1) Sets

[1/1] Let $A = \{1, 2, 3, 4\}$, $B = 2$, $C = \varnothing$, $D = \{1, 4, 5\}$, $E = \{3, \varnothing\}$, $F = \{\{1\}, 2, \{3\}, 4\}$. For each pair $X, Y \in \{A, B, C, D, E, F\}$ determine whether this statements are true or false.
(a) $X \subseteq Y$
(b) $X \in Y$

[2/1] Is there a set that has no proper subset?

[3/1] Is it true that if $A \subseteq B \cup C$ then $A \subseteq B$ or $A \subseteq C$. ?

[4/1] List the members of these set $\bigcup_{P(i)} A_i \cap \bigcup_{0<i<20} \{i\}$, where $P(n)$ if true for prime n, and $A_i = \{i, i + 1, 2i\}$. (Mind that $\{i, i + 1,..., 2i\}$ is something different).

[5/1] Determine the power of following sets:
(a) $\{0, 1\}^3 \cup \{0, 1, 2\}^2$
(b) $\{\{0,1\}\}^3$
(c) $\{\varnothing, \{\varnothing\}\} \times \varnothing$
(d) $\{0, \{0, \{0, \{0\}\}\}\} \times \{1, \{9, \{9, \{7\}\}\}\}$

[6/1] Prove in four ways. 1) using definition, 2) using 01-membership table, 3) with Venn diagrams, 4) using axioms:
(a) $A \cap B \subseteq A \cup B$
(b) $A \setminus (A \setminus B) = A \cap B$
(c) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$
(d) $\mathrm{A} \setminus (\mathrm{B} \setminus (\mathrm{C} \setminus \mathrm{D})) = (\mathrm{A} \setminus \mathrm{B}) \cup ((\mathrm{A} \cap \mathrm{C}) \setminus \mathrm{D})$

[7/1] Can Venn diagrams be constructed for arbitrary finite number of sets $A_1, A_2,..., A_n$?

[8/1] Which of the following equations holds for arbitrary A, B, C:
$A \times (B \cup C) = (A \times B) \cup (A \times C)$
$A \cap (B \times C) = (A \cap B) \times (A \cap C)$
$A \times (B \setminus C) = (A \times B) \setminus (A \times C)$
$A \setminus (B \times C) = (A \setminus B) \times (A \setminus C)$
$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

[9/1] Determine $\bigcup_{t\in R^+} A_t$ and $\bigcap_{t\in R^+} A_t$ for a family of sets A_i (assume $0 \in R^+$):

$A_t = \{x : 0 < x \leq \frac{1}{t+1}\}$

$A_t = \{x : \frac{-1}{t+1} < x < \frac{1}{t+1}\}$

$A_t = \{(x, y) : x^2 + y^2 \leq t^2\}$

$A_t = \{(x, y) : x^2 \leq t^2 y^2$

[10/1] Which of this implications is true? What can be told about reveres implications?

$A = B \Rightarrow A \cap C = B \cap C$

$A = B \Rightarrow A \cup C = B \cup C$

$A = B \Rightarrow A \setminus C = B \setminus C$

$A = B \Rightarrow A^c = B^c$

$A \subseteq B \Rightarrow A \cap C \subseteq B \cap C$

$A \subseteq B \Rightarrow A \cup C \subseteq B \cup C$

$A \subseteq B \Rightarrow A \setminus C \subseteq B \setminus C$

$A \subseteq B \Rightarrow A^c \subseteq B^c$

[11/1] Using results from previous exercises and the axioms of set algebra prove:

$A \cap B = A \Leftrightarrow A \cup B = B \Leftrightarrow A \setminus B = \varnothing \Leftrightarrow A^c \cup B = X$

$((\mathrm{A} \subseteq \mathrm{B}) \wedge (\mathrm{C} \subseteq \mathrm{D})) \Rightarrow \mathrm{A} \setminus \mathrm{D} \subseteq \mathrm{B} \setminus \mathrm{C}$

[12/1] Prove that:

$$\bigcup_{i=1}^{n} A_i \cap \bigcup_{i=1}^{m} B_i = \bigcup_{i=1}^{n} \bigcup_{j=1}^{m} (A_i \cap B_j)$$

$$\bigcup_{i=0}^{n-1} A_i = \bigcup_{i=0}^{n-1} (A_i \setminus A_{(i+1)\bmod n}) \cup \bigcap_{i=0}^{n-1} A_i$$

[13/1] Which of the conditional statements and equivalences are true? What can you say about the reverse implication?

$[(A \cap B) \setminus C = \varnothing] \Rightarrow [(A \cup B) \setminus (A \cup C) = B \setminus C]$

$[A \setminus (A \setminus B) = B] \Rightarrow [C \subseteq B \rightarrow C \subseteq A]$

$[A^C \cap B^C = A \cap B] \Leftrightarrow [A^C = B]$