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p1 = {x1, y1, z1};
v1 = {vx1, vy1, vz1};
r1 = 1;
p2 = {x2, y2, z2};
v2 = {vx2, vy2, vz2};
r2 = 1;
abs ((p1 + v1 t) - (p2 + v2 t))

{abs (t vx1 - t vx2 + x1 - x2), abs (t vy1 - t vy2 + y1 - y2), abs (t vz1 - t vz2 + z1 - z2)}

((p1 + v1 t) - (p2 + v2 t)) ^ 2

{(t vx1 - t vx2 + x1 - x2)^2, (t vy1 - t vy2 + y1 - y2)^2, (t vz1 - t vz2 + z1 - z2)^2}
Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]

(t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2
sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]]

sqrt[(t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2]
sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]] == r1 + r2

sqrt[(t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2] == r1 + r2
Solve[sqrt[Total[ ((p1 + v1 t) - (p2 + v2 t)) ^ 2]] == r1 + r2, t]

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

{{t ->
  (-2 vx1 x1 + 2 vx2 x1 + 2 vx1 x2 - 2 vx2 x2 - 2 vy1 y1 + 2 vy2 y1 + 2 vy1 y2 - 2 vy2 y2 - 2 vz1 z1 + 2 vz2
    z1 + 2 vz1 z2 - 2 vz2 z2 - Sqrt[(2 vx1 x1 - 2 vx2 x1 - 2 vx1 x2 + 2 vx2 x2 + 2 vy1 y1 - 2 vy2 y1 - 2
      vy1 y2 + 2 vy2 y2 + 2 vz1 z1 - 2 vz2 z1 - 2 vz1 z2 + 2 vz2 z2)^2 -
      4 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2)
      (x1^2 - 2 x1 x2 + x2^2 + y1^2 - 2 y1 y2 + y2^2 + z1^2 - 2 z1 z2 + z2^2 - Sqrt[(-1) [r1 + r2]])]) /
    (2 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2))},
  {t -> (-2 vx1 x1 + 2 vx2 x1 + 2 vx1 x2 - 2 vx2 x2 - 2 vy1 y1 + 2 vy2 y1 +
    2 vy1 y2 - 2 vy2 y2 - 2 vz1 z1 + 2 vz2 z1 + 2 vz1 z2 - 2 vz2 z2 +
    Sqrt[(2 vx1 x1 - 2 vx2 x1 - 2 vx1 x2 + 2 vx2 x2 + 2 vy1 y1 - 2 vy2 y1 - 2 vy1 y2 +
      2 vy2 y2 + 2 vz1 z1 - 2 vz2 z1 - 2 vz1 z2 + 2 vz2 z2)^2 -
      4 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2)
      (x1^2 - 2 x1 x2 + x2^2 + y1^2 - 2 y1 y2 + y2^2 + z1^2 - 2 z1 z2 + z2^2 - Sqrt[(-1) [r1 + r2]])]) /
    (2 (vx1^2 - 2 vx1 vx2 + vx2^2 + vy1^2 - 2 vy1 vy2 + vy2^2 + vz1^2 - 2 vz1 vz2 + vz2^2))}}

(t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2 == (r1 + r2) ^ 2

(t vx1 - t vx2 + x1 - x2)^2 + (t vy1 - t vy2 + y1 - y2)^2 + (t vz1 - t vz2 + z1 - z2)^2 == (r1 + r2) ^ 2

(t (vx1 - vx2) + x1 - x2)^2 + (t (vy1 - vy2) + y1 - y2)^2 + (t (vz1 - vz2) + z1 - z2)^2 == (r1 + r2) ^ 2

(t (vx1 - vx2) + x1 - x2)^2 + (t (vy1 - vy2) + y1 - y2)^2 + (t (vz1 - vz2) + z1 - z2)^2 == (r1 + r2) ^ 2

dvx = vx1 - vx2;
dvy = vy1 - vy2;
dvz = vz1 - vz2;
dx = x1 - x2;
dy = y1 - y2;
dz = z1 - z2;
r0 = r1 + r2;
(t dvx + dx)^2 + (t dvy + dy)^2 + (t dvz + dz)^2 == r0^2

(t (vx1 - vx2) + x1 - x2)^2 + (t (vy1 - vy2) + y1 - y2)^2 + (t (vz1 - vz2) + z1 - z2)^2 == (r1 + r2) ^ 2

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**Clear**[**dx**, **dvx**, **dy**, **dvy**, **dz**, **dvz**, **r0**]

$$(t \, dvx + dx)^2 + (t \, dvy + dy)^2 + (t \, dvz + dz)^2 == r0^2$$

$$(dx + dvx \, t)^2 + (dy + dvy \, t)^2 + (dz + dvz \, t)^2 == r0^2$$

**Solve**[(**dx** + **dvx** **t**)<sup>2</sup> + (**dy** + **dvy** **t**)<sup>2</sup> + (**dz** + **dvz** **t**)<sup>2</sup> == **r0**<sup>2</sup>, **t**]

$$\left\{ \left\{ t \rightarrow \frac{1}{2 \left( dvx^2 + dvy^2 + dvz^2 \right)} \left( -2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz - \sqrt{\left( (2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz)^2 - 4 \left( dvx^2 + dvy^2 + dvz^2 \right) \left( dx^2 + dy^2 + dz^2 - r0^2 \right) \right)} \right) \right\}, \right.$$

$$\left. \left\{ t \rightarrow \frac{1}{2 \left( dvx^2 + dvy^2 + dvz^2 \right)} \left( -2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz + \sqrt{\left( (2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz)^2 - 4 \left( dvx^2 + dvy^2 + dvz^2 \right) \left( dx^2 + dy^2 + dz^2 - r0^2 \right) \right)} \right) \right\} \right\}$$

**Solve**[**dx**<sup>2</sup> + 2 **dx** **dvx** **t** + (**dvx** **t**)<sup>2</sup> + **dy**<sup>2</sup> + 2 **dy** **dvy** **t** + (**dvy** **t**)<sup>2</sup> + **dz**<sup>2</sup> + 2 **dz** **dvz** **t** + (**dvz** **t**)<sup>2</sup> == **r0**<sup>2</sup>, **t**]

$$\left\{ \left\{ t \rightarrow \frac{1}{2 \left( dvx^2 + dvy^2 + dvz^2 \right)} \left( -2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz - \sqrt{\left( (2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz)^2 - 4 \left( dvx^2 + dvy^2 + dvz^2 \right) \left( dx^2 + dy^2 + dz^2 - r0^2 \right) \right)} \right) \right\}, \right.$$

$$\left. \left\{ t \rightarrow \frac{1}{2 \left( dvx^2 + dvy^2 + dvz^2 \right)} \left( -2 \, dvx \, dx - 2 \, dvy \, dy - 2 \, dvz \, dz + \sqrt{\left( (2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz)^2 - 4 \left( dvx^2 + dvy^2 + dvz^2 \right) \left( dx^2 + dy^2 + dz^2 - r0^2 \right) \right)} \right) \right\} \right\}$$

$$dx^2 + 2 \, dx \, dvx \, t + (dvx \, t)^2 + dy^2 + 2 \, dy \, dvy \, t + (dvy \, t)^2 + dz^2 + 2 \, dz \, dvz \, t + (dvz \, t)^2 == r0^2$$

$$dx^2 + dy^2 + dz^2 + 2 \, dvx \, dx \, t + 2 \, dvy \, dy \, t + 2 \, dvz \, dz \, t + dvx^2 \, t^2 + dvy^2 \, t^2 + dvz^2 \, t^2 == r0^2$$

$$a = dvx^2 + dvy^2 + dvz^2;$$

$$b = 2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz;$$

$$c = dx^2 + dy^2 + dz^2 - r0^2;$$

$$c + b \, t + a \, t^2 == 0$$

$$dx^2 + dy^2 + dz^2 - r0^2 + (2 \, dvx \, dx + 2 \, dvy \, dy + 2 \, dvz \, dz) \, t + (dvx^2 + dvy^2 + dvz^2) \, t^2 == 0$$

**Clear**[**a**, **b**, **c**];

**Solve**[**c** + **b** **t** + **a** **t**<sup>2</sup> == 0, **t**]

$$\left\{ \left\{ t \rightarrow \frac{-b - \sqrt{b^2 - 4 \, a \, c}}{2 \, a} \right\}, \left\{ t \rightarrow \frac{-b + \sqrt{b^2 - 4 \, a \, c}}{2 \, a} \right\} \right\}$$