```
(*sphere/sphere collision resolver*)
p1 = {p1.x, p1.y, p1.z};(*position*)
u1 = {u1.x, u1.y, u1.z};(*velocity*)
r1; (*radius*)
p2 = {p2.x, p2.y, p2.z};
u2 = {u2.x, u2.y, u2.z};
r2;
Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t]
```

$$\left\{\left\{t \to \left(-2\,p_{1.x}\,u_{1.x} + 2\,p_{2.x}\,u_{1.x} + 2\,p_{1.x}\,u_{2.x} - 2\,p_{2.x}\,u_{2.x} - 2\,p_{1.y}\,u_{1.y} + 2\,p_{2.y}\,u_{1.y} + 2\,p_{1.y}\,u_{2.y} - 2\,p_{2.y}\,u_{2.y} - 2\,p_{1.z}\,u_{1.z} + 2\,p_{2.z}\,u_{1.z} + 2\,p_{1.z}\,u_{2.z} - 2\,p_{2.z}\,u_{2.z} - \sqrt{\left(\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right)^2 - 4\left(p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2\right)\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)}\right) \Big/ \left(2\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)\right\},$$

$$\left\{t \to \left(-2\,p_{1.x}\,u_{1.x} + 2\,p_{2.x}\,u_{1.x} + 2\,p_{1.x}\,u_{2.x} - 2\,p_{2.x}\,u_{2.x} - 2\,p_{1.y}\,u_{1.y} + 2\,p_{2.y}\,u_{1.y} + 2\,p_{1.y}\,u_{2.y} - 2\,p_{2.y}\,u_{2.y} - 2\,p_{1.z}\,u_{1.z} + 2\,p_{2.z}\,u_{1.z} + 2\,p_{1.z}\,u_{2.z} - 2\,p_{2.z}\,u_{2.z} + \sqrt{\left(\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right)^2 - 4\left(p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2\right)\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)}\right) \Big/ \left(2\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)\right\}\right\}$$

```
EuclideanDistance[p1 + u1 t, p2 + u2 t]
```

$$\sqrt{\left(\text{Abs}\left[p_{1.x} - p_{2.x} + t\,u_{1.x} - t\,u_{2.x}\right]^2 + \text{Abs}\left[p_{1.y} - p_{2.y} + t\,u_{1.y} - t\,u_{2.y}\right]^2 + \text{Abs}\left[p_{1.z} - p_{2.z} + t\,u_{1.z} - t\,u_{2.z}\right]^2\right)}$$

```
Sqrt[Total[(p1 + u1 t - (p2 + u2 t))^2]] == r1 + r2
```

$$\sqrt{\left(p_{1.x} - p_{2.x} + t\,u_{1.x} - t\,u_{2.x}\right)^2 + \left(p_{1.y} - p_{2.y} + t\,u_{1.y} - t\,u_{2.y}\right)^2 + \left(p_{1.z} - p_{2.z} + t\,u_{1.z} - t\,u_{2.z}\right)^2} == r_1 + r_2$$

```
(Sqrt[Total[(p1 + u1 t - (p2 + u2 t))^2]])^2 == (r1 + r2)^2
```

$$\left(p_{1.x} - p_{2.x} + t\,u_{1.x} - t\,u_{2.x}\right)^2 + \left(p_{1.y} - p_{2.y} + t\,u_{1.y} - t\,u_{2.y}\right)^2 + \left(p_{1.z} - p_{2.z} + t\,u_{1.z} - t\,u_{2.z}\right)^2 == \left(r_1 + r_2\right)^2$$

```
Total[(p1 + u1 t - (p2 + u2 t))^2] == (r1 + r2)^2
```

$$\left(p_{1.x} - p_{2.x} + t\,u_{1.x} - t\,u_{2.x}\right)^2 + \left(p_{1.y} - p_{2.y} + t\,u_{1.y} - t\,u_{2.y}\right)^2 + \left(p_{1.z} - p_{2.z} + t\,u_{1.z} - t\,u_{2.z}\right)^2 == \left(r_1 + r_2\right)^2$$

```
Δp = p1 - p2
```

$$\left\{p_{1.x} - p_{2.x},\ p_{1.y} - p_{2.y},\ p_{1.z} - p_{2.z}\right\}$$

```
Collect[Expand[Total[(Δp + u1 t - u2 t)^2] - (r1 + r2)^2 == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
Collect[Expand[Total[(Δp + (u1 - u2) t)^2] - (r1 + r2)^2 == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
Δu = u1 - u2
```

$$\{u_{1.x} - u_{2.x},\ u_{1.y} - u_{2.y},\ u_{1.z} - u_{2.z}\}$$

```
Collect[Expand[Total[(Δp + Δu t)^2] - (r1 + r2)^2 == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
Collect[Expand[Total[Δp^2 + 2 Δp Δu t + Δu^2 t^2] - (r1 + r2)^2 == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
Collect[Total[Δp^2 + 2 Δp Δu t + Δu^2 t^2] - (r1 + r2)^2 == 0, t]
```

$$(p_{1.x} - p_{2.x})^2 + \left(p_{1.y} - p_{2.y}\right)^2 + (p_{1.z} - p_{2.z})^2 - (r_1 + r_2)^2 + t\,\left(2\,(p_{1.x} - p_{2.x})\,(u_{1.x} - u_{2.x}) + 2\,\left(p_{1.y} - p_{2.y}\right)\,\left(u_{1.y} - u_{2.y}\right) + 2\,(p_{1.z} - p_{2.z})\,(u_{1.z} - u_{2.z})\right) + t^2\,\left((u_{1.x} - u_{2.x})^2 + \left(u_{1.y} - u_{2.y}\right)^2 + (u_{1.z} - u_{2.z})^2\right) == 0$$

```
Total[Δp^2]
```

$$(p_{1.x} - p_{2.x})^2 + \left(p_{1.y} - p_{2.y}\right)^2 + (p_{1.z} - p_{2.z})^2$$

```
Collect[
 Expand[Total[Δp^2] - (r1 + r2)^2 + t (2 (p1.`x - p2.`x) (u1.`x - u2.`x) + 2 (p1.`y - p2.`y) (u1.`y - u2.`y) +
       2 (p1.`z - p2.`z) (u1.`z - u2.`z)) +
    t^2 ((u1.`x - u2.`x)^2 + (u1.`y - u2.`y)^2 + (u1.`z - u2.`z)^2) == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
2 Δp.Δu t
```

$$2\,t\,\left((p_{1.x} - p_{2.x})\,(u_{1.x} - u_{2.x}) + \left(p_{1.y} - p_{2.y}\right)\,\left(u_{1.y} - u_{2.y}\right) + (p_{1.z} - p_{2.z})\,(u_{1.z} - u_{2.z})\right)$$

```
Collect[Expand[
  Total[Δp^2] - (r1 + r2)^2 + 2 Δp.Δu t + t^2 ((u1.`x - u2.`x)^2 + (u1.`y - u2.`y)^2 + (u1.`z - u2.`z)^2) == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
Total[Δu^2]
```

$$(u_{1.x} - u_{2.x})^2 + \left(u_{1.y} - u_{2.y}\right)^2 + (u_{1.z} - u_{2.z})^2$$

```
Collect[Expand[Total[Δp^2] - (r1 + r2)^2 + 2 Δp.Δu t + t^2 Total[Δu^2] == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
a = Total[Δu^2];
b = 2 Δp.Δu;
c = Total[Δp^2] - (r1 + r2)^2;
Collect[Expand[a t^2 + b t + c == 0], t]
```

$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

```
Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t] == Solve[a t^2 + b t + c == 0, t]
```

True