```
(*sphere/sphere collision resolver*)
p_1 = \{p_{1.x}, p_{1.y}, p_{1.z}\}; (*position*)
 u_1 = \{u_{1.x}, u_{1.y}, u_{1.z}\}; (*velocity*)
r<sub>1</sub>; (*radius*)
p_2 = \{p_{2.x}, p_{2.y}, p_{2.z}\};
 u_2 = \{u_{2.x}, u_{2.y}, u_{2.z}\};
r2;
 Solve [EuclideanDistance [p_1 + u_1 t, p_2 + u_2 t] = r_1 + r_2, t]
   \left\{\left.\left\{\,t\,\to\,\left(\,-\,2\;p_{1\,.\,x}\;u_{1\,.\,x}\,+\,2\;p_{2\,.\,x}\;u_{1\,.\,x}\,+\,2\;p_{1\,.\,x}\;u_{2\,.\,x}\,-\,2\;p_{2\,.\,x}\;u_{2\,.\,x}\,-\,2\;p_{1\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_
                                                                          2\;p_{1.\;y}\;u_{2.\;y}-2\;p_{2.\;y}\;u_{2.\;y}-2\;p_{1.\;z}\;u_{1.\;z}+2\;p_{2.\;z}\;u_{1.\;z}+2\;p_{1.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2
                                                                          \sqrt{\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{2,y} u_{2,y} - 
                                                                                                                                                  2\,p_{1.\,y}\,u_{2.\,y}+2\,p_{2.\,y}\,u_{2.\,y}+2\,p_{1.\,z}\,u_{1.\,z}-2\,p_{2.\,z}\,u_{1.\,z}-2\,p_{1.\,z}\,u_{2.\,z}+2\,p_{2.\,z}\,u_{2.\,z}\big)^{2}-\\
                                                                                                               4 \left( p_{1.\,x}^2 - 2\,p_{1.\,x}\,p_{2.\,x} + p_{2.\,x}^2 + p_{1.\,y}^2 - 2\,p_{1.\,y}\,p_{2.\,y} + p_{2.\,y}^2 + p_{1.\,z}^2 - 2\,p_{1.\,z}\,p_{2.\,z} + p_{2.\,z}^2 - r_1^2 - 2\,r_1\,r_2 - 2\,r_1^2 + 2\,r_2^2\,p_{2.\,z}^2 + 2\,r_2^2\,p_{2.\,z}^
                                                                                                                                                  (u_{1,x}^2) (u_{1,x}^2 - 2u_{1,x}u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2u_{1,y}u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2u_{1,z}u_{2,z} + u_{2,z}^2))
                                                   \left(2\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)\right)\right\}
            \left\{t \to \left(-2 \, p_{1 \, . \, x} \, u_{1 \, . \, x} + 2 \, p_{2 \, . \, x} \, u_{1 \, . \, x} + 2 \, p_{1 \, . \, x} \, u_{2 \, . \, x} - 2 \, p_{2 \, . \, x} \, u_{2 \, . \, x} - 2 \, p_{1 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \,
                                                                          2 p_{1, y} u_{2, y} - 2 p_{2, y} u_{2, y} - 2 p_{1, z} u_{1, z} + 2 p_{2, z} u_{1, z} + 2 p_{1, z} u_{2, z} - 2 p_{2, z} u_{2, z} +
                                                                         \sqrt{\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{2,y} u_{2,y} - 
                                                                                                                                                  2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                                                                                                              4 \left( p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - 2 r_1 r_2 \right)
                                                                                                                                                  (u_{1,x}^2) (u_{1,x}^2 - 2u_{1,x}u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2u_{1,y}u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2u_{1,z}u_{2,z} + u_{2,z}^2))
                                                   \left\{2\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)\right\}\right\}
EuclideanDistance [p_1 + u_1 t, p_2 + u_2 t]
   \sqrt{\left(\text{Abs}[p_{1.x} - p_{2.x} + t u_{1.x} - t u_{2.x}]^2 + \right)}
                                    Abs \left[ p_{1.y} - p_{2.y} + t u_{1.y} - t u_{2.y} \right]^{2} + Abs \left[ p_{1.z} - p_{2.z} + t u_{1.z} - t u_{2.z} \right]^{2}
 \sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]} == r_1 + r_2
 \sqrt{\left(\,\left(p_{1.\,x}-p_{2.\,x}+t\,u_{1.\,x}-t\,u_{2.\,x}\right)^{\,2}+\left(p_{1.\,y}-p_{2.\,y}+t\,u_{1.\,y}-t\,u_{2.\,y}\right)^{\,2}+\left(p_{1.\,z}-p_{2.\,z}+t\,u_{1.\,z}-t\,u_{2.\,z}\right)^{\,2}\right)}\,=\,
    \left(\sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]}\right)^2 == (r_1 + r_2)^2
    \left( p_{1.\,x} - p_{2.\,x} + t\,u_{1.\,x} - t\,u_{2.\,x} \right)^{\,2} + \, \left( p_{1.\,y} - p_{2.\,y} + t\,u_{1.\,y} - t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{1.\,z} - p_{2.\,z} + t\,u_{1.\,z} - t\,u_{2.\,z} \right)^{\,2} = \, \left( r_1 + r_2 \right)^{\,2} + \, \left( p_{1.\,y} - p_{2.\,y} + t\,u_{1.\,y} - t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{1.\,z} - p_{2.\,z} + t\,u_{1.\,z} - t\,u_{2.\,z} \right)^{\,2} = \, \left( p_{1.\,y} - p_{2.\,y} + t\,u_{1.\,y} - t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{1.\,z} - p_{2.\,z} + t\,u_{1.\,z} - t\,u_{2.\,z} \right)^{\,2} = \, \left( p_{1.\,y} - p_{2.\,y} + t\,u_{1.\,y} - t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{1.\,y} - p_{2.\,z} + t\,u_{2.\,z} \right)^{\,2} = \, \left( p_{1.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2} + \, \left( p_{2.\,y} - p_{2.\,y} + t\,u_{2.\,y} \right)^{\,2
 Total [(p_1 + u_1 t - (p_2 + u_2 t))^2] = (r_1 + r_2)^2
     (p_{1,x} - p_{2,x} + t u_{1,x} - t u_{2,x})^2 + (p_{1,y} - p_{2,y} + t u_{1,y} - t u_{2,y})^2 + (p_{1,z} - p_{2,z} + t u_{1,z} - t u_{2,z})^2 = (r_1 + r_2)^2 
\Delta p = p_1 - p_2
    \{p_{1.x} - p_{2.x}, p_{1.y} - p_{2.y}, p_{1.z} - p_{2.z}\}
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Collect Expand Total \left[ \left( \Delta p + u_1 t - u_2 t \right)^2 \right] - \left( r_1 + r_2 \right)^2 = 0 \right], t
p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                        r_{2}^{2} + t \left(2\,p_{1.\,x}\,u_{1.\,x} - 2\,p_{2.\,x}\,u_{1.\,x} - 2\,p_{1.\,x}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{1.\,y}\,u_{1.\,y} - 2\,p_{2.\,y}\,u_{1.\,y} - 2\,p_{2.\,y}\,u_{1.\,y} - 2\,p_{2.\,y}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{2.\,y}\,u_{2.\,x} + 2\,p_{2.\,y}\,u_{2.\,x} + 2\,p_{2.\,y}\,u_{2.\,y} + 2\,
                                                           2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z}
                        t^{2}\left(u_{1,x}^{2}-2 u_{1,x} u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2 u_{1,y} u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2 u_{1,z} u_{2,z}+u_{2,z}^{2}\right)=0
Collect \left[ \text{Expand} \left[ \text{Total} \left[ (\Delta p + (u_1 - u_2) t)^2 \right] - (r_1 + r_2)^2 = 0 \right], t \right]
p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_2^2 - r_1^2 - r_2^2 - r_2^
                      r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                           2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z} + 
                        t^{2}\left(u_{1}^{2} + 2u_{1} + u_{2} + u_{2}^{2} + u_{1}^{2} + u_{1}^{2} + u_{1}^{2} + u_{2}^{2} + u_{1}^{2} + u_{2}^{2} + u_{2}^{2} + u_{1}^{2} + u_{2}^{2} + u_{2}^{2} \right) = 0
\Delta \mathbf{u} = \mathbf{u}_1 - \mathbf{u}_2
   \{u_{1.x} - u_{2.x}, u_{1.y} - u_{2.y}, u_{1.z} - u_{2.z}\}
Collect \left[ \text{Expand} \left[ \text{Total} \left[ (\Delta p + \Delta u t)^2 \right] - (r_1 + r_2)^2 = 0 \right], t \right]
p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                      r_2^2 + t \left( 2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} \right)
                                                           2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                        t^{2}\left(u_{1,x}^{2}-2\,u_{1,x}\,u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2\,u_{1,y}\,u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2\,u_{1,z}\,u_{2,z}+u_{2,z}^{2}\right)=0
Collect Expand Total [\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2] - (r_1 + r_2)^2 = 0], t
p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2
                      r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                           2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                        \mathsf{t}^2 \left( \mathsf{u}^2_{1.\,x} - 2\,\mathsf{u}_{1.\,x}\,\mathsf{u}_{2.\,x} + \mathsf{u}^2_{2.\,x} + \mathsf{u}^2_{1.\,y} - 2\,\mathsf{u}_{1.\,y}\,\mathsf{u}_{2.\,y} + \mathsf{u}^2_{2.\,y} + \mathsf{u}^2_{1.\,z} - 2\,\mathsf{u}_{1.\,z}\,\mathsf{u}_{2.\,z} + \mathsf{u}^2_{2.\,z} \right) \, = \, 0
Collect \left[ \text{Total} \left[ \Delta p^2 + 2 \Delta p \Delta u \, t + \Delta u^2 \, t^2 \right] - (r_1 + r_2)^2 = 0, t \right]
   (p_{1,x}-p_{2,x})^2 + (p_{1,y}-p_{2,y})^2 + (p_{1,z}-p_{2,z})^2 - (r_1+r_2)^2 +
                         \text{t} \, \left( 2 \, \left( p_{1.\,x} - p_{2.\,x} \right) \, \left( u_{1.\,x} - u_{2.\,x} \right) \, + \, 2 \, \left( p_{1.\,y} - p_{2.\,y} \right) \, \left( u_{1.\,y} - u_{2.\,y} \right) \, + \, 2 \, \left( p_{1.\,z} - p_{2.\,z} \right) \, \left( u_{1.\,z} - u_{2.\,z} \right) \right) \, + \, 2 \, \left( p_{2.\,x} - p_{2.\,z} \right) \, \left( u_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - p_{2.\,z} \right) \, \left( u_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - p_{2.\,z} \right) \, \left( u_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - p_{2.\,z} \right) \, \left( u_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + 
                        t^{2} \left( (u_{1.x} - u_{2.x})^{2} + (u_{1.y} - u_{2.y})^{2} + (u_{1.z} - u_{2.z})^{2} \right) = 0
Total[\Delta p^2]
    (p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2
             \text{Expand} \left[ \text{Total} \left[ \Delta p^2 \right] - (r_1 + r_2)^2 + \text{t} \left( 2 \left( p_{1, \ x} - p_{2, \ x} \right) \left( u_{1, \ x} - u_{2, \ x} \right) + 2 \left( p_{1, \ y} - p_{2, \ y} \right) \left( u_{1, \ y} - u_{2, \ y} \right) + 2 \left( p_{1, \ y} - p_{2, \ y} \right) \right) \right] 
                                                                                  2(p_{1,z} - p_{2,z})(u_{1,z} - u_{2,z}) +
                                                t^{2} \left( \left( u_{1, x} - u_{2, x} \right)^{2} + \left( u_{1, y} - u_{2, y} \right)^{2} + \left( u_{1, z} - u_{2, z} \right)^{2} \right) = 0 \right], t
p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                        r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} - 2 p_{2.y} u_
                                                            2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z} + 
                        2 Δρ.Δu t
   2 t \left( (p_{1.x} - p_{2.x}) (u_{1.x} - u_{2.x}) + (p_{1.y} - p_{2.y}) (u_{1.y} - u_{2.y}) + (p_{1.z} - p_{2.z}) (u_{1.z} - u_{2.z}) \right)
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Collect Expand
                                                                                                Total \left[ \Delta p^2 \right] - (r_1 + r_2)^2 + 2 \Delta p. \Delta u t + t^2 \left( (u_1 \cdot x - u_2 \cdot x)^2 + (u_1 \cdot y - u_2 \cdot y)^2 + (u_1 \cdot z - u_2 \cdot z)^2 \right) = 0 \right], t
                                                                p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                                                                                                r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_
                                                                                                                                                2\;p_{1.\;y}\;u_{2.\;y}+2\;p_{2.\;y}\;u_{2.\;y}+2\;p_{1.\;z}\;u_{1.\;z}-2\;p_{2.\;z}\;u_{1.\;z}-2\;p_{1.\;z}\;u_{2.\;z}+2\;p_{2.\;z}\;u_{2.\;z}\right)\;+
                                                                                                t^{2}\left(u_{1,x}^{2}-2\,u_{1,x}\,u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2\,u_{1,y}\,u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2\,u_{1,z}\,u_{2,z}+u_{2,z}^{2}\right) = 0
                                                                  Total \left[\Delta u^2\right]
                                                                      (u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2
                                                                Collect \left[ \text{Expand} \left[ \text{Total} \left[ \Delta p^2 \right] - (r_1 + r_2)^2 + 2 \Delta p \cdot \Delta u \, t + t^2 \, \text{Total} \left[ \Delta u^2 \right] = 0 \right], t \right]
                                                                p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_2^2 - r_1^2 - r_2^2 - r_2^
                                                                                              r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                                                                                                              2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                                                                                                 \mathsf{t}^2 \, \left( \mathsf{u}^2_{1.\,\, \mathsf{x}} \, - \, 2 \, \mathsf{u}_{1.\,\, \mathsf{x}} \, \mathsf{u}_{2.\,\, \mathsf{x}} \, + \, \mathsf{u}^2_{2.\,\, \mathsf{x}} \, + \, \mathsf{u}^2_{1.\,\, \mathsf{y}} \, - \, 2 \, \mathsf{u}_{1.\,\, \mathsf{y}} \, \mathsf{u}_{2.\,\, \mathsf{y}} \, + \, \mathsf{u}^2_{2.\,\, \mathsf{y}} \, + \, \mathsf{u}^2_{1.\,\, \mathsf{z}} \, - \, 2 \, \mathsf{u}_{1.\,\, \mathsf{z}} \, \mathsf{u}_{2.\,\, \mathsf{z}} \, + \, \mathsf{u}^2_{2.\,\, \mathsf{z}} \, \right) \, = \, 0 
                                                                  a = Total[\Delta u^2];
                                                                b = 2 \Delta p. \Delta u;
                                                                  c = Total[\Delta p^2] - (r_1 + r_2)^2;
                                                                Collect [Expand[at^2 + bt + c = 0], t]
                                                                  Solve[at^2+bt+c=0,t]
                                                                p_{1.\,x}^{2}-2\,p_{1.\,x}\,p_{2.\,x}+p_{2.\,x}^{2}+p_{1.\,y}^{2}-2\,p_{1.\,y}\,p_{2.\,y}+p_{2.\,y}^{2}+p_{1.\,z}^{2}-2\,p_{1.\,z}\,p_{2.\,z}+p_{2.\,z}^{2}-r_{1}^{2}-2\,r_{1}\,r_{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_{1}^{2}-r_
                                                                                              r_{2}^{2} + t (2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} - 2 p_{2.
                                                                                                                                              2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                                                                                                \left\{\left\{t \to \left(-2 \, p_{1.\,\,x} \, u_{1.\,\,x} + 2 \, p_{2.\,\,x} \, u_{1.\,\,x} + 2 \, p_{1.\,\,x} \, u_{2.\,\,x} - 2 \, p_{2.\,\,x} \, u_{2.\,\,x} - 2 \, p_{1.\,\,y} \, u_{1.\,\,y} + 2 \, p_{2.\,\,y} \, u_{1.\,\,y} + 2 \, p_{2.\,\,y} \, u_{1.\,\,y} + 2 \, p_{2.\,\,y} \, u_{2.\,\,x} + 2 \, p_{2.\,\,y} \, u_{2.\,\,x} - 2 \, p_{2.\,\,x} \, u_{2.\,\,x} - 2 \, p_{2.\,\,y} \, u_{2.\,\,y} + 2 \, p_{2.\,\,y} \, u
                                                                                                                                                            2\,p_{1.\,y}\,u_{2.\,y}-2\,p_{2.\,y}\,u_{2.\,y}-2\,p_{1.\,z}\,u_{1.\,z}+2\,p_{2.\,z}\,u_{1.\,z}+2\,p_{1.\,z}\,u_{2.\,z}-2\,p_{2.\,z}\,u_{2.\,z}-2\,p_{2.\,z}\,u_{2.\,z}
                                                                                                                                                            \sqrt{\left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} - 
                                                                                                                                                                                                                                                         2\;p_{1,\;y}\;u_{2,\;y}+2\;p_{2,\;y}\;u_{2,\;y}+2\;p_{1,\;z}\;u_{1,\;z}-2\;p_{2,\;z}\;u_{1,\;z}-2\;p_{1,\;z}\;u_{2,\;z}+2\;p_{2,\;z}\;u_{2,\;z}\Big)^{2}-\\
                                                                                                                                                                                                             4 \left( p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - 2 r_1 r_2 + r_2^2 r_1 r_2 - r_1^2 - r_1^2 r_2 r_1 r_2 - r_1^2 r_1 r_2 r_1 r_2 - r_1^2 r_1 r_2 r_2 r_1 r
                                                                                                                                                                                                                                                       (u_{1,x}^2) (u_{1,x}^2 - 2u_{1,x}u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2u_{1,y}u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2u_{1,z}u_{2,z} + u_{2,z}^2))
                                                                                                                                \left(2\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)\right)\right\}
                                                                                  \left\{ t \, \rightarrow \, \left( -\, 2\, p_{1\, .\, x}\, u_{1\, .\, x} \, +\, 2\, p_{2\, .\, x}\, u_{1\, .\, x} \, +\, 2\, p_{1\, .\, x}\, u_{2\, .\, x} \, -\, 2\, p_{2\, .\, x}\, u_{2\, .\, x} \, -\, 2\, p_{1\, .\, y}\, u_{1\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{1\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{1\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, x} \, -\, 2\, p_{2\, .\, x}\, u_{2\, .\, x} \, -\, 2\, p_{2\, .\, x}\, u_{2\, .\, x} \, -\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y}\, u_{2\, .\, y} \, +\, 2\, p_{2\, .\, y}\, u_{2\, .\, y}\,
                                                                                                                                                              2 p_{1, y} u_{2, y} - 2 p_{2, y} u_{2, y} - 2 p_{1, z} u_{1, z} + 2 p_{2, z} u_{1, z} + 2 p_{1, z} u_{2, z} - 2 p_{2, z} u_{2, z} +
                                                                                                                                                              \sqrt{\,\left(\,\left(\,2\;p_{1.\;x}\;u_{1.\;x}\,-\,2\;p_{2.\;x}\;u_{1.\;x}\,-\,2\;p_{1.\;x}\;u_{2.\;x}\,+\,2\;p_{2.\;x}\;u_{2.\;x}\,+\,2\;p_{1.\;y}\;u_{1.\;y}\,-\,2\;p_{2.\;y}\;u_{1.\;y}\,-\,2\;p_{2.\;y}\;u_{1.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\;u_{2.\;y}\,-\,2\;p_{2.\;y}\,-\,2\;p_{2.\;y}\,-\,2\;p_{2.\;y}\,-\,2\;p_{2.\;y}\,-\,2\;p_{2.\;y}\,-\,2\;p_{2.\;y}\,-
                                                                                                                                                                                                                                                         2p_{1,y}u_{2,y} + 2p_{2,y}u_{2,y} + 2p_{1,z}u_{1,z} - 2p_{2,z}u_{1,z} - 2p_{1,z}u_{2,z} + 2p_{2,z}u_{2,z}
                                                                                                                                                                                                             4 \left( p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - 2 r_1 r_2 + r_2^2 r_1 r_2 - r_1^2 - r_1^2 r_2 r_1 r_2 - r_1^2 r_1 r_2 r_1 r_2 - r_1^2 r_1 r_2 r_2 r_1 r
                                                                                                                                                                                                                                                         (u_{1}^{2}) (u_{1}^{2}) (u_{1}^{2}) (u_{1}^{2}) (u_{1}^{2}) (u_{2}^{2}) (u_{1}^{2}) (u_{2}^{2}) (u_{1}^{2}) (u_{2}^{2}) (u_{1}^{2}) (u_{2}^{2}) 
                                                                                                                                \left.\left(2\left(u_{1,\,x}^{2}-2\,u_{1,\,x}\,u_{2,\,x}+u_{2,\,x}^{2}+u_{1,\,y}^{2}-2\,u_{1,\,y}\,u_{2,\,y}+u_{2,\,y}^{2}+u_{1,\,z}^{2}-2\,u_{1,\,z}\,u_{2,\,z}+u_{2,\,z}^{2}\right)\right)\right\}\right\}
\ln[30] =  Solve [EuclideanDistance [p_1 + u_1 t, p_2 + u_2 t] == r_1 + r_2, t] == Solve [at^2 + bt + c == 0, t]
```

Out[30]= True