

Final Project: Milk Bank Case Study

API 530 - Optimization in Practice

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1 Problem Description

The Oklahoma Mothers' Milk Bank operates a network of 36 depots where donors drop off frozen breast milk. A single driver must collect milk from a subset of these depots before their freezers reach capacity. The objective is to minimize total travel time while ensuring that each depot is visited exactly once per route.

This problem is formulated as a Traveling Salesman Problem (TSP) for each pickup route. We solve two separate TSP instances: one for the Local (OKC Metro) depots and one for the Tulsa depots. The remaining 22 depots are serviced by courier and are not included in the routing model.

2 Compact mathematical formulation

2.1 Decision Variables

$$\text{depots of Routes} = \{0, 1, \dots, n-1\},$$

where depot 0 denotes the Oklahoma Mothers' Milk Bank (the depot / start&end location) and the remaining indices denote pickup depots.

$$x_{ij} = \begin{cases} 1 & \text{if the driver travels directly from depot } i \text{ to depot } j \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \in N$$

$$u_i = \text{Position of depot } i \text{ in the route (for subtour elimination)}, \quad \forall i \in N \setminus \{0\}$$

$$u_i \quad \forall i \in \mathcal{N}$$

$$u_i \in [0, n-1] \quad (\text{continuous}).$$

u_i are auxiliary (order) variables used in the Miller–Tucker–Zemlin (MTZ) subtour elimination formulation: they capture the position of depot i in the tour (except that u_0 is unconstrained by the MTZ inequalities below).

2.2 Objective function:

Minimize total travel time:

$$\min \sum_{i \in N} \sum_{j \in N} t_{ij} \cdot x_{ij} \tag{1}$$

2.3 Parameters:

$$TravelTime_{ij} \quad \forall i, j \in N$$

$$n = |N| = \text{Total number of depots in the route}$$

- Local route: $|N| = 9$ (HQ + 8 depots)
- Tulsa route: $|N| = 7$ (HQ + 6 depots)

2.4 Constraints

Leave each depot exactly once

$$\sum_{j \in N, j \neq i} x_{ij} = 1 \quad \forall i \in N \quad (2)$$

(Each depot has exactly one outgoing arc.)

Enter each depot exactly once

$$\sum_{i \in N, i \neq j} x_{ij} = 1 \quad \forall j \in N \quad (3)$$

(Each depot has exactly one incoming arc.)

No self-loops

$$x_{ii} = 0 \quad \forall i \in N \quad (4)$$

MTZ subtour elimination (Miller–Tucker–Zemlin) For all $i, j \in N$ with $i \neq 0, j \neq 0, i \neq j$:

$$u_i - u_j + n x_{ij} \leq n - 1 \quad (5)$$

These inequalities forbid nontrivial subtours by enforcing a consistent ordering u_i for visited depots (the factor n is the usual MTZ multiplier for a model with n depots).

Variable Bounds

$$x_{ij} \in \{0, 1\}, \quad \forall i, j \in N \quad (6)$$

$$u_i \geq 0, \quad \forall i \in N \setminus \{0\} \quad (7)$$

2.5 Notes on instances

We solve two separate instances of the above model:

- **Local (OKC Metro)** with $n_{\text{local}} = 9$ nodes (node indices 0 to 8).
- **Tulsa** with $n_{\text{tulsa}} = 8$ nodes (node indices 0 to 7).

Below we list the node index mapping used in the Local instance and the Tulsa instance (index 0 = depot).

2.6 Local instance: node list

0 : Oklahoma Mothers' Milk Bank (depot)
1 : Variety Care- Lafayette
2 : Cleveland County Moore
3 : Edmond Hope Center
4 : Integris Southwest
5 : Integris Baptist
6 : St. Anthony Shawnee Hospital
7 : Canadian Valley
8 : OU Children's Hospital

2.7 Tulsa instance: node list

0 : Oklahoma Mothers' Milk Bank (depot)
1 : YWCA South
2 : Tulsa Hillcrest Hospital
3 : Jane Phillips Medical Center
4 : St. John Owasso
5 : YWCA East
6 : OBI
7 : (implicit return to depot)

2.8 Results (from the Pyomo)

Local route (optimal):

0: Oklahoma Mothers' Milk Bank (Start)
1: Edmond Hope Center
2: Integris Baptist
3: Canadian Valley
4: Integris Southwest
5: Variety Care- Lafayette
6: Cleveland County Moore
7: St. Anthony Shawnee Hospital
8: OU Children's Hospital
9: Oklahoma Mothers' Milk Bank (Return)

Total driving time: 167.8 minutes (2.8 hours).

Tulsa route (optimal):

0: Oklahoma Mothers' Milk Bank (Start)
1: YWCA South
2: Tulsa Hillcrest Hospital
3: Jane Phillips Medical Center
4: St. John Owasso
5: YWCA East
6: OBI
7: Oklahoma Mothers' Milk Bank (Return)

Total driving time: 300.3 minutes (5.0 hours).