AREA OF CONCENTRATION...

LOGARITHM.

Mr. Notty (4)

HINT: (1) whenever we wish to find a variable and this variable is an index/power, we introduce to Logarithm function to bring down the power before solving for x.

(2) Recall (i) $\log_y x = \dfrac{\log x}{\log y}$ where necessary

Solve the following equations

a. $4^{2x+1} = 3^x$ (2016/2017) Subjective

b. $2^{x+1} = 4^x$ (2015/2016)

c. $3^{2x+1} = 5^x$ (A. $\dfrac{-1}{2+\log_3 5}$ B. $\dfrac{1}{2+\log_3 5}$ C. $\dfrac{-1}{1-\log_3 5}$ D. $\dfrac{-1}{2-\log_3 5}$)

d. $2^{2x} - 2^{x+3} + 15 = 0$

(A. $\log_2 3, -\log_2 5$ B. 2, 7.5 C. $\log_2 3, \log_2 5$ D. $-\log_2 3, \log_2 5$)

SOLUTIONS

(a) $4^{2x+1} = 3^x$ (Take Log on bothsides)

$\log 4^{2x+1} = \log 3^x$

$(2x+1)\log 4 = x\log 3 \Rightarrow 2x\log 4 + \log 4 = x\log 3$

$\log 4 = x\log 3 - 2x\log 4$ (factor out x on R.H.S)

$\log 4 = x(\log 3 - 2\log 4)$

$$\therefore x = \frac{\log 4}{\log 3 - 2\log 4}$$ (divide both numerator and denominator by Log4)

$$\Rightarrow x = \frac{\log 4/\log 4}{\dfrac{\log 3}{\log 4} - \dfrac{2\log 4}{\log 4}} = \frac{1}{\log_4 3 - 2}$$

$$\therefore x = \frac{1}{\log_4 3 - 2} \text{ or } \frac{-1}{2 - \log_4 3}$$

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(b) $2^{x+1} = 4^{x}$ (Since we can write 4 in base 2, we proceed writing the L.H.S and R.H.S in the same base)

$\therefore 2^{x+1} = (2^2)^x$

$\cancel{2}^{x+1} = \cancel{2}^{2x} \Rightarrow x+1 = 2x \Rightarrow 2x - x = 1$

$\Rightarrow x = 1$

(c) $3^{2x+1} = 5^{x}$ (Since we cannot write 5 in base 3 and vice-versa, we take Log on both sides)

$\therefore \operatorname{Log} 3^{2x+1} = \operatorname{Log} 5^{x}$

$(2x+1)\operatorname{Log}3 = x\operatorname{Log}5 \Rightarrow 2x\operatorname{Log}3 + \operatorname{Log}3 = x\operatorname{Log}5$

$\operatorname{Log}3 = x\operatorname{Log}5 - 2x\operatorname{Log}3 \Rightarrow \operatorname{Log}3 = x(\operatorname{Log}5 - 2\operatorname{Log}3)$

$\therefore x = \dfrac{\operatorname{Log}3}{\operatorname{Log}5 - 2\operatorname{Log}3}$ ——— ⊛

Note: Eqn⊛ above can lead us to various forms of solutions for x.

Form 1: $x = \dfrac{\operatorname{Log}3}{\operatorname{Log}5 - 2\operatorname{Log}3} = \dfrac{\operatorname{Log}3}{\operatorname{Log}5 - \operatorname{Log}9} = \dfrac{\operatorname{Log}3}{\operatorname{Log}5/9}$

recall: $\dfrac{\operatorname{Log}x}{\operatorname{Log}y} = \operatorname{Log}\frac{x}{y}$

$\therefore x = \dfrac{\operatorname{Log}3}{\operatorname{Log}5/9} = \operatorname{Log}\frac{3}{5/9}$

Form 2: $x = \dfrac{\operatorname{Log}3}{\operatorname{Log}5 - 2\operatorname{Log}3}$ (Since Log3 is found at the numerator, we divide both the numerator and denominator by Log3)

$\therefore x = \dfrac{\operatorname{Log}3/\operatorname{Log}3}{\frac{\operatorname{Log}5}{\operatorname{Log}3} - \frac{2\operatorname{Log}3}{\operatorname{Log}3}} = \dfrac{1}{\operatorname{Log}\frac{5}{3} - 2}$

$\therefore x = \dfrac{1}{\operatorname{Log}\frac{5}{3} - 2}$ or $\dfrac{-1}{2 - \operatorname{Log}\frac{5}{3}}$

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(d) $2^{2x} - 2^{x+3} + 15 = 0$

Hint: clearly, we see that we can rewrite the 1st and 2nd terms of the expression on the left hand side of the equation in terms of 2^x by exploring the idea of indices.

$\therefore (2^x)^2 - 2^x \cdot 2^3 + 15 = 0 \Rightarrow (2^x)^2 - (2^x) \cdot 8 + 15 = 0$ ——— (*)

Let $2^x = p$; eqn (*) becomes

$p^2 - 8p + 15 = 0 \Rightarrow p^2 - 3p - 5p + 15 = 0$

$\Rightarrow p(p-3) - 5(p-3) = 0 \Rightarrow (p-3)(p-5) = 0$

$\Rightarrow p - 3 = 0$ or $p - 5 = 0 \Rightarrow p = 3$ or $p = 5$

Recall that $2^x = p$

$\therefore$ When $p = 3 \Rightarrow 2^x = 3 \Rightarrow \log 2^x = \log 3$

$\Rightarrow x \log 2 = \log 3 \Rightarrow x = \frac{\log 3}{\log 2} = \log_2 3$

When $p = 5 \Rightarrow 2^x = 5 \Rightarrow \log 2^x = \log 5$

$\Rightarrow x \log 2 = \log 5 \Rightarrow x = \frac{\log 5}{\log 2} = \log_2 5$

$\therefore x = \log_2 3$ or $\log_2 5$

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SURDS:

HINT: The square root of a surd is obtained by

i - equating the square root of the surd to $\sqrt{x} \pm \sqrt{y}$ as the case may be.

ii - Next, we square bothsides of (i) above where the L.H.S reduces to the expression without a square root symbol, and the R.H.S transforms into $x + y \pm 2\sqrt{xy}$ as the case may be.

iii - Compare the L.H.S and R.H.S of (ii) above to have an equation of the form:

$x + y = a$

$2\sqrt{xy} = b\sqrt{c}$

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iv- The system of equation deduced from (iii) above is evaluated using the ~~Simultaneous~~ Simultaneous equation to find the values of x and y.

v. Always remember that $x > y$ (the value of x is always greater than the value of y)

Find the square root of the following:

(a) $4+2\sqrt{3}$ A. $1+\sqrt{3}$ B. $3-\sqrt{3}$ C. $2-\sqrt{3}$ D. $\sqrt{3}-1$

(b) $3+2\sqrt{3}$ A. $3-\sqrt{2}$ B. $2-\sqrt{2}$ C. $1+\sqrt{2}$ D. $\sqrt{2}-1$

(c) $5-2\sqrt{2}$ (subjective)

(d) $3-2\sqrt{2}$ A. $\sqrt{3}-\sqrt{2}$ B. $\sqrt{2}-1$ C. $1-\sqrt{2}$ D. $\sqrt{3}+\sqrt{2}$

(e) $4-2\sqrt{3}$ A. $\sqrt{3}-1$ B. $1-\sqrt{3}$ C. $2-\sqrt{3}$ D. $4-\sqrt{3}$

SOLUTIONS

(a) Let $\sqrt{4+2\sqrt{3}} = \sqrt{x}+\sqrt{y}$

Square bothsides above

$(\sqrt{4+2\sqrt{3}})^2 = (\sqrt{x}+\sqrt{y})^2$

$\Rightarrow 4+2\sqrt{3} = (\sqrt{x}+\sqrt{y})(\sqrt{x}+\sqrt{y})$

$4+2\sqrt{3} = x+\sqrt{xy}+\sqrt{xy}+y$

$4+2\sqrt{3} = x+y+2\sqrt{xy}$

Comparing bothsides

$x+y = 4$ ——— (i)

$\cancel{2}\sqrt{xy} = \cancel{2}\sqrt{3}$

$\therefore xy = 3$

$y = 3/x$ ——— (ii)

put $y = 3/x$ into (i)

$\therefore x + \frac{3}{x} = 4$

multiply bothsides by x

$x\left(x+\frac{3}{x}\right) = 4x$

$x^2 + 3 = 4x$

$x^2 - 4x + 3 = 0$

$x^2 - x - 3x + 3 = 0$

$x(x-1) - 3(x-1) = 0$

$(x-1)(x-3) = 0$

$x-1 = 0$ or $x-3 = 0$

$x = 1$ or $x = 3$

But $y = 3/x$

$\therefore$ when $x = 1$

$y = 3/1 = 3$

when $x = 3$

$y = 3/3 = 1$

$\therefore x=1, y=3$ or $x=3, y=1$

Since $x > y \Rightarrow x=3, y=1$

$\therefore \sqrt{4+2\sqrt{3}} = \sqrt{3}+\sqrt{1}$

$= \sqrt{3}+1$ or $1+\sqrt{3}$

2017/2016 EXAMS SOLUTION

① A standard football team consist of 11 players. Therefore, we wish to select 9 players (since 2 particular players must be in the team) out of 11 players.

using $\boxed{{}^{n-k}C_{r-k}}$ $n = 13,\ r = 11,\ k = 2$

$\therefore\ {}^{13-2}C_{11-2} = {}^{11}C_9 = \dfrac{11!}{(11-9)!\,9!} = \dfrac{11 \cdot \cancel{10}^{5} \cdot \cancel{9!}}{\cancel{2}! \cdot \cancel{9!}} = 55$ ways (D)

2) $x^4 - 2x^3 + 3x^2 - 4x + 2 = x^4 - 2x^3 + (x^2 + 2x^2) - 4x + 2$

$= (x^4 - 2x^3 + x^2) + (2x^2 - 4x + 2) = x^2(x^2 - 2x + 1) + 2(x^2 - 2x + 1)$

$= (x^2+2)(x^2 - 2x + 1) = (x^2+2)(x-1)^2$

$\therefore\ \dfrac{x^2+x-1}{x^4 - 2x^3 + 3x^2 - 4x + 2} = \dfrac{x^2+x-1}{(x^2+2)(x-1)^2} = \dfrac{Ax+B}{x^2+2} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2}$

$x^2 + x - 1 = (Ax+B)(x-1)^2 + C(x^2+2)(x-1) + D(x^2+2)$

Let $x = 1$

$1^2 + \cancel{1} - \cancel{1} = D(1^2+2) = 3D$

$\therefore\ 1 = 3D \Rightarrow D = \frac{1}{3}$

Let $x = 0$

$0^2 + 0 - 1 = (A(0)+B)(0-1)^2 + C(0^2+2)(0-1) + D(0^2+2)$

$-1 = B - 2C + 2D$

$\therefore\ 2C - B = 2D + 1 = 2(\frac{1}{3}) + 1 = \frac{2}{3} + 1 = 5/3$

$2C - B = \frac{5}{3}$ ——— (i)

Let $x = -1$

$(-1)^2 + (-1) - 1 = (A(-1)+B)(-1-1)^2 + C((-1)^2+2)(-1-1) + D((-1)^2+2)$

$1 - 1 - 1 = -4A + 4B - 6C + 3D$

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$$-1 = -4A + 4B - 6C + 3\left(\tfrac{1}{3}\right) = -4A + 4B - 6C + 1$$

$$\therefore 4A - 4B + 6C = 2 \Rightarrow 2A - 2B + 3C = 1 \quad \text{--- (ii)}$$

Let $x = 2$

$$2^2 + 2 - 1 = (A(2) + B)(2-1)^2 + C(2^2+2)(2-1) + D(2^2+2)$$

$$5 = 2A + B + 6C + 5D \Rightarrow 5 = 2A + B + 6C + 5\left(\tfrac{1}{3}\right)$$

$$5 = 2A + B + 6C + \tfrac{5}{3} \Rightarrow 2A + B + 6C = 5 - \tfrac{5}{3} = \tfrac{10}{3}$$

$$2A + B + 6C = \frac{10}{3} \Rightarrow 6A + 3B + 18C = 10 \quad \text{--- (iii)}$$

From (i) $2C - B = \frac{5}{3} \Rightarrow 6C - 3B = 5 \Rightarrow 3B = 6C - 5$ --- (iv)

put (iv) into (iii) : $6A + (6C - 5) + 18C = 10$

$$6A + 24C = 15 \Rightarrow 2A + 8C = 5 \Rightarrow 2A = 5 - 8C \quad \text{--- (v)}$$

put (v) into (ii) : $(5 - 8C) - 2B + 3C = 1$

$$-2B - 5C = -4 \Rightarrow 2B + 5C = 4 \quad \left(\text{But } B = 2C - \tfrac{5}{3}\right)$$

$$\Rightarrow 2\left(2C - \frac{5}{3}\right) + 5C = 4 \Rightarrow 2\left(\frac{6C - 5}{3}\right) + 5C = 4$$

$$\Rightarrow \frac{12C - 10}{3} + 5C = 4 \Rightarrow 12C - 10 + 15C = 12$$

$$27C = 22$$

③ $6x^3 + 11x^2 - 3x - 2$ (Testing $x = -2$)

$\therefore 6(-2)^3 + 11(-2)^2 - 3(-2) - 2 = -48 + 44 + 6 - 2 = 0$

$\therefore x = -2$ is a root and $x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array} \Rightarrow \frac{6x^3+11x^2-3x-2}{x+2} = 6x^2 - x - 1$$

$6x^2 - x - 1 = 6x^2 - 3x + 2x - 1 = 0$

$3x(2x-1) + 1(2x-1) = 0 \quad (2x-1)(3x+1)$

$\therefore$ The factors are $(x+2)(2x-1)(3x+1)$ ⓒ

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④ $x \in [(A \cap B) - C] \Rightarrow x \in [(A \cap B) \cap C^c]$

$\Rightarrow x \in (A \cap B) \cap x \in C^c$

$\Rightarrow x \in A \cap x \in B \cap x \in C^c$

$\Rightarrow x \in A$ and $x \in B$ and $x \in C^c$ Ⓑ

⑤ $\frac{1}{1-2x} = (1-2x)^{-1}$

⑫ The coefficient of x^5 will occur at the 6th term:

$\frac{n(n-1)(n-2)(n-3)(n-4)}{5!} x^5 = \frac{-1(-2)(-3)(-4)(-5)}{5!}(-2x)^5$

$= \frac{-120}{120} \cdot -2^5 x^5 = \frac{-120}{120} \times -32 \times x^5$

$= 32x^5 = 32$ Ⓐ

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⑥ $\left(\frac{3n^3+2}{n-5n^3}\right)_{n=1}^{\infty} = \underline{\frac{-3}{5}}$ ⑧

⑦ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$n = 20,\ S_n = 200;\ a = 40$

$\therefore 200 = \frac{20}{2}\left[2(40) + (20-1)d\right] \Rightarrow 200 = 10\left[80 + 19d\right]$

$\Rightarrow 20 = 80 + 19d \Rightarrow 19d = -60 \Rightarrow d = \underline{\frac{-60}{19}}$

⑧ $S_n = \frac{a(r^n - 1)}{r-1}$

$;\ n = 10,\ r = 2,\ a = 5$

$S_n = \frac{5(2^{10}-1)}{2-1} = \frac{5(2^{10}-1)}{1}$

$S_n = 5(1024-1) = 5 \times 1023$

$S_n = \underline{5115}$ ⓒ

⑨ $S_\infty = \frac{a}{1-r}$

$a = 2,\ r = \frac{19}{20}$

$S_\infty = \frac{2}{1-\frac{19}{20}} = \frac{2}{1/20} = 40$

$\therefore S_\infty = \underline{40}$ ⓒ

(10) $2\sin^2\theta - 3\sin\theta + 1 = 0$

Let $\sin\theta = t$

$2t^2 - 3t + 1 = 0 \Rightarrow 2t^2 - 2t - t + 1 = 0 \Rightarrow (2t-1)(t-1) = 0$

$2t - 1 = 0$ or $t - 1 = 0 \Rightarrow t = \frac{1}{2}$ or $t = 1$.

Recall $\sin\theta = t$

when $t = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ, 150^\circ$

when $t = 1$

$\sin\theta = 1$

$\theta = 90^\circ$

$\therefore \theta = \underline{30^\circ, 90^\circ, 150}$ (D)

(11) Let the disc be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

Suppose an odd no is picked, then the no of ways of picking any odd no btw 1 to 11 = 6 ways

The outcomes obtained when a coin is tossed = 2ways

$\therefore 6 \times 2 = 12$ ways

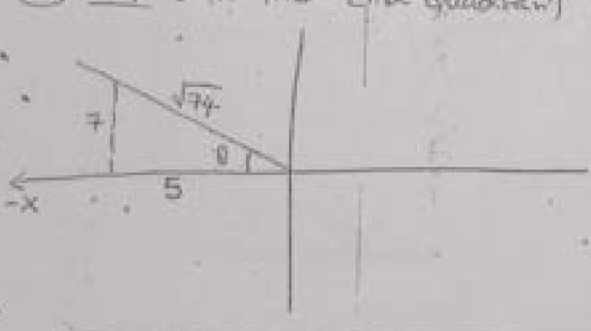
or Suppose an even no is picked, then the no of ways of picking any even no btw 1 to 11 = 5 ways

The outcomes obtained when a die is thrown = 6ways

$\therefore 6 \times 5 = 30$ ways

Hence, the total outcome = 12 + 30 = 42 ways (A)

(12) Note: In the 2nd Quadrant $\tan\theta$ is negative.



$H^2 = O^2 + A^2$ (Pythagoras thm).

$= 7^2 + 5^2$

$H = \sqrt{74}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{74}}{-5} = -\frac{\sqrt{74}}{5}$ (B)

(7)

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(13) $\tan^2 x\left(1+\cot^2 x\right) = \tan^2 x\left(1+\frac{1}{\tan^2 x}\right)$

$\tan^2 x + \cancel{\tan^2 x}\cdot\frac{1}{\cancel{\tan^2 x}} = \tan^2 x + 1$

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But $\tan^2 x + 1 = \sec^2 x = \frac{1}{\cos^2 x} = \frac{1}{1-\sin^2 x}$ (A)

(14) $\frac{1}{0.99} = (0.99)^{-1} = (1-0.01)^{-1}$

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$= 1 + (-1)(-0.01) + \frac{(-1)(-1-1)}{2!}(-0.01)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}(-0.01)^3 + \cdots$

$= 1 + 0.01 + \frac{2}{2\times1}(0.0001) + \frac{-6}{3\times2\times1}(-0.000001) + \cdots$

$= 1 + 0.01 + 0.0001 + 0.000001 + \cdots$

$= \underline{1.010101}$ (D)

(15) $\frac{4}{3}\cos^{-1}\left(\frac{y}{4}\right) = \pi \Rightarrow \cos^{-1}\left(\frac{y}{4}\right) = \frac{3\pi}{4}$

$\therefore \frac{y}{4} = \cos\left(\frac{3\pi}{4}\right) = \cos\left(\frac{3\times180}{4}\right) = \cos(135^\circ)$

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$\therefore \cos 135^\circ \equiv \cos 45^\circ$

Since cos is negative in 2nd quadrant

$\therefore \cos 135^\circ = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$

$\therefore \frac{y}{4} = \frac{-1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow y = \frac{-4\sqrt{2}}{2} = \underline{-2\sqrt{2}}$ (C)

(16) $\cos 3\theta = \cos(2\theta+\theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$= (\cos^2\theta - \sin^2\theta)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3\sin^2\theta\cos\theta$ (B)

(17) Let α^2+1 and β^2+1 be the roots of $x^2-4x+5=0$

$\therefore a=1,\ b=-4,\ c=5$

$\alpha^2+1+\beta^2+1 = \frac{-b}{a} = \frac{4}{1} = 4$

$\alpha^2+\beta^2+2 = 4$

$\alpha^2+\beta^2 = 2$ —— (i)

$(\alpha^2+1)(\beta^2+1) = \frac{c}{a} = \frac{5}{1} = 5$

$\alpha^2\beta^2+\alpha^2+\beta^2+1 = 5$

$(\alpha\beta)^2 + (\alpha^2+\beta^2) = 4 \Rightarrow (\alpha\beta)^2+2=4$

$(\alpha\beta)^2 = 2 \Rightarrow \alpha\beta = \pm\sqrt{2}$

from (i). $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 2$

$\Rightarrow (\alpha+\beta)^2 - (2\cdot\pm\sqrt{2}) = 2 \Rightarrow (\alpha+\beta)^2 \mp 2\sqrt{2} = 2$

$(\alpha+\beta)^2 = 2\pm 2\sqrt{2} \Rightarrow \alpha+\beta = \pm\sqrt{2\pm 2\sqrt{2}}$

$\therefore x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - (\pm\sqrt{2\pm 2\sqrt{2}})x + (\pm\sqrt{2}) = 0$

$x^2 \mp x(\sqrt{2\pm 2\sqrt{2}}) \pm \sqrt{2} = 0$

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(18) $[(A^c \cup B) \cap (B \cup C^c)]^c \Rightarrow (A^c \cup B)^c \cup (B \cup C^c)^c$

$\Rightarrow (A \cap B^c) \cup (B^c \cap C) \Rightarrow (B^c \cap A) \cup (B^c \cap C)$

$\Rightarrow \underline{B^c \cap (A \cup C)}$

(19) $2 \cdot 3^{2x} - 3^{x+1} + 1 = 0 \Rightarrow 2 \cdot (3^x)^2 - 3^x \cdot 3 + 1 = 0$

Let $3^x = t \Rightarrow 2t^2 - 3t + 1 = 0$

$2t^2 - 2t - t + 1 = 0 \Rightarrow 2t(t-1) - 1(t-1) = 0$

$(t-1)(2t-1) = 0 \Rightarrow t = 1$ or $t = \frac{1}{2}$

when $t = 1$

$3^x = 1$

$3^x = 3^0$

$x = 0$

when $t = \frac{1}{2}$

$3^x = \frac{1}{2}$

$3^x = 2^{-1}$

$x \log 3 = \log \frac{1}{2}$

$x = \dfrac{\log \frac{1}{2}}{\log 3}$

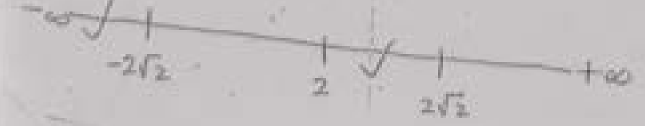
$x = \log_3 \frac{1}{2}$

$\therefore \underline{x = 0 \text{ or } x = \log_3 \frac{1}{2}}$ (B)

(20) $\frac{2x}{x-2} > x + 4 \Rightarrow \frac{2x}{x-2} - x - 4 > 0$

$\frac{2x - x(x-2) - 4(x-2)}{x-2} > 0 \Rightarrow \frac{\cancel{2x} - x^2 + \cancel{2x} - \cancel{4x} + 8}{x-2} > 0$

$\frac{8 - x^2}{x-2} > 0 \Rightarrow x \neq 2$



$\therefore$ The solution set is

$\underline{(-\infty, -2\sqrt{2}) \cup (2, 2\sqrt{2})}$

(21) $\sqrt{3-2\sqrt{2}} = \sqrt{x} - \sqrt{y}$

$\Rightarrow 3 - 2\sqrt{2} = x + y - 2\sqrt{xy}$

$x+y = 3$ —— (i)

$\cancel{2}\sqrt{xy} = \cancel{2}\sqrt{2}$

$\therefore xy = 2$

$y = \frac{2}{x}$ —— (ii)

put (ii) into (i)

$x + \frac{2}{x} = 3$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

we choose $x = 2$

$\therefore y = \frac{2}{x} = \frac{2}{2} = 1$

$\therefore x = 2,\ y = 1$

$\sqrt{3-2\sqrt{2}} = \sqrt{2} - \sqrt{1}$

$= \sqrt{2} - 1$ (C)

(22) $x+y = 3$ —— (i)

$x^2 - y^2 = 4 \Rightarrow (x-y)(x+y) = 4 \Rightarrow 3(x-y) = 4$

$x - y = \frac{4}{3} \Rightarrow x = \frac{4}{3} + y$ —— (ii)

put (ii) into (i):

$\frac{4}{3} + y + y = 3 \Rightarrow \frac{4}{3} + 2y = 3$

$2y = 3 - \frac{4}{3} = \frac{9-4}{3} = \frac{5}{3} \Rightarrow 2y = \frac{5}{3}$

$y = 5/6$

$\therefore x = \frac{4}{3} + \frac{5}{6} = \frac{8+5}{6} = \frac{13}{6}$

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$\therefore x = 13/6,\ y = 5/6$ (D)

(23) Let $p(x)$ be the polynomial;

$\therefore p(x) = 2x^7 + x^6 - x^4 - x^3 + 5$

when divided by $x+2$, we have $x = -2$

$\therefore p(-2) = 2(-2)^7 + (-2)^6 - (-2)^4 - (-2)^3 + 5$

$= -256 + 64 - 16 + 8 + 5$

$= \underline{-195}$ (C)

EXAMS FOCUS

Notty ①

TRIGONOMETRY :

1. $\sin^2x(\sec^2x + \csc^2x)$ is identically equal to ? (2016/2017)

Soln:

Note $\sec^2x = \frac{1}{\cos^2x}$ and $\csc^2x = \frac{1}{\sin^2x}$

$\therefore \sin^2x(\sec^2x + \csc^2x) = \sin^2x \cdot \frac{1}{\cos^2x} + \cancel{\sin^2x} \cdot \frac{1}{\cancel{\sin^2x}}$

$= \frac{\sin^2x}{\cos^2x} + 1 = \tan^2x + 1$

But $\tan^2x + 1 = \sec^2x$

Note: $\sec^2x = \frac{1}{\cos^2x} = \frac{1}{1-\sin^2x}$

2. If $\sin\theta = \frac{1}{3}$, find $\cos 2\theta$. (2016/2017)

Soln:

Note: $\cos 2\theta = \cos^2\theta - \sin^2\theta$ —— (i)

$2\cos^2\theta - 1$ —— (ii)

$1 - 2\sin^2\theta$ —— (iii)

$\therefore$ for quick result we use formula (iii)

$\Rightarrow \cos 2\theta = 1 - 2\sin^2\theta = 1 - 2(\sin\theta)^2$

$= 1 - 2(\frac{1}{3})^2 = 1 - \frac{2}{9}$

$\cos 2\theta = \frac{7}{9}$

3. Solve the equation $\frac{3}{2}\sin^{-1}(\frac{y}{7}) = \pi$ (2016/2017)

Soln:

$\frac{3}{2}\sin^{-1}(\frac{y}{7}) = \pi \quad = \sin^{-1}(\frac{y}{7}) = \frac{2\pi}{3}$

$\therefore \frac{y}{7} = \sin(\frac{2\pi}{3})$

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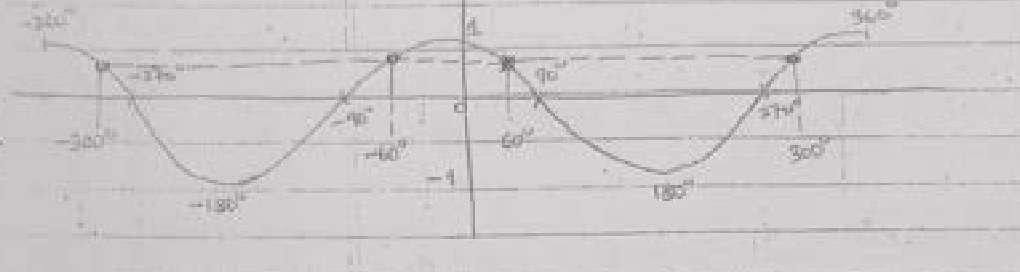
Nothy (3)



$\therefore \theta = -300°, -60°, 0°, 60°, 300°$

6- $Tan^2x\,(1 + cot^2x)$ is identically equal to? (2017/2018)

Soln:

Note : $cot^2x = \frac{1}{Tan^2x}$

$\therefore Tan^2x\,(1 + cot^2x) = Tan^2x\left(1 + \frac{1}{Tan^2x}\right)$

$= Tan^2x + \cancel{Tan^2x} \cdot \frac{1}{\cancel{Tan^2x}} = Tan^2x + 1$

But $Tan^2x + 1 = Sec^2x$

$\therefore Sec^2x = \frac{1}{1 - Sin^2x}$

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7. $\cos 3\theta$ is equivalent to? (2017/2018)

Soln:

$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

But $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$

$\therefore \cos 3\theta = (\cos^2\theta - \sin^2\theta)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

Netty (4)

$\cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$

8. If $\tan\theta = \frac{-7}{5}$ and θ is in the second quadrant, find $\sec\theta$. (2017/2013)

Soln:

Note: In the second quadrant, $\sec\theta$ is negative.

$\tan^2\theta + 1 = \sec^2\theta \Rightarrow \sec\theta = \pm\sqrt{\tan^2\theta + 1}$

$\sec\theta = \pm\sqrt{\left(\frac{-7}{5}\right)^2 + 1} = \pm\sqrt{\frac{49}{25} + 1} = \pm\sqrt{\frac{74}{25}}$

$\sec\theta = \pm\frac{\sqrt{74}}{5} \Rightarrow \sec\theta = \frac{\sqrt{74}}{5}$ or $\sec\theta = \frac{-\sqrt{74}}{5}$

$\therefore \sec\theta = \frac{-\sqrt{74}}{5}$

9. Solve the equation $\frac{4}{3}\cos^{-1}\left(\frac{y}{4}\right) = \pi$

Soln:

$\frac{4}{3}\cos^{-1}\left(\frac{y}{4}\right) = \pi \Rightarrow \cos^{-1}\left(\frac{y}{4}\right) = \frac{3\pi}{4}$

$\frac{y}{4} = \cos\left(\frac{3\pi}{4}\right)$

$\frac{3\pi}{4} = 135° \Rightarrow \cos 135° = -\cos 45° = \frac{-\sqrt{2}}{2}$

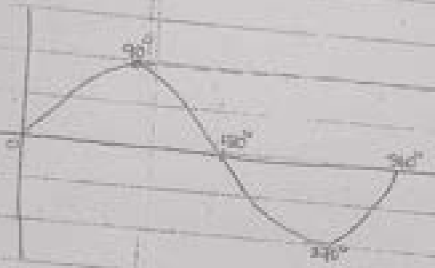
$\therefore \frac{y}{4} = \frac{-\sqrt{2}}{2} \Rightarrow y = \frac{-4\sqrt{2}}{2}$

$y = -2\sqrt{2}$

10. Find the value of θ in the range $0° < \theta < 360°$ for which $\sin\theta - 1 = 0$. (2017/2018) Nitty©

Soln.

$\sin\theta - 1 = 0 \Rightarrow \sin\theta = 1 \Rightarrow \theta = 90°$



$\therefore \theta = 90°$

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SEQUENCE AND SERIES

11. The sum to infinity of a Geometric Series with first term is 3 is $7/2$. (2016/2017)

Soln.

$a = 3, \; S_\infty = 7/2, \; r = ?$

$S_\infty = \frac{a}{1-r} = \frac{3}{1-r} = \frac{7}{2} \Rightarrow 6 = 7(1-r)$

$6 = 7 - 7r \Rightarrow 7r = 7 - 6 \Rightarrow 7r = 1$

$r = \frac{1}{7}$

12. The sum of the first 10 terms of a Geometric progression with common ratio 2 is 5,115. Find the first term. (2016/2017)

Soln.

$r = 2, \; S_{10} = 5115, \; n = 10, \; a = ?$

Notty ⑥

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(2^{10} - 1)}{2-1} = 5115$$

$$S_{10} = \frac{a(1024 - 1)}{1} \Rightarrow 5115 = 1023a$$

$$a = \frac{5115}{1023} = 5$$

$$\therefore \underline{a = 5}$$

13. Find the Limit of the sequence $\left(\frac{1+7n^4}{3n^4+n^2-n+4}\right)_{n=1}^{\infty}$ (2016/2017)

Soln:

Note: The ratio of the coefficient of the variable at the numerator and denominator gives the solution of the above problem.

$\therefore \left(\frac{1+7n^4}{3n^4+n^2-n+4}\right)$ clearly, we see that n^4 is at both numerator and denominator. So the solution is the coefficient of the variable.

$$\Rightarrow \left(\frac{1+7n^4}{3n^4+n^2-n+4}\right) = \frac{7}{3} \text{ or } 2\tfrac{1}{3}$$

14. Find the Sum of the first 4 terms of a Geometric progression whose first term is 5 and the common ratio is 2. (2017/2018)

Soln:

$a = 5$, $r = 2$, $S_4 = ?$ $n = 4$

$$S_n = \frac{a(r^n - 1)}{r-1} \Rightarrow S_4 = \frac{5(2^4 - 1)}{2-1} = \frac{5(16-1)}{1}$$

$$S_4 = 5 \times 15 = \underline{75}$$

Notty (7)

15. If the sum of the first 10 terms of an Arithmetic progression is 100, and the first term 1, find the common difference. (2017/2018)

Soln:

$S_{10} = 100$, $a = 1$, $r = ?$, $n = 10$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore 100 = \frac{10}{2}\left[2(1) + (10-1)d\right]$$

$$100 = 5\left[2 + 9d\right] \Rightarrow \frac{100}{5} = 2 + 9d$$

$$20 = 2 + 9d \Rightarrow 20 - 2 = 9d$$

$$18 = 9d \Rightarrow d = \frac{18}{9} = 2$$

$$\therefore \underline{d = 2}$$

16. Find the sum to infinity of a Geometric Series, whose first term is 2 and common ratio is $^{19}/_{20}$. (2017/2018)

Soln:

$a = 2$, $r = {}^{19}/_{20}$, $S_\infty = ?$

$$S_\infty = \frac{a}{1-r} = \frac{2}{1 - {}^{19}/_{20}} = \frac{2}{\frac{20-19}{20}} = \frac{2}{{}^{1}/_{20}}$$

$$S_\infty = 2 \times 20 = \underline{40}$$

17. Find the limit of the sequence $\left(\frac{n+7}{n-5}\right)_{n=1}^{\infty}$ (2017/2018)

Soln:

Hint: The ~~variable~~ variable with highest power is n and it is found at both numerator and denominator. Hence, the

Solution is the coefficient of this variable

$\therefore \left(\frac{n+7}{n-5}\right)_{n=1}^{\infty} = \frac{1}{1} = 1$

18. Find the limit of the sequence $\left(\frac{3n^3+2}{n-5n^3}\right)$. (2015/2016)

Soln:

Hint: Again, n^3 is present at both numerator and denominator and as such their coefficients becomes the solution.

$\therefore \left(\frac{3n^3+2}{n-5n^3}\right) = \frac{3}{-5} = -\frac{3}{5}$

BINOMIAL THEOREM

19. What is the coefficient of x^5 in the expansion of $\frac{1}{1-2x}$, for $|x|<1$. (2015/2016)

Soln:

$\frac{1}{1-2x} = (1-2x)^{-1}$

using $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$

Hint: Since $x = -2x$ from the above, then it implies that the variable x^5 will occur at $\frac{n(n-1)(n-2)(n-3)(n-4)}{5!}x^5$.

$\therefore$ The coefficient of x^5 will occur at: $\frac{n(n-1)(n-2)(n-3)(n-4)}{5!}x^5$

where $n=-1$, $x=-2x$, $5! = 120$

$= \frac{(-1)(-1-1)(-1-2)(-1-3)(-1-4)}{5!}(-2x)^5$

$= \frac{-120}{120} \times -32x^5 = 32x^5$

Nutty (9)

$\therefore$ The coefficient of x^5 is 32

20. what is the coefficient of x^2 in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{10}$ (2016/2017)

Soln.

Hint: separate the x-variable from everyother terms and equate to x^2 (this is how you obtain a value for r)

$x = x^2$, $y = \frac{-1}{2x}$, $n = 10$, $r = ?$

$$\therefore \left(x^2 - \frac{1}{2x}\right)^{10} = {}^{10}C_r \, (x^2)^{10-r} \left(\frac{-1}{2x}\right)^r$$

$$= {}^{10}C_r \cdot x^{20-2r} \cdot \left(\frac{-1}{2}\right)^r \cdot \left(\frac{1}{x}\right)^r$$

$$ {}^{10}C_r \, x^{20-2r} \cdot \frac{1}{x^r} \cdot \left(\frac{-1}{2}\right)^r \quad \text{——— ①}$$

$$ {}^{10}C_r \cdot \left(\frac{-1}{2}\right)^r \cdot x^{20-3r} \quad \text{——— (i)}$$

$$\therefore x^{20-3r} = x^2$$

$$\Rightarrow 20 - 3r = 2$$

$$20 - 2 = 3r$$

$$18 = 3r$$

$$r = \frac{18}{3} = 6$$

Put $r = 6$ into (i)

$$\therefore {}^{10}C_6 \cdot \left(\frac{-1}{2}\right)^6 \cdot x^{20-3(6)}$$

$$= \frac{10!}{(10-6)!\,6!} \cdot \frac{1}{2^6} \cdot x^{20-18}$$

$$= \frac{10!}{4!\,6!} \cdot \frac{1}{64} \cdot x^2$$

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Notty 10

$$= \frac{10 \cdot \cancel{9}^{3} \cdot \cancel{8} \cdot 7 \cdot \cancel{6!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{6!}} \times \frac{1}{64} \cdot x^2$$

$$210 \times \frac{1}{64} x^2 \Rightarrow \frac{210}{64} x^2$$

$$= \frac{105}{32} x^2$$

$\therefore$ The coefficient of x^2 is $\frac{105}{32}$

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UNIVERSITY OF UYO
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS & STATISTICS
FIRST SEMESTER EXAMINATION 2015/2016 SESSION
MTH 111: GENERAL MATHEMATICS I (B) TIME: $1\frac{1}{4}$ HOURS 1hr 15min

NAME OF STUDENT: .... SEX

REG. NO: .... GROUP

DEPARTMENT: .... SERIAL NUMBER

VENUE .... SIGNATURE ....

INSTRUCTION: ATTEMPT ALL QUESTIONS BY WRITING THE LETTER TO THE CORRECT OPTION IN THE BOXES PROVIDED ON THE RIGHT

WARNINGS:

(i) USE OF MANUAL CALCULATORS AND TELEPHONES NOT ALLOWED.

(ii) THIS QUESTION PAPER IS FOR STUDENTS OF BIOLOGICAL AND MEDICAL SCIENCES; PHARMACY, AGRICULTURE, ENVIRONMENTAL AND EDUCATION (WITH THE EXCEPTION OF PHYSICS, CHEMISTRY AND MATHEMATICS EDUCATION).

1. In how many ways can a standard football team be selected from 13 players, if two particular players must be in the team? D
   A. 35 B. 15 C. 5 D. 55

2. Resolve [illegible] into partial fractions.
   A. [illegible] B. [illegible]
   C. [illegible] D. [illegible]

3. Factorise the expression $6x^3 - 11x^2 - 3x - 2$
   A. [illegible] B. $(x-2)(3x+1)(2x+1)$
   C. [illegible]$(2x-1)$ D. $(x-2)(3x+1)(2x+1)$

4. $x \in [(A \cap B) - C]$ implies that
   A. $x \in A$ and $x \notin B$ or $x \in C^c$ B. $x \in A$ and $x \in B$ and $x \in C^c$
   C. $x \notin A$ and $x \notin B$ or $x \in C$ D. $x \in A$ and $x \in B$ and $x \notin C^c$

5. What is the coefficient of x^3 in the expansion of $\frac{1}{1-2x}$, for $|x| < 1$.
   A. 32 B. -16 C. 1 D. 16

6. Find the limit of the sequence [illegible] B
   A. [illegible] B. [illegible] C. -1 D. ∞

7. If the sum of the first 20 terms of an Arithmetic Progression is 200, and the first term 40, find the common difference.
   A. [illegible] B. [illegible] C. [illegible] D. [illegible]

8. Find the sum of the first 10 terms of a Geometric Progression whose first term is 5 and the common ratio is 2.
   A. 2,555 B. 5,120 C. 5,115 D. 70

9. Find the sum to infinity of a Geometric Series, whose first term is 2 and common ratio is $\frac{19}{20}$.
   A. [illegible] B. [illegible] C. 40 D. 20

10. Find the values of θ in the range $0 < \theta < 360°$ for which $2\sin^2\theta - 3\sin\theta + 1 = 0$.

A. $30°, 90°, 180°$ B. $30°, 220°, 150°$ C. $40°, 90°, 150°$ D. $30°, 90°, 150°$ D

11. A bag contains eleven discs, numbered from 1 to 11. A disc is drawn from the bag. If the number is odd, then a coin is tossed. If the number is even, then a die is thrown. How many outcomes are possible?

A. 42 B. 30 C. 88 D. 46

12. If $\tan\theta = -\frac{7}{3}$ and θ is in the second quadrant, find $\sec\theta$.

A. [illegible] B. $-\frac{\sqrt{58}}{3}$ C. $\frac{\sqrt{7}}{6}$ D. [illegible] B

13. $\tan^2 x(1 + \cot^2 x)$ is identically equal to

A. $\frac{1}{1-\sin^2 x}$ B. $\sec x$ C. $\csc x$ D. [illegible]

14. Using the binomial expansion, find the approximate value of [illegible] correct to 6 decimal places.

A. 1.000001 B. 1.010010 C. 1.000001 D. 1.010101 D

15. Solve the equation $\frac{3}{2}\cos^{-1}\frac{x}{4} = \pi$.

A. $-4\sqrt{2}$ B. $\sqrt{2}$ C. $-2\sqrt{2}$ D. $-\sqrt{2}$ C

16. $\cos 3\theta$ is equivalent to

A. $\cos^3\theta + 3\cos\theta\sin^2\theta$ B. $\cos^3\theta - 3\cos\theta\sin^2\theta$
C. $\cos^3\theta + \cos\theta\sin^2\theta$ D. $\cos^3\theta - 3\sin\theta\cos^2\theta$ B

17. If $\alpha^2 - 1$ and $\beta^2 + 1$ are the roots of the equation $x^2 - 4x + 5 = 0$, find equation whose roots are α and β.

A. $x^2 \mp x\sqrt{6} + 2$ B. $x^2 \mp x\sqrt{3} + 2$
C. $x^2 \mp x\sqrt{7} + 2$ D. $x^2 \mp x\sqrt{5} + 2$

18. The set $[(A^c \cup B) \cap (B \cup C^c)]^c$ is equal to

A. $B^c \cap (A \cup C)$ B. $B^c \cap (A \cup C)$ C. $C^c \cup (A \cap B)$ D. [illegible]

19. Solve the equation $2.3^{2x} - 3^{x+1} + 1 = 0$.

A. [illegible] or $-\log_2 3$ B. $x = 0$ or $\log_3\left(\frac{1}{2}\right)$
C. [illegible] $\log_3 2$ or $\log_3\left(\frac{1}{2}\right)$ D. $x = -\log_2 3$ or $-\log_3 2$ B

20. Solve the following inequality: $\frac{2x}{x-2} > x + 4$.

A. $(1, 2\sqrt{2})$ B. $(-\infty, 2\sqrt{2}) \cup (1, 2\sqrt{2})$
C. $(-\infty, -2\sqrt{2}) \cup (0, 2\sqrt{2})$ D. $(-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty)$

21. Find the square root of $3 - 2\sqrt{2}$.

A. $\sqrt{3} - \sqrt{2}$ B. $1 - \sqrt{2}$ C. $\sqrt{2} - 1$ D. $\sqrt{3} + \sqrt{2}$

22. Solve the simultaneous equations: $x + y = 3$; $x^2 - y^2 = 4$.

A. $x = 1, y = 2$ B. $x = 6, y = -3$ C. $x = \frac{18}{6}; y = -\frac{5}{6}$ D. $x = \frac{13}{6}, y = \frac{5}{6}$ D

23. Find the remainder when $2x^7 + x^6 - x^4 - x^3 + 5$ is divided by $x + 2$.

A. -185 B. -135 C. -195 D. -115

10. Find the values of θ in the range $0 < \theta < 360°$ for which $2\sin^2\theta - 3\sin\theta + 1 = 0$.

A. $30°, 90°, 180°$ B. $30°, 120°, 150°$ C. $40°, 90°, 150°$ D. $30°, 90°, 150°$

11. A bag contains eleven discs, numbered from 1 to 11. A disc is drawn from the bag. If the number is odd, then a coin is tossed. If the number is even, then a die is thrown. How many outcomes are possible?

A. 42 B. 30 C. 88 D. 46

12. If $\tan\theta = -\frac{7}{5}$ and θ is in the second quadrant, find $\sec\theta$.

A. $-\frac{\sqrt{7}}{5}$ B. $-\frac{\sqrt{74}}{5}$ C. $\frac{\sqrt{7}}{5}$ D. $\frac{\sqrt{74}}{5}$

13. $\mathrm{Tan}^2 x(1+\cot^2 x)$ is identically equal to

A. $\frac{1}{1-\sin^2 x}$ B. $\sec x$ C. $\csc x$ D. $\frac{1}{\sin^2 x}$

14. Using the binomial expansion, find the approximate value of $\frac{1}{0.99}$ correct to 6 decimal places

A. 1.000001 B. 1.010010 C. 1.000001 D. 1.010101

15. Solve the equation $\frac{4}{3}\cos^{-1}\frac{x}{4} = \pi$.

A. $-4\sqrt{2}$ B. $\sqrt{2}$ C. $-2\sqrt{2}$ D. $-\sqrt{2}$

16. $\mathrm{Cos}\,3\theta$ is equivalent to

A. $\cos^3\theta + 3\cos\theta\sin^2\theta$ B. $\cos^3\theta - 3\cos\theta\sin^2\theta$
C. $\cos^3\theta + \cos\theta\sin^2\theta$ D. $\cos^3\theta - 3\sin\theta\cos^2\theta$

17. If α^2+1 and β^2+1 are the roots of the equation $x^2-4x+5=0$, find equation whose roots are α and β.

A. $x^2 \mp x\sqrt{6}+2$ B. $x^2 \mp x\sqrt{3}+2$
C. $x^2 \mp x\sqrt{2}+2$ D. $x^2 \mp x\sqrt{5}+2$

18. The set $[(A^c \cup B) \cap (B \cup C^c)]^c$ is equal to

A. $A^c \cap (B \cup C)$ B. $B^c \cap (A \cup C)$ C. $C^c \cup (A \cap B)$ D. $A^c \cup (B \cap C)$

19. Solve the equation $2.3^{2x} - 3^{x+1} + 1 = 0$.

A. $x = 0$ or $-\log_2 3$ B. $x = 0$ or $\log_3\left(\frac{1}{2}\right)$
C. $x = -\log_3 2$ or $\log_3\left(\frac{1}{2}\right)$ D. $x = -\log_2 3$ or $\log_3 2$

20. Solve the following inequality: $\frac{2x}{x-2} > x+4$.

A. $(1, 2\sqrt{2})$ B. $(-\infty, 2\sqrt{2}) \cup (1, 2\sqrt{2})$
C. $(-\infty, -2\sqrt{2}) \cup (0, 2\sqrt{2})$ D. $(-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty)$

21. Find the square root of $3-2\sqrt{2}$.

A. $\sqrt{3}-\sqrt{2}$ B. $1-\sqrt{2}$ C. $\sqrt{2}-1$ D. $\sqrt{3}+\sqrt{2}$

22. Solve the simultaneous equations: $x+y=3$, $x^2-y^2=4$.

A. $x=1$, $y=2$ B. $x=6$, $y=-3$ C. $x=\frac{13}{6}$, $y=-\frac{5}{6}$ D. $x=\frac{13}{6}$, $y=\frac{5}{6}$

23. Find the remainder when $2x^7+x^6-x^4-x^3+5$ is divided by $x+2$.

A. -185 B. -135 C. -195 D. -115

Mc. Norty (6)

$^5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!}$

$= 5!$

$= 5\times4\times3\times2\times1$

$=$ 120 ways

(b) we wish to arrange 8 people with the condition that 3 of them must always sit together

$\therefore n = 8 ; m = 3$

$\Rightarrow m! \times (n-m+1)!$

$3! \times (8-3+1)!$

$3! \times 6!$

$(3\times2\times1) \times (6\times5\times4\times3\times2\times1)$

6×720

$=$ 4320 ways

(c) we wish to arrange 10 people with the condition that 4 of them must always sit together

$\therefore n = 10 ; m = 4$

$\Rightarrow m! \times (n-m+1)!$

$4! \times (10-4+1)!$

$4! \times 7!$

$(4\times3\times2\times1) \times (7\times6\times5\times4\times3\times2\times1)$

24×5040

$=$ 120960 ways

(d) we wish to arrange 8 people with the condition that 4 of them must not sit together

$\therefore n = 8 ; m = 4$

$\Rightarrow n! - [m! \times (n-m+1)!]$

$= 8! - [4! \times (8-4+1)!]$

$= 8! - [4! \times 5!]$

$= 40320 - [24 \times 120]$

$= 40320 - 2880$

$=$ 37440 ways

GoodLuck!!!

can decide to use the "rational root test" method to find the roots of the cubic function.

$\therefore RRT = \pm\left[\dfrac{\text{factors of constant term}}{\text{factors of leading coeff}}\right]$

$RRT = \pm\left[\dfrac{1, 2}{1, 2, 3, 6}\right]$

$= \pm\left[1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 2, \frac{2}{3}\right]$

Let $x = -2$

$\therefore 6(-2)^3 + 11(-2)^2 - 3(-2) - 2$

$6(-8) + 11(4) + 6 - 2$

$-48 + 44 + 4 = -4 + 4 = 0$

$\therefore (x+2)$ is a factor

Next, we use the "synthetic division algorithm" to find other factors.

$\therefore -2$	6	11	-3	-2
	↓	+	+	+
		-12	2	2
	6	-1	-1	0

$\therefore 6x^3 + 11x^2 - 3x - 2 = (x-2)(6x^2 - x - 1)$

We now factorize the quadratic function $6x^2 - x - 1$

$6x^2 - x - 1 = 6x^2 - 3x + 2x - 1$

$= 3x(2x-1) + 1(2x-1)$

$= (3x+1)(2x-1)$

$\therefore$ The factors of $6x^3 + 11x^2 - 3x - 2$ are $(x-2)(3x+1)(2x-1)$

NOTTY

PERMUTATION:

HINT: If we wish to arrange 'n' objects with the condition that

(i) m objects must always be together.

(ii) m objects must never be together.

we use the formula:

$m! \times (n-m+1)!$ whenever the objects must be together.

$n! - [m! \times (n-m+1)!]$ whenever the objects must never be together.

ⓐ In how many ways can 5 people be seated on a bench?

ⓑ In how many ways can 8 people be seated on a bench if 3 particular people must sit together?

ⓒ In how many ways can 10 people be seated on a bench if 4 particular people must sit together?

ⓓ In how many ways can 8 people be seated on a bench if particular people must not sit together?

SOLUTIONS

ⓐ We wish to arrange 5 people taken all at a time, we use

${}^5P_5 = 5!$

$\therefore \quad B + 2C = A - A$

$B + 2C = 0$ ——— (iv)

from (ii) $C - 3B = -7$

$\Rightarrow C = 3B - 7$ put into (iv)

$\therefore \quad B + 2(3B - 7) = 0$

$B + 6B - 14 = 0$

$7B = 14$

$\therefore \quad B = 14/5$

But $B + 2C = 0$

$\Rightarrow C = -B/2$

$C = -(\frac{14}{5}) \div 2$

$= \frac{-14}{5} \times \frac{1}{2} = -7/5$

$\therefore \quad C = -7/5$

Again $A + B = 14$

$\therefore \quad A = 14 - B$

$A = 14 - \frac{14}{5}$

$A = \frac{70 - 14}{5} = \frac{56}{5}$

$\therefore \quad A = 56/5$

$$\frac{7x^3}{x^3 - 2x^2 + x - 2} = 7 + \frac{56/5}{x-2} + \frac{\frac{14x}{5} - \frac{7}{5}}{x^2+1}$$

$$= 7 + \frac{56}{5(x-2)} + \frac{14x - 7}{5(x^2+1)}$$

FACTORIZATION :

HINT : Recall we said that a cubic function of the form

(i) $x^3 + ax^2 - x - a$ or

(ii) $x^3 - ax^2 + x - a$

is factored by grouping the first two terms and the next two terms and then factoring them properly.

Factorize the following functions

(a) $x^3 + 2x^2 - x - 2$ (b) $3x^3 + 7x^2 - 3x - 7$

(c) $x^3 + 10x^2 - x - 10$ (d) $6x^3 + 11x^2 - 3x - 2$

(e) $6x^4 - 10x^3 - 11x^2 + 13x + 2$

SOLUTIONS

Note: Problems (a) – (c) takes the form of $x^3 + ax^2 - x - a$, so we apply the "grouping form" of factorization.

(a) $x^3 + 2x^2 - x - 2$

$= x^2(x+2) - 1(x+2)$

$= (x+2)(x^2 - 1)$

$= (x+2)(x-1)(x+1)$

(b) $3x^3 + 7x^2 - 3x - 7$

$= x^2(3x+7) - 1(3x+7)$

$= (3x+7)(x^2 - 1)$

$= (3x+7)(x-1)(x+1)$

(c) $x^3 + 10x^2 - x - 10$

$= x^2(x+10) - 1(x+10)$

$= (x+10)(x^2 - 1)$

$= (x+10)(x-1)(x+1)$

(d) This problem is evaluated using the "trial and error" method or we

$\therefore x^3 - 2x^2 + x - 2$

$= x^2(x-2) + 1(x-2)$

$= (x-2)(x^2+1)$

$\therefore \dfrac{1}{x^3-2x^2+x-2} = \dfrac{1}{(x-2)(x^2+1)}$

$\dfrac{1}{(x-2)(x^2+1)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+1}$

$\therefore 1 = A(x^2+1) + (Bx+C)(x-2)$

$1 = Ax^2 + A + Bx^2 - 2Bx + Cx - 2C$

Comparing the LHS & RHS,

$A + B = 0$ —— (i)

$C - 2B = 0$ —— (ii)

$A - 2C = 1$ —— (iii)

from (iii): $A = 1 + 2C$

from (ii): $C = 2B$

from (i): $A = -B$

$\therefore$ from (i) $A + B = 0$

$1 + 2C + B = 0$

$1 + 2(2B) + B = 0$

$1 + 4B + B = 0$

$5B = -1$

$B = -\frac{1}{5}$

But $A = -B \Rightarrow A = -(-\frac{1}{5})$

$A = \frac{1}{5}$.

$C = 2B \Rightarrow C = 2(-\frac{1}{5}) = -\frac{2}{5}$

$\therefore A = \frac{1}{5},\ B = -\frac{1}{5},\ C = -\frac{2}{5}$

$\therefore \dfrac{1}{x^3-2x^2+x-2} = \dfrac{\frac{1}{5}}{x-2} + \dfrac{-\frac{1}{5}x - \frac{2}{5}}{x^2+1}$

$= \dfrac{1}{5(x-2)} + \dfrac{-(x+2)}{5(x^2+1)}$

$= \dfrac{1}{5(x-2)} - \dfrac{x+2}{5(x^2+1)}$ Ans

(4) $\dfrac{7x^3}{x^3-2x^2+x-2}$

Note: since the variable with the highest power is found at both the numerator and denominator, we go through the long division method first before resolving the solution obtained from the long division into its partial fractions.

$$\begin{array}{r|l} & 7 \\ \hline x^3-2x^2+x-2 & 7x^3+0x^2+0x+0 \\ (-) & 7x^3-14x^2+7x-14 \\ \hline & 14x^2-7x+14 \end{array}$$

$\therefore \dfrac{7x^3}{x^3-2x^2+x-2} = 7 + \dfrac{14x^2-7x+14}{x^3-2x^2+x-2}$

Next, we resolve the fractional term on the RHS into its partials.

$\therefore \dfrac{14x^2-7x+14}{x^3-2x^2+x-2} = \dfrac{14x^2-7x+14}{(x-2)(x^2+1)}$

$\dfrac{14x^2-7x+14}{(x-2)(x^2+1)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+1}$

$14x^2 - 7x + 14 = A(x^2+1) + (Bx+C)(x-2)$

$= Ax^2 + A + Bx^2 - 2Bx + Cx - 2C$

Comparing LHS & RHS

$A + B = 14$ —— (i)

$C - 2B = -7$ —— (ii)

$A - 2C = 14$ —— (iii)

observe that the RHS of (i) & (iii) are equal, then we equate their LHS

$\therefore A + B = A - 2C$

equal to ?
A. $B^c \cap (A \cup C)$ B. $A^c \cap (B \cup C)$
C. $C^c \cup (A \cap B)$ D. $A^c \cup (B^c \cap C)$

SOLUTIONS

(b) $x \in [(A \cap B) - C]$

Note that $X - Y' = X \cap Y'$

$\therefore x \in [(A \cap B) - C] \equiv x \in [(A \cap B) \cap C']$

$\Rightarrow x \in (A \cap B) \cap x \in C'$

$\Rightarrow x \in A \cap x \in B \cap x \in C'$

where intersection translates to and (ie $\cap \equiv$ and)

$\Rightarrow x \in A$ and $x \in B$ and $x \in C'$

(d) $[(A^c \cup B) \cap (B \cup C^c)]^c$

$\Rightarrow (A^c \cup B)^c \cup (B \cup C^c)^c$

$\Rightarrow (A \cap B^c) \cup (B^c \cap C)$

observe closely that inside the two brackets above B^c and $\cap$ is common, so we factor it out.

$\Rightarrow B^c \cap (A \cup C)$

PARTIAL FRACTION:

HINT: Attention should be paid on factorization as it determines the starting point. A cubic function and other higher power functions can be factorized using the "Rational Root Test".

$$RRT = \pm \boxed{\frac{\text{Factors of Constant term}}{\text{Factors of Leading Coefficient}}}$$

Resolve the following into partial fractions;

(a) $\dfrac{1}{x^2+x-2}$ (b) $\dfrac{1}{x^3-2x^2+x-2}$

(c) $\dfrac{x^2}{x^2+x-2}$ (d) $\dfrac{7x^3}{x^3-2x^2+x-2}$

(e) $\dfrac{x^2+x-1}{x^4-2x^3+3x^2-4x+2}$

(f) $\dfrac{2x^3+1}{x^3-x^2+x-1}$

SOLUTIONS

NOTE: If a cubic function takes the form (i) x^3+ax^2-x-a or (ii) x^3-ax^2+x-a, then the factors are obtained by factoring the first two terms then the next two terms.

ie x^3+ax^2-x-a
$\equiv x^2(x+a)-1(x+a)$
$\equiv (x+a)(x^2-1)$
$\equiv (x+a)(x-1)(x+1)$

ie x^3-ax^2+x-a
$\equiv x^2(x-a)+1(x-a)$
$\equiv (x-a)(x^2+1)$

(b) $\dfrac{1}{x^3-2x^2+x-2}$

we will begin by first factoring x^3-2x^2+x-2

replace the inequality sign with the equality sign and solve the numerator for x

$$\frac{-x^2+8}{x+4} = 0$$

$$\Rightarrow -x^2+8=0$$

$$\cancel{-}x^2 = \cancel{-}8$$

$$x^2 = 8$$

$$x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$$x = 2\sqrt{2} \text{ or } x = -2\sqrt{2}$$

Next, we insert all the values of x obtained both at the numerator and denominator on the number line so as to get the range of values of x.

$-\infty$ T -4 F $-2\sqrt{2}$ T $2\sqrt{2}$ F ∞

for the interval $(-\infty, -4)$: Let $x = -5$

$$\therefore \frac{-(-5)^2+8}{-5+4} = \frac{-25+8}{-1} = \frac{-17}{-1}$$

$$= 17 \text{ (True)}$$

for the interval $(-4, -2\sqrt{2})$: Let $x = -3$

$$\therefore \frac{-(-3)^2+8}{-3+4} = \frac{-9+8}{1} = \frac{-1}{1}$$

$$= 1 \text{ (False)}$$

for the interval $(-2\sqrt{2}, 2\sqrt{2})$: Let $x = 0$

$$\frac{-(0)^2+8}{0+4} = \frac{8}{4} = 2 \text{ (True)}$$

for the interval $(2\sqrt{2}, \infty)$: Let $x = 4$

$$\frac{-(4)^2+8}{4+4} = \frac{-16+8}{8} = \frac{-8}{8} = -1 \text{ (False)}$$

$\therefore$ The solution is the union of the intervals with "True" solutions

$$(-\infty, -4) \cup (-2\sqrt{2}, 2\sqrt{2})$$

SET THEORY:

HINT: Basically, you will be tested on how much you understand the translation of Union of a set ($\cup$), the intersection of a set ($\cap$) and the complementation law of de-Morgan. The Keyword for the union of a set is OR while the Keyword for the intersection of a set is AND. On the other hand, the complement symbol reverses the union to intersect and vice-versa.

(a) The set $(A^c \cap B)^c$ is equal to?
A. $B^c \cup A$ B. $A^c \cup B$ C. $A \cap B$ D. $B^c \cap A$

(b) $x \in [(A \cap B) - C]$ implies that?
A. $x \in A$ and $x \in B$ or $x \in C^c$
B. $x \in A$ and $x \in B$ and $x \in C^c$
C. $x \in A$ and $x \in B$ or $x \in C$
D. $x \in A$ and $x \in B$ and $x \notin C^c$

(c) The set $[(A^c \cap B) \cup (B \cap C^c)]^c$ is equal to?
A. $A^c \cup (B \cap C)$ B. $C^c \cup (A \cap B)$
C. $A^c \cup (B^c \cap C)$ D. $B^c \cup (A \cap C)$

(d) The set $[(A^c \cup B) \cap (B \cup C^c)]^c$ is

ⓑ $\frac{2x}{x-2} > x+4$

ⓒ $x^2-2x+1 > x+7$

ⓓ $\frac{x}{x-1} < x+2$

ⓔ $\frac{2x}{x+4} > x-2$

ⓕ $\frac{x}{x-1} > x+2$

ⓖ $x^2-4 < 0$

SOLUTIONS

ⓐ $x^2+x+1 > x+2$

$x^2+x+1-x-2 > 0$

$x^2-1 > 0$ ——— ⊛

replace the inequality sign with the equality

$\therefore x^2-1=0$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x=1$ or $x=-1$

$-\infty$ ——— -1 ——— 1 ——— $+\infty$

T✓ -1 F 1 T✓

Next, we wish to pick random values from the intervals on the above number line and test in eqn ⊛ to see which of the intervals is greater than zero, that is which of the intervals satisfy the inequality symbol at ⊛.

$\therefore$ Between the interval $(-\infty,-1)$:

Let $x=-3$

$\therefore (-3)^2-1>0$

$9-1>0$

$8>0$ (True)

Between the interval $(-1,1)$

Let $x=0$

$\therefore 0^2-1>0$

$-1>0$ (False)

Between the interval $(1,\infty)$

Let $x=5$

$\therefore 5^2-1>0$

$25-1>0$

$24>0$ (True)

The solution is the union of the intervals with TRUE SOLUTIONS

$(-\infty,-1)\cup(1,\infty)$

ⓔ $\frac{2x}{x+4} > x+2$

$\Rightarrow \frac{2x}{x+4} - x+2 > 0$

Take L.C.M

$\frac{2x - x(x+4)+2(x+4)}{x+4} > 0$

$\frac{\cancel{2x}-x^2-\cancel{4x}+\cancel{2x}+8}{x+4} > 0$

$\frac{-x^2+8}{x+4} > 0$ ——— ⊛

Next, equate the denominator to zero and solve for x

$x+4=0$

$x=-4$

(b) $2x^7+x^6-x^4-x^3+5$ is divided by $x+2$.

(c) $3x^{10}+2x^7+x^6-x^4-2x^3+5$ is divided by $x+1$.

(d) $x^{10}+2x^7+x^6-x^4-x^3+5$ is divided by $x+1$.

~~(e) Find the ...~~

(e) $x^6-x^4-x^3+5$ is divided by $x+2$.

(f) If the remainder when x^3+x^2-bx+7 is divided by $x+3$ is 1, find the value of b.

SOLUTIONS

(a) $x+1=0 \Rightarrow x=-1$

Put $x=-1$ into the polynomial

$(-1)^6-(-1)^4-(-1)^3+5$

$\cancel{1}-\cancel{1}+1+5$

$= \underline{6}$

(b) $x+2=0 \Rightarrow x=-2$

Put $x=-2$ into the polynomial

$2(-1)^7+(-1)^6-(-1)^4-(-1)^3+5$

$\cancel{-2}+\cancel{1}-1+\cancel{1}+5$

$= \underline{4}$

(f) HINT: we shall equate the divisor to zero and then solve for x. Next, we will put the value of x into the polynomial and equate it to the remainder given (this will help to find b)

$\therefore x+3=0 \Rightarrow x=-3$

Put $x=-3$ into the polynomial and equate to the remainder

$\therefore (-3)^3+(-3)^2-b(-3)+7=1$

$-27+9+3b+7=1$

$-27+16+3b=1$

$-11+3b=1$

$3b=1+11$

$3b=12$

$b=12/3$

$\underline{b=4}$

INEQUALITY

HINT: (i) A quadratic inequality is solved by making sure that the R.H.S of the inequality is zero, next we replace the inequality sign with an equality sign and solve the resultant quadratic equation.

(ii) while a rational inequality is also solved by making sure that the R.H.S of the inequality is zero, next the L.H.S is simplified properly and the values of x at the denominator and numerator is obtained and inscribed on the number line.

Solve the following inequality;

(a) $x^2+x+1 > x+2$

SOLUTIONS

MR. NOTTY (8)

(c) $a = 1, b = 4, c = 3$

$\alpha+\beta = \frac{-b}{a} = \frac{-4}{1} = -4$

$\alpha\beta = c/a = 3/1 = 3$

Since the new roots are α^2/β, β^2/α, then:

* Sum of new roots $= \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

$= \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{(-4)^3 - 3(3)(-4)}{3}$

$= \frac{-64+36}{3} = \frac{-28}{3}$

* Product of new roots $= \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha}$

$\Rightarrow \alpha\beta = 3$

$\therefore$ The new quadratic equation is of the form:

$x^2 - (\text{Sum of roots})x + \text{product of roots} = 0$

$\Rightarrow x^2 - \left(\frac{-28}{3}\right)x + 3 = 0$

$x^2 + \frac{28x}{3} + 3 = 0$

multiply through by 3

$3x^2 + 28x + 9 = 0$

(d) $a = 1, b = -4, c = 5$

$\alpha+\beta = \frac{-b}{a} = \frac{-(-4)}{1} = 4$

$\alpha\beta = c/a = 5/1 = 5$

Since the new roots are $\alpha - 2$ and $\beta - 2$, then

* Sum of new roots $= (\alpha-2)+(\beta-2)$

$= \alpha - 2 + \beta - 2$

$= \alpha+\beta - 4$

$= 4 - 4$

$= 0$

* Product of new roots $= (\alpha-2)(\beta-2)$

$= \alpha\beta - 2\alpha - 2\beta + 4$

$= \alpha\beta - 2(\alpha+\beta) + 4$

$= 5 - 2(4) + 4$

$= 5 - 8 + 4$

$= +1$

The new equation is of the form;

$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

$\Rightarrow x^2 - (0)x + 1 = 0$

$x^2 + 1 = 0$

POLYNOMIAL FUNCTION:

HINT: The remainder function is obtained by equating the divisor to zero and finding the value of x. This value of x is then substituted into the polynomial (the expression at the numerator) and evaluated.

Find the remainder when

(a) $x^6 - x^4 - x^3 + 5$ is divided by $x+1$.

MR. NOTTY (7)

$x = 3 - \frac{4}{5} = \frac{15-4}{5} = \frac{11}{5}$

$\therefore x = -1, y = 2$ and $x = \frac{11}{5}, y = 2$

from (iii), $x = \frac{5}{3} + y$

$x = \frac{5}{3} + \frac{2}{3} = \frac{7}{3}$

$\therefore x = \frac{7}{3}, y = \frac{2}{3}$

(c) $x + y = 3$ ——— (i)

$x^2 - y^2 = 5$ ——— (ii)

HINT: we can make x or y the subject of formula in (i) and put into (ii) and then solve the resultant quadratic equation. But we shall be explaining a different method by using the "idea from the difference of two squares"

from (ii) $x^2 - y^2 = 5$

$\Rightarrow (x-y)(x+y) = 5$

Recall from (i) $x+y = 3$

$\therefore (x-y) \cdot 3 = 5$

$x - y = \frac{5}{3}$

$x = \frac{5}{3} + y$ ——— (iii)

Put (iii) into (i) and solve for y.

$x + y = 3$

$(\frac{5}{3} + y) + y = 3$

$\frac{5}{3} + 2y = 3$

$\therefore 2y = 3 - \frac{5}{3}$

$2y = \frac{9-5}{3} = \frac{4}{3}$

$2y = \frac{4}{3}$

$y = \frac{2}{3}$

QUADRATIC EQUATION.

HINT: The new quadratic equation is formed by:

(i) finding a, b and c.

(ii) finding $\alpha+\beta$ and $\alpha\beta$.

(iii) Finding the sum of new roots.

(iv) finding the product of new roots.

(v) inserting the values in (iii) and (iv) into the equation:

$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

(vi) Recall that:

$(\alpha+\beta)^2 \neq \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$(\alpha-\beta)^3 \neq \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$\alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

If α and β are the roots of the quadratic equation

(a) $x^2 + 4x + 3 = 0$, find equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$

(b) $x^2 - 3x + 4 = 0$, find equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$

(c) $3x^2 - 4x + 1 = 0$, find equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$

(d) $x^2 - 4x + 5 = 0$, find equation whose roots are $\alpha - 2$ and $\beta - 2$

$\frac{\alpha}{\beta}$ $\frac{\beta}{\alpha}$ $\frac{\beta^2}{\alpha}$ $(\alpha+\beta)$ $(\alpha\beta)$

SIMULTANEOUS EQUATIONS.

HINT: we will be using the substitution method in solving the problem. Usually, we make x or y the subject of formula from (i) and inserted into eq. (ii) and then solving the resultant quadratic equation.

Solve the following simultaneous equations:

a. $x+y=3$ and $x^2+y^2=5$

b. $x+2y=3$ and $x^2+y^2=5$

c. $x+y=3$ and $x^2-y^2=5$

d. $x+y=3$ and $x^2-y^2=4$

e. $x+2y=3$ and $2x^2+y^2=5$

f. $x-y=3$ and $x^2+y^2=5$

SOLUTIONS

(a) $x+y=3$ —— (i)

$x^2+y^2=5$ —— (ii)

from (i) $x+y=3$

$y=3-x$

from (ii) $x^2+y^2=5$

$\Rightarrow x^2+(3-x)^2=5$

$x^2+(9+x^2-6x)=5$

$x^2+9+x^2-6x=5$

$2x^2-6x+9-5=0$

$2x^2-6x+4=0$

$2x^2-4x-2x+4=0$

$2x(x-2)-2(x-2)=0$

$(x-2)(2x-2)=0$

$x-2=0$ or $2x-2=0$

$x=2$ or $x=1$

But $y=3-x$

when $x=2$

$y=3-2=1$

when $x=1$

$y=3-1=2$

$\therefore x=2, y=1$ and $x=1, y=2$

(b) $x+2y=3$ —— (i)

$x^2+y^2=5$ —— (ii)

from (i) $x+2y=3$

$x=3-2y$

from (ii) $x^2+y^2=5$

$(3-2y)^2+y^2=5$

$(9+4y^2-12y)+y^2=5$

$9+4y^2-12y+y^2-5=0$

$5y^2-12y+4=0$

$5y^2-10y-2y+4=0$

$5y(y-2)-2(y-2)=0$

$(y-2)(5y-2)=0$

$y-2=0$ or $5y-2=0$

$y=2$ or $y=2/5$

But $x=3-2y$

when $y=2$

$x=3-2(2)=3-4$

$x=-1$

when $y=2/5$

$x=3-2(2/5)$

$x = \frac{5}{3} + y$

ⓑ Let $\sqrt{3+2\sqrt{2}} = \sqrt{x}+\sqrt{y}$ —— (i)

and $\sqrt{3-2\sqrt{2}} = \sqrt{x}-\sqrt{y}$ —— (ii)

Square bothsides of (i)

$(\sqrt{3+2\sqrt{2}})^2 = (\sqrt{x}+\sqrt{y})^2$

$\Rightarrow 3+2\sqrt{2} = x+y+2\sqrt{xy}$

∴ Comparing the non-surdic part,

$x+y = 3$ —— ⓐ

Multiplying (i) and (ii)

$(\sqrt{3+2\sqrt{2}})(\sqrt{3-2\sqrt{2}}) = (\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$

$\sqrt{(3+2\sqrt{2})(3-2\sqrt{2})} = (\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$

employing the idea of difference of two squares above

$(a+b)(a-b) = a^2-b^2$

$\therefore \sqrt{3^2-(2\sqrt{2})^2} = (\sqrt{x})^2-(\sqrt{y})^2$

$\sqrt{9-8} = x-y$

$\therefore x-y = 1$ —— ⓑ

Solving ⓐ and ⓑ Simultaneously

$x+y=3$

$x-y=1$

$2x = 4$

$x = 4/2 = 2$

from ⓐ: $x+y=3$

$y = 3-x$

$y = 3-2$

$y = 1$

$\therefore x=2,\ y=1$

$\sqrt{3+2\sqrt{2}} = \sqrt{x}+\sqrt{y}$

$= \sqrt{2}+\sqrt{1}$

$= \sqrt{2}+1$ or $1+\sqrt{2}$

ⓒ $\sqrt{5-2\sqrt{2}} = \sqrt{x}-\sqrt{y}$ —— (i)

$\sqrt{5+2\sqrt{2}} = \sqrt{x}+\sqrt{y}$ —— (ii)

Square bothsides of (i)

$(\sqrt{5-2\sqrt{2}})^2 = (\sqrt{x}-\sqrt{y})^2$

$5-2\sqrt{2} = x+y-2\sqrt{xy}$

Comparing non-surdic part,

$x+y = 5$ —— ⓐ

Multiplying (i) and (ii)

$(\sqrt{5-2\sqrt{2}})(\sqrt{5+2\sqrt{2}}) = (\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$

$\sqrt{(5-2\sqrt{2})(5+2\sqrt{2})} = (\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$

Recall: $(a+b)(a-b) = a^2-b^2$

$\therefore \sqrt{5^2-(2\sqrt{2})^2} = (\sqrt{x})^2-(\sqrt{y})^2$

$\sqrt{25-8} = x-y$

$\sqrt{17} = x-y$

$x-y = \sqrt{17}$ —— ⓑ

Solving ⓐ and ⓑ Simultaneously

$x+y = 5$

$x-y = \sqrt{17}$

$2x = 5+\sqrt{17}$

$x = \frac{5+\sqrt{17}}{2}$

from ⓐ: $x+y = 5$

$y = 5-x$

$y = 5-\frac{(5+\sqrt{17})}{2}$

$y = \frac{10-(5+\sqrt{17})}{2}$

$y = \frac{10-5-\sqrt{17}}{2} = \frac{5-\sqrt{17}}{2}$

$\therefore \sqrt{5-2\sqrt{2}} = \sqrt{x}-\sqrt{y}$

$= \sqrt{\frac{5+\sqrt{17}}{2}} - \sqrt{\frac{5-\sqrt{17}}{2}}$

iv- The system of equation deduced from (iii) above is evaluated using the simultaneous equation to find the values of x and y.

v. Always remember that $x > y$ (the value of x is always greater than the value of y)

Find the square root of the following:

(a) $\sqrt{4+2\sqrt{3}}$ A. $1+\sqrt{3}$ B. $3-\sqrt{3}$ C. $2-\sqrt{3}$ D. $\sqrt{3}-1$

(b) $3+2\sqrt{3}$ A. $3-\sqrt{2}$ B. $2-\sqrt{2}$ C. $1+\sqrt{2}$ D. $\sqrt{2}-1$

(c) $5-2\sqrt{2}$ - (Subjective)

(d) $3-2\sqrt{2}$ A. $\sqrt{3}-\sqrt{2}$ B. $\sqrt{2}-1$ C. $1-\sqrt{2}$ D. $\sqrt{3}+\sqrt{2}$

(e) $4-2\sqrt{3}$ A. $\sqrt{3}-1$ B. $1-\sqrt{3}$ C. $2-\sqrt{3}$ D. $4-\sqrt{3}$

$\sqrt{x} \pm \sqrt{y}$

SOLUTIONS

(a) Let $\sqrt{4+2\sqrt{3}} = \sqrt{x}+\sqrt{y}$

Square bothsides above:

$(\sqrt{4+2\sqrt{3}})^2 = (\sqrt{x}+\sqrt{y})^2$

$\Rightarrow 4+2\sqrt{3} = (\sqrt{x}+\sqrt{y})(\sqrt{x}+\sqrt{y})$

$4+2\sqrt{3} = x+\sqrt{xy}+\sqrt{xy}+y$

$4+2\sqrt{3} = x+y+2\sqrt{xy}$

Comparing bothsides

$x+y = 4$ ——— (i)

$\cancel{2}\sqrt{xy} = \cancel{2}\sqrt{3}$

$\therefore xy = 3$

$y = 3/x$ ——— (ii)

put $y = 3/x$ into (i)

$\therefore x + \frac{3}{x} = 4$

multiply bothsides by x

$x(x+\frac{3}{x}) = 4x$

$x^2+3 = 4x$

$x^2-4x+3=0$

$x^2-x-3x+3=0$

$x(x-1)-3(x-1)=0$

$(x-1)(x-3)=0$

$x-1=0$ or $x-3=0$

$x=1$ or $x=3$

But $y = 3/x$

$\therefore$ when $x=1$

$y = 3/1 = 3$

when $x=3$

$y = 3/3 = 1$

$\therefore x=1, y=3$ or $x=3, y=1$

Since $x>y \Rightarrow x=3, y=1$

$\therefore \sqrt{4+2\sqrt{3}} = \sqrt{3}+\sqrt{1}$

$= \sqrt{3}+1$ or $1+\sqrt{3}$

(d) $2^{2x} - 2^{x+3} + 15 = 0$

Hint: clearly, we see that we can rewrite the 1st and 2nd terms of the expression on the left hand side of the equation in terms of 2^x by exploring the idea of indices.

$\therefore (2^x)^2 - 2^x \cdot 2^3 + 15 = 0 \Rightarrow (2^x)^2 - (2^x) \cdot 8 + 15 = 0$ ——— (*)

Let $2^x = p$; eqn (*) becomes

$p^2 - 8p + 15 = 0 \Rightarrow p^2 - 3p - 5p + 15 = 0$

$\Rightarrow p(p-3) - 5(p-3) = 0 \Rightarrow (p-3)(p-5) = 0$

$\Rightarrow p - 3 = 0$ or $p - 5 = 0 \Rightarrow p = 3$ or $p = 5$

Recall that $2^x = p$

$\therefore$ when $p = 3 \Rightarrow 2^x = 3 \Rightarrow \log 2^x = \log 3$

$\Rightarrow x\log 2 = \log 3 \Rightarrow x = \frac{\log 3}{\log 2} = \log_2 3$

when $p = 5 \Rightarrow 2^x = 5 \Rightarrow \log 2^x = \log 5$

$\Rightarrow x\log 2 = \log 5 \Rightarrow x = \frac{\log 5}{\log 2} = \log_2 5$

$\therefore x = \log_2 3$ or $\log_2 5$

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SURDS:

HINT: The square root of a surd is obtained by

i- equating the square root of the surd to $\sqrt{x} \pm \sqrt{y}$ as the case may be.

ii- Next, we square bothsides of (i) above where the L.H.S reduces to the expression without a square root symbol, and the R.H.S transforms into $x + y \pm 2\sqrt{xy}$ as the case may be.

iii- Compare the L.H.S and R.H.S of (ii) above to have an equation of the form:

$x + y = a$

$2\sqrt{xy} = b\sqrt{c}$

$\sqrt{xy} = \sqrt{3}$

Square both sides;

$(\sqrt{xy})^2 = (\sqrt{3})^2 \Rightarrow xy = 3$

$y = \frac{3}{x}$ ——— (ii)

put $\frac{3}{x}$ for y in (i)

$x + \frac{3}{x} = 4 \Rightarrow x^2 + 3 = 4x$

$x^2 - 4x + 3 = 0 \Rightarrow (x-3)(x-1) = 0$

$x - 3 = 0$ or $x - 1 = 0 \Rightarrow x = 3$ or $x = 1$

HINT: Between $x=3$ and $x=1$, we choose the larger value of x as our desired value; $x = 3$

$\therefore y = \frac{3}{x} = \frac{3}{3} = 1 \quad (x=3, y=1)$

$\sqrt{4+2\sqrt{3}} = \sqrt{x} + \sqrt{y} = \sqrt{3} + \sqrt{1}$

$= \sqrt{3} + 1$

Since addition is Commutative, we have

$\sqrt{4+2\sqrt{3}} = \boxed{1 + \sqrt{3}}$ Ans

(6) $x + 2y = 3$ ——— (i)

$x^2 + y^2 = 5$ ——— (ii)

from (i) $x = 3 - 2y$ put into (ii)

$(3-2y)^2 + y^2 = 5$

$9 - 12y + 4y^2 + y^2 = 5$

$5y^2 - 12y + 9 - 5 = 0$

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(4)

$5y^2 - 12y + 4 = 0 \Rightarrow 5y^2 - 10y - 2y + 4 = 0$

$5y(y-2) - 2(y-2) = 0 \rightarrow (y-2)(5y-2) = 0$

$y - 2 = 0$ or $5y - 2 = 0 \Rightarrow y = 2$ or $y = 2/5$

Recall : $x = 3 - 2y$

when $y = 2$; $x = 3 - 2(2) = 3 - 4 = -1$

$(-1, 2)$

when $y = 2/5$; $x = 3 - 2(\frac{2}{5}) = 3 - \frac{4}{5} = \frac{11}{5}$

$(\frac{11}{5}, \frac{2}{5})$

$\therefore$ $\boxed{x = -1, y = 2 \text{ and } x = \frac{11}{5}, y = \frac{2}{5}}$ Ans

(7) $x^6 - x^4 - x^3 + 5 \div x + 1$

using the synthetic division algorithm,

HINT : For proper division we introduce place holders $0x^0$ for missing terms.

$\therefore x^6 - x^4 - x^3 + 5 = x^6 + 0x^5 - x^4 - x^3 + 0x^2 + 0x + 5$

$$\begin{array}{r|rrrrrrr} -1 & 1 & 0 & -1 & -1 & 0 & 0 & 5 \\ & \downarrow & -1 & 1 & 0 & 1 & -1 & 1 \\ \hline & 1 & -1 & 0 & -1 & 1 & -1 & 6 \end{array}$$

(6) = Rem

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$\therefore$ $\boxed{\text{Remainder} = 6}$

(8) The number of ways of arranging 5 people on a bench is 5! ways $= 5 \times 4 \times 3 \times 2 \times 1 = \boxed{120 \text{ ways}}$

(9) $\frac{1}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$

(5)

$1 = A(x-1) + B(x+2)$

Let $x = -2$

$1 = A(-2-1) + B(-2+2) = A(-3) + \cancel{B(0)}$

$1 = -3A \Rightarrow A = \frac{-1}{3}$

Let $x = 1$

$1 = A(1-1) + B(1+2) = \cancel{A(0)} + B(3)$

$1 = 3B \Rightarrow B = \frac{1}{3}$

$\therefore A = \frac{-1}{3}, \; B = \frac{1}{3}$

$$\frac{1}{x^2+x-2} = \frac{\frac{-1}{3}}{x+2} + \frac{\frac{1}{3}}{x-1}$$

$$= \boxed{\frac{1}{3(x-1)} - \frac{1}{3(x+2)}} \text{ Ans}$$

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(10) using the Rational Root Test $= \pm\left(\frac{\text{Factors of constant term}}{\text{Factors of Leading Coefficient}}\right)$

Constant term $= 2 = 1, 2$

Leading coefficient $= 1 = 1$

$\therefore RRT = \pm\left(\frac{1, 2}{1}\right) = \pm(1, 2)$

Testing $x = -2$: $x^3 + 2x^2 - x - 2 = (-2)^3 + 2(-2)^2 - (-2) - 2$

$= -8 + 8 + 2 - 2 = 0$

$\therefore x = -2$ is a root $\Rightarrow x+2$ is a factor

HINT: From all possible options, only option C fits into the RRT.

$\therefore \boxed{(x-1)(x+1)(x+2)}$ Ans

SOLUTIONS TO 2016/2017 EXAMS

1. Using the formula $\boxed{{}^{n-k}C_{r-k}}$

n = Total number of players available = 13 players
r = Standard football team to be selected = 11 players
k = number of particular players that MUST be selected = 3.

$\therefore {}^{13-3}C_{11-3} = {}^{10}C_8 = \frac{10!}{(10-8)!\,8!} = \frac{10!}{2!\,8!}$

$= \frac{10 \times 9 \times \cancel{8!}}{2 \times 1 \cancel{8!}} = 5 \times 9 = 45$ ways (C)

2. $\frac{2x^3+1}{x^3-x^2+x-1}$

Since the variable with the highest power is found both at the numerator and denominator, we employ the LONG DIVISION METHOD before applying any of CaseI to Case III to resolve the P.F

$$\begin{array}{r|l} & \quad 2 \\ x^3-x^2+x-1 & 2x^3+0x^2+0x+1 \\ (-) & 2x^3-2x^2+2x-2 \\ \hline & \qquad 2x^2-2x+3 \end{array}$$

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$\therefore \frac{2x^3+1}{x^3-x^2+x-1} \equiv 2 + \frac{2x^2-2x+3}{x^3-x^2+x-1}$

Next, we resolve the rational part into its partials using Case I to Case III

$\therefore x^3-x^2+x-1 = (x^3-x^2)+(x-1) = x^2(x-1)+1(x-1)$
$= (x-1)(x^2+1)$

$\frac{2x^2-2x+3}{x^3-x^2+x-1} = \frac{2x^2-2x+3}{(x-1)(x^2+1)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$\therefore 2x^2 - 2x + 3 = A(x^2+1) + (Bx+C)(x-1)$

Let $x = 1$

$2(1)^2 - 2(1) + 3 = A(1^2+1) + (B(1)+C)(1-1)$

$\cancel{2} - \cancel{2} + 3 = A(2) + \cancel{(B+C)(0)}$

$\therefore 3 = 2A \Rightarrow A = \frac{3}{2}$

Let $x = 0$

$\cancel{2(0)^2} - \cancel{2(0)} + 3 = A(0^2+1) + (B(0)+C)(0-1)$

$3 = A - C$

$C = A - 3 = \frac{3}{2} - 3 = \frac{3-6}{2}$

$C = \frac{-3}{2}$

Let $x = 2$

$2(2)^2 - 2(2) + 3 = A(2^2+1) + (B(2)+C)(2-1)$

$8 - 4 + 3 = 5A + 2B + C$

$7 = 5\left(\frac{3}{2}\right) + 2B - \frac{3}{2}$

$7 = \frac{15}{2} + 2B - \frac{3}{2}$

$14 = 15 + 4B - 3$

$14 = 12 + 4B$

$2 = 4B$

$B = \frac{1}{2}$

$\boxed{A = \frac{3}{2},\ B = \frac{1}{2},\ C = \frac{-3}{2}}$

$\therefore \frac{2x^3 - 1}{x^3 - x^2 + x - 1} = 2 + \frac{3}{2(x-1)} + \frac{x-3}{2(x^2+1)}$ D

Mr. Nottty ⑦

③ using the Rational Root/Zero Test,

$$RRT = \pm\left(\frac{\text{Factors of Constant term}}{\text{Factors of Leading coefficient}}\right)$$

$\therefore 6x^4 - 10x^3 - 11x^2 + 13x + 2$

Factors of constant term (2) = 1, 2

Factors of Leading Coefficient (6) = 1, 2, 3, 6

$\therefore RRT = \pm\left(\frac{1,2}{1,2,3,6}\right) = \pm\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 2, \frac{2}{3}\right)$

Testing $x = 1$,

$6(1)^4 - 10(1)^3 - 11(1)^2 + 13(1) + 2$

$= 6 - 10 - 11 + 13 + 2 = 0$

$\therefore x = 1$ is a root and $x - 1 = 0$ is a factor.

$$\begin{array}{r|rrrrr} 1 & 6 & -10 & -11 & 13 & 2 \\ & & 6 & -4 & -15 & -2 \\ \hline & 6 & -4 & -15 & -2 & 0 \\ & x^3 & x^2 & x & K & \end{array}$$

$\Rightarrow \frac{6x^4 - 10x^3 - 11x^2 + 13x + 2}{x - 1}$

$= 6x^3 - 4x^2 - 15x - 2$

Testing $x = 2$

$6(2)^3 - 4(2)^2 - 15(2) - 2$

$= 48 - 16 - 30 - 2 = 0$

$\therefore x_1 = 2$ is a root and $x - 2$ is a factor

$$\begin{array}{r|rrrr} 2 & 6 & -4 & -15 & -2 \\ & & 12 & 16 & 2 \\ \hline & 6 & 8 & 1 & 0 \end{array}$$

$\Rightarrow \frac{6x^3 - 4x^2 - 15x - 2}{x - 2}$

$= 6x^2 + 8x + 1$

Using the formula method =

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 6,\ b = 8,\ c = 1$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(6)(1)}}{2(6)} = \frac{-8 \pm \sqrt{64 - 24}}{12}$$

$$x = \frac{-8 \pm \sqrt{40}}{12} = \frac{-8 \pm 2\sqrt{10}}{12}$$

$$x = \frac{-8 + 2\sqrt{10}}{12} \text{ or } \frac{-8 - 2\sqrt{10}}{12}$$

$$x = \frac{-4 + \sqrt{10}}{6} \text{ or } \frac{-4 - \sqrt{10}}{6}$$

$\therefore$ the factors are $\underline{(x-1)(x-2)(6x+4-\sqrt{10})(6x+4+\sqrt{10})}$

(4) $x \notin A \cap B^c \Rightarrow x \notin A \ \cap \ x \notin B^c$

$\Rightarrow x \notin A$ and $x \notin B^c$ (D)

(5) using $(x+y)^n = {}^nC_r x^{n-r} y^r$ NOTTY
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$\left(x^2 - \frac{1}{2x}\right)^{10} \Rightarrow x = x^2,\ y = \frac{-1}{2x},\ n = 10$

$\therefore\ {}^{10}C_r (x^2)^{10-r} \left(\frac{-1}{2x}\right)^r \Rightarrow {}^{10}C_r\, x^{20-2r} \cdot \frac{-1^r}{(2x)^r}$

PTO

Mr. NOTTY (7)

$$^{10}C_r\, x^{20-2r} \cdot \frac{-1^r}{2^r x^r} \Rightarrow {}^{10}C_r \frac{-1^r}{2^r} \cdot \frac{x^{20-2r}}{x^r}$$

$$\therefore {}^{10}C_r \frac{-1^r}{2^r} \cdot x^{20-2r-r} = {}^{10}C_r \frac{-1^r}{2^r} \cdot x^{20-3r} \quad \text{——— } (*)$$

Equating x^{20-3r} to x^2

$$\Rightarrow x^{20-3r} = x^2$$

$$20-3r = 2$$
$$20-2 = 3r$$
$$3r = 18$$
$$r = 6$$

put $r=6$ into $(*)$.

$$^{10}C_6 \frac{(-1)^6}{2^6} \cdot x^{20-18} = \frac{10!}{4!\,6!} \cdot \frac{1}{64} \cdot x^2$$

$$\frac{10 \cdot \cancel{9}^{3} \cdot \cancel{8} \cdot 7 \cdot \cancel{6!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{6!}} \cdot \frac{x^2}{64} = \frac{210 \cdot x^2}{64} = \frac{105}{32} x^2$$

$\therefore$ the coefficient of x^2 is $\frac{105}{32}$ (C)

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(6) The solution is obtained by taking the ratio of the [illegible] (usually at both the numerator and [illegible] highest power /index.

$$\therefore \left(\frac{1+7n^4}{3n^4+n^2-n+4}\right)_{n=1}^{\infty} = \frac{7}{3} = 2\tfrac{1}{3} \quad (D)$$

⑦ Let the sum be: $a-2d + a-d + a + a+d + a+2d = 37$

$$\cancel{a-2d} + \cancel{a-d} + a + \cancel{a+d} + \cancel{a+2d} = 37$$

$$5a = 37 \Rightarrow a = \frac{37}{5}$$

Let the sum of squares be $(a-2d)^2 + (a-d)^2 + a^2 + (a+d)^2 + (a+2d)^2 = 3$

$$= (a^2 - 4ad + 4d^2) + (a^2 - 2ad + d^2) + a^2 + (a^2 + 2ad + d^2) + (a^2 + 4ad + 4d^2) = 3$$

$$= 5a^2 + 10d^2 = 347 \Rightarrow 10d^2 = 347 - 5a^2$$

$$10d^2 = 347 - 5\left(\frac{37}{5}\right)^2 = 347 - 5\left(\frac{1369}{25}\right) = 347 - \frac{1369}{5}$$

$$10d^2 = \frac{1735 - 1369}{5} = \frac{366}{5}$$

$$d^2 = \frac{366}{50} = \frac{183}{25}$$

$$d = \pm\sqrt{\frac{183}{25}} = \pm\frac{\sqrt{183}}{5}$$

$$\therefore a = \frac{37}{5};\ d = \frac{\sqrt{183}}{5} \text{ or } -\frac{\sqrt{183}}{5}$$

⑧ $\therefore S_n = \dfrac{a(r^n - 1)}{r - 1}$ (since $r > 1$)

$r = 2,\ S_n = 5115,\ n = 10$

$$\therefore 5115 = \frac{a(2^{10} - 1)}{2 - 1} = \frac{a(2^{10} - 1)}{1} = a(2^{10} - 1)$$

$$\therefore a = \frac{5115}{1024 - 1} = \frac{5115}{1023} = 5$$

$\therefore a = 5$ Ⓐ

$S_\infty = \frac{a}{1-r}$, $a = 3$, $S_\infty = \frac{7}{2}$

$\therefore \frac{7}{2} = \frac{3}{1-r} \Rightarrow 7(1-r) = 6 \Rightarrow 7 - 7r = 6$

$7 - 6 = 7r \Rightarrow 1 = 7r \Rightarrow r = \frac{1}{7}$ (D)

$2\cos^2\theta - 3\cos\theta + 1 = 0$

Let $\cos\theta = t$:

$2t^2 - 3t + 1 = 0 \Rightarrow 2t^2 - 2t - t + 1 = 0$

$2t(t-1) - 1(t-1) = 0 \Rightarrow (t-1)(2t-1) = 0$

$t = 1$ or $t = \frac{1}{2}$

Recall $\cos\theta = t$

when $t = 1$

$\cos\theta = 1$

$\Rightarrow \theta = 0°$

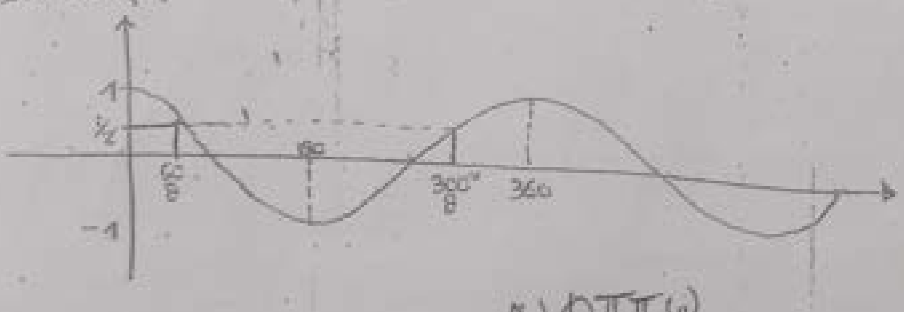
when $t = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60°, 300°$

$\therefore \theta = -300°, -60°, 0°, 60°, 300°$ (C)

Cosine Graph



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1) $\frac{1}{1\cdot01} = (1\cdot01)^{-1} = (1+0\cdot01)^{-1}$

Recall $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$

$\therefore n = -1, \; x = 0\cdot01$

$$(1+0\cdot01)^{-1} = 1 + (-1)(0\cdot01) + \frac{(-1)(-1-1)}{2!}(0\cdot01)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}(0\cdot01)^3 + \cdots$$

$$= 1 - 0\cdot01 + \frac{\cancel{2}}{\cancel{2}\times1}(0\cdot0001) + \frac{-\cancel{6}}{\cancel{3}\times\cancel{2}\times1}(0\cdot000001) + \cdots$$

$$= 1 - 0\cdot01 + 0\cdot0001 - 0\cdot000001 + \cdots$$

$$= (1+0\cdot0001) - (0\cdot01 + 0\cdot000001) + \cdots$$

$$= 1\cdot0001 - 0\cdot010001$$

$$= \underline{0\cdot990099} \quad \text{D}$$

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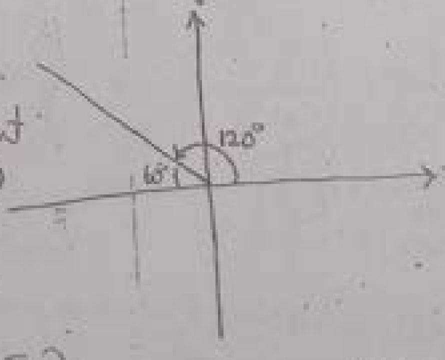
(15) $\frac{3}{2}\sin^{-1}\left(\frac{y}{7}\right) = \pi \implies \sin^{-1}\left(\frac{y}{7}\right) = \frac{2\pi}{3}$

$\implies \frac{y}{7} = \sin\left(\frac{2\pi}{3}\right) \implies \frac{y}{7} = \sin\left(\frac{2\times180}{3}\right)$

$\implies \frac{y}{7} = \sin(120^\circ)$



NOTE: We can obtain the sine equivalent of 120° using the quadrant (2nd Quad) by traversing about the horizontal axis. Hence, $\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$.

$\therefore \frac{y}{7} = \frac{\sqrt{3}}{2} \implies y = \frac{7}{2}\sqrt{3}$ (A)

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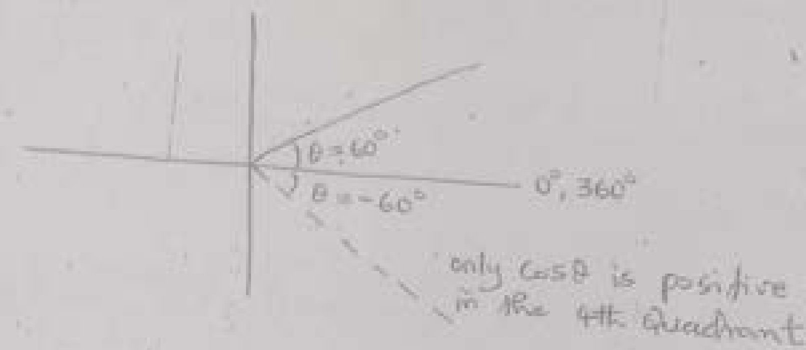
We can equally use the quadrant to find θ



(11) If a coin is tossed once, we will have 2 possible outcomes (Head or Tail)

Also, if a die is thrown once, we will have 6 possible outcomes (1, 2, 3, 4, 5 or 6)

$\therefore$ The total no of possible outcomes $= 2 \times 6 = \underline{12 \text{ out}}\text{comes}$

(12) $\sin\theta = \frac{1}{3}$

$\therefore \cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{1}{3}\right)^2 = 1 - \frac{2}{9}$

$= \frac{7}{9}$ (B)

(13) $\sin^2 x\,(\sec^2 x + \operatorname{cosec}^2 x)$

Note: $\sec x = \frac{1}{\cos x}$ and $\operatorname{cosec} x = \frac{1}{\sin x}$

$\Rightarrow \sin^2 x\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) = \frac{\sin^2 x}{\cos^2 x} + \frac{\sin^2 x}{\sin^2 x}$

$= \frac{\sin^2 x}{\cos^2 x} + 1 = \tan^2 x + 1 = \sec^2 x$

But $\sec^2 x = \frac{1}{\cos^2 x} = \frac{1}{1 - \sin^2 x}$ (D)

(16) $\sin 4\theta = \sin(2\theta + 2\theta) = 2\sin 2\theta \cos 2\theta$

$= 2[2\sin\theta\cos\theta \cdot (\cos^2\theta - \sin^2\theta)]$

$= 2[2\cos^3\theta\sin\theta - 2\sin^3\theta\cos\theta]$

$= \underline{\underline{4\cos^3\theta\sin\theta - 4\sin^3\theta\cos\theta}}$ (C)

(17) Let α^2+1 and β^2+1 be the roots of $3x^2-4x+1=0$

then $\alpha^2+1 + \beta^2+1 = \frac{-b}{a} = \frac{4}{3}$

$\alpha^2+\beta^2+2 = \frac{4}{3} \Rightarrow \alpha^2+\beta^2 = \frac{4}{3} - 2 = -\frac{2}{3}$ —(i)

also, $(\alpha^2+1)(\beta^2+1) = \frac{c}{a} = \frac{1}{3}$

$\alpha^2\beta^2 + \alpha^2 + \beta^2 + 1 = \frac{1}{3} \Rightarrow (\alpha\beta)^2 + (\alpha^2+\beta^2) = \frac{1}{3} - 1 = -$

$\therefore (\alpha\beta)^2 + (\alpha^2+\beta^2) = \frac{-2}{3} \Rightarrow (\alpha\beta)^2 = \frac{-2}{3} - (\alpha^2+\beta^2)$

$(\alpha\beta)^2 = \frac{-2}{3} - \left(\frac{-2}{3}\right) = \frac{-2}{3} + \frac{2}{3} = 0$

$\alpha\beta = 0$ —(ii)

from (i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{-2}{3}$

$(\alpha+\beta)^2 - 2(0) = -2/3 \Rightarrow \alpha+\beta = \pm\sqrt{\frac{-2}{3}} = \pm\frac{\sqrt{-2}}{\sqrt{3}}$

$\therefore$ The new equation is $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

$\therefore x^2 - (\alpha+\beta)x + \alpha\beta = 0 \Rightarrow x^2 - \left(\pm\frac{\sqrt{-2}}{\sqrt{3}}\right)x + 0 = 0$

$\therefore x^2 - \frac{x\sqrt{-2}}{3} + 0 = 0$

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$\Rightarrow x^2\sqrt{3} - x\sqrt{-2} = 0$ (B)

(18) $[(A^c \cap B) \cup (B \cap C^c)]^c = (A^c \cap B)^c \cap (B \cap C^c)^c$

$= (A \cup B^c) \cap (B^c \cup C) = (B^c \cup A) \cap (B^c \cup C)$

$= B^c \cup (A \cap C)$ (C)

(19) $2^{2x} - 2^{x+3} + 15 = 0 \Rightarrow (2^x)^2 - 2^x \cdot 2^3 + 15 = 0$

Let $2^x = t$

$\therefore t^2 - 8t + 15 = 0 \Rightarrow t^2 - 5t - 3t + 15 = 0$

$t(t-5) - 3(t-5) = 0 \Rightarrow (t-3)(t-5) = 0$

$t = 3$ or $t = 5$

$\boxed{\text{Recall } 2^x = t}$

When $t = 3$

$2^x = 3$

$x \log 2 = \log 3$

$x = \frac{\log 3}{\log 2}$

$x = \log_2 3$

When $t = 5$

$2^x = 5$

$x \log 2 = \log 5$

$x = \frac{\log 5}{\log 2}$

$x = \log_2 5$

$\therefore x = \log_2 3$ or $\log_2 5$

(20) $\dfrac{2x}{x+4} > x-2 \Rightarrow \dfrac{2x}{x+4} - x + 2 > 0$

$\Rightarrow \dfrac{2x - x(x+4) + 2(x+4)}{x+4} > 0 \Rightarrow \dfrac{\cancel{2x} - x^2 - \cancel{4x} + \cancel{2x} + 8}{x+4} >$

$\Rightarrow \dfrac{8-x^2}{x+4} > 0$ —— (*)

We will solve for x in x+4, then exclude its value ($x \neq -4$) from the solution set. Next, we equate the numerator ($8-x^2$) zero and solve also for x

$\therefore x+4 = 0 \Rightarrow x = -4$

Eqn (*) is satisfied whenever $x \neq -4$.

$\therefore 8 - x^2 = 0 \Rightarrow x^2 = 8 \Rightarrow x = \pm\sqrt{8} \Rightarrow x = \pm 2\sqrt{2}$

$\therefore x = 2\sqrt{2}$ or $-2\sqrt{2}$

$-\infty$ —— -4 —— $-2\sqrt{2}$ —— $2\sqrt{2}$ —— $+\infty$

The intervals $(-\infty, -4)$ and $(-2\sqrt{2}, 2\sqrt{2})$ will satisfy the condition for eqn (*), as its values show that the L.H.S is greater than the R.H.S

$\therefore$ The solution set is $(-\infty, -4) \cup (-2\sqrt{2}, 2\sqrt{2})$ (D)

(21) $\sqrt{4-2\sqrt{3}} = \sqrt{x} - \sqrt{y}$

$\therefore 4 - 2\sqrt{3} = x + y - 2\sqrt{xy}$

$x + y = 4$ —— (i)

$2\sqrt{xy} = 2\sqrt{3}$

$\Rightarrow xy = 3$

$y = \dfrac{3}{x}$ —— (ii)

from (i) $x + \dfrac{3}{x} = 4$

$\therefore x^2 - 4x + 3 = 0$

$x^2 - x - 3x + 3 = 0$

$x(x-1) - 3(x-1) = 0$

$(x-1)(x-3) = 0$

$\therefore x = 1$ or $x = 3$

We choose $x = 3$, since $x > y$.

$\therefore y = \dfrac{3}{x} = \dfrac{3}{3} = 1$

$\therefore x = 3, y = 1$

$\sqrt{4-2\sqrt{3}} = \sqrt{x} - \sqrt{y}$

$= \sqrt{3} - 1$ (D)

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(22) $x + 2y = 3 \Rightarrow x = 3 - 2y$

$2x^2 + y^2 = 5 \Rightarrow 2(3-2y)^2 + y^2 = 5 \Rightarrow 2(9 - 12y + 4y^2) + y^2 = 5$

$= 18 - 24y + 8y^2 + y^2 = 5 \Rightarrow 9y^2 - 24y + 13 = 0$

(18) $a = 9,\ b = -24,\ c = 13$

$$\therefore y = \frac{24 \pm \sqrt{(-24)^2 - 4(9)(13)}}{2(9)} = \frac{24 \pm \sqrt{576 - 468}}{18} = \frac{24 \pm \sqrt{108}}{18}$$

$$y = \frac{24 \pm 6\sqrt{3}}{18} = \frac{6(4 \pm \sqrt{3})}{18} = \frac{4 \pm \sqrt{3}}{3}$$

$$y = \frac{4+\sqrt{3}}{3} \text{ or } y = \frac{4-\sqrt{3}}{3}.$$

when $y = \frac{4+\sqrt{3}}{3}$

$\Rightarrow x = 3 - 2\left(\frac{4+\sqrt{3}}{3}\right)$

$= \frac{9 - 8 - 2\sqrt{3}}{3}$

$x = \frac{1 - 2\sqrt{3}}{3}$

when $y = \frac{4-\sqrt{3}}{3}$

$\Rightarrow x = 3 - 2\left(\frac{4-\sqrt{3}}{3}\right)$

$= \frac{9 - 8 + 2\sqrt{3}}{3}$

$x = \frac{1 + 2\sqrt{3}}{3}$

$\therefore \left(\frac{1+2\sqrt{3}}{3}, \frac{4-\sqrt{3}}{3}\right)$ and $\left(\frac{1-2\sqrt{3}}{3}, \frac{4+\sqrt{3}}{3}\right)$

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(23) Let $p(x)$ be the polynomial

$\therefore p(x) = x^3 + x^2 - bx + 7$

put $x = -3$ into $p(x)$ and equate to 1

$\therefore p(-3) = (-3)^3 + (-3)^2 - b(-3) + 7 = 1$

$= -27 + 9 + 3b + 7 = 1$

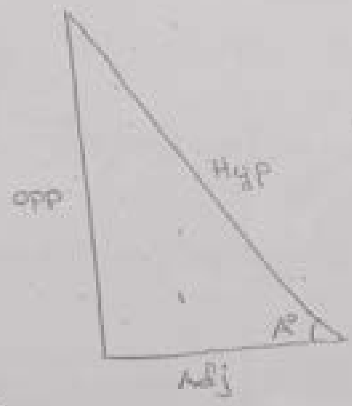
$3b = 1 + 27 - 7 - 9$

$3b = 12$

$\therefore b = 4$ (B)

TRIGONOMETRY AND RELATED FORMULA $\longrightarrow$

BASIC TRIGONOMETRIC RATIO :



1. $\sin A = \dfrac{opp}{Hyp}$ 2. $\cos A = \dfrac{Adj}{Hyp}$

3. $\tan A = \dfrac{opp}{Adj}$ 4. $\sec A = \dfrac{1}{\cos A}$

5. $\operatorname{cosec} A = \dfrac{1}{\sin A}$ 6. $\cot A = \dfrac{1}{\tan A}$

NOTE : 1π RADIAN $\equiv 180^\circ$
2π RADIAN $\equiv 360^\circ$

TRIGONOMETRIC IDENTITIES :

1 2 2 1
1 5 4 3 1

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1. $\cos^2 A + \sin^2 A = 1$
2. $1 + \tan^2 A = \sec^2 A$
3. $\cot^2 A + 1 = \operatorname{cosec}^2 A$
4. $\sin(-A) = -\sin A$
5. $\cos(-A) = \cos A$
6. $\tan(-A) = -\tan A$

NOTE : $\cos^2 A = (\cos A)^2$, WHICH IMPLIES THAT $\cos^2 A \neq \cos A^2$

ADDITION FORMULA OR DOUBLE ANGLE FORMULA

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$

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3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$

4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$

5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

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NEXT: $\boxed{\text{SUPPOSE } A = B \Rightarrow A+B = A+A = 2A}$

7. $\sin(A+A) = \sin 2A = \sin A\cos A + \cos A\sin A$
$= 2\sin A\cos A$

8. $\sin(A-A) = \sin 0 = \sin A\cos A - \cos A\sin A$
$= 0$

9. $\cos(A+A) = \cos 2A = \cos A\cos A - \sin A\sin A$
$= \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

10. $\cos(A-A) = \cos 0 = \cos A\cos A + \sin A\sin A$
$= \cos^2 A + \sin^2 A$
$= 1$

11. $\tan(A+A) = \tan 2A = \dfrac{\tan A + \tan A}{1 - \tan A\tan A}$
$= \dfrac{2\tan A}{1 - \tan^2 A}$

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12. $\tan(A - A) = \tan 0 = \dfrac{\tan A - \tan A}{1 + \tan A \tan A}$

$$= \frac{0}{1 + \tan^2 A}$$

$$= 0$$

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HALF ANGLE FORMULA:

SUPPOSE $A = \frac{A}{2} + \frac{A}{2}$, THEN

1. $\sin A = \sin\left(\frac{A}{2} + \frac{A}{2}\right) = \sin\frac{A}{2}\cos\frac{A}{2} + \cos\frac{A}{2}\sin\frac{A}{2}$

$$= 2\sin\frac{A}{2}\cos\frac{A}{2}$$

2. $\cos A = \cos\left(\frac{A}{2} + \frac{A}{2}\right) = \cos\frac{A}{2}\cos\frac{A}{2} - \sin\frac{A}{2}\sin\frac{A}{2}$

$$2^* \begin{cases} = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} \\ = 2\cos^2\frac{A}{2} - 1 \\ = 1 - 2\sin^2\frac{A}{2} \end{cases}$$

THE FOLLOWING CAN BE DEDUCED FROM 2* ABOVE:

3*. $\sin^2\frac{A}{2} = \frac{1}{2}\left[1 - \cos A\right]$

4*. $\cos^2\frac{A}{2} = \frac{1}{2}\left[1 + \cos A\right]$

5*. $\tan^2\frac{A}{2} = \dfrac{1 - \cos A}{1 + \cos A}$

6. $\tan A = \tan\left(\frac{A}{2} + \frac{A}{2}\right) = \dfrac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$

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PRODUCT FORMULA:

1. $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$
2. $\sin A \sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$
3. $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
4. $\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

SUPPOSE $A+B = x$ AND $A-B = y$, THEN FROM SIMULTANEOUS EQUATION WE GET THAT $A = \frac{x+y}{2}$, $B = \frac{x-y}{2}$

5. $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
6. $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$
7. $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
8. $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

NOTE: $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{2t}{1+t^2}$ (WHERE $t = \tan\theta$)

<u>SPECIAL ANGLE FORMULA FOR</u> $0°, 30°, 45°, 60°$ AND $90°$

FOR $0°$: $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$

$\text{cosec}\, 0°$ = UNDEFINED, $\sec 0° = 0$, $\cot 0°$ = UNDEFINED

<u>FOR $30°$</u>: $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$

$\text{cosec}\, 30° = 2$, $\sec 30° = \frac{2}{\sqrt{3}}$, $\cot 30° = \sqrt{3}$

For 45°: $\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

$\text{cosec } 45° = \sqrt{2}$, $\sec 45° = \sqrt{2}$, $\cot 45° = 1$

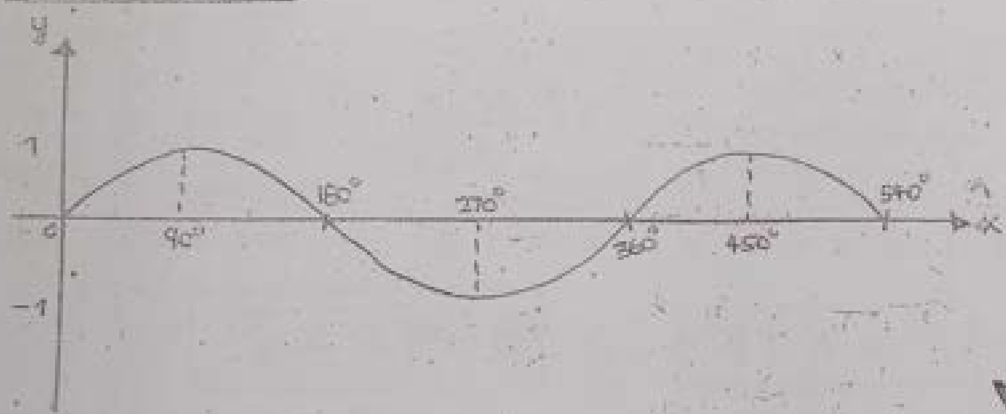
For 60°: $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

$\text{cosec } 60° = 2$, $\sec 60° = 2$, $\cot 60° = \frac{1}{\sqrt{3}}$

For 90°: $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ = UNDEFINED

$\text{cosec } 90° = 1$, $\sec 90°$ = UNDEFINED, $\cot 90° = 0$

SINE GRAPH:



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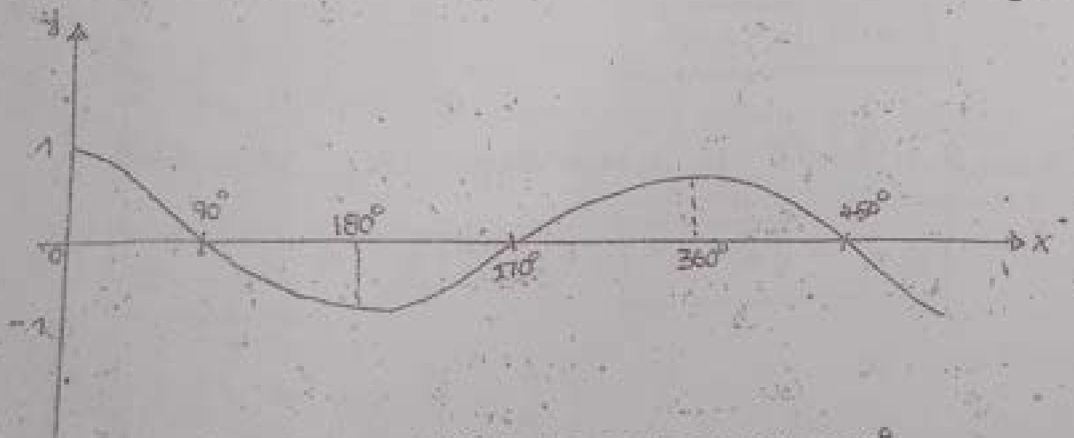
COSINE GRAPH:



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FOR ANGLES ABOVE 90° BUT LESS THAN 360°

$\sin 180° = 0$ $\cos 180° = -1$ $\tan 180° = 0$

$\sin 270° = -1$ $\cos 270° = 0$ $\tan 270°$ = UNDEFINED

$\sin 360° = 0$ $\cos 360° = 1$ $\tan 360° = 0$

EQUATION OF THE FORM: $A\cos\theta + B\sin\theta = R\cos(\theta - \alpha)$

$A\cos\theta + B\sin\theta = R\cos(\theta - \alpha)$

$\boxed{A\cos\theta + B\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha}$ — (*)

COMPARING THE L.H.S AND R.H.S OF (*):

$\boxed{R\cos\alpha = A}$ AND $\boxed{R\sin\alpha = B}$

Also, $\frac{R\sin\alpha}{R\cos\alpha} = \frac{B}{A} \Rightarrow \boxed{\tan\alpha = \frac{B}{A}} \Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{B}{A}\right)}$

Also, $R^2\cos^2\alpha = A^2$ AND $R^2\sin^2\alpha = B$

Adding: $R^2\cos^2\alpha + R^2\sin^2\alpha = A^2 + B^2$

$R^2\left[\cos^2\alpha + \sin^2\alpha\right] = A^2 + B^2$ (WHERE $\cos^2\alpha + \sin^2\alpha =$

$\therefore \boxed{R^2 = A^2 + B^2} \Rightarrow \boxed{R = \pm\sqrt{A^2 + B^2}}$

HINT: $A\cos\theta + B\sin\theta$ IS A MAXIMUM IF

$\boxed{R = +\sqrt{A^2 + B^2}}$

$A\cos\theta + B\sin\theta$ IS A MINIMUM IF

$\boxed{R = -\sqrt{A^2 + B^2}}$

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PROBLEMS AND SOLUTIONS ON TRIGONOMETRY

① Find the value of $\cos\left(\frac{\pi}{12}\right)$ using the known values of sine and cosine of $\frac{\pi}{3}$ and $\frac{\pi}{4}$. Leave your answer as a surdic expression. (2010/2011)

Solution

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

Also recall that $\cos(A-B) = \cos A\cos B + \sin A \sin B$

where $A = \frac{\pi}{3} = 60^\circ$ and $B = \frac{\pi}{4} = 45^\circ$

$$\therefore \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ \quad \text{——— (i)}$$

NOTE: $\cos 60^\circ = \frac{1}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$\therefore$ eqn (i) becomes:

$$\left(\frac{1}{2} \times \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{1}{4}\left(\sqrt{2} + \sqrt{6}\right)$$

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② Find the value of $\tan\left(\frac{7\pi}{12}\right)$ using the known values of tangents of $\frac{\pi}{3}$ and $\frac{\pi}{4}$. Leave your answer as a surd expression. (2010/2011)

Solution

$$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$

Recall: $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

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...ATION OF THE FORM: $A\cos\theta + B\sin\theta = R\cos(\theta \ldots)$

where $A = \frac{\pi}{3}$ and $B = \frac{\pi}{4}$

$\therefore \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \dfrac{\tan\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{4}\right)}{1 - \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}$

NOTE: $\tan\left(\frac{\pi}{3}\right) = \tan 60^\circ = \sqrt{3}$

$\tan\left(\frac{\pi}{4}\right) = \tan 45^\circ = 1$

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$\Rightarrow \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \dfrac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)} = \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}}$ ——— (i)

We will solve eqn (i) by rationalizing the surdic expression using its conjugate:

$\therefore \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}} \times \dfrac{1 + \sqrt{3}}{1 + \sqrt{3}} = \dfrac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 + \cancel{\sqrt{3}} - \cancel{\sqrt{3}} - 3}$

$= \dfrac{4 + 2\sqrt{3}}{-2} = \dfrac{\cancel{2}(2 + \sqrt{3})}{-\cancel{2}} = \underline{-(2 + \sqrt{3})}$

(3) If $\sin\theta = \frac{3}{5}$ and $\cos\beta = \frac{12}{13}$, find $\sin(\theta + \beta)$. (2008/200...

solution

$\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$

HINT: clearly, we see that we only know values for $\sin\theta$ and $\cos\beta$, so that from the expansion above we must also find the values of $\sin\beta$ and $\cos\theta$ using any known trigonometric identity.

Recall: $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$

$\frac{9}{25} + \cos^2\theta = 1$

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(2)

$\therefore \cos^2\theta = 1 - \frac{9}{25} = \frac{25-9}{25} = \frac{16}{25}$

$\therefore \cos\theta = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

$\Rightarrow \cos\theta = \frac{4}{5}$

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Also, $\cos^2\beta + \sin^2\beta = 1$

$\left(\frac{12}{13}\right)^2 + \sin^2\beta = 1$

$\frac{144}{169} + \sin^2\beta = 1$

$\therefore \sin^2\beta = 1 - \frac{144}{169} = \frac{169-144}{169} = \frac{25}{169}$

$\sin\beta = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$

$\Rightarrow \sin\beta = \frac{5}{13}$

$\therefore \sin(\theta+\beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$

$= \left(\frac{3}{5}\times\frac{12}{13}\right) + \left(\frac{4}{5}\times\frac{5}{13}\right)$

$= \frac{36}{65} + \frac{20}{65} = \frac{56}{65}$

$\therefore \sin(\theta+\beta) = \frac{56}{65}$ //

4. Given that $\tan\theta = 2$, what is $\frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta}$? (2008/2009).

solution

Recall, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2}{1} = 2$

By observation, $\sin\theta = 2$ and $\cos\theta = 1$

$$\therefore \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta} = \frac{2+1}{1-2} = \frac{3}{-1} = \underline{-3}$$

(5) If $3\sin A - 5\cos A = R\sin(A-B)$, then $\cos B$ and $R\sin B$ are respectively?

solution.

$R\sin(A-B) = R[\sin A\cos B - \cos A\sin B]$

$= R\sin A\cos B - R\cos A\sin B$ ——— (*)

$\therefore 3\sin A - 5\cos A = R\sin A\cos B - R\cos A\sin B$ ——— (**)

comparing bothsides of (*)(**), we have

$R\cancel{\sin A}\cos B = 3\cancel{\sin A}$

$\Rightarrow R\cos B = 3 \Rightarrow \cos B = \frac{3}{R}$

Also, $R\cancel{\cos A}\sin B = 5\cancel{\cos A}$

$\Rightarrow R\sin B = 5$ ——— (ii)

But $R = \pm\sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

From (i) $R\cos B = 3$

$\Rightarrow \sqrt{34}\cos B = 3$

$\Rightarrow \cos B = \frac{3}{\sqrt{34}}$

$\therefore \cos B$ and $R\sin B = \frac{3}{\sqrt{34}}$ and 5 respectively.

(6) If $5\cos\theta + 12\sin\theta = R\cos(\theta-\alpha)$. Find $\sin^{-1}\alpha$.

Solution

$5\cos\theta + 12\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

comparing the R.H.S and L.H.S of the above equation:

$R\cos\alpha = 5$ and $R\sin\alpha = 12$

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$\therefore \sin\alpha = \frac{12}{R} \Rightarrow \sin^2\alpha = \frac{12^2}{R^2}$

But $R^2 = A^2 + B^2 = 5^2 + 12^2 = 25 + 144 = 169$

$\therefore \sin^2\alpha = \frac{144}{169}$

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(7) If $\cos\theta + \sin\theta = R\cos(\theta - \alpha)$, what is α?

Solution

$A = 1$ and $B = 1$

$\therefore \alpha = \tan^{-1}\left(\frac{B}{A}\right) \Rightarrow \alpha = \tan^{-1}\left(\frac{1}{1}\right)$

$\therefore \alpha = \tan^{-1}(1) \Rightarrow \alpha = 45^\circ$

(8) Express $2\cos x \cos 3x$ as the sum of sines and cosines.

Solution

Recall: $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\Rightarrow 2\cos A \cos B = \cos(A+B) + \cos(A-B)$

where $A = x$ and $B = 3x$

$\therefore 2\cos x \cos 3x = \cos(x+3x) + \cos(x-3x)$

$= \cos 4x + \cos(-2x)$

Recall: $\cos(-\theta) = \cos\theta \Rightarrow \cos(-2x) = \cos 2x$

$2\cos x \cos 3x = \cos 4x + \cos 2x$

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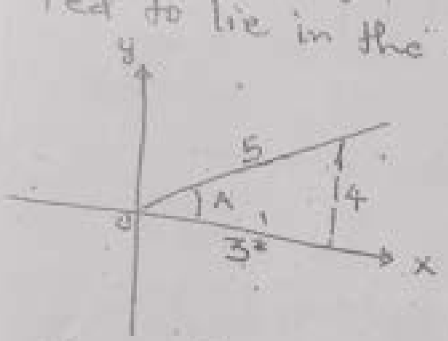
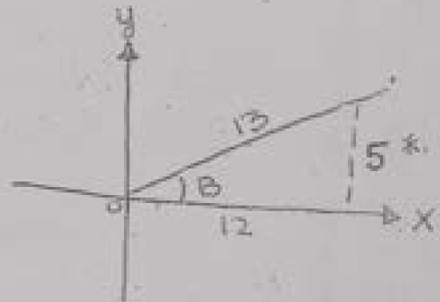
(9) If $\sin A = \frac{4}{5}$, $0 \le A \le \frac{\pi}{2}$ and $\cos B = \frac{12}{13}$; $0 \le B \le \frac{\pi}{2}$. Find the value of $\cos(A-B)$.

Solution

Hint: $0 \le T \le \frac{\pi}{2}$ implies that T lies in the 1st quadrant (angle ranging from $0°$ to $90°$). Hence, A and B are restricted to lie in the 1st quadrant.





$\sin A = \frac{4}{5}$, then using the Pythagoras theorem:

$5^2 = 4^2 + x^2$

$x^2 = 5^2 - 4^2$

$x^2 = 25 - 16$

$x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

$\therefore x = 3$ (1st quadrant)

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$\cos B = \frac{12}{13}$, then using the Pythagoras theorem

$13^2 = 12^2 + y^2$

$y^2 = 13^2 - 12^2$

$y^2 = 169 - 144$

$y^2 = 25$

$y = \pm\sqrt{25} = \pm 5$

$\therefore y = 5$ (1st quadrant).

$\Rightarrow \sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$ $\quad \cos B = \frac{12}{13}$, $\sin B = \frac{5}{13}$

$\therefore \cos(A-B) = \cos A \cos B + \sin A \sin B$

$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13}$

$= \frac{36}{65} + \frac{20}{65} = \frac{36+20}{65}$

$\cos(A-B) = \frac{56}{65}$

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AE CONCENTRATION...

(1)

(10) $Tan^2 x\,(1 + \cot^2 x)$ is identically equal to?

Solution:

NOTE: $1 + \cot^2 x = \operatorname{cosec}^2 x = \frac{1}{\sin^2 x}$

Also, $Tan^2 x = \frac{\sin^2 x}{\cos^2 x}$

$$\therefore Tan^2 x\,(1 + \cot^2 x) = \frac{\sin^2 x}{\cos^2 x}\left(\frac{1}{\sin^2 x}\right) = \frac{\cancel{\sin^2 x}}{\cos^2 x} \cdot \frac{1}{\cancel{\sin^2 x}}$$

$$= \frac{1}{\cos^2 x} = \frac{1}{1 - \sin^2 x} = \sec^2 x = 1 + \tan^2 x$$

(11) Find the values of θ in the range $0 < \theta < 360°$ for which $2\sin^2\theta - 3\sin\theta + 1 = 0$.

Solutions:

$2\sin^2\theta - 3\sin\theta + 1 = 0 \Rightarrow 2(\sin\theta)^2 - 3(\sin\theta) + 1 = 0$

Let $\sin\theta = p$

$\therefore 2p^2 - 3p + 1 = 0 \Rightarrow 2p^2 - 2p - p + 1 = 0$

$2p(p-1) - 1(p-1) = 0 \Rightarrow (p-1)(2p-1) = 0$

$p - 1 = 0$ or $2p - 1 = 0 \Rightarrow p = 1$ or $p = \frac{1}{2}$

Recall: $\sin\theta = p$

NOTTP (7)

when $p = 1$

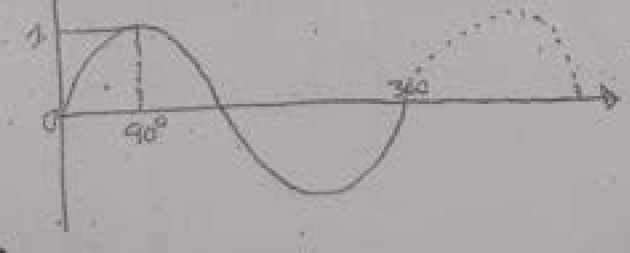
$\sin\theta = 1$

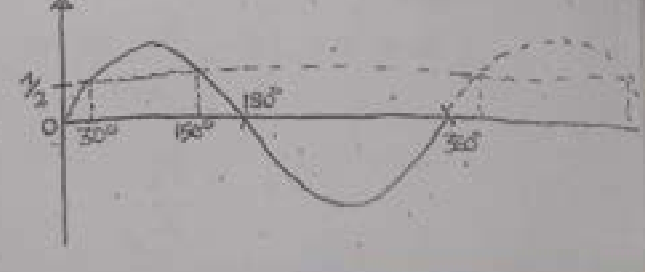
$\therefore \theta = 90°$

when $p = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$ and $150°$





(12) Find $\tan B$ if $\tan(A+B) = 2$, and $\tan A = \frac{1}{4}$.

Solution

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore 2 = \frac{\frac{1}{4} + \tan B}{1 - \frac{1}{4}\tan B} \implies 2\left(1 - \frac{1}{4}\tan B\right) = \frac{1}{4} + \tan B$$

$$2 - \frac{1}{2}\tan B = \frac{1}{4} + \tan B \implies 2 - \frac{1}{4} = \tan B + \frac{1}{2}\tan B$$

$$\frac{7}{4} = \frac{3}{2}\tan B \implies \tan B = \frac{7}{4} \times \frac{2}{3} = \frac{7}{6}$$

$$\therefore \tan B = \frac{7}{6}$$

(13) Given that $75^\circ \equiv 45^\circ + 30^\circ$, given in surd form the value of $\sin 75^\circ - \cos 75^\circ$

Solution

$$\sin 75^\circ - \cos 75^\circ = \sin(45^\circ + 30^\circ) - \cos(45^\circ + 30^\circ)$$

$$= \left[\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ\right] - \left[\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ\right]$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$$

$$= \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$$

$$\cancel{\frac{\sqrt{3}}{2\sqrt{2}}} + \frac{1}{2\sqrt{2}} - \cancel{\frac{\sqrt{3}}{2\sqrt{2}}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

(14) Find all solutions to the equation $2\sin 2A + 1 = 0$.

ANS: Since we have only one trigonometric ratio (sine) from the above problem, then the problem is easily solved by just knowing where A falls into whether 1st, 2nd, 3rd or 4th quadrant.

$\therefore$ $2\sin 2A + 1 = 0 \Rightarrow 2\sin 2A = -1$

$\Rightarrow \sin 2A = -1/2$ (Let $2A = t$)

$\therefore \sin(2A) = \sin t = -1/2$ ——— (*)

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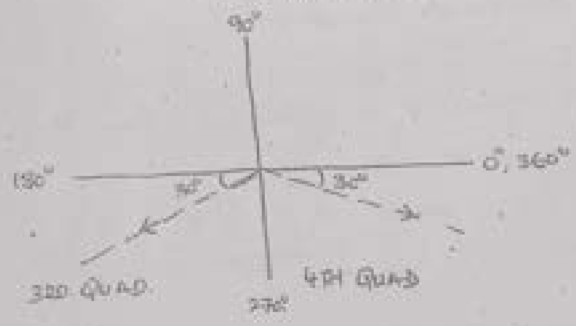
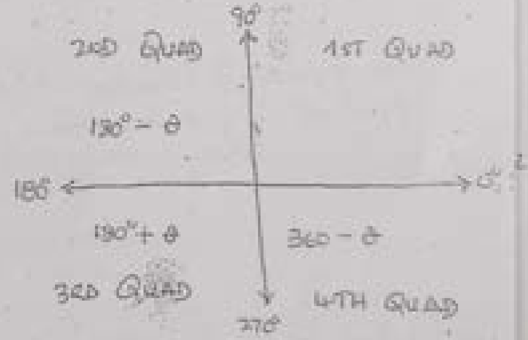
NOTE: Eqn (*) is valid if and only if t belongs to the 3rd or 4th quadrant, since it is only in these quadrants that $\sin t$ is negat

HINT: we wish to find the angle for t for which $\sin t = -1/2$. This can be done if we draw the quadrant and employ the necess formula in solving for t.





$\therefore$ Since $\frac{1}{2} = 30°$, therefore $-\frac{1}{2} = -30°$. Hence we want to find $t = -\frac{1}{2}$ in the 3rd and 4th quadrant by adjusting the arrow from the 2nd quadrant into the 3rd quadrant by $30°$ $(180° + 30°)$ and also adjusting the arrow from the 1st quadrant into the 4th quadrant by $30°$ (clockwise movement which results into $-\theta$, $360° - 30°$).

$\rightarrow$ In the 3rd Quadrant: $t = 180° + 30° = 210°$

$\therefore t = 210°$

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(9)

But $t = 2A \Rightarrow 2A = 210^\circ \Rightarrow A = 105^\circ$

Note: $1\pi = 180^\circ$

$\Rightarrow 105^\circ = \frac{105\pi}{180^\circ} = \frac{7\pi}{12}$

$\therefore A = \frac{7\pi}{12}$ (in the third Quadrant)

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Also, in the 4th Quadrant: $t = 360^\circ - 30^\circ = 330^\circ$

$\therefore t = 330^\circ$

But $t = 2A \Rightarrow 2A = 330^\circ \Rightarrow A = 165^\circ$

Note $= 180^\circ = 1\pi$ radian

$\Rightarrow 165^\circ = \frac{165\pi}{180} = \frac{11\pi}{12}$

$\therefore A = \frac{11\pi}{12}$ (in the fourth Quadrant)

The general solution to the equation is obtained from the periodicity principle:

$$A = \frac{7}{12}\pi + \pi K \quad \text{and} \quad A = \frac{11}{12}\pi + \pi K$$

(15) Find all solutions of the equation $\cos 4A = \frac{1}{2}$ in the interval $[0, 2\pi]$

#ANS: Just like example 14 above, we will rewrite 4A into a single variable, say t.

$\therefore$ Let $4A = t \Rightarrow \cos 4A = \cos t = \frac{1}{2}$

Hint: Since $t = \frac{1}{2}$ means that $\cos t = \frac{1}{2}$ is found either in the 1st Quadrant or 4th Quadrant since cosine is positive in the 1st and 4th Quadrants respectively.

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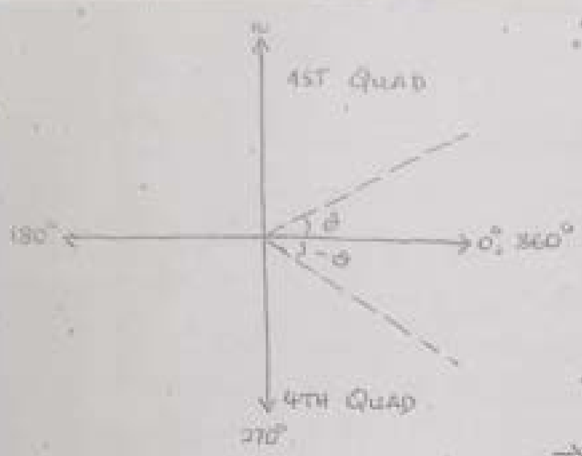

$\therefore \cos t = \frac{1}{2} \Rightarrow t = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

Since $\cos(-\theta) = \cos(\theta) = \frac{1}{2}$

$\therefore -60° \equiv 60°$ (for cosine)

$\Rightarrow t = 60° = \frac{\pi}{3}$ and $\frac{-\pi}{3}$

But $t = 4A$

$\therefore 4A = \frac{\pi}{3} + 2\pi K$ and $4A = \frac{-\pi}{3} + 2\pi$

$\Rightarrow A = \frac{\pi}{12} + \frac{\pi K}{2}$ and $A = \frac{-\pi}{12} + \frac{\pi K}{2}$

Note: To get all the solutions (not general solution), we put $K = 0, 1, 2, 3$

when $K = 0$: $A = \frac{\pi}{12} + \frac{\pi(0)}{2}$ and $\frac{-\pi}{12} + \frac{\pi(0)}{2}$

$A = \frac{\pi}{12}$ and $\frac{-\pi}{12}$ (we choose $\frac{\pi}{12}$ since $\frac{\pi}{12} \in [0, 2\pi]$)

when $K = 1$: $A = \frac{\pi}{12} + \frac{\pi(1)}{2}$ and $\frac{-\pi}{12} + \frac{\pi(1)}{2}$

$= \frac{\pi}{12} + \frac{\pi}{2}$ and $\frac{-\pi}{12} + \frac{\pi}{2}$

$= \frac{7\pi}{12}$ and $\frac{5\pi}{12}$ (we choose both since $\frac{7\pi}{12}, \frac{5\pi}{12} \in [0, 2$

when $K = 2$: $A = \frac{\pi}{12} + \frac{\pi(2)}{2}$ and $\frac{-\pi}{12} + \frac{\pi(2)}{2}$

$= \frac{\pi}{12} + \pi$ and $\frac{-\pi}{12} + \pi$

$= \frac{13\pi}{12}$ and $\frac{11\pi}{12}$ (we choose both since $\frac{13\pi}{12}, \frac{11\pi}{12} \in [0, 2$

when $K = 3$: $A = \frac{\pi}{12} + \frac{\pi(3)}{2}$ and $\frac{-\pi}{12} + \frac{\pi(3)}{2}$

$= \frac{\pi}{12} + \frac{3\pi}{2}$ and $\frac{-\pi}{12} + \frac{3\pi}{2}$

$= \frac{19\pi}{12}$ and $\frac{17\pi}{12}$ (we choose both since $\frac{13\pi}{12}, \frac{17\pi}{12} \in [0, 2$

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$\Rightarrow 2A = 210° \Rightarrow A = 105°$.

when $K=4$: $A = \frac{\pi}{12} + \frac{\pi(4)}{2}$ and $\frac{-\pi}{12} + \frac{\pi(4)}{2}$

$= \frac{\pi}{12} + 2\pi$ and $-\frac{\pi}{12} + 2\pi$

$= \frac{49\pi}{12}$ and $\frac{47\pi}{12}$ ($\frac{49\pi}{12}$ and $\frac{47\pi}{12}$ are not choosen because they donot belong to $[0, 2\pi]$)

$\therefore$ The solutions of the equation are $\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12} \text{ and } \frac{23\pi}{12}\right\}$.

(16) Find all the solutions of the equation $3\tan^3 A - \tan A = 0$ that lie in the interval $[0, 2\pi]$.

#ANS: The value of A is obtained by simplifying (factorizing) the above trigonometric equation.

$\therefore 3\tan^3 A - \tan A = 0 \Rightarrow \tan A(3\tan^2 A - 1) = 0$

$\Rightarrow \tan A = 0$ or $3\tan^2 A - 1 = 0$

$\Rightarrow \tan A = 0$ or $3\tan^2 A = 1$

$\Rightarrow \tan A = 0$ or $\tan^2 A = \frac{1}{3}$

$\Rightarrow \tan A = 0$ or $\tan A = \pm\sqrt{1/3} = \pm\frac{1}{\sqrt{3}}$

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$\therefore \tan A = 0$ or $\tan A = \frac{1}{\sqrt{3}}$ or $\tan A = \frac{-1}{\sqrt{3}}$

for $\tan A = 0 \Rightarrow A = 0°, \pi, 2\pi$

for $\tan A = \frac{1}{\sqrt{3}} \Rightarrow A = \frac{\pi}{6}, \frac{7\pi}{6}$

for $\tan A = \frac{-1}{\sqrt{3}} \Rightarrow A = \frac{5\pi}{6}, \frac{11\pi}{6}$

$\therefore$ The solutions of the above equation are $\left\{0, \frac{\pi}{6}, \frac{\pi}{6}, \pi, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi\right\}$

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(17) Solve $\cot x \cos^2 x = 2\cot x$.

ANS: Because of the presence of square on cosine, the above equation will be simplified by factorization and not using direct cancellation.

Note: The fact that we have $\cot x$ on both sides of the equation does not guarantee direct cancellation. Rather, we subtract $2\cot$ from both sides of the equation so that the RHS of the equation is zero.

Note: $\cot x = \frac{\cos x}{\sin x}$

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Note: $\frac{A}{B} = 0$ if and only if $A = 0$

$\therefore \cot x \cos^2 x = 2\cot x \Rightarrow \cot x \cos^2 x - 2\cot x = 0$

$\Rightarrow \cot x[\cos^2 x - 2] = 0 \Rightarrow \cot x = 0$ or $\cos^2 x - 2 = 0$

$\Rightarrow \cot x = 0$ or $\cos^2 x = 2$

$\Rightarrow \cot x = 0$ or $\cos x = \pm\sqrt{2}$

$\Rightarrow \cot x = 0$ or $\cos x = \sqrt{2}$ or $\cos x = -\sqrt{2}$

$\boxed{\text{for } \cot x = 0}$: $\cot x = \frac{\cos x}{\sin x} = 0$ if and only if $\cos x = 0$

$\therefore \cot x = 0$ if and only if $\cos x = 0$

Next, $\cos x = 0$ when $x = \pi/2$ and $\frac{3\pi}{2}$.

$\Rightarrow$ The general solution for $\cot x = 0$ is $\frac{\pi}{2} + 2\pi k$ and $\frac{3\pi}{2} + 2\pi k$

$\boxed{\text{for } \cos^2 x - 2 = 0}$: where $\cos x = -\sqrt{2}$ and $\sqrt{2}$

We know that the range of values for the solution of a cosine function lies btw -1 and 1. But here $-\sqrt{2}$ and $\sqrt{2}$ lie outside this range, and hence do not form part of our

(18) if $\cos 40^\circ \cos x + \sin 40^\circ \sin x = \frac{\sqrt{3}}{2}$, find x for $0^\circ \le x \le 360^\circ$

#ANS: The above trigonometric problem is solved by recalling the appropriate trigonometric identity.

$\therefore$ Recall that $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$$\therefore \cos 40^\circ \cos x = \frac{1}{2}[\cos(40^\circ+x) + \cos(40^\circ-x)] \quad \text{—— (i)}$$

Also, $\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$

$$\therefore \sin 40^\circ \sin x = -\frac{1}{2}[\cos(40^\circ+x) - \cos(40^\circ-x)] \quad \text{—— (ii)}$$

$\Rightarrow \therefore \cos 40^\circ \cos x + \sin 40^\circ \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{1}{2}[\cos(40^\circ+x) + \cos(40^\circ-x)] - \frac{1}{2}[\cos(40^\circ+x) - \cos(40^\circ-x)] = \frac{\sqrt{3}}{2}$

$= \cancel{\frac{1}{2}\cos(40^\circ+x)} + \frac{1}{2}\cos(40^\circ-x) - \cancel{\frac{1}{2}\cos(40^\circ+x)} + \frac{1}{2}\cos(40^\circ-x) = \frac{\sqrt{3}}{2}$

$= \frac{1}{2}\cos(40^\circ-x) + \frac{1}{2}\cos(40^\circ-x) = \frac{\sqrt{3}}{2}$

$= \cos(40^\circ-x) = \frac{\sqrt{3}}{2}$

Let $40^\circ - x = \theta$ —— (*)

$\therefore \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$

$\theta = 30^\circ$

But $40^\circ - x = \theta$ from (*)

$\therefore 40 - x = 30^\circ$

$x = 40^\circ - 30^\circ$

$x = 10^\circ$

The general solution for x in the given interval : $x = \theta \pm 2\pi K$

$\therefore x = \{10^\circ, 350^\circ\}$

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TRIGONOMETRY

Trigonometry is the art of doing "ALGEBRA" over a circle. He
It is a mixture of "algebra and geometry". The sine and cosine functions are just the coordinates of a point on the unit circle. Hence, the study of angles with respect its sides.

Note:

i) angles are measured in Degrees, Minutes and Sec

ii) Degrees are divided into 60 parts, each known minutes.

iii) Minutes are further divided into 60 parts, each known as seconds.

iv) One full "rotation" is equal to 360° or four right angles (4 × 90°).

v) Angles are measured in Degrees or Radians

vi) ~~The~~ The unit of angle is x^{0} or x(rad) as the case may be.

$$\therefore 1\text{rad} = \pi = 180^\circ$$

$$\therefore 2\pi = 360^\circ$$

$$\frac{\pi}{3} = 90^\circ$$

$$\frac{\pi}{4} = 45^\circ \;;\; \frac{\pi}{6} = 3$$

$$\frac{\pi}{3} = 60^\circ$$

THE RIGHT ANGLED TRIANGLE.

A Right-Angled Triangle is a triangle which ~~have~~ has one of its three angles as 90°.

Lets consider the right angled triangle below, with $A\hat{C}B$ then, the following are deductions for all SIX BASIC TRIGONOMETRIC FUNCTIONS of θ from SOH CAH TOA.

Sin

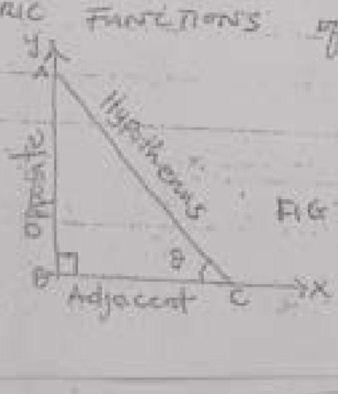


FIG I

Let Hypothenus = AC = t

opposite = AB = y

Adjacent = BC = x

SOH : $\sin\theta = \frac{Opp}{Hyp} = \frac{y}{t}$; ——— (i)

CAH : $\cos\theta = \frac{Adj}{Hyp} = \frac{x}{t}$; ——— (ii)

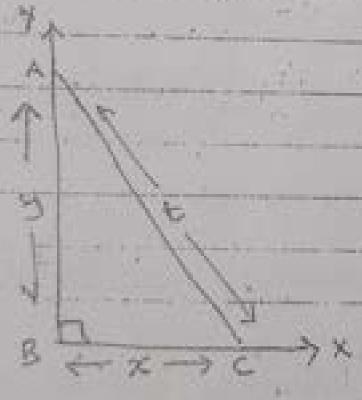
TOA : $\tan\theta = \frac{Opp}{Adj} = \frac{y}{x}$; ——— (iii)

Others include :

$\sec\theta = \frac{1}{\cos\theta} = \frac{t}{x}$ ——— (iv) ; $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{t}{y}$ ——— (v)

cotangent $\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$ ——— (vi)

From the diagram above (FIG I), we observe that the



hypothenus is the longest side. The Pythagoras theorem states that " the square of the hypothenus is the sum of the square of the opposite and the adjacent

i.e $t^2 = y^2 + x^2$ ——— (*)

Equation (*) is the mathematical form of the Pythagoras theorem. Other deductions can be obtained from this identity.

PYTHAGOREAN IDENTITIES

$$t^2 = y^2 + x^2 \text{ ——— } (*)$$

A: Eliminating x^2, we have :

$$\frac{t^2}{x^2} = \frac{y^2}{x^2} + \frac{x^2}{x^2}$$

$$\left(\frac{t}{x}\right)^2 = \left(\frac{y}{x}\right)^2 + 1$$

but $\frac{t}{x} = \sec\theta$ and $\frac{y}{x} = \tan\theta$

$$\therefore \left(\frac{t}{x}\right)^2 = \left(\frac{y}{x}\right)^2 + 1 = (\sec\theta)^2 = (\tan\theta)^2 + 1$$

$$= \tan^2\theta + 1 = \sec^2\theta \text{ ——— (a)}$$

B: Eliminating y^2, we have:

$$\frac{t^2}{y^2} = \frac{y^2}{y^2} + \frac{x^2}{y^2} = \left(\frac{t}{y}\right)^2 = 1 + \left(\frac{x}{y}\right)^2$$

but $\frac{t}{y} = \operatorname{cosec}\theta$ and $\frac{x}{y} = \cot\theta$

$$\therefore \left(\frac{t}{y}\right)^2 = 1 + \left(\frac{x}{y}\right)^2 = (\operatorname{cosec}\theta)^2 = 1 + (\cot\theta)^2$$

$$= 1 + \cot^2\theta = \operatorname{cosec}^2\theta \text{ ——— (b)}$$

C: Eliminating t^2, we get:

$$\frac{t^2}{t^2} = \frac{y^2}{t^2} + \frac{x^2}{t^2} = 1 = \left(\frac{y}{t}\right)^2 + \left(\frac{x}{t}\right)^2$$

but: $\frac{y}{t} = \sin\theta$ and $\frac{x}{t} = \cos\theta$

$$\therefore 1 = \left(\frac{y}{t}\right)^2 + \left(\frac{x}{t}\right)^2 = 1 = (\sin\theta)^2 + (\cos\theta)^2$$

$$= \sin^2\theta + \cos^2\theta = 1 \text{ ——— (c)}$$

In general we have that :-

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

Basic Pythagorean Identity.

TRIGONOMETRIC RATIOS FOR NEGATIVE ARGUMENTS :-

Whenever the angle is rotated anti-clockwise, then the angle is POSITIVE (ie $+\theta$) but when the angle is rotated clockwise, then the angle is NEGATIVE (ie $-\theta$) as shown in the diagram below :

Diagram

The following are the trigonometric ratios for negative arguments :

$$\sin(-\theta) = -\sin(\theta)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\tan(-\theta) = -\tan(\theta)$$
$$\csc(-\theta) = -\csc(\theta)$$
$$\sec(-\theta) = \sec(\theta)$$
$$\cot(-\theta) = -\cot(\theta)$$

Trigonometric ratios for negative argument.

TABLE AND SIGNS OF TRIGONOMETRIC RATIOS :

$90°$, $\frac{\pi}{2}$

QUADRANT II (Sine only)	QUADRANT I (All)
Sine is +ve , cosec is +ve	Sine is +ve , cosec is +ve
cosine is -ve , sec is -ve	cosine is +ve , sec is +ve
tan is -ve , cot is -ve	tan is +ve , cot is +ve

$(180°)$, π — $0°$, 2π $(360°)$

QUADRANT III (tan only)	QUADRANT IV (cosine only)
Sine is -ve , cosec is -ve	Sine is -ve , cosec is -ve
cosine is -ve , sec is -ve	cosine is +ve , sec is +ve
tan is +ve , cot is +ve	tan is -ve , cot is -ve

$\frac{3\pi}{2}$, $270°$

PERIODICITY:

This is simply the repetition of a noted value over a particular interval.

Illustration: We know that $360^\circ = 2\pi$ (which implies a complete revolution), and suppose $\theta = \pi/2$, then $\sin\theta = \sin\frac{\pi}{2}$ ($\sin\pi/2 = \sin 90^\circ = 1$). observe at this point that as θ increases by 2π ($\sin(\theta + 2\pi) = \sin(\frac{\pi}{2} + 2\pi)$) then the value 1 repeats which is a mere repetition of $\sin\pi/2$. clearly, $\sin\frac{\pi}{2} = \sin(\frac{\pi}{2} + 2\pi) = \sin(\frac{\pi}{2} + 2\pi + 2\pi) = \sin(\frac{\pi}{2} + 2\pi + 2\pi + 2\pi)$; this continues until $\sin(\frac{\pi}{2} + 2\pi K)$ and it is still a mere repetition of $\sin(\pi/2)$ for any value of K.

In general $f(t)$ is periodic if there is a constant p such that $f(t+p) = f(T)$ $\forall T$ in the domain of f

Note: Sine, cosine, cosecant and secant have the period $p = 2\pi$ but tangent and cotangent have a period of $p = \pi$.

$$\sin(\theta + 2\pi) = \sin\theta$$
$$\cos(\theta + 2\pi) = \cos\theta$$
$$\tan(\theta + \pi) = \tan\theta$$
$$\operatorname{cosec}(\theta + 2\pi) = \operatorname{cosec}\theta$$
$$\sec(\theta + 2\pi) = \sec\theta$$
$$\cot(\theta + \pi) = \cot\theta$$

PROPERTIES OF ANGLES

The property of any angle is dependent on where the particular angle falls on the trigonometrical argand diagram ie

Quadrant II (180° − Angle)	Quadrant I (Angle = β)
Quadrant III (180 + Angle)	Quadrant IV (360 − Angle)

1. COMPLEMENTARY PROPERTY: Two angles α and β are said to be complementary if and only if they sum up to 90° (ie $\alpha + \beta = 90^\circ$).

Note: α and β need not be both positive (ie 30° and 60°) only but could also be positive and negative (ie 120° and -
if $\alpha + \beta = 90^\circ$, then $\alpha = 90^\circ - \beta$. The complement of α is $90^\circ - \alpha$ (which is β).

$\therefore$ The complement of $\sin\alpha = \cos(90^\circ - \alpha) = \cos\beta$

The complement of $\tan\alpha = \cot(90^\circ - \alpha) = \cot\beta$

The complement of $\sec\alpha = \operatorname{cosec}(90^\circ - \alpha) = \csc\beta$

ie $\sin 30^\circ = \cos(90^\circ - 30^\circ) = \cos 60^\circ$

2. SUPPLEMENTARY PROPERTY: Let α and β be two angles, the $\alpha + \beta = 180^\circ$ is called supplementary if and only if $\alpha = 180 - \beta$ where $90^\circ < \beta < 180^\circ$ and α is acute and β is obtuse.

$$\left.\begin{aligned} \sin\beta &= \sin(180 - \beta) = \sin\alpha \\ \cos\beta &= \cos(180 - \beta) = -\cos\alpha \\ \tan\beta &= \tan(180 - \beta) = -\tan\alpha \end{aligned}\right\}$$

only sine is positive [in] the 2nd Quadrant. Hence cosine and tangent are negative

3. REFLEX PROPERTY: ~~It~~ Two cases will be considered and they are (i) 3rd Quadrant case (ii) 4th Quadrant case

(i) 3RD QUADRANT CASE: if β is obtuse and $\beta > 180$ (lie in the 3rd quadrant) and α is acute then

$$\alpha \equiv \beta - 180^\circ$$

$$\left.\begin{aligned} \sin\beta &= -\sin(\beta - 180^\circ) = -\sin\alpha \\ \cos\beta &= -\cos(\beta - 180^\circ) = -\cos\alpha \\ \tan\beta &= \tan(\beta - 180^\circ) = \tan\alpha \end{aligned}\right\} \quad 180^\circ < \beta < 270^\circ$$

ie $\tan 210^\circ = \tan(210^\circ - 180^\circ) = \tan 30^\circ$

ii) 4th QUADRANT CASE : if β is obtuse and $\beta > 270^\circ$ and α is acute then
$$\alpha = 360^\circ - \beta$$

$$\left.\begin{aligned} \sin\beta &= -\sin(360^\circ - \beta) = -\sin\alpha \\ \cos\beta &= \cos(360^\circ - \beta) = \cos\alpha \\ \tan\beta &= -\tan(360^\circ - \beta) = -\tan\alpha \end{aligned}\right\} 270^\circ < \beta < 360^\circ$$

ie $\cos 290^\circ = \cos(360^\circ - 290^\circ) = \cos 70^\circ$

Hint : finally, if $\beta > 360^\circ$ the periodicity property is used to evaluate such a trigonometric function.

EXAMPLES ON PYTHAGOREAN IDENTITY.

Example 1 : Simplify the expression $\dfrac{\sec^2 A + \operatorname{cosec}^2 A}{\sec A \operatorname{cosec} A} - \tan A$

Solution :

Recall that $\sec A = \dfrac{1}{\cos A}$ and $\operatorname{cosec} A = \dfrac{1}{\sin A}$

also, $\tan A = \dfrac{\sin A}{\cos A}$

$$\therefore \frac{\sec^2 A + \operatorname{cosec}^2 A}{\sec A \operatorname{cosec} A} - \tan A = \frac{\left(\frac{1}{\cos A}\right)^2 + \left(\frac{1}{\sin A}\right)^2}{\sec A \operatorname{cosec} A} - \frac{\sin A}{\cos A}$$

$$\frac{\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}}{\frac{1}{\cos A} \cdot \frac{1}{\sin A}} - \frac{\sin A}{\cos A} = \frac{\frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}}{\frac{1}{\cos A \sin A}} - \frac{\sin A}{\cos A}$$

But $\sin^2 A + \cos^2 A = \cos^2 A + \sin^2 A = 1$

$$\therefore \frac{\frac{1}{\cos^2 A \sin^2 A}}{\frac{1}{\cos A \sin A}} - \frac{\sin A}{\cos A} = \frac{\cos A \sin A}{\cos^2 A \sin^2 A} - \frac{\sin A}{\cos A}$$

$$= \frac{1}{\cos A \sin A} - \frac{\sin A}{\cos A} = \frac{1}{\cos A}\left(\frac{1}{\sin A} - \frac{\sin A}{1}\right)$$

$$= \frac{1}{\cos A}\left(\frac{1 - \sin^2 A}{\sin A}\right) = \frac{1}{\cos A}\left(\frac{\cos^2 A}{\sin A}\right) \quad (1 - \sin^2 A = \cos$$

$$= \frac{\cos A}{\sin A} = \underline{\cot A}$$

Example 2 : Simplify the expression $\frac{(\cos A - \sin A)^2}{\cos A + \sin A} + \frac{1}{\frac{\cos A + \sin A}{2\cos A \sin A}}$

Solution:

$(\cos A - \sin A)^2 = \cos^2 A - 2\cos A \sin A + \sin^2 A$ (from binomial expansion)

$= \cos^2 A + \sin^2 A - 2\cos A \sin A$

$= 1 - 2\cos A \sin A$ (Since $\cos^2 A + \sin^2 A = 1$)

$\therefore$ The expression above simplifies as:

$$\frac{1 - 2\cos A \sin A}{\cos A + \sin A} + \frac{2\cos A \sin A}{\cos A + \sin A}$$

$$= \frac{1 - \cancel{2\cos A \sin A} + \cancel{2\cos A \sin A}}{\cos A + \sin A}$$

$$= \underline{\frac{1}{\cos A + \sin A}}$$

EXAMPLES ON PROPERTIES OF ANGLES

Example 3 : Evaluate $\sin(-930^\circ)$

Solution:

Recall that $\sin(-\theta) = -\sin\theta$

$\therefore \sin(-930^\circ) = -\sin 930^\circ$

Again since $930^\circ > 360^\circ$

then from the periodicity principle, we have that:

$\sin(-930^\circ) = -\sin 930^\circ = -\sin(2\times 360^\circ + 210^\circ)$

$= -\sin 210^\circ$

Again 210° is found in the 3rd quadrant, and in this quadrant sine is negative.

$\therefore -\sin 210^\circ = -(-\sin 210^\circ) = \sin 210^\circ$

$\Rightarrow 210^\circ = 180^\circ + \theta$

$\therefore \theta = 210^\circ - 180^\circ$

$\theta = 30^\circ$

$\therefore \sin 210^\circ = \sin(180^\circ + \theta)$

$\sin 210^\circ = \sin(180^\circ + 30^\circ)$

$\sin 210^\circ = \sin 30^\circ$

But $\sin 30^\circ = 1/2$

$\therefore \sin 210^\circ = 1/2$

finally, $\sin(-930^\circ) = \sin 210^\circ = \underline{\underline{1/2}}$

Example 4: Find in surdic form $\cos 150^\circ$.

Solution

observe that 150° is found in the 2nd quadrant, and cosine is negative in this quadrant.

$\therefore \cos 150^\circ = -\cos(180^\circ - \theta)$

$\cos 150^\circ = -\cos(180^\circ - 30^\circ)$

$\therefore \cos 150^\circ = -\cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$

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Attempt this: Find in surdic form $\tan 300^\circ$.

Example 5: ~~If $\sin A = \cos A$, find A.~~

find θ if $\cos\theta = \sin\theta$.

Solution:

Recall that $\sin\beta = \cos(90^\circ - \beta) = \cos\alpha$ (where $\alpha = 90^\circ - \beta$

$\therefore \sin\theta = \cos(90^\circ - \theta)$

if $\cos\theta = \sin\theta$

then $\cos\theta = \cos(90^\circ - \theta)$

$\Rightarrow \theta = 90^\circ - \theta \qquad \Rightarrow \theta + \theta = 90^\circ$

$\Rightarrow 2\theta = 90^\circ \qquad \Rightarrow \theta = \underline{45^\circ}$

Example 6 : if $\tan(\alpha - 10^\circ) = \cot\alpha$, find α

Solution

Recall that $\tan\theta = \cot(90^\circ - \theta)$

$\therefore \tan(\alpha - 10^\circ) = \cot\alpha$

$\Rightarrow \tan(\alpha - 10^\circ) = \tan(90^\circ - \alpha)$

$\Rightarrow \alpha - 10^\circ = 90^\circ - \alpha$

$\Rightarrow \alpha + \alpha = 90^\circ + 10^\circ$

$2\alpha = 100^\circ$

$\alpha = \underline{50^\circ}$

Example 7 : Solve for x if $\tan(45^\circ + x) + \cot(45^\circ + x) = 4$, an
$0 < x < 360^\circ$

Solution:

Recall that $\cot\theta = \frac{1}{\tan\theta} \Rightarrow \cot(45^\circ + x) = \frac{1}{\tan(45^\circ + x)}$

$\therefore \tan(45^\circ + x) + \cot(45^\circ + x) = 4$

$\Rightarrow \tan(45^\circ + x) + \frac{1}{\tan(45^\circ + x)} = 4$

Let $\tan(45^\circ + x) = p$

$p + \frac{1}{p} = 4$

$\Rightarrow p^2 + 1 = 4p$

$\Rightarrow p^2 + 4p + 1 = 0$

Solving the above quadratic using formula method, we get

$a = 1, \quad b = -4, \quad c = 1$

$p = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$p = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \dfrac{4 \pm \sqrt{16 - 4}}{2}$

$p = \dfrac{4 \pm \sqrt{12}}{2} = \dfrac{4 \pm 2\sqrt{3}}{2} = \dfrac{\cancel{2}(2 \pm \sqrt{3})}{\cancel{2}}$

$p = 2 \pm \sqrt{3}$

$\therefore p = 2 + \sqrt{3}$ or $p = 2 - \sqrt{3}$

Note: $\sqrt{3} = 1.732$

$\therefore p = 2 + 1.732 = 3.732$

or $p = 2 - 1.732 = 0.268$

Recall $\tan(45° + x) = p$

when $p = 2 + \sqrt{3} = 3.732$

$\tan(45° + x) = 3.732$

Hint: the positive sign indicates that $45 + x$ lie in the 1st quadrant or 3rd quadrant

$\therefore \tan(45° + x) = 3.732$

$45° + x = \tan^{-1}(3.732)$

$\therefore 45° + x = 75°$

for $p = 2+\sqrt{3}$; $45° + x = 75°$ (in the 1st quadrant
also $45° + x = 180° + 75° = 255°$ (in the 3rd quadrant

$\therefore$ if $45° + x = 75° \Rightarrow x = 30°$
if $45° + x = 255° \Rightarrow x = 210°$

when $p = 2 - \sqrt{3} = 0.268$
$\therefore$ ~~45°+x = 0.268~~
$\tan(45° + x) = 0.268$

Hint: This is positive again, which means that $45 + x$ will lie on the 1st quadrant or 3rd quadrant.

$\therefore \tan(45° + x) = 0.268$
$45° + x = \tan^{-1}(0.268)$
$45° + x = 15°$

$\therefore 45° + x = 15°$ (in the 1st quadrant)
$45° + x = 180 + 15° = 195°$ (in the 3rd quadrant)

if $45° + x = 15 \Rightarrow x = -30°$
if $45° + x = 195° \Rightarrow x = 150°$

But x should lie between $0 < x < 360°$
$\therefore x = 30°, \underline{\underline{150°}}$ and $210°$

ADDITION FORMULA:

Let A and B be two given angles, where (A + B) is

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad \text{—— (i)}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \quad \text{—— (ii)}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B \quad \text{—— (iii)}$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B \quad \text{—— (iv)}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad \text{—— (v)}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \quad \text{—— (vi)}$$

$$\cot(A+B) = \frac{\cos(A+B)}{\sin(A+B)} = \frac{\cos A \cos B - \sin A \sin B}{\sin A \cos B + \cos A \sin B}$$

$$\therefore \cot(A+B) = \frac{\cos A \cos B - \sin A \sin B}{\sin A \cos B + \cos A \sin B}$$

We want to rewrite the R.H.S to take the form which includes cotA, cotB, cotA cotB and 1. So, we divide all terms by sinA sinB.

$$\cot(A+B) = \frac{\frac{\cos A \cos B}{\sin A \sin B} - \frac{\sin A \sin B}{\sin A \sin B}}{\frac{\sin A \cos B}{\sin A \sin B} + \frac{\cos A \sin B}{\sin A \sin B}}$$

$$= \frac{\frac{\cos A}{\sin A}\cdot\frac{\cos B}{\sin B} - 1}{\frac{\cancel{\sin A}}{\cancel{\sin A}}\cdot\frac{\cos B}{\sin B} + \frac{\cos A}{\sin A}\cdot\frac{\cancel{\sin B}}{\cancel{\sin B}}}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A} = \frac{\cot A \cot B - 1}{\cot A + \cot B} \quad \text{—— (vii)}$$

Also, $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A} = \frac{1 + \cot A \cot B}{\cot B - \cot A}$ — viii

But, when $A = B$

$\therefore \sin(A+B) = \sin(A+A) = \sin 2A$

$$\sin 2A = \sin(A+A) = \sin A \cos A + \cos A \sin A$$
$$= 2\sin A \cos A \quad \text{— ix}$$

$$\cos 2A = \cos(A+A) = \cos A \cos A - \sin A \sin A$$
$$= \cos^2 A - \sin^2 A \quad \text{— x}$$

But $\sin^2 A = 1 - \cos^2 A$ (Since $\cos^2 A + \sin^2 A = 1$)

$\therefore$ Eqn x becomes :

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= \cos^2 A - (1 - \cos^2 A)$$
$$= \cos^2 A - 1 + \cos^2 A$$
$$= 2\cos^2 A - 1 \quad \text{— xi}$$

also, $\cos^2 A = 1 - \sin^2 A$

$\therefore$ Eqn x becomes :

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$(1 - \sin^2 A) - \sin^2 A$$
$$1 - \sin^2 A - \sin^2 A$$
$$1 - 2\sin^2 A \quad \text{— xii}$$

$$\tan 2A = \tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$= \frac{2\tan A}{1-\tan^2 A} \quad \text{———} \quad (xiii)$$

HALF ANGLE FORMULA:

Suppose $A = \frac{A}{2} + \frac{A}{2}$ and $B = \frac{\beta}{2} + \frac{\beta}{2}$

then $\sin A = \sin\left(\frac{A}{2} + \frac{A}{2}\right) = \sin\frac{A}{2}\cos\frac{A}{2} + \cos\frac{A}{2}\sin\frac{A}{2}$.

$$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} \quad \text{———} \quad (xiv)$$

$$\cos A = \cos\left(\frac{A}{2} + \frac{A}{2}\right) = \cos\frac{A}{2}\cos\frac{A}{2} - \sin\frac{A}{2}\sin\frac{A}{2}.$$

$$= \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$$

$\therefore$ $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$ ——— (**)

$$\cos A = 2\cos^2\frac{A}{2} - 1$$

$\therefore$ $2\cos^2\frac{A}{2} = \cos A + 1$

$$\cos^2 A/2 = \frac{\cos A + 1}{2} \quad \text{———} \quad (xv)$$

Also, from eqn (**) we have

$$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$$

$$\cos A = 1 - 2\sin^2\frac{A}{2}$$

$$2\sin^2\frac{A}{2} = 1 - \cos A$$

Finally $\tan^2\frac{A}{2} = \frac{\sin^2\frac{A}{2}}{\cos^2\frac{A}{2}} = \frac{\frac{1}{2}(1-\cos A)}{\frac{1}{2}(1+\cos A)}$

$$\tan^2\frac{A}{2} = \frac{1-\cos A}{1+\cos A} \quad \text{——— (xvii)}$$

PRODUCT FORMULA:

Let α and β be two given angles, then

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right] \quad \text{——— (xviii)}$$

$$\sin\alpha\sin\beta = -\frac{1}{2}\left[\cos(\alpha+\beta) - \cos(\alpha-\beta)\right] \quad \text{——— (xix)}$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right] \quad \text{——— (xx)}$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right] \quad \text{——— (xxi)}$$

SUM AND DIFFERENCE FORMULA:

Suppose $\alpha+\beta = X$ and $\alpha-\beta = y$, solving simultaneous we have:

$$\alpha + \beta = x$$
$$\alpha - \beta = y$$
$$2\alpha = x+y$$

$$\alpha = \frac{x+y}{2} \quad \text{——— (I)}$$

Also, $\alpha+\beta = X \implies \frac{x+y}{2} + \beta = x$

$$\beta = x - \frac{(x+y)}{2} \Rightarrow \beta = \frac{2x - x - y}{2}$$

$$\beta = \frac{x-y}{2} \quad \text{———} \quad (II)$$

Next, inserting I and II for α and β in xviii, xi
XX and XXi, we get:

$$\frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right] = \cos\alpha\cos\beta$$

$$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$$

$$\therefore \cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \quad \text{———} \quad (xx$$

$$-\frac{1}{2}\left[\cos(\alpha+\beta) - \cos(\alpha-\beta)\right] = \sin\alpha\sin\beta$$

$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2\sin\alpha\sin\beta$$

$$\cos y - \cos x = 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \quad \text{———} \quad (xxii$$

$$\frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right] = \sin\alpha\cos\beta$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \quad \text{———} \quad (xxiv)$$

$$\frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right] = \cos\alpha\sin\beta$$

$$\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$$

$$\therefore \sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \quad \text{———} \quad (xxv)$$

Example 8: Find $\sin 150^\circ + \sin 60^\circ$

Solution:

Recall: $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

Put $x = 150^\circ$ and $y = 60^\circ$

$$\therefore \sin 150^\circ + \sin 60^\circ = 2\sin\left(\frac{150^\circ + 60^\circ}{2}\right)\cos\left(\frac{150^\circ - 60^\circ}{2}\right)$$

$$= 2\sin\left(\frac{210^\circ}{2}\right)\cos\left(\frac{90^\circ}{2}\right)$$

$$= 2\sin 105^\circ \cos 45^\circ$$

Simplifying the above, we have

$\cos 45^\circ = \frac{1}{\sqrt{2}}$, $\sin 105^\circ = \sin(60^\circ + 45^\circ)$

$$\sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\therefore 2\sin 105^\circ \cos 45^\circ = 2\cdot\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\cdot\frac{1}{\sqrt{2}}$$

$$= \cancel{2}\cdot\frac{(\sqrt{3}+1)}{\cancel{2}\sqrt{2}}\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{3}+1}{2}$$

$$\therefore \sin 150^\circ + \sin 60^\circ = \frac{1}{2}(\sqrt{3}+1)$$

Example 9: find $\cos 90^\circ + \cos 60^\circ$

Solution

Recall: $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

where $x = 90^\circ$ and $y = 60^\circ$

$$\therefore \cos 90^\circ + \cos 60^\circ = 2\cos\left(\frac{90^\circ + 60^\circ}{2}\right)\cos\left(\frac{90^\circ - 60^\circ}{2}\right)$$

$$= 2\cos\left(\frac{150^\circ}{2}\right)\cos\left(\frac{30^\circ}{2}\right)$$

$$= 2\cos 75^\circ \cos 15^\circ$$

Simplifying the above, we have

$$\cos 75^\circ = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

where $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 30^\circ = \frac{1}{2}$, $\cos 45^\circ = \frac{1}{\sqrt{2}}$ and $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$$\cos 75^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{\cancel{\sqrt{3}}}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$\therefore \cos 90^\circ + \cos 60^\circ = 2\cos 75^\circ \cos 15^\circ$$

$$= 2 \times \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

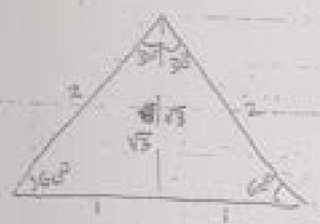
$$= \frac{\cancel{2} \times (\sqrt{3} - 1)(\sqrt{3} + 1)}{4 \times \cancel{2}}$$

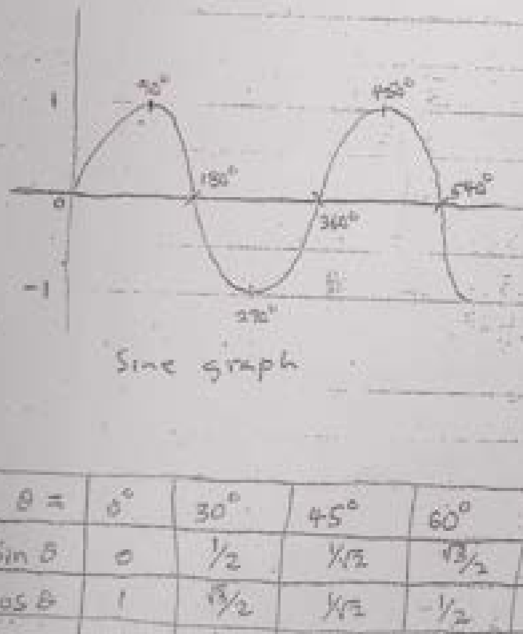
$$= \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$\therefore \cos 90^\circ + \cos 60^\circ = \frac{1}{2}$

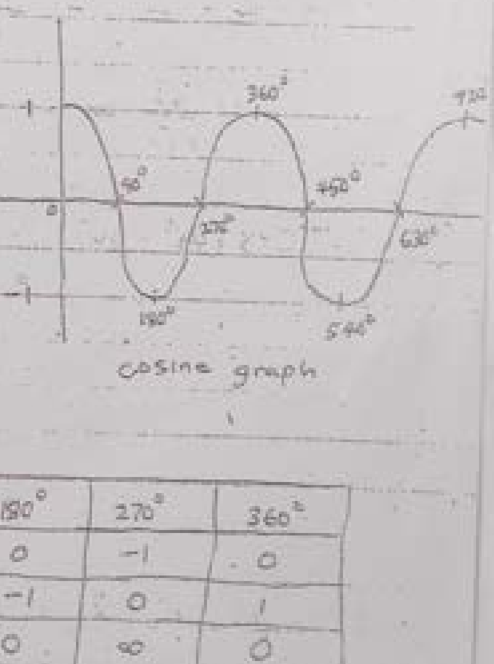
DIAGRAMS AND TABLE OF SPECIAL ANGLES:

The special angles in trigonometry are those angles that have standard values. The values can be generated without the aid of a calculating machine. Below are the diagrams used in generating these values:





Sine graph



cosine graph

$\theta =$	0°	30°	45°	60°	90°	180°	270°	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0

2015/2016 TEST SOLUTIONS

1

(1) Let α, β be the roots of $x^2 - 3x + 4 = 0$, then

$a = 1,\ b = -3,\ c = 4$

α, β

$\therefore \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3$

$\alpha\beta = \frac{c}{a} = \frac{4}{1} = 4$

Since we wish to form a new quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ then we find its sum and Product.

$$\text{Sum} = \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{(3)^3 - 3(4)(3)}{4} = \frac{27 - 36}{4} = \frac{-9}{4}$$

$$\text{Product} = \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = 4$$

$\therefore$ The new quadratic equation thus formed is

$x^2 - (\text{Sum of roots})x + \text{product of roots} = 0$

$$x^2 - \left(\frac{-9}{4}\right)x + 4 = 0$$

$$x^2 + \frac{9x}{4} + 4 = 0$$

$$\boxed{4x^2 + 9x + 16 = 0} \text{ Ans}$$

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(2) $(A^c \cap B)^c = (A^c)^c \cup B^c = A \cup B^c$

$= \boxed{B^c \cup A}$ Ans

③ $2^{x+1} = 4^{x}$ ②

$\cancel{2}^{x+1} = \cancel{2}^{2x} \Rightarrow x+1 = 2x \Rightarrow 1 = 2x - x$

$\therefore \boxed{x = 1}$ Ans

④ $x^2+x+1 > x+2$

$x^2+x+1-x-2 > 0$

$x^2 - 1 > 0$ ——— ①

$\therefore x^2 - 1 = 0 \Rightarrow (x-1)(x+1) = 0$

$x-1 = 0$ or $x+1 = 0 \Rightarrow x = 1$ or $x = -1$

$-\infty$ ——— -1 ——— 1 ——— $+\infty$

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
$x-1$	$-ve$	$-ve$	$+ve$
$x+1$	$-ve$	$+ve$	$+ve$
Product	$+ve$	$-ve$	$+ve$

Since, the product of the factors should be greater than zero (positive), then the intervals $(-\infty,-1) \cup (1,\infty)$ is the solution set.

$\boxed{(-\infty,-1) \cup (1,\infty)}$

⑤ $\sqrt{4+2\sqrt{3}} = \sqrt{x} + \sqrt{y}$

Square bothsides, we have ;

$\left(\sqrt{4+2\sqrt{3}}\right)^2 = \left(\sqrt{x}+\sqrt{y}\right)^2$

$4 + 2\sqrt{3} = x+y+2\sqrt{xy}$

comparing bothsides, we get :

$x+y = 4$ ——— (i)

$\cancel{2}\sqrt{xy} = \cancel{2}\sqrt{3}$

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AREA OF CONCENTRATION...

Mr. Notty (1).

LOGARITHM.

HINT : (1) whenever we wish to find a variable and this variable is an index/power, we introduce to Logarithm function to bring down the power before solving for x.

(2) Recall (i) $\log_y x = \frac{\log x}{\log y}$ where necessary

Solve the following equations

a. $4^{2x+1} = 3^x$ (2016/2017) Subjective

b. $2^{x+1} = 4^x$ (2015/2016)

c. $3^{2x+1} = 5^x$ (A. $\frac{-1}{2+\log_3 5}$ B. $\frac{1}{2+\log_3 5}$ C. $\frac{-1}{-1-\log_3 5}$ D. $\frac{-1}{2-\log_3 5}$)

d. $2^{2x} - 2^{x+3} + 15 = 0$

(A. $\log_2 3, -\log_2 5$ B. $2, 7.5$ C. $\log_2 3, \log_2 5$ D. $-\log_2 3, \log_2 5$)

SOLUTIONS

(a) $4^{2x+1} = 3^x$ (Take Log on bothsides)

$\log 4^{2x+1} = \log 3^x$

$(2x+1)\log 4 = x\log 3 \Rightarrow 2x\log 4 + \log 4 = x\log 3$

$\log 4 = x\log 3 - 2x\log 4$ (factor out x on R.H.S)

$\log 4 = x(\log 3 - 2\log 4)$

$\therefore x = \frac{\log 4}{\log 3 - 2\log 4}$ (divide both numerator and denominator by Log4)

$\Rightarrow x = \frac{\log 4/\log 4}{\frac{\log 3}{\log 4} - \frac{2\log 4}{\log 4}} = \frac{1}{\log_4 3 - 2}$

NOTTY

$\therefore x = \frac{1}{\log_4 3 - 2}$ or $\frac{-1}{2 - \log_4 3}$

MR. NUTTY ②

(b) $2^{x+1} = 4^x$ (Since we can write 4 in base 2, we proceed making the L.H.S and R.H.S in the same base)

$\therefore 2^{x+1} = (2^2)^x$

$\cancel{2}^{x+1} = \cancel{2}^{2x} \Rightarrow x+1 = 2x \Rightarrow 2x - x = 1$

$\Rightarrow \underline{x = 1}$

(c) $3^{2x+1} = 5^x$ (Since we cannot write 5 in base 3 and vice-versa, we take Log on both sides)

$\therefore \text{Log}\,3^{2x+1} = \text{Log}\,5^x$

$(2x+1)\text{Log}3 = x\text{Log}5 \Rightarrow 2x\text{Log}3 + \text{Log}3 = x\text{Log}5$

$\text{Log}3 = x\text{Log}5 - 2x\text{Log}3 \Rightarrow \text{Log}3 = x(\text{Log}5 - 2\text{Log}3)$

$\therefore x = \dfrac{\text{Log}3}{\text{Log}5 - 2\text{Log}3}$ ——— ⊛

Note: Eqn ⊛ above can lead us to various forms of solutions for x.

Form 1: $x = \dfrac{\text{Log}3}{\text{Log}5 - 2\text{Log}3} = \dfrac{\text{Log}3}{\text{Log}5 - \text{Log}9} = \dfrac{\text{Log}3}{\text{Log}5/9}$

recall: $\dfrac{\text{Log}\,x}{\text{Log}\,y} = \text{Log}_y x$

$\therefore x = \dfrac{\text{Log}3}{\text{Log}5/9} = \text{Log}_{5/9} 3$

Form 2: $x = \dfrac{\text{Log}3}{\text{Log}5 - 2\text{Log}3}$ (Since Log3 is found at the numerator, we divide both the numerator and denominator by Log3)

$\therefore x = \dfrac{\text{Log}3/\text{Log}3}{\dfrac{\text{Log}5}{\text{Log}3} - \dfrac{2\text{Log}3}{\text{Log}3}} = \dfrac{1}{\text{Log}_3 5 - 2}$

$\therefore x = \dfrac{1}{\text{Log}_3 5 - 2}$ or $\dfrac{-1}{2 - \text{Log}_3 5}$ //

16. Acceleration = 2m/s2 , vel = 10m/s and time = 5s what is F? (a) 5N (b) 20N (c)10N (d) 2N

17. When Δf = TsinΘ the situation is known as _____ (a)centrifugal acc (b) centripetal (c) centripetal acc (d)centrifugal

18. Centripetal force is given as _______ (a)mv/r (b)mv2/a (c)mv2/r (d)mv2/t

19. In the absence of net force a moving object will (a) slow down (b) stop immediately (c) move with constant speed (d) turn left

20. When a cat sleeps on a table the net force is on -------------(a) directed downward (b) directed upward (c) Zero(d)horizontal

21. For a cone in a neutral equilibrium, the height of the centre of gravity and the potential energy is .....? (A) unchanged (B) lowered (C) raised (D) balanced

22. The angular position of a point on a rotating wheel is given by: $\theta = 4 + 8t^2 + 4t^3$. Calculate its angular acceleration at t = 2.0s? (A) 64 rad/s^2 (B) 8 rad/s^2 (C) 12 rad/s^2 (D) 38 rad/s^2

23. A body whose parts have fixed position relative to each other is called.....?
(A) rigid (B) rotational (C) microscopic (D) fluid

24. A force F = 4i + j N acts at the point x=0 and y=2m. The torque about the origin as pivot is...?
(A) -8Nm (B) +4Nm (C) -3Nm (D) +6Nm

25. .... is regarded as translational inertia. (A) mass (B) density (C) pressure (D) weight

26. A particle has an average velocity of 108kmh^{-1}. How far does it travel in the time interval of 1 minute? .
(a) 1.8km (b) 2.3km (c) 18m (d) 3.2m

27. A car travels with a certain average velocity in 30 seconds, and covers 900m. Find the average velocity.
(a) 30ms^{-1} (b) 60ms^{-1} (c) 27000ms^{-1} (d) 27km

28. The equation of a displacement – time curve of a particle is given by $x = 10t^2$. Find the instantaneous velocity at t = 5s: (a) 100m/s (b) 40m/s (c) 50m/s (d) 2m/s

29. Find the average acceleration of a car which accelerates from 0km/h to 180km/h in 10s
(a) 5ms^{-2} (b) 18ms^{-2} (c) 1800km (d) 170ms^{-1}

30. An electron enters a region with a speed of 5 x 10^{6}m/s and is slowed down at the rate of 1.25x10^{14}m/s^2. How far does the electron travel? (a) 0.1m (b) 0.8m (c) 1.8m (d) 2.0m

31. $i.i = j.j = k.k$ is. (A) perpendicular (B) unit vector (C) unity (D) vector

Use this to answer questions 32 – 35. $a = 2i - j + 4k$, $b = i + 2j + 3k$

32. $b + a$ (A) $3i - j + 7k$ (B) $3i + j + 7k$ (C) $i + j + k$ (D) $-i + j - 7k$

33. $b - a$ (A) $2i + 2j + 8k$ (B) $i - j + 5k$ (C) $5i + 11k$ (D) $3i + j + 7k$

34. $2a + b$ (A) $4i - k$ (B) $4i - j + k$ (C) $-4i + j - k$ (D) $2i - j + k$

35. $b.a$ (A) -6 (B) 6 (C) 12 (D) 4