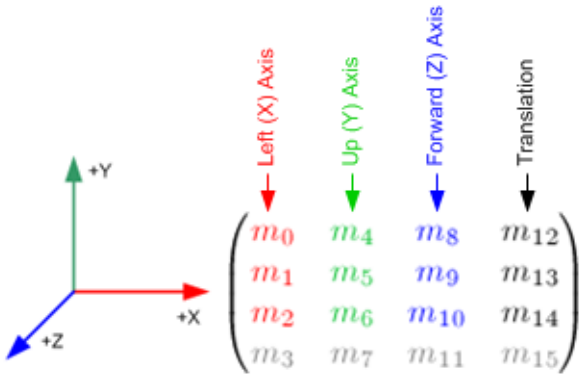
```
(*matrix transformations, rotation, translation, scaling*)
```




```
Clear["Global`*"];

M = IdentityMatrix[4](*axes*)
```

```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

```
M // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
{1, 0, 0, 0} // Column
```

```
1
0
0
0
```

```
M[[3, 4]] = -1;
M
M // MatrixForm
M = IdentityMatrix[3];
```

```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, -1}, {0, 0, 0, 1}}
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
x = M[[All, 1]]
y = M[[All, 2]]
z = M[[All, 3]]
```

```
{1, 0, 0}
```

```
{0, 1, 0}
```

```
{0, 0, 1}
```

```
α;(*rotation of x-axis*)
RotationMatrix[α, x] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\alpha] & -\text{Sin}[\alpha] \\ 0 & \text{Sin}[\alpha] & \text{Cos}[\alpha] \end{pmatrix}$$

```
β;(*rotation of y-axis*)
RotationMatrix[β, y] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\beta] & 0 & \text{Sin}[\beta] \\ 0 & 1 & 0 \\ -\text{Sin}[\beta] & 0 & \text{Cos}[\beta] \end{pmatrix}$$

```
γ;(*rotation of z-axis*)
RotationMatrix[γ, z] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\gamma] & -\text{Sin}[\gamma] & 0 \\ \text{Sin}[\gamma] & \text{Cos}[\gamma] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
RotationMatrix[α, x].RotationMatrix[β, y].RotationMatrix[γ, z] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\beta]\,\text{Cos}[\gamma] & -\text{Cos}[\beta]\,\text{Sin}[\gamma] & \text{Sin}[\beta] \\ \text{Cos}[\gamma]\,\text{Sin}[\alpha]\,\text{Sin}[\beta] + \text{Cos}[\alpha]\,\text{Sin}[\gamma] & \text{Cos}[\alpha]\,\text{Cos}[\gamma] - \text{Sin}[\alpha]\,\text{Sin}[\beta]\,\text{Sin}[\gamma] & -\text{Cos}[\beta]\,\text{Sin}[\alpha] \\ -\text{Cos}[\alpha]\,\text{Cos}[\gamma]\,\text{Sin}[\beta] + \text{Sin}[\alpha]\,\text{Sin}[\gamma] & \text{Cos}[\gamma]\,\text{Sin}[\alpha] + \text{Cos}[\alpha]\,\text{Sin}[\beta]\,\text{Sin}[\gamma] & \text{Cos}[\alpha]\,\text{Cos}[\beta] \end{pmatrix}$$

```
RotationMatrix[γ, z].RotationMatrix[α, x].RotationMatrix[β, y] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\beta]\,\text{Cos}[\gamma] - \text{Sin}[\alpha]\,\text{Sin}[\beta]\,\text{Sin}[\gamma] & -\text{Cos}[\alpha]\,\text{Sin}[\gamma] & \text{Cos}[\gamma]\,\text{Sin}[\beta] + \text{Cos}[\beta]\,\text{Sin}[\alpha]\,\text{Sin}[\gamma] \\ \text{Cos}[\gamma]\,\text{Sin}[\alpha]\,\text{Sin}[\beta] + \text{Cos}[\beta]\,\text{Sin}[\gamma] & \text{Cos}[\alpha]\,\text{Cos}[\gamma] & -\text{Cos}[\beta]\,\text{Cos}[\gamma]\,\text{Sin}[\alpha] + \text{Sin}[\beta]\,\text{Sin}[\gamma] \\ -\text{Cos}[\alpha]\,\text{Sin}[\beta] & \text{Sin}[\alpha] & \text{Cos}[\alpha]\,\text{Cos}[\beta] \end{pmatrix}$$

```
t = {xo, yo, zo};(*translation*)
TranslationTransform[t]
```

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & \text{xo} \\ 0 & 1 & 0 & \text{yo} \\ 0 & 0 & 1 & \text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

```
s = {sx, sy, sz};(*scale*)
ScalingTransform[s]
```

$$\text{Null}^2\,\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & 0 \\ 0 & \text{sy} & 0 & 0 \\ 0 & 0 & \text{sz} & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

```
TranslationTransform[t].ScalingTransform[s].RotationTransform[α, x]
```

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & \text{xo} \\ 0 & \text{sy}\,\text{Cos}[\alpha] & -\text{sy}\,\text{Sin}[\alpha] & \text{yo} \\ 0 & \text{sz}\,\text{Sin}[\alpha] & \text{sz}\,\text{Cos}[\alpha] & \text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

```
TranslationTransform[t].RotationTransform[α, x].ScalingTransform[s]
```

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & \text{xo} \\ 0 & \text{sy}\,\text{Cos}[\alpha] & -\text{sz}\,\text{Sin}[\alpha] & \text{yo} \\ 0 & \text{sy}\,\text{Sin}[\alpha] & \text{sz}\,\text{Cos}[\alpha] & \text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

```
ScalingTransform[s].TranslationTransform[t].RotationTransform[α, x]
```

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} \text{sx} & 0 & 0 & \text{sx}\,\text{xo} \\ 0 & \text{sy}\,\text{Cos}[\alpha] & -\text{sy}\,\text{Sin}[\alpha] & \text{sy}\,\text{yo} \\ 0 & \text{sz}\,\text{Sin}[\alpha] & \text{sz}\,\text{Cos}[\alpha] & \text{sz}\,\text{zo} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

```
Clear["Global`*"];
M = {{xx, yx, zx, ox}, {xy, yy, zy, oy}, {xz, yz, zz, oz}, {xo, yo, zo, oo}}
```

```
{{xx, yx, zx, ox}, {xy, yy, zy, oy}, {xz, yz, zz, oz}, {xo, yo, zo, oo}}
```

```
M // MatrixForm
```

$$\begin{pmatrix} xx & yx & zx & ox \\ xy & yy & zy & oy \\ xz & yz & zz & oz \\ xo & yo & zo & oo \end{pmatrix}$$

```
A = Flatten[M]
```

```
{xx, yx, zx, ox, xy, yy, zy, oy, xz, yz, zz, oz, xo, yo, zo, oo}
```

```
A[[13]]
A[[14]]
A[[15]]
```

```
xo
```

```
yo
```

```
zo
```

```
M = M[[1 ;; 3, 1 ;; 3]]
```

```
{{xx, yx, zx}, {xy, yy, zy}, {xz, yz, zz}}
```

```
RotationMatrix[α, {1, 0, 0}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\alpha] & -\text{Sin}[\alpha] \\ 0 & \text{Sin}[\alpha] & \text{Cos}[\alpha] \end{pmatrix}$$

```
M.{1, 0, 0}
M.{0, 1, 0}
M.{0, 0, 1}
```

```
{xx, xy, xz}
```

```
{yx, yy, yz}
```

```
{zx, zy, zz}
```

```
M.M // MatrixForm
```

$$\begin{pmatrix} xx^2 + xy\,yx + xz\,zx & xx\,yx + yx\,yy + yz\,zx & xx\,zx + yx\,zy + zx\,zz \\ xx\,xy + xy\,yy + xz\,zy & xy\,yx + yy^2 + yz\,zy & xy\,zx + yy\,zy + zy\,zz \\ xx\,xz + xy\,yz + xz\,zz & xz\,yx + yy\,yz + yz\,zz & xz\,zx + yz\,zy + zz^2 \end{pmatrix}$$

```
M.RotationMatrix[α, x] // MatrixForm
```

$$\begin{pmatrix} xx & yx\,\text{Cos}[\alpha] + zx\,\text{Sin}[\alpha] & zx\,\text{Cos}[\alpha] - yx\,\text{Sin}[\alpha] \\ xy & yy\,\text{Cos}[\alpha] + zy\,\text{Sin}[\alpha] & zy\,\text{Cos}[\alpha] - yy\,\text{Sin}[\alpha] \\ xz & yz\,\text{Cos}[\alpha] + zz\,\text{Sin}[\alpha] & zz\,\text{Cos}[\alpha] - yz\,\text{Sin}[\alpha] \end{pmatrix}$$

```
RotationMatrix[α, x].M // MatrixForm
```

$$\begin{pmatrix} xx & yx & zx \\ xy\,\text{Cos}[\alpha] - xz\,\text{Sin}[\alpha] & yy\,\text{Cos}[\alpha] - yz\,\text{Sin}[\alpha] & zy\,\text{Cos}[\alpha] - zz\,\text{Sin}[\alpha] \\ xz\,\text{Cos}[\alpha] + xy\,\text{Sin}[\alpha] & yz\,\text{Cos}[\alpha] + yy\,\text{Sin}[\alpha] & zz\,\text{Cos}[\alpha] + zy\,\text{Sin}[\alpha] \end{pmatrix}$$

RotationMatrix[β, y].M // MatrixForm

$$\begin{pmatrix} xx\,\text{Cos}[\beta] + xz\,\text{Sin}[\beta] & yx\,\text{Cos}[\beta] + yz\,\text{Sin}[\beta] & zx\,\text{Cos}[\beta] + zz\,\text{Sin}[\beta] \\ xy & yy & zy \\ xz\,\text{Cos}[\beta] - xx\,\text{Sin}[\beta] & yz\,\text{Cos}[\beta] - yx\,\text{Sin}[\beta] & zz\,\text{Cos}[\beta] - zx\,\text{Sin}[\beta] \end{pmatrix}$$

RotationMatrix[γ, z].M // MatrixForm

$$\begin{pmatrix} xx\,\text{Cos}[\gamma] - xy\,\text{Sin}[\gamma] & yx\,\text{Cos}[\gamma] - yy\,\text{Sin}[\gamma] & zx\,\text{Cos}[\gamma] - zy\,\text{Sin}[\gamma] \\ xy\,\text{Cos}[\gamma] + xx\,\text{Sin}[\gamma] & yy\,\text{Cos}[\gamma] + yx\,\text{Sin}[\gamma] & zy\,\text{Cos}[\gamma] + zx\,\text{Sin}[\gamma] \\ xz & yz & zz \end{pmatrix}$$

Inverse[M] // MatrixForm

$$\begin{pmatrix} \frac{-yz\,zy+yy\,zz}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-xx\,yz\,zy-xy\,yx\,zz+xx\,yy\,zz} & \frac{yz\,zx-yx\,zz}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-xx\,yz\,zy-xy\,yx\,zz+xx\,yy\,zz} & \frac{-yy\,zx+}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-} \\ \frac{xz\,zy-xy\,zz}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-xx\,yz\,zy-xy\,yx\,zz+xx\,yy\,zz} & \frac{-xz\,zx+xx\,zz}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-xx\,yz\,zy-xy\,yx\,zz+xx\,yy\,zz} & \frac{xy\,zx-}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-} \\ \frac{-xz\,yy+xy\,yz}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-xx\,yz\,zy-xy\,yx\,zz+xx\,yy\,zz} & \frac{xz\,yx-xx\,yz}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-xx\,yz\,zy-xy\,yx\,zz+xx\,yy\,zz} & \frac{-xy\,yx+}{-xz\,yy\,zx+xy\,yz\,zx+xz\,yx\,zy-} \end{pmatrix}$$