

This algorithm is described in section 13.7 of **Combinatorial Optimization (Korte-Vygen)**

Input: Two matroids $(E, \mathcal{F}_1), (E, \mathcal{F}_2)$ given by independence oracles. Weights $c : E \rightarrow \mathbb{R}$.

Output: A set $X \in \mathcal{F}_1 \cap \mathcal{F}_2$ whose weight $c(X)$ is maximum.

1. Set $k := 0$ and $X_0 := \emptyset$. Set $c_1(e) := c(e)$ and $c_2(e) := 0$ for all $e \in E$.
2. For each $y \in E \setminus X_k$ and $i \in \{1, 2\}$ compute

$$C_i(X_k, y) := \{x \in X_k \cup \{y\} : X_k \cup \{y\} \notin \mathcal{F}_i, (X_k \cup \{y\}) \setminus \{x\} \in \mathcal{F}_i\}$$

3. Compute

$$A^{(1)} := \{(x, y) : y \in E \setminus X_k, x \in C_1(X_k, y) \setminus \{y\}\}$$

$$A^{(2)} := \{(y, x) : y \in E \setminus X_k, x \in C_2(X_k, y) \setminus \{y\}\}$$

$$S := \{y \in E \setminus X_k : X_k \cup \{y\} \in \mathcal{F}_1\}$$

$$T := \{y \in E \setminus X_k : X_k \cup \{y\} \in \mathcal{F}_2\}$$

4. Compute

$$m_1 := \max\{c_1(y) : y \in S\}$$

$$m_2 := \max\{c_2(y) : y \in T\}$$

$$\bar{S} := \{y \in S : c_1(y) = m_1\}$$

$$\bar{T} := \{y \in T : c_2(y) = m_2\}$$

$$\bar{A}^{(1)} := \{(x, y) \in A^{(1)} : c_1(x) = c_1(y)\}$$

$$\bar{A}^{(2)} := \{(y, x) \in A^{(2)} : c_2(x) = c_2(y)\}$$

$$\bar{G} := (E, \bar{A}^{(1)} \cup \bar{A}^{(2)})$$

5. Apply BFS to compute the set R of vertices reachable from \bar{S} in \bar{G} .
6. If $R \cap \bar{T} \neq \emptyset$, then find an $\bar{S} - \bar{T}$ path P in \bar{G} with a minimum number of edges, set $X_{k+1} := X_k \triangle V(P)$ and $k := k + 1$ and go to 2.
7. Compute

$$\varepsilon_1 := \min\{c_1(x) - c_1(y) : (x, y) \in \delta_{A^{(1)}}^+(R)\}$$

$$\varepsilon_2 := \min\{c_2(x) - c_2(y) : (y, x) \in \delta_{A^{(2)}}^+(R)\}$$

$$\varepsilon_3 := \min\{m_1 - c_1(y) : y \in S \setminus R\}$$

$$\varepsilon_4 := \min\{m_2 - c_2(y) : y \in T \cap R\}$$

$$\varepsilon := \min\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$$

where $\min \emptyset := \infty$.

8. If $\varepsilon < \infty$ then set $c_1(x) := c_1(x) - \varepsilon$ and $c_2(x) := c_2(x) + \varepsilon$ for all $x \in R$ and go to 4. If $\varepsilon = \infty$, then among X_0, X_1, \dots, X_k , let X be the one with maximum weight. Stop and output X .