In[1]:=
```
Clear["*"];
```

In[1]:=
```
(*sphere/sphere collision resolver*)
p1 = {p1.x, p1.y, p1.z};(*position*)
u1 = {u1.x, u1.y, u1.z};(*velocity*)
r1; (*radius*)
p2 = {p2.x, p2.y, p2.z};
u2 = {u2.x, u2.y, u2.z};
r2;
Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t]
```

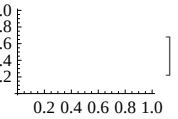
Out[7]=
$$\left\{\left\{t \to \left(-2\,p_{1.x}\,u_{1.x}+2\,p_{2.x}\,u_{1.x}+2\,p_{1.x}\,u_{2.x}-2\,p_{2.x}\,u_{2.x}-2\,p_{1.y}\,u_{1.y}+2\,p_{2.y}\,u_{1.y}+2\,p_{1.y}\,u_{2.y}-2\,p_{2.y}\,u_{2.y}-2\,p_{1.z}\,u_{1.z}+2\,p_{2.z}\,u_{1.z}+2\,p_{1.z}\,u_{2.z}-2\,p_{2.z}\,u_{2.z}-\sqrt{\left(\left(2\,p_{1.x}\,u_{1.x}-2\,p_{2.x}\,u_{1.x}-2\,p_{1.x}\,u_{2.x}+2\,p_{2.x}\,u_{2.x}+2\,p_{1.y}\,u_{1.y}-2\,p_{2.y}\,u_{1.y}-2\,p_{1.y}\,u_{2.y}+2\,p_{2.y}\,u_{2.y}+2\,p_{1.z}\,u_{1.z}-2\,p_{2.z}\,u_{1.z}-2\,p_{1.z}\,u_{2.z}+2\,p_{2.z}\,u_{2.z}\right)^2-4\left(p_{1.x}^2-2\,p_{1.x}\,p_{2.x}+p_{2.x}^2+p_{1.y}^2-2\,p_{1.y}\,p_{2.y}+p_{2.y}^2+p_{1.z}^2-2\,p_{1.z}\,p_{2.z}+p_{2.z}^2-r_1^2-2\,r_1\,r_2-r_2^2\right)\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)}\right)\Big/\left(2\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)\right\},$$
$$\left\{t \to \left(-2\,p_{1.x}\,u_{1.x}+2\,p_{2.x}\,u_{1.x}+2\,p_{1.x}\,u_{2.x}-2\,p_{2.x}\,u_{2.x}-2\,p_{1.y}\,u_{1.y}+2\,p_{2.y}\,u_{1.y}+2\,p_{1.y}\,u_{2.y}-2\,p_{2.y}\,u_{2.y}-2\,p_{1.z}\,u_{1.z}+2\,p_{2.z}\,u_{1.z}+2\,p_{1.z}\,u_{2.z}-2\,p_{2.z}\,u_{2.z}+\sqrt{\left(\left(2\,p_{1.x}\,u_{1.x}-2\,p_{2.x}\,u_{1.x}-2\,p_{1.x}\,u_{2.x}+2\,p_{2.x}\,u_{2.x}+2\,p_{1.y}\,u_{1.y}-2\,p_{2.y}\,u_{1.y}-2\,p_{1.y}\,u_{2.y}+2\,p_{2.y}\,u_{2.y}+2\,p_{1.z}\,u_{1.z}-2\,p_{2.z}\,u_{1.z}-2\,p_{1.z}\,u_{2.z}+2\,p_{2.z}\,u_{2.z}\right)^2-4\left(p_{1.x}^2-2\,p_{1.x}\,p_{2.x}+p_{2.x}^2+p_{1.y}^2-2\,p_{1.y}\,p_{2.y}+p_{2.y}^2+p_{1.z}^2-2\,p_{1.z}\,p_{2.z}+p_{2.z}^2-r_1^2-2\,r_1\,r_2-r_2^2\right)\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)}\right)\Big/\left(2\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)\right\}\right\}$$

In[8]:=
```
(*if overlapping pick nearest t<0, for collision detection check t>0 and t<1*)
np1 = p1 + u1 t;(*move out of collision*)
np2 = p2 + u2 t;
nml = Normalize[np1 - np2];(*collision plane normal*)
m1;(*mass*)
m2;
(*     velocity given    velocities received along collision plane normal*)
v1 = u1 - (u1.nml) nml + (u1.nml) nml ((m1 - m2) / (m1 + m2)) + (u2.nml) nml (2 m2 / (m1 + m2));
v2 = u2 - (u2.nml) nml + (u2.nml) nml ((m2 - m1) / (m1 + m2)) + (u1.nml) nml (2 m1 / (m1 + m2));
np1 = np1 + v1 (1 - t);(*perform remaining dt with new velocities*)
np2 = np2 + v2 (1 - t);
```

In[17]:=
```
v1 = (u1.nml) nml ((m1 - m2) / (m1 + m2)) + (u2.nml) nml (2 m2 / (m1 + m2));
v2 = (u2.nml) nml ((m2 - m1) / (m1 + m2)) + (u1.nml) nml (2 m1 / (m1 + m2));
m1 u1 + m2 u2 == m1 v1 + m2 v2;
```

In[20]:=
```
(*sphere/plane collision resolver*)
p_g = {0, 0, 0};(*dot on plane*)
n_g = Normalize[{0, 1, 0}];(*normal*)
p = {0, 1.5, 0};(*sphere position*)
u = {0, -1, 0};(*velocity*)
r = 1;(*radius*)
Remove[t];
f = (p + u t - p_g).n_g - r == 0;(*signed distance to plane*)
Plot[f, {t, 0, 1}]
Solve[f, t]
t = .5;
(*pick nearest t<0, t>1 or t<0 collision is in future or past*)
np = p + u t (*move sphere out of collision*)
v = u - 2 (u.n_g) n_g(*reflect*)
np = np + v (1 - t)(*second part of dt*)
```

Out[27]= $DisplayFunction



Out[28]= $\{\{t \to -1.\ (-0.5 + 1.\ \{0, 1.5, 0\}_g)\}\}$

Out[30]= {0., 1., 0.}

Out[31]= {0, 1, 0}

Out[32]= {0., 1.5, 0.}

```
(*simplified*)
```

In[34]:=
```
Clear["*"];
```

In[1]:=
```
(*sphere/sphere collision resolver*)
p1 = {p1.x, p1.y, p1.z};(*position*)
u1 = {u1.x, u1.y, u1.z};(*velocity*)
r1; (*radius*)
p2 = {p2.x, p2.y, p2.z};
u2 = {u2.x, u2.y, u2.z};
r2;
Solve[EuclideanDistance[p1 + u1 t, p2 + u2 t] == r1 + r2, t]
```

Out[7]=
$$\left\{\left\{t \to \left(-2\,p_{1.x}\,u_{1.x}+2\,p_{2.x}\,u_{1.x}+2\,p_{1.x}\,u_{2.x}-2\,p_{2.x}\,u_{2.x}-2\,p_{1.y}\,u_{1.y}+2\,p_{2.y}\,u_{1.y}+2\,p_{1.y}\,u_{2.y}-2\,p_{2.y}\,u_{2.y}-2\,p_{1.z}\,u_{1.z}+2\,p_{2.z}\,u_{1.z}+2\,p_{1.z}\,u_{2.z}-2\,p_{2.z}\,u_{2.z}-\sqrt{\left(\left(2\,p_{1.x}\,u_{1.x}-2\,p_{2.x}\,u_{1.x}-2\,p_{1.x}\,u_{2.x}+2\,p_{2.x}\,u_{2.x}+2\,p_{1.y}\,u_{1.y}-2\,p_{2.y}\,u_{1.y}-2\,p_{1.y}\,u_{2.y}+2\,p_{2.y}\,u_{2.y}+2\,p_{1.z}\,u_{1.z}-2\,p_{2.z}\,u_{1.z}-2\,p_{1.z}\,u_{2.z}+2\,p_{2.z}\,u_{2.z}\right)^2-4\left(p_{1.x}^2-2\,p_{1.x}\,p_{2.x}+p_{2.x}^2+p_{1.y}^2-2\,p_{1.y}\,p_{2.y}+p_{2.y}^2+p_{1.z}^2-2\,p_{1.z}\,p_{2.z}+p_{2.z}^2-r_1^2-2\,r_1\,r_2-r_2^2\right)\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)}\right)\Big/\left(2\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)\right\},$$
$$\left\{t \to \left(-2\,p_{1.x}\,u_{1.x}+2\,p_{2.x}\,u_{1.x}+2\,p_{1.x}\,u_{2.x}-2\,p_{2.x}\,u_{2.x}-2\,p_{1.y}\,u_{1.y}+2\,p_{2.y}\,u_{1.y}+2\,p_{1.y}\,u_{2.y}-2\,p_{2.y}\,u_{2.y}-2\,p_{1.z}\,u_{1.z}+2\,p_{2.z}\,u_{1.z}+2\,p_{1.z}\,u_{2.z}-2\,p_{2.z}\,u_{2.z}+\sqrt{\left(\left(2\,p_{1.x}\,u_{1.x}-2\,p_{2.x}\,u_{1.x}-2\,p_{1.x}\,u_{2.x}+2\,p_{2.x}\,u_{2.x}+2\,p_{1.y}\,u_{1.y}-2\,p_{2.y}\,u_{1.y}-2\,p_{1.y}\,u_{2.y}+2\,p_{2.y}\,u_{2.y}+2\,p_{1.z}\,u_{1.z}-2\,p_{2.z}\,u_{1.z}-2\,p_{1.z}\,u_{2.z}+2\,p_{2.z}\,u_{2.z}\right)^2-4\left(p_{1.x}^2-2\,p_{1.x}\,p_{2.x}+p_{2.x}^2+p_{1.y}^2-2\,p_{1.y}\,p_{2.y}+p_{2.y}^2+p_{1.z}^2-2\,p_{1.z}\,p_{2.z}+p_{2.z}^2-r_1^2-2\,r_1\,r_2-r_2^2\right)\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)}\right)\Big/\left(2\left(u_{1.x}^2-2\,u_{1.x}\,u_{2.x}+u_{2.x}^2+u_{1.y}^2-2\,u_{1.y}\,u_{2.y}+u_{2.y}^2+u_{1.z}^2-2\,u_{1.z}\,u_{2.z}+u_{2.z}^2\right)\right)\right\}\right\}$$

In[8]:=
```
EuclideanDistance[p1 + u1 t, p2 + u2 t]
```

Out[8]=
$$\sqrt{\left(\text{Abs}\left[p_{1.x}-p_{2.x}+t\,u_{1.x}-t\,u_{2.x}\right]^2+\text{Abs}\left[p_{1.y}-p_{2.y}+t\,u_{1.y}-t\,u_{2.y}\right]^2+\text{Abs}\left[p_{1.z}-p_{2.z}+t\,u_{1.z}-t\,u_{2.z}\right]^2\right)}$$

In[9]:=
```
Sqrt[Total[(p1 + u1 t - (p2 + u2 t))^2]] == r1 + r2
```

Out[9]=
$$\sqrt{\left(p_{1.x}-p_{2.x}+t\,u_{1.x}-t\,u_{2.x}\right)^2+\left(p_{1.y}-p_{2.y}+t\,u_{1.y}-t\,u_{2.y}\right)^2+\left(p_{1.z}-p_{2.z}+t\,u_{1.z}-t\,u_{2.z}\right)^2}==r_1+r_2$$

In[10]:=
```
(Sqrt[Total[(p1 + u1 t - (p2 + u2 t))^2]])^2 == (r1 + r2)^2
```

Out[10]=
$$\left(p_{1.x}-p_{2.x}+t\,u_{1.x}-t\,u_{2.x}\right)^2+\left(p_{1.y}-p_{2.y}+t\,u_{1.y}-t\,u_{2.y}\right)^2+\left(p_{1.z}-p_{2.z}+t\,u_{1.z}-t\,u_{2.z}\right)^2==\left(r_1+r_2\right)^2$$

In[11]:=
```
Total[(p1 + u1 t - (p2 + u2 t))^2] == (r1 + r2)^2
```

Out[11]=
$$\left(p_{1.x}-p_{2.x}+t\,u_{1.x}-t\,u_{2.x}\right)^2+\left(p_{1.y}-p_{2.y}+t\,u_{1.y}-t\,u_{2.y}\right)^2+\left(p_{1.z}-p_{2.z}+t\,u_{1.z}-t\,u_{2.z}\right)^2==\left(r_1+r_2\right)^2$$

In[12]:=
```
Δp = p1 - p2
```

Out[12]=
$$\left\{p_{1.x}-p_{2.x},\ p_{1.y}-p_{2.y},\ p_{1.z}-p_{2.z}\right\}$$

In[13]:=
```
Collect[Expand[Total[(Δp + u1 t - u2 t)^2] - (r1 + r2)^2 == 0], t]
```

Out[13]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

In[14]:=
```
Collect[Expand[Total[(Δp + (u1 - u2) t)^2] - (r1 + r2)^2 == 0], t]
```

Out[14]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

In[15]:=
```
Δu = u1 - u2
```

Out[15]=
$$\{u_{1.x} - u_{2.x},\ u_{1.y} - u_{2.y},\ u_{1.z} - u_{2.z}\}$$

In[16]:=
```
Collect[Expand[Total[(Δp + Δu t)^2] - (r1 + r2)^2 == 0], t]
```

Out[16]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

In[17]:=
```
Collect[Expand[Total[Δp^2 + 2 Δp Δu t + Δu^2 t^2] - (r1 + r2)^2 == 0], t]
```

Out[17]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

In[18]:=
```
Collect[Total[Δp^2 + 2 Δp Δu t + Δu^2 t^2] - (r1 + r2)^2 == 0, t]
```

Out[18]=
$$(p_{1.x} - p_{2.x})^2 + (p_{1.y} - p_{2.y})^2 + (p_{1.z} - p_{2.z})^2 - (r_1 + r_2)^2 + t\,\left(2\,(p_{1.x} - p_{2.x})\,(u_{1.x} - u_{2.x}) + 2\,(p_{1.y} - p_{2.y})\,(u_{1.y} - u_{2.y}) + 2\,(p_{1.z} - p_{2.z})\,(u_{1.z} - u_{2.z})\right) + t^2\,\left((u_{1.x} - u_{2.x})^2 + (u_{1.y} - u_{2.y})^2 + (u_{1.z} - u_{2.z})^2\right) == 0$$

In[19]:=
```
Total[Δp^2]
```

Out[19]=
$$(p_{1.x} - p_{2.x})^2 + (p_{1.y} - p_{2.y})^2 + (p_{1.z} - p_{2.z})^2$$

In[20]:=
```
Collect[
 Expand[Total[Δp^2] - (r1 + r2)^2 + t (2 (p1.`x - p2.`x) (u1.`x - u2.`x) + 2 (p1.`y - p2.`y) (u1.`y - u2.`y) +
       2 (p1.`z - p2.`z) (u1.`z - u2.`z)) +
   t^2 ((u1.`x - u2.`x)^2 + (u1.`y - u2.`y)^2 + (u1.`z - u2.`z)^2) == 0], t]
```

Out[20]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,\left(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right) + t^2\,\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right) == 0$$

In[21]:=
```
2 Δp.Δu t
```

Out[21]=
$$2\,t\,\left((p_{1.x} - p_{2.x})\,(u_{1.x} - u_{2.x}) + (p_{1.y} - p_{2.y})\,(u_{1.y} - u_{2.y}) + (p_{1.z} - p_{2.z})\,(u_{1.z} - u_{2.z})\right)$$

In[22]:=
```
Collect[Expand[
  Total[Δp^2] - (r1 + r2)^2 + 2 Δp.Δu t + t^2 ((u1.`x - u2.`x)^2 + (u1.`y - u2.`y)^2 + (u1.`z - u2.`z)^2) == 0], t]
```

Out[22]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}) + t^2\,(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2) == 0$$

In[23]:=
```
Total[Δu^2]
```

Out[23]=
$$(u_{1.x} - u_{2.x})^2 + (u_{1.y} - u_{2.y})^2 + (u_{1.z} - u_{2.z})^2$$

In[24]:=
```
Collect[Expand[Total[Δp^2] - (r1 + r2)^2 + 2 Δp.Δu t + t^2 Total[Δu^2] == 0], t]
```

Out[24]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}) + t^2\,(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2) == 0$$

In[29]:=
```
a = Total[Δu^2];
b = 2 Δp.Δu;
c = Total[Δp^2] - (r1 + r2)^2;
Collect[Expand[a t^2 + b t + c == 0], t]
Solve[a t^2 + b t + c == 0, t]
```

Out[32]=
$$p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2 + t\,(2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}) + t^2\,(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2) == 0$$

Out[33]=
$$\left\{\left\{t \to \left(-2\,p_{1.x}\,u_{1.x} + 2\,p_{2.x}\,u_{1.x} + 2\,p_{1.x}\,u_{2.x} - 2\,p_{2.x}\,u_{2.x} - 2\,p_{1.y}\,u_{1.y} + 2\,p_{2.y}\,u_{1.y} + 2\,p_{1.y}\,u_{2.y} - 2\,p_{2.y}\,u_{2.y} - 2\,p_{1.z}\,u_{1.z} + 2\,p_{2.z}\,u_{1.z} + 2\,p_{1.z}\,u_{2.z} - 2\,p_{2.z}\,u_{2.z} - \sqrt{\left((2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right)^2 - 4\left(p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2\right)\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)}\right) \Big/ \left(2\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)\right\},$$

$$\left\{t \to \left(-2\,p_{1.x}\,u_{1.x} + 2\,p_{2.x}\,u_{1.x} + 2\,p_{1.x}\,u_{2.x} - 2\,p_{2.x}\,u_{2.x} - 2\,p_{1.y}\,u_{1.y} + 2\,p_{2.y}\,u_{1.y} + 2\,p_{1.y}\,u_{2.y} - 2\,p_{2.y}\,u_{2.y} - 2\,p_{1.z}\,u_{1.z} + 2\,p_{2.z}\,u_{1.z} + 2\,p_{1.z}\,u_{2.z} - 2\,p_{2.z}\,u_{2.z} + \sqrt{\left((2\,p_{1.x}\,u_{1.x} - 2\,p_{2.x}\,u_{1.x} - 2\,p_{1.x}\,u_{2.x} + 2\,p_{2.x}\,u_{2.x} + 2\,p_{1.y}\,u_{1.y} - 2\,p_{2.y}\,u_{1.y} - 2\,p_{1.y}\,u_{2.y} + 2\,p_{2.y}\,u_{2.y} + 2\,p_{1.z}\,u_{1.z} - 2\,p_{2.z}\,u_{1.z} - 2\,p_{1.z}\,u_{2.z} + 2\,p_{2.z}\,u_{2.z}\right)^2 - 4\left(p_{1.x}^2 - 2\,p_{1.x}\,p_{2.x} + p_{2.x}^2 + p_{1.y}^2 - 2\,p_{1.y}\,p_{2.y} + p_{2.y}^2 + p_{1.z}^2 - 2\,p_{1.z}\,p_{2.z} + p_{2.z}^2 - r_1^2 - 2\,r_1\,r_2 - r_2^2\right)\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)}\right) \Big/ \left(2\left(u_{1.x}^2 - 2\,u_{1.x}\,u_{2.x} + u_{2.x}^2 + u_{1.y}^2 - 2\,u_{1.y}\,u_{2.y} + u_{2.y}^2 + u_{1.z}^2 - 2\,u_{1.z}\,u_{2.z} + u_{2.z}^2\right)\right)\right\}\right\}$$