Problem 1 Editorial: she shells book shelves by the she shelf
3 Points

Problem ID: `bookshelf`
Rank: `1`

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Overview

Problem Statement:
https://calico.berkeley.edu/files/calico-fa22/contest/bookshelf/bookshelf.pdf

As the first problem of this contest, we open by doing some geometry! There are a couple different ways to solve for the volume of these bookshelves—we go over two below.

Main Test Set

The Sum of Its Parts *Solutions available in Java, Python*

One straightforward approach is to sum the volume of all boards used to construct the bookshelf.

$\mathbf{N}$ + 1 boards are needed to construct the bookshelf's $\mathbf{N}$ shelves. Each board has a height of $\mathbf{B}$, a width of $\mathbf{W}$, and a depth of $\mathbf{D}$, and thus a volume of $\mathbf{BWD}$. Altogether, these boards have a volume of $(\mathbf{N} + 1)\mathbf{BWD}$.

Next, we find the volume of the boards that make up the sides of the bookshelf. These boards have a width of $\mathbf{B}$ and a depth of $\mathbf{D}$. The height of these boards comes from the total height of all shelves $\mathbf{H} = \mathbf{H_1} + \mathbf{H_2} + \ldots + \mathbf{H_N}$, plus the $\mathbf{N}$ + 1 boards of height $\mathbf{B}$ between the shelves. Adding these values together gives us a height of $\mathbf{H} + \mathbf{B}(\mathbf{N} + 1)$, meaning each of these boards has a volume of $(\mathbf{H} + \mathbf{B}(\mathbf{N} + 1))\mathbf{BD}$. Since there are two of these boards, the total volume of the side boards is $2(\mathbf{H} + \mathbf{B}(\mathbf{N} + 1))\mathbf{BD}$.

Adding together the volume of all boards gives us the total volume of the bookshelf: $(\mathbf{N} + 1)\mathbf{BWD} + 2(\mathbf{H} + \mathbf{B}(\mathbf{N} + 1))\mathbf{BD}$.

Negative Space *Solutions available in C++, Java, Python (main | alt)*

Another approach involves us finding the total volume of the rectangular prism the bookshelf occupies, minus the volume cut out by the shelves.

The height of the bookshelf, as determined earlier, is $2(\mathbf{H} + \mathbf{B}(\mathbf{N} + 1))\mathbf{BD}$, where $\mathbf{H} = \mathbf{H_1} + \mathbf{H_2} + \ldots + \mathbf{H_N}$. The width of the bookshelf is $\mathbf{W} + 2\mathbf{B}$, and the depth of the bookshelf is $\mathbf{D}$. Altogether, the rectangular prism occupied by the bookshelf has a volume of $2(\mathbf{H} + \mathbf{B}(\mathbf{N} + 1))\mathbf{BD}(\mathbf{W} + 2\mathbf{B})\mathbf{D} = 2\mathbf{BD}^2(\mathbf{H} + \mathbf{B}(\mathbf{N} + 1))(\mathbf{W} + 2\mathbf{B})$.

The i^{th} shelf cuts out a space of width $\mathbf{W}$, depth $\mathbf{D}$, and height $\mathbf{H_i}$. Collectively, the space cut out has a width $\mathbf{W}$, depth $\mathbf{D}$, and height $\mathbf{H} = \mathbf{H_1} + \mathbf{H_2} + \ldots + \mathbf{H_N}$, giving us a volume of $\mathbf{WDH}$.

Subtracting the space cut out by the shelves from the volume of the rectangular prism gives us the bookshelf's volume: $2\mathbf{BD}^2(\mathbf{H} + \mathbf{B}(\mathbf{N} + 1))(\mathbf{W} + 2\mathbf{B}) - \mathbf{WDH}$.