```
(*sphere/sphere collision resolver*)
p_1 = \{p_{1.x}, p_{1.y}, p_{1.z}\}; (*position*)
  u_1 = \{u_{1.x}, u_{1.y}, u_{1.z}\}; (*velocity*)
r<sub>1</sub>; (*radius*)
p_2 = \{p_{2.x}, p_{2.y}, p_{2.z}\};
u_2 = \{u_{2.x}, u_{2.y}, u_{2.z}\};
Solve [EuclideanDistance [p_1 + u_1 t, p_2 + u_2 t] == r_1 + r_2, t]
Solve::ifun: Inverse functions are being used by Solve, so
                                   some solutions may not be found; use Reduce for complete solution information. >>>
  \left\{ t \rightarrow \left( -2\,p_{1.\,x}\,u_{1.\,x} + 2\,p_{2.\,x}\,u_{1.\,x} + 2\,p_{1.\,x}\,u_{2.\,x} - 2\,p_{2.\,x}\,u_{2.\,x} - 2\,p_{1.\,y}\,u_{1.\,y} + 2\,p_{2.\,y}\,u_{1.\,y} + 2\,p_{2.\,y}\,u_{1.\,y} + 2\,p_{2.\,y}\,u_{2.\,x} \right\} \right\} = 0
                                                    2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} - 2 p_
                                                    \sqrt{\left(2 p_{1..x} u_{1..x} - 2 p_{2..x} u_{1..x} - 2 p_{1..x} u_{2..x} + 2 p_{2..x} u_{2..x} + 2 p_{1..y} u_{1..y} - 2 p_{2..y} u_{1..y} - 2 p_{2..y} u_{1..y} - 2 p_{2..y} u_{2..x} + 2 p_{2..x} u_{2..x} + 2 p_{2..x} u_{2..x} + 2 p_{2..x} u_{2..x} + 2 p_{2..y} u_{2..y} - 2 p_{2..y} u_{2.
                                                                                                      2\;p_{1.\;y}\;u_{2.\;y}\;+\;2\;p_{2.\;y}\;u_{2.\;y}\;+\;2\;p_{1.\;z}\;u_{1.\;z}\;-\;2\;p_{2.\;z}\;u_{1.\;z}\;-\;2\;p_{1.\;z}\;u_{2.\;z}\;+\;2\;p_{2.\;z}\;u_{2.\;z}\;\rangle^{\;2}\;-\;2\;p_{2.\;z}\;u_{2.\;z}\;+\;2\;p_{2.\;z}\;u_{2.\;z}\;\varphi^{\;2}\;
                                                                            4 \left( p_{1}^{2} + 2 p_{1,x} p_{2,x} + p_{2}^{2} + p_{1,y}^{2} - 2 p_{1,y} p_{2,y} + p_{2,y}^{2} + p_{1,z}^{2} - 2 p_{1,z} p_{2,z} + p_{2,z}^{2} - r_{1}^{2} - 2 r_{1} r_{2} - 2 r_{1} r_{2} - r_{1}^{2} - r_{
                                                                                                      (u_{1.x}^2) (u_{1.x}^2 - 2u_{1.x}u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2u_{1.y}u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2u_{1.z}u_{2.z} + u_{2.z}^2))
                                   \left(2\left(u_{1}^{2} + 2u_{1} + u_{2} + u_{2}^{2} + u_{1}^{2} + u_{1}^{2} + u_{1}^{2} - 2u_{1} + u_{2}^{2} + u_{2}^{2} + u_{2}^{2} + u_{1}^{2} - 2u_{1} + u_{2}^{2} + u_{2}^{2}\right)\right)\right\}
         \left\{\,t\,\rightarrow\,\left(-\,2\;p_{1\,.\,x}\;u_{1\,.\,x}\,+\,2\;p_{2\,.\,x}\;u_{1\,.\,x}\,+\,2\;p_{1\,.\,x}\;u_{2\,.\,x}\,-\,2\;p_{2\,.\,x}\;u_{2\,.\,x}\,-\,2\;p_{1\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}
                                                     2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} +
                                                    \sqrt{\left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} - 
                                                                                                      2\;p_{1.\;y}\;u_{2.\;y}\;+\;2\;p_{2.\;y}\;u_{2.\;y}\;+\;2\;p_{1.\;z}\;u_{1.\;z}\;-\;2\;p_{2.\;z}\;u_{1.\;z}\;-\;2\;p_{1.\;z}\;u_{2.\;z}\;+\;2\;p_{2.\;z}\;u_{2.\;z}\;\rangle^2\;-
                                                                             4(p_1^2 \times -2p_{1,x}p_{2,x} + p_2^2 \times +p_1^2 \times -2p_{1,y}p_{2,y} + p_2^2 \times +p_1^2 \times -2p_{1,z}p_{2,z} + p_2^2 \times -p_1^2 -2r_1r_2 -
                                                                                                    (u_{1,x}^2) (u_{1,x}^2 - 2u_{1,x}u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2u_{1,y}u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2u_{1,z}u_{2,z} + u_{2,z}^2))
                                   \left(2\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)\right)\right\}
EuclideanDistance[p_1 + u_1 t, p_2 + u_2 t]
  \sqrt{\left(\text{Abs}\left[p_{1.x} - p_{2.x} + \text{tu}_{1.x} - \text{tu}_{2.x}\right]^2 + \right]}
                         Abs [p_{1.y} - p_{2.y} + t u_{1.y} - t u_{2.y}]^2 + Abs [p_{1.z} - p_{2.z} + t u_{1.z} - t u_{2.z}]^2
 \sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]} == r_1 + r_2
\sqrt{\left(\left(p_{1.\,x}-p_{2.\,x}+t\,u_{1.\,x}-t\,u_{2.\,x}\right)^{\,2}+\left(p_{1.\,y}-p_{2.\,y}+t\,u_{1.\,y}-t\,u_{2.\,y}\right)^{\,2}+\left(p_{1.\,z}-p_{2.\,z}+t\,u_{1.\,z}-t\,u_{2.\,z}\right)^{\,2}\right)}}\;=\;
   \left(\sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]}\right)^2 = (r_1 + r_2)^2
   (p_{1.x} - p_{2.x} + t u_{1.x} - t u_{2.x})^2 + (p_{1.y} - p_{2.y} + t u_{1.y} - t u_{2.y})^2 + (p_{1.z} - p_{2.z} + t u_{1.z} - t u_{2.z})^2 = (r_1 + r_2)^2 
 Total [(p_1 + u_1 t - (p_2 + u_2 t))^2] == (r_1 + r_2)^2
  (p_{1.x} - p_{2.x} + t u_{1.x} - t u_{2.x})^2 + (p_{1.y} - p_{2.y} + t u_{1.y} - t u_{2.y})^2 + (p_{1.z} - p_{2.z} + t u_{1.z} - t u_{2.z})^2 = (r_1 + r_2)^2
 \Delta p = p_1 - p_2
   \{p_{1.x} - p_{2.x}, p_{1.y} - p_{2.y}, p_{1.z} - p_{2.z}\}
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Clear["Global`*"];

$$t^{2}\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)=0$$

$$Collect\left[\text{Expand}\left[\text{Total}\left[\Delta p^{2}+2\Delta p \Delta u t+\Delta u^{2} t^{2}\right]-\left(r_{1}+r_{2}\right)^{2}=0\right], t\right]$$

 $p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_2^2 - r_1^2 - r_2^2 - r_2^$ $r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_$ $2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}$ $\mathsf{t}^2 \left(\mathsf{u}^2_{1.\,x} - 2\,\mathsf{u}_{1.\,x}\,\mathsf{u}_{2.\,x} + \mathsf{u}^2_{2.\,x} + \mathsf{u}^2_{1.\,y} - 2\,\mathsf{u}_{1.\,y}\,\mathsf{u}_{2.\,y} + \mathsf{u}^2_{2.\,y} + \mathsf{u}^2_{1.\,z} - 2\,\mathsf{u}_{1.\,z}\,\mathsf{u}_{2.\,z} + \mathsf{u}^2_{2.\,z} \right) \, = \, 0$

Collect [Total $[\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2] - (r_1 + r_2)^2 = 0, t$]

$$\begin{array}{l} \left(p_{1.\,x}-p_{2.\,x}\right)^{\,2} + \, \left(p_{1.\,y}-p_{2.\,y}\right)^{\,2} + \, \left(p_{1.\,z}-p_{2.\,z}\right)^{\,2} - \, \left(r_{1}+r_{2}\right)^{\,2} + \\ \\ t \, \left(2 \, \left(p_{1.\,x}-p_{2.\,x}\right) \, \left(u_{1.\,x}-u_{2.\,x}\right) + 2 \, \left(p_{1.\,y}-p_{2.\,y}\right) \, \left(u_{1.\,y}-u_{2.\,y}\right) + 2 \, \left(p_{1.\,z}-p_{2.\,z}\right) \, \left(u_{1.\,z}-u_{2.\,z}\right) \right) + \\ \\ t^{\,2} \, \left(\left(u_{1.\,x}-u_{2.\,x}\right)^{\,2} + \, \left(u_{1.\,y}-u_{2.\,y}\right)^{\,2} + \, \left(u_{1.\,z}-u_{2.\,z}\right)^{\,2} \right) \ = \ 0 \end{array}$$

 $Total[\Delta p^2]$

$$(p_{1.x} - p_{2.x})^2 + (p_{1.y} - p_{2.y})^2 + (p_{1.z} - p_{2.z})^2$$

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$$(p_{1.\,x} - p_{2.\,x})^2 + (p_{1.\,y} - p_{2.\,y})^2 + (p_{1.\,z} - p_{2.\,z})^2$$

Total
$$[\Delta p^2] == \Delta p \cdot \Delta p$$

True

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Collect
             Expand \left[\Delta p \cdot \Delta p - (r_1 + r_2)^2 + t \left(2 \left(p_1 \cdot x - p_2 \cdot x\right) \left(u_1 \cdot x - u_2 \cdot x\right) + 2 \left(p_1 \cdot y - p_2 \cdot y\right) \left(u_1 \cdot y - u_2 \cdot y\right) + 2 \left(p_1 \cdot y - p_2 \cdot y\right) \right]
                                                                                 2(p_{1,z} - p_{2,z})(u_{1,z} - u_{2,z}) +
                                                t^{2} \left( \left( u_{1, x} - u_{2, x} \right)^{2} + \left( u_{1, y} - u_{2, y} \right)^{2} + \left( u_{1, z} - u_{2, z} \right)^{2} \right) = 0 \right], t
p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2
                      r_2^2 + t(2p_1 \cdot u_1 \cdot v - 2p_2 \cdot u_1 \cdot v - 2p_1 \cdot u_2 \cdot v + 2p_2 \cdot u_2 \cdot v + 2p_1 \cdot v_1 \cdot v - 2p_2 \cdot v_1 
                                                          2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                        t^{2}\left(u_{1,x}^{2}-2\,u_{1,x}\,u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2\,u_{1,y}\,u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2\,u_{1,z}\,u_{2,z}+u_{2,z}^{2}\right)=0
 2 Δρ. Δu t
   2t\left(\left(p_{1.x}-p_{2.x}\right)\left(u_{1.x}-u_{2.x}\right)+\left(p_{1.y}-p_{2.y}\right)\left(u_{1.y}-u_{2.y}\right)+\left(p_{1.z}-p_{2.z}\right)\left(u_{1.z}-u_{2.z}\right)\right)
Collect
            Expand \left[\Delta p \cdot \Delta p - (r_1 + r_2)^2 + 2 \Delta p \cdot \Delta u t + t^2 \left( (u_1 \cdot x - u_2 \cdot x)^2 + (u_1 \cdot y - u_2 \cdot y)^2 + (u_1 \cdot z - u_2 \cdot z)^2 \right) = 0 \right]
p_{1-x}^2 - 2p_{1,x}p_{2,x} + p_{2-x}^2 + p_{1-y}^2 - 2p_{1,y}p_{2,y} + p_{2-y}^2 + p_{1-z}^2 - 2p_{1,z}p_{2,z} + p_{2-z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                      r_2^2 + t(2p_1 \times u_1 \times -2p_2 \times u_1 \times -2p_1 \times u_2 \times +2p_2 \times u_2 \times +2p_1 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_2 \times +2p_2 \times u_2 \times +2p_2 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_2 \times +2p_2 \times u_2 \times +2p_2 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_2 \times +2p_2 \times u_2 \times +2p_2 \times u_2 \times +2p_2 \times u_2 \times -2p_2 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_2 \times +2p_2 \times u_2 \times +2p_2 \times u_2 \times -2p_2 \times u_2 \times -2p_2 \times u_1 \times -2p_2 \times u_1 \times -2p_2 \times u_2 \times -2p_2 \times -2p_2 \times u_2 \times -2p_2 \times u_2 \times -2p_2 \times u_2 \times -2p_2 \times
                                                          2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z}
                        t^{2}\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)=0
Total \left[\Delta u^2\right]
    (u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2
   (u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2
Collect Expand [\Delta p \cdot \Delta p - (r_1 + r_2)^2 + 2 \Delta p \cdot \Delta u t + t^2 \Delta u \cdot \Delta u = 0], t
p_{1-x}^2 - 2p_{1,x}p_{2,x} + p_{2-x}^2 + p_{1-y}^2 - 2p_{1,y}p_{2,y} + p_{2-y}^2 + p_{1-z}^2 - 2p_{1,z}p_{2,z} + p_{2-z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                      r_2^2 + t \left(2 p_{1..x} u_{1..x} - 2 p_{2..x} u_{1..x} - 2 p_{1..x} u_{2..x} + 2 p_{2..x} u_{2..x} + 2 p_{1..v} u_{1..v} - 2 p_{2..v} u_{1..v} - 2 p_{2..v} u_{1..v} - 2 p_{2..v} u_{2..x} + 2 p_{2..x} u_{2..x} + 2 p_{2..x} u_{2..x} + 2 p_{2..v} u_{2..x} + 2 p_{2..
                                                            2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                        t^{2}\left(u_{1,x}^{2}-2\,u_{1,x}\,u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2\,u_{1,y}\,u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2\,u_{1,z}\,u_{2,z}+u_{2,z}^{2}\right)=0
 a = \Delta u \cdot \Delta u;
b = 2 \Delta p. \Delta u;
 c = \Delta p \cdot \Delta p - (r_1 + r_2)^2;
Collect [Expand[at^2 + bt + c = 0], t]
p_{1}^{2} + 2p_{1} + p_{2} + p_{2}^{2} + p_{1}^{2} + p_{1}^{2} + p_{1}^{2} + p_{2}^{2} + p_{1}^{2} + p_{2}^{2} + p_{2}^{2} + p_{1}^{2} + p_{2}^{2} +
                        r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} - 2 p_{2.y} u_
                                                            2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z}
                        t^{2}\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)=0
 Solve [EuclideanDistance [p_1 + u_1 t, p_2 + u_2 t] = r_1 + r_2, t] = Solve [a t^2 + b t + c = 0, t]
Solve::ifun: Inverse functions are being used by Solve, so
                                                some solutions may not be found; use Reduce for complete solution information. >>>
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>>

True