Problem Set 3

ECON 304 – Intermediate Macroeconomics

Sean Callahan
Department of Economics • Fall 2025

2025-11-01

Assignment Information

Due Date: November 3, 2025
Percent of Final Grade: 6.0 %
Student Name: _____
Instructions: Show all work clearly. Partial credit may be awarded for correct methodology even if the final answer is incorrect. Groups of no more than three are permitted.

Submission Instructions

1. Show all mathematical work clearly
2. Include graphs where requested
3. Explain economic intuition behind your answers
4. Submit by **November 3, 2025** at the beginning of class.
5. Late submissions will be penalized 10% per day

Office Hours: Wednesday, 12:00-2:00 PM; Friday, 12:00-1:00 PM; or by appointment

This problem set covers topics related to the long run capital accumulation, economic growth, and technological innovation.

Problem 1: The Canonical Solow Model

Points: 40

Suppose aggregate output in the economy is characterized by $Y_t = F(L_t, K_t) = L_t^{\alpha} K_t^{\beta}$, where labor L and capital K are the only inputs into production. Further, assume no population growth and constant participation and employment rates.

Question 1

Points: 5

Mathematically define constant (CRS), decreasing (DRS) and increasing returns to scale (IRS) with the given production function.

Question 2

Points: 10

Under CRS, DRS, and IRS, show whether output is characterized by diminishing marginal products of labor and capital or not. If it is not, explain.

Question 3

Points: 5

For the questions 3 and 4, assume $F(L, K_t) = L^{3/4} K^{1/4}$, $s = 0.2$ *and* $\delta = 0.1$.

Given what you know about aggregate output, derive the capital accumulation equation (or capital motion equation) by relating the future capital stock to today's investment/saving decision s and the depreciation of capital δ. Express this in terms of labor (i.e., per worker). Assume the government runs a balanced budget.

Question 4

Points: 10

Solve for the steady-state level of capital per worker k^* and output per worker y^*. What is the output-capital ratio and is it constant? What is the growth rate of output and the growth rate of capital? How, if at all, do they differ? Finally, what does the level of economic growth mean for workers in the long run?

Question 5

Points: 10

Suppose the production function is $F(H, K_t) = H^{3/4} K_t^{1/4}$, where $H = hL$ and h represents human capital or education/skills per worker. Solve for the steady-state level of output per worker y^*. How does this result differ from that in the previous question? Can h explain some of the variation in incomes across countries? *You may use the same values of* s *and* δ *as before.*

Problem 2: The Solow Model with technological progress

Points: 80

Suppose the economy is characterized by the production function $Y_t = F(A_t L_t, K_t) = (A_t L_t)^{1/2} K_t^{1/2}$, where A_t represents total factor productivity (TFP) or the level of technology in the economy. Assume the population and labor force grow at the rate $g_N = 0.02$ and TFP grows at a rate $g_A > 0$.

Further assume there is some savings rate $0 \leq s \leq 1$ and depreciation rate $\delta = 0.1$.

Question 1

Points: 10

Assuming a balanced government budget, derive the capital accumulation equation in effective labor terms.

Question 2

Points: 15

Solve for the balanced growth path (BGP) level of capital per effective worker $\tilde{k}^*$ and output per effective worker $\tilde{y}^*$.

Question 3

Points: 15

Solve for the output-capital ratio, what does it depend on? What must the growth rates of output and capital be? Finally, what is the growth rate of output per *worker*?

Question 4

Points: 40

Suppose instead that $g_A = a\frac{Y_t}{A_t L_t} = a\tilde{y}_t$, where $a > 0$ and g_A represents a Kaldor-Verdoorn effect.

Derive the modified capital accumulation equation and solve for $\tilde{k}^*$ and $\tilde{y}^*$ on the BGP. Show how the savings rate affects long-run growth and compare this to the Solow model when g_A was exogenous. Might this modified model explain why some countries are rich and others are poor? Does this model predict convergence or divergence? Explain all of your answers.