```
(*sphere/sphere collision resolver*)
p_1 = \{p_{1.x}, p_{1.y}, p_{1.z}\}; (*position*)
 u_1 = \{u_{1.x}, u_{1.y}, u_{1.z}\}; (*velocity*)
r<sub>1</sub>; (*radius*)
p_2 = \{p_{2.x}, p_{2.y}, p_{2.z}\};
 u_2 = \{u_{2.x}, u_{2.y}, u_{2.z}\};
r2;
  Solve [EuclideanDistance [p_1 + u_1 t, p_2 + u_2 t] = r_1 + r_2, t]
  \left\{\left\{\,t\,\rightarrow\,\left(\,-\,2\;p_{1\,.\,x}\;u_{1\,.\,x}\,+\,2\;p_{2\,.\,x}\;u_{1\,.\,x}\,+\,2\;p_{1\,.\,x}\;u_{2\,.\,x}\,-\,2\;p_{2\,.\,x}\;u_{2\,.\,x}\,-\,2\;p_{1\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{1\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,+\,2\;p_{2\,.\,y}\;u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,u_{2\,.\,y}\,
                                                         2\;p_{1.\;y}\;u_{2.\;y}-2\;p_{2.\;y}\;u_{2.\;y}-2\;p_{1.\;z}\;u_{1.\;z}+2\;p_{2.\;z}\;u_{1.\;z}+2\;p_{1.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}\;u_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2\;p_{2.\;z}-2
                                                         \sqrt{\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{1,y}}\right)}
                                                                                                                2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                                                                                     4 \left( p_{1.\,x}^2 - 2\,p_{1.\,x}\,p_{2.\,x} + p_{2.\,x}^2 + p_{1.\,y}^2 - 2\,p_{1.\,y}\,p_{2.\,y} + p_{2.\,y}^2 + p_{1.\,z}^2 - 2\,p_{1.\,z}\,p_{2.\,z} + p_{2.\,z}^2 - r_1^2 - 2\,r_1\,r_2 - 2\,r_1^2 + 2\,r_2^2\,p_{2.\,z}^2 + 2\,r_2^2\,p_{2.\,z}^
                                                                                                                r_2^2) \left(u_{1,x}^2 - 2u_{1,x}u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2u_{1,y}u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2u_{1,z}u_{2,z} + u_{2,z}^2\right)\right)
                                       \left(2\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)\right)\right\}
         \left\{t \to \left(-2 \, p_{1 \, . \, x} \, u_{1 \, . \, x} + 2 \, p_{2 \, . \, x} \, u_{1 \, . \, x} + 2 \, p_{1 \, . \, x} \, u_{2 \, . \, x} - 2 \, p_{2 \, . \, x} \, u_{2 \, . \, x} - 2 \, p_{1 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{1 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \, u_{2 \, . \, y} + 2 \, p_{2 \, . \, y} \,
                                                         2 p_{1, y} u_{2, y} - 2 p_{2, y} u_{2, y} - 2 p_{1, z} u_{1, z} + 2 p_{2, z} u_{1, z} + 2 p_{1, z} u_{2, z} - 2 p_{2, z} u_{2, z} +
                                                        \sqrt{\left(2 p_{1,x} u_{1,x} - 2 p_{2,x} u_{1,x} - 2 p_{1,x} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{1,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{1,y} - 2 p_{2,y} u_{2,x} + 2 p_{2,x} u_{2,x} + 2 p_{2,y} u_{2,y} - 
                                                                                                                2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                                                                                    4 \left( p_{1,x}^2 - 2 p_{1,x} p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2 p_{1,y} p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2 p_{1,z} p_{2,z} + p_{2,z}^2 - r_1^2 - 2 r_1 r_2 - 2 r_1 r_2 \right)
                                                                                                                (u_{1,x}^2) (u_{1,x}^2 - 2u_{1,x}u_{2,x} + u_{2,x}^2 + u_{1,y}^2 - 2u_{1,y}u_{2,y} + u_{2,y}^2 + u_{1,z}^2 - 2u_{1,z}u_{2,z} + u_{2,z}^2))
                                       \left\{2\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)\right\}\right\}
 EuclideanDistance [p_1 + u_1 t, p_2 + u_2 t]
  \sqrt{\left(\text{Abs}[p_{1.x} - p_{2.x} + t u_{1.x} - t u_{2.x}]^2 + \right)}
                            Abs \left[ p_{1.y} - p_{2.y} + t u_{1.y} - t u_{2.y} \right]^{2} + Abs \left[ p_{1.z} - p_{2.z} + t u_{1.z} - t u_{2.z} \right]^{2}
 \sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]} == r_1 + r_2
  \sqrt{(p_{1.x} - p_{2.x} + t u_{1.x} - t u_{2.x})^2 + (p_{1.y} - p_{2.y} + t u_{1.y} - t u_{2.y})^2 + (p_{1.z} - p_{2.z} + t u_{1.z} - t u_{2.z})^2} = r_1 + r_2
   \left(\sqrt{\text{Total}[(p_1 + u_1 t - (p_2 + u_2 t))^2]}\right)^2 = (r_1 + r_2)^2
   (p_{1.\,x} - p_{2.\,x} + t\,u_{1.\,x} - t\,u_{2.\,x})^{\,2} + \left(p_{1.\,y} - p_{2.\,y} + t\,u_{1.\,y} - t\,u_{2.\,y}\right)^{\,2} + \left(p_{1.\,z} - p_{2.\,z} + t\,u_{1.\,z} - t\,u_{2.\,z}\right)^{\,2} = (r_1 + r_2)^{\,2} 
 Total [(p_1 + u_1 t - (p_2 + u_2 t))^2] = (r_1 + r_2)^2
  (p_{1.x} - p_{2.x} + t u_{1.x} - t u_{2.x})^2 + (p_{1.y} - p_{2.y} + t u_{1.y} - t u_{2.y})^2 + (p_{1.z} - p_{2.z} + t u_{1.z} - t u_{2.z})^2 = (r_1 + r_2)^2
\Delta p = p_1 - p_2
  \{p_{1.x} - p_{2.x}, p_{1.y} - p_{2.y}, p_{1.z} - p_{2.z}\}
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Collect Expand Total \left[ \left( \Delta p + u_1 t - u_2 t \right)^2 \right] - \left( r_1 + r_2 \right)^2 = 0 \right], t
p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                        r_{2}^{2} + t \left(2\,p_{1.\,x}\,u_{1.\,x} - 2\,p_{2.\,x}\,u_{1.\,x} - 2\,p_{1.\,x}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{1.\,y}\,u_{1.\,y} - 2\,p_{2.\,y}\,u_{1.\,y} - 2\,p_{2.\,y}\,u_{1.\,y} - 2\,p_{2.\,y}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{2.\,x}\,u_{2.\,x} + 2\,p_{2.\,y}\,u_{2.\,x} + 2\,p_{2.\,y}\,u_{2.\,x} + 2\,p_{2.\,y}\,u_{2.\,y} + 2\,
                                                           2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z}
                        t^{2}\left(u_{1,x}^{2}-2 u_{1,x} u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2 u_{1,y} u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2 u_{1,z} u_{2,z}+u_{2,z}^{2}\right)=0
Collect \left[ \text{Expand} \left[ \text{Total} \left[ (\Delta p + (u_1 - u_2) t)^2 \right] - (r_1 + r_2)^2 = 0 \right], t \right]
p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_2^2 - r_1^2 - r_2^2 - r_2^
                      r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                           2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z} + 
                        t^{2}\left(u_{1}^{2} + 2u_{1} + u_{2} + u_{2}^{2} + u_{1}^{2} + u_{1}^{2} + u_{1}^{2} + u_{2}^{2} + u_{1}^{2} + u_{2}^{2} + u_{2}^{2} + u_{1}^{2} + u_{2}^{2} + u_{2}^{2} \right) = 0
\Delta \mathbf{u} = \mathbf{u}_1 - \mathbf{u}_2
   \{u_{1.x} - u_{2.x}, u_{1.y} - u_{2.y}, u_{1.z} - u_{2.z}\}
Collect \left[ \text{Expand} \left[ \text{Total} \left[ (\Delta p + \Delta u t)^2 \right] - (r_1 + r_2)^2 = 0 \right], t \right]
p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                      r_2^2 + t \left( 2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} \right)
                                                           2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                        t^{2}\left(u_{1,x}^{2}-2\,u_{1,x}\,u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2\,u_{1,y}\,u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2\,u_{1,z}\,u_{2,z}+u_{2,z}^{2}\right)=0
Collect Expand Total \left[\Delta p^2 + 2 \Delta p \Delta u t + \Delta u^2 t^2\right] - (r_1 + r_2)^2 = 0, t
p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2
                      r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                           2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                        \mathsf{t}^2 \left( \mathsf{u}^2_{1.\,x} - 2\,\mathsf{u}_{1.\,x}\,\mathsf{u}_{2.\,x} + \mathsf{u}^2_{2.\,x} + \mathsf{u}^2_{1.\,y} - 2\,\mathsf{u}_{1.\,y}\,\mathsf{u}_{2.\,y} + \mathsf{u}^2_{2.\,y} + \mathsf{u}^2_{1.\,z} - 2\,\mathsf{u}_{1.\,z}\,\mathsf{u}_{2.\,z} + \mathsf{u}^2_{2.\,z} \right) \, = \, 0
Collect \left[ \text{Total} \left[ \Delta p^2 + 2 \Delta p \Delta u \, t + \Delta u^2 \, t^2 \right] - (r_1 + r_2)^2 = 0, t \right]
   (p_{1,x}-p_{2,x})^2 + (p_{1,y}-p_{2,y})^2 + (p_{1,z}-p_{2,z})^2 - (r_1+r_2)^2 +
                         \text{t} \, \left( 2 \, \left( p_{1.\,x} - p_{2.\,x} \right) \, \left( u_{1.\,x} - u_{2.\,x} \right) \, + \, 2 \, \left( p_{1.\,y} - p_{2.\,y} \right) \, \left( u_{1.\,y} - u_{2.\,y} \right) \, + \, 2 \, \left( p_{1.\,z} - p_{2.\,z} \right) \, \left( u_{1.\,z} - u_{2.\,z} \right) \right) \, + \, 2 \, \left( p_{2.\,x} - p_{2.\,z} \right) \, \left( u_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - p_{2.\,z} \right) \, \left( u_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - p_{2.\,z} \right) \, \left( u_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z} \right) \, + \, 2 \, \left( p_{2.\,x} - u_{2.\,z}
                        t^{2} \left( (u_{1.x} - u_{2.x})^{2} + (u_{1.y} - u_{2.y})^{2} + (u_{1.z} - u_{2.z})^{2} \right) = 0
Total[\Delta p^2]
    (p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2 + (p_{1,z} - p_{2,z})^2
             \text{Expand} \left[ \text{Total} \left[ \Delta p^2 \right] - (r_1 + r_2)^2 + \text{t} \left( 2 \left( p_{1, \ x} - p_{2, \ x} \right) \left( u_{1, \ x} - u_{2, \ x} \right) + 2 \left( p_{1, \ y} - p_{2, \ y} \right) \left( u_{1, \ y} - u_{2, \ y} \right) + 2 \left( p_{1, \ y} - p_{2, \ y} \right) \right) \right] 
                                                                                  2(p_{1,z} - p_{2,z})(u_{1,z} - u_{2,z}) +
                                                t^{2} \left( \left( u_{1, x} - u_{2, x} \right)^{2} + \left( u_{1, y} - u_{2, y} \right)^{2} + \left( u_{1, z} - u_{2, z} \right)^{2} \right) = 0 \right], t
p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                        r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                             2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z} + 
                        2 Δρ.Δu t
   2 t \left( (p_{1.x} - p_{2.x}) (u_{1.x} - u_{2.x}) + (p_{1.y} - p_{2.y}) (u_{1.y} - u_{2.y}) + (p_{1.z} - p_{2.z}) (u_{1.z} - u_{2.z}) \right)
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Collect [Expand]
                                  Total \left[ \Delta p^2 \right] - (r_1 + r_2)^2 + 2 \Delta p. \Delta u t + t^2 \left( (u_1 \cdot x - u_2 \cdot x)^2 + (u_1 \cdot y - u_2 \cdot y)^2 + (u_1 \cdot z - u_2 \cdot z)^2 \right) = 0 \right], t
p_{1.x}^2 - 2p_{1.x}p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2p_{1.y}p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2p_{1.z}p_{2.z} + p_{2.z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_1^
                                r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_
                                                                                    2\;p_{1.\;y}\;u_{2.\;y}+2\;p_{2.\;y}\;u_{2.\;y}+2\;p_{1.\;z}\;u_{1.\;z}-2\;p_{2.\;z}\;u_{1.\;z}-2\;p_{1.\;z}\;u_{2.\;z}+2\;p_{2.\;z}\;u_{2.\;z}\Big)\;+
                                  t^{2}\left(u_{1,x}^{2}-2\,u_{1,x}\,u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2\,u_{1,y}\,u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2\,u_{1,z}\,u_{2,z}+u_{2,z}^{2}\right) = 0
Total[\Delta u^2]
      (u_{1,x} - u_{2,x})^2 + (u_{1,y} - u_{2,y})^2 + (u_{1,z} - u_{2,z})^2
\texttt{Collect}\big[\texttt{Expand}\big[\texttt{Total}\big[\Delta p^2\big] - (r_1 + r_2)^2 + 2\Delta p.\Delta u \, t + t^2\,\texttt{Total}\big[\Delta u^2\big] = 0\big],\,t\big]
p_{1,x}^2 - 2p_{1,x}p_{2,x} + p_{2,x}^2 + p_{1,y}^2 - 2p_{1,y}p_{2,y} + p_{2,y}^2 + p_{1,z}^2 - 2p_{1,z}p_{2,z} + p_{2,z}^2 - r_1^2 - 2r_1r_2 - r_2^2 - r_1^2 - r_2^2 - r_1^2 - r_2^2 - r_2^
                                r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                                                    2 p_{1,y} u_{2,y} + 2 p_{2,y} u_{2,y} + 2 p_{1,z} u_{1,z} - 2 p_{2,z} u_{1,z} - 2 p_{1,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z} + 2 p_{2,z} u_{2,z}
                                   \mathsf{t}^2 \, \left( \mathsf{u}^2_{1.\,\, \mathsf{x}} \, - \, 2 \, \mathsf{u}_{1.\,\, \mathsf{x}} \, \mathsf{u}_{2.\,\, \mathsf{x}} \, + \, \mathsf{u}^2_{2.\,\, \mathsf{x}} \, + \, \mathsf{u}^2_{1.\,\, \mathsf{y}} \, - \, 2 \, \mathsf{u}_{1.\,\, \mathsf{y}} \, \mathsf{u}_{2.\,\, \mathsf{y}} \, + \, \mathsf{u}^2_{2.\,\, \mathsf{y}} \, + \, \mathsf{u}^2_{1.\,\, \mathsf{z}} \, - \, 2 \, \mathsf{u}_{1.\,\, \mathsf{z}} \, \, \mathsf{u}_{2.\,\, \mathsf{z}} \, + \, \mathsf{u}^2_{2.\,\, \mathsf{z}} \, \right) \, = \, 0 
a = Total[\Delta u^2];
b = 2 \Delta p. \Delta u;
c = Total[\Delta p^2] - (r_1 + r_2)^2;
Collect [Expand[at^2 + bt + c = 0], t]
p_{1.\,x}^2 - 2\,p_{1.\,x}\,p_{2.\,x} + p_{2.\,x}^2 + p_{1.\,y}^2 - 2\,p_{1.\,y}\,p_{2.\,y} + p_{2.\,y}^2 + p_{1.\,z}^2 - 2\,p_{1.\,z}\,p_{2.\,z} + p_{2.\,z}^2 - r_1^2 - 2\,r_1\,r_2 - 2\,r_2^2 + r_2^2\,p_{2.\,z}^2 - r_1^2 - 2\,r_2^2 + r_2^2\,p_{2.\,z}^2 - r_2^2 
                                r_2^2 + t \left(2 p_{1.x} u_{1.x} - 2 p_{2.x} u_{1.x} - 2 p_{1.x} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{1.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{1.y} - 2 p_{2.y} u_{2.x} + 2 p_{2.x} u_{2.x} + 2 p_{2.y} u_{2.y} + 2 p_{2.y} u_
                                                                                      2 p_{1.y} u_{2.y} + 2 p_{2.y} u_{2.y} + 2 p_{1.z} u_{1.z} - 2 p_{2.z} u_{1.z} - 2 p_{1.z} u_{2.z} + 2 p_{2.z} u_{2.z} + + 2 p_{2.z
```

 $t^{2}\left(u_{1,x}^{2}-2u_{1,x}u_{2,x}+u_{2,x}^{2}+u_{1,y}^{2}-2u_{1,y}u_{2,y}+u_{2,y}^{2}+u_{1,z}^{2}-2u_{1,z}u_{2,z}+u_{2,z}^{2}\right)=0$ 

True

Solve[EuclideanDistance[ $p_1 + u_1 t, p_2 + u_2 t$ ] ==  $r_1 + r_2, t$ ] == Solve[at<sup>2</sup> + bt + c == 0, t]