

Chapter 2

Type-2 Fuzzy Logic Systems

In this chapter, a brief overview of the basic concepts of type-2 fuzzy systems is presented. This overview is intended to provide the basic concepts needed to understand the methods and algorithms presented later in this book [1–3]. The basic concepts that are covered in this chapter are: type-2 fuzzy sets, membership functions, type-2 inference, type reduction and defuzzification.

We begin by defining type-2 fuzzy sets and their corresponding membership functions. If for a type-1 membership function, as in Fig. 2.1, we blur it to the left and to the right, as illustrated in Fig. 2.2, then a type-2 membership function is produced. In this case, for a specific value x' , the membership function (u'), takes on different values, which are not all weighted the same, so we can assign membership grades to all of those points.

By doing this for all $x \in X$, we form a three-dimensional membership function—a type-2 membership function—that characterizes a type-2 fuzzy set [2, 3]. A type-2 fuzzy set \tilde{A} , is characterized by the membership function:

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \quad (2.1)$$

in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$. In fact $J_x \subseteq [0, 1]$ represents the primary membership of x , and $\mu_{\tilde{A}}(x, u)$ is a type-1 fuzzy set known as the secondary set. Hence, a type-2 membership grade can be any subset in $[0, 1]$, the primary membership, and corresponding to each primary membership, there is a secondary membership (which can also be in $[0, 1]$) that defines the possibilities for the primary membership. Uncertainty is represented by a region, which is called the footprint of uncertainty (FOU). When $\mu_{\tilde{A}}(x, u) = 1, \forall u \in J_x \subseteq [0, 1]$ we have an interval type-2 membership function, as shown in Fig. 2.3. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\bar{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

A fuzzy logic system (FLS) described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties,

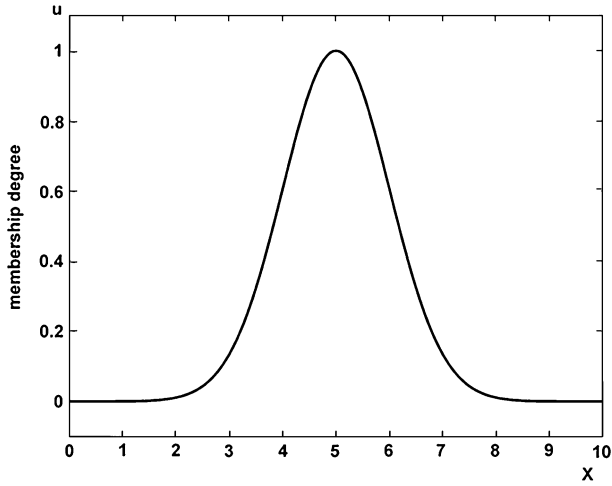


Fig. 2.1 An example of a type-1 membership function

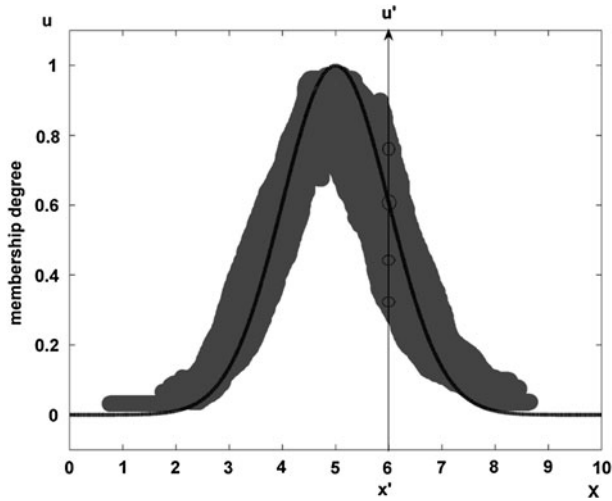


Fig. 2.2 Blurred type-1 membership function

because they use type-1 fuzzy sets that are certain (viz, fully described by single numeric values). On the other hand, type-2 FLSs, are useful in circumstances where it is difficult to determine an exact numeric membership function, and there are measurement uncertainties [3].

A type-2 FLS is characterized by IF-THEN rules, where their antecedent or consequent sets are now of type-2. Type-2 FLSs, can be used when the circumstances are too uncertain to determine exact membership grades such as when the training

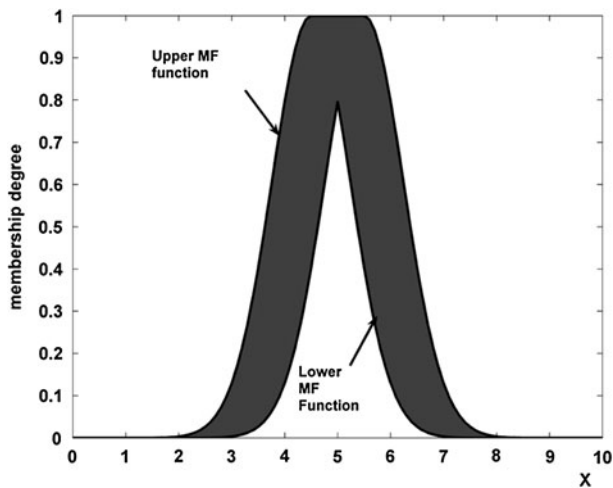


Fig. 2.3 Interval type-2 membership function

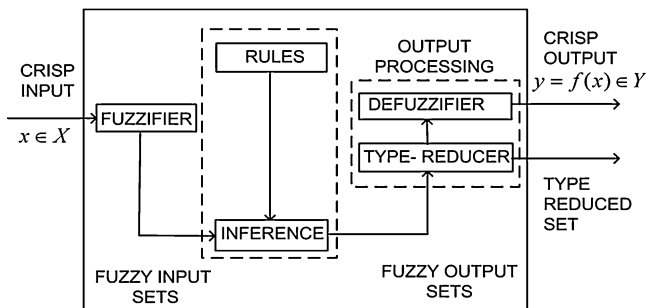


Fig. 2.4 Type-2 fuzzy logic system

data is affected by noise. Similarly, to the type-1 FLS, a type-2 FLS includes a fuzzifier, a rule base, fuzzy inference engine, and an output processor, as we can see in Fig. 2.4 for a Mamdani model. The output processor includes type-reducer and defuzzifier; it generates a type-1 fuzzy set output (from the type-reducer) or a number (from the defuzzifier) [2]. Now we explain each of the blocks shown in Fig. 2.4.

2.1 Fuzzifier

The fuzzifier maps a numeric vector $\mathbf{x} = (x_1, \dots, x_p)^T \in X_1 \times X_2 \times \dots \times X_p \equiv \mathbf{X}$ into a type-2 fuzzy set \tilde{A}_x in \mathbf{X} [3], an interval type-2 fuzzy set in this case. We use type-2 singleton fuzzifier, in a singleton fuzzification, the input fuzzy set has only a

single point on nonzero membership. \tilde{A}_x is a type-2 fuzzy singleton if $\mu_{\tilde{A}_x}(x) = 1/1$ for $\mathbf{x} = \mathbf{x}'$ and $\mu_{\tilde{A}_x}(x) = 1/0$ for all other $\mathbf{x} \neq \mathbf{x}'$.

2.2 Rules

The structure of rules in a type-1 FLS and a type-2 FLS is the same, but in the latter the antecedents and the consequents is represented by type-2 fuzzy sets. So for a type-2 FLS with p inputs (linguistic variables) $x_1 \in X_1, \dots, x_p \in X_p$ and one output $y \in Y$, Multiple Input Single Output (MISO), if we assume there are M rules, the l th rule in the type-2 FLS can be written down as follows (where the F 's and G are appropriate fuzzy sets for each rule):

$$R^l : \text{IF } x_1 \text{ is } \tilde{F}_1^l \text{ and } \dots \text{ and } x_p \text{ is } \tilde{F}_p^l, \text{ THEN } y \text{ is } \tilde{G}^l \quad l = 1, \dots, M \quad (2.2)$$

2.3 Inference

In the type-2 FLS, the inference engine combines rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. It is necessary to compute the join \sqcup , (unions) and the meet Π (intersections), as well as the extended sup-star compositions (sup star compositions) of type-2 relations. If $\tilde{F}_1^l \times \dots \times \tilde{F}_p^l = \tilde{A}^l$, then (2.2) can be re-written as follows

$$R^l : \tilde{F}_1^l \times \dots \times \tilde{F}_p^l \rightarrow \tilde{G}^l = \tilde{A}^l \rightarrow \tilde{G}^l \quad l = 1, \dots, M \quad (2.3)$$

R^l is described by the membership function $\mu_{R^l}(\mathbf{x}, y) = \mu_{R^l}(x_1, \dots, x_p, y)$, where

$$\mu_{R^l}(\mathbf{x}, y) = \mu_{\tilde{A}^l \rightarrow \tilde{G}^l}(\mathbf{x}, y) \quad (2.4)$$

can be written as:

$$\begin{aligned} \mu_{R^l}(\mathbf{x}, y) &= \mu_{\tilde{A}^l \rightarrow \tilde{G}^l}(\mathbf{x}, y) = \mu_{\tilde{F}_1^l}(x_1) \Pi \dots \Pi \mu_{\tilde{F}_p^l}(x_p) \Pi \mu_{\tilde{G}^l}(y) \\ &= [\Pi_{i=1}^p \mu_{\tilde{F}_i^l}(x_i)] \Pi \mu_{\tilde{G}^l}(y) \end{aligned} \quad (2.5)$$

In general, the p -dimensional input to R^l is given by the type-2 fuzzy set \tilde{A}_x whose membership function becomes

$$\mu_{\tilde{A}_x}(\mathbf{x}) = \mu_{\tilde{x}_1}(x_1) \Pi \dots \Pi \mu_{\tilde{x}_p}(x_p) = \Pi_{i=1}^p \mu_{\tilde{x}_i}(x_i) \quad (2.6)$$

where $\tilde{x}_i (i = 1, \dots, p)$ are the labels of the fuzzy sets describing the inputs. Each rule R^l determines a type-2 fuzzy set $\tilde{B}^l = \tilde{A}_x \circ R^l$ such that:

$$\mu_{\tilde{B}^l}(y) = \mu_{\tilde{A}_x \circ R^l} = \sqcup_{x \in X} \left[\mu_{\tilde{A}_x}(\mathbf{x}) \cap \mu_{R^l}(\mathbf{x}, y) \right] \quad y \in Y \quad l = 1, \dots, M \quad (2.7)$$

This dependency is the input/output relation shown in Fig. 2.3, which holds between the type-2 fuzzy set that activates a certain rule in the inference engine and the type-2 fuzzy set at the output of that engine [3].

In the FLS, we used interval type-2 fuzzy sets and intersection under product t-norm, so the result of the input and antecedent operations, which are contained in the firing set $\cap_{i=1}^p \mu_{\tilde{F}_{i_i}}^l(x'_i \equiv F^l(\mathbf{x}'))$, is an interval type-1 set,

$$F^l(\mathbf{x}') = [\underline{f}^l(\mathbf{x}'), \bar{f}^l(\mathbf{x}')] \equiv [\underline{f}^l, \bar{f}^l] \quad (2.8)$$

where

$$\underline{f}^l(\mathbf{x}') = \underline{\mu}_{\tilde{F}_1^l}(x'_1) * \dots * \underline{\mu}_{\tilde{F}_p^l}(x'_p) \quad (2.9)$$

and

$$\bar{f}^l(\mathbf{x}') = \bar{\mu}_{\tilde{F}_1^l}(x'_1) * \dots * \bar{\mu}_{\tilde{F}_p^l}(x'_p) \quad (2.10)$$

here $*$ stands for the product operation.

2.4 Type Reducer

The type-reducer generates a type-1 fuzzy set output, which is then converted in a numeric output through running the defuzzifier. This type-1 fuzzy set is also an interval set, for the case of our FLS we used center of sets (cos) type reduction, Y_{cos} , which is expressed as [3]

$$\begin{aligned} Y_{\text{cos}}(x) &= [y_l, y_r] \\ &= \int_{y^1 \in [y_1^l, y_1^r]} \dots \int_{y^M \in [y_M^l, y_M^r]} \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \dots \int_{f^M \in [\underline{f}^M, \bar{f}^M]} 1 / \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \end{aligned} \quad (2.11)$$

This interval set is determined by its two end points, y_l and y_r , which corresponds to the centroid of the type-2 interval consequent set \tilde{G}^i ,

$$C_{\tilde{G}^i} = \int_{\theta_1 \in J_{y_1}} \dots \int_{\theta_N \in J_{y_N}} 1 / \frac{\sum_{i=1}^N y_i \theta_i}{\sum_{i=1}^N \theta_i} = [y_l^i, y_r^i] \quad (2.12)$$

before the computation of $Y_{\text{cos}}(\mathbf{x})$, we must evaluate equation (2.12), and its two end points, y_l and y_r . If the values of f_i and y_i that are associated with y_l are denoted f_l^i and y_l^i , respectively, and the values of f_i and y_i that are associated with y_r are denoted f_r^i and y_r^i , respectively, from equation (2.13), we have [3]

$$y_l = \frac{\sum_{i=1}^M f_l^i y_l^i}{\sum_{i=1}^M f_l^i} \quad (2.13)$$

$$y_r = \frac{\sum_{i=1}^M f_r^i y_r^i}{\sum_{i=1}^M f_r^i} \quad (2.14)$$

The values of y_l and y_r define the output interval of the type-2 fuzzy system, which can be used to verify if training or testing data are contained in the output of the fuzzy system. This measure of covering the data is considered as one of the design criteria in finding an optimal interval type-2 FS. The other optimization criteria, is that the length of this output interval should be as small as possible.

2.5 Defuzzifier

From the type-reducer, we obtain an interval set Y_{\cos} , to defuzzify it we use the average of y_l and y_r , so the defuzzified output of an interval singleton type-2 FLS is [3]

$$y(\mathbf{x}) = \frac{y_l + y_r}{2} \quad (2.15)$$

To the moment, most of the interval type-2 fuzzy systems that have been developed for the applications follow the architecture of Fig. 2.4 and the definitions presented in this Chapter. In this sense, what has been presented constitutes a good basis for understanding the rest of the chapters of this book.

References

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