

```
In[927]:= p1 = {x1, y1, z1};
          v1 = {vx1, vy1, vz1};
          p2 = {x2, y2, z2};
          v2 = {vx2, vy2, vz2};
          abs ((p1 + v1 t) - (p2 + v2 t))
```

```
Out[931]= {abs (t vx1 - t vx2 + x1 - x2), abs (t vy1 - t vy2 + y1 - y2), abs (t vz1 - t vz2 + z1 - z2)}
```

```
In[932]:= ((p1 + v1 t) - (p2 + v2 t)) ^ 2
```

```
Out[932]= {(t vx1 - t vx2 + x1 - x2)^2, (t vy1 - t vy2 + y1 - y2)^2, (t vz1 - t vz2 + z1 - z2)^2}
```

```
In[933]:= Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]
```

```
Out[933]= (t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2
```

```
In[934]:= sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]]
```

```
Out[934]= sqrt[(t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2]
```

```
In[935]:= sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]] == r1 + r2
```

```
Out[935]= sqrt[(t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2] == r1 + r2
```

```
In[936]:= Solve[sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]] == r1 + r2, t]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

```
Out[936]= {{t ->
  (- 2 vx1 x1 + 2 vx2 x1 + 2 vx1 x2 - 2 vx2 x2 - 2 vy1 y1 + 2 vy2 y1 + 2 vy1 y2 - 2 vy2 y2 - 2 vz1 z1 + 2 vz2
    z1 + 2 vz1 z2 - 2 vz2 z2 - sqrt[(2 vx1 x1 - 2 vx2 x1 - 2 vx1 x2 + 2 vx2 x2 + 2 vy1 y1 - 2 vy2 y1 - 2
      vy1 y2 + 2 vy2 y2 + 2 vz1 z1 - 2 vz2 z1 - 2 vz1 z2 + 2 vz2 z2)^2 -
      4 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2)
      (x1^2 - 2 x1 x2 + x2^2 + y1^2 - 2 y1 y2 + y2^2 + z1^2 - 2 z1 z2 + z2^2 - sqrt[(-1) [r1 + r2]])]) /
    (2 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2))},
  {t -> (- 2 vx1 x1 + 2 vx2 x1 + 2 vx1 x2 - 2 vx2 x2 - 2 vy1 y1 + 2 vy2 y1 +
    2 vy1 y2 - 2 vy2 y2 - 2 vz1 z1 + 2 vz2 z1 + 2 vz1 z2 - 2 vz2 z2 +
    sqrt[(2 vx1 x1 - 2 vx2 x1 - 2 vx1 x2 + 2 vx2 x2 + 2 vy1 y1 - 2 vy2 y1 - 2 vy1 y2 +
      2 vy2 y2 + 2 vz1 z1 - 2 vz2 z1 - 2 vz1 z2 + 2 vz2 z2)^2 -
      4 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2)
      (x1^2 - 2 x1 x2 + x2^2 + y1^2 - 2 y1 y2 + y2^2 + z1^2 - 2 z1 z2 + z2^2 - sqrt[(-1) [r1 + r2]])]) /
    (2 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2))}}
```

```
In[937]:=
```

```
In[938]:= (t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2 == (r1 + r2) ^ 2
```

```
Out[938]= (t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2 == (r1 + r2) ^ 2
```

```
In[939]:= (t (vx1 - vx2) + x1 - x2)^2 + (t (vy1 - vy2) + y1 - y2)^2 + (t (vz1 - vz2) + z1 - z2)^2 == (r1 + r2) ^ 2
```

```
Out[939]= (t (vx1 - vx2) + x1 - x2)^2 + (t (vy1 - vy2) + y1 - y2)^2 + (t (vz1 - vz2) + z1 - z2)^2 == (r1 + r2) ^ 2
```

```
In[940]:= dvx = vx1 - vx2;
          dvy = vy1 - vy2;
          dvz = vz1 - vz2;
          dx = x1 - x2;
          dy = y1 - y2;
          dz = z1 - z2;
          r0 = r1 + r2;
```

$$(t \, dvx + dx)^2 + (t \, dvy + dy)^2 + (t \, dvz + dz)^2 == r0^2$$

$$\text{Out[947]}= (t \, (vx1 - vx2) + x1 - x2)^2 + (t \, (vy1 - vy2) + y1 - y2)^2 + (t \, (vz1 - vz2) + z1 - z2)^2 == (r1 + r2)^2$$

```
In[948]:= Clear[dx, dvx, dy, dvy, dz, dvz, r0]
```

$$\text{In[949]}= (t \, dvx + dx)^2 + (t \, dvy + dy)^2 + (t \, dvz + dz)^2 == r0^2$$

$$\text{Out[949]}= (dx + dvx \, t)^2 + (dy + dvy \, t)^2 + (dz + dvz \, t)^2 == r0^2$$

```
In[950]:= Solve[(dx + dvx t)^2 + (dy + dvy t)^2 + (dz + dvz t)^2 == r0^2, t]
```

$$\text{Out[950]}= \left\{ \left\{ t \rightarrow \frac{1}{2 \left(dvx^2 + dvy^2 + dvz^2 \right)} \left(-2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz - \sqrt{\left(2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz \right)^2 - 4 \left(dvx^2 + dvy^2 + dvz^2 \right) \left(dx^2 + dy^2 + dz^2 - r0^2 \right)} \right) \right\}, \right. \\ \left. \left\{ t \rightarrow \frac{1}{2 \left(dvx^2 + dvy^2 + dvz^2 \right)} \left(-2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz + \sqrt{\left(2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz \right)^2 - 4 \left(dvx^2 + dvy^2 + dvz^2 \right) \left(dx^2 + dy^2 + dz^2 - r0^2 \right)} \right) \right\} \right\}$$

```
In[951]:= Solve[dx^2 + 2 dx dvx t + (dvx t)^2 + dy^2 + 2 dy dvy t + (dvy t)^2 + dz^2 + 2 dz dvz t + (dvz t)^2 == r0^2, t]
```

$$\text{Out[951]}= \left\{ \left\{ t \rightarrow \frac{1}{2 \left(dvx^2 + dvy^2 + dvz^2 \right)} \left(-2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz - \sqrt{\left(2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz \right)^2 - 4 \left(dvx^2 + dvy^2 + dvz^2 \right) \left(dx^2 + dy^2 + dz^2 - r0^2 \right)} \right) \right\}, \right. \\ \left. \left\{ t \rightarrow \frac{1}{2 \left(dvx^2 + dvy^2 + dvz^2 \right)} \left(-2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz + \sqrt{\left(2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz \right)^2 - 4 \left(dvx^2 + dvy^2 + dvz^2 \right) \left(dx^2 + dy^2 + dz^2 - r0^2 \right)} \right) \right\} \right\}$$

```
In[952]:= dx^2 + 2 dx dvx t + (dvx t)^2 + dy^2 + 2 dy dvy t + (dvy t)^2 + dz^2 + 2 dz dvz t + (dvz t)^2 == r0^2
```

$$\text{Out[952]}= dx^2 + dy^2 + dz^2 + 2 \, dvx \, dx \, t + 2 \, dvy \, dy \, t + 2 \, dvz \, dz \, t + dvx^2 \, t^2 + dvy^2 \, t^2 + dvz^2 \, t^2 == r0^2$$

```
In[953]:= a = dvx^2 + dvy^2 + dvz^2;
          b = 2 dvx dx + 2 dvy dy + 2 dvz dz;
          c = dx^2 + dy^2 + dz^2 - r0^2;
          c + b t + a t^2 == 0
```

$$\text{Out[956]}= dx^2 + dy^2 + dz^2 - r0^2 + (2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz) \, t + (dvx^2 + dvy^2 + dvz^2) \, t^2 == 0$$

```
In[957]:= Clear[a, b, c];
```

```
Solve[c + b t + a t^2 == 0, t]
```

$$\text{Out[958]}= \left\{ \left\{ t \rightarrow \frac{-b - \sqrt{b^2 - 4 \, a \, c}}{2 \, a} \right\}, \left\{ t \rightarrow \frac{-b + \sqrt{b^2 - 4 \, a \, c}}{2 \, a} \right\} \right\}$$

In[959]:=