Does the Markov decision process fit the data

— Testing for the Markov property in sequential decision making

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Chengchun Shi [1] and Runzhe Wan [2] and Rui Song [2] and Wenbin Lu [2] and Ling Leng [3]

[1]London School of Economics and Political Science

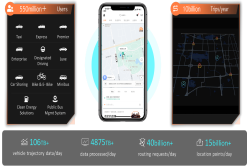
[2]North Carolina State University

[3]Amazon

Developing AI with Reinforcement Learning

In this talk, we will focus on...

- Reinforcement learning in **offline real-world applications**.
  - Most works consider developing AI in games (online).



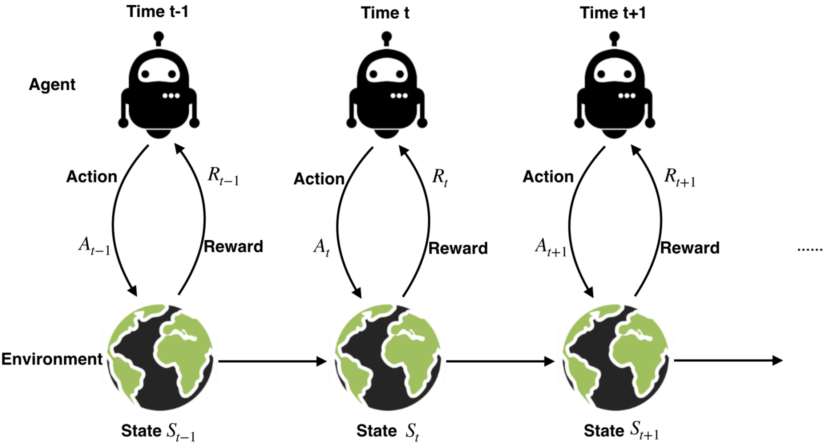
(a) Ridesharing

(b) Mobile health

(c) Auto driving

- **Statistical inference** in reinforcement learning.
  - Is statistical inference useful in reinforcement leaning?

Sequential decision making



Objective: find an optimal policy that maximizes the cumulative reward

The agent's policy

- The agent implements a **mapping** from the observed data to a probability distribution over actions at each time step

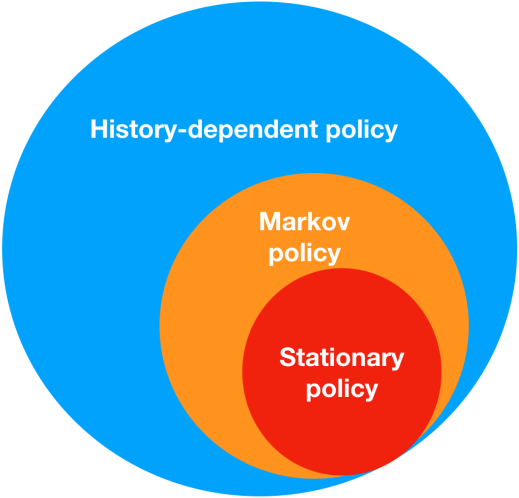
- The collection of these mappings $\pi = \{\pi_t\}_t$ is called **the agent's policy**:

$$\pi_t(a, \bar{s}) = \Pr(A_t = a | \bar{S}_t = \bar{s}),$$

  where $\bar{S}_t = (S_t, A_{t-1}, S_{t-1}, \cdots, A_0, S_0)$ is the set of observed state-action history up to time t

- **History-dependent** policy: π_t depends on $\bar{S}_t$
- **Markov** policy: π_t depends on $\bar{S}_t$ only through S_t, $\forall t$
- **Stationary** policy: π is Markov & π_t is homogeneous in t, $\forall t$

The Agent's Policy (Cont'd)



Reinforcement learning

- **RL algorithms**: trust region policy optimization (Schulman et al., 2015), deep Q-network (DQN, Mnih et al., 2015), asynchronous advantage actor-critic (Minh et al., 2016), quantile regression DQN (Dabney et al., 2018).
- **Foundations** of RL:
  - **Markov decision process** (MDP, Puterman, 1994): ensures the optimal policy is *stationary*, and is *not* history-dependent.
  - **Markov assumption** (MA): conditional on the present, the future and the past are independent,

    $$\boldsymbol{S}_{t+1}, \boldsymbol{R}_t \perp\!\!\!\perp \{(\boldsymbol{S}_j, \boldsymbol{A}_j, \boldsymbol{R}_j)\}_{j<t} | \boldsymbol{S}_t, \boldsymbol{A}_t.$$

    When $\boldsymbol{R}_t$ is a deterministic function of $(\boldsymbol{S}_t, \boldsymbol{A}_t, \boldsymbol{S}_{t+1})$:

    $$\boldsymbol{S}_{t+1} \perp\!\!\!\perp \{(\boldsymbol{S}_j, \boldsymbol{A}_j)\}_{j<t} | \boldsymbol{S}_t, \boldsymbol{A}_t.$$

    The Markov transition kernel is homogeneous in time.

RL models

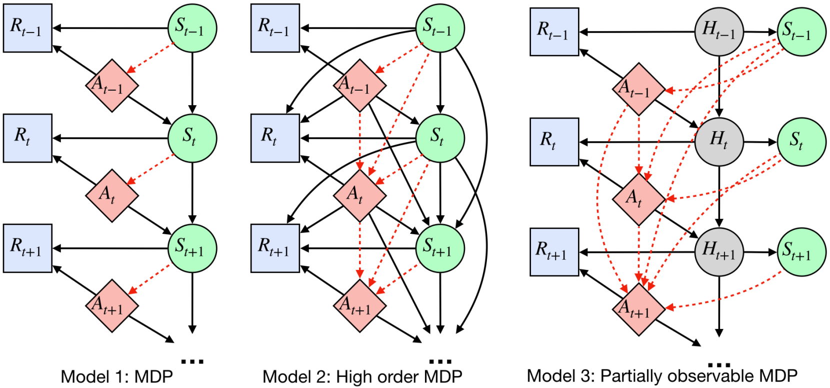


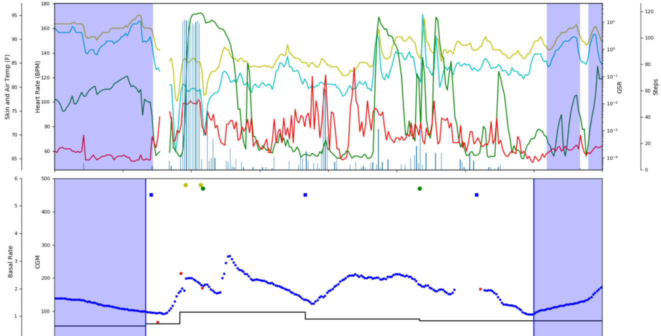
Figure: Causal diagrams for MDPs, HMDPs and POMDPs. The solid lines represent the causal relationships and the dashed lines indicate the information needed to implement the optimal policy. $\{H_t\}_t$ denotes latent variables.

Contributions

- **Methodologically**
  - propose a **forward-backward learning** procedure to test MA;
  - **first** work on developing consistent tests for MA in RL;
  - sequentially apply the proposed test for RL **model selection**;
  - critical to **offline** domains:
    - For **under-fitted** models, any stationary policy is not optimal;
    - For **over-fitted** models, the estimated policy might be very noisy due to the inclusion of many irrelevant lagged variables.
- **Empirically**
  - identify the optimal policy in **high-order** MDPs;
  - detect **partially observable** MDPs.
- **Theoretically**
  - prove our test **controls type-I error** under a **bidirectional** asymptotic framework.

Applications in high-order MDPs

- **Data**: the OhioT1DM dataset (Marling & Bunescu, 2018).
- Measurements for 6 patients with type I diabetes over 8 weeks.
- One-hour interval as a time unit.
- **State**: patients' time-varying variables, e.g., glucose levels, food intake, exercise intensity.
- **Action**: to inject insulin or not.
- **Reward**: the Index of Glycemic Control (Rodbard, 2009).

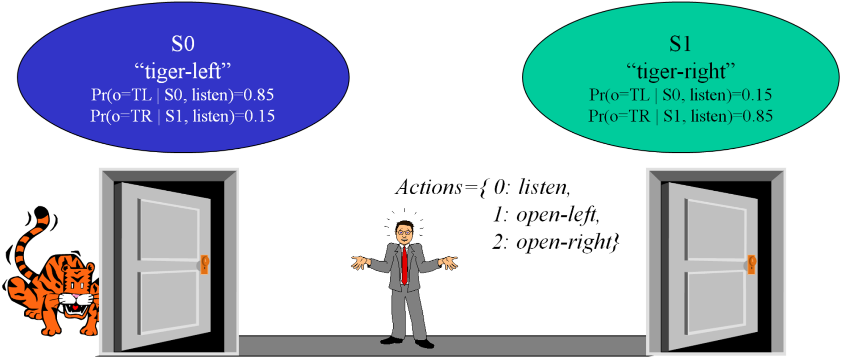


Applications in high-order MDPs (Cont'd)

- **Analysis I**:
  - sequentially apply our test to determine the order of MDP;
  - conclude it is a **fourth-order** MDP.
- **Analysis II**:
  - split the data into training/testing samples;
  - policy optimization based on fitted-Q iteration (Ernst et al., 2005), by assuming it is a k-th order MDP for $k = 1, \cdots, 10$;
  - policy evaluation based on fitted-Q evaluation (Le et al., 2019);
  - use random forest to model the Q-function;
  - repeat the above procedure to compute the average value of policies computed under each MDP model assumption.

order	1	2	3	4	5	6	7	8	9	10
value	-90.8	-57.5	-63.8	**-52.6**	-56.2	-60.1	-63.7	-54.9	-65.1	-59.6

Applications in partially observable MDPs



Reward Function

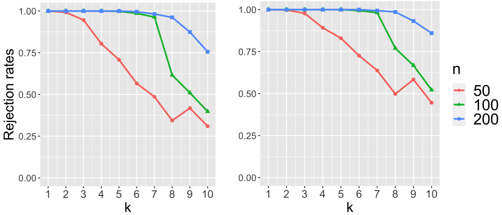
- *Penalty for wrong opening: -100*
- *Reward for correct opening: +10*
- *Cost for listening action: -1*

Observations

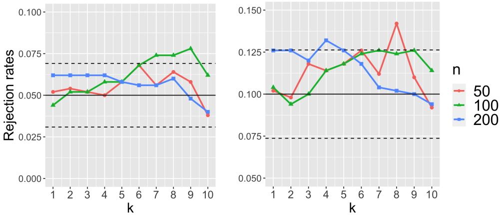
- *to hear the tiger on the left (TL)*
- *to hear the tiger on the right(TR)*

Applications in partially observable MDPs (Cont'd)

- Empirical rejection rates under the alternative hypothesis (MA is violated). $\alpha = (0.05, 0.1)$ from left to right.



- Empirical rejection rates under the null hypothesis (MA holds). $\alpha = (0.05, 0.1)$ from left to right.



Methodology

- **First work** to test MA in sequential decision making
- Existing approach in time series: Cheng and Hong (2012)
  - characterize MA based on the notion of **conditional characteristic function** (CCF);
  - use local polynomial regression to estimate CCF.
- Challenge:
  - develop a valid test for MA in **moderate or high-dimensions**
  - the dimension of the state increases as we concatenate measurements over multiple time points in order to test for a high-order MDP.
- This motivates our **forward-backward learning** procedure.

Methodology (Cont'd)

Some key components of our algorithm:

- To deal with moderate or high-dimensional state space, employ modern machine learning (ML) algorithms to estimate CCF:
    - Learn CCF of $\boldsymbol{S}_{t+1}$ given $\boldsymbol{A}_t$ and $\boldsymbol{S}_t$ (**forward learner**);
    - Learn CCF of $(\boldsymbol{S}_t, \boldsymbol{A}_t)$ given $(\boldsymbol{S}_{t+1}, \boldsymbol{A}_{t+1})$ (**backward learner**);
    - Develop a random forest-based algorithm to estimate CCF;
    - Borrow ideas from the quantile random forest algorithm (Meinshausen, 2006) to facilitate the computation.
- To alleviate the bias of ML algorithms, construct **doubly-robust** estimating equations by integrating forward and backward learners;
- To improve the power, construct a **maximum-type** test statistic;
- To control the type-I error, approximate the distribution of our test via **multiplier bootstrap** (Chernozhukov, et al., 2014).

Bidirectional theory

- N the number of trajectories;
- T the number of decision points in each trajectory;
- bidirectional asymptotics: a framework where either N or T grows to ∞;
- large T, small N (mobile health)

- large N, small T (some medical studies)

- large N, large T (games)

Bidirectional theory (cont'd)

(C1) Actions are generated by a fixed behavior policy.
(C2) The process $\{S_t\}_{t \geq 0}$ is exponentially β-mixing.
(C3) The ℓ_2 prediction errors of forward and backward learners converge at a rate faster than $(NT)^{-1/4}$.

Theorem

Assume (C1)-(C3) hold. Then under some other mild conditions, our test controls the type-I error asymptotically as either N or T diverges to ∞.

- Paper: http://proceedings.mlr.press/v119/shi20c/shi20c.pdf
- Code: https://github.com/RunzheStat/TestMDP

Thank you! ☺