# Athemath Fall 2025 Diagnostic

Athemath Staff

*Due August 31st, 2025*

## §1 Instructions

**This is not a quiz!** It is simply for us to get a sense of your current math level, so that we can determine whether or not you will benefit from Athemath classes, and if so, which Athemath classes would be appropriate for you. You are not required to solve all or even most of the problems, although we do encourage you to try your best on all of them! While later problems will generally be harder, they also play to different strengths and you may find one particularly easy.

For all problems, **proof-based solutions are encouraged**. We would like you to explain all of your steps instead of just giving an answer. If you don't have experience with proofs, just try to explain your answer as much as you can. LaTeX submissions and *neat*, dark handwriting submissions are both allowed.

Please do not use computer programs, Google, WolframAlpha, etc. to help you find solutions (however, GeoGebra is allowed). Additionally, please do not discuss this quiz with anyone else until after the application deadline has passed. If you find the test difficult, that's because it's designed to be! If you get stuck, take a walk, try a different problem, or try a strategy you dismissed at first. And remember that you don't have to solve all of the problems to get in. Historically, the average admitted student solved around two or three problems (admissions problem sets have varied in difficulty over the years though).

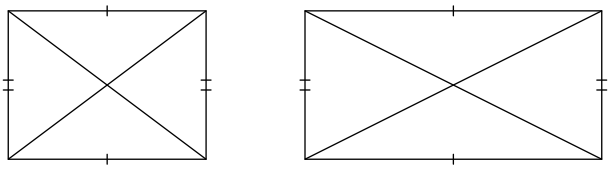
Ask for clarifications by emailing contact@athemath.org. Submit your completed solutions to the application form by the end of the day on **August 31st**. However, the sooner you reply, the sooner you might hear back from us! As a reminder, only students of underrepresented genders can apply. Have fun!

Please justify your answers and show your work!

# § 2 The Problems!

## Problem 1

Points $A$, $B$, $C$, $D$ are on the plane such that $AB = CD$, $AC = BD$, and $AD = BC$. Show that they form a rectangle.



## Problem 2

Greta wants to join the Fish Action Party. However, the Party requires that Greta puts her life savings of $n$ fish in a row from left to right and randomly assign one label from $1, 2, \ldots, n$ to each fish, so that each number appears exactly once. Let $a_i$ be the label on the $i$-th fish from the left. The Party's Doctrine states that if there does not exist an integer $k$ such that $2 \leq k \leq n-1$ and $a_k$ is less than both $a_{k-1}$ and $a_{k+1}$, then Greta is allowed to join the Fish Action Party. In terms of $n$, what is the probability that Greta is admitted?

## Problem 3

Let $n$ be a positive integer. There are $2n$ cards, with 2 of each label from 1 through $n$. These cards are dealt at random to Catherine and Rachel, each receiving $n$ cards.

They play the following game. On each turn, there is a "leader," with Catherine being the leader on the first turn. On each turn, the leader plays a card, and the other player does so after. The player who plays the higher card becomes the leader, and the current leader wins if there is a tie. The player who wins the last turn wins.

If both players play optimally, what is the probability that Catherine wins?

## Problem 4

Vera the pufferfish is running for Prime Minister of Singapore as the leader of the Fish Action Party. To campaign, she wants to visit every house numbered from 1 to 1000, starting at house 1. At each step, she may:

i) Visit a house whose number is a multiple of the previous house number (e.g. 1, 8, 16, **48**)

i) Revisit a house with a smaller number she has already been to (e.g. 3, **1**, 8, 16, 48, **3**)

She may not visit any house numbered above 1000. Given that she has to visit every house from 1 to 1000 at least once starting from house 1, find the minimal number of visits she has to make in total (including house 1 at the start).

## Problem 5

In acute triangle $\triangle ABC$ with circumcenter $O$, $D$ is on segment $BC$. The perpendicular bisectors of $BD$ and $DC$ intersect minor arc $\widehat{BC}$ at $K$ and $L$, respectively. Points $S$ and $T$ are drawn on major arc $\widehat{BAC}$ defined by $S = KD \cap (ABC)$ and $T = LD \cap (ABC)$. Prove that the line through $S$ perpendicular to $OB$, the line through $T$ perpendicular to $OC$, and the line through $D$ perpendicular to $BC$ are concurrent.