Sphere sphere collision

In[209]:=
```
p1 = {p1x, p1y, p1z};
v1 = {v1x, v1y, v1z};
r1 = r1;
p2 = {p2x, p2y, p2z};
v2 = {v2x, v2y, v2z};
r2 = r2;
(p1 + v1 t) - (p2 + v2 t)
```

Out[215]= $\{p_x - p_{2x} + t\, v_x - t\, v_{2x},\ p_y - p_{2y} + t\, v_y - t\, v_{2y},\ p_z - p_{2z} + t\, v_z - t\, v_{2z}\}$

In[216]:=
```
EuclideanDistance[{0, 0, 0}, (p1 + v1 t) - (p2 + v2 t)] == r1 + r2
```

Out[216]= $\sqrt{\left(\text{Abs}\left[-p_x + p_{2x} - t\, v_x + t\, v_{2x}\right]^2 + \text{Abs}\left[-p_y + p_{2y} - t\, v_y + t\, v_{2y}\right]^2 + \text{Abs}\left[-p_z + p_{2z} - t\, v_z + t\, v_{2z}\right]^2\right)} == 0$

In[217]:=
```
Sqrt[Total[((p2 + v2 t) - (p1 + v1 t))^2]] == r1 + r2
```

Out[217]= $\sqrt{\left((-p_x + p_{2x} - t\, v_x + t\, v_{2x})^2 + \left(-p_y + p_{2y} - t\, v_y + t\, v_{2y}\right)^2 + (-p_z + p_{2z} - t\, v_z + t\, v_{2z})^2\right)} == 0$

In[218]:=
```
Solve[Sqrt[Total[((p2 + v2 t) - (p1 + v1 t))^2]] == r1 + r2, t]
```

Out[218]=
$$\left\{\left\{t \to \left(-2\, p_x\, v_x + 2\, p_{2x}\, v_x + 2\, p_x\, v_{2x} - 2\, p_{2x}\, v_{2x} - 2\, p_y\, v_y + 2\, p_{2y}\, v_y + 2\, p_y\, v_{2y} - 2\, p_{2y}\, v_{2y} - 2\, p_z\, v_z + 2\, p_{2z}\, v_z + 2\, p_z\, v_{2z} - 2\, p_{2z}\, v_{2z} - \sqrt{\left((2\, p_x\, v_x - 2\, p_{2x}\, v_x - 2\, p_x\, v_{2x} + 2\, p_{2x}\, v_{2x} + 2\, p_y\, v_y - 2\, p_{2y}\, v_y - 2\, p_y\, v_{2y} + 2\, p_{2y}\, v_{2y} + 2\, p_z\, v_z - 2\, p_{2z}\, v_z - 2\, p_z\, v_{2z} + 2\, p_{2z}\, v_{2z})^2 - 4\left(p_x^2 - 2\, p_x\, p_{2x} + p_{2x}^2 + p_y^2 - 2\, p_y\, p_{2y} + p_{2y}^2 + p_z^2 - 2\, p_z\, p_{2z} + p_{2z}^2\right)\left(v_x^2 - 2\, v_x\, v_{2x} + v_{2x}^2 + v_y^2 - 2\, v_y\, v_{2y} + v_{2y}^2 + v_z^2 - 2\, v_z\, v_{2z} + v_{2z}^2\right)\right)}\right) \Big/ \left(2\left(v_x^2 - 2\, v_x\, v_{2x} + v_{2x}^2 + v_y^2 - 2\, v_y\, v_{2y} + v_{2y}^2 + v_z^2 - 2\, v_z\, v_{2z} + v_{2z}^2\right)\right)\right\},$$
$$\left\{t \to \left(-2\, p_x\, v_x + 2\, p_{2x}\, v_x + 2\, p_x\, v_{2x} - 2\, p_{2x}\, v_{2x} - 2\, p_y\, v_y + 2\, p_{2y}\, v_y + 2\, p_y\, v_{2y} - 2\, p_{2y}\, v_{2y} - 2\, p_z\, v_z + 2\, p_{2z}\, v_z + 2\, p_z\, v_{2z} - 2\, p_{2z}\, v_{2z} + \sqrt{\left((2\, p_x\, v_x - 2\, p_{2x}\, v_x - 2\, p_x\, v_{2x} + 2\, p_{2x}\, v_{2x} + 2\, p_y\, v_y - 2\, p_{2y}\, v_y - 2\, p_y\, v_{2y} + 2\, p_{2y}\, v_{2y} + 2\, p_z\, v_z - 2\, p_{2z}\, v_z - 2\, p_z\, v_{2z} + 2\, p_{2z}\, v_{2z})^2 - 4\left(p_x^2 - 2\, p_x\, p_{2x} + p_{2x}^2 + p_y^2 - 2\, p_y\, p_{2y} + p_{2y}^2 + p_z^2 - 2\, p_z\, p_{2z} + p_{2z}^2\right)\left(v_x^2 - 2\, v_x\, v_{2x} + v_{2x}^2 + v_y^2 - 2\, v_y\, v_{2y} + v_{2y}^2 + v_z^2 - 2\, v_z\, v_{2z} + v_{2z}^2\right)\right)}\right) \Big/ \left(2\left(v_x^2 - 2\, v_x\, v_{2x} + v_{2x}^2 + v_y^2 - 2\, v_y\, v_{2y} + v_{2y}^2 + v_z^2 - 2\, v_z\, v_{2z} + v_{2z}^2\right)\right)\right\}\right\}$$

In[219]:=
```
(Sqrt[Total[((p2 + v2 t) - (p1 + v1 t))^2]])^2 == (r1 + r2)^2
```

Out[219]= $(-p_x + p_{2x} - t\, v_x + t\, v_{2x})^2 + \left(-p_y + p_{2y} - t\, v_y + t\, v_{2y}\right)^2 + (-p_z + p_{2z} - t\, v_z + t\, v_{2z})^2 == 0$

In[220]:=

In[221]:=
```
Collect[
  (-px + p2x - t vx + t v2x)^2 + (-py + p2y - t vy + t v2y)^2 + (-pz + p2z - t vz + t v2z)^2 == (r1 + r2)^2, t]
```

Out[221]= $p_x^2 - 2\, p_x\, p_{2x} + p_{2x}^2 + p_y^2 - 2\, p_y\, p_{2y} + p_{2y}^2 + p_z^2 - 2\, p_z\, p_{2z} + p_{2z}^2 + t\left(2\, p_x\, v_x - 2\, p_{2x}\, v_x - 2\, p_x\, v_{2x} + 2\, p_{2x}\, v_{2x} + 2\, p_y\, v_y - 2\, p_{2y}\, v_y - 2\, p_y\, v_{2y} + 2\, p_{2y}\, v_{2y} + 2\, p_z\, v_z - 2\, p_{2z}\, v_z - 2\, p_z\, v_{2z} + 2\, p_{2z}\, v_{2z}\right) + t^2\left(v_x^2 - 2\, v_x\, v_{2x} + v_{2x}^2 + v_y^2 - 2\, v_y\, v_{2y} + v_{2y}^2 + v_z^2 - 2\, v_z\, v_{2z} + v_{2z}^2\right) == 0$

In[222]:=
$$p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(v_x^2 - 2\,v_x\,v_{2x} + v_{2x}^2 + v_y^2 - 2\,v_y\,v_{2y} + v_{2y}^2 + v_z^2 - 2\,v_z\,v_{2z} + v_{2z}^2) - (r1 + r2)^2 == (r1 + r2)^2 - (r1 + r2)^2$$

Out[222]=
$$p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(v_x^2 - 2\,v_x\,v_{2x} + v_{2x}^2 + v_y^2 - 2\,v_y\,v_{2y} + v_{2y}^2 + v_z^2 - 2\,v_z\,v_{2z} + v_{2z}^2) == 0$$

In[223]:=
$$\text{Total}[v_1^2]$$

Out[223]=
$$v_x^2 + v_y^2 + v_z^2$$

In[224]:=
$$-(r1 + r2)^2 + p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(\text{Total}[v_1^2] - 2\,v_x\,v_{2x} + v_{2x}^2 - 2\,v_y\,v_{2y} + v_{2y}^2 - 2\,v_z\,v_{2z} + v_{2z}^2) == 0$$

Out[224]=
$$p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(v_x^2 - 2\,v_x\,v_{2x} + v_{2x}^2 + v_y^2 - 2\,v_y\,v_{2y} + v_{2y}^2 + v_z^2 - 2\,v_z\,v_{2z} + v_{2z}^2) == 0$$

In[225]:=
$$\text{Total}[v_2^2]$$

Out[225]=
$$v_{2x}^2 + v_{2y}^2 + v_{2z}^2$$

In[226]:=
$$-(r1 + r2)^2 + p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(\text{Total}[v_1^2] + \text{Total}[v_2^2] - 2\,v_x\,v_{2x} - 2\,v_y\,v_{2y} - 2\,v_z\,v_{2z}) == 0$$

Out[226]=
$$p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(v_x^2 - 2\,v_x\,v_{2x} + v_{2x}^2 + v_y^2 - 2\,v_y\,v_{2y} + v_{2y}^2 + v_z^2 - 2\,v_z\,v_{2z} + v_{2z}^2) == 0$$

In[227]:=
$$-2\,v_1 . v_2$$

Out[227]=
$$-2\,(v_x\,v_{2x} + v_y\,v_{2y} + v_z\,v_{2z})$$

In[228]:=
$$-(r1 + r2)^2 + p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(\text{Total}[v_1^2] + \text{Total}[v_2^2] - 2\,v_1 . v_2) == 0$$

Out[228]=
$$p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_x\,v_x - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} + 2\,p_y\,v_y - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} + 2\,p_z\,v_z - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(v_x^2 + v_{2x}^2 + v_y^2 + v_{2y}^2 + v_z^2 + v_{2z}^2 - 2\,(v_x\,v_{2x} + v_y\,v_{2y} + v_z\,v_{2z})) == 0$$

In[229]:=
$$2\,p_1 . v_1$$

Out[229]=
$$2\,(p_x\,v_x + p_y\,v_y + p_z\,v_z)$$

In[230]:=
$$-(r1 + r2)^2 + p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(2\,p_1 . v_1 - 2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} - 2\,p_{2z}\,v_z - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(\text{Total}[v_1^2] + \text{Total}[v_2^2] - 2\,v_1 . v_2) == 0$$

Out[230]=
$$p_x^2 - 2\,p_x\,p_{2x} + p_{2x}^2 + p_y^2 - 2\,p_y\,p_{2y} + p_{2y}^2 + p_z^2 - 2\,p_z\,p_{2z} + p_{2z}^2 + t\,(-2\,p_{2x}\,v_x - 2\,p_x\,v_{2x} + 2\,p_{2x}\,v_{2x} - 2\,p_{2y}\,v_y - 2\,p_y\,v_{2y} + 2\,p_{2y}\,v_{2y} - 2\,p_{2z}\,v_z + 2\,(p_x\,v_x + p_y\,v_y + p_z\,v_z) - 2\,p_z\,v_{2z} + 2\,p_{2z}\,v_{2z}) + t^2\,(v_x^2 + v_{2x}^2 + v_y^2 + v_{2y}^2 + v_z^2 + v_{2z}^2 - 2\,(v_x\,v_{2x} + v_y\,v_{2y} + v_z\,v_{2z})) == 0$$

In[231]:= $\mathbf{2\,p_2 . v_2}$

Out[231]= $2\left(p_{2x}\,v_{2x}+p_{2y}\,v_{2y}+p_{2z}\,v_{2z}\right)$

In[232]:= $\mathbf{-(r1+r2)^2+p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-2\,p_z\,p_{2z}+p_{2z}^2+}$
$\mathbf{t\left(2\,p_1 . v_1-2\,p_{2x}\,v_x-2\,p_x\,v_{2x}+2\,p_2 . v_2-2\,p_{2y}\,v_y-2\,p_y\,v_{2y}-2\,p_{2z}\,v_z-2\,p_z\,v_{2z}\right)+}$
$\mathbf{t^2\left(\text{Total}\left[v_1^2\right]+\text{Total}\left[v_2^2\right]-2\,v_1 . v_2\right)==0}$

Out[232]= $p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-2\,p_z\,p_{2z}+p_{2z}^2+$
$t\left(-2\,p_{2x}\,v_x-2\,p_x\,v_{2x}-2\,p_{2y}\,v_y-2\,p_y\,v_{2y}-2\,p_{2z}\,v_z+2\left(p_x\,v_x+p_y\,v_y+p_z\,v_z\right)-2\,p_z\,v_{2z}+\right.$
$\left.2\left(p_{2x}\,v_{2x}+p_{2y}\,v_{2y}+p_{2z}\,v_{2z}\right)\right)+t^2\left(v_x^2+v_{2x}^2+v_y^2+v_{2y}^2+v_z^2+v_{2z}^2-2\left(v_x\,v_{2x}+v_y\,v_{2y}+v_z\,v_{2z}\right)\right)==0$

In[233]:= $\mathbf{-2\,p_2 . v_1}$

Out[233]= $-2\left(p_{2x}\,v_x+p_{2y}\,v_y+p_{2z}\,v_z\right)$

In[234]:= $\mathbf{-(r1+r2)^2+p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-}$
$\mathbf{2\,p_z\,p_{2z}+p_{2z}^2+t\left(2\,p_1 . v_1-2\,p_2 . v_1-2\,p_x\,v_{2x}+2\,p_2 . v_2-2\,p_y\,v_{2y}-2\,p_z\,v_{2z}\right)+}$
$\mathbf{t^2\left(\text{Total}\left[v_1^2\right]+\text{Total}\left[v_2^2\right]-2\,v_1 . v_2\right)==0}$

Out[234]= $p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-2\,p_z\,p_{2z}+p_{2z}^2+$
$t\left(-2\,p_x\,v_{2x}-2\,p_y\,v_{2y}+2\left(p_x\,v_x+p_y\,v_y+p_z\,v_z\right)-2\left(p_{2x}\,v_x+p_{2y}\,v_y+p_{2z}\,v_z\right)-2\,p_z\,v_{2z}+\right.$
$\left.2\left(p_{2x}\,v_{2x}+p_{2y}\,v_{2y}+p_{2z}\,v_{2z}\right)\right)+t^2\left(v_x^2+v_{2x}^2+v_y^2+v_{2y}^2+v_z^2+v_{2z}^2-2\left(v_x\,v_{2x}+v_y\,v_{2y}+v_z\,v_{2z}\right)\right)==0$

In[235]:= $\mathbf{-2\,p_1 . v_2}$

Out[235]= $-2\left(p_x\,v_{2x}+p_y\,v_{2y}+p_z\,v_{2z}\right)$

In[236]:= $\mathbf{-(r1+r2)^2+p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-2\,p_z\,p_{2z}+p_{2z}^2+}$
$\mathbf{t\,(2\,p_1 . v_1-2\,p_2 . v_1-2\,p_1 . v_2+2\,p_2 . v_2)+t^2\left(\text{Total}\left[v_1^2\right]+\text{Total}\left[v_2^2\right]-2\,v_1 . v_2\right)==0}$

Out[236]= $p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-2\,p_z\,p_{2z}+p_{2z}^2+$
$t\left(2\left(p_x\,v_x+p_y\,v_y+p_z\,v_z\right)-2\left(p_{2x}\,v_x+p_{2y}\,v_y+p_{2z}\,v_z\right)-2\left(p_x\,v_{2x}+p_y\,v_{2y}+p_z\,v_{2z}\right)+\right.$
$\left.2\left(p_{2x}\,v_{2x}+p_{2y}\,v_{2y}+p_{2z}\,v_{2z}\right)\right)+t^2\left(v_x^2+v_{2x}^2+v_y^2+v_{2y}^2+v_z^2+v_{2z}^2-2\left(v_x\,v_{2x}+v_y\,v_{2y}+v_z\,v_{2z}\right)\right)==0$

In[237]:= $\mathbf{\text{Total}\left[p_1^2\right]}$

Out[237]= $p_x^2+p_y^2+p_z^2$

In[238]:= $\mathbf{-(r1+r2)^2+\text{Total}\left[p_1^2\right]-2\,p_x\,p_{2x}+p_{2x}^2-2\,p_y\,p_{2y}+p_{2y}^2-2\,p_z\,p_{2z}+p_{2z}^2+}$
$\mathbf{t\,(2\,p_1 . v_1-2\,p_2 . v_1-2\,p_1 . v_2+2\,p_2 . v_2)+t^2\left(\text{Total}\left[v_1^2\right]+\text{Total}\left[v_2^2\right]-2\,v_1 . v_2\right)==0}$

Out[238]= $p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-2\,p_z\,p_{2z}+p_{2z}^2+$
$t\left(2\left(p_x\,v_x+p_y\,v_y+p_z\,v_z\right)-2\left(p_{2x}\,v_x+p_{2y}\,v_y+p_{2z}\,v_z\right)-2\left(p_x\,v_{2x}+p_y\,v_{2y}+p_z\,v_{2z}\right)+\right.$
$\left.2\left(p_{2x}\,v_{2x}+p_{2y}\,v_{2y}+p_{2z}\,v_{2z}\right)\right)+t^2\left(v_x^2+v_{2x}^2+v_y^2+v_{2y}^2+v_z^2+v_{2z}^2-2\left(v_x\,v_{2x}+v_y\,v_{2y}+v_z\,v_{2z}\right)\right)==0$

In[239]:= $\mathbf{\text{Total}\left[p_2^2\right]}$

Out[239]= $p_{2x}^2+p_{2y}^2+p_{2z}^2$

In[240]:= $\mathbf{-(r1+r2)^2+\text{Total}\left[p_1^2\right]-2\,p_x\,p_{2x}+\text{Total}\left[p_2^2\right]-2\,p_y\,p_{2y}-2\,p_z\,p_{2z}+}$
$\mathbf{t\,(2\,p_1 . v_1-2\,p_2 . v_1-2\,p_1 . v_2+2\,p_2 . v_2)+t^2\left(\text{Total}\left[v_1^2\right]+\text{Total}\left[v_2^2\right]-2\,v_1 . v_2\right)==0}$

Out[240]= $p_x^2-2\,p_x\,p_{2x}+p_{2x}^2+p_y^2-2\,p_y\,p_{2y}+p_{2y}^2+p_z^2-2\,p_z\,p_{2z}+p_{2z}^2+$
$t\left(2\left(p_x\,v_x+p_y\,v_y+p_z\,v_z\right)-2\left(p_{2x}\,v_x+p_{2y}\,v_y+p_{2z}\,v_z\right)-2\left(p_x\,v_{2x}+p_y\,v_{2y}+p_z\,v_{2z}\right)+\right.$
$\left.2\left(p_{2x}\,v_{2x}+p_{2y}\,v_{2y}+p_{2z}\,v_{2z}\right)\right)+t^2\left(v_x^2+v_{2x}^2+v_y^2+v_{2y}^2+v_z^2+v_{2z}^2-2\left(v_x\,v_{2x}+v_y\,v_{2y}+v_z\,v_{2z}\right)\right)==0$

In[241]:=
```
-2 p1.p2
```

Out[241]=
$$-2\left(p_x\,p_{2x}+p_y\,p_{2y}+p_z\,p_{2z}\right)$$

In[242]:=
```
f = -(r1 + r2)^2 + Total[p1^2] + Total[p2^2] - 2 p1.p2 +
    t (2 p1.v1 - 2 p2.v1 - 2 p1.v2 + 2 p2.v2) + t^2 (Total[v1^2] + Total[v2^2] - 2 v1.v2) == 0
```

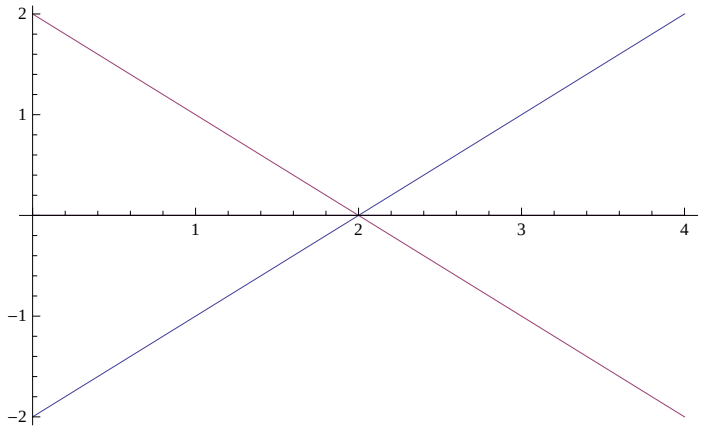
Out[242]=
$$p_x^2+p_{2x}^2+p_y^2+p_{2y}^2+p_z^2+p_{2z}^2-2\left(p_x\,p_{2x}+p_y\,p_{2y}+p_z\,p_{2z}\right)+t\left(2\left(p_x\,v_x+p_y\,v_y+p_z\,v_z\right)-2\left(p_{2x}\,v_x+p_{2y}\,v_y+p_{2z}\,v_z\right)-2\left(p_x\,v_{2x}+p_y\,v_{2y}+p_z\,v_{2z}\right)+2\left(p_{2x}\,v_{2x}+p_{2y}\,v_{2y}+p_{2z}\,v_{2z}\right)\right)+t^2\left(v_x^2+v_{2x}^2+v_y^2+v_{2y}^2+v_z^2+v_{2z}^2-2\left(v_x\,v_{2x}+v_y\,v_{2y}+v_z\,v_{2z}\right)\right)==0$$

In[243]:=
```
p1 = {-2, 0, 0};
v1 = {1, 0, 0};
r1 = 1;
p2 = {2, 0, 0};
v2 = {-1, 0, 0};
r2 = 1;
f = -(r1 + r2)^2 + Total[p1^2] + Total[p2^2] - 2 p1.p2 +
    t (2 p1.v1 - 2 p2.v1 - 2 p1.v2 + 2 p2.v2) + t^2 (Total[v1^2] + Total[v2^2] - 2 v1.v2) == 0
Solve[f, t]
Plot[{p1 + v1 t, p2 + v2 t}, {t, 0, 4}]
```

Out[249]=
$$12-16\,t+4\,t^2==0$$

Out[250]=
$$\{\{t\to 1\},\{t\to 3\}\}$$

Out[251]=



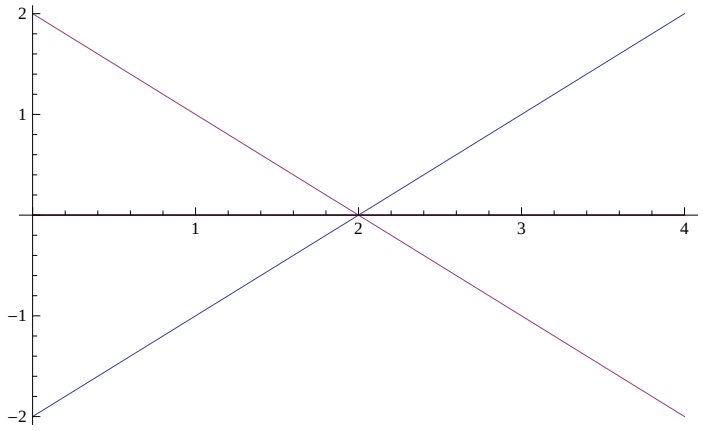
In[252]:=

```
p1 = {-2, 0, 0};
v1 = {1, 0, 0};
r1 = 0;
p2 = {2, 0, 0};
v2 = {-1, 0, 0};
r2 = 0;
f = -(r1 + r2)^2 + Total[p1^2] + Total[p2^2] - 2 p1.p2 +
    t (2 p1.v1 - 2 p2.v1 - 2 p1.v2 + 2 p2.v2) + t^2 (Total[v1^2] + Total[v2^2] - 2 v1.v2) == 0
Solve[f, t]
Plot[{p1 + v1 t, p2 + v2 t}, {t, 0, 4}]
```

Out[258]= $16 - 16\,t + 4\,t^2 == 0$

Out[259]= $\{\{t \to 2\}, \{t \to 2\}\}$

Out[260]=



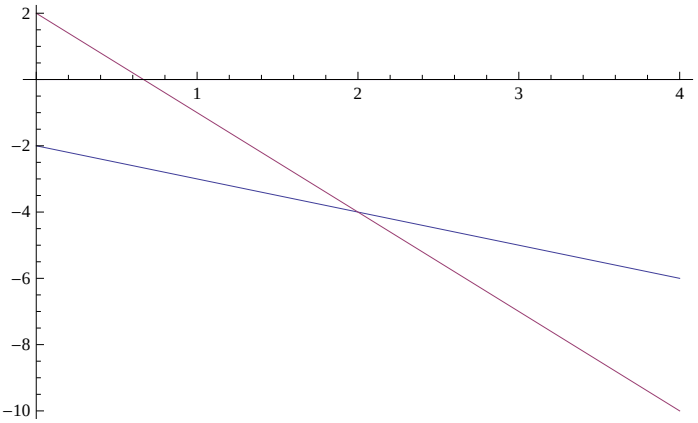
In[261]:=

```
p1 = {-2, 0, 0};
v1 = {-1, 0, 0};
r1 = 0;
p2 = {2, 0, 0};
v2 = {-3, 0, 0};
r2 = 0;
f = -(r1 + r2)^2 + Total[p1^2] + Total[p2^2] - 2 p1.p2 +
    t (2 p1.v1 - 2 p2.v1 - 2 p1.v2 + 2 p2.v2) + t^2 (Total[v1^2] + Total[v2^2] - 2 v1.v2) == 0
Solve[f, t]
Plot[{p1 + v1 t, p2 + v2 t}, {t, 0, 4}]
```

Out[267]= $16 - 16\,t + 4\,t^2 == 0$

Out[268]= $\{\{t \to 2\}, \{t \to 2\}\}$

Out[269]=



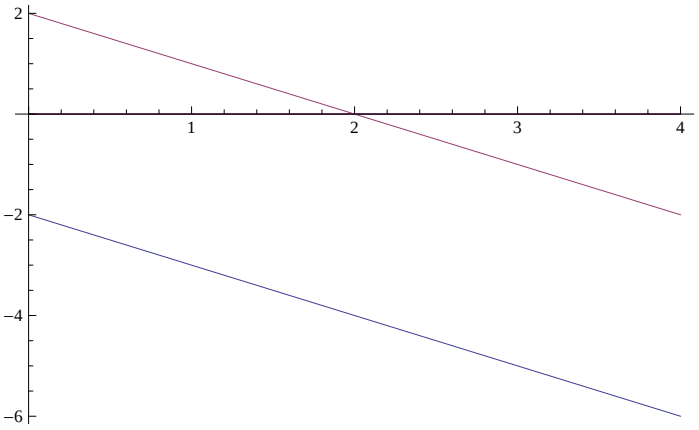
In[270]:=

```
p1 = {-2, 0, 0};
v1 = {-1, 0, 0};
r1 = 0;
p2 = {2, 0, 0};
v2 = {-1, 0, 0};
r2 = 0;
f = -(r1 + r2)^2 + Total[p1^2] + Total[p2^2] - 2 p1.p2 +
    t (2 p1.v1 - 2 p2.v1 - 2 p1.v2 + 2 p2.v2) + t^2 (Total[v1^2] + Total[v2^2] - 2 v1.v2) == 0
Solve[f, t]
Plot[{p1 + v1 t, p2 + v2 t}, {t, 0, 4}]
```

Out[276]= False

Out[277]= { }

Out[278]=



In[279]:=

```
p1 = {-2, 0, 0};
v1 = {-1, 0, 0};
r1 = 0;
p2 = {2, 0, 0};
v2 = {1, 0, 0};
r2 = 0;
f = -(r1 + r2)^2 + Total[p1^2] + Total[p2^2] - 2 p1.p2 +
    t (2 p1.v1 - 2 p2.v1 - 2 p1.v2 + 2 p2.v2) + t^2 (Total[v1^2] + Total[v2^2] - 2 v1.v2) == 0
Solve[f, t]
Plot[{p1 + v1 t, p2 + v2 t}, {t, 0, 4}]
```

Out[285]= $16 + 16\,t + 4\,t^2 == 0$

Out[286]= $\{\{t \to -2\}, \{t \to -2\}\}$

Out[287]=