Two Cells Numerical Algorithm

Let **index 1 denote leader cell** while **index 2 denotes trailer cell**.

Hmm… choice of parameters to get this going.
"Knowns": f, t1, t2, t0, tau, tcc, xi (drag or constant of proportionality in Step 9)

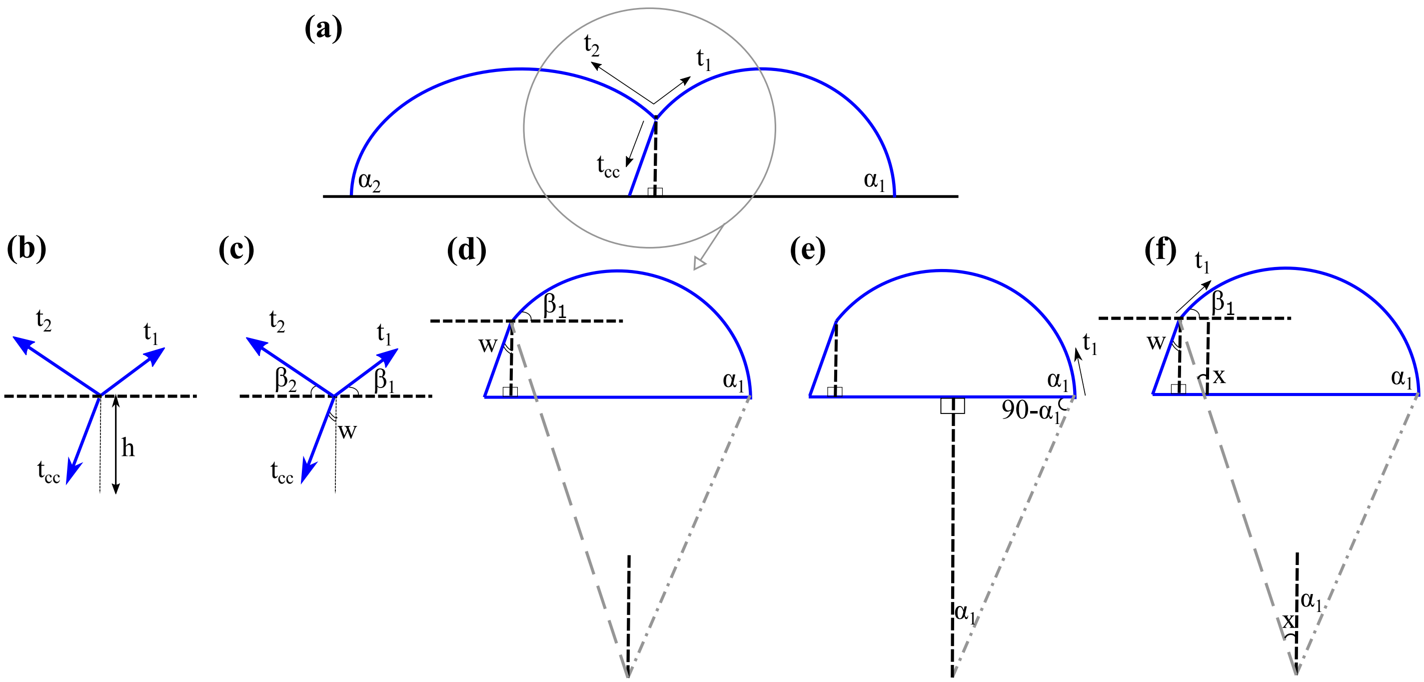


Figure 1.

Let $w = w + 0.001$

1) Given f, t1, t2, t0, tau, choose v — obtain alpha1, alpha2

Let $v = v + 0.001$

(1.a) $$(1+f)\left(1-\frac{t_1}{t_0+\tau}\right) - v = t_1 \cos\alpha_1 \text{ : leader}$$

(1.b) $$(1-f)\left(1-\frac{t_2}{t_0-\tau}\right) + v = t_2 \cos\alpha_2 \text{ : trailer}$$

(1.a) $$\alpha_1 = \cos^{-1}\left[\frac{(1+f)}{t_1}\left(1-\frac{t_1}{t_0+\tau}\right) - \frac{v}{t_1}\right]$$

(1.b) $\alpha_2 = \cos^{-1}\left[\frac{(1-f)}{t_2}\left(1-\frac{t_2}{t_0-\tau}\right)+\frac{v}{t_2}\right]$

Solvability conditions:

- $-1 \le \frac{(1+f)}{t_1}\left(1-\frac{t_1}{t_0+\tau}\right)-\frac{v}{t_1} \le 1 \Rightarrow -t_1 \le (1+f)\left(1-\frac{t_1}{t_0+\tau}\right)-v \le t_1$

$-t_1 \le (1+f)\frac{t_0+\tau-t_1}{t_0+\tau}-\frac{v(t_0+\tau)}{t_0+\tau} \le t_1 \Rightarrow -t_1(t_0+\tau) \le (1+f)(t_0+\tau-t_1)-v(t_0+\tau) \le t_1(t_0+\tau)$

- Similarly, $-t_2(t_0-\tau) \le (1-f)(t_0-\tau-t_2)-v(t_0-\tau) \le t_2(t_0-\tau)$

Conditions:

- $t_0 \neq \tau$
- $\beta_1 < \alpha_1$ and $\beta_2 < \alpha_2$

2) Given t1, t2, tcc, choose w — obtain beta1, beta2 from force balance at the top of the cell-cell interaction region.

(2.a) $t_1 \cos\beta_1 = t_2 \cos\beta_2 + t_{cc}\sin w$ (x-direction)
(2.b) $t_1 \sin\beta_1 + t_2 \sin\beta_2 = t_{cc}\cos w$ (y-direction)

(2.a) $\beta_1 = \cos^{-1}(t_2/t_1 \cos\beta_2 + t_{cc}/t_1 \sin w)$
(2.b) $t_1 \sin\left[\cos^{-1}(t_2/t_1 \cos\beta_2 + t_{cc}/t_1 \sin w)\right] + t_2 \sin\beta_2 = t_{cc}\cos w$

Solve 2.b with fzero in Matlab for beta2.

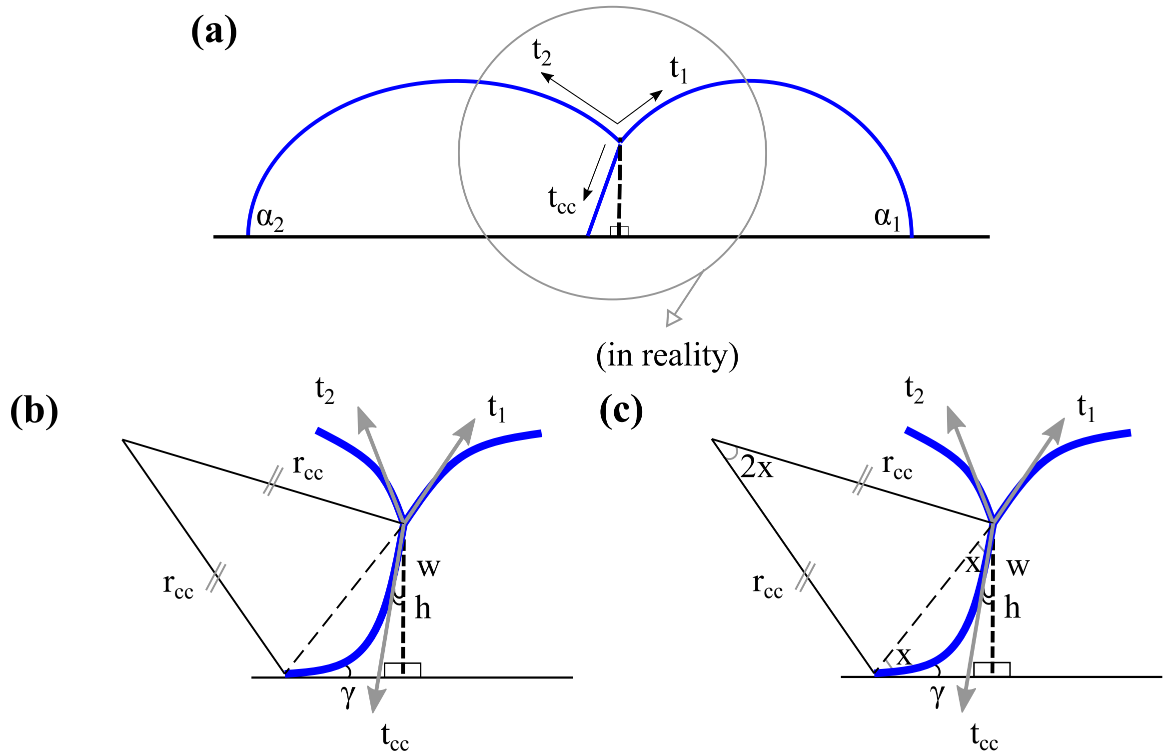
3) Given alpha1, alpha2, beta1, beta2, find r1 and r2:

(3.a) $r_1 = \left[\left(\frac{1}{2}-\frac{1}{4}\sin(2\alpha_1)\right)+\left(\frac{1}{2}-\frac{1}{4}\sin(2\beta_1)\right)\right]^{-1/2}$
(3.b) $r_2 = \left[\left(\frac{1}{2}-\frac{1}{4}\sin(2\alpha_2)\right)+\left(\frac{1}{2}-\frac{1}{4}\sin(2\beta_2)\right)\right]^{-1/2}$

4) Find the equation for the height of the cell-cell junction:
(4.a) $h = h_1 = r_1\left(\cos\beta_1 - \cos\alpha_1\right)$
(4.b) $h = h_2 = r_2\left(\cos\beta_2 - \cos\alpha_2\right)$
or alternatively $r_1\left(\cos\beta_1 - \cos\alpha_1\right) = r_2\left(\cos\beta_2 - \cos\alpha_2\right)$

One suggestion is to loop over steps 1-4 and adjust v until $\delta h = |h_1 - h_2| \le$ tolerance.

5) Intracellular pressure is:



$p_1 = \dfrac{t_1}{r_1}$ and $p_2 = \dfrac{t_2}{r_2}$

Figure 2.

6) Find the radius of curvature for the cell-cell junction:

$$|p_2 - p_1| = \frac{t_{cc}}{r_{cc}} \rightarrow r_{cc} = \frac{t_{cc}}{|t_2/r_2 - t_1/r_1|}$$

7) Knowing h, w, rcc, we find what is the angle gamma at the ventral end of the cell-cell boundary.

(1) $\cos(x + w) = \dfrac{h}{s}$

(2) $\sin(x + \gamma) = \dfrac{h}{s}$

(3) $s^2 = 2r_{cc}^2\,(1 - \cos(2x))$

Combining (1) and (3), we obtain:

$\dfrac{h^2}{\cos^2(x + w)} = 2r_{cc}^2\,\big(1 - \cos(2x)\big)$ fzero for $x \in [0, \pi/4]$.

Then: $\gamma = \dfrac{\pi}{2} - w - 2x$.

8) Gamma is a function of v, or a constant. In any case, the force balance condition at the ventral end of the cell-cell boundary closes the system.

$t_{cc}\cos(\gamma) = \xi v_{\text{end}} \Rightarrow v_{\text{end}} = \dfrac{t_{cc}}{\xi}\cos(\gamma)$

Now loop over steps 1-8 and adjust w until $\delta v = |v - v_{\text{end}}| \leq$ tolerance.

Could also loop over steps 1-8 and adjust w until $\delta\gamma = |\gamma - \cos^{-1}(\xi v/t_{cc})| \leq$ tolerance.