

- Force balance @ nodes:

$$(1) F_p - \xi_1 v_1 - t_L \cos \alpha_L = 0 \text{ where } F_p = (1 + f) \left(1 - \frac{t_L}{t_0 + \tau} \right)$$

$$(3) F_r + \xi_3 v_3 - t_T \cos \alpha_T = 0 \text{ where } F_r = (1 - f) \left(1 - \frac{t_T}{t_0 + \tau} \right)$$

$$(2) F_c \cos \gamma - \xi_2 v_2 = 0$$

$$(4) t_L \sin \alpha_L + t_T \sin \alpha_T + t_{cc} \cos(x + w) = \xi_4 x_{4y}$$

$$t_L \cos \alpha_L = t_T \cos \alpha_T + t_{cc} \sin(x + w) + \xi_4 x_{4x}$$

Move the points according to their velocity from the force balance

This is the vertex model with 8 unknowns | if $v_1 = v_2 = v_3 = v_4 = \sqrt{v_{4x}^2 + v_{4y}^2} = v$ then 5 unknowns.

- Find area of triangles defined by nodes 1,2,4 (leader) and 2,3,4 (trailer) – done numerically.
- Due to area conservation, loop over θ, r combinations until $A_\Delta + A_{segment} > 1$

$$A_{segment} = \frac{\theta - \sin \theta}{2} r^2$$

Result of the loop: $r_L, r_T, (\alpha_L + \beta_L)$, and $(\alpha_T + \beta_T)$

- Find center of circles:

Formula based on rhombus enclosed for two points (x_1, y_1) and (x_2, y_2) lie on a circle of radius r .

$$x_a = \frac{1}{2}(x_2 - x_1)$$

$$y_a = \frac{1}{2}(y_2 - y_1)$$

$$x_0 = x_1 + x_a$$

$$y_0 = y_1 + y_a$$

$$a = \sqrt{x_a^2 + y_a^2}$$

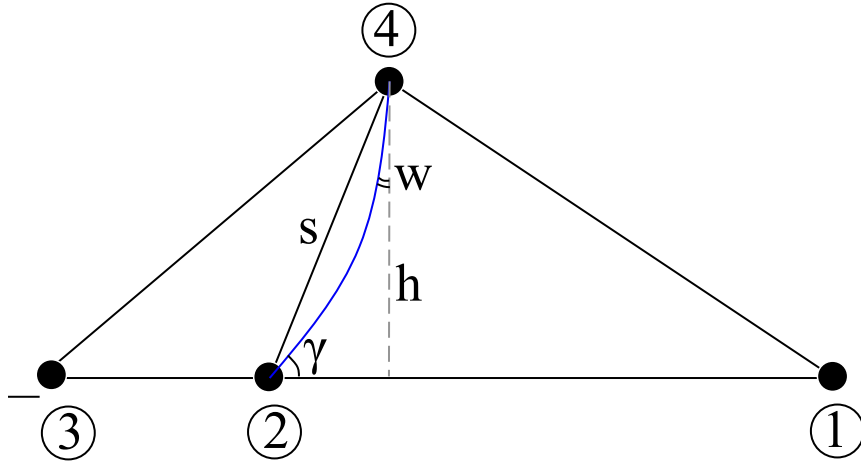
$$b = \sqrt{r^2 - a^2}$$

$$x_c = x_0 \pm \frac{by_a}{a} \text{ and } y_c = y_0 \mp \frac{bx_a}{a}$$

- Find and isolate angles $\alpha_L, \alpha_T, \beta_L, \beta_T$ since

$$\sin \alpha_L = \frac{x_c - x_1}{r_L}$$

$$\sin \alpha_T = \frac{x_3 - x_c}{r_T}$$



- Compute the cell-cell interface geometry:

$$r_{cc} = \frac{t_{cc}}{t_T/r_T - t_L/r_L}$$

As above find the center of the circle defined by radius r_{cc} and nodes 2 and 4.

Lastly find the angle of opening (aka $2x$):

$$s^2 = 2r_{cc}^2 - 2r_{cc}^2 \cos(2x) = 2r_{cc}^2 (1 - \cos(2x))$$

$$x = \frac{1}{2} \cos^{-1} \left(1 - \frac{s^2}{2r_{cc}^2} \right)$$

$$\gamma = \dots \text{ since } \sin(\gamma + x) = \frac{h}{s}$$

$$w = \dots \text{ since } \frac{\pi}{2} = \gamma + w + 2x$$

At this point the morphology is known.

- Proceed with the routine.

Example

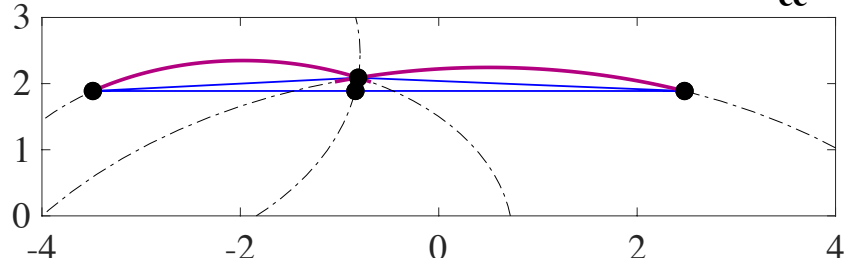
T = 0:

$$r_L = 5.64, r_T = 2.72, r_{cc} = 3.4108$$

$$t_L = 1, \alpha_L = 0.35753, \beta_L = 0.27847$$

$$t_T = 1.2, \alpha_T = 0.59052, \beta_T = 0.49348$$

$$x = 0.029604, \gamma = 1.4034, w = 0.10822, h = 0.2, t_{cc} = 0.9$$



$$r_L = 6.84, r_T = 2.99, r_{cc} = 3.5275$$

$$t_L = 1, \alpha_L = 0.31845, \beta_L = 0.21755$$

$$t_T = 1.2, \alpha_T = 0.55406, \beta_T = 0.42994$$

$$x = 0.03523, \gamma = 1.4574, w = 0.042919, h = 0.24774, t_{cc} = 0.9$$

