

```

Clear["*"];

(*sphere/sphere collision resolver*)
p1 = {p1.x, p1.y, p1.z}; (*position*)
u1 = {u1.x, u1.y, u1.z}; (*velocity*)
r1; (*radius*)
p2 = {p2.x, p2.y, p2.z};
u2 = {u2.x, u2.y, u2.z};
r2;
Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t]

{ {t -> (-2 p1.x u1.x + 2 p2.x u1.x + 2 p1.x u2.x - 2 p2.x u2.x - 2 p1.y u1.y + 2 p2.y u1.y +
2 p1.y u2.y - 2 p2.y u2.y - 2 p1.z u1.z + 2 p2.z u1.z + 2 p1.z u2.z - 2 p2.z u2.z -
sqrt((2 p1.x u1.x - 2 p2.x u1.x - 2 p1.x u2.x + 2 p2.x u2.x + 2 p1.y u1.y - 2 p2.y u1.y -
2 p1.y u2.y + 2 p2.y u2.y + 2 p1.z u1.z - 2 p2.z u1.z - 2 p1.z u2.z + 2 p2.z u2.z)^2 -
4 (p1.x^2 - 2 p1.x p2.x + p2.x^2 + p1.y^2 - 2 p1.y p2.y + p2.y^2 + p1.z^2 - 2 p1.z p2.z + p2.z^2 - r1^2 - 2 r1 r2 -
r2^2) (u1.x^2 - 2 u1.x u2.x + u2.x^2 + u1.y^2 - 2 u1.y u2.y + u2.y^2 + u1.z^2 - 2 u1.z u2.z + u2.z^2))) /
(2 (u1.x^2 - 2 u1.x u2.x + u2.x^2 + u1.y^2 - 2 u1.y u2.y + u2.y^2 + u1.z^2 - 2 u1.z u2.z + u2.z^2)) } ,
{t -> (-2 p1.x u1.x + 2 p2.x u1.x + 2 p1.x u2.x - 2 p2.x u2.x - 2 p1.y u1.y + 2 p2.y u1.y +
2 p1.y u2.y - 2 p2.y u2.y - 2 p1.z u1.z + 2 p2.z u1.z + 2 p1.z u2.z - 2 p2.z u2.z +
sqrt((2 p1.x u1.x - 2 p2.x u1.x - 2 p1.x u2.x + 2 p2.x u2.x + 2 p1.y u1.y - 2 p2.y u1.y -
2 p1.y u2.y + 2 p2.y u2.y + 2 p1.z u1.z - 2 p2.z u1.z - 2 p1.z u2.z + 2 p2.z u2.z)^2 -
4 (p1.x^2 - 2 p1.x p2.x + p2.x^2 + p1.y^2 - 2 p1.y p2.y + p2.y^2 + p1.z^2 - 2 p1.z p2.z + p2.z^2 - r1^2 - 2 r1 r2 -
r2^2) (u1.x^2 - 2 u1.x u2.x + u2.x^2 + u1.y^2 - 2 u1.y u2.y + u2.y^2 + u1.z^2 - 2 u1.z u2.z + u2.z^2))) /
(2 (u1.x^2 - 2 u1.x u2.x + u2.x^2 + u1.y^2 - 2 u1.y u2.y + u2.y^2 + u1.z^2 - 2 u1.z u2.z + u2.z^2)) } }

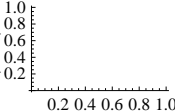
(*if overlapping pick nearest t<0, for collision detection check t>0 and t<1*)
np1 = p1 + u1 t; (*move out of collision*)
np2 = p2 + u2 t;
nml = Normalize[np1 - np2]; (*collision plane normal*)
m1; (*mass*)
m2;
(*      velocity given      velocities received along collision plane normal*)
v1 = u1 - (u1.nml) nml + (u1.nml) nml ((m1 - m2) / (m1 + m2)) + (u2.nml) nml (2 m2 / (m1 + m2));
v2 = u2 - (u2.nml) nml + (u2.nml) nml ((m2 - m1) / (m1 + m2)) + (u1.nml) nml (2 m1 / (m1 + m2));
np1 = np1 + v1 (1 - t); (*perform remaining dt with new velocities*)
np2 = np2 + v2 (1 - t);

v1 = (u1.nml) nml ((m1 - m2) / (m1 + m2)) + (u2.nml) nml (2 m2 / (m1 + m2));
v2 = (u2.nml) nml ((m2 - m1) / (m1 + m2)) + (u1.nml) nml (2 m1 / (m1 + m2));
m1 u1 + m2 u2 == m1 v1 + m2 v2;

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(*sphere/plane collision resolver*)
p_g = {0, 0, 0}; (*dot on plane*)
n_g = Normalize[{0, 1, 0}]; (*normal*)
p = {0, 1.5, 0}; (*sphere position*)
u = {0, -1, 0}; (*velocity*)
r = 1; (*radius*)
Remove[t];
f = (p + u t - p_g).n_g - r == 0; (*signed distance to plane*)
Plot[f, {t, 0, 1}]
Solve[f, t]
t = .5;
(*pick nearest t<0, t>1 or t<0 collision is in future or past*)
np = p + u t (*move sphere out of collision*)
v = u - 2 (u.n_g) n_g (*reflect*)
np = np + v (1 - t) (*second part of dt*)

$DisplayFunction[

]

{{t -> -1. (-0.5 + 1. {0, 1.5, 0}_g)}}
{0., 1., 0.}
{0, 1, 0}
{0., 1.5, 0.}

(*simplified*)
Clear["*"];

```

(*sphere/sphere collision resolver*)

$\mathbf{p}_1 = \{\mathbf{p}_{1.x}, \mathbf{p}_{1.y}, \mathbf{p}_{1.z}\};$ (*position*)

$\mathbf{u}_1 = \{\mathbf{u}_{1.x}, \mathbf{u}_{1.y}, \mathbf{u}_{1.z}\};$ (*velocity*)

$\mathbf{r}_1;$ (*radius*)

$\mathbf{p}_2 = \{\mathbf{p}_{2.x}, \mathbf{p}_{2.y}, \mathbf{p}_{2.z}\};$

$\mathbf{u}_2 = \{\mathbf{u}_{2.x}, \mathbf{u}_{2.y}, \mathbf{u}_{2.z}\};$

$\mathbf{r}_2;$

$\text{Solve}[\text{EuclideanDistance}[\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t}, \mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}] == \mathbf{r}_1 + \mathbf{r}_2, \mathbf{t}]$

$$\left\{ \left\{ \mathbf{t} \rightarrow \left(-2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} + 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} - \sqrt{\left(\left(2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} - 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} \right)^2 - 4 \left(\mathbf{p}_{1.x}^2 - 2 \mathbf{p}_{1.x} \mathbf{p}_{2.x} + \mathbf{p}_{2.x}^2 + \mathbf{p}_{1.y}^2 - 2 \mathbf{p}_{1.y} \mathbf{p}_{2.y} + \mathbf{p}_{2.y}^2 + \mathbf{p}_{1.z}^2 - 2 \mathbf{p}_{1.z} \mathbf{p}_{2.z} + \mathbf{p}_{2.z}^2 - \mathbf{r}_1^2 - 2 \mathbf{r}_1 \mathbf{r}_2 - \mathbf{r}_2^2 \right) \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right)} \right) \right\} \right\} / \left(2 \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right) \right) \right\},$$

$$\left\{ \left\{ \mathbf{t} \rightarrow \left(-2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} + 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} + \sqrt{\left(\left(2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} - 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} \right)^2 - 4 \left(\mathbf{p}_{1.x}^2 - 2 \mathbf{p}_{1.x} \mathbf{p}_{2.x} + \mathbf{p}_{2.x}^2 + \mathbf{p}_{1.y}^2 - 2 \mathbf{p}_{1.y} \mathbf{p}_{2.y} + \mathbf{p}_{2.y}^2 + \mathbf{p}_{1.z}^2 - 2 \mathbf{p}_{1.z} \mathbf{p}_{2.z} + \mathbf{p}_{2.z}^2 - \mathbf{r}_1^2 - 2 \mathbf{r}_1 \mathbf{r}_2 - \mathbf{r}_2^2 \right) \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right)} \right) \right\} \right\} / \left(2 \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right) \right) \right\} \right\}$$

$\text{EuclideanDistance}[\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t}, \mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}]$

$$\sqrt{\left(\text{Abs}[\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x}]^2 + \text{Abs}[\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y}]^2 + \text{Abs}[\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z}]^2 \right)}$$

$$\sqrt{\text{Total}[(\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t} - (\mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}))^2]} == \mathbf{r}_1 + \mathbf{r}_2$$

$$\sqrt{\left((\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x})^2 + (\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y})^2 + (\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z})^2 \right)} == \mathbf{r}_1 + \mathbf{r}_2$$

$$\left(\sqrt{\text{Total}[(\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t} - (\mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}))^2]} \right)^2 == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$$(\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x})^2 + (\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y})^2 + (\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z})^2 == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$$\text{Total}[(\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t} - (\mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}))^2] == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$$(\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x})^2 + (\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y})^2 + (\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z})^2 == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$\Delta \mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2$

$$\{\mathbf{p}_{1.x} - \mathbf{p}_{2.x}, \mathbf{p}_{1.y} - \mathbf{p}_{2.y}, \mathbf{p}_{1.z} - \mathbf{p}_{2.z}\}$$

Collect[**Expand**[**Total**[($\Delta p + u_1 t - u_2 t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Expand**[**Total**[($\Delta p + (u_1 - u_2) t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

$$\Delta u = u_1 - u_2$$

$$\{u_{1,x} - u_{2,x}, u_{1,y} - u_{2,y}, u_{1,z} - u_{2,z}\}$$

Collect[**Expand**[**Total**[($\Delta p + \Delta u t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Expand**[**Total**[($\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2$) - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Total**[($\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2$) - ($r_1 + r_2$)² == 0], t]

$$(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2 - (r_1 + r_2)^2 + t \left(2 (p_{1,x} - p_{2,x}) (u_{1,x} - u_{2,x}) + 2 (p_{1,y} - p_{2,y}) (u_{1,y} - u_{2,y}) + 2 (p_{1,z} - p_{2,z}) (u_{1,z} - u_{2,z}) \right) + t^2 \left((u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2 \right) == 0$$

Total[(Δp^2)

$$(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2$$

Collect[

$$\text{Expand}[\text{Total}[\Delta p^2] - (r_1 + r_2)^2 + t \left(2 (p_{1,x} - p_{2,x}) (u_{1,x} - u_{2,x}) + 2 (p_{1,y} - p_{2,y}) (u_{1,y} - u_{2,y}) + 2 (p_{1,z} - p_{2,z}) (u_{1,z} - u_{2,z}) \right) + t^2 \left((u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2 \right) == 0], t]$$

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

2 $\Delta p \cdot \Delta u t$

$$2 t \left((p_{1,x} - p_{2,x}) (u_{1,x} - u_{2,x}) + (p_{1,y} - p_{2,y}) (u_{1,y} - u_{2,y}) + (p_{1,z} - p_{2,z}) (u_{1,z} - u_{2,z}) \right)$$

Collect[Expand[
Total[Δp^2] - $(r_1 + r_2)^2 + 2 \Delta p \cdot \Delta u t + t^2 ((u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2) = 0]$, t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - \\ r_2^2 + t (2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - \\ 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}) + \\ t^2 (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) = 0$$

Total[Δu^2]

$$(u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2$$

Collect[Expand[Total[Δp^2] - $(r_1 + r_2)^2 + 2 \Delta p \cdot \Delta u t + t^2$ Total[Δu^2] = 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - \\ r_2^2 + t (2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - \\ 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}) + \\ t^2 (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) = 0$$

a = Total[Δu^2];

b = 2 $\Delta p \cdot \Delta u$;

c = Total[Δp^2] - $(r_1 + r_2)^2$;

Collect[Expand[a t² + b t + c = 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - \\ r_2^2 + t (2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - \\ 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}) + \\ t^2 (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) = 0$$

Solve[EuclideanDistance[p₁ + u₁ t, p₂ + u₂ t] == r₁ + r₂, t] == Solve[a t² + b t + c == 0, t]

Solve[a t² + b t + c == 0, t]

True

$$\left\{ \left\{ t \rightarrow \left(-2 p_{1,x} u_{1,x} + 2 p_{2,x} u_{1,x} + 2 p_{1,x} u_{2,x} - 2 p_{2,x} u_{2,x} - 2 p_{1,y} u_{1,y} + 2 p_{2,y} u_{1,y} + \right. \right. \\ \left. \left. 2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} - \right. \right. \\ \left. \sqrt{\left(\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - \right. \right. \right. \\ \left. \left. \left. 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right)^2 - \right. \right. \\ \left. \left. 4 \left(p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - \right. \right. \right. \\ \left. \left. \left. r_2^2 \right) \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) \right) \right) \right\} / \\ \left(2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) \right) \left. \right\}, \\ \left\{ t \rightarrow \left(-2 p_{1,x} u_{1,x} + 2 p_{2,x} u_{1,x} + 2 p_{1,x} u_{2,x} - 2 p_{2,x} u_{2,x} - 2 p_{1,y} u_{1,y} + 2 p_{2,y} u_{1,y} + \right. \right. \\ \left. \left. 2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} + \right. \right. \\ \left. \sqrt{\left(\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - \right. \right. \right. \\ \left. \left. \left. 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right)^2 - \right. \right. \\ \left. \left. 4 \left(p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - \right. \right. \right. \\ \left. \left. \left. r_2^2 \right) \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) \right) \right) \right\} / \\ \left(2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) \right) \left. \right\} \left. \right\}$$