[DM 01]

[1/01] Use induction to prove:
(1) the formula for sum of elements of arithmetic and geometric progression.
(2) prove that $n \in \boldsymbol{N} \Rightarrow 6 \mid n(n + 1)(2n + 1)$ (a | b meens "a divides b" or "b is divisible by a")
(3) prove that $n \in \boldsymbol{NP} \Rightarrow 4 \mid 1 + 3^n$ (NP – set of all odd positive integers)
(4) prove that $n \in \boldsymbol{NP} \Rightarrow 5 \mid 2^n + 3^n$
(5) prove that for a sequence a_n defined: $a_1 = 1$, $a_2 = 2$, $a_n = a_{n-1} + a_{n-2}$, formula $a_n < (7/4)^n$ holds.
(6) sequence a_n defined: $a_n = a_{n-1} / (2a_{n-1} + 1)$, satisfy $a_n = a_0 / (2na_0 + 1)$.
(7) $m^5 / 5 + m^3 / 3 + 7m / 15$ is integer for arbitrary integer m.

[2/01] For arbitrary sequence of positive real numbers $a_1,...,a_n$ prove:

- $a_n/a_1 + \sum_{i=1}^{n-1} a_i / a_{i+1} \geq n$,
- if $a_1 \cdots a_n = 1$, then $\sum_{i=1}^{n} a_i \geq n$,
- $(a_1 \cdots a_n)^{1/n} \leq (a_1 + ... + a_n)/n$. When the equality holds?

[3/01]
```
const   k = ...;
type  T = array[1..k] of integer;
for i = 1 to k – 1 do
  if T[i] > T[i+1] then "swap T[i] and T[i + 1]"
writeln(T[k]);
```

Prove that the above algorithm prints maximal value of array T. (Tip: use induction with respect to i where predicate is formulated: $P(i) \Leftrightarrow$ "after i loop step maximal element is at index greater that i”).

[4/01]
```
const
  k = ...;
type
  T = array[1..k] of integer;
for i = 1 to k – 1 do
  for j = 1 to k – i do
    if T[j] < T[j+1] then "zamień T[j] z T[j + 1]"
```

Prove that the above algorithm sorts the array T.

[5/01] In the town Crosstown there are no dead end and there are only two way roads. Each crossroad has even number of road. from each crossroad can get to each other. Prove that there is a walk that traverses each road exactly once and the starting point is also a terminal point. (tip: use the strong version of induction and induce with respect to number of roads)

[6/01] For arbitrary integer n prove that there exists c_n such that:
In arbitrary regular polygon on nodes number greater or equal than c_n if you color line segments connecting nodes of polygon with n colors. There will exist a triangle in the same color.

[7/01] Which of the following implications hold? Give counterexample if not.

- ❑ $[P(1) \wedge \forall_{n \in N}(P(n^2) \rightarrow P(n^2 + 1)) \wedge \forall_{n \in N}(P(n + 1) \rightarrow P(n))] \Rightarrow \forall_{n \in N}(P(n))$
- ❑ $[P(125) \wedge \forall_{n \in N}(P(n^2) \rightarrow P(n^2 + 2n + 2)) \wedge \forall_{n \in N}(P(n + 1) \rightarrow P(n))] \Rightarrow \forall_{n \in N}(P(n))$

[8/01]

(a) Prove that the following program prints only integers.

```
x := 1;
while (1 < 2) do
  begin
    writeln(x);
    x := x + √(12(x−1)) + 3;
  end;
```

(b) Prove that the following program prints only integers.

```
x = 1;
while (1 > 0)
{
        print(x);
        x = 3 + x + 2√(3x−2);
}
```

[9/01] Given a recursive sequence:

$$a_1 = 1, a_2 = 2$$
$$a_n = a_{n-1}^2 + a_{n-2}^2 + a_{n-2} + 1, \text{dla } n \geq 3$$

Prove that, $\sim\exists_{n \in N} (3 \mid a_n)$.

[10/01] Given a recursive sequence:

$$a_1 = 4, a_2 = 2002$$
$$a_n = a_{n-1}^2 + a_{n-2}^2 + a_{n-1} + 2, \text{dla } n \geq 3$$

Prove that, $\forall_{n \in N} (4 \mid (a_n + 1)^2 - 1)$.

[11/01] Given a_n defined recursively:
$a_1 = 1$, $a_2 = 1$,
$a_{n+2} = a_{n+1} + a_n$, for $n \in \boldsymbol{N}$.

Prove that, $\forall_{n \in N}(5 \mid a_{5n})$.