Statistical Inference in Reinforcement Learning

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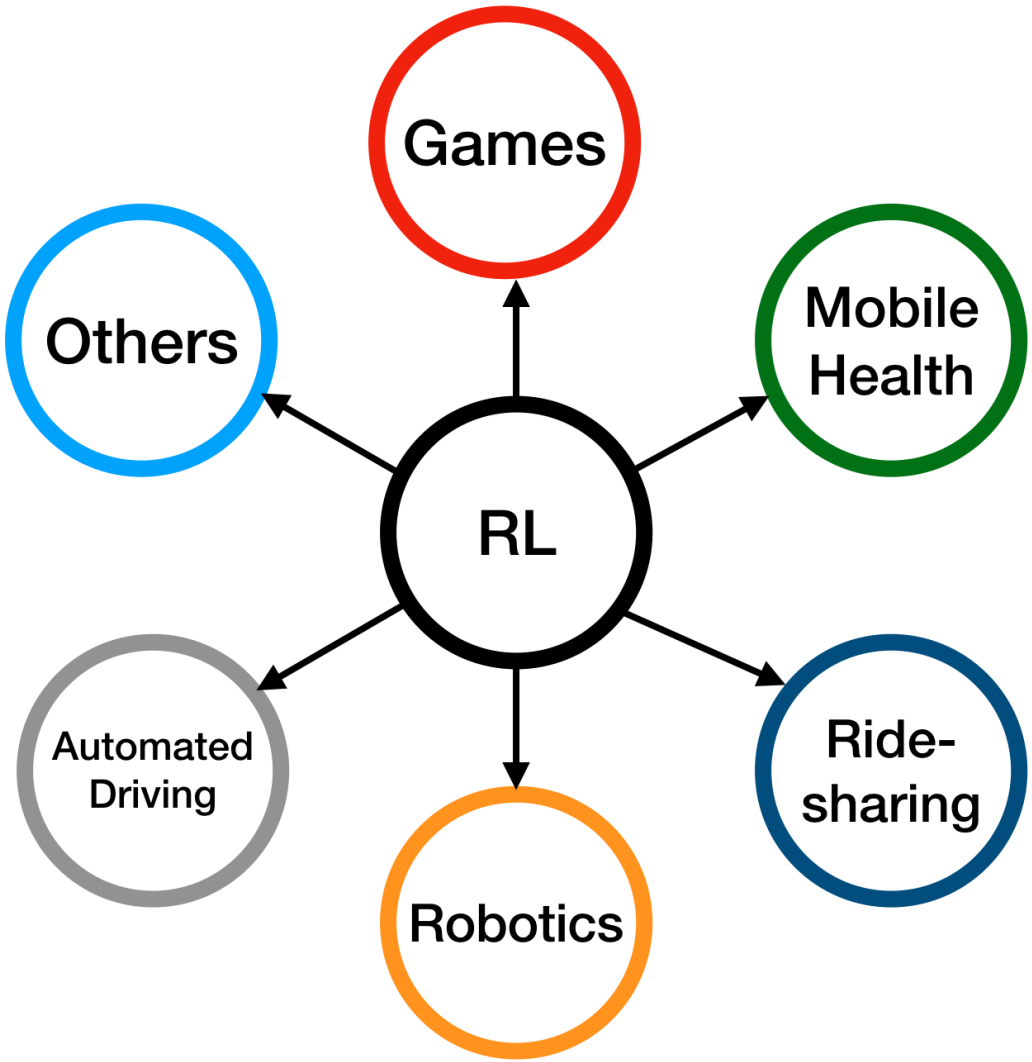
Developing AI with reinforcement learning

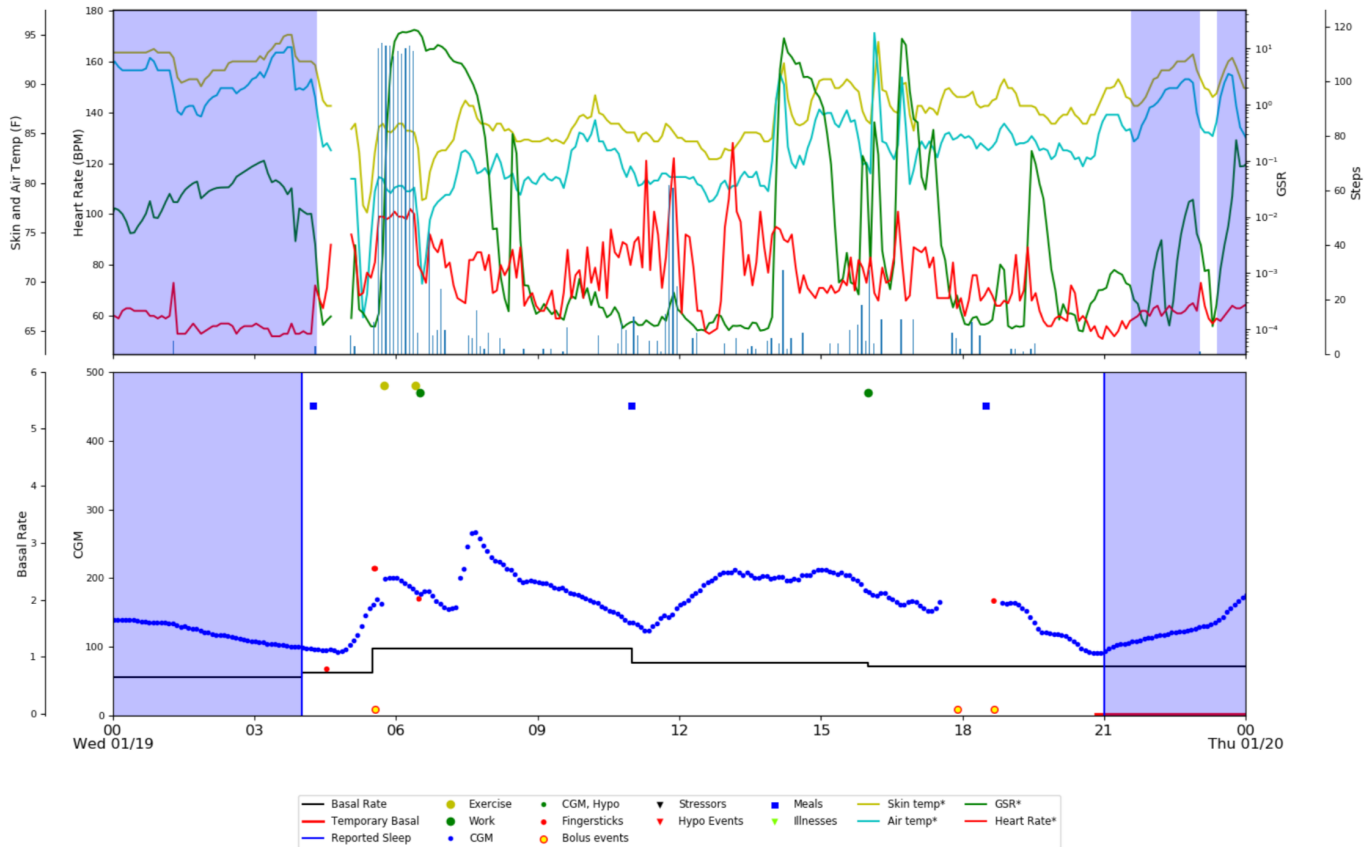
Reinforcement learning applications



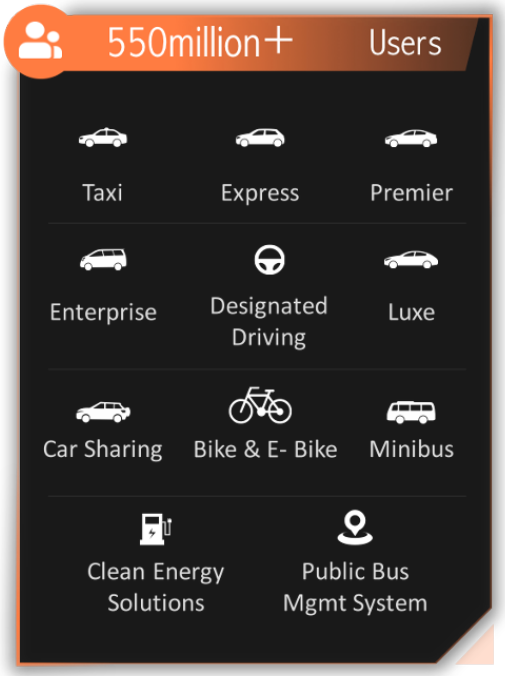
We focus on applications in **mobile health** and **ride-sharing**.

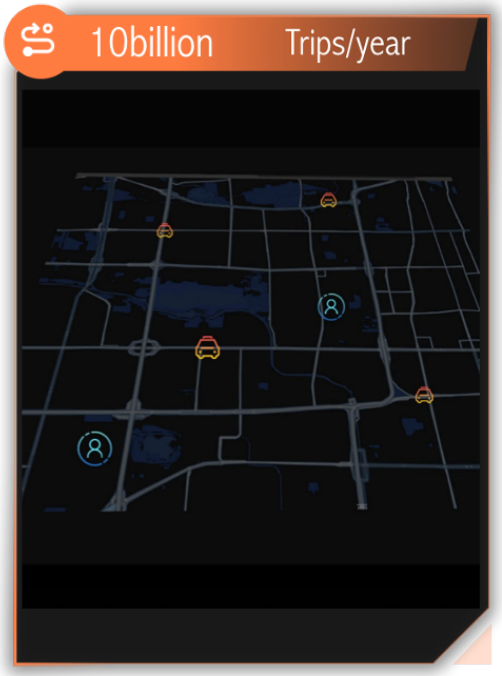
Applications in mobile health (mHealth)

- Management of **Type-I diabetes**.
- **Subject**: Patients with Type-I diabetes.
- **Intervention**: Determine whether a patient needs to inject insulin or not based on their glucose levels, food intake, etc.
- **Data**: OhioT1DM dataset (Marling and Bunescu, 2018)



Applications in ridesharing





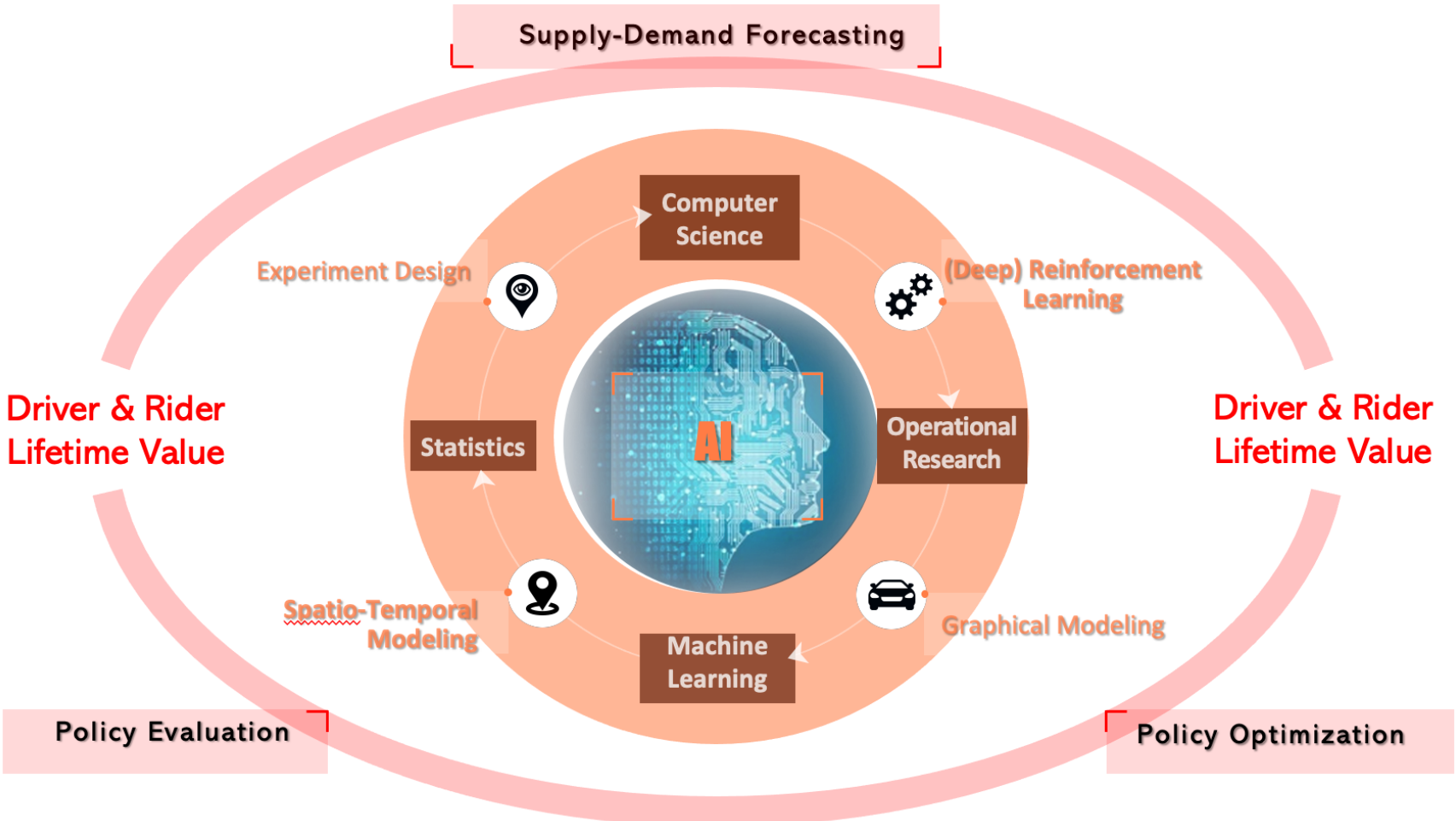
106TB+ vehicle trajectory data/day

4875TB+ data processed/day

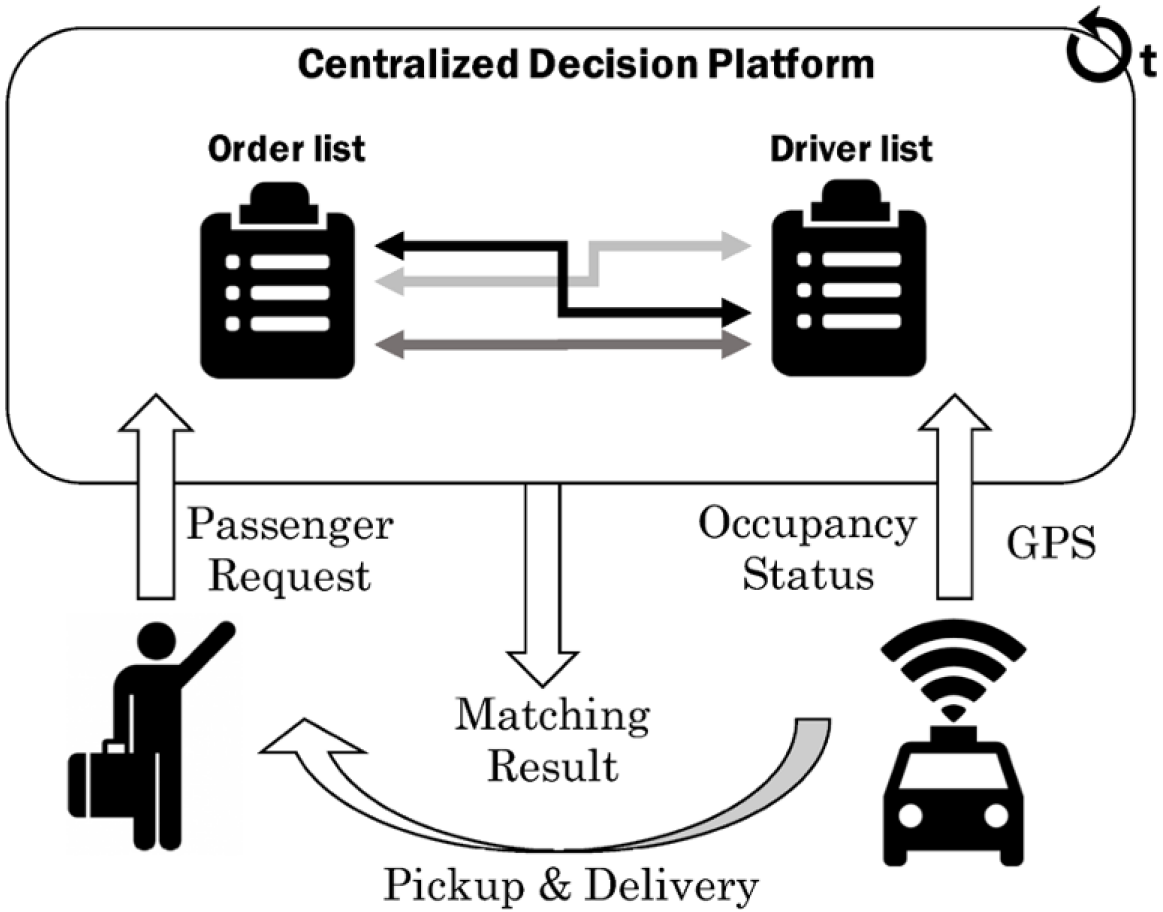
40billion+ routing requests/day

15billion+ location points/day

Applications in ridesharing (Cont'd)



Order dispatch



In this talk, we will focus on...

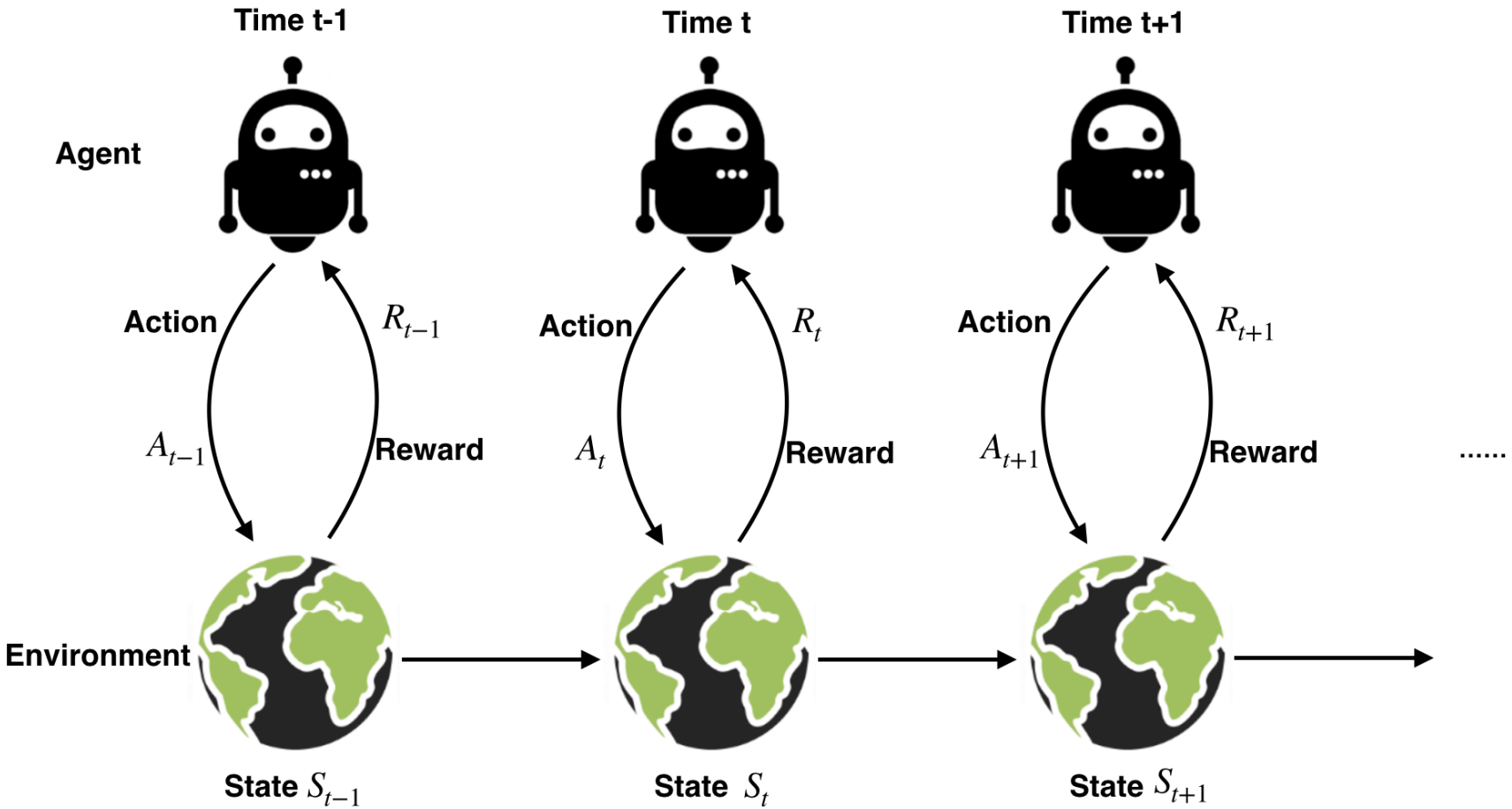
- **Statistical inference** in **reinforcement learning** (RL).
- Is statistical inference useful for RL?

Does the Markov Decision Process Fit the Data: Testing for the Markov Property in Sequential Decision Making

joint work with Runzhe Wan, Rui Song, Wenbin Lu (NCSU) and Ling Leng (Amazon)

—*ICML (2020).*

Sequential decision making



Objective: find an optimal policy that maximizes the cumulative reward

The agent's policy

- The agent implements a **mapping** from the observed data to a probability distribution over actions at each time step

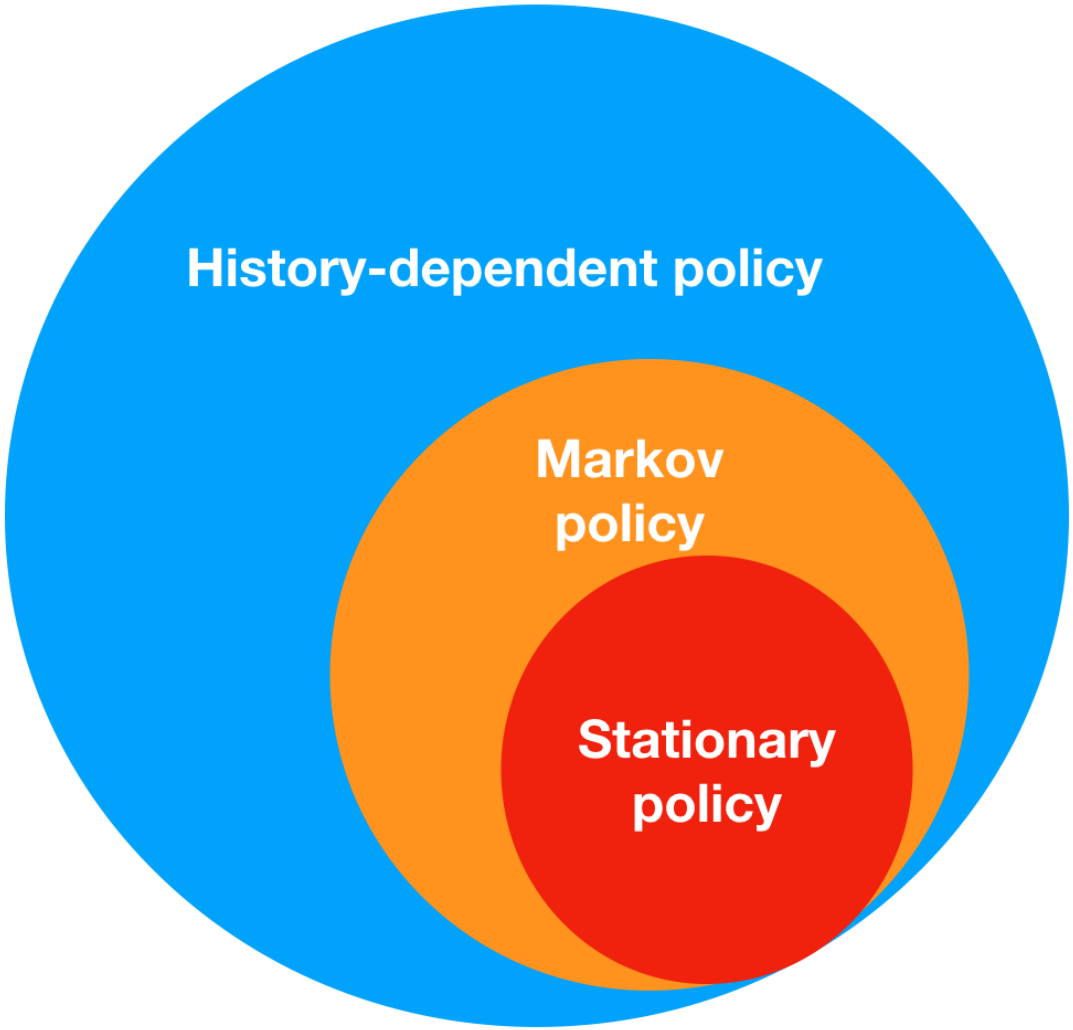
- The collection of these mappings $\pi = \{\pi_t\}_t$ is called **the agent's policy**:

$$\pi_t(a, \bar{s}) = \Pr(A_t = a | \bar{S}_t = \bar{s}),$$

  where $\bar{S}_t = (S_t, A_{t-1}, S_{t-1}, \cdots, A_0, S_0)$ is the set of observed state-action history up to time t

- **History-dependent** policy: π_t depends on $\bar{S}_t$
- **Markov** policy: π_t depends on $\bar{S}_t$ only through S_t, $\forall t$
- **Stationary** policy: π is Markov & π_t is homogeneous in t, $\forall t$

The Agent's Policy (Cont'd)



Reinforcement learning

- **RL algorithms**: trust region policy optimization (Schulman et al., 2015), deep Q-network (DQN, Mnih et al., 2015), asynchronous advantage actor-critic (Minh et al., 2016), quantile regression DQN (Dabney et al., 2018).
- **Foundations** of RL:
  - **Markov decision process** (MDP, Puterman, 1994): ensures the optimal policy is *stationary*, and is *not* history-dependent.
  - **Markov assumption** (MA): conditional on the present, the future and the past are independent,

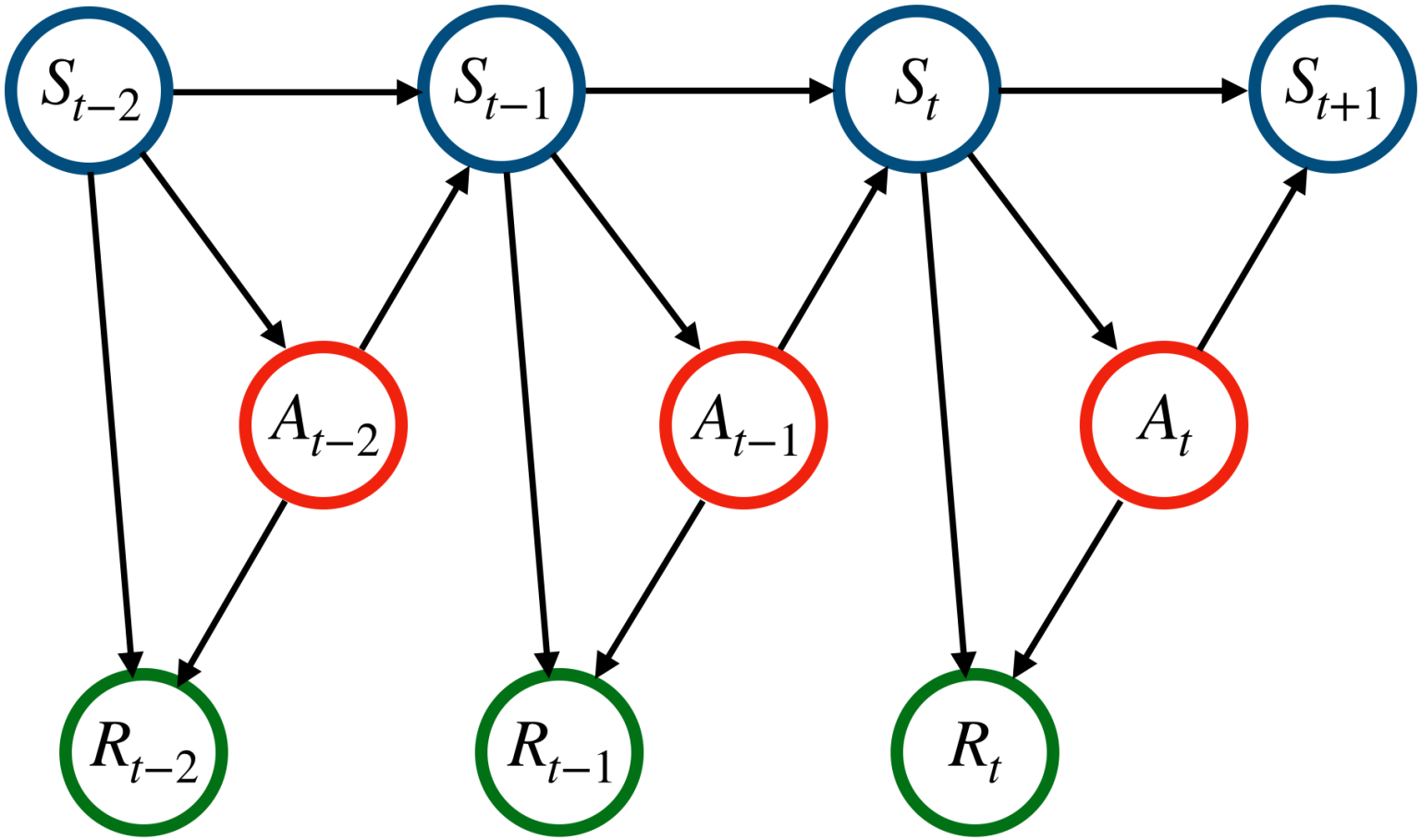
$$S_{t+1}, R_t \perp\!\!\!\perp \{(S_j, A_j, R_j)\}_{j<t} | S_t, A_t.$$

When R_t is a deterministic function of (S_t, A_t, S_{t+1}):

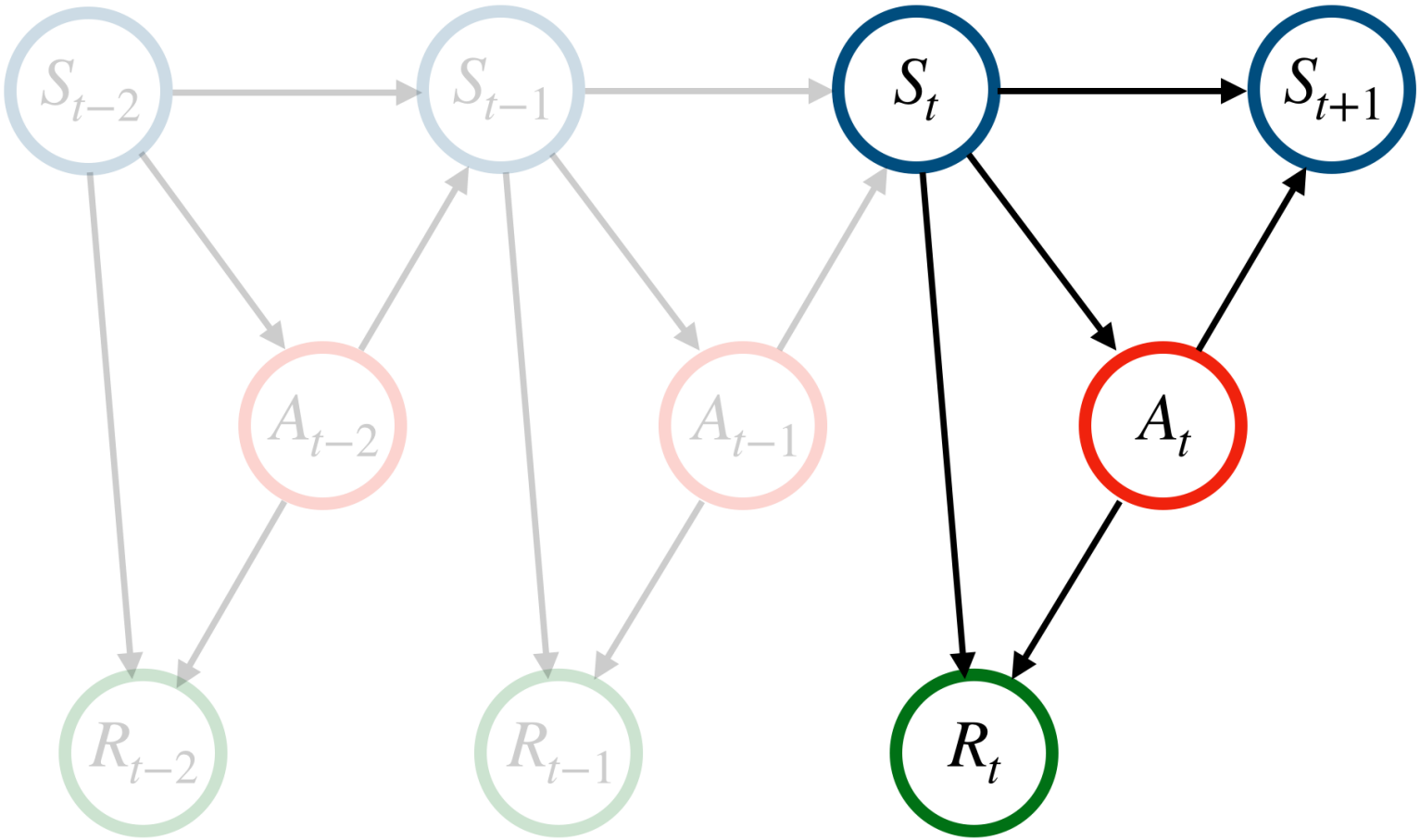
$$S_{t+1} \perp\!\!\!\perp \{(S_j, A_j)\}_{j<t} | S_t, A_t.$$

The Markov transition kernel is homogeneous in time.

Markov Decision Process



Markov Decision Process



RL models

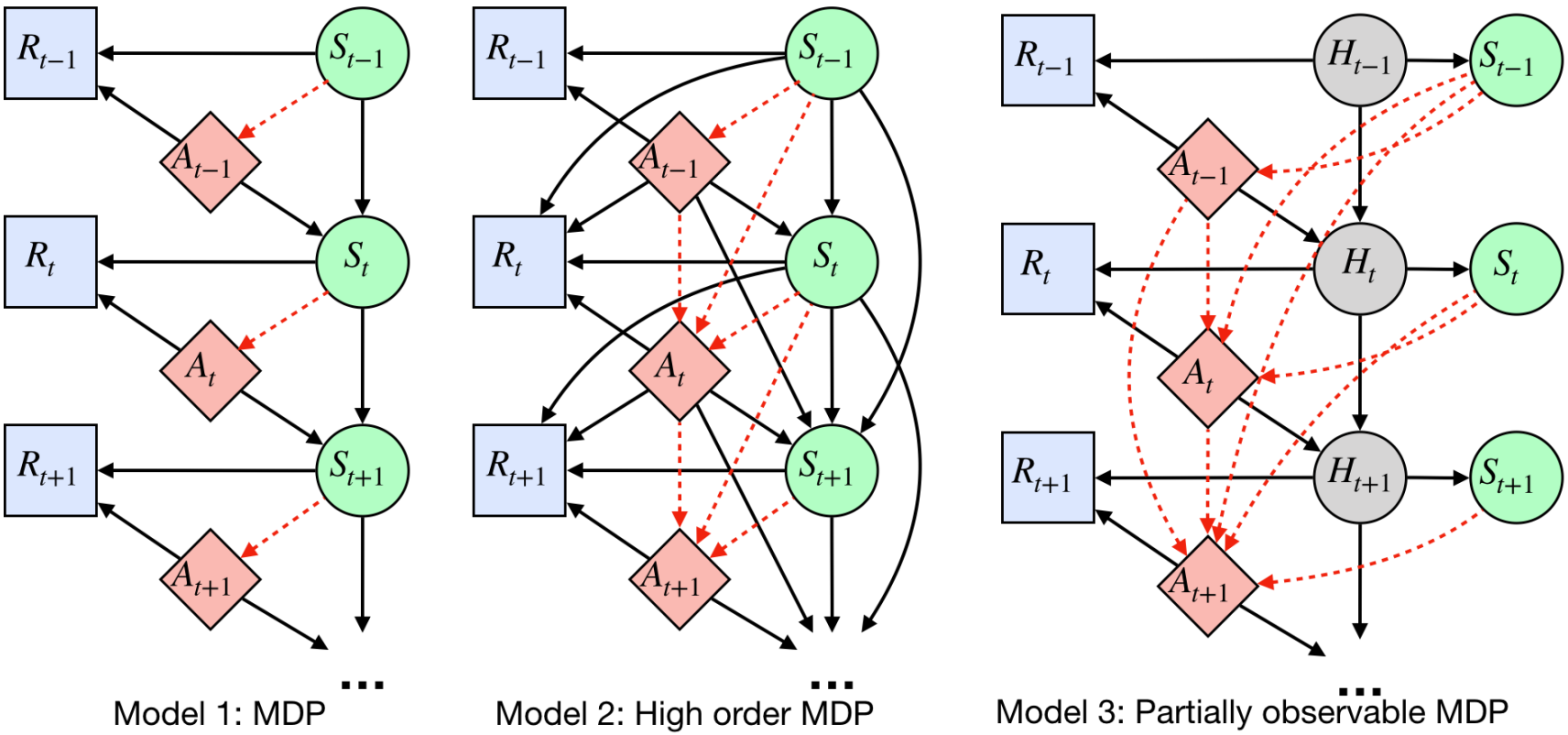


Figure: Causal diagrams for MDPs, HMDPs and POMDPs. The solid lines represent the causal relationships and the dashed lines indicate the information needed to implement the optimal policy. $\{H_t\}_t$ denotes latent variables.

Contributions

- **Methodologically**
  - propose a **forward-backward learning** procedure to test MA;
  - **first** work on developing consistent tests for MA in RL;
  - sequentially apply the proposed test for RL **model selection**;
  - critical to **offline** domains:
    - For **under-fitted** models, any stationary policy is not optimal;
    - For **over-fitted** models, the estimated policy might be very noisy due to the inclusion of many irrelevant lagged variables.
- **Empirically**
  - identify the optimal policy in **high-order** MDPs;
  - detect **partially observable** MDPs.
- **Theoretically**
  - prove our test **controls type-I error** under a **bidirectional** asymptotic framework.

Applications in high-order MDPs

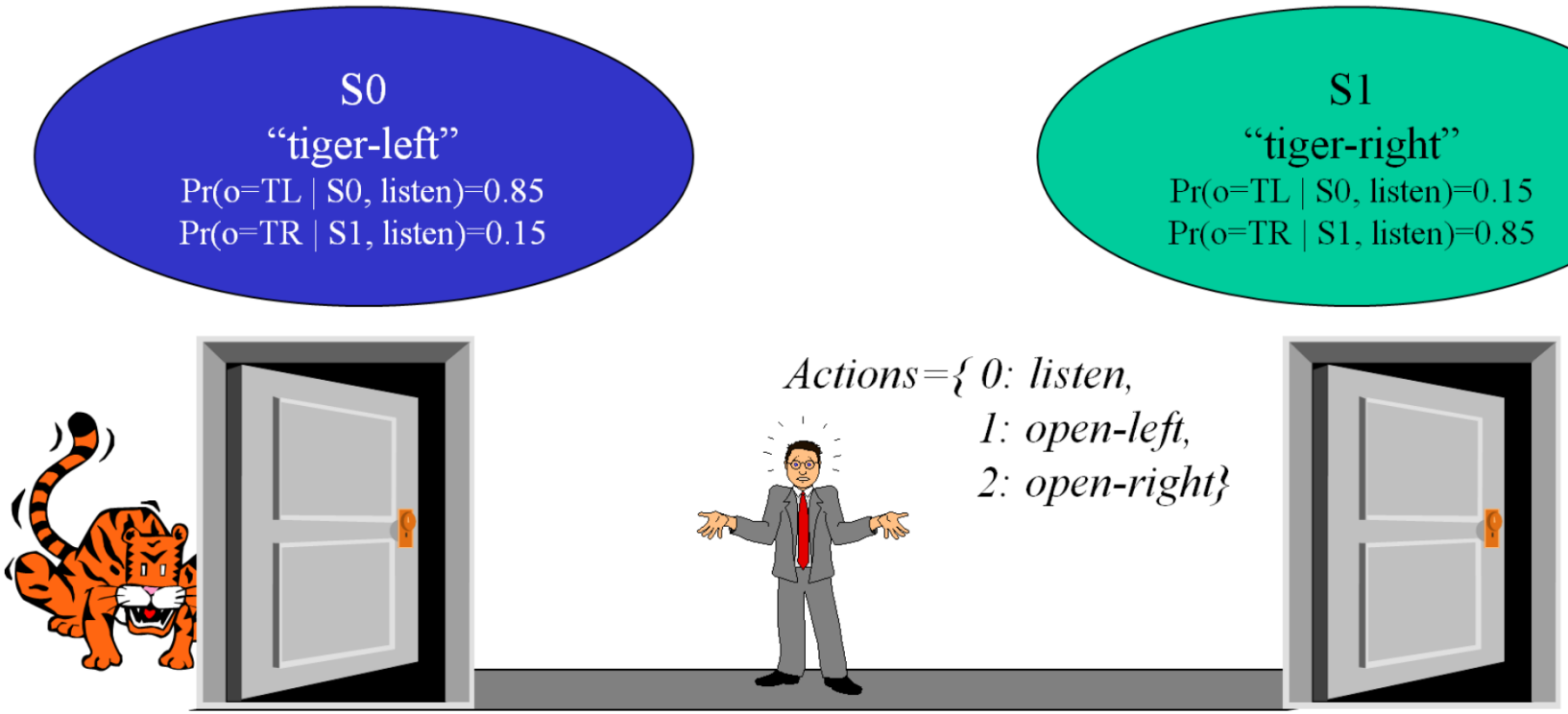
- **Data**: the OhioT1DM dataset (Marling & Bunescu, 2018).
- Measurements for 6 patients with type I diabetes over 8 weeks.
- One-hour interval as a time unit.
- **State**: patients' time-varying variables, e.g., glucose levels, food intake, exercise intensity.
- **Action**: to inject insulin or not.
- **Reward**: the Index of Glycemic Control (Rodbard, 2009).

Applications in high-order MDPs (Cont'd)

- **Analysis I**:
  - sequentially apply our test to determine the order of MDP;
  - conclude it is a **fourth-order** MDP.
- **Analysis II**:
  - split the data into training/testing samples;
  - policy optimization based on fitted-Q iteration (Ernst et al., 2005), by assuming it is a k-th order MDP for $k = 1, \cdots, 10$;
  - policy evaluation based on fitted-Q evaluation (Le et al., 2019);
  - use random forest to model the Q-function;
  - repeat the above procedure to compute the average value of policies computed under each MDP model assumption.

order	1	2	3	4	5	6	7	8	9	10
value	-90.8	-57.5	-63.8	**-52.6**	-56.2	-60.1	-63.7	-54.9	-65.1	-59.6

Applications in partially observable MDPs



Reward Function

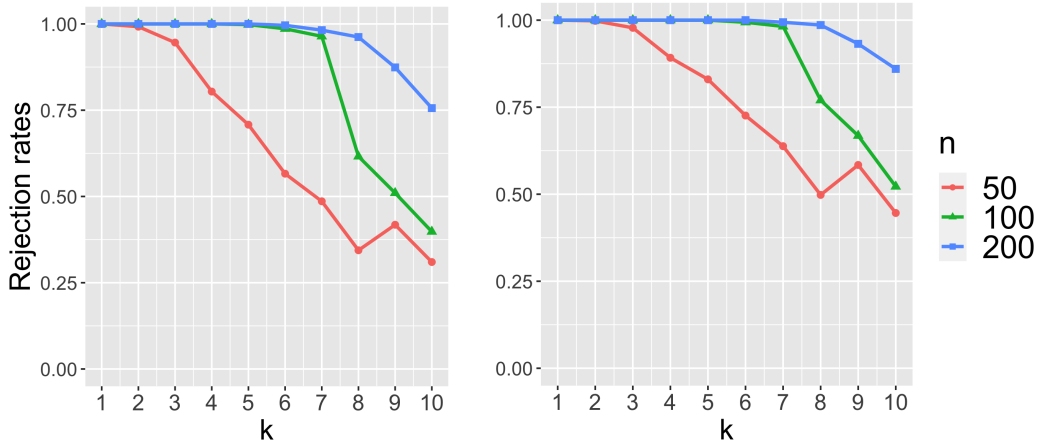
- *Penalty for wrong opening: -100*
- *Reward for correct opening: +10*
- *Cost for listening action: -1*

Observations

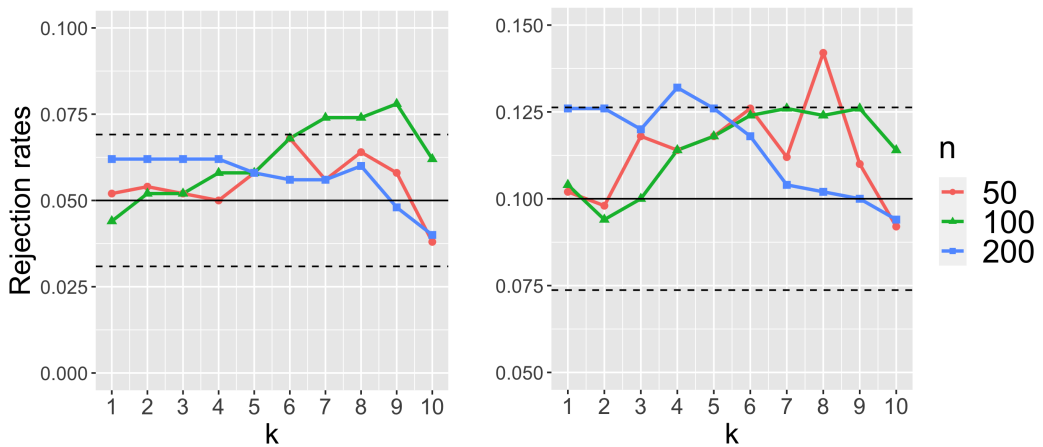
- *to hear the tiger on the left (TL)*
- *to hear the tiger on the right(TR)*

Applications in partially observable MDPs (Cont'd)

- Empirical rejection rates under the alternative hypothesis (MA is violated). $\alpha = (0.05, 0.1)$ from left to right.



- Empirical rejection rates under the null hypothesis (MA holds). $\alpha = (0.05, 0.1)$ from left to right.



Methodology

- **First work** to test MA in sequential decision making
- Existing approach in time series: Cheng and Hong (2012)
  - characterize MA based on the notion of **conditional characteristic function** (CCF);
  - use kernel smoother to estimate CCF.
- Challenge:
  - develop a valid test for MA in **moderate or high-dimensions**
  - the dimension of the state increases as we concatenate measurements over multiple time points in order to test for a high-order MDP.
- This motivates our **forward-backward learning** procedure.

Methodology (Cont'd)

Some key components of our algorithm:

- To deal with moderate or high-dimensional state space, employ modern machine learning (ML) algorithms to estimate CCF:
  - Learn CCF of $\boldsymbol{S}_{t+1}$ given $\boldsymbol{A}_t$ and $\boldsymbol{S}_t$ (**forward learner**);
  - Learn CCF of $(\boldsymbol{S}_t, \boldsymbol{A}_t)$ given $(\boldsymbol{S}_{t+1}, \boldsymbol{A}_{t+1})$ (**backward learner**);
  - Develop a random forest-based algorithm to estimate CCF;
  - Borrow ideas from the quantile random forest algorithm (Meinshausen, 2006) to facilitate the computation.
- To alleviate the bias of ML algorithms, construct **doubly-robust** estimating equations by integrating forward and backward learners;
- To improve the power, construct a **maximum-type** test statistic;
- To control the type-I error, approximate the distribution of our test via **multiplier bootstrap** (Chernozhukov, et al., 2014).

Characterization of the Markov assumption (MA)

- Suppose the process is stationary. MA is equivalent to

$$\boldsymbol{S}_{t+q+1} \perp\!\!\!\perp (\boldsymbol{S}_{t-1}, \boldsymbol{A}_{t-1}) | \{(\boldsymbol{S}_j, \boldsymbol{A}_j)\}_{t \le j \le t+q}, \quad \forall q \ge 0.$$

- Adopting the CCF approach, MA is equivalent to

$$\begin{aligned} &\underbrace{\mathrm{E}\{\exp(i\mu^\top \boldsymbol{S}_{t+q+1}) | \boldsymbol{S}_{t+q}, \boldsymbol{A}_{t+q}\}}_{\text{Forward learner } \varphi(\mu | \boldsymbol{S}_{t+q}, \boldsymbol{A}_{t+q})} \\ &\quad \times \mathrm{E}\{\exp(i\nu^\top (\boldsymbol{S}_{t-1}, \boldsymbol{A}_{t-1})) | \{(\boldsymbol{S}_j, \boldsymbol{A}_j)\}_{t \le j \le t+q}\} \\ &= \mathrm{E}\{\exp(i\mu^\top \boldsymbol{S}_{t+q+1} + i\nu^\top (\boldsymbol{S}_{t-1}, \boldsymbol{A}_{t-1})) | \{(\boldsymbol{S}_j, \boldsymbol{A}_j)\}_{t \le j \le t+q}\} \end{aligned}$$

  for any μ, ν and q.

Forward and Backward Learning

- Take expectation on the expectation on both LHS and RHS,

$$\mathrm{E}\{\exp(i\mu^\top \boldsymbol{S}_{t+q+1}) - \varphi(\mu|\boldsymbol{S}_{t+q}, \boldsymbol{A}_{t+q})\}\exp(i\nu^\top(\boldsymbol{S}_{t-1}, \boldsymbol{A}_{t-1})) = 0.$$

  Chen and Hong (2012)'s test is built upon a similar equation, requiring bias of $\widehat{\varphi}$ to converge faster than the parametric-rate.

- The proposed test is built upon

$$\mathrm{E}\{\exp(i\mu^\top \boldsymbol{S}_{t+q+1}) - \varphi(\mu|\boldsymbol{S}_{t+q}, \boldsymbol{A}_{t+q})\}$$
$$\times[\exp(i\nu^\top(\boldsymbol{S}_{t-1}, \boldsymbol{A}_{t-1})) - \underbrace{\mathrm{E}\{\exp(i\nu^\top(\boldsymbol{S}_{t-1}, \boldsymbol{A}_{t-1}))|\boldsymbol{S}_t, \boldsymbol{A}_t\}}_{\text{Backward learner } \psi(\nu|\boldsymbol{S}_t,\boldsymbol{A}_t)}] = 0.$$

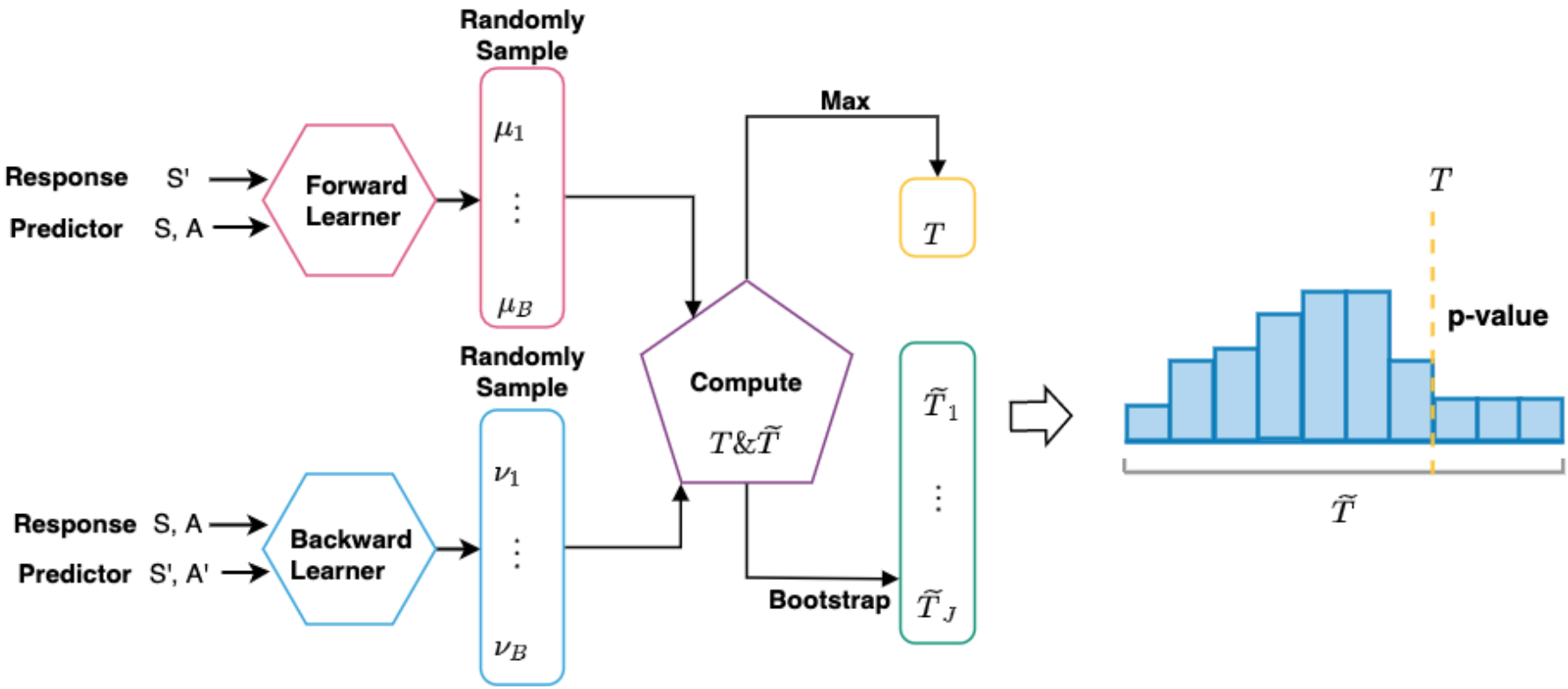
  The above equation is **doubly-robust**, allowing biases of $\widehat{\varphi}$ and $\widehat{\psi}$ to converge slower than the parametric-rate.

Algorithm

1. Grow trees as in random forest, with multivariate responses.

2. Compute the weight as a function of the **state**-**action** pair as an average over the individual weight for each tree.
3. Take weighted average of $\{\exp(i\mu^\top \boldsymbol{S}_{i,t})\}$ (forward learner) and $\{\exp(i\nu^\top(\boldsymbol{S}_{i,t}, \boldsymbol{A}_{i,t}))\}$ (backward learner) as output.

Algorithm (Cont'd)



Bidirectional theory

- N the number of trajectories;
- T the number of decision points per trajectory;
- bidirectional asymptotics: a framework allows either N or $T \to \infty$;
- large T, small N (OhioT1DM dataset)

- large N, small T (HeartStep)

- large N, large T (games)

Bidirectional theory (cont'd)

(C1) Actions are generated by a fixed behavior policy.
(C2) The process $\{S_t\}_{t\geq 0}$ is exponentially β-mixing.
(C3) The ℓ_2 prediction errors of forward and backward learners converge at a rate faster than $(NT)^{-1/4}$.

Theorem

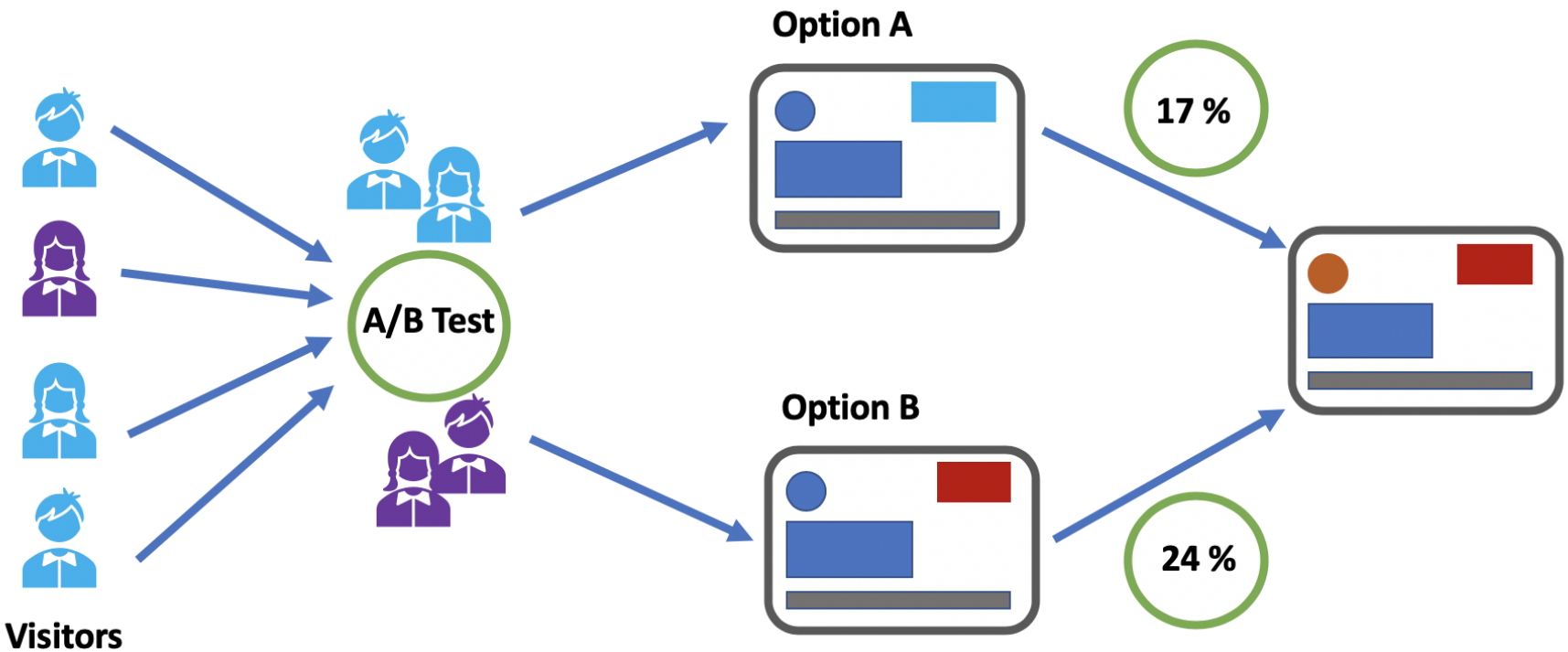
Assume (C1)-(C3) hold. Then under some other mild conditions, our test controls the type-I error asymptotically as either N or T diverges to ∞.

A Reinforcement Learning Framework for Time-Dependent Causal Effects Evaluation in A/B Testing

joint work with Xiaoyu Wang, Shikai Luo, Rui Song, Hongtu Zhu and Jieping Ye

——*JASA, in review.*

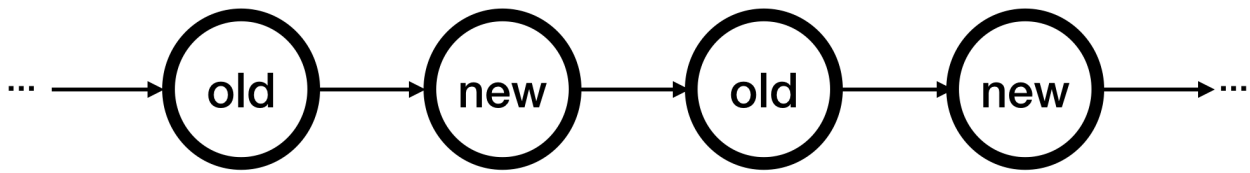
A/B testing



Taken from `https://towardsdatascience.com/how-to-conduct-a-b-testing-3076074a8458`.

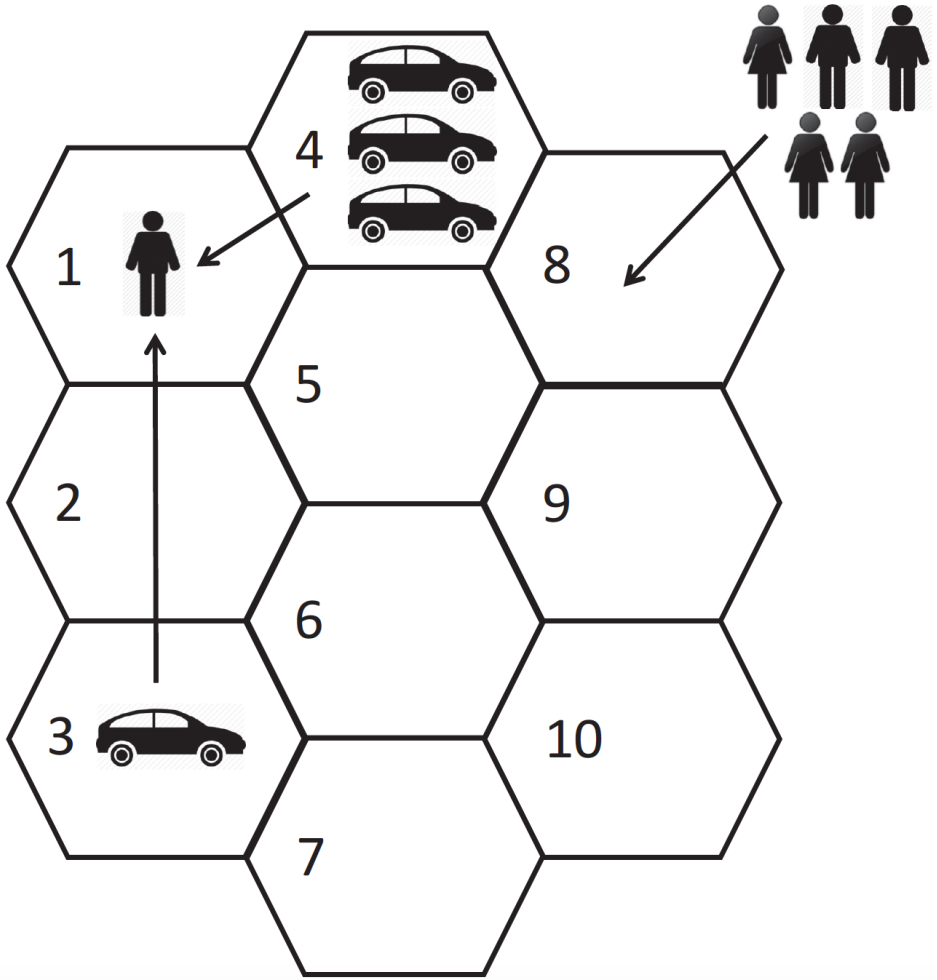
Motivation and challenges

- The project is motivated by the need for comparing the **long-term rewards** of different **order dispatching policies** in **ridesharing platforms**.
- Challenges:
  1. The existence of **carryover effects**:
     - Alternating time-interval design



     - Past actions will affect future outcomes.
  2. The need for **early termination**:
     - Each experiment takes a considerable time (at most 2 weeks);
     - Early termination to save time and budget.
  3. The need for **adaptive randomization**:
     - Maximize the total reward (e.g., epsilon-greedy);
     - Detect the alternative faster.
- No existing test has addressed three challenges simultaneously.

Illustration of carryover effects



Limitations of existing A/B tests

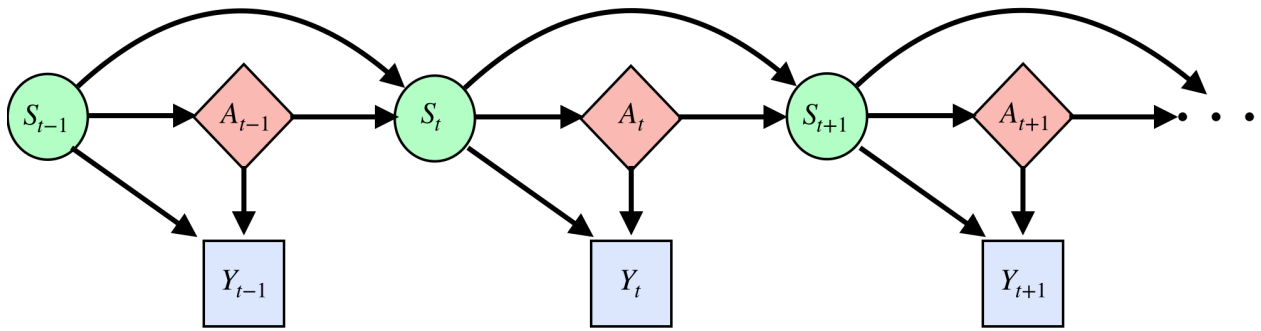
- Most existing tests **cannot** detect carryover effects.
- $\mathcal{H}_0$: The old policy ($\boldsymbol{A} = 0$) has larger cumulative rewards.
- **Example 1**. $\boldsymbol{S}_t \sim N(0, 0.25)$, $\boldsymbol{R}_t = \boldsymbol{S}_t + \delta \boldsymbol{A}_t$
- **Example 2**. $\boldsymbol{S}_t = 0.5\boldsymbol{S}_{t-1} + \delta \boldsymbol{A}_{t-1} + N(0, 0.25)$, $\boldsymbol{R}_t = \boldsymbol{S}_t$

Example 1	t-test 0.76	DML-based test **1.00**	our test **0.98**
Example 2	t-test 0.04	DML-based test 0.06	our test **0.73**

Table: Powers of t-test, DML-based test (Chernozhukov et al., 2018) and the proposed test with $T = 500$, $\delta = 0.1$.

Contributions and advances of our proposal

- Introduce an RL framework for A/B testing



  1. $\boldsymbol{A_{t-1}}$ impacts $\boldsymbol{R_t}$ indirectly through its effect on $\boldsymbol{S_t}$
  2. $\boldsymbol{S_t}$ shall include important mediators between $\boldsymbol{A_{t-1}}$ and $\boldsymbol{R_t}$

- Propose a test procedure for detecting value difference
  1. allows for **sequential monitoring**
  2. allows for **online updating**
  3. applicable to a wide range of designs, including the **Markov** design, **alternating-time-interval** design and **adaptive** design

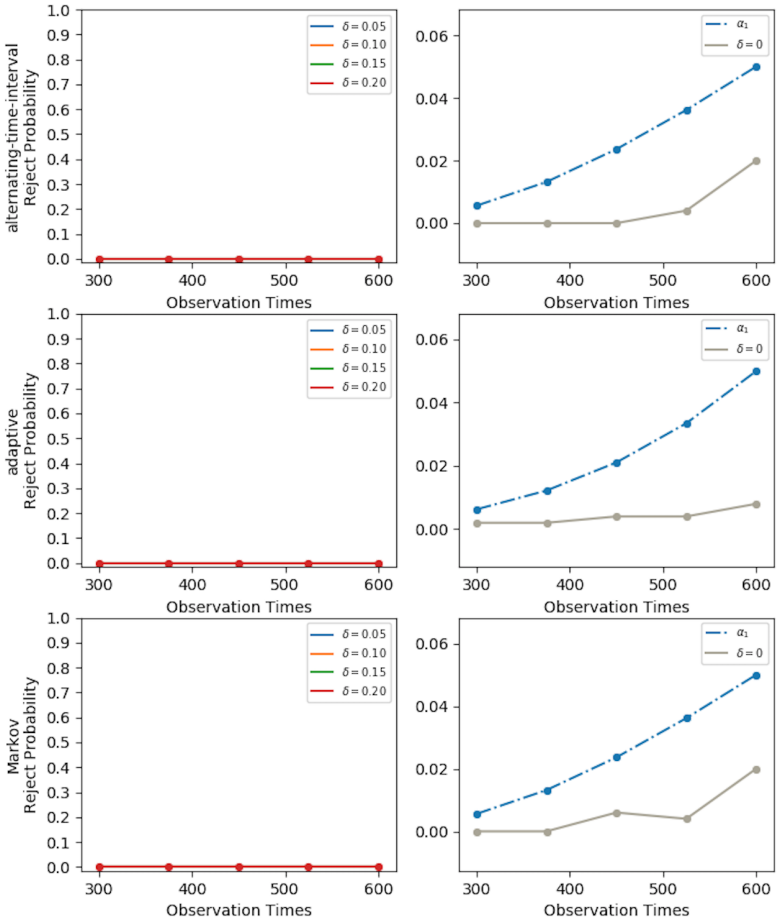
Methodology and theory

- Apply **temporal difference learning** to evaluate value difference and provide **uncertainty quantification**
- Adopt the α-**spending approach** (Lan & DeMets, 1983) for sequential monitoring
- Develop a **bootstrap-assisted procedure** for determine the stopping boundary
  - The numerical integration method designed for classical sequential tests is **not** applicable in adaptive design, due to the carryover effects

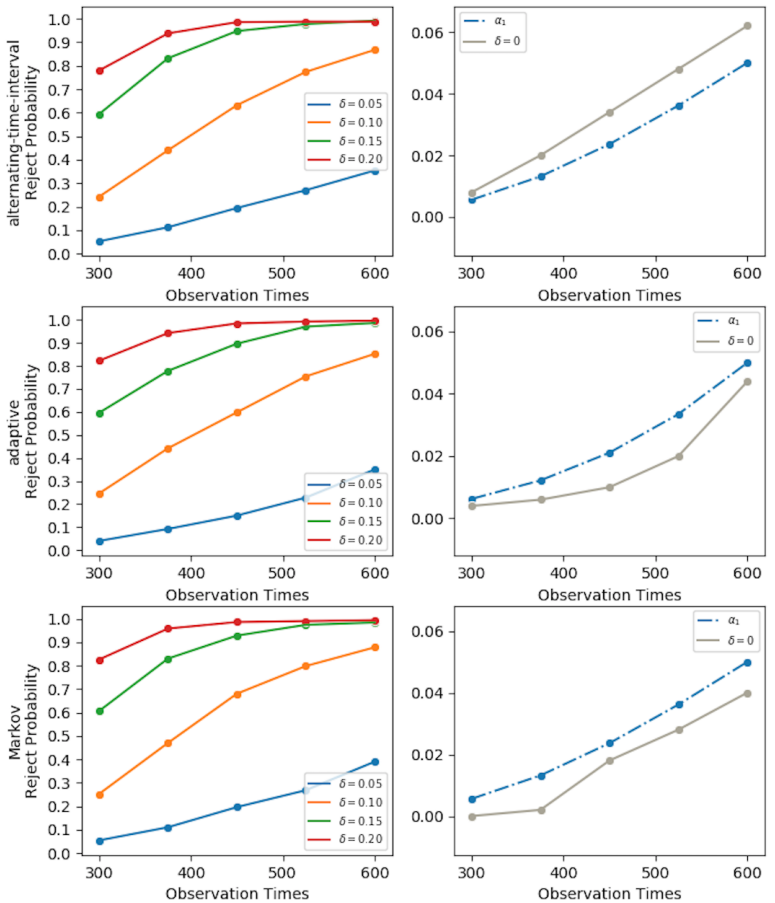
Theorem (Informal statement)

Under the Markov, alternating-time-interval or adaptive design, the proposed test can control type-I error and is consistent against alternatives that converge to the null at the $T^{-1/2}$ *rate.*

Simulation



(a) Power and size of t-test



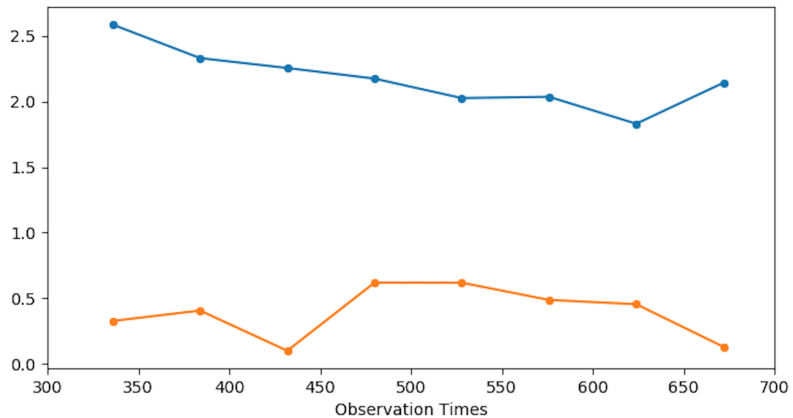
(b) Power and size of our test

Application to ridesharing platform

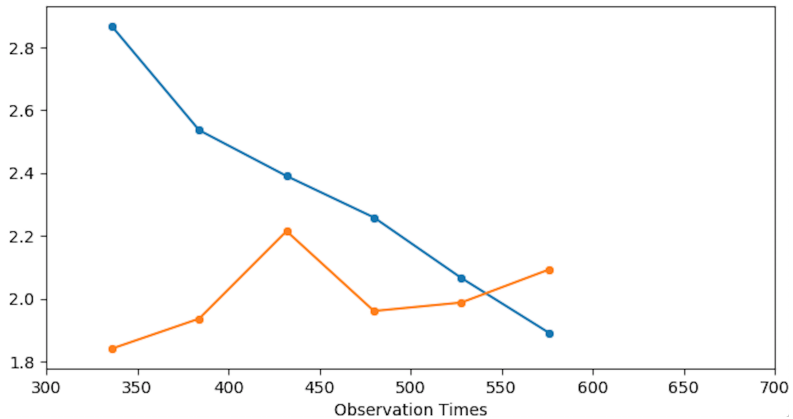
- **Data**: a given city from December 3rd to 16th (two weeks)
- **30 minutes** as one time unit, sample size = **672**
- **State**:
  1. number of drivers (supply)
  2. number of requests (demand)
  3. supply and demand equilibrium metric (mediator)
- **Action**: new policy $\boldsymbol{A} = 1$ v.s. old $\boldsymbol{A} = 0$
- **Reward**: drivers' income
- The new policy is expected to have **better** performance

Application to ridesharing platform (Cont'd)

- The proposed test



(a) AA experiment
Blue line: stopping boundary



(b) AB experiment
Orange line: test statistics

- T-test: **fail** to reject $\mathcal{H}_0$ in A/B experiment with p-value 0.18

Thanks!