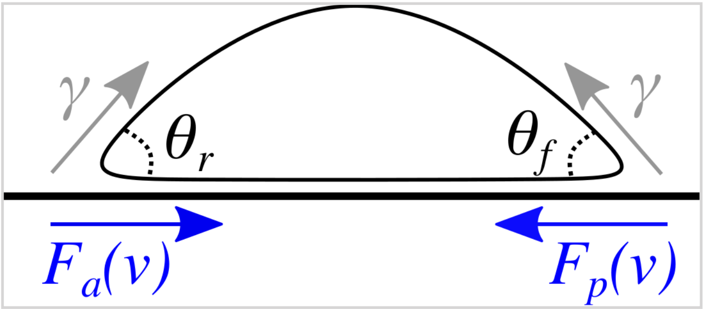
Morphodynamics

Single cell bubble model:



γ : membrane/cortex tension
$F_a(v)$: adhesive forces
$F_p(v)$: protrusive forces
θ_r : rear internal angle
θ_f : front internal angle

By Young's equation between three "phases": cell, exterior/air (endoderm), surface (epidermis), we obtain the following force balance expression in the horizontal direction:

$$\gamma \cos(\theta_r) + F_a(v) = 0$$
$$\theta_r = \arccos\left(-\frac{F_a(v)}{\gamma}\right)$$

$$F_p(v) - \gamma \cos(\theta_f) = 0$$
$$\theta_f = \arccos\left(\frac{F_p(v)}{\gamma}\right)$$

To introduce dynamics, we assume the following functional forms for the adhesive and protrusive forces as functions of cell velocity:

(1) Protrusive: $F_p = -\beta_p \log\left(\frac{v(e-1)}{\alpha_p e} + \frac{1}{e}\right)$

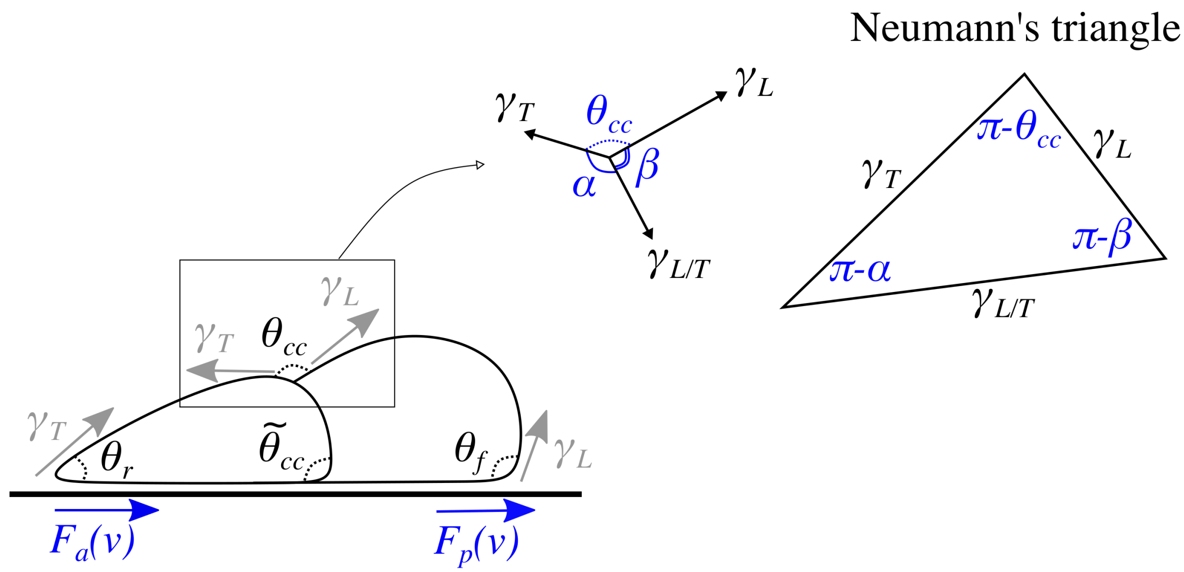
- chosen such that $F_p(v=0) = \beta_p$ and $F_p(v=\alpha_p) = 0$.

(2) Adhesive: $F_a = \epsilon_1 - (\epsilon_2 - \epsilon_1)\log\left(\frac{-v(1-1/e)}{\alpha_a} + 1\right)$

- chosen such that $F_a(v=0) = \epsilon_1$ and $F_a(v=\alpha_a) = \epsilon_2$

Can find steady-state solution for the velocity when the two forces are balanced.
Can find emergent shape by using the resulting velocity to compute forces and then together with membrane/cortex tension, deduce the rear and front angles.

A pair of cells:



By Young's equation between four "phases": trailer cell, leader cell, exterior/air (endoderm), surface (epidermis), we obtain the following force balance expressions in the horizontal direction:

(1) Rear region in trailer cell: $\gamma_T \cos(\theta_r) + F_a(v) = 0$, and then $\theta_r = \arccos\left(-\frac{F_a(v)}{\gamma_T}\right)$

(2) Front region in leader cell: $F_p(v) - \gamma_L \cos(\theta_f) = 0$, and then $\theta_f = \arccos\left(\frac{F_p(v)}{\gamma_L}\right)$

(3) Cell-cell region in all directions:

$$\gamma_L + \gamma_{L/T}\cos(\beta) + \gamma_T \cos(\theta_{cc}) = 0$$

$$\gamma_L \cos(\beta) + \gamma_{L/T} + \gamma_T \cos(\alpha) = 0$$

$$\gamma_L \cos(\theta_{cc}) + \gamma_{L/T}\cos(\alpha) + \gamma_T = 0$$

From Neumann's triangle:

$$\theta_{cc} = \pi - \arccos\left(\frac{\gamma_L^2 + \gamma_T^2 - \gamma_{L/T}^2}{2\gamma_L\gamma_T}\right)$$

What is the interpretation of $\gamma_{L/T}$? A combination of tensions in each cell plus cell-cell forces?

Introducing dynamics, again, we assume the following functional forms for the adhesive and protrusive forces as functions of cell velocity:

(1) Protrusive: $F_p = -\beta_p \log\left(\frac{v(e-1)}{\alpha_p e} + \frac{1}{e}\right)$

- chosen such that $F_p(v = 0) = \beta_p$ and $F_p(v = \alpha_p) = 0$.

(2) Adhesive: $F_a = \epsilon_1 - (\epsilon_2 - \epsilon_1)\log\left(\dfrac{-v(1 - 1/e)}{\alpha_a} + 1\right)$

- chosen such that $F_a(v = 0) = \epsilon_1$ and $F_a(v = \alpha_a) = \epsilon_2$

We can find the steady state velocity where the two forces are balanced: $F_a(v) = F_p(v)$.
From the known steady-state velocity, we can yet again compute the front and rear angles.

What about the cell-cell contact angle?