In[21]:=
```
(*matrix transformations, rotation, translation, scaling*)
```

In[22]:=
```
Clear["*"];
```

In[1]:=
```
I = IdentityMatrix[3] // MatrixForm(*identity matrix*)
```

Out[1]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[2]:=
```
x = IdentityMatrix[3][[All, 1]]
y = IdentityMatrix[3][[All, 2]]
z = IdentityMatrix[3][[All, 3]]
```

Out[2]= {1, 0, 0}

Out[3]= {0, 1, 0}

Out[4]= {0, 0, 1}

In[5]:=
```
α;(*rotation of x-axis*)
RotationMatrix[α, x] // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\alpha] & -\text{Sin}[\alpha] \\ 0 & \text{Sin}[\alpha] & \text{Cos}[\alpha] \end{pmatrix}$$

In[7]:=
```
β;(*rotation of y-axis*)
RotationMatrix[β, y] // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\beta] & 0 & \text{Sin}[\beta] \\ 0 & 1 & 0 \\ -\text{Sin}[\beta] & 0 & \text{Cos}[\beta] \end{pmatrix}$$

In[9]:=
```
γ;(*rotation of z-axis*)
RotationMatrix[γ, z] // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\gamma] & -\text{Sin}[\gamma] & 0 \\ \text{Sin}[\gamma] & \text{Cos}[\gamma] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[11]:=
```
RotationMatrix[α, x].RotationMatrix[β, y].RotationMatrix[γ, z] // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\beta]\,\text{Cos}[\gamma] & -\text{Cos}[\beta]\,\text{Sin}[\gamma] & \text{Sin}[\beta] \\ \text{Cos}[\gamma]\,\text{Sin}[\alpha]\,\text{Sin}[\beta] + \text{Cos}[\alpha]\,\text{Sin}[\gamma] & \text{Cos}[\alpha]\,\text{Cos}[\gamma] - \text{Sin}[\alpha]\,\text{Sin}[\beta]\,\text{Sin}[\gamma] & -\text{Cos}[\beta]\,\text{Sin}[\alpha] \\ -\text{Cos}[\alpha]\,\text{Cos}[\gamma]\,\text{Sin}[\beta] + \text{Sin}[\alpha]\,\text{Sin}[\gamma] & \text{Cos}[\gamma]\,\text{Sin}[\alpha] + \text{Cos}[\alpha]\,\text{Sin}[\beta]\,\text{Sin}[\gamma] & \text{Cos}[\alpha]\,\text{Cos}[\beta] \end{pmatrix}$$

In[12]:=
```
RotationMatrix[γ, z].RotationMatrix[α, x].RotationMatrix[β, y] // MatrixForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\beta]\,\text{Cos}[\gamma] - \text{Sin}[\alpha]\,\text{Sin}[\beta]\,\text{Sin}[\gamma] & -\text{Cos}[\alpha]\,\text{Sin}[\gamma] & \text{Cos}[\gamma]\,\text{Sin}[\beta] + \text{Cos}[\beta]\,\text{Sin}[\alpha]\,\text{Sin}[\gamma] \\ \text{Cos}[\gamma]\,\text{Sin}[\alpha]\,\text{Sin}[\beta] + \text{Cos}[\beta]\,\text{Sin}[\gamma] & \text{Cos}[\alpha]\,\text{Cos}[\gamma] & -\text{Cos}[\beta]\,\text{Cos}[\gamma]\,\text{Sin}[\alpha] + \text{Sin}[\beta]\,\text{Sin}[\gamma] \\ -\text{Cos}[\alpha]\,\text{Sin}[\beta] & \text{Sin}[\alpha] & \text{Cos}[\alpha]\,\text{Cos}[\beta] \end{pmatrix}$$

In[13]:=
```
t = {xo, yo, zo};(*translation*)
TranslationTransform[t]
```

Out[14]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & \text{xo} \\ 0 & 1 & 0 & \text{yo} \\ 0 & 0 & 1 & \text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

In[15]:=
```
s = {sx, sy, sz};(*scale*)
ScalingTransform[s]
```

Out[15]=
$$\text{Null}^2\ \text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & 0 \\ 0 & \text{sy} & 0 & 0 \\ 0 & 0 & \text{sz} & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

In[16]:=

In[17]:=
```
TranslationTransform[t].ScalingTransform[s].RotationTransform[α, x]
```

Out[17]=
$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & \text{xo} \\ 0 & \text{sy}\,\text{Cos}[\alpha] & -\text{sy}\,\text{Sin}[\alpha] & \text{yo} \\ 0 & \text{sz}\,\text{Sin}[\alpha] & \text{sz}\,\text{Cos}[\alpha] & \text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

In[18]:=
```
TranslationTransform[t].RotationTransform[α, x].ScalingTransform[s]
```

Out[18]=
$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & \text{xo} \\ 0 & \text{sy}\,\text{Cos}[\alpha] & -\text{sz}\,\text{Sin}[\alpha] & \text{yo} \\ 0 & \text{sy}\,\text{Sin}[\alpha] & \text{sz}\,\text{Cos}[\alpha] & \text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

In[19]:=
```
ScalingTransform[s].TranslationTransform[t].RotationTransform[α, x]
```

Out[19]=
$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & \text{sx}\,\text{xo} \\ 0 & \text{sy}\,\text{Cos}[\alpha] & -\text{sy}\,\text{Sin}[\alpha] & \text{sy}\,\text{yo} \\ 0 & \text{sz}\,\text{Sin}[\alpha] & \text{sz}\,\text{Cos}[\alpha] & \text{sz}\,\text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$