Problem 3 Editorial: This problem was not brought to you by jane street
3 Points

Problem ID: `paint`
Rank: `1`

Special thanks to Joshc (Joshua Chen) for contributing to this editorial.

Overview

Problem statement: https://calico.berkeley.edu/files/calico-fa22/contest/paint/paint.pdf

Although this problem seems pretty straightforward as we can just try converting all the paint into each of the three colors to pick the minimum cost, be careful as there are multiple ways of converting all the colors into a single color! If we don't consider all of these conversions, we may end up with a suboptimal and incorrect cost!

Main Test Set

Try All Options *Solutions available in C++, Java, Python*

If we can find the minimum cost to convert all of the paint to white, then we can use the same approach to find the minimum cost for orange and brown as well. We can then compute the final answer by taking the minimum of the three options. With this in mind, let's focus on trying to convert all of the paint to white.

What's the minimum cost to convert a bucket of orange paint to a bucket of white paint? We should never convert a bucket of paint to a color it was previously, since this would be a waste of money. So, there are only two options: either we directly convert it to white, or we convert it first to brown and then to white. The minimum cost to convert a bucket of orange paint to white is hence $\min(\mathbf{C_{OW}}, \mathbf{C_{BO}} + \mathbf{C_{BW}})$, and so the minimum cost to convert all $\mathbf{O}$ buckets of orange paint to white is $\mathbf{O} \times \min(\mathbf{C_{OW}}, \mathbf{C_{BO}} + \mathbf{C_{BW}})$. Similarly, the minimum cost to convert from a bucket of brown paint to white is $\min(\mathbf{C_{BW}}, \mathbf{C_{BO}} + \mathbf{C_{OW}})$ and $\mathbf{B} \times \min(\mathbf{C_{BW}}, \mathbf{C_{BO}} + \mathbf{C_{OW}})$ for all brown paint. Hence, the minimum cost to convert everything to white paint is $\mathbf{O} \times \min(\mathbf{C_{OW}}, \mathbf{C_{BO}} + \mathbf{C_{BW}}) + \mathbf{B} \times \min(\mathbf{C_{BW}}, \mathbf{C_{BO}} + \mathbf{C_{OW}})$.

Similarly, it costs a minimum of $\mathbf{W} \times \min(\mathbf{C_{OW}}, \mathbf{C_{BW}} + \mathbf{C_{BO}}) + \mathbf{B} \times \min(\mathbf{C_{BO}}, \mathbf{C_{BW}}+\mathbf{C_{OW}})$ to convert all paint to orange, and likewise $\mathbf{W} \times \min(\mathbf{C_{BW}}, \mathbf{C_{OW}} + \mathbf{C_{BO}}) + \mathbf{O} \times \min(\mathbf{C_{BO}}, \mathbf{C_{OW}} + \mathbf{C_{BW}})$ to convert all paint to brown. The final answer is the minimum of the three expressions.

Challenge

This problem naturally motivates the following generalized problem:

You start with $\mathbf{N}$ ($2 \le \mathbf{N} \le 200$) different colors of paint, with a certain number of buckets of each paint color. You can convert a bucket of paint from one color to another, where the cost to convert between each unordered pair of colors is given to you in the input. Find the minimum cost to make all of the buckets have the same color of paint.

Note that the original problem is equivalent to the generalized problem with $\mathbf{N} = 3$. Can you solve this harder problem? Feel free to share any ideas you have in our CALICO Community Discord server!