

(*sphere/sphere collision resolver*)

$\mathbf{p}_1 = \{\mathbf{p}_{1.x}, \mathbf{p}_{1.y}, \mathbf{p}_{1.z}\}; (*\text{position}*)$

$\mathbf{u}_1 = \{\mathbf{u}_{1.x}, \mathbf{u}_{1.y}, \mathbf{u}_{1.z}\}; (*\text{velocity}*)$

$\mathbf{r}_1; (*\text{radius}*)$

$\mathbf{p}_2 = \{\mathbf{p}_{2.x}, \mathbf{p}_{2.y}, \mathbf{p}_{2.z}\};$

$\mathbf{u}_2 = \{\mathbf{u}_{2.x}, \mathbf{u}_{2.y}, \mathbf{u}_{2.z}\};$

$\mathbf{r}_2;$

$\text{Solve}[\text{EuclideanDistance}[\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t}, \mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}] == \mathbf{r}_1 + \mathbf{r}_2, \mathbf{t}]$

$$\left\{ \left\{ \mathbf{t} \rightarrow \left(-2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} + \right. \right. \\ \left. \left. 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} - \right. \right. \\ \left. \left. \sqrt{\left(\left(2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} - \right. \right. \right. \\ \left. \left. \left. 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} \right)^2 - \right. \right. \\ \left. \left. 4 \left(\mathbf{p}_{1.x}^2 - 2 \mathbf{p}_{1.x} \mathbf{p}_{2.x} + \mathbf{p}_{2.x}^2 + \mathbf{p}_{1.y}^2 - 2 \mathbf{p}_{1.y} \mathbf{p}_{2.y} + \mathbf{p}_{2.y}^2 + \mathbf{p}_{1.z}^2 - 2 \mathbf{p}_{1.z} \mathbf{p}_{2.z} + \mathbf{p}_{2.z}^2 - \mathbf{r}_1^2 - 2 \mathbf{r}_1 \mathbf{r}_2 - \right. \right. \right. \\ \left. \left. \left. \mathbf{r}_2^2 \right) \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right) \right) \right) \right\} / \\ \left(2 \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right) \right) \left\{ \right. \\ \left. \left\{ \mathbf{t} \rightarrow \left(-2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} + 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} - 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} + \right. \right. \right. \\ \left. \left. \left. 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} - 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} + 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} + \right. \right. \right. \\ \left. \left. \left. \sqrt{\left(\left(2 \mathbf{p}_{1.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{2.x} \mathbf{u}_{1.x} - 2 \mathbf{p}_{1.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{2.x} \mathbf{u}_{2.x} + 2 \mathbf{p}_{1.y} \mathbf{u}_{1.y} - 2 \mathbf{p}_{2.y} \mathbf{u}_{1.y} - \right. \right. \right. \right. \\ \left. \left. \left. 2 \mathbf{p}_{1.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{2.y} \mathbf{u}_{2.y} + 2 \mathbf{p}_{1.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{2.z} \mathbf{u}_{1.z} - 2 \mathbf{p}_{1.z} \mathbf{u}_{2.z} + 2 \mathbf{p}_{2.z} \mathbf{u}_{2.z} \right)^2 - \right. \right. \\ \left. \left. 4 \left(\mathbf{p}_{1.x}^2 - 2 \mathbf{p}_{1.x} \mathbf{p}_{2.x} + \mathbf{p}_{2.x}^2 + \mathbf{p}_{1.y}^2 - 2 \mathbf{p}_{1.y} \mathbf{p}_{2.y} + \mathbf{p}_{2.y}^2 + \mathbf{p}_{1.z}^2 - 2 \mathbf{p}_{1.z} \mathbf{p}_{2.z} + \mathbf{p}_{2.z}^2 - \mathbf{r}_1^2 - 2 \mathbf{r}_1 \mathbf{r}_2 - \right. \right. \right. \\ \left. \left. \left. \mathbf{r}_2^2 \right) \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right) \right) \right) \right\} / \\ \left(2 \left(\mathbf{u}_{1.x}^2 - 2 \mathbf{u}_{1.x} \mathbf{u}_{2.x} + \mathbf{u}_{2.x}^2 + \mathbf{u}_{1.y}^2 - 2 \mathbf{u}_{1.y} \mathbf{u}_{2.y} + \mathbf{u}_{2.y}^2 + \mathbf{u}_{1.z}^2 - 2 \mathbf{u}_{1.z} \mathbf{u}_{2.z} + \mathbf{u}_{2.z}^2 \right) \right) \left\{ \right\} \left. \right\}$$

$\text{EuclideanDistance}[\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t}, \mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}]$

$$\sqrt{\left(\text{Abs}[\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x}]^2 + \right. \\ \left. \text{Abs}[\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y}]^2 + \text{Abs}[\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z}]^2 \right)}$$

$$\sqrt{\text{Total}[(\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t} - (\mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}))^2]} == \mathbf{r}_1 + \mathbf{r}_2$$

$$\sqrt{\left((\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x})^2 + (\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y})^2 + (\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z})^2 \right)} == \\ \mathbf{r}_1 + \mathbf{r}_2$$

$$\left(\sqrt{\text{Total}[(\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t} - (\mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}))^2]} \right)^2 == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$$(\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x})^2 + (\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y})^2 + (\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z})^2 == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$$\text{Total}[(\mathbf{p}_1 + \mathbf{u}_1 \mathbf{t} - (\mathbf{p}_2 + \mathbf{u}_2 \mathbf{t}))^2] == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$$(\mathbf{p}_{1.x} - \mathbf{p}_{2.x} + \mathbf{t} \mathbf{u}_{1.x} - \mathbf{t} \mathbf{u}_{2.x})^2 + (\mathbf{p}_{1.y} - \mathbf{p}_{2.y} + \mathbf{t} \mathbf{u}_{1.y} - \mathbf{t} \mathbf{u}_{2.y})^2 + (\mathbf{p}_{1.z} - \mathbf{p}_{2.z} + \mathbf{t} \mathbf{u}_{1.z} - \mathbf{t} \mathbf{u}_{2.z})^2 == (\mathbf{r}_1 + \mathbf{r}_2)^2$$

$\Delta \mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2$

$$\{\mathbf{p}_{1.x} - \mathbf{p}_{2.x}, \mathbf{p}_{1.y} - \mathbf{p}_{2.y}, \mathbf{p}_{1.z} - \mathbf{p}_{2.z}\}$$

Collect[**Expand**[**Total**[($\Delta p + u_1 t - u_2 t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Expand**[**Total**[($\Delta p + (u_1 - u_2) t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

$$\Delta u = u_1 - u_2$$

$$\{u_{1,x} - u_{2,x}, u_{1,y} - u_{2,y}, u_{1,z} - u_{2,z}\}$$

Collect[**Expand**[**Total**[($\Delta p + \Delta u t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Expand**[**Total**[($\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2$) - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Total**[($\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2$) - ($r_1 + r_2$)² == 0], t]

$$(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2 - (r_1 + r_2)^2 + t \left(2 (p_{1,x} - p_{2,x}) (u_{1,x} - u_{2,x}) + 2 (p_{1,y} - p_{2,y}) (u_{1,y} - u_{2,y}) + 2 (p_{1,z} - p_{2,z}) (u_{1,z} - u_{2,z}) \right) + t^2 \left((u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2 \right) == 0$$

Total[(Δp^2)]

$$(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2$$

Collect[

$$\text{Expand}[\text{Total}[\Delta p^2] - (r_1 + r_2)^2 + t \left(2 (p_{1,x} - p_{2,x}) (u_{1,x} - u_{2,x}) + 2 (p_{1,y} - p_{2,y}) (u_{1,y} - u_{2,y}) + 2 (p_{1,z} - p_{2,z}) (u_{1,z} - u_{2,z}) \right) + t^2 \left((u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2 \right) == 0], t]$$

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

2 $\Delta p \cdot \Delta u t$

$$2 t \left((p_{1,x} - p_{2,x}) (u_{1,x} - u_{2,x}) + (p_{1,y} - p_{2,y}) (u_{1,y} - u_{2,y}) + (p_{1,z} - p_{2,z}) (u_{1,z} - u_{2,z}) \right)$$

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Collect[Expand[
  Total[Δp2] - (r1 + r2)2 + 2 Δp.Δu t + t2 ((u1.x - u2.x)2 + (u1.y - u2.y)2 + (u1.z - u2.z)2) == 0], t]
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$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t (2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}) + t^2 (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) == 0$$

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Total[Δu2]
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$$(u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2$$

```
Collect[Expand[Total[Δp2] - (r1 + r2)2 + 2 Δp.Δu t + t2 Total[Δu2] == 0], t]
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$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t (2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}) + t^2 (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) == 0$$

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a = Total[Δu2];
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b = 2 Δp.Δu;
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c = Total[Δp2] - (r1 + r2)2;
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Collect[Expand[a t2 + b t + c == 0], t]
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Solve[a t2 + b t + c == 0, t]
```

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t (2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}) + t^2 (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) == 0$$

$$\left\{ \left\{ t \rightarrow \left(-2 p_{1,x} u_{1,x} + 2 p_{2,x} u_{1,x} + 2 p_{1,x} u_{2,x} - 2 p_{2,x} u_{2,x} - 2 p_{1,y} u_{1,y} + 2 p_{2,y} u_{1,y} + 2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} - \sqrt{\left((2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z})^2 - 4 (p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2) (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) \right)} \right) \right\},$$

$$\left\{ t \rightarrow \left(-2 p_{1,x} u_{1,x} + 2 p_{2,x} u_{1,x} + 2 p_{1,x} u_{2,x} - 2 p_{2,x} u_{2,x} - 2 p_{1,y} u_{1,y} + 2 p_{2,y} u_{1,y} + 2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} + \sqrt{\left((2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z})^2 - 4 (p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2) (u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2) \right)} \right) \right\}$$

```
In[30]:= Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t] == Solve[a t2 + b t + c == 0, t]
```

```
Out[30]= True
```