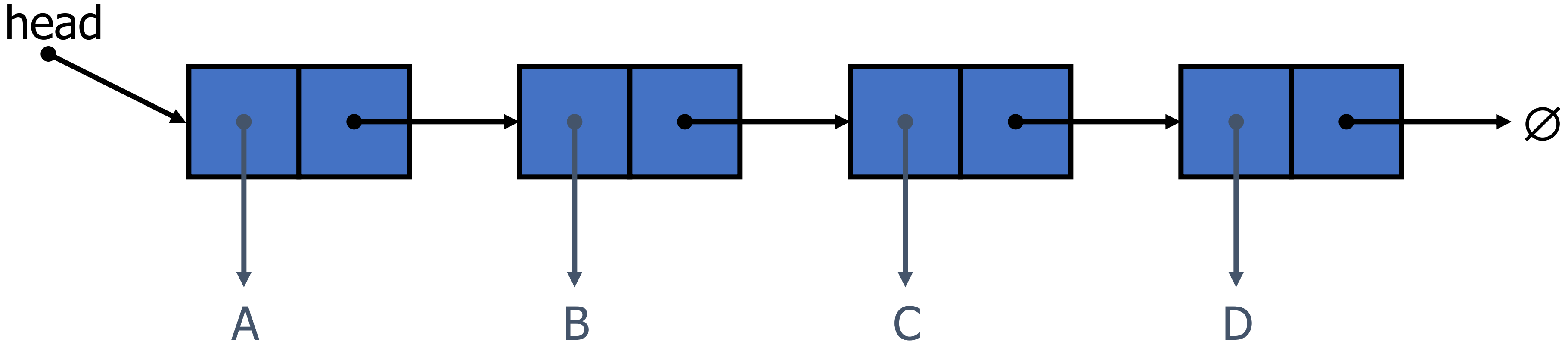
Linked Lists



Singly Linked List

- singly linked list: data structure consisting of
  - a sequence of nodes,
  - starting from a head pointer
- each node stores
  - element
  - **link to the next node** (use pointers with C/C++)
- Nodes are not adjacent in memory like an array
- **Elements** of nodes can be combination of multiple data-types unlike arrays!





Implementing a Singly Linked List

```
class SNode:
    def __init__(self, elem=None, next=None):
        self.elem = elem
        self.next  = next

class SList:
    def __init__(self):
        self.head = None
```

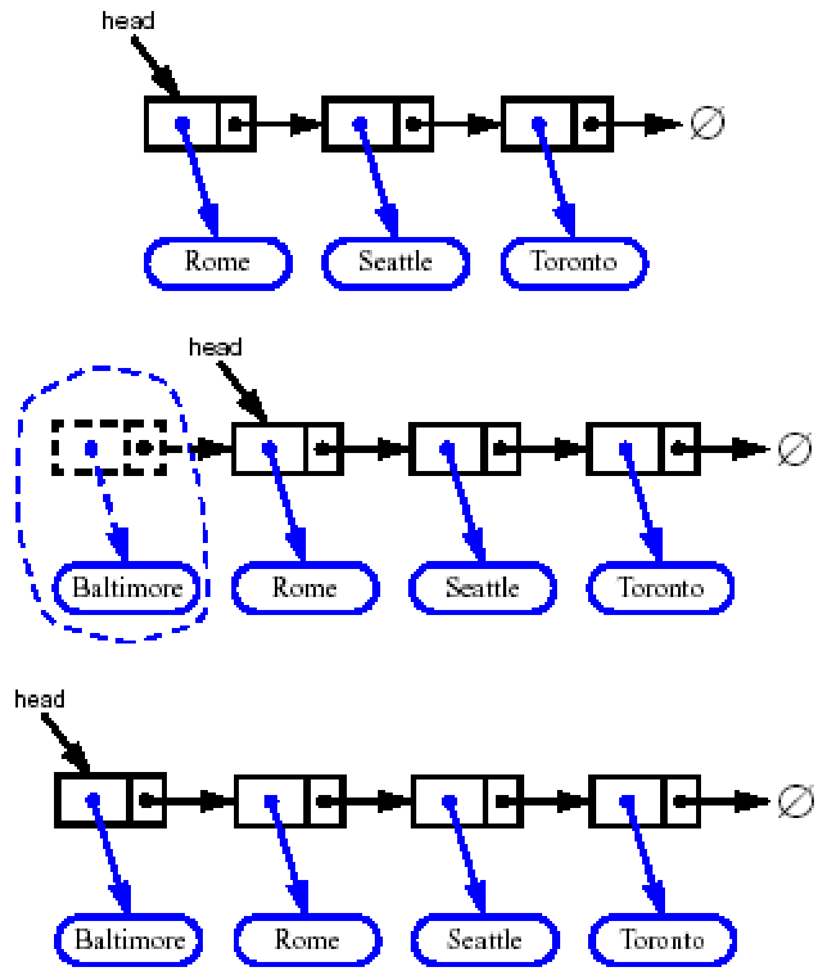
```
node3 = SNode("Toronto")
node2 = SNode("Seattle", node3)
node1 = SNode("Rome", node2)

list = SList()
list.head = node1

currentNode = list.head
while (currentNode):
    print(currentNode.elem)
    currentNode = currentNode.next
```

```
Rome
Seattle
Toronto
```

How do we insert/delete efficiently in a Singly Linked List?

Inserting at the Head

1. Allocate a new node
2. Insert new element
3. Have new node point to old head
4. Update head to point to new node




```
newNode = SNode("Baltimore")
newNode.next = list.head
list.head = newNode

printList(list)
```

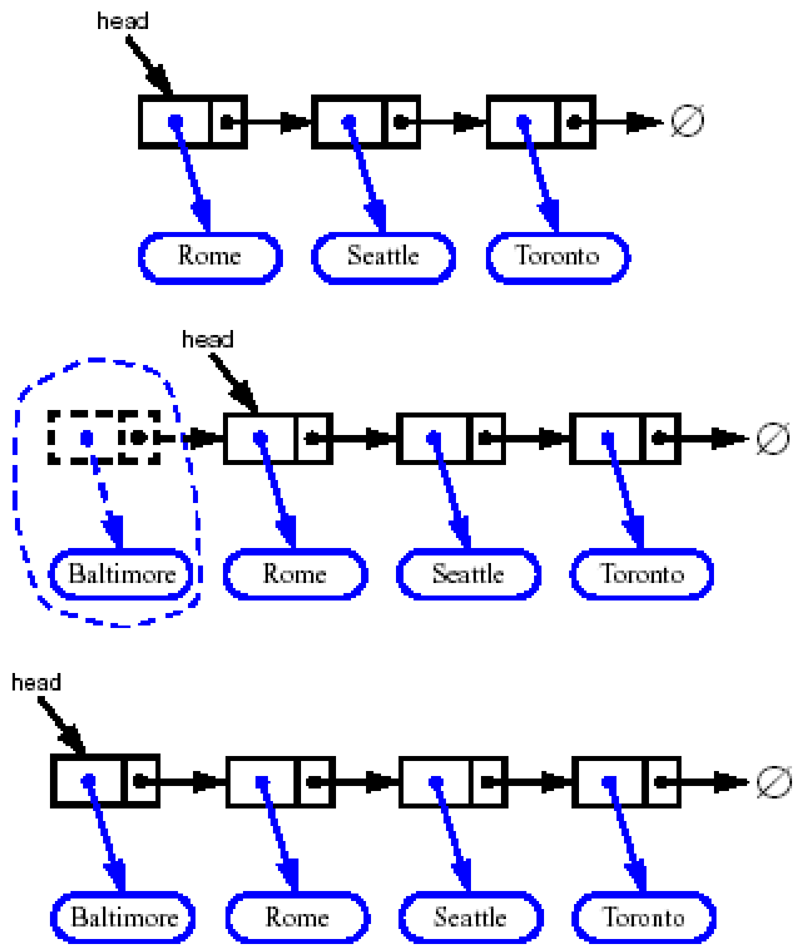
```
Baltimore
Rome
Seattle
Toronto
```

Removing at the Head

1. Update head to point to next node

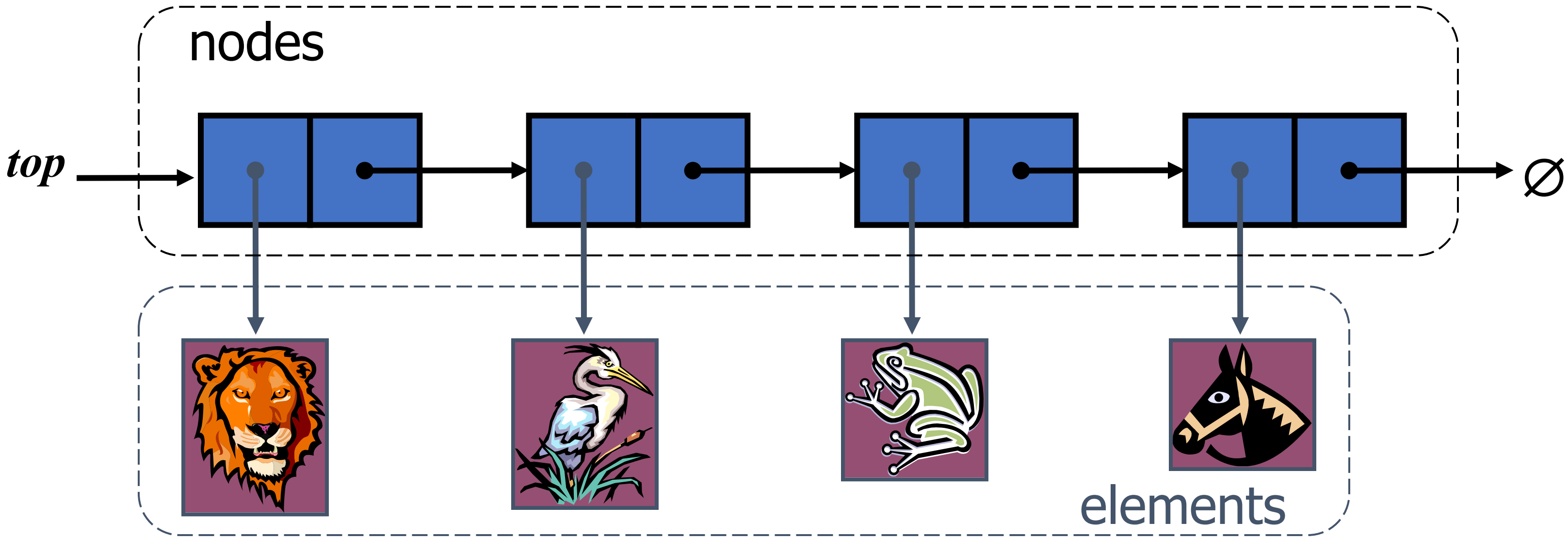
```
list.head = list.head.next
printList(list)
```

```
Rome
Seattle
Toronto
```



Stack as Singly Linked List

- **top element at head (i.e., the head is called top, when we are dealing with stack)**



- The space used is $O(n)$ and each operation of the Stack ADT takes $O(1)$ time

Stack as a Singly Linked List

```
class Stack:
     def __init__(self):
         self.list = SList()
         self.count = 0

     def isEmpty(self):
         return self.count == 0

     def push(self, item):
         newNode = SNode(item, self.list.head)
         self.list.head = newNode
         self.count += 1

     def pop(self):
         if self.isEmpty():
             raise Exception('stack is empty.')
         else:
             item = self.list.head.elem
             self.list.head = self.list.head.next
             self.count -= 1
             return item

     def size(self):
         return self.count
```

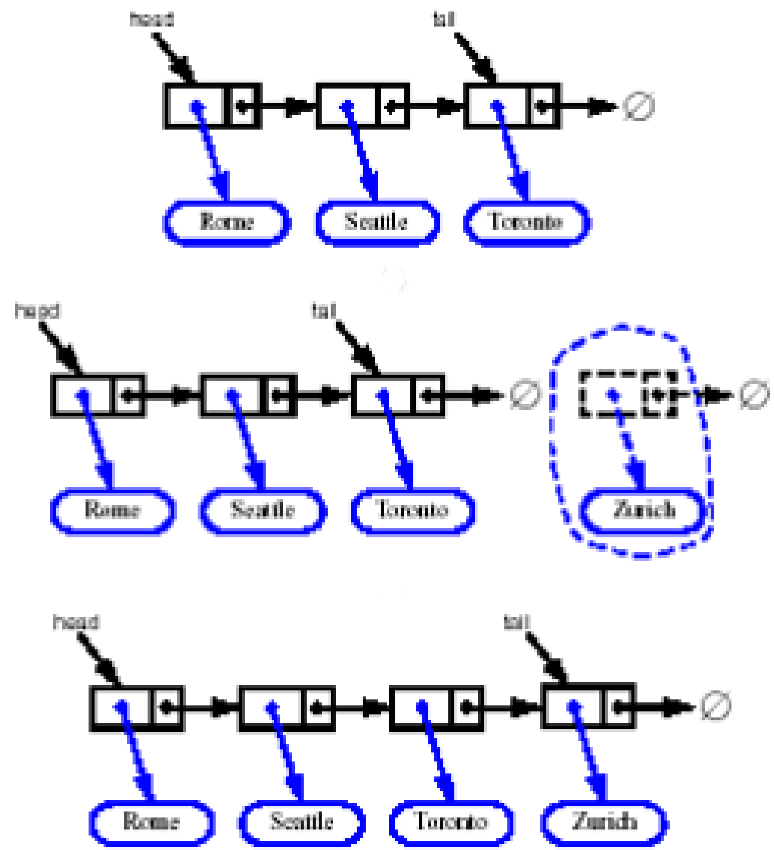
Stack: Defined in terms of the Singly Linked List

Push: Add to the head of the list

Pop: Remove to the head of the list
Move the head

Inserting at the Tail (at the End)

1. Allocate a new node
2. Insert new element
3. Have new node point to null
4. Have old last node point to new node
5. Update tail to point to new node

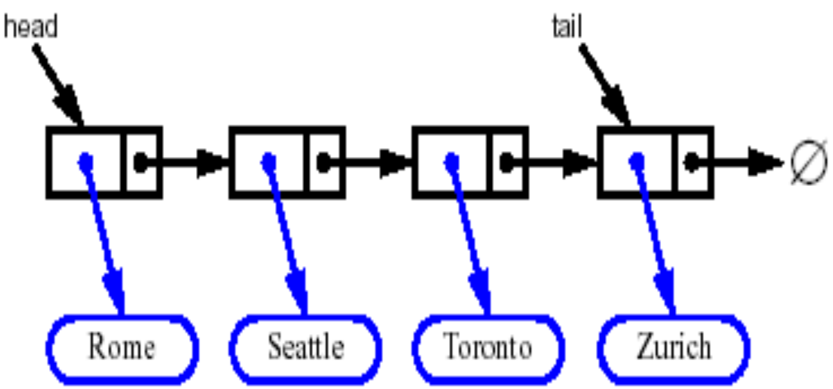


- **requires pointer to tail: list.tail**
  - Complexity of appending to tail = O(1)

Removing at the Tail ?!

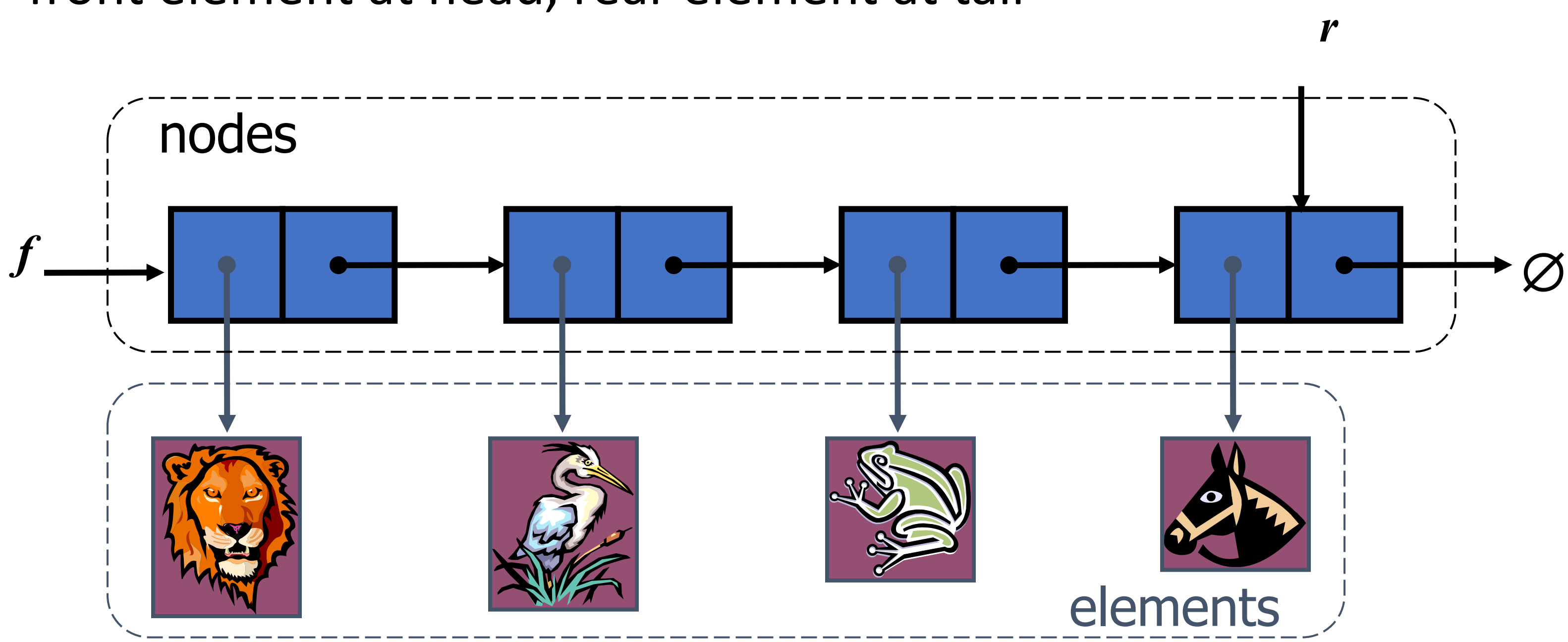
- **no constant-time way to update the tail to point to the previous node. WHY?**

- removing at the tail of a singly linked list is not efficient!
  - Once has to traverse the whole length of the list
  - Complexity O(n)



Queue as Singly Linked List

- front element at head, rear element at tail



- The space used is $O(n)$ and each operation of the Queue ADT takes $O(1)$ time

Queue as Singly Linked List

```
class Queue:
    def __init__(self):
        self.head = None
        self.tail = None
        self.count = 0

    def isEmpty(self):
        return self.count == 0

    def enqueue(self, elem):
        newNode = SNode(elem)
        if self.isEmpty():
            self.head = newNode
            self.tail = newNode
        else:
            self.tail.next = newNode
            self.tail = newNode
        self.count += 1
```

How do we implement a Deque (pop/push at both ends)?

```
    def dequeue(self):
        if self.isEmpty():
            raise Exception('Queue is empty.')
        else:
            elem = self.head.elem
            if self.head.next:
                self.head = self.head.next
            else:
                self.head = None
                self.tail = None
            self.count -= 1
            return elem

    def size(self):
        return self.count
```

```
Q = Queue()
Q.isEmpty()
```

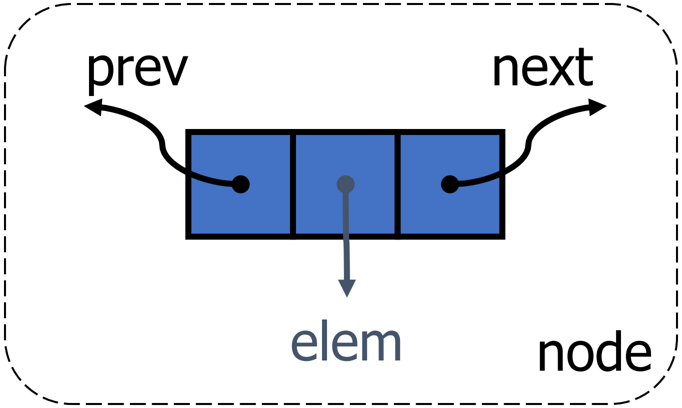
```
True
```

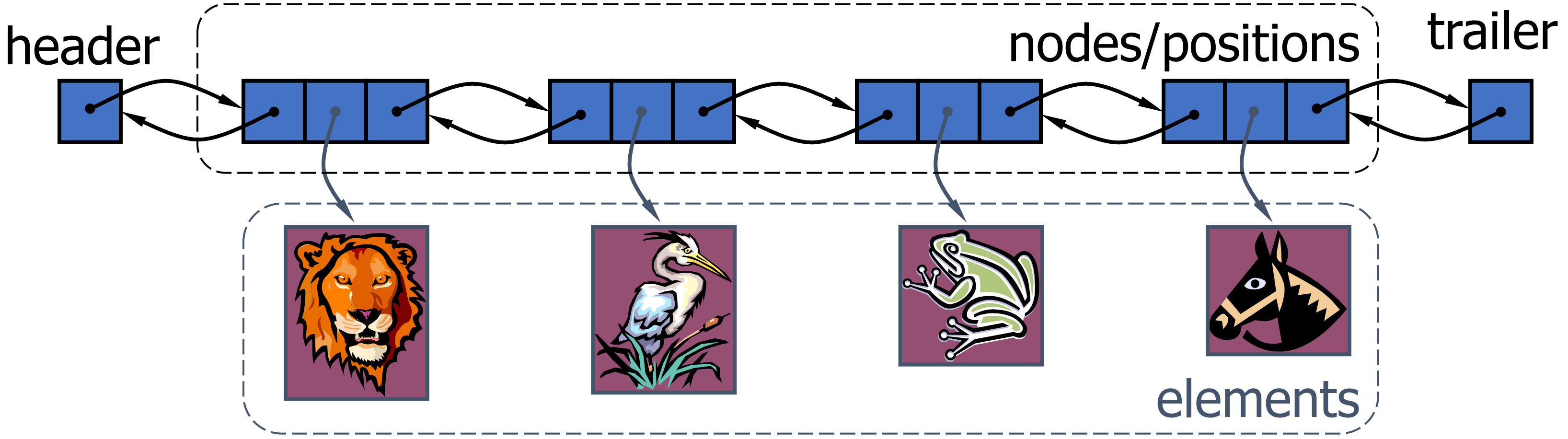
```
Q.enqueue('lion')
Q.enqueue('bird')
Q.enqueue('frog')
Q.enqueue('horse')
Q.dequeue()
```

```
'lion'
```

Doubly Linked List

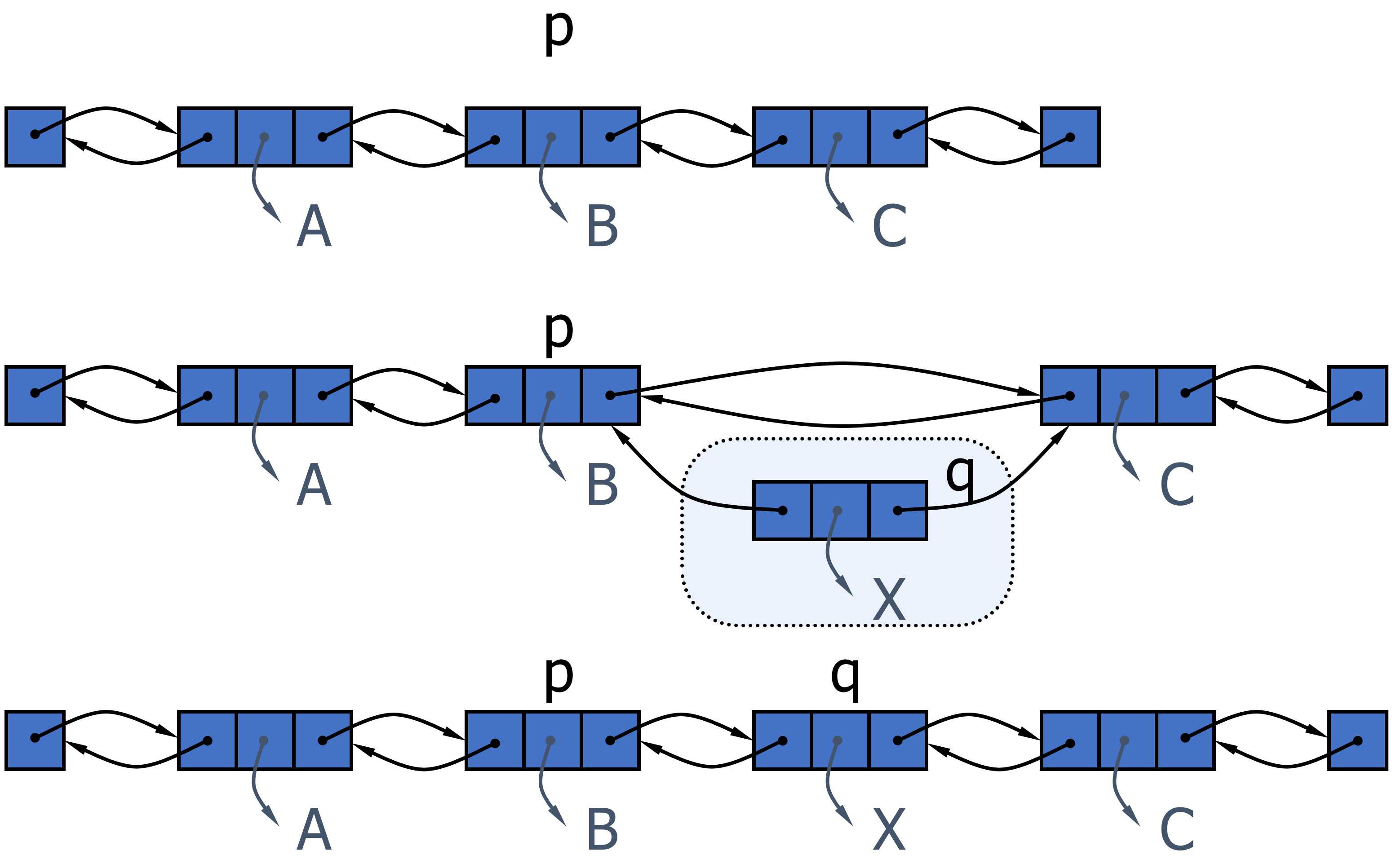
- each node stores
  - element
  - link to next node
  - **link to previous node**
- special trailer and header nodes





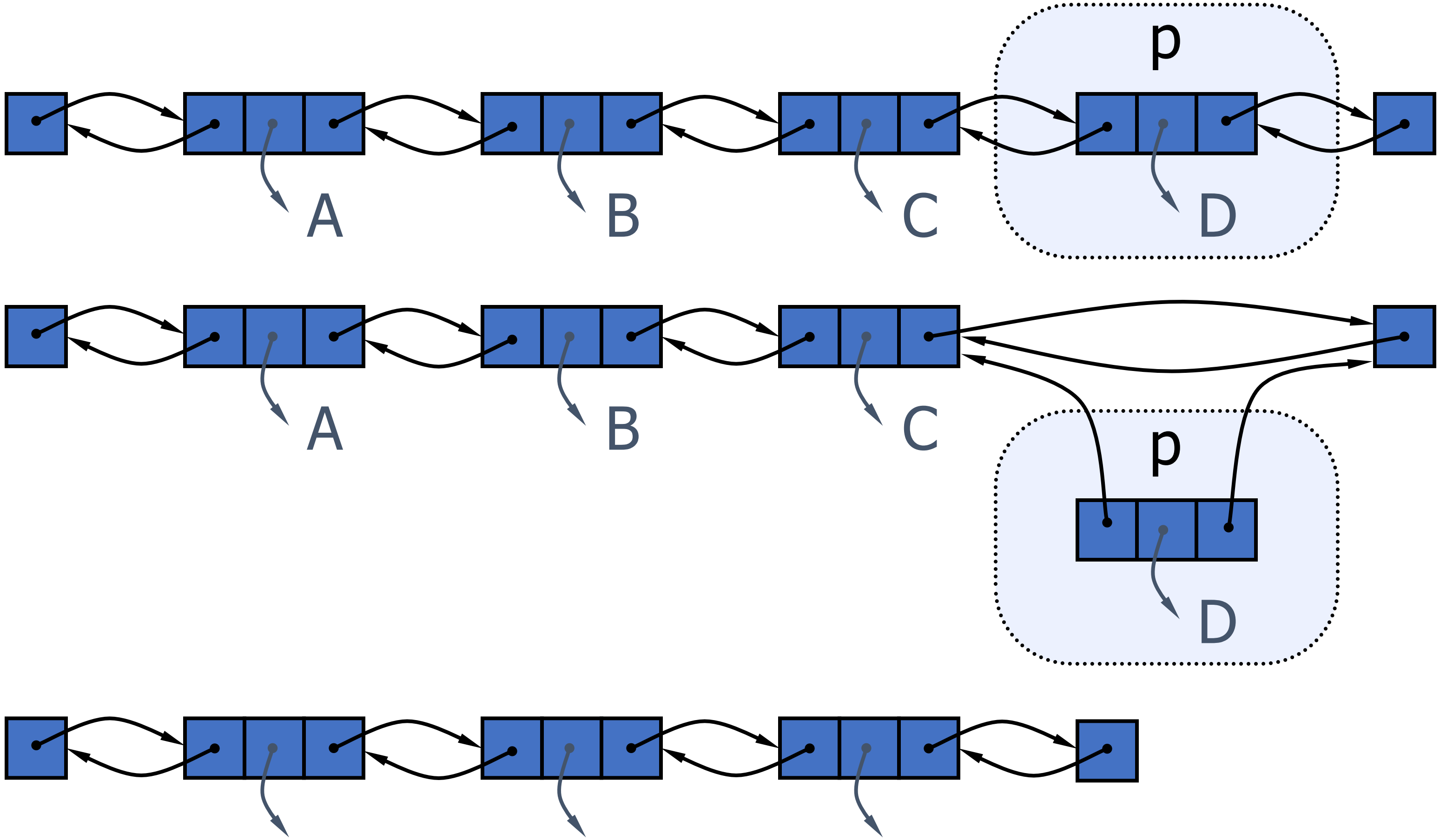
Insertion

- **Insert a new node, q, between p and its successor.**



Deletion

- Remove a node, p, from a doubly-linked list.



Summary

- ADTs we have seen so far
  - Stacks, Queues, (and Lists and Sets in Python)

- data structures for Stacks and Queues:
  - array, but fixed capacity
  - linked lists

- Singly Linked Lists vs Doubly Linked Lists


That's all Folks!
Any Question?