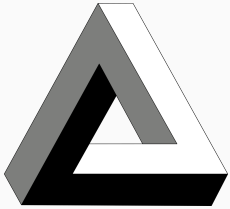
Stating the Obvious

boxy

September 2022

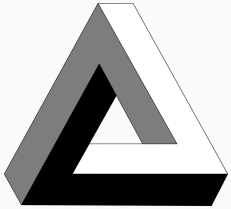
Paradoxes and Fallacies

- What is the smallest positive integer not definable in under hundred letters?



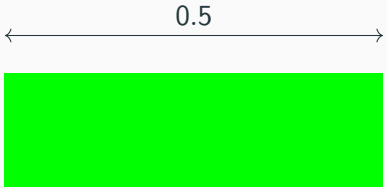
Paradoxes and Fallacies

- What is the smallest positive integer not definable in under hundred letters?
- It is impossible to run any distance



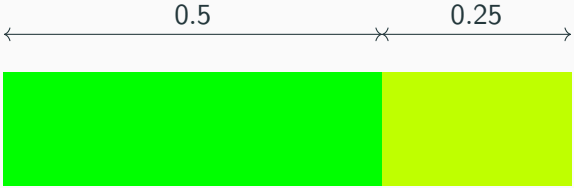
Zeno's Paradox

First run half a meter.



Zeno's Paradox

Then a quarter meter.



Zeno's Paradox

And so on.

Project Background

Programming Languages

Calculating $5 + 3 * 2$.

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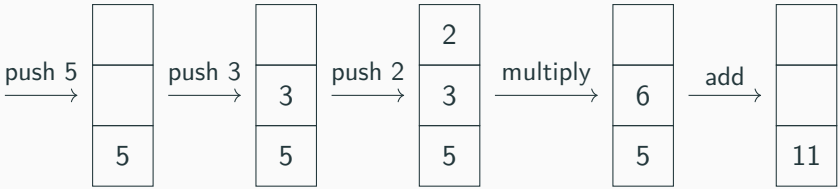

Programming Languages

```
static void cmd_backup(struct userrec *u, int idx, char *par)
{
        putlog(LOG_CMDS, "*", "#%s# backup", dcc[idx].nick);
        dprintf(idx, "Backing up the channel & user files...\n");
        call_hook(HOOK_BACKUP);
}
```

Programming Languages

```haskell
powerset = filterM (\_ -> [True, False])
```

The Obvious

Some of our fellow sinners are among the most careful and competent logicians on the contemporary scene.

Haskell B. Curry and Robert Feys

The Obvious

- “Roses are red and violets are blue” is logically equivalent to “Violets are blue and roses are red”

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The Obvious

- "Roses are red and violets are blue" is logically equivalent to "Violets are blue and roses are red"
- $2 + 3 = 5$
- $a + b = b + a$
- $1 \neq 2$

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Applications

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- New languages can reduce labour

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- New languages can reduce labour
- New languages and theorem provers can eliminate bugs
  - $59 billion loss per year in US
  - Death

Foundations of Mathematics

Meta-Languages

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$$\frac{A \quad B}{A \wedge B}\ (\wedge I)$$

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λ-Calculi

Proof languages are often based off the λ-calculus.

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$$T = \underbrace{\lambda x.}_{\text{Abstraction}} \quad \lambda y.x$$

$$F = \lambda x.\lambda y.y$$

$$N = \lambda b. \underbrace{b\ F\ T}_{\text{Application}}$$

Proof languages are often based off the λ-calculus.

$$
\begin{aligned}
& (\lambda b.b\ F\ \ T)(\lambda x.\lambda y.y) \\
\rightarrow\ & (\lambda x.\lambda y.y)\ F\ \ T \\
\rightarrow\ & T
\end{aligned}
$$

The Language

Formal Language Theory

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$$\langle add \rangle \rightarrow \text{<var> "+" <add> | <mult>}$$

$$\langle mult \rangle \rightarrow \text{<var> "*" <mult> | "(" <add> ")" | <var>}$$

$$\langle var \rangle \rightarrow \text{"a" | "b" | "c"}$$

Formal Language Theory

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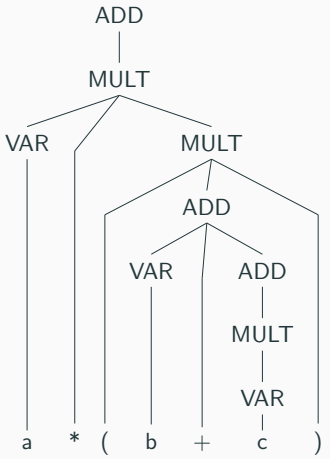

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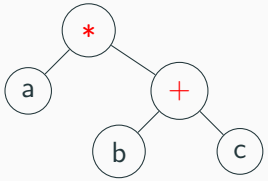
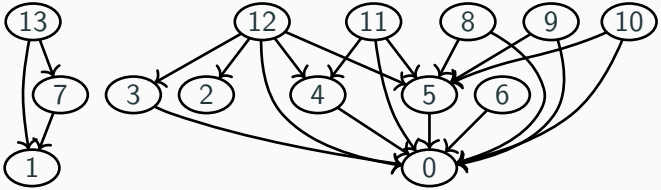
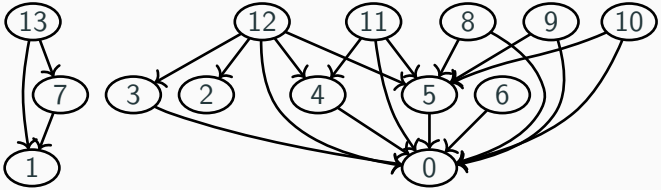


Implementation Details



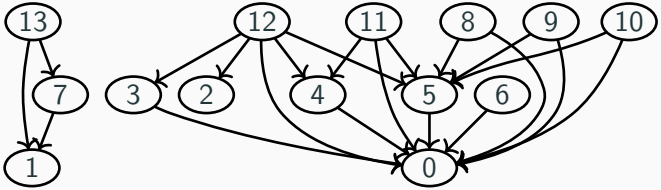
Implementation Details

- The language is parsed



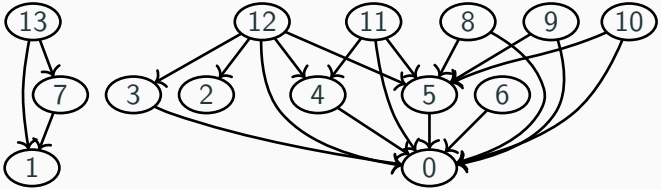
Implementation Details

- The language is parsed
- Definitions across files are resolved



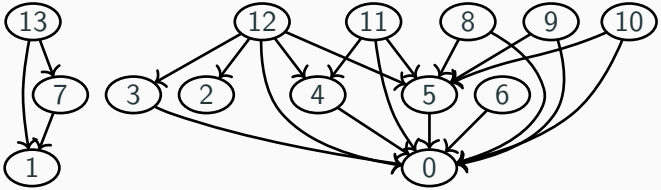
Implementation Details

- The language is parsed
- Definitions across files are resolved
- Definitions are topologically sorted



Implementation Details

- The language is parsed
- Definitions across files are resolved
- Definitions are topologically sorted
- Type-checking is done



One is Not Two

Logic definitions:

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bottom = forall a : Prop, a;
not = fun a : Prop => forall b : a, bottom;
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Definition of natural numbers:

```
nat : Set;
0 : nat;
succ : forall a : nat, nat;
1 = succ 0; 2 = succ 1;
```

One is Not Two

Axioms:

```
equal : forall a : nat, forall b : nat, Prop;
lower : forall a : nat,
        forall b : nat,
        forall p : equal (succ a) (succ b), equal a b;
0_ne_succ : forall a : nat, not (equal 0 (succ a));
```

One is Not Two

The proof:

```
1_ne_2 :  not (equal 1 2)
       =  fun p : equal 1 2
       => let 0_eq_1 = lower 0 1 p
       in 0_ne_succ 0 0_eq_1;
```

Tada!

> The point of philosophy is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it.
>
> Bertrand Russell