

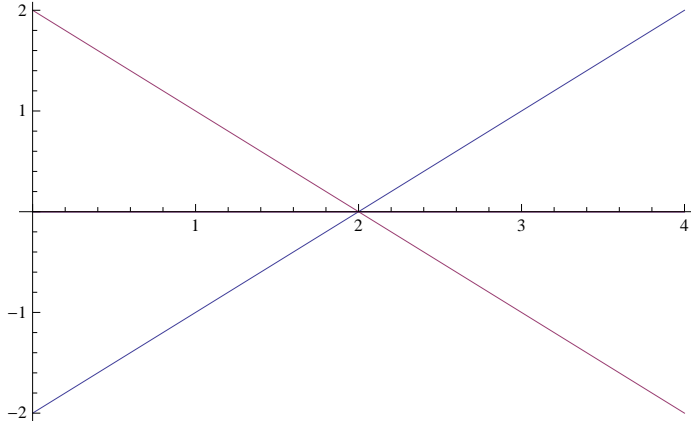
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p1 = {px1, py1, pz1};
v1 = {vx1, vy1, vz1};
r1 = a;
p2 = {px2, py2, pz2};
v2 = {vx2, vy2, vz2};
r2 = b;
(p1 + v1 t) - (p2 + v2 t) == 0

{px1 - px2 + t vx1 - t vx2, py1 - py2 + t vy1 - t vy2, pz1 - pz2 + t vz1 - t vz2} == 0

((p1 + v1 t) - (p2 + v2 t))^2
{ (px1 - px2 + t vx1 - t vx2)^2, (py1 - py2 + t vy1 - t vy2)^2, (pz1 - pz2 + t vz1 - t vz2)^2 }
Total[ ((p1 + v1 t) - (p2 + v2 t))^2 ]
(px1 - px2 + t vx1 - t vx2)^2 + (py1 - py2 + t vy1 - t vy2)^2 + (pz1 - pz2 + t vz1 - t vz2)^2
Sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t))^2 ] ]
Sqrt[(px1 - px2 + t vx1 - t vx2)^2 + (py1 - py2 + t vy1 - t vy2)^2 + (pz1 - pz2 + t vz1 - t vz2)^2]
Sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t))^2 ] ] == r1 + r2
Sqrt[(px1 - px2 + t vx1 - t vx2)^2 + (py1 - py2 + t vy1 - t vy2)^2 + (pz1 - pz2 + t vz1 - t vz2)^2] == a + b
(px1 - px2 + t vx1 - t vx2)^2 + (py1 - py2 + t vy1 - t vy2)^2 + (pz1 - pz2 + t vz1 - t vz2)^2 == (a + b)^2
(px1 - px2 + t vx1 - t vx2)^2 + (py1 - py2 + t vy1 - t vy2)^2 + (pz1 - pz2 + t vz1 - t vz2)^2 == (a + b)^2
dp = p1 - p2;
dv = v1 - v2;
(dp[[1]] + t dv[[1]])^2 + (dp[[2]] + t dv[[2]])^2 + (dp[[3]] + t dv[[3]])^2 == (r1 + r2)^2
(px1 - px2 + t (vx1 - vx2))^2 + (py1 - py2 + t (vy1 - vy2))^2 + (pz1 - pz2 + t (vz1 - vz2))^2 == (a + b)^2
p1 = {-2, 0, 0};
v1 = {1, 0, 0};
r1 = 2;
p2 = {2, 0, 0};
v2 = {-1, 0, 0};
r2 = 1;
dp = p1 - p2;
dv = v1 - v2;
Solve[(dp[[1]] + t dv[[1]])^2 + (dp[[2]] + t dv[[2]])^2 + (dp[[3]] + t dv[[3]])^2 - (r1 + r2)^2 == 0, t]
{{t -> 1/2}, {t -> 7/2}}
{{t -> 1}, {t -> 3}}
Plot[{p1 + v1 t, p2 + v2 t}, {t, 0, 4}]
{{t -> 1}, {t -> 3}}

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Expand $\left[(dp[[1]] + t dv[[1]])^2 + (dp[[2]] + t dv[[2]])^2 + (dp[[3]] + t dv[[3]])^2 - (r_1 + r_2)^2, t\right]$

$$16 - (a + b)^2 - 16 t + 4 t^2$$

$p_1 = \{px_1, py_1, pz_1\};$

$v_1 = \{vx_1, vy_1, vz_1\};$

$r_1 = a;$

$p_2 = \{px_2, py_2, pz_2\};$

$v_2 = \{vx_2, vy_2, vz_2\};$

$r_2 = b;$

$dp = p_1 - p_2;$

$dv = v_1 - v_2;$

Collect $\left[(dp[[1]] + t dv[[1]])^2 + (dp[[2]] + t dv[[2]])^2 + (dp[[3]] + t dv[[3]])^2 - (r_1 + r_2)^2, t\right]$

$$-(a + b)^2 + px_1^2 - 2 px_1 px_2 + px_2^2 + py_1^2 - 2 py_1 py_2 + py_2^2 + pz_1^2 - 2 pz_1 pz_2 +$$

$$pz_2^2 + t (2 px_1 (vx_1 - vx_2) - 2 px_2 (vx_1 - vx_2) + 2 py_1 (vy_1 - vy_2) - 2 py_2 (vy_1 - vy_2) +$$

$$2 pz_1 (vz_1 - vz_2) - 2 pz_2 (vz_1 - vz_2)) + t^2 ((vx_1 - vx_2)^2 + (vy_1 - vy_2)^2 + (vz_1 - vz_2)^2)$$

dv^2

$$\{(vx_1 - vx_2)^2, (vy_1 - vy_2)^2, (vz_1 - vz_2)^2\}$$

Total $[dv^2]$

$$(vx_1 - vx_2)^2 + (vy_1 - vy_2)^2 + (vz_1 - vz_2)^2$$

$2 p_1 \cdot dv$

$$2 (px_1 (vx_1 - vx_2) + py_1 (vy_1 - vy_2) + pz_1 (vz_1 - vz_2))$$

$-2 p_1 \cdot p_2$

$$-2 (px_1 px_2 + py_1 py_2 + pz_1 pz_2)$$

$$-(a + b)^2 + px_1^2 - 2 px_1 px_2 + px_2^2 + py_1^2 - 2 py_1 py_2 + py_2^2 + pz_1^2 -$$

$$2 pz_1 pz_2 + pz_2^2 + t (2 px_1 (vx_1 - vx_2) - 2 px_2 (vx_1 - vx_2) + 2 py_1 (vy_1 - vy_2) -$$

$$2 py_2 (vy_1 - vy_2) + 2 pz_1 (vz_1 - vz_2) - 2 pz_2 (vz_1 - vz_2)) + t^2 \text{Total}[dv^2]$$

$$-(a + b)^2 + px_1^2 - 2 px_1 px_2 + px_2^2 + py_1^2 - 2 py_1 py_2 + py_2^2 + pz_1^2 - 2 pz_1 pz_2 +$$

$$pz_2^2 + t (2 px_1 (vx_1 - vx_2) - 2 px_2 (vx_1 - vx_2) + 2 py_1 (vy_1 - vy_2) - 2 py_2 (vy_1 - vy_2) +$$

$$2 pz_1 (vz_1 - vz_2) - 2 pz_2 (vz_1 - vz_2)) + t^2 ((vx_1 - vx_2)^2 + (vy_1 - vy_2)^2 + (vz_1 - vz_2)^2)$$

$$\begin{aligned}
& - (a + b)^2 + p_{x_1}^2 - 2 \mathbf{p}_1 \cdot \mathbf{p}_2 + p_{x_2}^2 + p_{y_1}^2 + p_{y_2}^2 + p_{z_1}^2 + p_{z_2}^2 + \\
& \quad t (2 \mathbf{p}_1 \cdot \mathbf{dv} - 2 p_{x_2} (v_{x_1} - v_{x_2}) - 2 p_{y_2} (v_{y_1} - v_{y_2}) - 2 p_{z_2} (v_{z_1} - v_{z_2})) + t^2 \text{Total}[\mathbf{dv}^2] \\
& - (a + b)^2 + p_{x_1}^2 + p_{x_2}^2 + p_{y_1}^2 + p_{y_2}^2 + p_{z_1}^2 + p_{z_2}^2 - 2 (p_{x_1} p_{x_2} + p_{y_1} p_{y_2} + p_{z_1} p_{z_2}) + \\
& \quad t (-2 p_{x_2} (v_{x_1} - v_{x_2}) - 2 p_{y_2} (v_{y_1} - v_{y_2}) + 2 (p_{x_1} (v_{x_1} - v_{x_2}) + p_{y_1} (v_{y_1} - v_{y_2}) + p_{z_1} (v_{z_1} - v_{z_2})) - \\
& \quad 2 p_{z_2} (v_{z_1} - v_{z_2})) + t^2 ((v_{x_1} - v_{x_2})^2 + (v_{y_1} - v_{y_2})^2 + (v_{z_1} - v_{z_2})^2) \\
& \text{Total}[\mathbf{p}_1^2 + \mathbf{p}_2^2] \\
& p_{x_1}^2 + p_{x_2}^2 + p_{y_1}^2 + p_{y_2}^2 + p_{z_1}^2 + p_{z_2}^2 \\
& - (a + b)^2 - 2 \mathbf{p}_1 \cdot \mathbf{p}_2 + \text{Total}[\mathbf{p}_1^2 + \mathbf{p}_2^2] + \\
& \quad t (2 \mathbf{p}_1 \cdot \mathbf{dv} - 2 p_{x_2} (v_{x_1} - v_{x_2}) - 2 p_{y_2} (v_{y_1} - v_{y_2}) - 2 p_{z_2} (v_{z_1} - v_{z_2})) + t^2 \text{Total}[\mathbf{dv}^2] \\
& - (a + b)^2 + p_{x_1}^2 + p_{x_2}^2 + p_{y_1}^2 + p_{y_2}^2 + p_{z_1}^2 + p_{z_2}^2 - 2 (p_{x_1} p_{x_2} + p_{y_1} p_{y_2} + p_{z_1} p_{z_2}) + \\
& \quad t (-2 p_{x_2} (v_{x_1} - v_{x_2}) - 2 p_{y_2} (v_{y_1} - v_{y_2}) + 2 (p_{x_1} (v_{x_1} - v_{x_2}) + p_{y_1} (v_{y_1} - v_{y_2}) + p_{z_1} (v_{z_1} - v_{z_2})) - \\
& \quad 2 p_{z_2} (v_{z_1} - v_{z_2})) + t^2 ((v_{x_1} - v_{x_2})^2 + (v_{y_1} - v_{y_2})^2 + (v_{z_1} - v_{z_2})^2) \\
& - \text{Total}[2 \mathbf{p}_2 \mathbf{dv}] \\
& - 2 p_{x_2} (v_{x_1} - v_{x_2}) - 2 p_{y_2} (v_{y_1} - v_{y_2}) - 2 p_{z_2} (v_{z_1} - v_{z_2}) \\
& - (a + b)^2 - 2 \mathbf{p}_1 \cdot \mathbf{p}_2 + \text{Total}[\mathbf{p}_1^2 + \mathbf{p}_2^2] + t (2 \mathbf{p}_1 \cdot \mathbf{dv} - \text{Total}[2 \mathbf{p}_2 \mathbf{dv}]) + t^2 \text{Total}[\mathbf{dv}^2] \\
& - (a + b)^2 + p_{x_1}^2 + p_{x_2}^2 + p_{y_1}^2 + p_{y_2}^2 + p_{z_1}^2 + p_{z_2}^2 - 2 (p_{x_1} p_{x_2} + p_{y_1} p_{y_2} + p_{z_1} p_{z_2}) + \\
& \quad t (-2 p_{x_2} (v_{x_1} - v_{x_2}) - 2 p_{y_2} (v_{y_1} - v_{y_2}) + 2 (p_{x_1} (v_{x_1} - v_{x_2}) + p_{y_1} (v_{y_1} - v_{y_2}) + p_{z_1} (v_{z_1} - v_{z_2})) - \\
& \quad 2 p_{z_2} (v_{z_1} - v_{z_2})) + t^2 ((v_{x_1} - v_{x_2})^2 + (v_{y_1} - v_{y_2})^2 + (v_{z_1} - v_{z_2})^2)
\end{aligned}$$