Practice Exam 2: The Medium Run

ECON 304 Fall 2025

Problem 1: The 3-equation Model

Points: 100

Suppose conflict between workers and firms is expressed by the following relations:

$$\text{WS Relation: } w_r^{ws} = 1.0 - 2u$$
$$\text{PS Relation: } w_r^{ps} = \frac{1}{1+m}$$

Assume the markup $m = 0.25$.

a. (5 point)

State the condition for which the wage demands of workers are consistent with the price decision of firms and estimate the NAIRU.

b. (10 point)

If nominal wage growth is given by $\frac{\Delta w_n}{w_{-1,n}} = \pi^e + \frac{\Delta w_r^{ws}}{w}$, where $w_r = 1.0 - 2u_n$ and assuming the markup m is constant, then firms increase prices according to $\pi = \frac{\Delta w_n}{w_{-1,n}}$. With this information and the results of **part a**, derive the Phillips Curve.

c. (5 points)

Take the derivative of the Phillips Curve to mathematically illustrate the sensitivity of the inflation rate to the unemployment rate. When is inflation stable?

d. (5 points)

Consider the following relations that form the foundation of the 3-equation model:

$$\text{IS: } Y_t = 1.2 - 1.5r_{t-1}$$
$$\text{PC: } \pi_t = \pi_{t-1} + 2.5(Y_t - Y_n)$$

Specify the IS relation as the output gap when $Y_n = 1.2 - 1.5r_n$.

e. (10 points)

Suppose the central bank's loss function takes the following form:

$$L_t^{CB} = (Y_t - Y_n)^2 + 1.5(\pi_t - \bar{\pi})^2$$

Set up the central bank's loss minimization problem with its constraint.

f. (10 points)

Derive the central bank's monetary rule (MR). Briefly explain the intuition of the first order condition.

g. (20 points)

Part A (Points: 10)

Derive the Interest Rate Rule from the 3-equation model. Show each step *clearly*. It may be beneficial to pick up at a convenient point in **part f**.

Part B (Points: 10)

Using the same parameter values $\alpha = 2.5$, $\mu = 1.5$, and $\beta = 1.5$. Suppose there is a permanent positive demand shock that shifts the IS curve to $Y_t = 1.025 - 1.5r_{t-1}$ from $Y_t = 0.975 - 1.5r_{t-1}$. The economy was initially in equilibrium at $Y_n = 0.975 - 1.5r_n$, where $r_n = r_0$ is the natural real interest rate and the initial real interest rate.

Suppose, $A = 0.975$, $A' = 1.025$, $u_n = 0.10$, $r_0 = 0.05$, $\pi_0 = \bar{\pi} = 0.02$, and the labor force is normalized to $N = 1$. Note that $Y = 1 - u_t$.

With this information, calculate output Y_t, equilibrium output Y_n, unemployment u_t, the inflation rate π_t, the real interest rate r_{t-1} and the new natural real interest rate r_n' (this changes with the shock) across a sequence of three time periods ($t = 1, 2, 3$) where the shock occurs in period 1.

The order in which the variables are listed does not translate to the order they necessarily need to be calculated.

h. (35 points)

The standard WS-PS model treats the natural rate of unemployment as fixed. But suppose wage-setting exhibits **hysteresis** - current wages depend on past unemployment through insider-outsider effects:

$$w_r^{WS} = z - au_t + \lambda u_{t-1}$$

where $z = 1.0$, $a = 2$, and $\lambda > 0$ captures hysteresis strength (high past unemployment raises current wage demands as long-term unemployed become ineffective competitors). The price-setting curve remains:

$$w_r^{PS} = \frac{1}{1+m}$$

where $m = 0.25$ as before. When wage demands are consistent with firm pricing behavior when **hysteresis** is present, we obtain the time-varying natural rate of unemployment $u_{n,t}$.

Part a (10 points)

Solve for the time-varying natural rate of unemployment $u_{n,t}$ with the WS-PS relations.

Part b (10 points)

Take the derivative of $u_{n,t}$ with respect to u_{t-1}, what is its sign and interpretation?

Part c (15 points)

Consider an economy with labor market hysteresis where the time-varying natural rate evolves according to $u_{n,t} = \bar{u} + \frac{\lambda}{a} u_{t-1}$, with $\bar{u} = 0.10$, $\lambda = 0.5$ and $a = 2$.

Suppose the economy experiences a temporary negative demand shock that raises unemployment above its initial natural rate for one period.

Discuss how the adjustment process back to equilibrium differs from the standard model without hysteresis. In your answer, address:

- Whether temporary shocks have permanent effects and why. Assume $u_{n,0} = 0.13$ and calculate one period into the future $t = 1$ to help assist your explanation.
- How the central bank's stabilization policy effectiveness is affected by hysteresis
- The policy implications for responding to recessions when hysteresis is present

---