Concept Review

Geometric Lateral Controllers

Why Explore Geometric Lateral Controllers?

One of the two primary controllers used in a self-driving context, the lateral controller is responsible for lane-keeping under steady conditions. A variety of technologies can be used to estimate the current position of the vehicle. The pose measurements are eventually passed to a lateral controller to determine the steering output. This is one of the core components that makes a self-driving vehicle autonomous.

A geometric pure pursuit controller is relatively simple to understand, relatively intuitive to implement and finds its uses in parking lots and low-speed navigation scenarios. Stanley controllers work across a wider range of speeds for city and highway driving scenarios. Model based controllers, which are not in the scope of this example, are also adaptable to various types of vehicles, and take system dynamics into account. Pure pursuit and the Stanley controller will be covered in this document.

Error Terms

There are two central error parameters that need to be eliminated by an effective lateral controller – the cross-track error and the heading error. These are visually described in Figure 1. Use Figure 1a. as a reference for a vehicle being correctly centered in the lane with the correct heading.

a. Correct position and heading

b. Correct position incorrect heading

c. Correct heading incorrect position

Figure 1: Lane-keeping errors in the context of self-driving and lateral steering control

Although the vehicle in figure 1b is located relatively in the middle of the lane, it is not pointed in the correct direction. If it continues along its current heading, it will eventually leave the lane. This is called a **heading error ψ**, and is measured with respect to the reference lane direction. The reference can be calculated from a vector pointing towards the next desired waypoint, or from a camera-based solution detecting lanes. Figure 1c. demonstrates a different error. The vehicle is pointed in the correct direction while not centered in the lane. This is called a **cross-track error e**, and can lead to long run drift if not corrected.

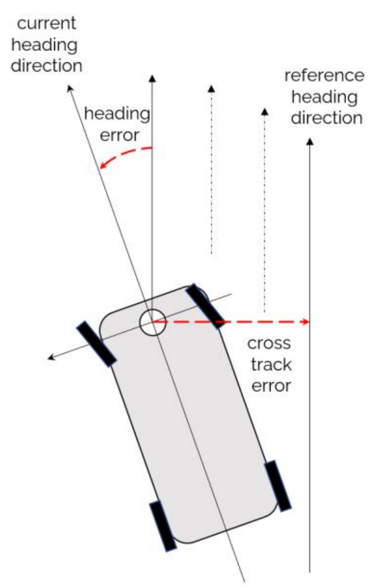

Figure 2: heading error (ψ) and cross-track error (e)

Keeping the car centered in the lane is also vital for safety, and in conjunction with the longitudinal speed controllers can help prevent over or under steering. These 2 error terms are also demonstrated in Figure 2 from a top view.

Geometric lateral controllers do not take vehicle dynamics into account. Model based controllers are outside the scope of this document. when it comes to self-driving vehicles the most popular options lateral control strategies are the pure-pursuit and Stanley controllers.

Pure-pursuit Controller

This is a geometric path-tracking controller that only uses vehicle kinematics and a reference point on the desired trajectory of the vehicle. A **target point** at a fixed distance ahead of the vehicle is taken from the desired trajectory. The goal of this controller is to always steers the vehicle to the **target point**. The target point is continuously resampled as the car moves forward towards a goal location.

The task at hand is to use vehicle kinematics and geometry to compute the steering angle. Assume a bicycle model represents the vehicle kinematics in this case. Refer to the **bicycle model concept review** for more information. In this model, the rear axle is used as reference point on the vehicle. The target point and the instantaneous point of rotation form an isosceles triangle with two sides $\boldsymbol{R}$ and the target distance $\boldsymbol{P}$ as shown in Figure 3. below.

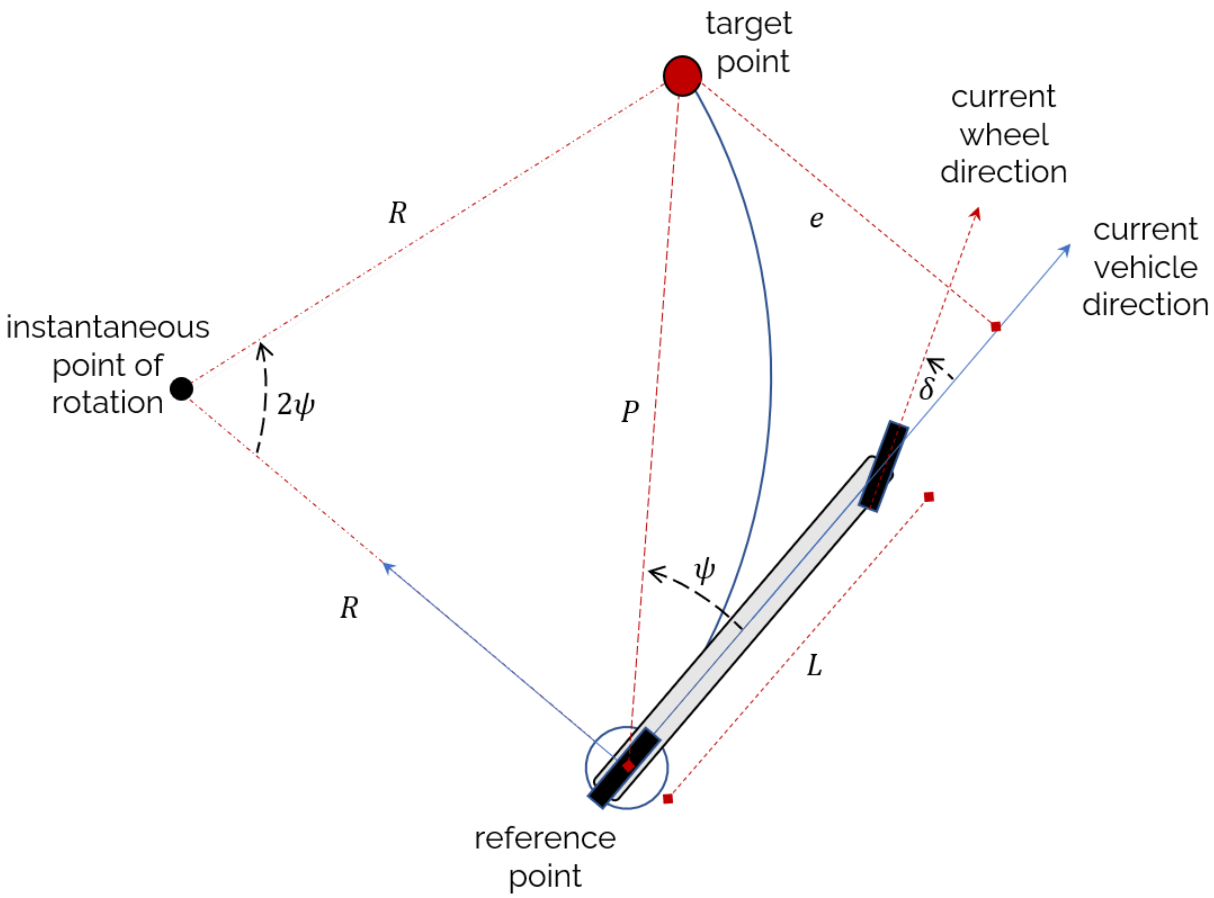

Figure 3: Geometric diagram for the pure pursuit controller

The current vehicle steering is set to $\boldsymbol{\delta}$ and the heading error is $\boldsymbol{\psi}$. It is expected that in the long run, $\boldsymbol{\psi}$ would approach 0.

From the law of sines in the isosceles triangle,

$$\frac{P}{\sin(2\psi)} = \frac{R}{\sin\left(\frac{\pi}{2} - \psi\right)}$$

$$\frac{P}{2\sin(\psi)\cos(\psi)} = \frac{R}{\cos(\psi)}$$

$$\frac{1}{\mathrm{R}} = \frac{2\sin(\psi)}{P}$$

The bicycle model provides the following relationship with the curvature and steering angle,

$$R = \frac{L}{\tan(\delta)}$$

Which leads to the pure pursuit control law,

$$\delta = \tan^{-1}\left(\frac{2L\sin(\psi)}{P}\right)$$

Measurements on the vehicle's position and heading, as well as the target point coordinates should yield all the information needed to calculate $\boldsymbol{\psi}$ and P. Knowing the wheel track parameter $\boldsymbol{L}$ should yield the desired steering angle $\boldsymbol{\delta}$. It is advantageous to adjust the target distance parameter $\boldsymbol{P}$ based on the speed $\boldsymbol{v}$ of the vehicle $\boldsymbol{P} = \boldsymbol{K} * \boldsymbol{v}$ via a gain $\boldsymbol{K}$. Without this, the controller can over-compensate, making it unstable at higher speeds than originally intended for.

The pure-pursuit controller equation takes the heading error $\boldsymbol{\psi}$ into account, but not the cross-track error $\boldsymbol{e}$. This term can be represented from the geometry as,

$$\sin(\psi) = \frac{e}{P}$$

This leads to the relationship,

$$\frac{\tan(\delta)}{L} = \frac{1}{R} = \frac{2\sin(\psi)}{P}$$

$$e = P\sin(\psi) = \frac{P^2\tan(\delta)}{2L}$$

which demonstrates that as the heading error dissipates due to steering, so does the cross-track error.

Stanley Controller

It's control law explicitly handles both cross-track and heading error simultaneously and guarantees convergence. This controller was developed by Stanford University's DARPA Grand Challenge team and uses the front axle as a reference point instead. Due to its' success in the DARPA Grand Challenge the Stanley Controller became very popular in lateral vehicle control.

In this situation, the heading error and cross-track error definitions change slightly, as illustrated in Figure 2
Figure 4. In the pure pursuit controller:

- Heading error $\boldsymbol{\psi_p}$ – the angle between the direction of the vehicle and a line joining the reference and target points
- Cross track error $\boldsymbol{e_p}$ – the shortest distance from the target point to the direction vector of the vehicle

However, in the Stanley controller,

- Heading error $\boldsymbol{\psi_s}$ – the angle between the direction of the vehicle and a line joining the previous/next targets waypoints.
- Cross track error $\boldsymbol{e_s}$ – the shortest distance from the front axle reference point to the line joining the previous/next target waypoints. It is vital to understand these differences, and as such, are illustrated in Figure 4 for clarity.

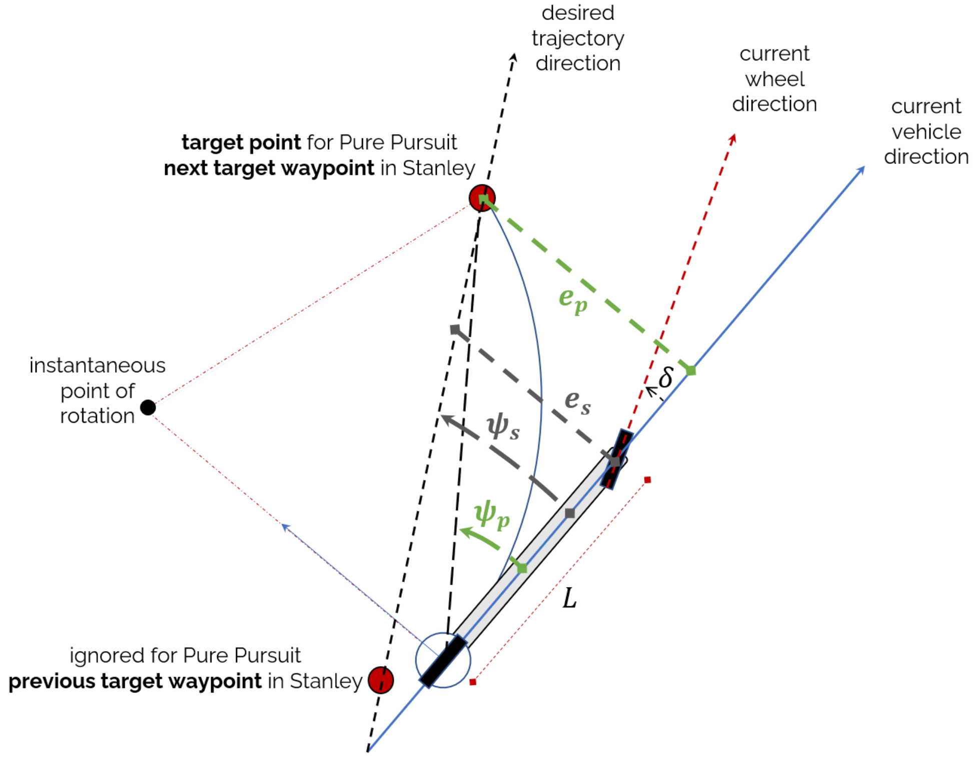

Figure 4: Geometric diagram comparing heading and cross track error definitions in a pure pursuit

controller versus a Stanley controller implementation

Once these terms are defined, the Stanley controller law can be defined with the following equations,

$$\delta_{\psi} = \psi_s$$

$$\delta_e = \tan^{-1}\left(\frac{k \;\, e_s}{v}\right)$$

$$\delta = \delta_{\psi} + \delta_e \quad , \quad \delta \in [\delta_{min}, \;\delta_{max}]$$

Where $\boldsymbol{\psi_s}$ is the heading error, $\boldsymbol{e_s}$ is the cross-track error, $\boldsymbol{v}$ is the forward velocity of the vehicle along the current wheel direction, and $\boldsymbol{k}$ is the Stanley controller cross track gain. The result steering action due to heading error $\boldsymbol{\delta_{\psi}}$ and steering action due to cross track error $\boldsymbol{\delta_e}$ are added together and saturated within the maximum and minimum allowed steering range to provide the steering output $\boldsymbol{\delta}$.

In the standard implementation, the Stanley controller can behave erratically at low speed as $\boldsymbol{v} \rightarrow \boldsymbol{0}$. To counter this, a softening constant $\boldsymbol{k_v}$ can be added to ensure the denominator never approaches 0.

$$\delta_e = \tan^{-1}\left(\frac{k \;\, e_s}{k_v + v}\right)$$

Consider some common scenarios as follows,

1. The cross-track error is small, but the heading error is large:

   The steering output mainly reflects the heading error, quickly navigating the car to align with the desired direction.

2. The cross-track error is large, but the heading error is small:

   The steering action due to cross track settles to either $\pi/2$ or $-\pi/2$ based on the range of the arctangent function. This turns the car dramatically to approach the desired trajectory. As the car settles closer and closer, the heading error term takes over, aligning the car as required.