In what follows we shall introduce a standard and widely applied method for estimatig volatility on the basis of historical data on returns, that is, we consider the second meaning of volatility.

Under the LRW hypothesis a sensible estimate of σ^2 is:

$$\sum_{i=0,\ldots,n} \left(r^*_{t-i} - \overline{r}^*\right)^2 / n$$

Where $\overline{r}^*$is the sample mean.

This is the standard unbiased estimate for the variance of uncorrelated random variables with identical expected values and variances (the simple empirical variance of the data, where the denominator its taken as the actual number of observations $n+1$, could be used without problems as in standard applications the sample size is quite big).

Notice that each data point is given the same weight: the hypothesis is such that any new observation should improve the estimate in the same way.

The log random walk would justify such an estimate.

In practice, nobody uses such estimate and a common choice is the exponential smoothing estimate, while already quite old when suggested by J. P. Morgan in the Riskmetrics context, this is commonly known in the field as the Riskmetrics estimate:

$$V_t = \frac{\sum_{i=0,\ldots,n} \lambda^i r^{*2}_{t-i}}{\sum_{i=0,\ldots,n} \lambda^i}$$

From a statistician's point of view this is an exponentially smoothed estimate with λ a smoothing parameter: $0 < \lambda < 1$.

Common values of the smoothing parameter are around 0.95.

Users of such an estimate do not consider sensible to consider each data point equally relevant. Old observations are less relevant than new ones.

Implicitly, then, while we "believe" the log random walk when "annualizing" volatility, we do not believe it when estimating volatility.

Moreover it shall be noticed that, in this estimate, the sampling mean of returns does not appear. This is a choice which can be justified in two ways: first we can assume the expected return μ over a small time interval to be very small. With a non negligible variance it is quite likely that an estimate of the expected value of returns could show an higher sampling variability than its likely size and so it could create problems to the statistical stability of the variance estimate[11]. Second, an estimate

[11]A simple 'back of the envelope' computation: say the standard deviation for stock returns over one year is in the range of 30%. Even in the simple case where data on returns are i.i.d., if we estimate the expected return over one year with the sample mean we need about 30 observations (years!) in order to reduce the sampling standard deviation of the mean to about 5.5% so to be able to estimate reliably risk premia (this is financial jargon: the expected value of return is commonly called 'risk