Statistical Inference in Reinforcement Learning

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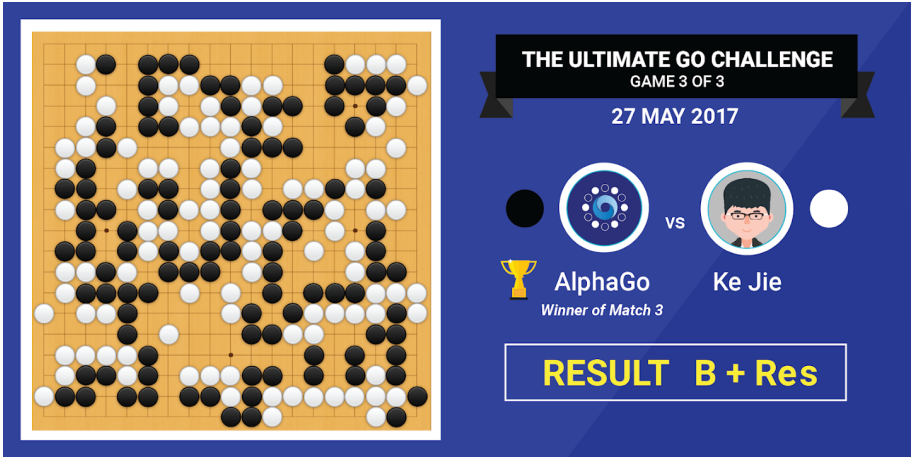
Developing AI with Reinforcement Learning



Reinforcement Learning Applications

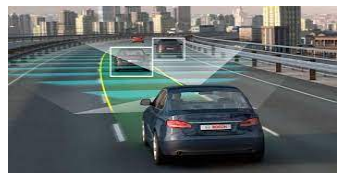
(a) Games

(b) Health Care

(c) Ridesharing

(d) Robotics

(e) Finance

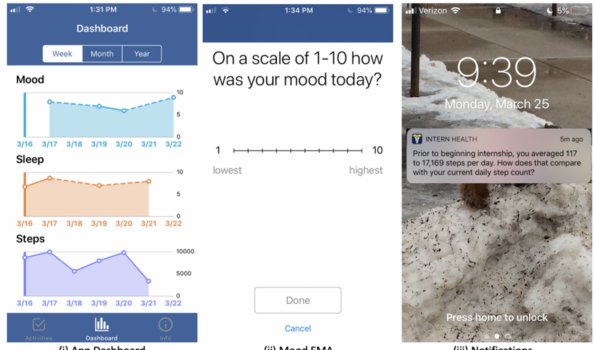


(f) Automated Driving

We focus on applications in **mobile health** (mHealth) and **ridesharing**

Applications in mHealth

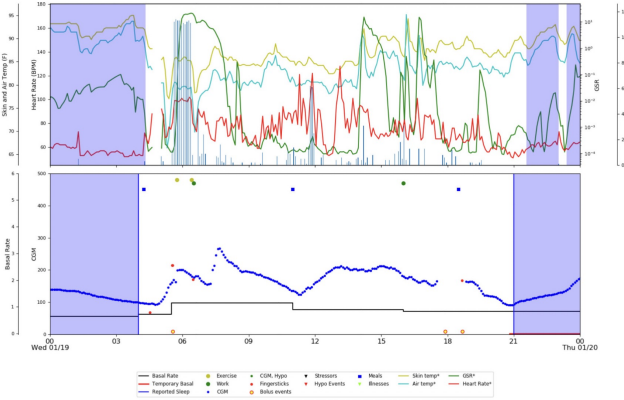
- Use of cellphones and wearable devices in healthcare
- **Data**: Intern Health Study (NeCamp et al., 2020)
- **Subject**: First-year medical interns working in stressful environments (e.g., long work hours and sleep deprivation)
- **Objective**: Promote physical and mental well-beings
- **Intervention**: Determine whether to send certain text message to a subject



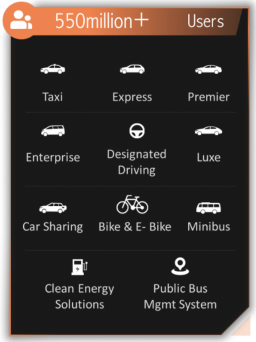
(i) App Dashboard (ii) Mood EMA (iii) Notifications

Applications in mHealth (Cont'd)

- Management of **Type-I diabetes**
- **Subject**: Patients with Type-I diabetes
- **Intervention**: Determine whether a patient needs to **inject insulin or not** based on their glucose levels, food intake, exercise intensity
- **Data**: OhioT1DM dataset (Marling and Bunescu, 2018)



Applications in Ridesharing



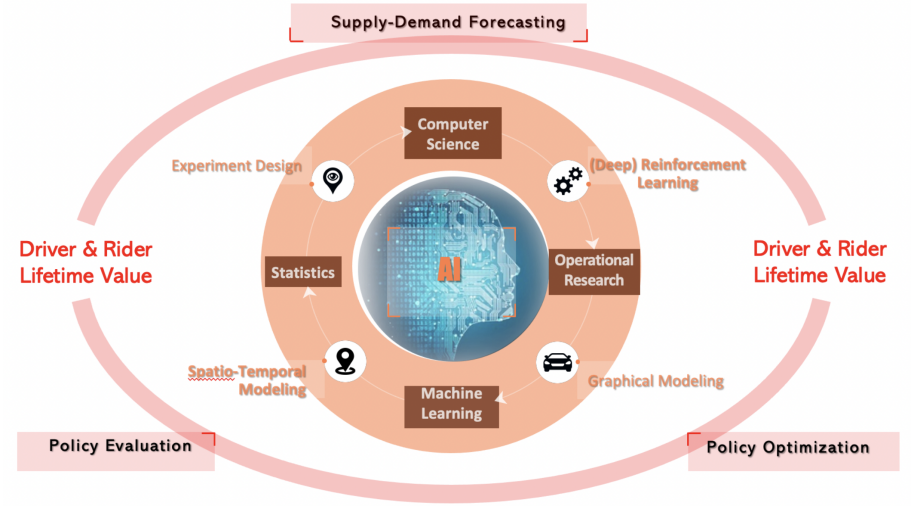
106TB+
vehicle trajectory data/day

4875TB+
data processed/day

40billion+
routing requests/day

15billion+
location points/day

Applications in Ridesharing (Cont'd)



In this talk, we will focus on ...

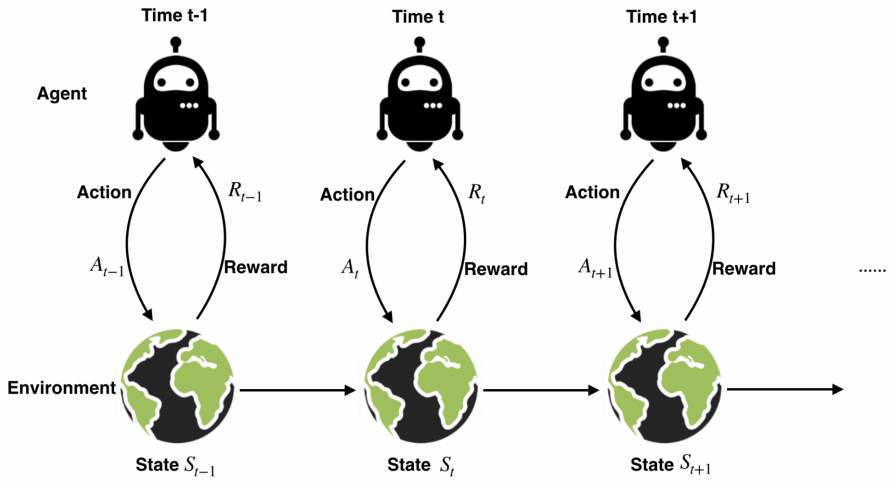
- **Statistical inference** in reinforcement learning (RL)
- Is statistical inference useful for RL?

Project I

Does the Markov Decision Process Fit the Data: Testing for the Markov Property in Sequential Decision Making

Joint work with Runzhe Wan, Wenbin Lu, Rui Song and Ling Leng
——ICML (2020)

Sequential Decision Making



Objective: find an optimal policy that maximizes the cumulative reward

The Agent's Policy

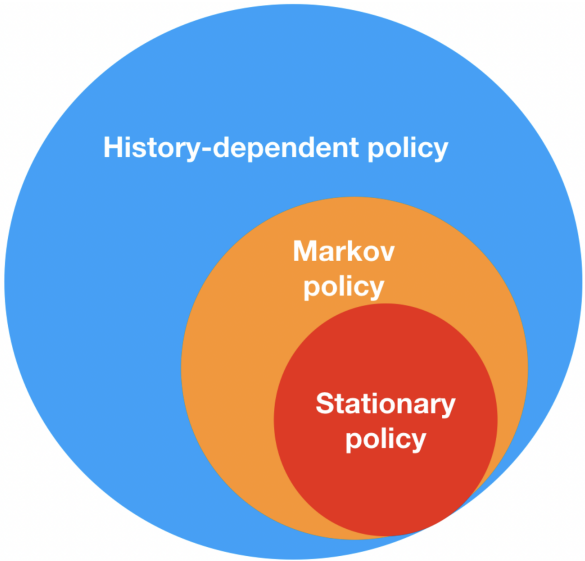
- The agent implements a **mapping** π_t from the observed data to a probability distribution over actions at each time step
- The collection of these mappings $\pi = \{\pi_t\}_t$ is called **the agent's policy**:

$$\pi_t(a|\bar{s}) = \Pr(A_t = a|\bar{S}_t = \bar{s}),$$

where $\bar{S}_t = (S_t, R_{t-1}, A_{t-1}, S_{t-1}, \cdots, R_0, A_0, S_0)$ is the set of **observed data history** up to time t.

- **History-Dependent** Policy: π_t depends on $\bar{S}_t$.
- **Markov** Policy: π_t depends on $\bar{S}_t$ only through S_t.
- **Stationary** Policy: π is Markov & π_t is **homogeneous** in t, i.e., $\pi_0 = \pi_1 = \cdots$.

The Agent's Policy (Cont'd)



Reinforcement Learning

- **RL algorithms**: trust region policy optimization (Schulman et al., 2015), deep Q-network (DQN, Mnih et al., 2015), asynchronous advantage actor-critic (Minh et al., 2016), quantile regression DQN (Dabney et al., 2018).
- **Foundations** of RL:
  - **Markov decision process** (MDP, Puterman, 1994): ensures the optimal policy is *stationary*, and is *not* history-dependent.
  - **Markov assumption** (MA): conditional on the present, the future and the past are independent,

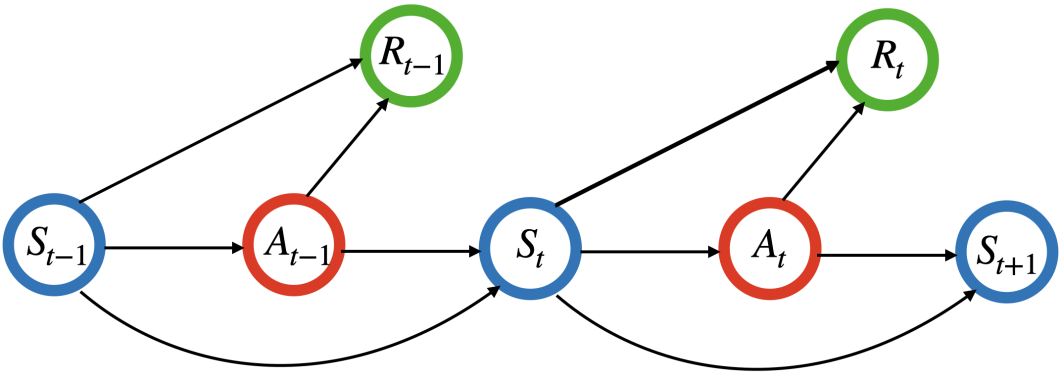
$$S_{t+1}, R_t \perp\!\!\!\perp \{(S_j, A_j, R_j)\}_{j<t} | S_t, A_t.$$

When R_t is a deterministic function of (S_t, A_t, S_{t+1})

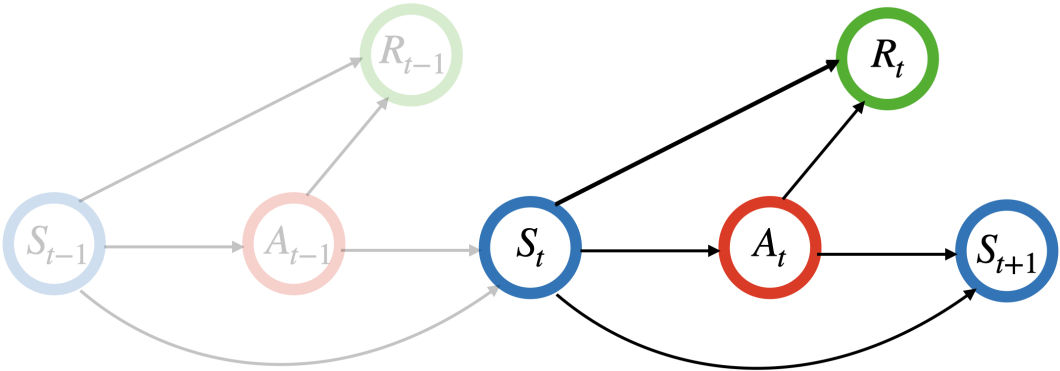
$$S_{t+1} \perp\!\!\!\perp \{(S_j, A_j)\}_{j<t} | S_t, A_t.$$

The Markov transition kernel is homogeneous in time

Markov Assumption



Markov Assumption



RL Models

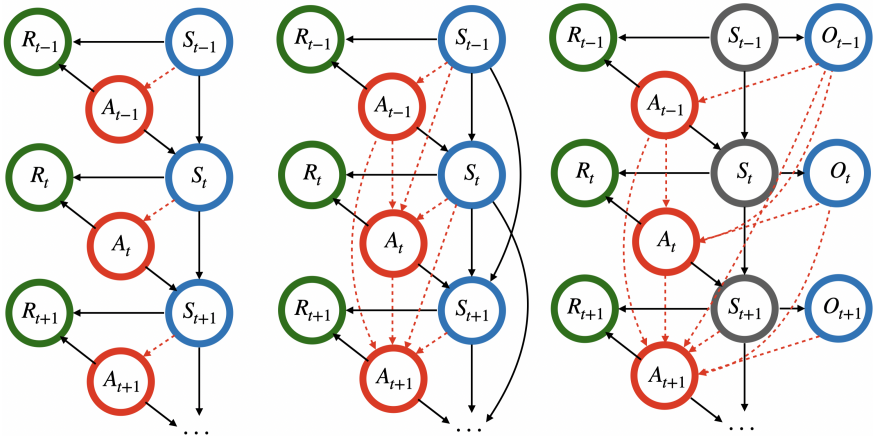


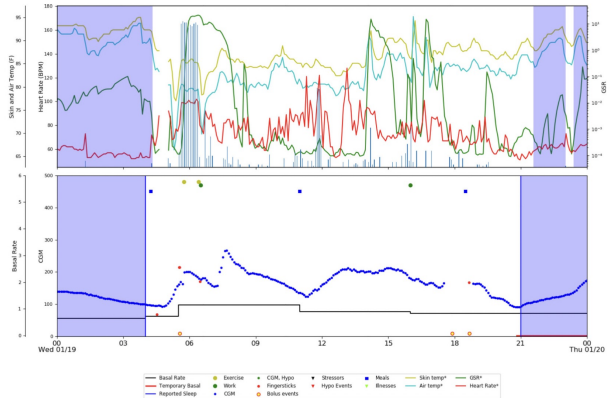
Figure: Causal diagrams for MDPs, HMDPs & POMDPs. The solid lines characterize the relationships among the variables and the dashed lines indicate the information needed to implement the optimal policy. In Model III, $\{\boldsymbol{S_t}\}_t$ denotes latent variables.

Contributions

- **Methodologically**
  - propose a **forward-backward learning** procedure to test MA
  - **first** work on developing consistent tests for MA in RL
  - sequentially apply the proposed test for RL **model selection** (e.g., test kth order MDP for $k = 1, 2, \cdots$)
  - critical to **offline** domains given a historical dataset **without online collection**:
    - For **under-fitted** models, any stationary policy is not optimal
    - For **over-fitted** models, the estimated policy might be very noisy due to the inclusion of many irrelevant lagged variables
- **Empirically**
  - identify the optimal policy in **high-order** MDPs
  - detect **partially observable** MDPs
- **Theoretically**
  - prove our test **controls type-I error** under a **bidirectional** asymptotic framework

Applications in High-Order MDPs

- **Data**: the OhioT1DM dataset
- Measurements for 6 patients with type I diabetes over 8 weeks.
- One-hour interval as a time unit.
- **State**: glucose levels, food intake, exercise intensity
- **Action**: to inject insulin or not.
- **Reward**: the Index of Glycemic Control (Rodbard, 2009).

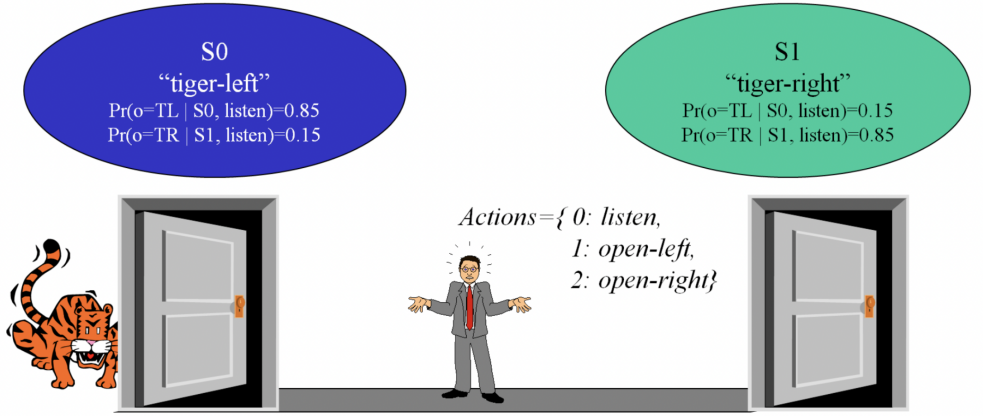


Applications in High-Order MDPs (Cont'd)

- **Analysis I**:
  - sequentially apply our test to determine the order of MDP
  - conclude it is a **fourth-order** MDP
- **Analysis II**:
  - split the data into training/testing samples
  - policy optimization based on **fitted-Q iteration**, by assuming it is a k-th order MDP for $k = 1, \cdots, 10$
  - policy evaluation based on **fitted-Q evaluation**
  - use **random forest** to model the Q-function
  - repeat the above procedure to compute the average value of policies computed under each MDP model assumption

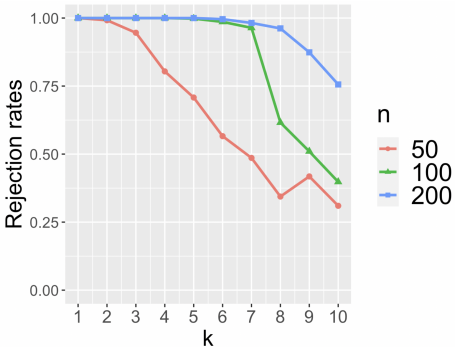
order	1	2	3	4	5	6	7	8	9	10
value	-90.8	-57.5	-63.8	**-52.6**	-56.2	-60.1	-63.7	-54.9	-65.1	-59.6

Applications in Partially Observable MDPs

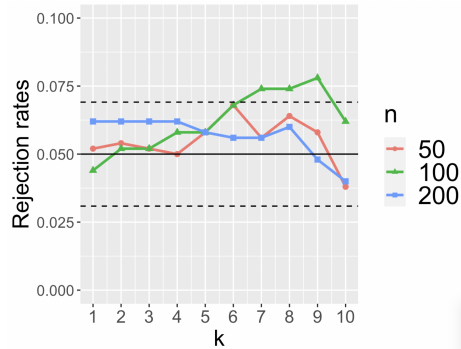


Applications in Partially Observable MDPs (Cont'd)

- Under $\mathcal{H}_1$ (MA is violated, alternative). Significance level = 0.05.



- Under $\mathcal{H}_0$ (MA holds, null). Significance level = 0.05.



Methodology

- **First** work to test MA in RL
- Existing approach in time series: Cheng and Hong (2012)
  - characterize MA based on the notion of **conditional characteristic function** (CCF)
  - use local polynomial regression to estimate CCF
- **Challenge**:
  - develop a valid test for MA in **moderate or high-dimensions**
  - the dimension of the state increases as we concatenate measurements over multiple time points in order to test for a high-order MDP.
- This motivates our **forward-backward learning** procedure.

Methodology (Cont'd)

Some key components of our algorithm:

- To deal with moderate or high-dimensional state space, employ modern machine learning (ML) algorithms to estimate CCF:
  - Learn CCF of S_{t+1} given A_t and S_t (**forward learner**)
  - Learn CCF of (S_t, A_t) given (S_{t+1}, A_{t+1}) (**backward learner**)
  - Develop a **random forest**-based algorithm to estimate CCF
  - Borrow ideas from the quantile random forest algorithm (Meinshausen, 2006) to facilitate the computation
- To alleviate the bias of ML algorithms, construct **doubly-robust** test statistics by integrating forward and backward learners;
- To improve the power, consider a **maximum-type** test statistic;
- To control the type-I error, approximate the distribution of our test via **high-dimensional multiplier bootstrap** (Chernozhukov, et al., 2014).

Bidirectional Theory

- $\boldsymbol{N}$ the number of trajectories
- $\boldsymbol{T}$ the number of decision points per trajectory
- **bidirectional asymptotics**: a framework allows either $\boldsymbol{N}$ or $\boldsymbol{T} \to \infty$
- large $\boldsymbol{N}$, small $\boldsymbol{T}$ (Intern Health Study)

- small $\boldsymbol{N}$, large $\boldsymbol{T}$ (OhioT1DM dataset)

- large $\boldsymbol{N}$, large $\boldsymbol{T}$ (games)

Bidirectional Theory (Cont'd)

(C1) Actions are generated by a fixed behavior policy.

(C2) The observed data is exponentially β-mixing.

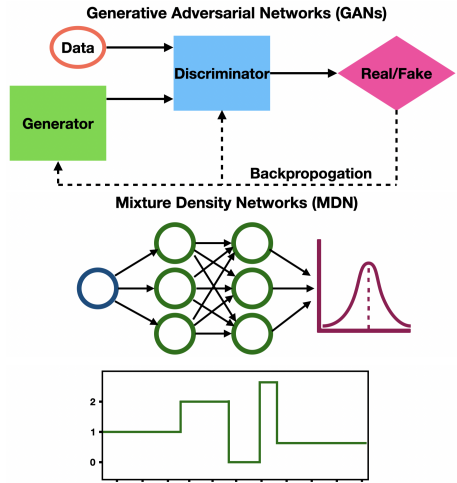
(C3) The ℓ_2 prediction errors of forward and backward learners converge at a rate faster than $(\boldsymbol{NT})^{-\mathbf{1/4}}$.

Theorem

Assume (C1)-(C3) hold. Then under some other mild conditions, our test controls the type-I error asymptotically as either $\boldsymbol{N}$ *or* $\boldsymbol{T}$ *diverges to* ∞.

Some Follow-ups

- **Double GANs** for conditional independence testing (*JMLR, 2021*)
- Testing DAGs via supervised, structural learning and **GANs** (*JASA, 2023+*)
- Testing Markovanity in time series via **deep generative learning** (*JRSSB, 2023+*)
  - Derive the convergence rate of **MDN**
- A robust test for the **stationarity** assumption in RL (*ICML, 2023*)
  - Our test helps identify a better policy in the **Intern Health Study**

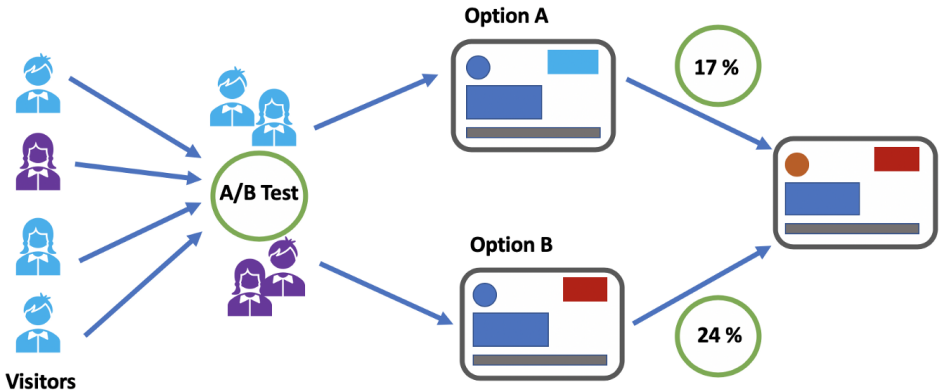


Project II

Dynamic Causal Effects Evaluation in A/B Testing with a Reinforcement Learning Framework

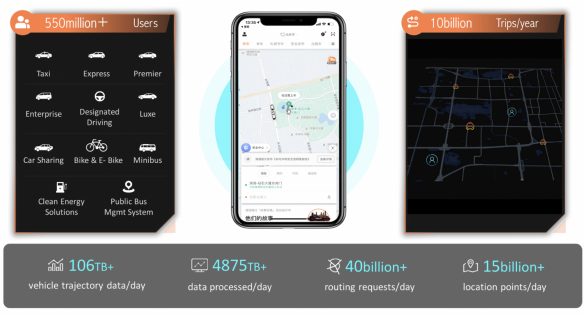
Joint work with Xiaoyu Wang, Shikai Luo, Hongtu Zhu, Jieping Ye and Rui Song
——JASA (2023)

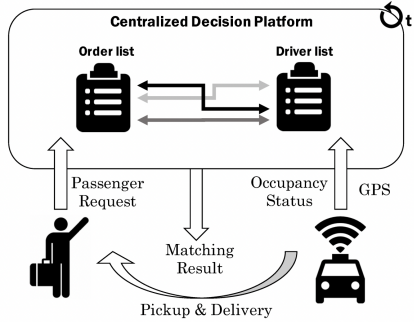
A/B Testing



Taken from
https://towardsdatascience.com/how-to-conduct-a-b-testing-3076074a8458

Motivation: Order Dispatch

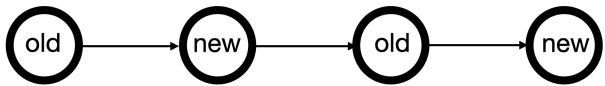




Our project is motivated by the need for comparing the **long-term rewards** of different **order dispatching** policies in **ridesharing platforms**

Challenges

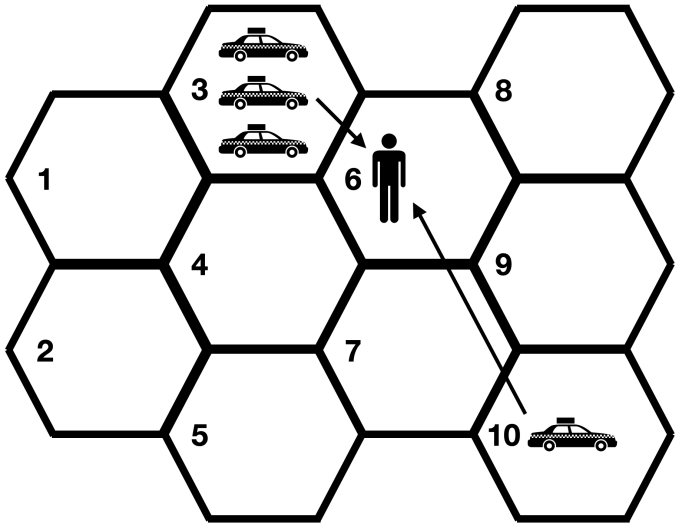
1. The existence of **carryover effects**:
   - Under the alternating-time-interval design



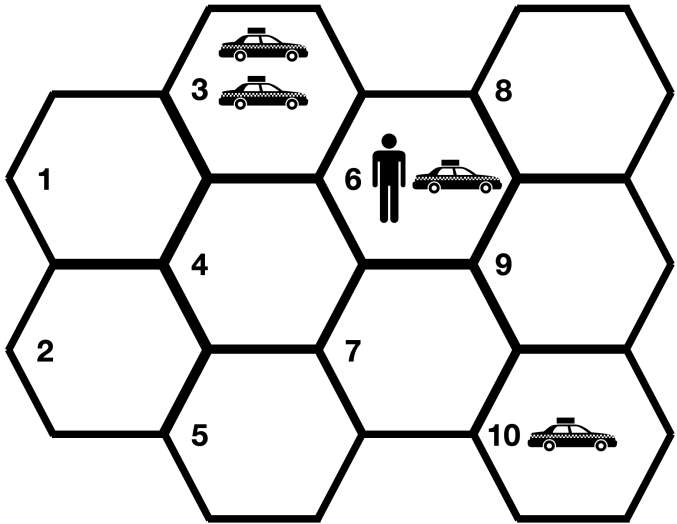
   - Past actions will affect future outcomes
2. The need for **early termination**:
   - Each experiment takes a considerable time (at most 2 weeks)
   - Early termination to save time and budget
3. The need for **adaptive randomization**:
   - Maximize the total reward (e.g., epsilon-greedy)
   - Detect the alternative faster

To our knowledge, **no** existing test has addressed three challenges simultaneously

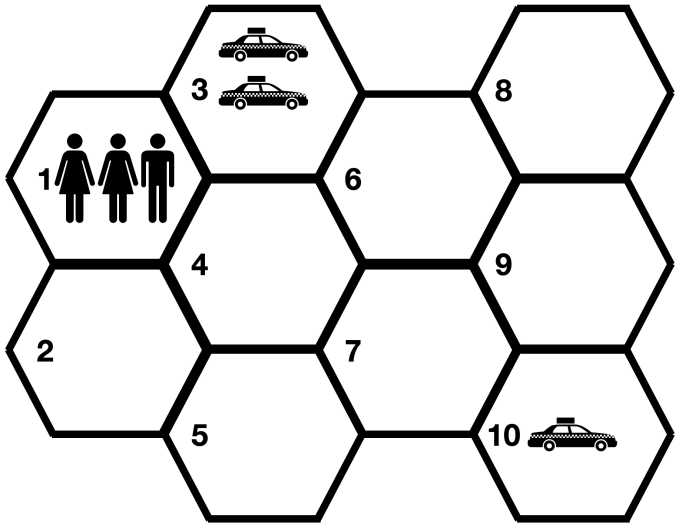
Illustration of the Carryover Effects



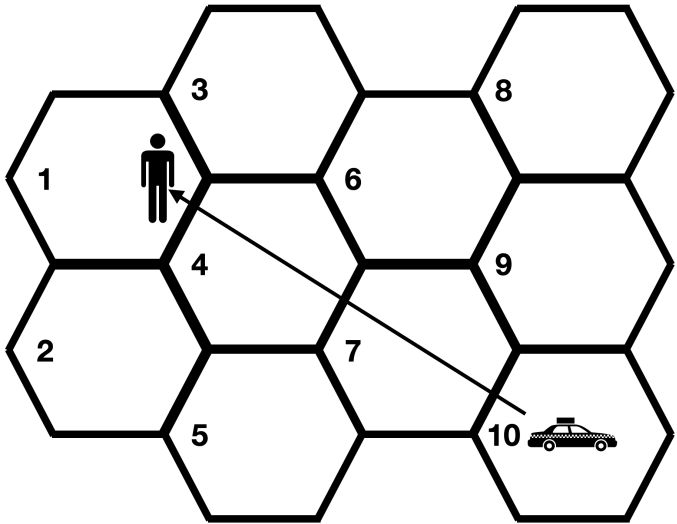
Adopting the Closest Driver Policy



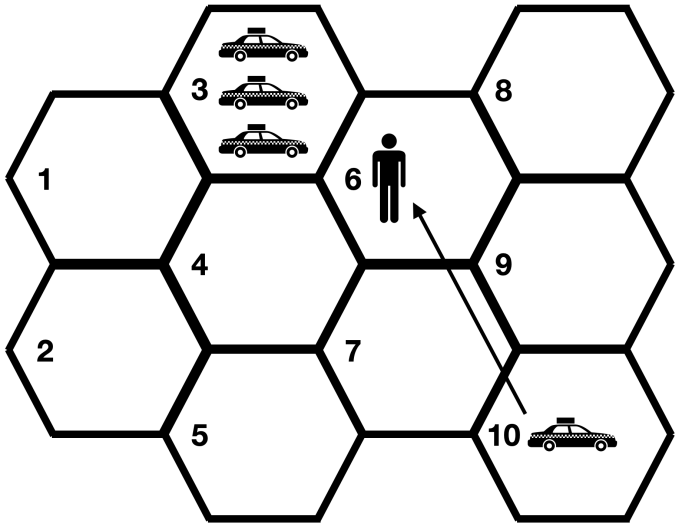
Some Time Later · · ·



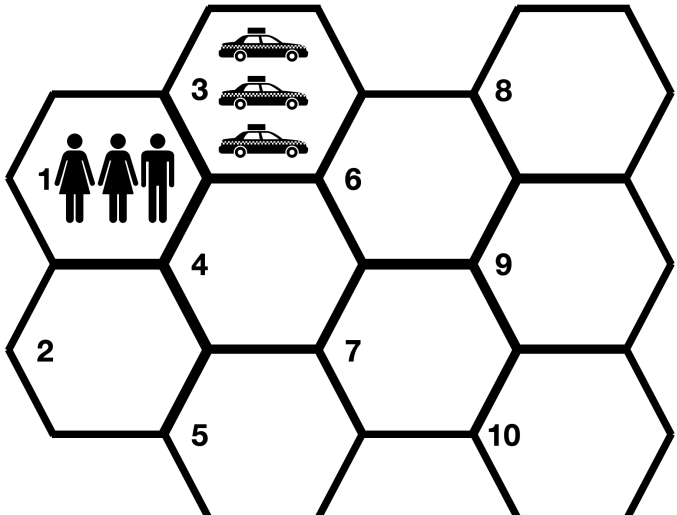
Miss One Order



Consider a Different Action



Able to Match All Orders



Existence of Carryover Effects

past actions $\rightarrow$ distribution of drivers $\rightarrow$ future rewards

Limitations of Existing A/B tests

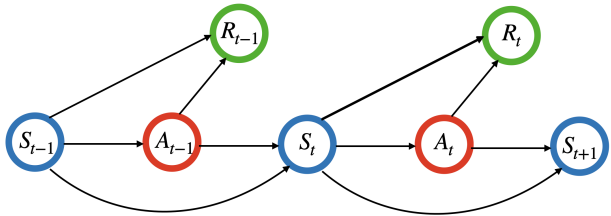
- Most existing tests **cannot** detect carryover effects
- **Example 1**. $S_t \sim N(0, 0.25)$, $R_t = S_t + \delta A_t$
- **Example 2**. $S_t = 0.5 S_{t-1} + \delta A_{t-1} + N(0, 0.25)$, $R_t = S_t$
- $\mathcal{H}_0$: The old policy ($A = 0$) has larger cumulative rewards ($\delta \leq 0$)
- $\mathcal{H}_1$: The new policy ($A = 1$) has larger cumulative rewards ($\delta > 0$)

Table: Powers of t-test, DML-based test (Chernozhukov et al., 2018) and the proposed test with $T = 500, \delta = 0.1$ ($\mathcal{H}_1$ holds in both examples)

Example 1	t-test 0.76	DML-based test **1.00**	our test **0.98**
Example 2	t-test 0.04	DML-based test 0.06	our test **0.73**

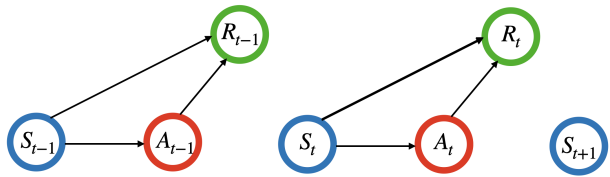
Contributions and Advances of Our Proposal

- Introduce an RL framework for A/B testing



1. A_{t-1} impacts R_t indirectly through its effect on S_t
2. S_t shall include important **mediators** between A_{t-1} and R_t

- Most existing works require the independence assumption



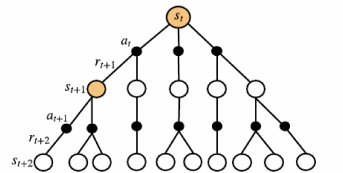
Contributions and Advances (Cont'd)

Propose a test procedure for comparing long-term rewards of two policies

1. allows for **sequential monitoring**
2. allows for **online updating**
3. applicable to a wide range of designs, including the **Markov** design, **alternating-time-interval** design and **adaptive** design

Methodology

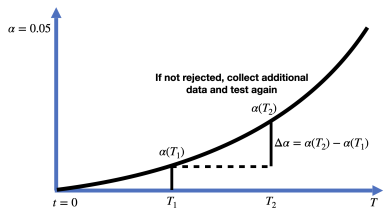
- Apply temporal difference (**TD**) **learning** with **sieve** method for value evaluation



$$TD: v(s_t) \leftarrow v(s_t) + \eta(r_{t+1} + \lambda v(s_{t+1}) - v(s_t))$$

- Provide **uncertainty quantification** (Shi et al., 2022, JRSSB)

- Adopt the α-**spending approach** (Lan & DeMets, 1983) for sequential monitoring



- Develop a **bootstrap-assisted procedure** for determining the stopping boundary[a]

[a] The numerical integration method designed for classical sequential tests is **not** applicable in adaptive design, due to the carryover effects

Theory

Theorem (Validity and Consistency)

Under the Markov, alternating-time-interval or adaptive design, the proposed test can **control type-I error** *and can* **detect** *local alternative hypotheses.*

Theorem (Undersmoothing and Efficiency)

Suppose **sieve** *method is used for function approximation in temporal difference learning.*

1. **Undersmoothing** *is not needed to guarantee that the policy value estimator has a tractable limiting distribution.*
2. *The final policy value estimator is* **semiparametrically efficient**.

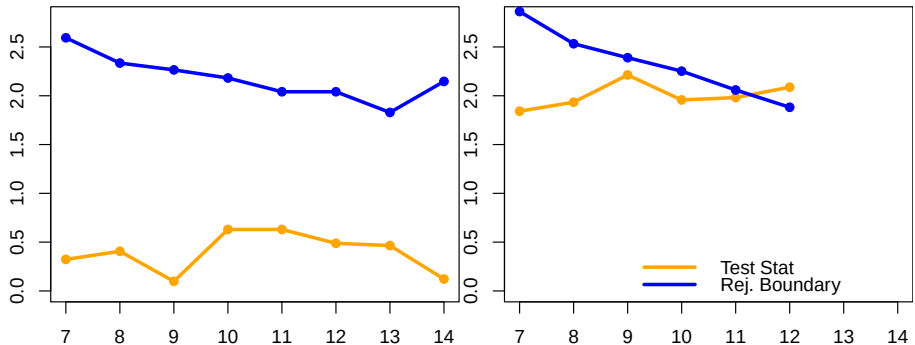
- The bias of the policy value estimator decays at a faster rate than the pointwise bias of the sieve estimator (Shen 1997; Newey et al, 1998)
- The proposed test will **not** be overly sensitive to the number of basis functions
- **Cross-validation** can be employed to select the basis functions

Application to Ridesharing Platform

- **Data**: a given city from December 3rd to 16th (two weeks)
- **30 minutes** as one time unit, sample size = **672**
- **State**:
  1. number of drivers (supply)
  2. number of requests (demand)
  3. supply and demand equilibrium metric (mediator)
- **Action**: new policy $\boldsymbol{A} = \mathbf{1}$ v.s. old $\boldsymbol{A} = \mathbf{0}$
- **Reward**: drivers' income
- The new policy is expected to have **better** performance

Application to Ridesharing Platform (Cont'd)

- The proposed test



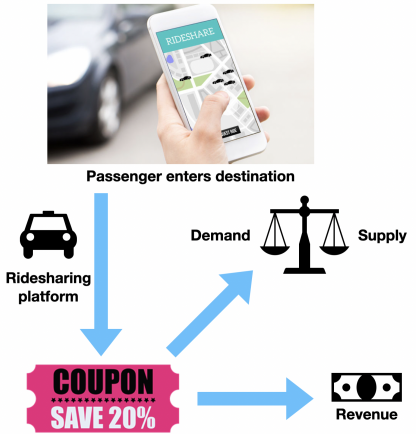
(a) AA Experiment: Day

(b) AB Experiment: Day

- t-test: **fail** to reject $\mathcal{H}_0$ in A/B experiment with p-value 0.18

Some Follow-ups

- A **multi-agent RL** framework for A/B testing (*AOAS, 2023*)



- **Optimal experimental designs** for A/B testing (*NeurIPS, 2023*)

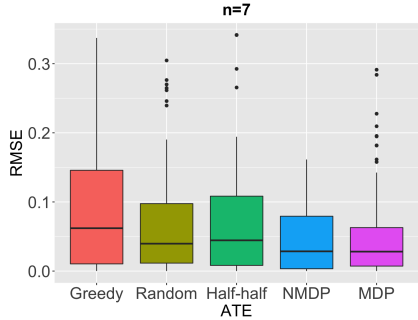


Figure: Boxplots of the RMSEs of various ATE estimators under different designs in a **real-data-based simulator**

Project III

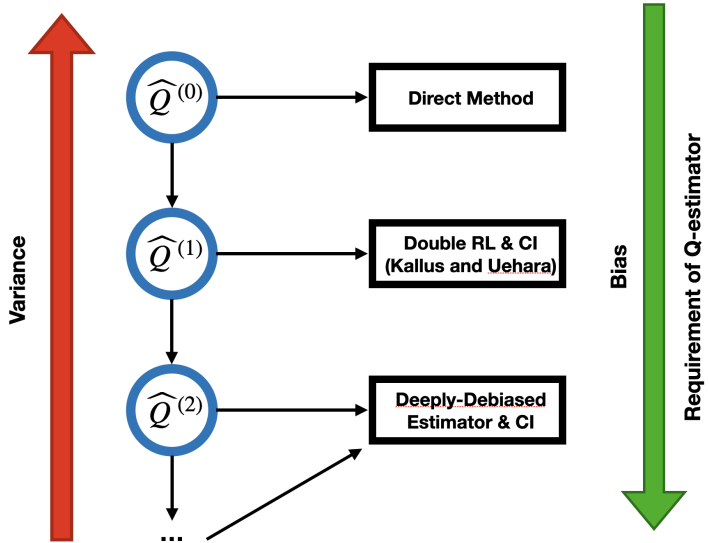
Deeply-Debiased Off-Policy Interval Estimation

joint work with Runzhe Wan, Victor Chernozhukov, and Rui Song
——ICML, 2021 (long talk, top 3% of submissions)

Off-Policy Evaluation

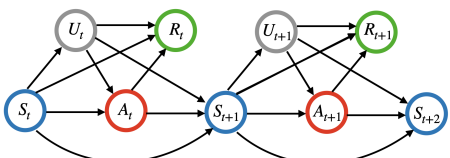
- **Objective**: Evaluate the impact of a target policy **offline** using historical data generated from a different behavior policy and provide rigorous **uncertainty quantification** (healthcare, automated driving, ridesharing, robotics, e.g.)
- Consider the reinforcement learning (e.g., MDP) setting
- Many existing methods focus on providing point estimators
- **Main idea**: Develop a **deeply-debiasing** process using higher order influence function (Robins et al., 2017)

Method


Variance
$\widehat{Q}^{(0)}$
$\widehat{Q}^{(1)}$
$\widehat{Q}^{(2)}$
...
Direct Method
Double RL & CI
(Kallus and Uehara)
Deeply-Debiased
Estimator & CI
Bias
Requirement of Q-estimator

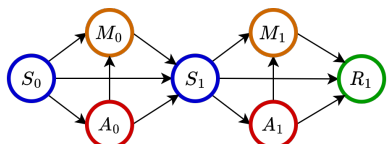
Some Follow-ups

- **Causal RL**: OPE in **confounded MDPs/POMDPs** (*JASA, 2022+; ICML 2022, 2023*)



- OPE in **doubly inhomogeneous environments** (Bian et al., 2023)

- An RL framework for **dynamic mediation analysis** (*ICML, 2023*)



  - IDE: $A_1 \to R_1$
  - IME: $A_1 \to M_1 \to R_1$
  - DDE: $A_0 \to S_1 \to R_1$
  - DME: $A_0 \to M_0 \to S_1 \to R_1$

Thank You!

☺Papers and softwares can be found on my personal website

`callmespring.github.io`