Practice Exam 2: The Medium Run Answer Key

ECON 304 Fall 2025

Problem 1: The 3-equation Model

Points: 100

Suppose conflict between workers and firms is expressed by the following relations:

$$\text{WS Relation: } w_r^{ws} = 1.0 - 2u$$
$$\text{PS Relation: } w_r^{ps} = \frac{1}{1+m}$$

Assume the markup $m = 0.25$.

a. (5 point)

State the condition for which the wage demands of workers are consistent with the price decision of firms and estimate the NAIRU.

Solution

Equilibrium occurs when $w_r^{ws} = w_r^{ps}$, so $1.0 - 2u_n = \frac{1}{1+0.25} = 0.8$. Solving for the NAIRU: $u_n = \frac{1.0-0.8}{2} = 0.10 = 10\%$.

b. (10 point)

If nominal wage growth is given by $\frac{\Delta w_n}{w_{-1,n}} = \pi^e + \frac{\Delta w_r^{ws}}{w}$, where $w_r = 1.0 - 2u_n$ and assuming the markup m is constant, then firms increase prices according to $\pi = \frac{\Delta w_n}{w_{-1,n}}$. With this information and the results of **part a**, derive the Phillips Curve.

Solution

Since $\pi = \pi^e + \frac{\Delta w_r^{ws}}{w}$ and $w_r^{ws} = 1.0 - 2u$, we get $\frac{\Delta w_r^{ws}}{w} = -\frac{2}{0.8}(u_t - u_n) = -2.5(u_t - 0.10)$. This gives us the Phillips Curve: $\pi = \pi^e + 2.5(0.1 - u_t)$ where $\alpha = 2.5$.

c. (5 points)

Take the derivative of the Phillips Curve to mathematically illustrate the sensitivity of the inflation rate to the unemployment rate. When is inflation stable?

Answer: $\frac{\partial \pi}{\partial u} = -2.5 < 0$. The sensitivity parameter is $\alpha = 2.5$, meaning a 1 percentage point increase in unemployment reduces inflation by 2.5 percentage points. Inflation is stable when $u = u_n = 0.10 = 10\%$ (unemployment at the natural rate).

d. (5 points)

Consider the following relations that form the foundation of the 3-equation model:

$$\text{IS: } Y_t = 1.2 - 1.5r_{t-1}$$
$$\text{PC: } \pi_t = \pi_{t-1} + 2.5(Y_t - Y_n)$$

Specify the IS relation as the output gap when $Y_n = 1.2 - 1.5r_n$.

Solution

$$Y_t - Y_n = 1.2 - 1.5r_{t-1} - 1.2 + 1.5r_n$$
$$Y_t - Y_n = 1.5(r_n - r_{t-1})$$

e. (10 points)

Suppose the central bank's loss function takes the following form:

$$L_t^{CB} = (Y_t - Y_n)^2 + 1.5(\pi_t - \bar{\pi})^2$$

Set up the central bank's loss minimization problem with its constraint.

Solution

Choose Y_t that minimizes

$$L_t^{CB} = (Y_t - Y_n)^2 + 1.5(\pi_t - \bar{\pi})^2 \text{ subject to } \pi_t = \pi_{t-1} + 2.5(Y_t - Y_n)$$
$$\text{or}$$
$$\min_{Y_t} \left\{(Y_t - Y_n)^2 + 1.5(\pi_t - \bar{\pi})^2 \mid \pi_t = \pi_{t-1} + 2.5(Y_t - Y_n)\right\}$$

f. (10 points)

Derive the central bank's monetary rule (MR). Briefly explain the intuition of the first order condition.

Solution

To minimize the central bank's loss function by choosing Y_t, derive its first order conditions

$$\frac{\partial L_t^{CB}}{\partial Y_t} = 2(Y_t - Y_n) + 2(2.5)(1.5)(\pi_{t-1} + 2.5(Y_t - Y_n) - \bar{\pi}) = 0$$

Substitute π_t back into the FOC. Isolate the output gap:

$$Y_t - Y_n = -3.75(\pi_t - \bar{\pi})$$

This is the MR relation.

g. (20 points)

Part A (Points: 10)

Derive the Interest Rate Rule from the 3-equation model. Show each step *clearly*. It may be beneficial to pick up at a convenient point in **part f**.

Solution

Solve for the output gap in the FOC without substituting π_t:

$$\begin{aligned}
-(Y_t - Y_n)(\frac{1}{1.5 \cdot 2.5} + 2.5) &= \pi_{t-1} - \bar{\pi} \\
-(Y_t - Y_n) &= \frac{1.5 \cdot 2.5(\pi_{t-1} - \bar{\pi})}{1 + 1.5 \cdot 2.5^2} \\
Y_t - Y_n &= -\frac{3.75(\pi_{t-1} - \bar{\pi})}{10.375}
\end{aligned}$$

Now substitute the IS relation by replacing the output gap. This will derive the interest rate rule:

$$r_{t-1} = r_n + \frac{3.75(\pi_{t-1} - \bar{\pi})}{1.5 \cdot 10.375} = r_n + \frac{3.75(\pi_{t-1} - \bar{\pi})}{15.56}$$

Part B (Points: 10)

Using the same parameter values $\alpha = 2.5$, $\mu = 1.5$, and $\beta = 1.5$. Suppose there is a permanent positive demand shock that shifts the IS curve to $Y_t = 1.025 - 1.5r_{t-1}$ from $Y_t = 0.975 - 1.5r_{t-1}$. The economy was initially in equilibrium at $Y_n = 0.975 - 1.5r_n$, where $r_n = r_0$ is the natural real interest rate and the initial real interest rate.

Suppose, $A = 0.975$, $A' = 1.025$, $u_n = 0.10$, $r_0 = 0.05$, $\pi_0 = \bar{\pi} = 0.02$, and the labor force is normalized to $N = 1$. Note that $Y = 1 - u_t$.

With this information, calculate output Y_t, equilibrium output Y_n, unemployment u_t, the inflation rate π_t, the real interest rate r_{t-1} and the new natural real interest rate r'_n (this changes with the shock) across a sequence of three time periods ($t = 1, 2, 3$) where the shock occurs in period 1.

The order in which the variables are listed does not translate to the order the y necessarily need to be calculated.

Solution

First, verify the initial equilibrium and estimate Y_n:

$$\begin{aligned}
Y_n &= 0.975 - 1.5(0.05) = 0.975 - 0.075 = 0.90 \\
u_n &= 1 - Y_n = 1 - 0.90 = 0.10 = 10\%
\end{aligned}$$

This confirms the economy starts in equilibrium with $Y_0 = Y_n = 0.90$ and $u_0 = u_n = 10\%$.

After the permanent demand shock, the new IS curve is $Y_t = 1.025 - 1.5r_{t-1}$. The new natural interest rate that maintains $Y_n = 0.90$ is:

$$\begin{aligned}
0.90 &= 1.025 - 1.5r'_n \\
r'_n &= \frac{1.025 - 0.90}{1.5} = \frac{0.125}{1.5} = 0.0833
\end{aligned}$$

With the new parameters, the interest rate rule becomes:

$$r_{t-1} = r'_n + \frac{3.75(\pi_{t-1} - \bar{\pi})}{15.56} = 0.0833 + \frac{3.75(\pi_{t-1} - 0.02)}{15.56}$$

Period $t = 1$:

$$\begin{aligned}
Y_1 &= 1.025 - 1.5(0.05) = 1.025 - 0.075 = 0.95 \\
u_1 &= 1 - 0.95 = 0.05 = 5\% \\
\pi_1 &= 0.02 + 2.5(0.95 - 0.90) = 0.02 + 2.5(0.05) = 0.145 \Rightarrow 14.5\% \\
r_0 &= 0.05 \text{ (initially)}
\end{aligned}$$

The real interest rate does not initially respond as the demand shock is sudden.

Period $t = 2$:

$$\begin{aligned}
r_1 &= 0.0833 + \frac{3.75(0.145 - 0.02)}{15.56} = 0.0833 + \frac{0.4688}{15.56} = 0.0833 + 0.0301 = 0.1134 \\
Y_2 &= 1.025 - 1.5(0.1134) = 1.025 - 0.1701 = 0.855 \\
u_2 &= 1 - 0.855 = 0.145 = 14.5\% \\
\pi_2 &= 0.145 + 2.5(0.855 - 0.90) = 0.145 + 2.5(-0.045) = 0.145 - 0.1125 = 0.0325 \Rightarrow 3.25\%
\end{aligned}$$

Period $t = 3$:

$$\begin{aligned}
r_2 &= 0.0833 + \frac{3.75(0.0325 - 0.02)}{15.56} = 0.0833 + \frac{0.0469}{15.56} = 0.0833 + 0.0030 = 0.0863 \\
Y_3 &= 1.025 - 1.5(0.0863) = 1.025 - 0.1295 = 0.8955 \\
u_3 &= 1 - 0.8955 = 0.1045 = 10.45\% \\
\pi_3 &= 0.0325 + 2.5(0.8955 - 0.90) = 0.0325 + 2.5(-0.0045) = 0.0325 - 0.0113 = 0.0212 \Rightarrow 2.12\%
\end{aligned}$$

Observations: The system exhibits realistic stabilization dynamics:

- Output jumps from 0.90 to 0.95 initially (unemployment falls from 10% to 5%) due to the positive demand shock
- Inflation spikes to 14.5% in period 1, prompting aggressive monetary tightening
- The central bank raises rates from 5% to 11.34%, overshooting the new natural rate (8.33%)
- This causes a contraction in period 2 where unemployment rises to 14.5%, bringing inflation down rapidly
- By period 3, the system is converging back toward equilibrium: $u = 10.45\%$, $\pi = 2.12\%$, and $r = 8.63\%$

h. (35 points)

The standard WS-PS model treats the natural rate of unemployment as fixed. But suppose wage-setting exhibits **hysteresis** - current wages depend on past unemployment through insider-outsider effects:

$$w_r^{WS} = z - au_t + \lambda u_{t-1}$$

where $\lambda > 0$ captures hysteresis strength (high past unemployment raises current wage demands as long-term unemployed become ineffective competitors). The price-setting curve remains:

$$w_r^{PS} = \frac{1}{1+m}$$

When wage demands are consistent with firm pricing behavior when **hysteresis** is present, we obtain the time-varying natural rate of unemployment $u_{n,t}$.

Part a (10 points)

Solve for the time-varying natural rate of unemployment $u_{n,t}$ with the WS-PS relations.

Solution

$$z - au_t + \lambda u_{t-1} = \frac{1}{1+m}$$
$$u_{n,t} = \frac{(z + \lambda u_{t-1})(1+m) - 1}{a(1+m)}$$

Part b (10 points)

Take the derivative of $u_{n,t}$ with respect to u_{t-1}, what is its sign and interpretation?

Solution

$$\frac{\partial u_{n,t}}{\partial u_{t-1}} = \frac{\lambda}{a} > 0$$

As the previous period's unemployment rate rises u_{t-1}, the time-varying natural rate of unemployment $u_{n,t}$ rises with it. This represents labor market hysteresis – the natural rate of unemployment is dependent on previous values of actual unemployment in the labor market.

Part c (15 points)

Consider an economy with labor market hysteresis where the time-varying natural rate evolves according to $u_{n,t} = \bar{u} + \frac{\lambda}{a} u_{t-1}$, with $\bar{u} = 0.10$, $\lambda = 0.5$ and $a = 2$.

Suppose the economy experiences a temporary negative demand shock that raises unemployment above its initial natural rate for one period.

Discuss how the adjustment process back to equilibrium differs from the standard model without hysteresis. In your answer, address:

- Whether temporary shocks have permanent effects and why. Assume $u_{n,0} = 0.13$ and calculate one period into the future $t = 1$ to help assist your explanation.
- How the central bank's stabilization policy effectiveness is affected by hysteresis
- The policy implications for responding to recessions when hysteresis is present

Solution

Adjustment Process with Hysteresis:

In the standard model without hysteresis, a temporary shock raises unemployment temporarily, but once the shock dissipates, the economy returns to the fixed natural rate u_n. The temporary increase in unemployment has no lasting effects.

With hysteresis ($\lambda > 0$), the adjustment process fundamentally changes. When unemployment rises to 13% in period 0 due to a shock, the natural rate itself increases in period 1: $u_{n,1} = 0.10 + \frac{0.5}{2}(0.13) = 0.1325 = 13.25\%$. Even after the demand shock disappears, the economy's equilibrium unemployment rate has permanently shifted upward. This creates **path dependence** – the economy's history matters for its long-run equilibrium.

Effectiveness of Stabilization Policy:

Hysteresis dramatically changes the stakes for monetary policy. In the standard model, temporary policy mistakes are simply temporary. With hysteresis, allowing unemployment to rise causes permanent scarring to the labor market. This occurs because:

- Long-term unemployed workers lose skills and exit the labor market
- Insider workers (the employed) have less competition, increasing their wage demands
- The natural rate "follows" actual unemployment upward

This makes **aggressive stabilization policy** much more important. The central bank should respond more forcefully to negative shocks to prevent unemployment from rising, as the costs are permanent rather than temporary.

Policy Implications:

Central banks should lean more aggressively against recessions than booms, since recessions cause permanent damage and make workers worse off. The traditional trade-off between inflation and unemployment changes where higher unemployment translates to a permanently higher natural rate of unemployment. This requires quick and decisive action from the central bank to prevent permanent damage to the labor market.

In summary, hysteresis transforms monetary policy from managing cyclical fluctuations to preventing permanent damage to the economy's productive capacity.
