

```
(*force, mass, acceleration, velocity,  
linear momentum, kinetic energy, elastic collision *)
```

```
Clear["*"];
```

```
m;(*mass*)
```

```
v;(*velocity*)
```

```
p = m v;(*linear momentum*)
```

```
a;(*acceleration*)
```

```
F = m a;(*force*)
```

```
u1;(*velocity during collision*)
```

```
u2;
```

```
v1;(*velocity after collision*)
```

```
v2;
```

```
m1 u1 + m2 u2 == m1 v1 + m2 v2;(*conservation of linear momentum*)
```

```
 $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 == \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ ;(*conservation of kinetic energy*)
```

```
eq1 = m1 u1 + m2 u2 == m1 v1 + m2 v2;
```

```
 $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 == \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ ;
```

```
Collect[Solve[{eq1, eq2}, {v1, v2}], {u1, u2}]
```

```
 $\left\{ \left\{ v_1 \rightarrow u_1, v_2 \rightarrow u_2 \right\}, \left\{ v_1 \rightarrow \frac{(m_1 - m_2) u_1}{m_1 + m_2} + \frac{2 m_2 u_2}{m_1 + m_2}, v_2 \rightarrow \frac{2 m_1 u_1}{m_1 + m_2} + \frac{(-m_1 + m_2) u_2}{m_1 + m_2} \right\} \right\}$ 
```

```
 $v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2 m_2}{m_1 + m_2} u_2$ ;
```

```
 $v_2 = \frac{m_2 - m_1}{m_1 + m_2} u_2 + \frac{2 m_1}{m_1 + m_2} u_1$ ;
```

```
nm = {nx, ny, nz};(*collision plane normal, where velocities are exchanged*)
```

```
u1 = {u1.x, u1.y, u1.z};
```

```
u2 = {u2.x, u2.y, u2.z};
```

```
 $v_1 = \frac{(m_1 - m_2) (u_1 \cdot nm) nm + 2 m_2 (u_2 \cdot nm) nm}{m_1 + m_2}$ ;
```

```
 $v_2 = \frac{(m_2 - m_1) (u_2 \cdot nm) nm + 2 m_1 (u_1 \cdot nm) nm}{m_1 + m_2}$ ;
```

```
u1 = u1 - (u1.nm) nm + v1;
```

```
u2 = u2 - (u2.nm) nm + v2;
```