Statistical Inference in Reinforcement Learning

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Developing AI with Reinforcement Learning

Reinforcement Learning Applications

(a) Games

(b) Mobile Health

(c) Ridesharing

(d) Robotics

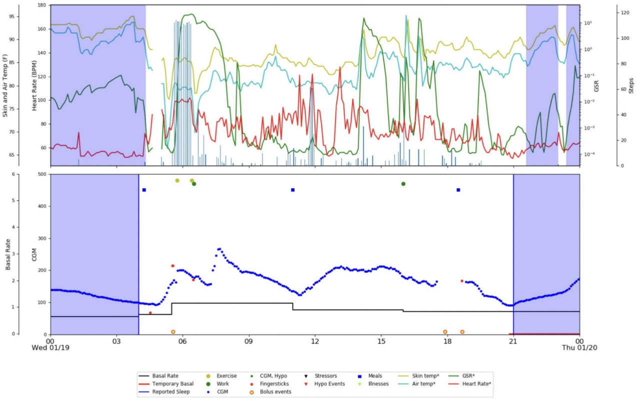
(e) Finance

(f) Automated Driving

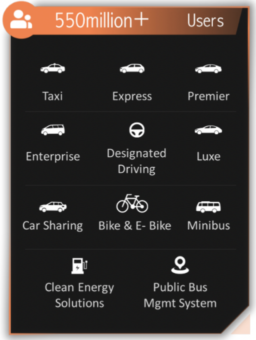
We focus on applications in **mobile health** (mHealth) and **ridesharing**

Applications in mHealth

- Management of **Type-I diabetes**
- **Subject**: Patients with Type-I diabetes.
- **Intervention**: Determine whether a patient needs to inject insulin or not based on their glucose levels, food intake, etc.
- **Data**: OhioT1DM dataset (Marling and Bunescu, 2018)



Applications in Ridesharing



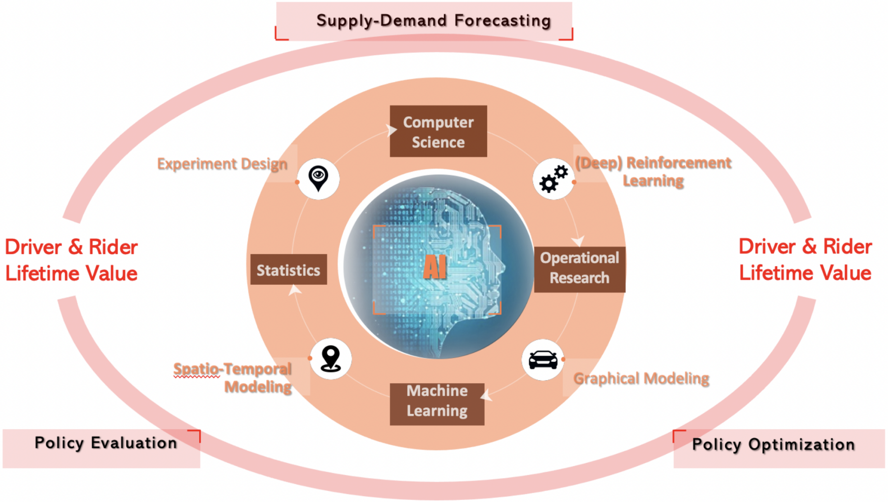
106TB+ vehicle trajectory data/day

4875TB+ data processed/day

40billion+ routing requests/day

15billion+ location points/day

Applications in Ridesharing (Cont'd)



In this talk, we will focus on ...

- **Statistical inference** in **reinforcement learning** (RL)

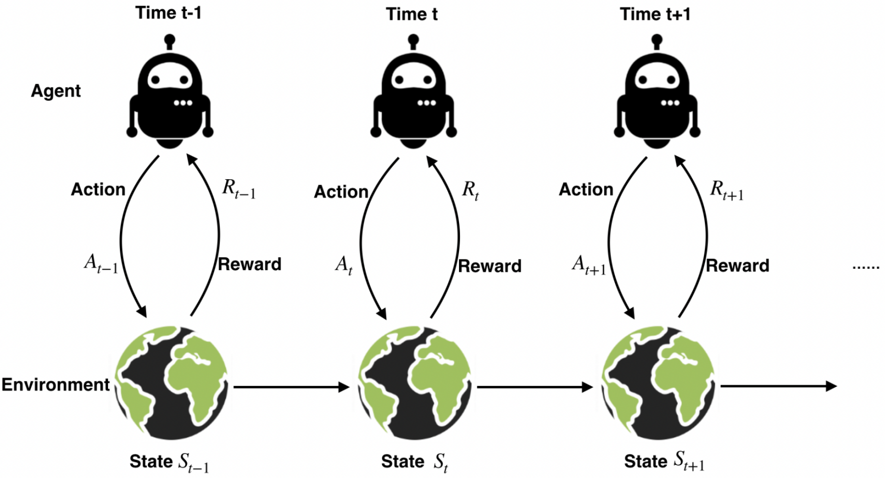
- Is statistical inference useful for RL?

Project I

Does the Markov Decision Process Fit the Data: Testing for the Markov Property in Sequential Decision Making

Joint work with Runzhe Wan, Rui Song, Wenbin Lu (NCSU) and Ling Leng (Amazon)
——ICML, 2020.

Sequential Decision Making



Objective: find an optimal policy that maximizes the cumulative reward

The Agent's Policy

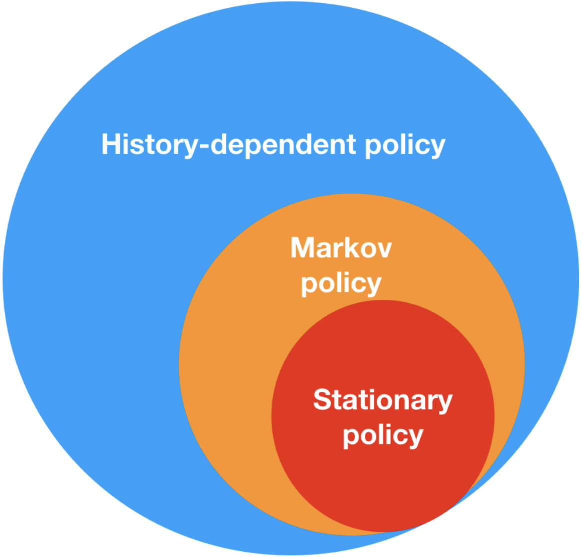
- The agent implements a **mapping** π_t from the observed data to a probability distribution over actions at each time step
- The collection of these mappings $\pi = \{\pi_t\}_t$ is called **the agent's policy**:

$$\pi_t(a|\bar{s}) = \Pr(A_t = a|\bar{S}_t = \bar{s}),$$

where $\bar{S}_t = (S_t, A_{t-1}, S_{t-1}, \cdots, A_0, S_0)$ is the set of **observed data history** up to time t.

- **History-Dependent** Policy: π_t depends on $\bar{S}_t$.
- **Markov** Policy: π_t depends on $\bar{S}_t$ only through S_t.
- **Stationary** Policy: π is Markov & π_t is **homogeneous** in t, i.e., $\pi_0 = \pi_1 = \cdots$.

The Agent's Policy (Cont'd)



Reinforcement Learning

- **RL algorithms**: trust region policy optimization (Schulman et al., 2015), deep Q-network (DQN, Mnih et al., 2015), asynchronous advantage actor-critic (Minh et al., 2016), quantile regression DQN (Dabney et al., 2018).
- **Foundations** of RL:
  - **Markov decision process** (MDP, Puterman, 1994): ensures the optimal policy is *stationary*, and is *not* history-dependent.
  - **Markov assumption** (MA): conditional on the present, the future and the past are independent,

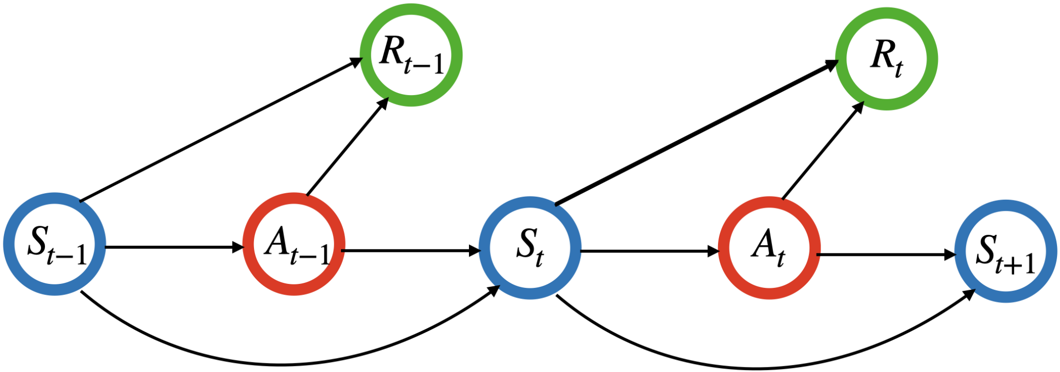
    $$S_{t+1}, R_t \perp\!\!\!\perp \{(S_j, A_j, R_j)\}_{j<t} | S_t, A_t.$$

    When R_t is a deterministic function of (S_t, A_t, S_{t+1}):

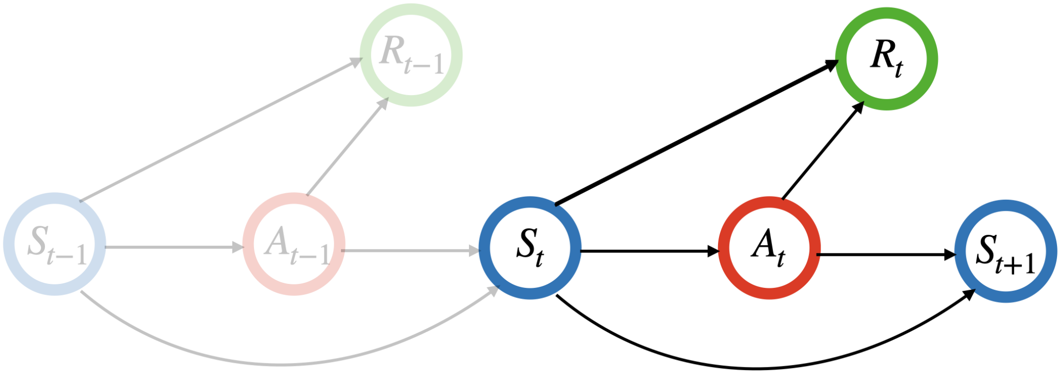
    $$S_{t+1} \perp\!\!\!\perp \{(S_j, A_j)\}_{j<t} | S_t, A_t.$$

    The Markov transition kernel is homogeneous in time.

Markov Property



Markov Property



RL Models

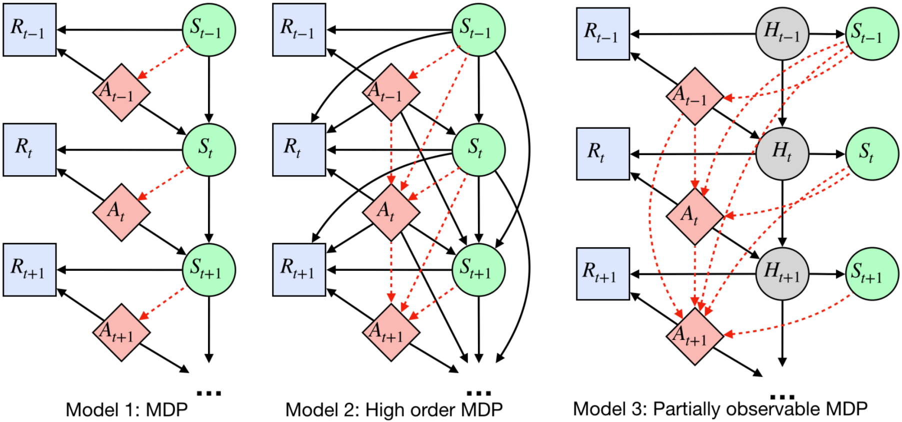


Figure: Causal diagrams for MDPs, HMDPs and POMDPs. The solid lines represent the causal relationships and the dashed lines indicate the information needed to implement the optimal policy. $\{\boldsymbol{H_t}\}_t$ denotes latent variables.

Contributions

- **Methodologically**
  - propose a **forward-backward learning** procedure to test MA
  - **first** work on developing consistent tests for MA in RL
  - sequentially apply the proposed test for RL **model selection**
  - critical to **offline** domains:
    - For **under-fitted** models, any stationary policy is not optimal
    - For **over-fitted** models, the estimated policy might be very noisy due to the inclusion of many irrelevant lagged variables
- **Empirically**
  - identify the optimal policy in **high-order** MDPs
  - detect **partially observable** MDPs
- **Theoretically**
  - prove our test **controls type-I error** under a **bidirectional** asymptotic framework

Applications in High-Order MDPs

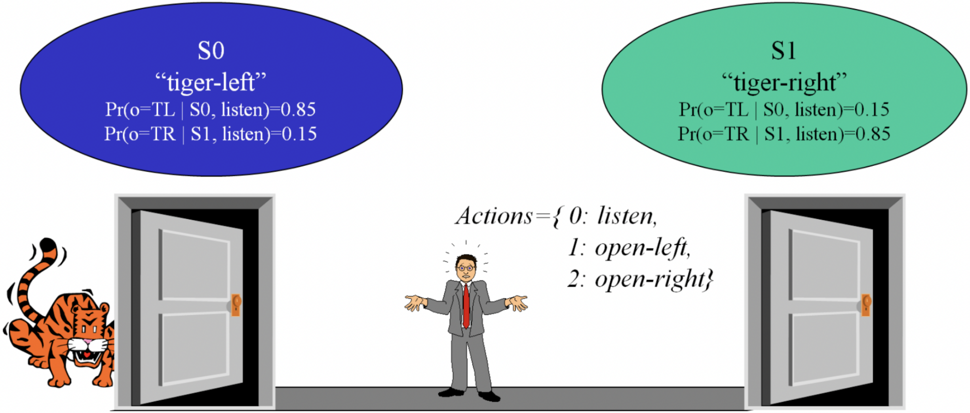
- **Data**: the OhioT1DM dataset (Marling & Bunescu, 2018)
- Measurements for 6 patients with type I diabetes over 8 weeks
- One-hour interval as a time unit
- **State**: patients' time-varying variables, e.g., glucose levels, food intake, exercise intensity.
- **Action**: to inject insulin or not.
- **Reward**: the Index of Glycemic Control (Rodbard, 2009).

Applications in High-Order MDPs (Cont'd)

- **Analysis I**:
  - sequentially apply our test to determine the order of MDP
  - conclude it is a **fourth-order** MDP
- **Analysis II**:
  - split the data into training/testing samples
  - policy optimization based on **fitted-Q iteration** (Ernst et al., 2005), by assuming it is a k-th order MDP for $k = 1, \cdots, 10$
  - policy evaluation based on **fitted-Q evaluation** (Le et al., 2019)
  - use **random forest** to model the Q-function
  - repeat the above procedure to compute the average value of policies computed under each MDP model assumption

order	1	2	3	4	5	6	7	8	9	10
value	-90.8	-57.5	-63.8	**-52.6**	-56.2	-60.1	-63.7	-54.9	-65.1	-59.6

Applications in Partially Observable MDPs



Reward Function

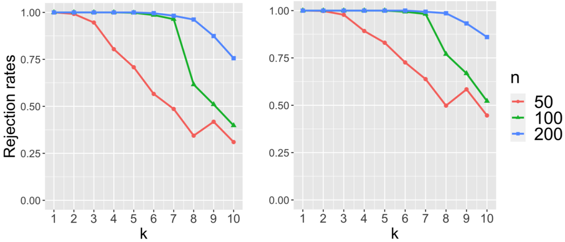
- *Penalty for wrong opening: -100*
- *Reward for correct opening: +10*
- *Cost for listening action: -1*

Observations

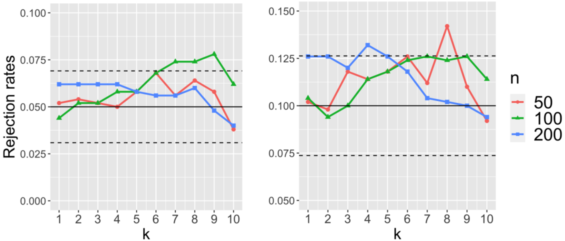
- *to hear the tiger on the left (TL)*
- *to hear the tiger on the right(TR)*

Applications in partially observable MDPs (Cont'd)

- Under the alternative hypothesis (MA is violated). $\alpha = (0.05, 0.1)$ from left to right.



- Under the null hypothesis (MA holds). $\alpha = (0.05, 0.1)$ from left to right.



Methodology

- **First** work to test MA in sequential decision making
- Existing approach in time series: Cheng and Hong (2012)
  - characterize MA based on the notion of **conditional characteristic function (CCF)**
  - use local polynomial regression to estimate CCF
- Challenge:
  - develop a valid test for MA in **moderate or high-dimensions**
  - the dimension of the state increases as we concatenate measurements over multiple time points in order to test for a high-order MDP.
- This motivates our **forward-backward learning** procedure.

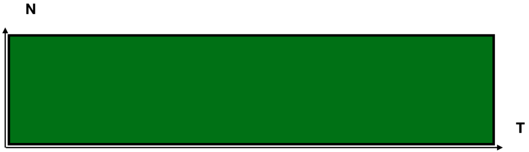
Methodology (Cont'd)

Some key components of our algorithm:

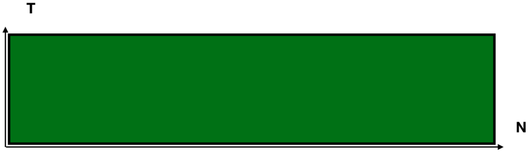
- To deal with moderate or high-dimensional state space, employ modern machine learning (ML) algorithms to estimate CCF:
  - Learn CCF of S_{t+1} given A_t and S_t (**forward learner**)
  - Learn CCF of (S_t, A_t) given (S_{t+1}, A_{t+1}) (**backward learner**)
  - Develop a random forest-based algorithm to estimate CCF
  - Borrow ideas from the quantile random forest algorithm (Meinshausen, 2006) to facilitate the computation
- To alleviate the bias of ML algorithms, construct **doubly-robust** estimating equations by integrating forward and backward learners;
- To improve the power, construct a **maximum-type** test statistic;
- To control the type-I error, approximate the distribution of our test via **multiplier bootstrap** (Chernozhukov, et al., 2014).

Bidirectional Theory

- N the number of trajectories;
- T the number of decision points per trajectory;
- **bidirectional asymptotics**: a framework allows either N or $T \to \infty$
- large T, small N (OhioT1DM dataset)



- large N, small T (Intern Health Study)



- large N, large T (Games)

Bidirectional Theory (Cont'd)

(C1) Actions are generated by a fixed behavior policy.

(C2) The observed data is exponentially β-mixing.

(C3) The ℓ_2 prediction errors of forward and backward learners converge at a rate faster than $(NT)^{-1/4}$.

Theorem

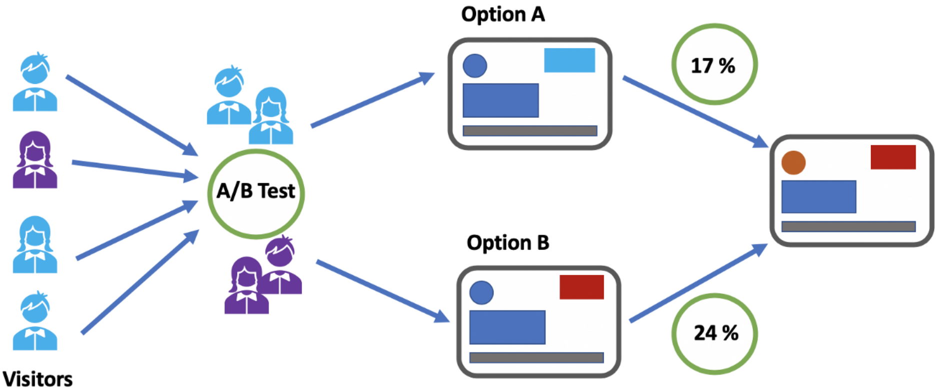
Assume (C1)-(C3) hold. Then under some other mild conditions, our test controls the type-I error asymptotically as either $\boldsymbol{N}$ or $\boldsymbol{T}$ diverges to ∞.

Project II

Dynamic Causal Effects Evaluation in A/B Testing with a Reinforcement Learning Framework

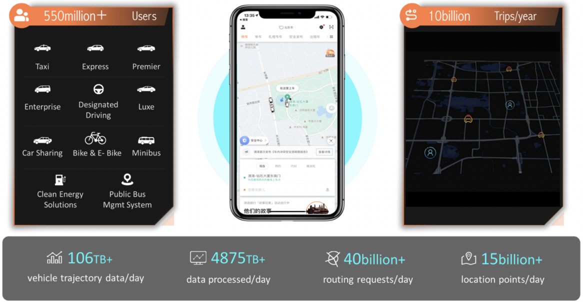
Joint work with Xiaoyu Wang, Shikai Luo, Hongtu Zhu, Jieping Ye and Rui Song ——JASA, accepted.

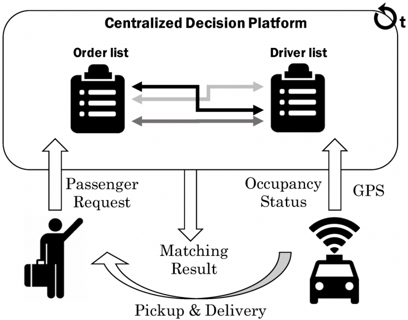
A/B Testing



Taken from
https://towardsdatascience.com/how-to-conduct-a-b-testing-3076074a8458

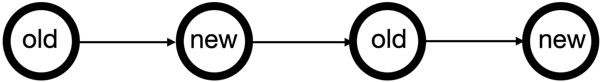
Motivations: Order Dispatch





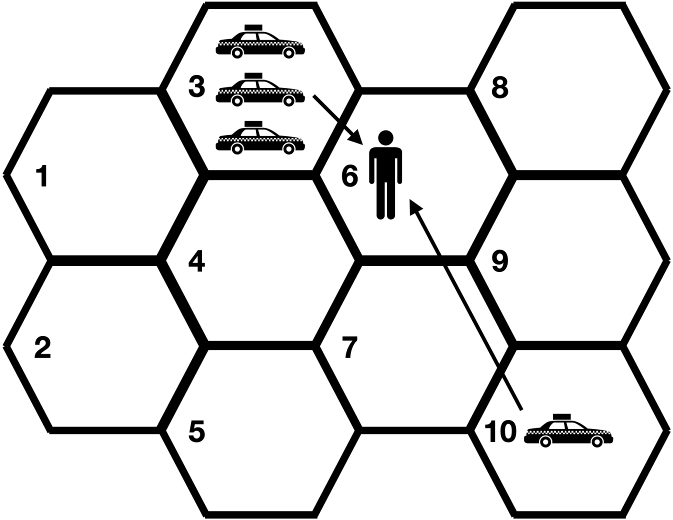
Motivations and Challenges

- The project is motivated by the need for comparing the **long-term rewards** of different **order dispatching** policies in **ridesharing platforms**
- Challenges:
  1. The existence of **carryover effects**:
     - Alternating-time-interval design

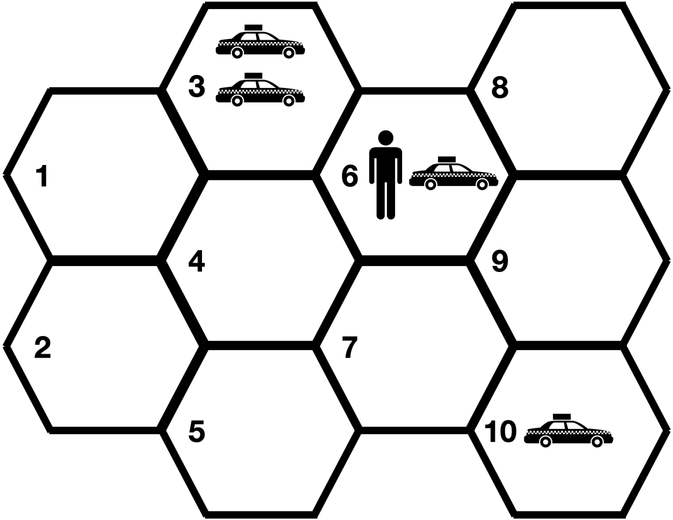


     - Past actions will affect future outcomes
  2. The need for **early termination**:
     - Each experiment takes a considerable time (at most 2 weeks)
     - Early termination to save time and budget
  3. The need for **adaptive randomization**:
     - Maximize the total reward (e.g., epsilon-greedy)
     - Detect the alternative faster
- No existing test has addressed three challenges simultaneously

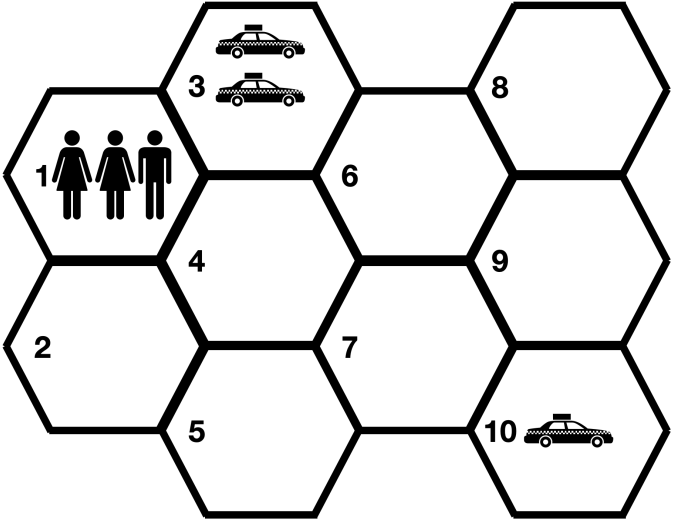
Illustration of the Carryover Effects



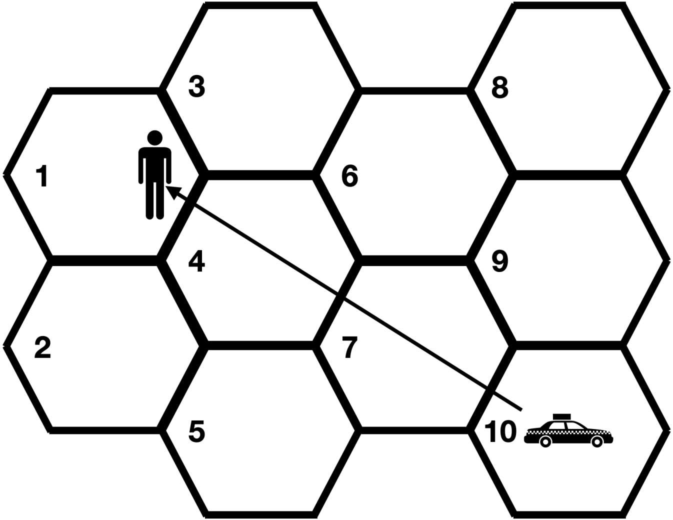
Adopting the Closest Driver Policy



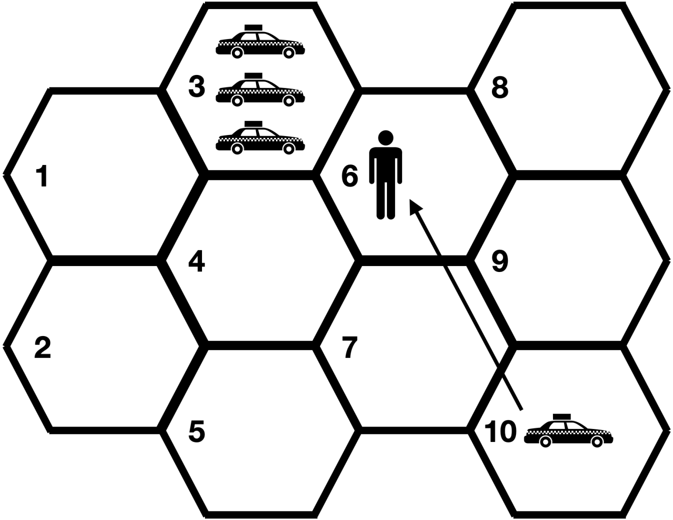
Some Time Later · · ·



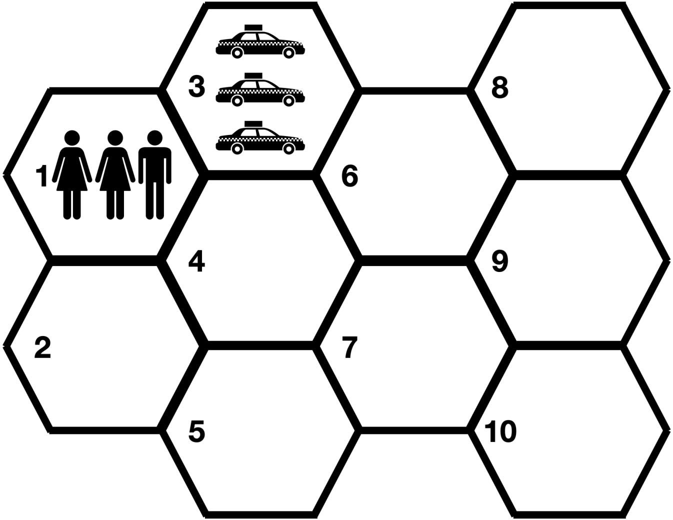
Miss One Order



Consider a Different Action



Able to Match All Orders



Existence of Carryover Effects

past actions $\rightarrow$ future distribution of drivers $\rightarrow$ future rewards

Limitations of Existing A/B tests

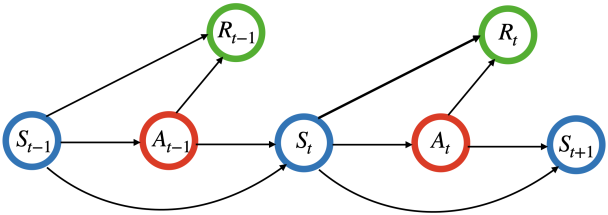
- Most existing tests **cannot** detect carryover effects
- $\mathcal{H}_0$: The old policy ($\boldsymbol{A} = 0$) has larger cumulative rewards
- $\mathcal{H}_1$: The new policy ($\boldsymbol{A} = 1$) has larger cumulative rewards
- **Example 1**. $\boldsymbol{S_t} \sim N(0, 0.25), \boldsymbol{R_t} = \boldsymbol{S_t} + \delta \boldsymbol{A_t}$
- **Example 2**. $\boldsymbol{S_t} = 0.5\boldsymbol{S_{t-1}} + \boldsymbol{A_{t-1}} + N(0, 0.25), \boldsymbol{R_t} = \boldsymbol{S_t}$

Example 1	t-test 0.76	DML-based test **1.00**	our test **0.98**
Example 2	t-test 0.04	DML-based test 0.06	our test **0.73**

Table: Powers of t-test, DML-based test (Chernozhukov et al., 2018) and the proposed test with $T = 500, \delta = 0.1$

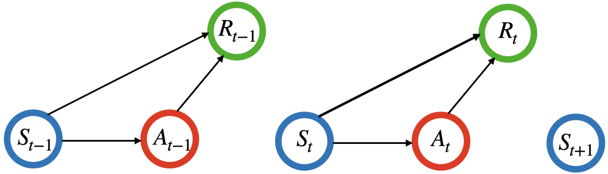
Contributions and Advances of Our Proposal

- Introduce an RL framework for A/B testing



  1. $\boldsymbol{A_{t-1}}$ impacts $\boldsymbol{R_t}$ indirectly through its effect on $\boldsymbol{S_t}$
  2. $\boldsymbol{S_t}$ shall include important mediators between $\boldsymbol{A_{t-1}}$ and $\boldsymbol{R_t}$

- Most existing works require the independence assumption



Contributions and Advances (Cont'd)

Propose a test procedure for detecting value difference

1. allows for **sequential monitoring**
2. allows for **online updating**
3. applicable to a wide range of designs, including the **Markov** design, **alternating-time-interval** design and **adaptive** design

Methodology

- Apply **temporal difference learning** to evaluate value difference and provide **uncertainty quantification** (Shi et al., 2021, JRSSB)
- Adopt the α-**spending approach** (Lan & DeMets, 1983) for sequential monitoring
- Develop a **bootstrap-assisted procedure** for determine the stopping boundary
  - The numerical integration method designed for classical sequential tests is **not** applicable in adaptive design, due to the carryover effects

Theory

Theorem (Validity and Consistency)

Under the Markov, alternating-time-interval or adaptive design, the proposed test can **control type-I error** *and is* **consistent** *against alternatives that converge to the null at the parametric rate*

Theory (Cont'd)

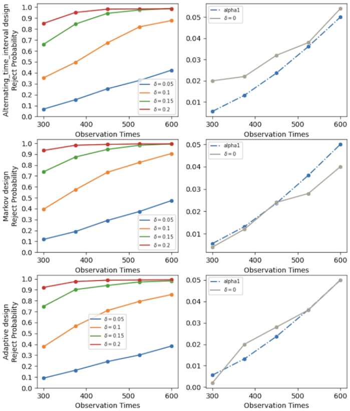
Theorem (Undersmoothing and Efficiency)

Suppose **sieve** *method is used for function approximation in temporal difference learning.*

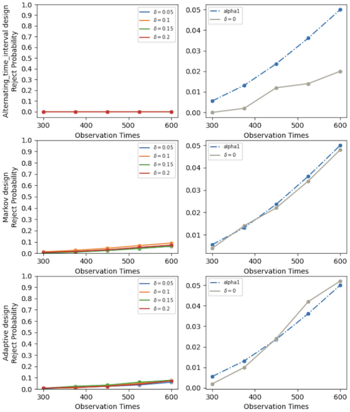
1. **Undersmoothing** *is not needed to guarantee that the resulting value estimator has a tractable limiting distribution.*
2. *The value estimator is* **semiparametrically efficient***.*

- Sieve estimators of conditional expectations are **idempotent**
- The proposed test will **not** be overly sensitive to the number of basis functions
- **Cross-validation** can be employed to select the basis functions

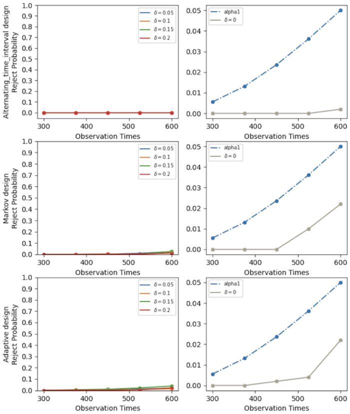
Simulation



(a) Power and size of our test



(b) Power and size of t test



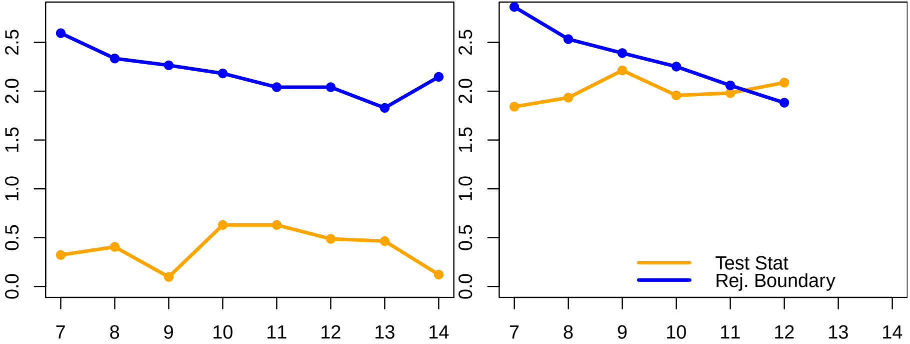
(c) Power and size of a version of the O'Brien Fleming sequential test

Application to Ridesharing Platform

- **Data**: a given city from December 3rd to 16th (two weeks)
- **30 minutes** as one time unit, sample size = **672**
- **State**:
  1. number of drivers (supply)
  2. number of requests (demand)
  3. supply and demand equilibrium metric (mediator)
- **Action**: new policy $\boldsymbol{A} = \mathbf{1}$ v.s. old $\boldsymbol{A} = \mathbf{0}$
- **Reward**: drivers' income
- The new policy is expected to have **better** performance

Application to Ridesharing Platform (Cont'd)

- The proposed test



(a) AA Experiment: Day

(b) AB Experiment: Day

- t-test: **fail** to reject $\mathcal{H}_0$ in A/B experiment with p-value 0.18

Project III

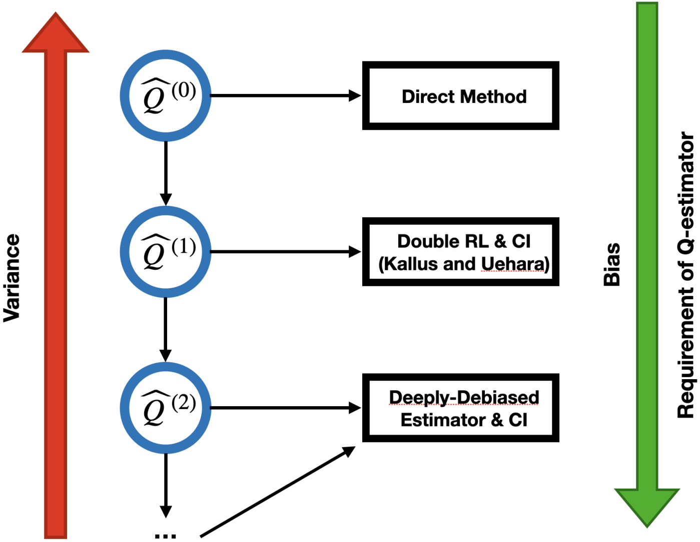
Deeply-Debiased Off-Policy Interval Estimation

joint work with Runzhe Wan, Victor Chernozhukov and Rui Song
——ICML 2021 (long talk, top 3% of the submissions).

Off-Policy Interval Estimation

- **Objective**: Evaluate the impact of a target policy **offline** using historical data generated from a different behavior policy and provide rigorous **uncertainty quantification** (healthcare, automated driving, ridesharing, robotics, e.g.)
- Most existing methods focus on providing point estimators
- **Main idea**: Develop a **deeply-debiasing** process using higher order influence function (Robins et al., 2017)

Off-Policy Interval Estimation (Cont'd)



Thank You!

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