

```

Clear["Global`*"];
(*sphere/sphere collision resolver*)
p1 = {p1.x, p1.y, p1.z}; (*position*)
u1 = {u1.x, u1.y, u1.z}; (*velocity*)
r1; (*radius*)
p2 = {p2.x, p2.y, p2.z};
u2 = {u2.x, u2.y, u2.z};
r2;
Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t]

```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{ \left\{ t \rightarrow \left(-2 p_{1,x} u_{1,x} + 2 p_{2,x} u_{1,x} + 2 p_{1,x} u_{2,x} - 2 p_{2,x} u_{2,x} - 2 p_{1,y} u_{1,y} + 2 p_{2,y} u_{1,y} + 2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} - \sqrt{\left(\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right)^2 - 4 \left(p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 \right) \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right)} \right) \right\}, \right. \\
\left. \left\{ t \rightarrow \left(-2 p_{1,x} u_{1,x} + 2 p_{2,x} u_{1,x} + 2 p_{1,x} u_{2,x} - 2 p_{2,x} u_{2,x} - 2 p_{1,y} u_{1,y} + 2 p_{2,y} u_{1,y} + 2 p_{1,y} u_{2,y} - 2 p_{2,y} u_{2,y} - 2 p_{1,z} u_{1,z} + 2 p_{2,z} u_{1,z} + 2 p_{1,z} u_{2,z} - 2 p_{2,z} u_{2,z} + \sqrt{\left(\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right)^2 - 4 \left(p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 \right) \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right)} \right) \right\} \right\}$$

```
EuclideanDistance[p1 + u1 t, p2 + u2 t]
```

$$\sqrt{\left(\text{Abs}[p_{1,x} - p_{2,x} + t u_{1,x} - t u_{2,x}]^2 + \text{Abs}[p_{1,y} - p_{2,y} + t u_{1,y} - t u_{2,y}]^2 + \text{Abs}[p_{1,z} - p_{2,z} + t u_{1,z} - t u_{2,z}]^2 \right)}$$

$$\sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]} == r_1 + r_2$$

$$\sqrt{\left((p_{1,x} - p_{2,x} + t u_{1,x} - t u_{2,x})^2 + (p_{1,y} - p_{2,y} + t u_{1,y} - t u_{2,y})^2 + (p_{1,z} - p_{2,z} + t u_{1,z} - t u_{2,z})^2 \right)} == r_1 + r_2$$

$$\left(\sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]} \right)^2 == (r_1 + r_2)^2$$

$$(p_{1,x} - p_{2,x} + t u_{1,x} - t u_{2,x})^2 + (p_{1,y} - p_{2,y} + t u_{1,y} - t u_{2,y})^2 + (p_{1,z} - p_{2,z} + t u_{1,z} - t u_{2,z})^2 == (r_1 + r_2)^2$$

$$\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2] == (r_1 + r_2)^2$$

$$(p_{1,x} - p_{2,x} + t u_{1,x} - t u_{2,x})^2 + (p_{1,y} - p_{2,y} + t u_{1,y} - t u_{2,y})^2 + (p_{1,z} - p_{2,z} + t u_{1,z} - t u_{2,z})^2 == (r_1 + r_2)^2$$

$$\Delta p = p_1 - p_2$$

$$\{p_{1,x} - p_{2,x}, p_{1,y} - p_{2,y}, p_{1,z} - p_{2,z}\}$$

Collect[**Expand**[**Total**[($\Delta p + u_1 t - u_2 t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Expand**[**Total**[($\Delta p + (u_1 - u_2) t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

$\Delta u = u_1 - u_2$

$$\{u_{1,x} - u_{2,x}, u_{1,y} - u_{2,y}, u_{1,z} - u_{2,z}\}$$

Collect[**Expand**[**Total**[($\Delta p + \Delta u t$)²] - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Expand**[**Total**[($\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2$) - ($r_1 + r_2$)² == 0], t]

$$p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - r_2^2 + t \left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} \right) + t^2 \left(u_{1,x}^2 - 2 u_{1,x} u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2 u_{1,y} u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2 u_{1,z} u_{2,z} + u_{2,z}^2 \right) == 0$$

Collect[**Total**[($\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2$) - ($r_1 + r_2$)² == 0], t]

$$(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2 - (r_1 + r_2)^2 + t \left(2 (p_{1,x} - p_{2,x}) (u_{1,x} - u_{2,x}) + 2 (p_{1,y} - p_{2,y}) (u_{1,y} - u_{2,y}) + 2 (p_{1,z} - p_{2,z}) (u_{1,z} - u_{2,z}) \right) + t^2 \left((u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2 \right) == 0$$

Total[(Δp^2)]

$$(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2$$

$\Delta p \cdot \Delta p$

$$(p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2$$

Total[(Δp^2)] == $\Delta p \cdot \Delta p$

True

```

Collect[
  Expand[Δp.Δp - (r1 + r2)2 + t (2 (p1,x - p2,x) (u1,x - u2,x) + 2 (p1,y - p2,y) (u1,y - u2,y) +
    2 (p1,z - p2,z) (u1,z - u2,z)) +
    t2 ((u1,x - u2,x)2 + (u1,y - u2,y)2 + (u1,z - u2,z)2) == 0], t]

p1,x2 - 2 p1,x p2,x + p2,x2 + p1,y2 - 2 p1,y p2,y + p2,y2 + p1,z2 - 2 p1,z p2,z + p2,z2 - r12 - 2 r1 r2 -
  r22 + t (2 p1,x u1,x - 2 p2,x u1,x - 2 p1,x u2,x + 2 p2,x u2,x + 2 p1,y u1,y - 2 p2,y u1,y -
    2 p1,y u2,y + 2 p2,y u2,y + 2 p1,z u1,z - 2 p2,z u1,z - 2 p1,z u2,z + 2 p2,z u2,z) +
  t2 (u1,x2 - 2 u1,x u2,x + u2,x2 + u1,y2 - 2 u1,y u2,y + u2,y2 + u1,z2 - 2 u1,z u2,z + u2,z2) == 0

2 Δp.Δu t

2 t ((p1,x - p2,x) (u1,x - u2,x) + (p1,y - p2,y) (u1,y - u2,y) + (p1,z - p2,z) (u1,z - u2,z))

Collect[
  Expand[Δp.Δp - (r1 + r2)2 + 2 Δp.Δu t + t2 ((u1,x - u2,x)2 + (u1,y - u2,y)2 + (u1,z - u2,z)2) == 0], t]

p1,x2 - 2 p1,x p2,x + p2,x2 + p1,y2 - 2 p1,y p2,y + p2,y2 + p1,z2 - 2 p1,z p2,z + p2,z2 - r12 - 2 r1 r2 -
  r22 + t (2 p1,x u1,x - 2 p2,x u1,x - 2 p1,x u2,x + 2 p2,x u2,x + 2 p1,y u1,y - 2 p2,y u1,y -
    2 p1,y u2,y + 2 p2,y u2,y + 2 p1,z u1,z - 2 p2,z u1,z - 2 p1,z u2,z + 2 p2,z u2,z) +
  t2 (u1,x2 - 2 u1,x u2,x + u2,x2 + u1,y2 - 2 u1,y u2,y + u2,y2 + u1,z2 - 2 u1,z u2,z + u2,z2) == 0

Total[Δu2]

(u1,x - u2,x)2 + (u1,y - u2,y)2 + (u1,z - u2,z)2

Δu.Δu

(u1,x - u2,x)2 + (u1,y - u2,y)2 + (u1,z - u2,z)2

Collect[Expand[Δp.Δp - (r1 + r2)2 + 2 Δp.Δu t + t2 Δu.Δu == 0], t]

p1,x2 - 2 p1,x p2,x + p2,x2 + p1,y2 - 2 p1,y p2,y + p2,y2 + p1,z2 - 2 p1,z p2,z + p2,z2 - r12 - 2 r1 r2 -
  r22 + t (2 p1,x u1,x - 2 p2,x u1,x - 2 p1,x u2,x + 2 p2,x u2,x + 2 p1,y u1,y - 2 p2,y u1,y -
    2 p1,y u2,y + 2 p2,y u2,y + 2 p1,z u1,z - 2 p2,z u1,z - 2 p1,z u2,z + 2 p2,z u2,z) +
  t2 (u1,x2 - 2 u1,x u2,x + u2,x2 + u1,y2 - 2 u1,y u2,y + u2,y2 + u1,z2 - 2 u1,z u2,z + u2,z2) == 0

a = Δu.Δu;
b = 2 Δp.Δu;
c = Δp.Δp - (r1 + r2)2;
Collect[Expand[a t2 + b t + c == 0], t]

p1,x2 - 2 p1,x p2,x + p2,x2 + p1,y2 - 2 p1,y p2,y + p2,y2 + p1,z2 - 2 p1,z p2,z + p2,z2 - r12 - 2 r1 r2 -
  r22 + t (2 p1,x u1,x - 2 p2,x u1,x - 2 p1,x u2,x + 2 p2,x u2,x + 2 p1,y u1,y - 2 p2,y u1,y -
    2 p1,y u2,y + 2 p2,y u2,y + 2 p1,z u1,z - 2 p2,z u1,z - 2 p1,z u2,z + 2 p2,z u2,z) +
  t2 (u1,x2 - 2 u1,x u2,x + u2,x2 + u1,y2 - 2 u1,y u2,y + u2,y2 + u1,z2 - 2 u1,z u2,z + u2,z2) == 0

Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t] == Solve[a t2 + b t + c == 0, t]

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The
answer was obtained by solving a corresponding exact system and numericizing the result. >>

True

```