F_K/F_π from Möbius Domain-Wall fermions solved on gradient-flowed dynamical HISQ ensembles

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We perform a lattice QCD calculation of F_K/F_π using Möbius Domain-Wall fermions computed on gradient-flowed $N_f = 2+1+1$ HISQ ensembles. The calculation is performed with five values of the pion mass ranging from $130 \lesssim m_\pi \lesssim 400$ MeV, three lattice spacings of $a \sim 0.09, 0.12$ and 0.15 fm and multiple values of the lattice volume. The interpolation to the physical point is performed with mixed-action effective field theory and discretization-enhanced continuum chiral perturbation theory with a sub-percent total uncertainty. The final resuls is consistent with the FLAG average at the 1σ level, providing an important benchmark point for our new lattice action.

I. INTRODUCTION

The leptonic decay constants of the pion and kaon are *gold plated* quantities [] that can be precisely determined using lattice QCD (LQCD). As such, they provide critical input to constraints on new physics by allowing a determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements when combined with the experimental leptonic decay rates of the pions and kaons. Within the Standard Model (SM), the CKM matrix is unitary matrix, providing strict constraints on various sums of the matrix elements. There is a substantial flavor physics program dedicated to searching indirectly for beyond the SM physics through violations of these constraints [].

Lattice QCD calculations of these and other gold plated quantities is now a mature endeavor, with the recent formation of the Flavionet Lattice Averaging Group (FLAG) which performs global averages

mature - now FLAG

universality of continuum limit

scale setting

OLD BELOW

The decay constants of light pseudoscalar mesons are 'gold-plated' quantities, where state-of-the-art lattice QCD post-dictions have determined this value consistently with sub-percent precision. In particular, the ratio F_k/F_π is a dimensionless quantity and provides a straightforward and precise quantity to demonstrate control over systematic uncertainties for specific implementations of lattice actions. When combined with the experimental pion and kaon leptonic decay widths, the ratio also yields one of the most precise determinations of the ratio $|V_{us}|/|V_{ud}|$.

We calculate F_k/F_π on the lattice ensembles generated by the MILC collaboration with $N_f = 2+1+1$ dynamical flavors using the highly improved staggered quark (HISQ) action, and the one-loop improved Symanzik improved gauge action. The gauge ensembles are further smeared using a 4-dimensional gradient-flow procedure, suppressing UV artifacts. For the valence sector, we employ the Möbius domain-wall action and tune the action such that the residual chiral symmetry breaking is less than 10% of the input light-quark mass. Further details of this mixed-action setup are discussed in detail in Ref. .

In the following sections we ...

II. THEORY BACKGROUND

The renormalized decay constants can be calculated directly from the five-dimensional Ward identity satisfied by the Domain-wall action,

$$F^{q_1 q_2} = z_p^{q_1 q_2} \frac{m^{q_1} + m_{\text{res}}^{q_1} + m^{q_2} + m_{\text{res}}^{q_2}}{(E_0^{q_1 q_2})^{3/2}}, \tag{2.1}$$

where z_p is the point-sink overlap factor, m is the input valence quark mass, m_{res} is the size of the residual chiral symmetry breaking, and E_0 is the ground state energy of the meson. This expression normalizes the pion decay constant to $F_\pi = 92.2$ MeV.

On the lattice, the point-sink overlap factor and the ground state energy is extracted from the zero-momentum two-point correlation function,

$$C_{\text{2pt}}^{q_1 q_2}(t) = \sum_{\mathbf{x}} \langle \pi(\mathbf{x}, t) \pi^\dagger(\mathbf{0}, 0) \rangle \tag{2.2}$$

where $\pi = \bar{q}_1 \gamma_5 q_2$ is the meson interpolating operator. Tithout loss of generality the source is defined to be at the origin and t is the source-sink separation time. Inserting the resolution of the identity, and moving to the Heisenburg picture yields the spectral decomposition of the two-point correlation function,

$$C_{\text{2pt}}^{q_1 q_2}(t) = \sum_n z_n^{q_1 q_2} z_n^{q_1 q_2 \dagger} \left(e^{-E_n^{q_1 q_2} t} + e^{-E_n^{q_1 q_2}(T-t)} \right), \tag{2.3}$$

where T is the temporal length of the lattice, and captures the backwards propagating signal due to periodic boundary conditions. As discussed in Ref. , gradient-flowed HISQ dampens the domain-wall oscillations that exponentially decay with the cutoff scale, and consequently, we do not include that term in the fit ansatz.

The residual chiral symmetry breaking m_{res} is revealed from the correlation function,

$$m_{\text{res}}(t) = \frac{\sum_{\mathbf{x}} \langle \overline{Q}(\mathbf{x}, t) \gamma_5 Q(\mathbf{x}, t) \bar{q}(\mathbf{0}, 0) \gamma_5 q(\mathbf{0}, 0) \rangle}{\sum_{\mathbf{x}} \langle \bar{q}(\mathbf{x}, t) \gamma_5 q(\mathbf{x}, t) \bar{q}(\mathbf{0}, 0) \gamma_5 q(\mathbf{0}, 0) \rangle}, \tag{2.4}$$

where Q is the quark field in the midpoint of the fifth dimension and q is a quark field bound on the domain wall. We extract the value of m_{res} by fitting the correlation function to a constant.

Additionally, because of the mixed-action setup, chiral perturbation theory predicts F_k/F_π to depend on the mass of a mixed-meson, a meson that is constructed out of a pair of domain-wall and HISQ propagators. The Wick contractions are carried out analogously to Eq. (2.2), but due to the remaining taste degrees-of-freedom of the HISQ propagator, the spectral decomposition has additional oscillating opposite parity states,

$$C_{\text{2pt}}^{\text{mix}}(t) = \sum_n (-1)^{n(t+1)} A_n^{\text{mix}} \left(e^{-E_n^{\text{mix}} t} + e^{-E_n^{\text{mix}}(T-t)} \right), \tag{2.5}$$

where odd values of n correspond to a second tower of oscillating states, and E_0^{mix} is the ground state energy of the mixed meson. The coefficient A_n^{mix} is analogous to the overlap factors of Eq. (2.3), however since the precise overlap factor is unused, we simplify the fit ansatz to improve numerical stability.

III. LATTICE CALCULATION

The calculation is performed on a subset of the $N_f = 2+1+1$ HISQ gauge configurations generated by the MILC collaboration. The pion masses are $m_\pi \sim \{135, 220, 310\}$ MeV, with lattice spacings of $a \sim \{0.09, 0.12, 0.15\}$ fm for the two heavier ensembles, while we use only the two coarser lattices for the physical-mass ensemble. Additionally, a dedicated volume study where $m_\pi L \sim \{5.4, 4.3, 3.2\}$ is generated with $m_\pi \sim 220$ MeV and $a \sim 0.12$ fm. As a result, the MILC HISQ ensembles allows control over the main systematic uncertainties from lattice calculations: physical mass, continuum limit, and infinite volume extrapolations. Details of the ensembles are listed in Table. . All correlation functions also reveal no autocorrelations between successive configurations.

Physical observables are inferred from lattice correlation functions under the Bayesian framework. In particular, we quote the posterior distributions for ground state quantities needed to construct the decay constants. Since only the path integral average yields well-defined observables, all distributions capture the uncertainty of the mean, and therefore must be Gaussian in the large statistics limit, guaranteed by the central limit theorem. Due to this simplification, a Monte Carlo evaluation of Bayes' Theorem can be approximated by a maximum likelihood estimation of the joint likelihood-prior distribution across an ensemble of bootstrap resamples. While invoking bootstrap resampling inescapably convolutes a pure Bayesian philosophy, this approximation is less demanding computationally, and is shown to work well in practice []. Bayesian inference is performed by `lsqfit` [].

A. Decay constants

In the following sections, we discuss the fit procedure and results for the $m_{\rm res}$ and two-point correlation functions. We account for correlations between all parameters by coordinating the bootstrap draws for each ensemble. This is guaranteed by pre-drawing a set of random numbers for each HISQ gauge configuration.

1. Residual chiral symmetry breaking

The parameter $m_{\rm res}$ is extracted by fitting the correlator constructed from Eq. (2.4) to a constant. The time ranges are chosen to be symmetric across the mid-point, and identical for any given lattice spacing. The prior distribution of $m_{\rm res}$ is determined by plotting the $m_{\rm res}$ correlator, where an approximate value is chosen for the central value, with a width that is commensurate to zero at one standard deviation. The prior width is approximately x orders-of-magnitude larger than the posterior width and is unconstraining. The complete table of fit ranges and prior distributions for $m_{\rm res}$ is given in Table. **??**.

2. Ground state energies and overlap factors

B. Mixed mesons

IV. CHIRAL-CONTINUUM EXTRAPOLATION

1. Mixed action EFT expression for F_K/F_π

$$\begin{aligned}
\frac{F_K}{F_\pi} &= 1 + \frac{\mathcal{I}(m_{ju})}{2F^2} + \frac{\mathcal{I}(m_\pi)}{8F^2} - \frac{\mathcal{I}(m_{rs})}{4F^2} + \frac{\mathcal{I}(m_{ru})}{4F^2} - \frac{\mathcal{I}(m_{sj})}{2F^2} + \frac{\mathcal{I}(m_{ss})}{4F^2} - \frac{3\mathcal{I}(m_X)}{8F^2} \\
&\quad + \Delta_{ju}^2\left[-\frac{d\mathcal{I}(m_\pi)}{8F^2} + \frac{\mathcal{K}(m_\pi, m_X)}{4F^2}\right] - \Delta_{ju}^4 \frac{\mathcal{K}_{21}(m_\pi, m_X)}{24F^2} \\
&\quad + \Delta_{ju}^2 \Delta_{rs}^2 \left[\frac{\mathcal{K}(m_\pi, m_{ss}, m_X)}{6F^2} + \frac{\mathcal{K}_{21}(m_{ss}, m_X)}{12F^2}\right] \\
&\quad + \Delta_{rs}^2 \left[\frac{\mathcal{K}(m_{ss}, m_X)}{4F^2} - \frac{\mathcal{K}_{21}(m_{ss}, m_X) m_K^2}{6F^2} + \frac{\mathcal{K}_{21}(m_{ss}, m_X) m_\pi^2}{6F^2}\right] \\
&\quad + 4L_5(\mu)\frac{m_K^2 - m_\pi^2}{F^2}
\end{aligned} \tag{4.1}$$

In this expression

$$\mathcal{I}(m) = \frac{m^2}{(4\pi)^2} \ln\left(\frac{m^2}{\mu^2}\right) + \frac{m^2}{4\pi^2} \sum_{|\mathbf{n}|\neq 0} \frac{c_n}{mL|\mathbf{n}|} K_1(mL|\mathbf{n}|) \tag{4.2}$$

$$\begin{aligned}
d\mathcal{I}(m) &= \frac{1 + \ln\left(\frac{m^2}{\mu^2}\right)}{(4\pi)^2} + \sum_{|\mathbf{n}|\neq 0} \frac{c_n}{(4\pi)^2}\left[\frac{2K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|} - K_0(mL|\mathbf{n}|) - K_2(mL|\mathbf{n}|)\right] \\
&= \frac{1}{(4\pi)^2} + \frac{\mathcal{I}(m)}{m^2} + \sum_{|\mathbf{n}|\neq 0} \frac{c_n}{(4\pi)^2}\left[\frac{2K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|} - K_0(mL|\mathbf{n}|) - K_2(mL|\mathbf{n}|)\right]
\end{aligned} \tag{4.3}$$

$$\mathcal{K}(m,M)=\frac{1}{M^2-m^2}\left[\mathcal{I}(M)-\mathcal{I}(m)\right] \tag{4.4}$$

$$\begin{aligned}\mathcal{K}_{21}(m,M)&=\int_R\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-m^2)^2}\frac{1}{k^2-M^2}\\&=\frac{\partial}{\partial m^2}\int_R\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}\frac{1}{k^2-M^2}\\&=\frac{\partial}{\partial m^2}\mathcal{K}(m,M)\\&=\frac{1}{(M^2-m^2)^2}\left[\mathcal{I}(M)-\mathcal{I}(m)\right]-\frac{1}{M^2-m^2}d\mathcal{I}(m)\end{aligned} \tag{4.5}$$

$$\begin{aligned}\mathcal{K}(m_1,m_2,m_3)=&\frac{1}{m_1^2-m_2^2}\frac{1}{m_1^2-m_3^2}\mathcal{I}(m_1)+\frac{1}{m_2^2-m_1^2}\frac{1}{m_2^2-m_3^2}\mathcal{I}(m_2)\\&+\frac{1}{m_3^2-m_1^2}\frac{1}{m_3^2-m_2^2}\mathcal{I}(m_3)\end{aligned} \tag{4.6}$$

We can estimate the FV corrections by tracking only the terms with π and ju meson loops. Introducing shorthand notation, we find

$$\begin{aligned}\delta\frac{F_K}{F_\pi}=&\frac{\delta\mathcal{I}(m_{ju})}{2F^2}+\frac{\delta\mathcal{I}(m_\pi)}{8F^2}+\Delta_{ju}^2\left[-\frac{\delta d\mathcal{I}(m_\pi)}{8F^2}+\frac{\delta\mathcal{K}(m_\pi,m_X)}{4F^2}\right]\\&-\Delta_{ju}^4\frac{\delta\mathcal{K}_{21}(m_\pi,m_X)}{24F^2}+\Delta_{ju}^2\Delta_{rs}^2\frac{\delta\mathcal{K}(m_\pi,m_{ss},m_X)}{6F^2}\\=&\sum_{|\mathbf{n}|\neq0}c_n\Bigg\{2\frac{\epsilon_{ju}^2K_1^{ju}}{x_{ju}^{Ln}}+\frac{1}{2}\frac{\epsilon_\pi^2K_1^\pi}{x_\pi^{Ln}}-\frac{1}{8}\frac{\Delta_{ju}^2}{(4\pi F)^2}\left[\frac{2K_1^\pi}{x_\pi^{Ln}}-K_0^\pi-K_2^\pi\right]\\&+\frac{\Delta_{ju}^2}{4F^2}\frac{-1}{(m_X^2-m_\pi^2)}\frac{m_\pi^2}{4\pi^2}\frac{K_1^\pi}{x_\pi^{Ln}}\\&-\frac{\Delta_{ju}^4}{24F^2}\left[\frac{1}{(m_X^2-m_\pi^2)^2}\frac{-m_\pi^2}{4\pi^2}\frac{K_1^\pi}{x_\pi^{Ln}}-\frac{1}{m_X^2-m_\pi^2}\frac{1}{(4\pi)^2}\left[\frac{2K_1^\pi}{x_\pi^{Ln}}-K_0^\pi-K_2^\pi\right]\right]\\&+\frac{\Delta_{ju}^2\Delta_{rs}^2}{6F^2}\frac{1}{(m_\pi^2-m_X^2)(m_\pi^2-m_{ss}^2)}\frac{m_\pi^2}{4\pi^2}\frac{K_1^\pi}{x_\pi^{Ln}}\Bigg\}\\=&\sum_{|\mathbf{n}|\neq0}c_n\Bigg\{2\epsilon_{ju}^2\frac{K_1^{ju}}{x_{ju}^{Ln}}+\frac{\epsilon_\pi^2}{2}\frac{K_1^\pi}{x_\pi^{Ln}}-\frac{\epsilon_{\Delta_{ju}}^2}{8}\left[\frac{2K_1^\pi}{x_\pi^{Ln}}-K_0^\pi-K_2^\pi\right]-\frac{\epsilon_{\Delta_{ju}}^2\epsilon_\pi^2}{\epsilon_X^2-\epsilon_\pi^2}\frac{K_1^\pi}{x_\pi^{Ln}}\\&+\frac{\epsilon_{\Delta_{ju}}^4}{24}(4\pi)^4F^2\left[\frac{4F^2\epsilon_\pi^2/(4\pi F)^4}{(\epsilon_X^2-\epsilon_\pi^2)^2}\frac{K_1^\pi}{x_\pi^{Ln}}+\frac{1/(4\pi F)^2}{\epsilon_X^2-\epsilon_\pi^2}\frac{1}{(4\pi)^2}\left[\frac{2K_1^\pi}{x_\pi^{Ln}}-K_0^\pi-K_2^\pi\right]\right]\\&+\frac{\epsilon_{\Delta_{ju}}^2\epsilon_{\Delta_{rs}}^2 4\epsilon_\pi^2}{6(\epsilon_\pi^2-\epsilon_X^2)(\epsilon_\pi^2-\epsilon_{ss}^2)}\frac{K_1^\pi}{x_\pi^{Ln}}\Bigg\}\\=&\sum_{|\mathbf{n}|\neq0}c_n\Bigg\{2\epsilon_{ju}^2\frac{K_1^{ju}}{x_{ju}^{Ln}}+\frac{\epsilon_\pi^2}{2}\frac{K_1^\pi}{x_\pi^{Ln}}-\frac{\epsilon_{\Delta_{ju}}^2}{8}\left[\frac{2K_1^\pi}{x_\pi^{Ln}}-K_0^\pi-K_2^\pi\right]-\frac{\epsilon_{\Delta_{ju}}^2\epsilon_\pi^2}{\epsilon_X^2-\epsilon_\pi^2}\frac{K_1^\pi}{x_\pi^{Ln}}\\&+\frac{\epsilon_{\Delta_{ju}}^4}{24}\left[\frac{4\epsilon_\pi^2}{(\epsilon_X^2-\epsilon_\pi^2)^2}\frac{K_1^\pi}{x_\pi^{Ln}}+\frac{1}{\epsilon_X^2-\epsilon_\pi^2}\left[\frac{2K_1^\pi}{x_\pi^{Ln}}-K_0^\pi-K_2^\pi\right]\right]\\&+\frac{2}{3}\frac{\epsilon_{\Delta_{ju}}^2\epsilon_{\Delta_{rs}}^2\epsilon_\pi^2}{(\epsilon_\pi^2-\epsilon_X^2)(\epsilon_\pi^2-\epsilon_{ss}^2)}\frac{K_1^\pi}{x_\pi^{Ln}}\Bigg\}\end{aligned} \tag{4.7}$$

where $K_i^\phi=K_i(m_\phi L|\mathbf{n}|)$, $x_\phi^{LN}=m_\phi L|\mathbf{n}|$ and $\epsilon_Y^2=m_Y^2/(4\pi F)^2$. The first 10 weights in the finite volume sum are listed in Table I.

$\lvert\mathbf{n}\rvert$	1	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{4}$	$\sqrt{5}$	$\sqrt{6}$	$\sqrt{7}$	$\sqrt{8}$	$\sqrt{9}$	$\sqrt{10}$
c_n	6	12	8	6	24	24	0	12	30	24

TABLE I. Finite volume weight factors for the first few finite volume modes.

V. UNCERTAINTY AND SENSITIVITY ANALYSIS

VI. CONCLUSIONS AND OUTLOOK

[1] just so tex doesn't crash. Remove when we actually cite something.

[1] G. Aad *et al.* (ATLAS Collaboration), Phys.Lett. **B716**, 1 (2012), arXiv:1207.7214 [hep-ex].