

## Two Cells Numerical Algorithm

Let **index 1** denote **leader cell** while **index 2** denotes **trailer cell**.

Hmm... choice of parameters to get this going.

“Knowns”:  $f$ ,  $t_1$ ,  $t_2$ ,  $t_0$ ,  $\tau$ ,  $t_{cc}$ ,  $\xi$  (drag or constant of proportionality in Step 9)

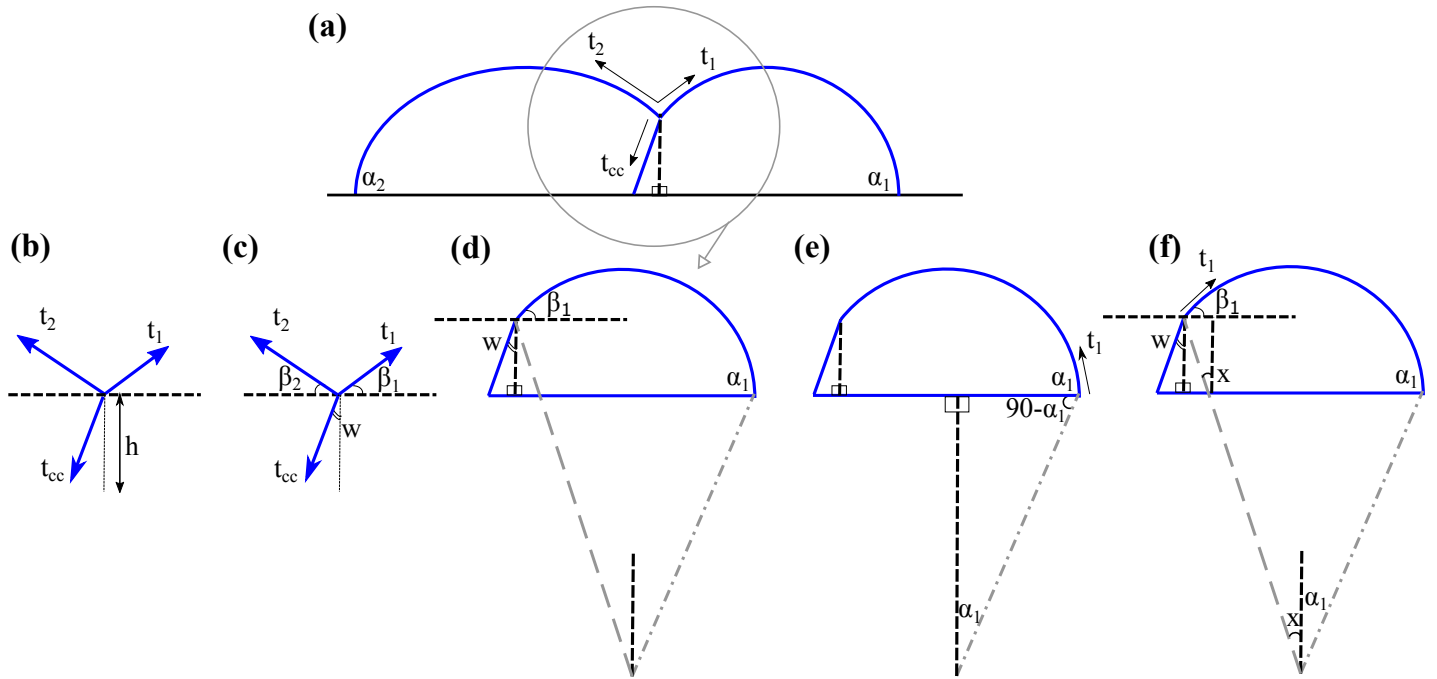


Figure 1.

Let  $w = w + 0.001$

1) Given  $f$ ,  $t_1$ ,  $t_2$ ,  $t_0$ ,  $\tau$ , choose  $v$  — obtain  $\alpha_1$ ,  $\alpha_2$

Let  $v = v + 0.001$

$$(1.a) \quad (1 + f) \left( 1 - \frac{t_1}{t_0 + \tau} \right) - v = t_1 \cos \alpha_1 : \text{leader}$$

$$(1.b) \quad (1 - f) \left( 1 - \frac{t_2}{t_0 - \tau} \right) + v = t_2 \cos \alpha_2 : \text{trailer}$$

$$(1.a) \quad \alpha_1 = \cos^{-1} \left[ \frac{(1 + f)}{t_1} \left( 1 - \frac{t_1}{t_0 + \tau} \right) - \frac{v}{t_1} \right]$$

$$(1.b) \alpha_2 = \cos^{-1} \left[ \frac{(1-f)}{t_2} \left( 1 - \frac{t_2}{t_0 - \tau} \right) + \frac{v}{t_2} \right]$$

Solvability conditions:

- $-1 \leq \frac{(1+f)}{t_1} \left( 1 - \frac{t_1}{t_0 + \tau} \right) - \frac{v}{t_1} \leq 1 \Rightarrow -t_1 \leq (1+f) \left( 1 - \frac{t_1}{t_0 + \tau} \right) - v \leq t_1$   
 $-t_1 \leq (1+f) \frac{t_0 + \tau - t_1}{t_0 + \tau} - \frac{v(t_0 + \tau)}{t_0 + \tau} \leq t_1 \Rightarrow -t_1(t_0 + \tau) \leq (1+f)(t_0 + \tau - t_1) - v(t_0 + \tau) \leq t_1(t_0 + \tau)$
- Similarly,  $-t_2(t_0 - \tau) \leq (1-f)(t_0 - \tau - t_2) - v(t_0 - \tau) \leq t_2(t_0 - \tau)$

Conditions:

- $t_0 \neq \tau$
- $\beta_1 < \alpha_1$  and  $\beta_2 < \alpha_2$

2) Given  $t_1, t_2, t_{cc}$ , choose  $w$  — obtain  $\beta_1, \beta_2$  from force balance at the top of the cell-cell interaction region.

$$(2.a) \quad t_1 \cos \beta_1 = t_2 \cos \beta_2 + t_{cc} \sin w \quad (\text{x-direction})$$

$$(2.b) \quad t_1 \sin \beta_1 + t_2 \sin \beta_2 = t_{cc} \cos w \quad (\text{y-direction})$$

$$(2.a) \quad \beta_1 = \cos^{-1}(t_2/t_1 \cos \beta_2 + t_{cc}/t_1 \sin w)$$

$$(2.b) \quad t_1 \sin [\cos^{-1}(t_2/t_1 \cos \beta_2 + t_{cc}/t_1 \sin w)] + t_2 \sin \beta_2 = t_{cc} \cos w$$

Solve 2.b with fzero in Matlab for  $\beta_2$ .

3) Given  $\alpha_1, \alpha_2, \beta_1, \beta_2$ , find  $r_1$  and  $r_2$ :

$$(3.a) \quad r_1 = \left[ \left( \frac{1}{2} - \frac{1}{4} \sin(2\alpha_1) \right) + \left( \frac{1}{2} - \frac{1}{4} \sin(2\beta_1) \right) \right]^{-1/2}$$

$$(3.b) \quad r_2 = \left[ \left( \frac{1}{2} - \frac{1}{4} \sin(2\alpha_2) \right) + \left( \frac{1}{2} - \frac{1}{4} \sin(2\beta_2) \right) \right]^{-1/2}$$

4) Find the equation for the height of the cell-cell junction:

$$(4.a) \quad h = h_1 = r_1 (\cos \beta_1 - \cos \alpha_1)$$

$$(4.b) \quad h = h_2 = r_2 (\cos \beta_2 - \cos \alpha_2)$$

$$\text{or alternatively } r_1 (\cos \beta_1 - \cos \alpha_1) = r_2 (\cos \beta_2 - \cos \alpha_2)$$

One suggestion is to loop over steps 1-4 and adjust  $v$  until  $\delta h = |h_1 - h_2| \leq \text{tolerance}$ .

5) Intracellular pressure is:

$$p_1 = \frac{t_1}{r_1} \text{ and } p_2 = \frac{t_2}{r_2}$$

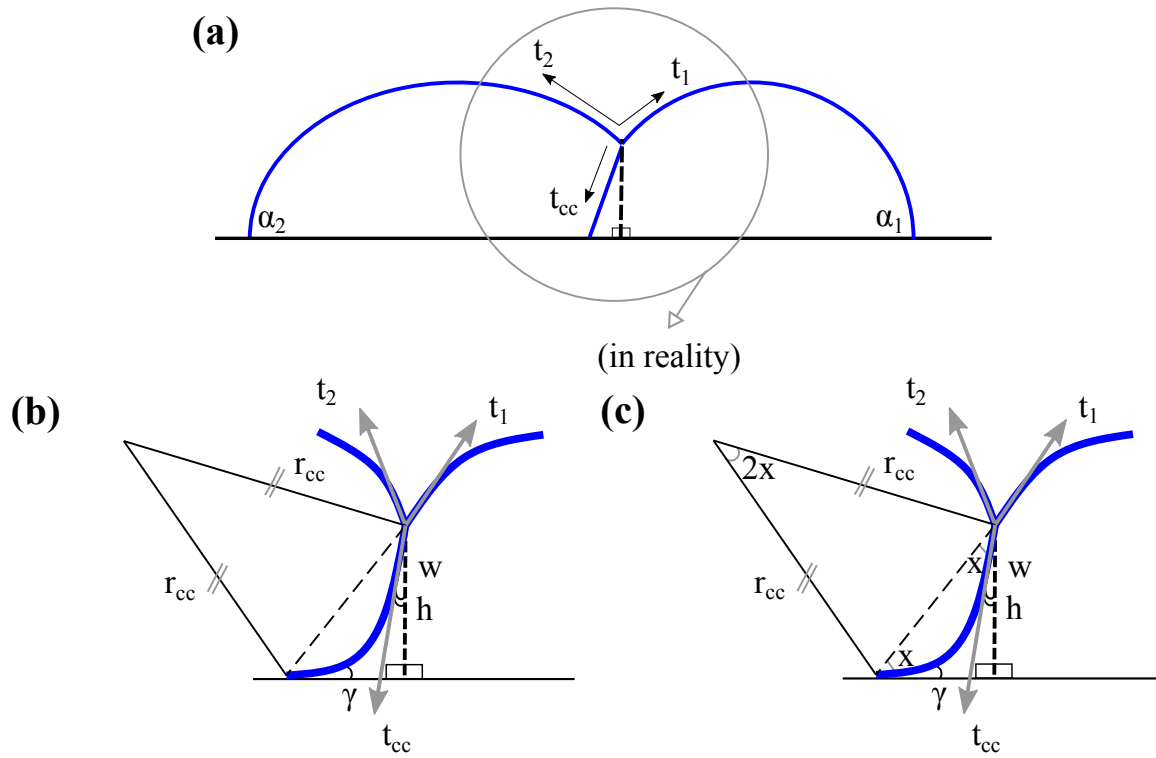


Figure 2.

6) Find the radius of curvature for the cell-cell junction:

$$|p_2 - p_1| = \frac{t_{cc}}{r_{cc}} \rightarrow r_{cc} = \frac{t_{cc}}{|t_2/r_2 - t_1/r_1|}$$

7) Knowing  $h$ ,  $w$ ,  $r_{cc}$ , we find what is the angle  $\gamma$  at the ventral end of the cell-cell boundary.

$$(1) \cos(x + w) = \frac{h}{s}$$

$$(2) \sin(x + \gamma) = \frac{h}{s}$$

$$(3) s^2 = 2r_{cc}^2 (1 - \cos(2x))$$

Combining (1) and (3), we obtain:

$$\frac{h^2}{\cos^2(x + w)} = 2r_{cc}^2 (1 - \cos(2x)) \text{ for } x \in [0, \pi/4].$$

$$\text{Then: } \gamma = \frac{\pi}{2} - w - 2x.$$

8) Gamma is a function of  $v$ , or a constant. In any case, the force balance condition at the ventral end of the cell-cell boundary closes the system.

$$t_{cc} \cos(\gamma) = \xi v_{\text{end}} \Rightarrow v_{\text{end}} = \frac{t_{cc}}{\xi} \cos(\gamma)$$

Now loop over steps 1-8 and adjust  $w$  until  $\delta v = |v - v_{\text{end}}| \leq \text{tolerance}$ .

Could also loop over steps 1-8 and adjust  $w$  until  $\delta \gamma = |\gamma - \cos^{-1}(\xi v/t_{cc})| \leq \text{tolerance}$ .