

Search: Linear and Binary

Instructor: Krishna Venkatasubramanian

CSC 212

Announcements

- No class next week, however the lab session will continue
- Assignment 2 release postponed to Nov 5, 2019
- Assignment 1 deadline extended to Oct 31, 6pm (NOTE THE TIME)
- MY office hours cancelled today

Searching

- Given a **collection** and an **element (key)** to find...
- Output
 - **Print a message** (“Found”, “Not Found”)
 - **Return a value** (position of key)
- **Don't modify** the collection in the search!

Searching Example (Linear Search)

Linear Search: A Simple Search

- A search **traverses** the collection until
 - The desired element is **found**
 - Or the collection is **exhausted**
- If the collection is **ordered**, I might not have to look at all elements
 - I can **stop looking when I know the element cannot be in the collection.**

Un-Ordered Iterative Array Search

Search(A, target)

Scan the array

```
for i = 1 ... N:  
    if A[i] = target:  
        return "Found"
```

```
return ("Not found")
```

Pseudocode

Un-Ordered Iterative Array Search

Search(A, target)

```
for i = 1 ... N:  
    if A[i] = target:  
        return "Found"  
  
return ("Not found")
```

Pseudocode

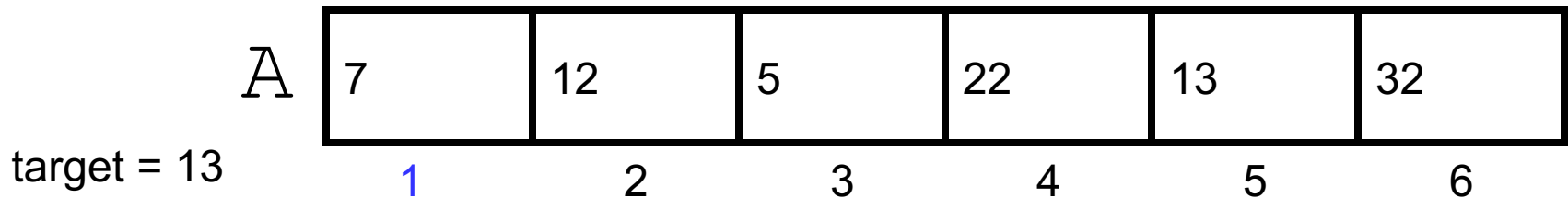
target = 13

A	7	12	5	22	13	32
	1	2	3	4	5	6

Un-Ordered Iterative Array Search

Search(A, target)

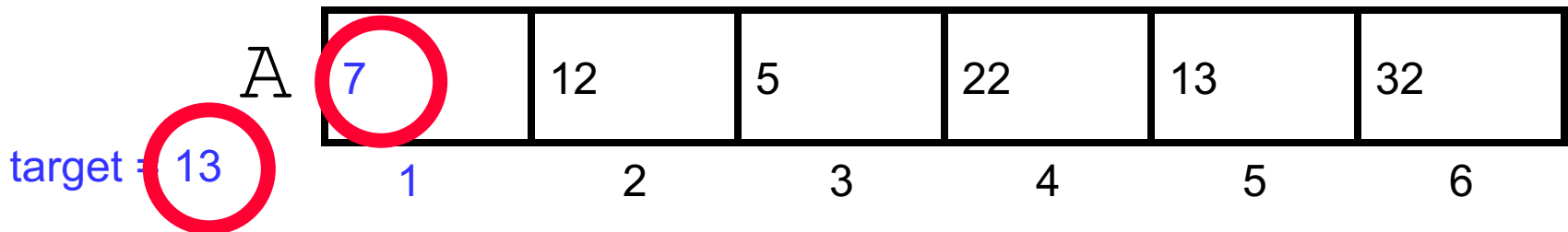
```
for i = 1 ... N: ←  
    if A[i] = target:  
        return "Found"  
  
return ("Not found")
```



Un-Ordered Iterative Array Search

Search(A, target)

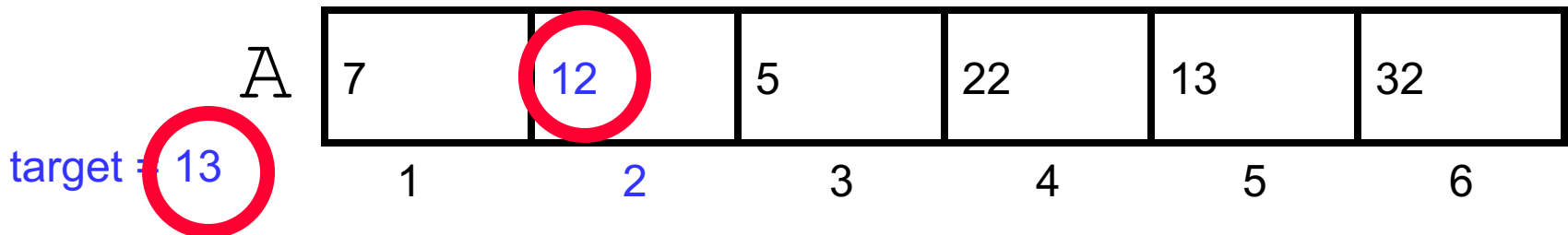
```
for i = 1 ... N:  
    if A[i] = target: ←  
        return "Found"  
  
return ("Not found")
```



Un-Ordered Iterative Array Search

Search(A, target)

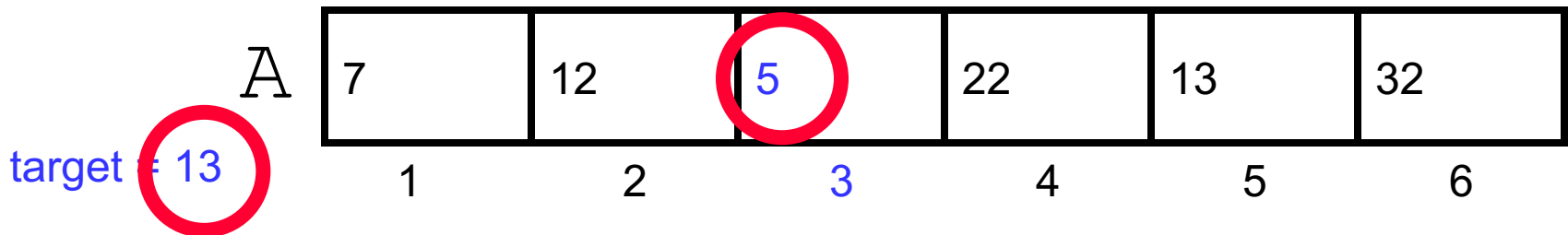
```
for i = 1 ... N:  
    if A[i] = target: ←  
        return "Found"  
  
return ("Not found")
```



Un-Ordered Iterative Array Search

Search(A, target)

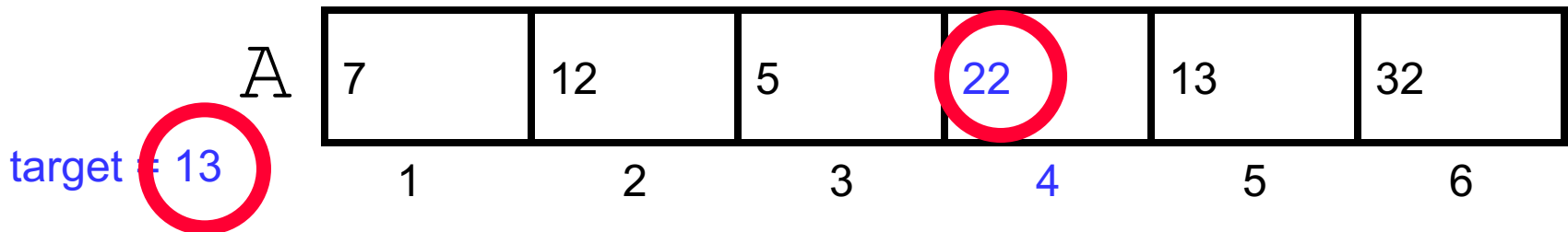
```
for i = 1 ... N:  
    if A[i] = target: ←  
        return "Found"  
  
return ("Not found")
```



Un-Ordered Iterative Array Search

Search(A, target)

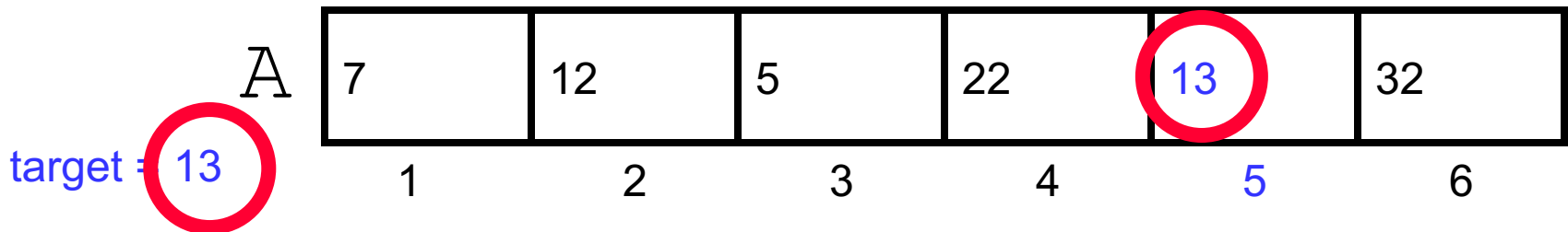
```
for i = 1 ... N:  
    if A[i] = target: ←  
        return "Found"  
  
return ("Not found")
```



Un-Ordered Iterative Array Search

Search(A, target)

```
for i = 1 ... N:  
    if A[i] = target: ←  
        return "Found"  
  
return ("Not found")
```



Un-Ordered Iterative Array Search

Search (A, target)

```
for i = 1 ... N:  
    if A[i] = target:  
        return "Found"
```



Target data found

target = 13

A	7	12	5	22	13	32
	1	2	3	4	5	6

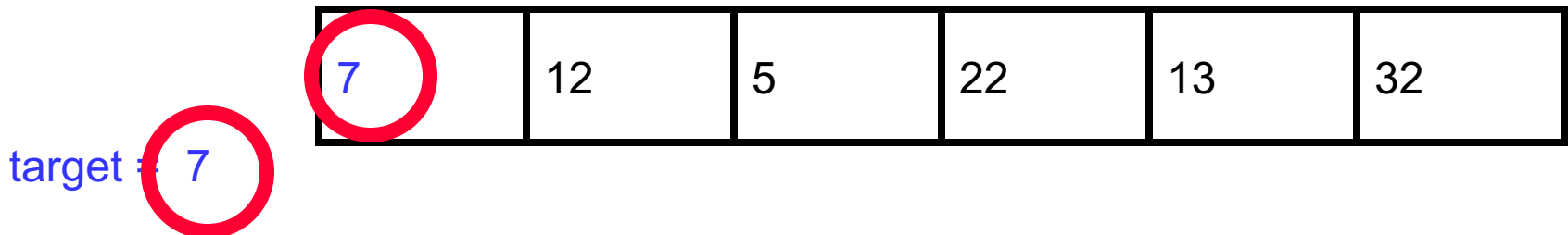
Linear Search Analysis: Best Case

Search(A, target)

```
for i = 1 ... N:  
    if A[i] = target:  
        return "Found"  
  
return ("Not found")
```

Best Case:
1 comparison

Best Case: match with the first item



Linear Search Analysis: Worst Case

Search(A, target)

```
for i = 1 ... N:  
    if A[i] = target:  
        return "Found"  
  
return ("Not found")
```

Worst Case:
N comparisons

Worst Case: match with the last item (or no match)

7	12	5	22	13	32
---	----	---	----	----	----

target = 32

Searching Example

(Binary Search on Sorted List)

The Scenario

- We have a **sorted array**
- We want to determine if a **particular element** is in the array
 - Once **found**, print or return (index, boolean, etc.)
 - If **not found**, indicate the element is not in the collection

7	12	42	59	71	86	104	212
---	----	----	----	----	----	-----	-----

A Better Search Algorithm

- Of course we **could use our simpler search** and traverse the array
- But we can use the fact that **the array is sorted** to our advantage
- This will allow us to **reduce the number of comparisons**

Binary Search

- Requires a **sorted array** or a **binary search tree**.

(We will see this later in the course)

- Cuts the “search space” **in half** each time.
- Keeps cutting the search space in half until the **target is found** or has **exhausted the all possible locations**.

Binary Search Algorithm

Searching for X

look at “middle” element

if no match then

look @ smaller half if $X < \text{middle}$

or @ larger half if $X > \text{middle}$

1	7	9	12	33	42	59	76	81	84	91	92	93	99
---	---	---	----	----	----	----	----	----	----	----	----	----	----

Look for 42

The Algorithm

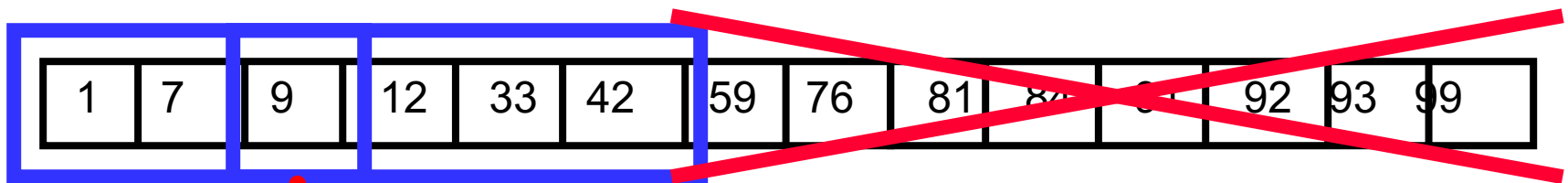
Searching for X

look at “middle” element

if no match then

look @ smaller half if $X < \text{middle}$

or @ larger half if $X > \text{middle}$



Look for 42

The Algorithm

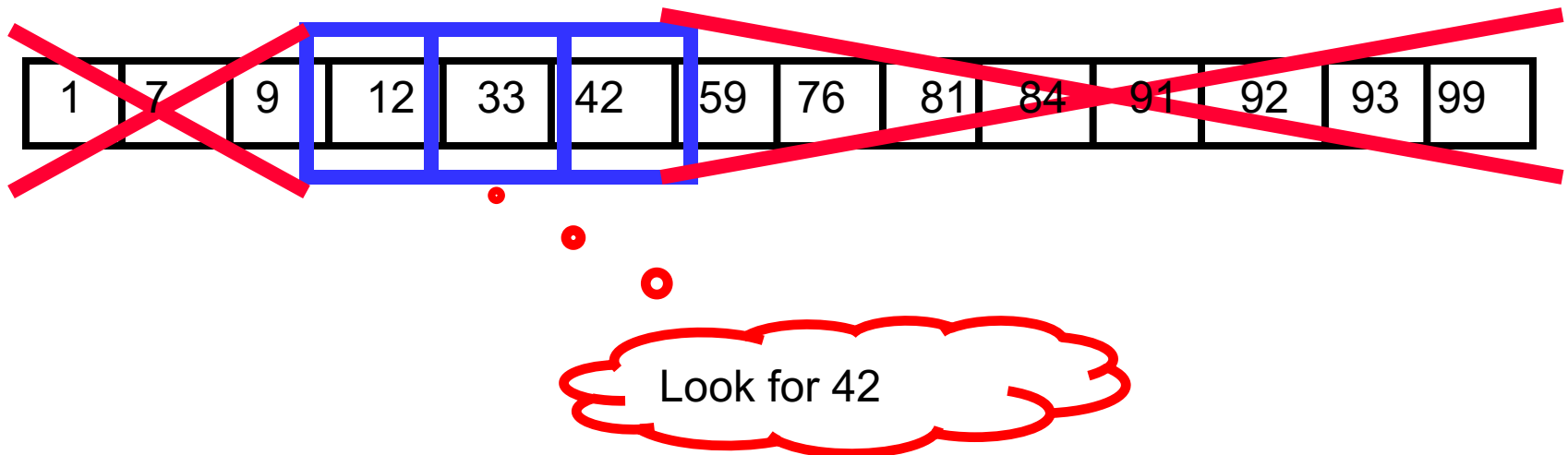
Searching for X

look at “middle” element

if no match then

look @ smaller half if $X < \text{middle}$

or @ larger half if $X > \text{middle}$



The Algorithm

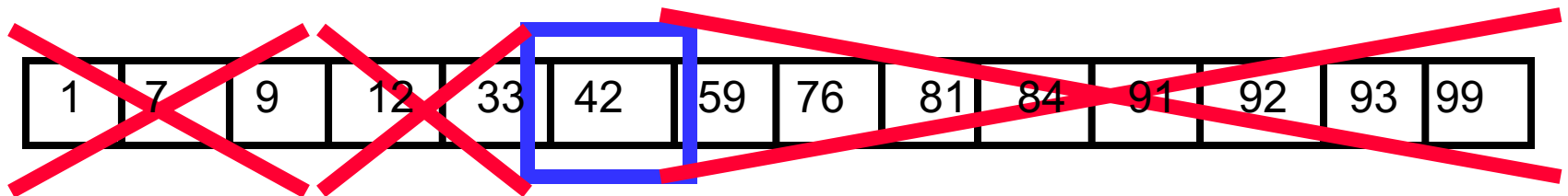
Searching for X

look at “middle” element

if no match then

look @ smaller half if $X < \text{middle}$

or @ larger half if $X > \text{middle}$



Look for 42

The Binary Search Algorithm (Pseudocode)

Searching for X

calculate middle position

if (Left-index and Right-index have “crossed”)
then

 “Item not found”

elseif (element at middle = to_find) **then**

 “Item Found”

elseif to_find < element at middle **then**

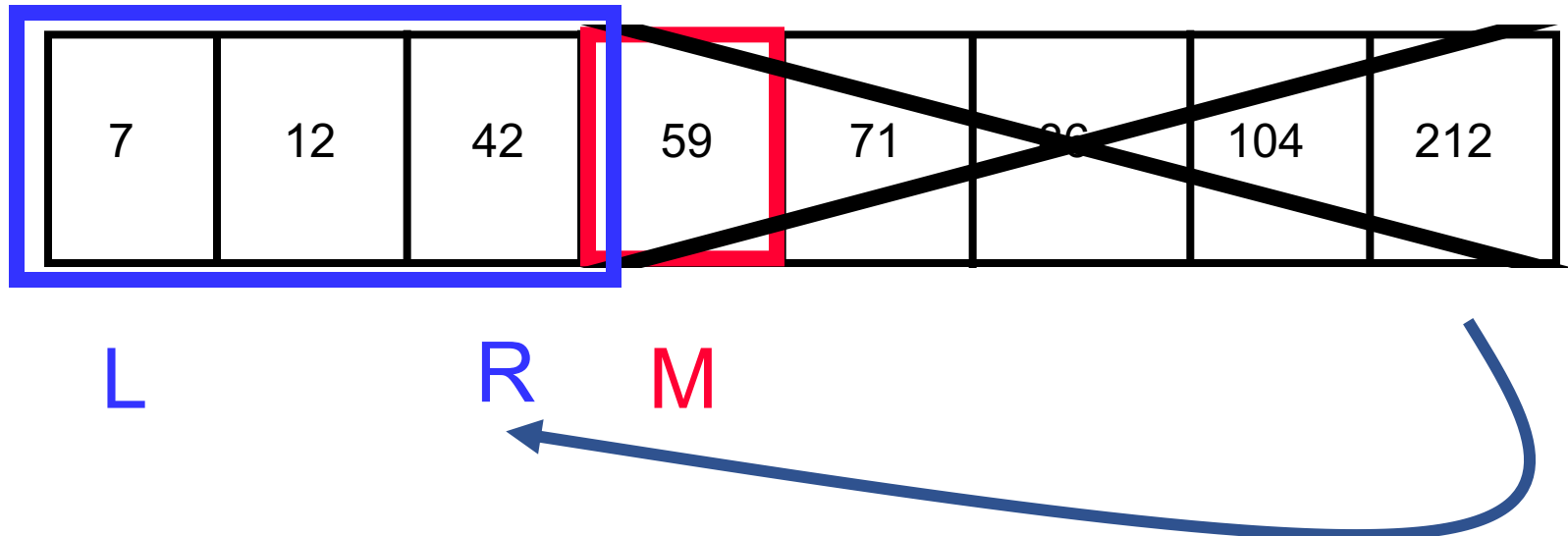
 Move Right-index to middle -1

else

 Move the Left-index to middle +1

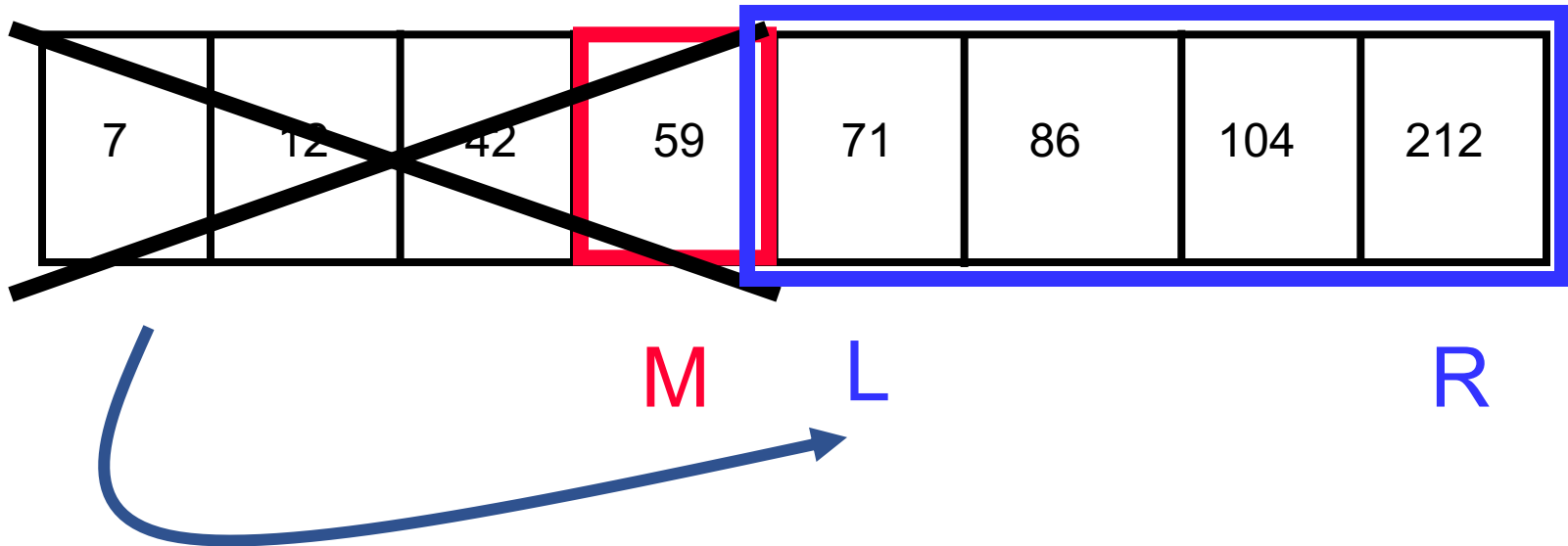
Visually: Moving Right Index

- Use indices “**Left (L)**” and “**Right(R)**” to keep track of where we are looking
- Set **Right = middle – 1**. Leave Left alone.

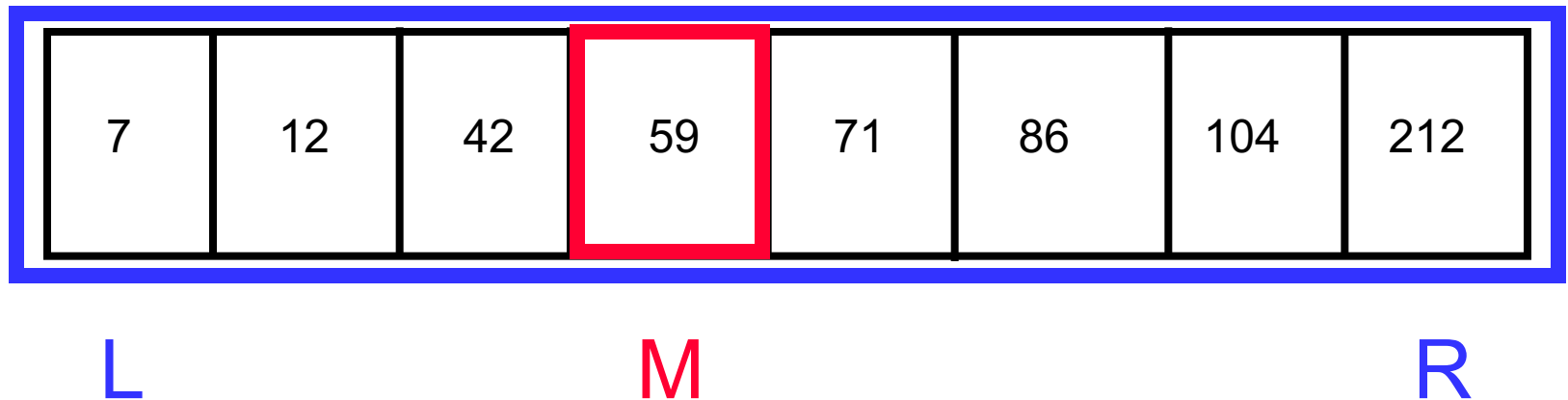


Visually: Moving Left Index

- Use indices “**Left**” and “**Right**” to keep track of where we are looking
- Set **Left = middle + 1**. Leave Right alone.

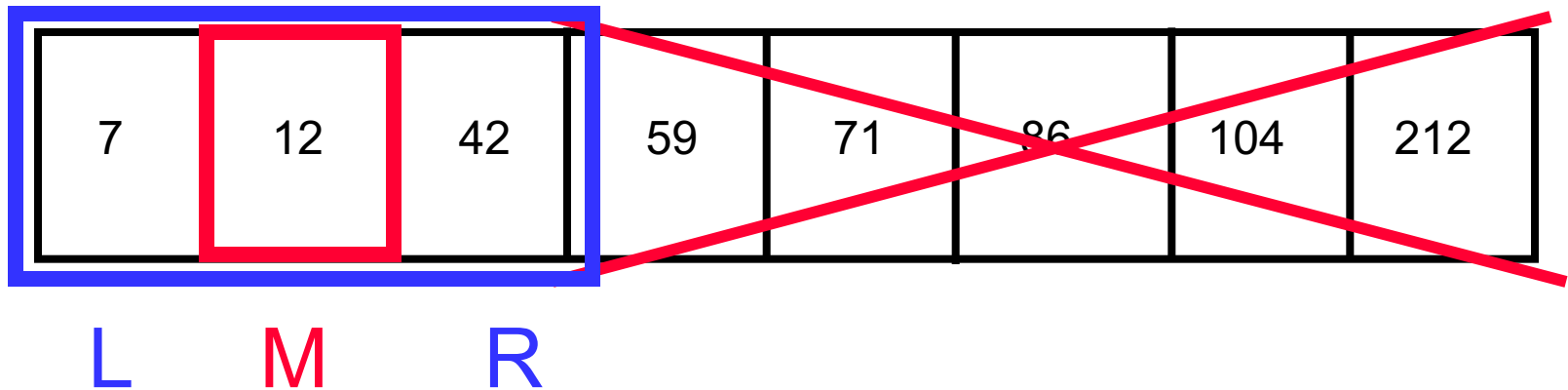


Binary Search Example – Found



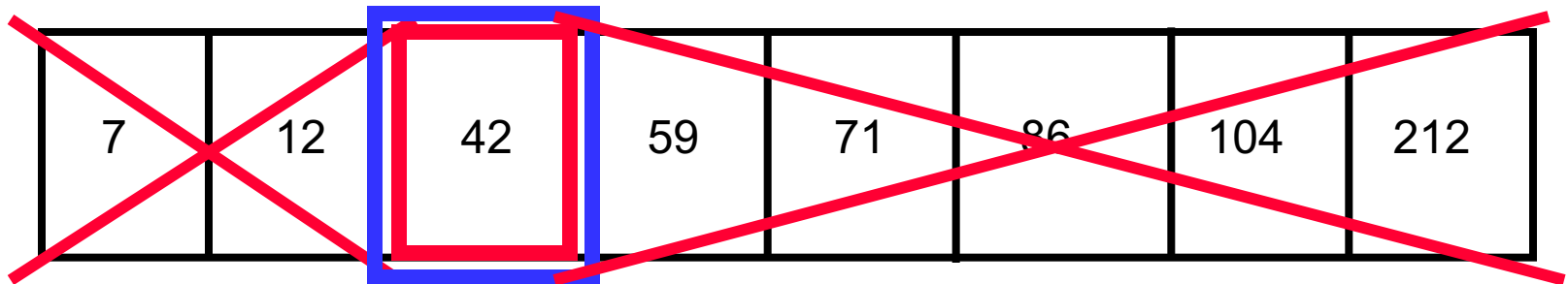
Looking for 42

Binary Search Example – Found



Looking for 42

Binary Search Example – Found



L
M
R

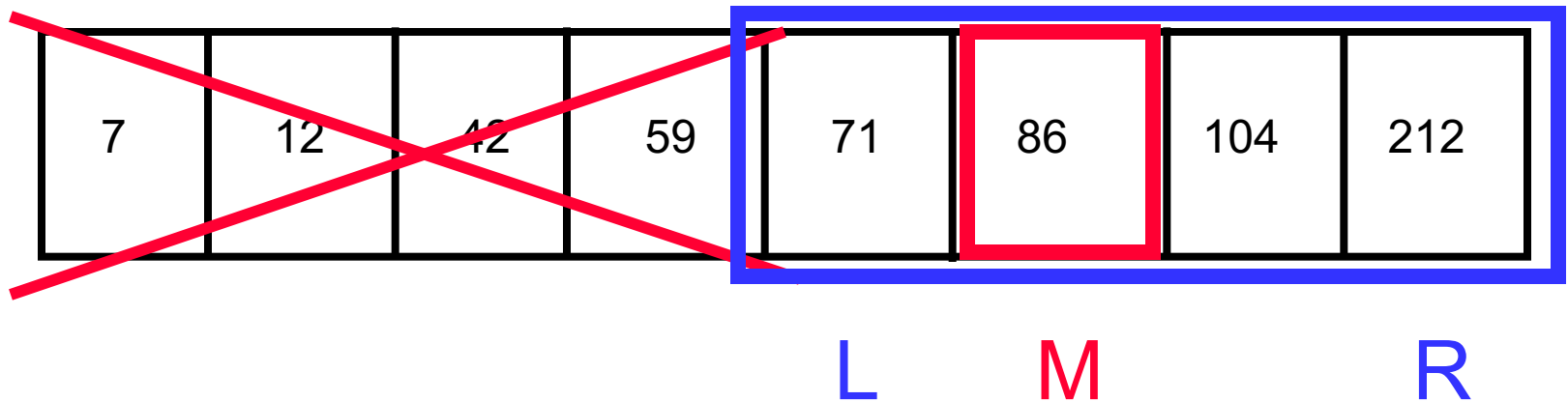
42 found – in 3 comparisons

Binary Search Example – Not Found

7	12	42	59	71	86	104	212
L			M				R

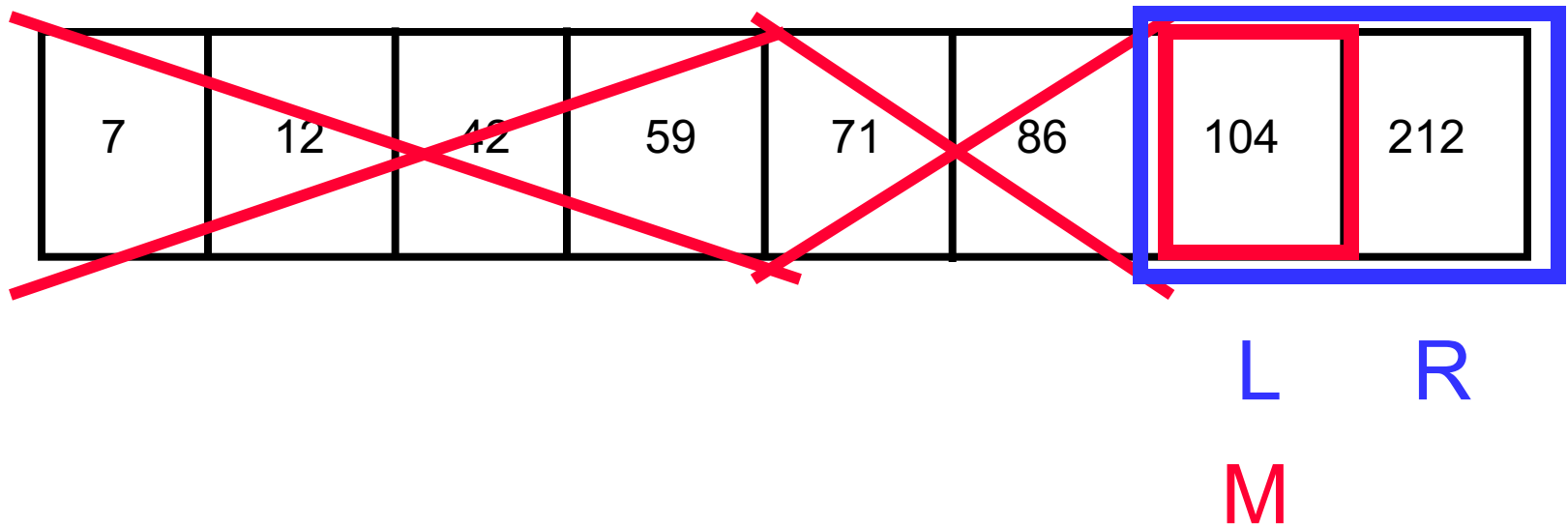
Looking for 89

Binary Search Example – Not Found



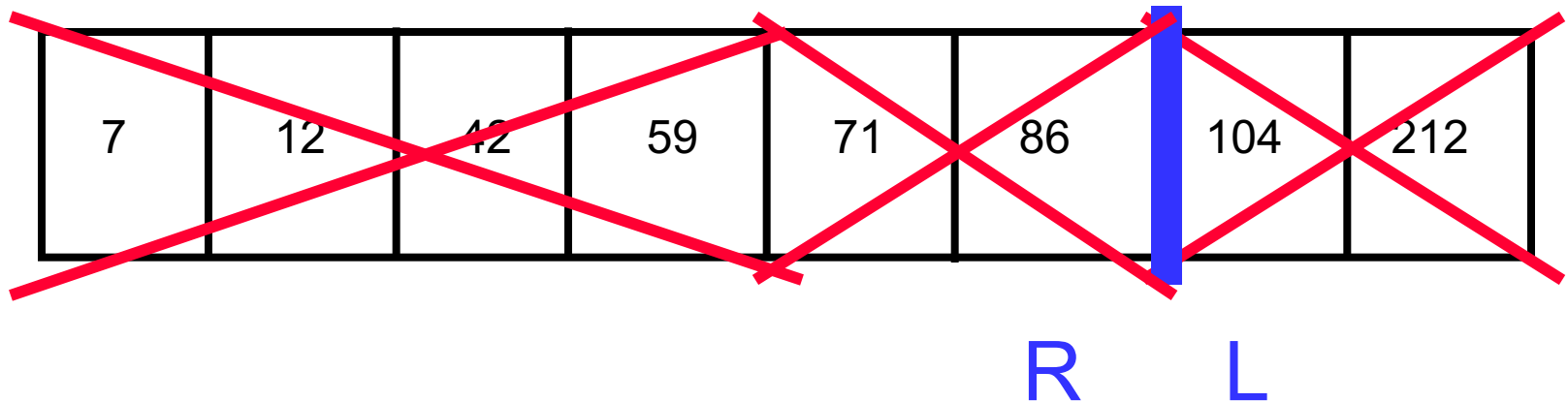
Looking for 89

Binary Search Example – Not Found



Looking for 89

Binary Search Example – Not Found



89 not found – 3 comparisons

Binary Search Function

```
def BinarySearch(A, first, last, target):  
    middle <- (first + last) / 2  
  
    if first > last:  
        return false  
    if A[middle] == target:  
        return true  
    if target < A[middle]:  
        return BinarySearch(A, first, middle-1, target)  
    else:  
        return BinarySearch(A, middle+1, last, target)
```

Binary Search Analysis: Best Case

```
def BinarySearch(A, first, last, target):  
    middle <- (first + last) / 2  
  
    if first > last:  
        return false  
    if A[middle] == target:  
        return true  
    if target < A[middle]:  
        return BinarySearch(A, first, middle-1, target)  
    else:  
        return BinarySearch(A, middle+1, last, target)
```

Best Case:
1 comparison

Best Case: match from the first comparison

1	7	9	12	33	42	59	81	84	91	92	93	99	
---	---	---	----	----	----	----	----	----	----	----	----	----	--

Target: 59

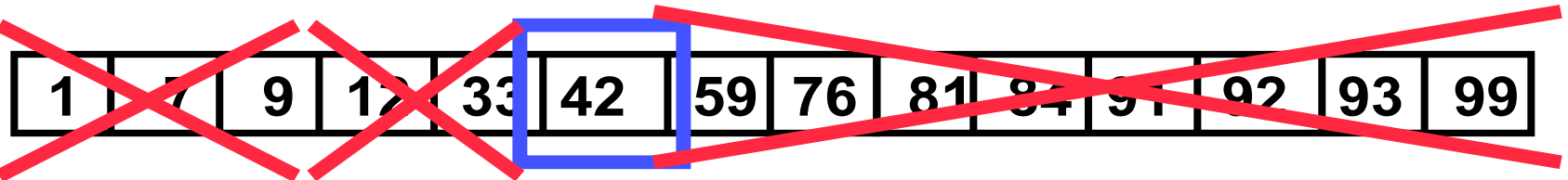
Binary Search Analysis: Worst Case

```
def BinarySearch(A, first, last, target):  
    middle <- (first + last) / 2  
  
    if first > last:  
        return false  
    if A[middle] = target:  
        return true  
    if target < A[middle]:  
        return BinarySearch(A, first, middle-1, target)  
    else:  
        return BinarySearch(A, middle+1, last, target)
```

**How many
comparisons??**



Worst Case: divide until reach one item, or no match.



Binary Search Analysis: Worst Case

- With each comparison we throw away $\frac{1}{2}$ of the list

N 1 comparison

$N/2$ 1 comparison

$N/4$ 1 comparison

$N/8$ 1 comparison

⋮

1 1 comparison

Number of steps is at
most $\rightarrow \log_2 N$

Summary

- Binary search **reduces the work by half** at each comparison
- If array is not sorted → Linear Search
 - Best Case $O(1)$
 - Worst Case $O(N)$
- If array is sorted → Binary search
 - Best Case $O(1)$
 - Worst Case $O(\log_2 N)$



That's all Folks!
Any Question?