

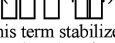
Cell's dorsal surface is Ω ; the boundary of this surface is $\partial\Omega$ (see figure). This flat surface is in the x-y-plane; x-axis is the direction of the tactic directional cue, and the cell is polarized in that direction.

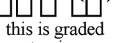
The ventral surface, $z = h(x, y)$, is given by the equation: $\Delta h(x, y) = -T / P$, where T [pN/um] is the cortex tension and is a given parameter, and P [pN/um²] is variable in time (see below). The boundary condition for eq. $\Delta h(x, y) = -T / P$ is $h(x, y) = 0$ on $\partial\Omega$. In addition, the volume of the cell is conserved: $\int_{\Omega} h dx dy = v$ where v is the constant model parameter – cell volume.

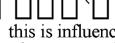
The boundary of the dorsal surface (we'll call it cell edge) is deforming in a locally normal direction (see fig) with local velocity, which is a function of 1) angle θ between the x-axis and polar angular coordinate of the point at the edge, and 2) of the contact angle $\varphi(\theta) = \arctan[\nabla h(x(\theta), y(\theta))]$ where $x(\theta), y(\theta)$ are Cartesian coordinates of the point at the cell edge with polar coordinate θ . To measure θ , we need to define the cell center (cross in the fig). One convenient way to define it is find the dashed line parallel to the x-axis which divides the dorsal surface in two equal halves (so that areas to the left and right from this line are the same: $A_1 = A_2$), and then take the center of the dashed line (see fig).

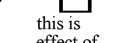
Let us try the following velocity of the boundary, which can be derived from a force balance combined

$$V(\theta) = \kappa_1 \left(\frac{A_0 - |\Omega|}{A_0} \right) + \kappa_2 \cos(\theta) - \kappa_3 T \cos(\varphi(\theta)) + \kappa_4 P$$


 this term stabilizes
the dorsal surface
area $|\Omega|$ to around
target area A_0


 this is graded
protrusion-
retraction


 this is influence
of contact angle
and cortex tension


 this is
effect of
pressure
pushing on
endoderm

with Young-Dupre eq.:

Here $\kappa_{1,2,3,4}$ are model parameters.

First, scale and non-dimensionalize the model. I would take $\text{volume}^{1/3}$ as length scale, $T^*(\text{length scale})$ as force scale; $(\text{length scale})/\kappa_2$ as time scale.

Then, think about the numerics. The algorithm probably should be similar to that in Hunter's paper:
At any time step,

- 1) On a given Ω , solve $\Delta h(x, y) = -T / P$ with $h(x, y) = 0$ on $\partial\Omega$. Find P from the condition:

$$\int_{\Omega} h dx dy = v . \text{ (In Hunter's paper it seems they have some neat trick for doing that)}$$

- 2) Find the cell center, compute $V(\theta)$, deform the cell edge.

