Optimal Designs for A/B Testing in Two-Sided Marketplaces

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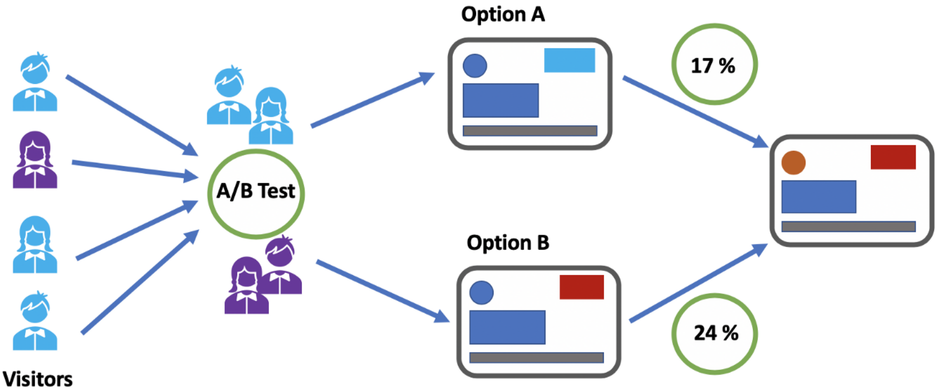
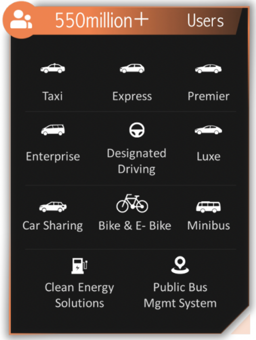
A/B Testing



Taken from
`https://towardsdatascience.com/how-to-conduct-a-b-testing-3076074a8458`

Ridesharing



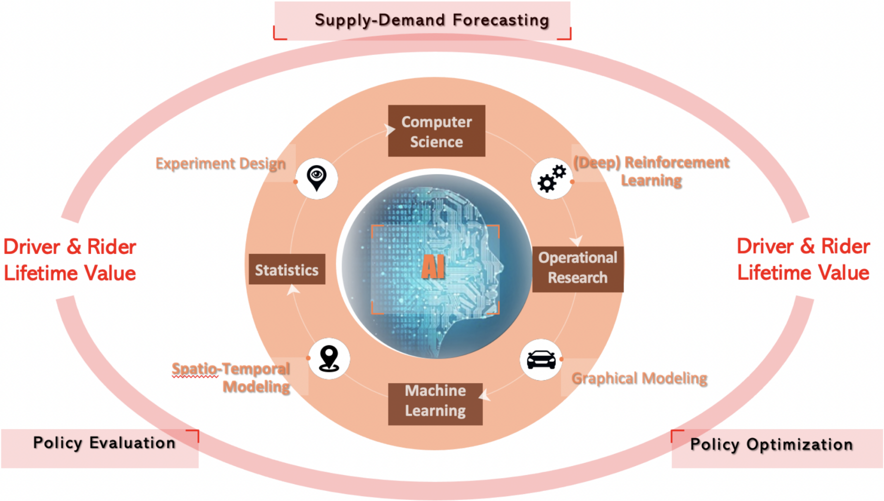
106TB+ vehicle trajectory data/day

4875TB+ data processed/day

40billion+ routing requests/day

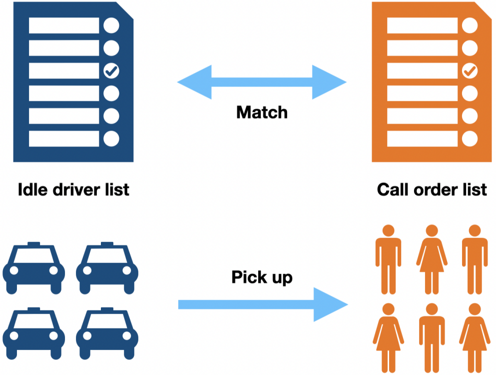
15billion+ location points/day

Ridesharing (Cont'd)



Policies of Interest

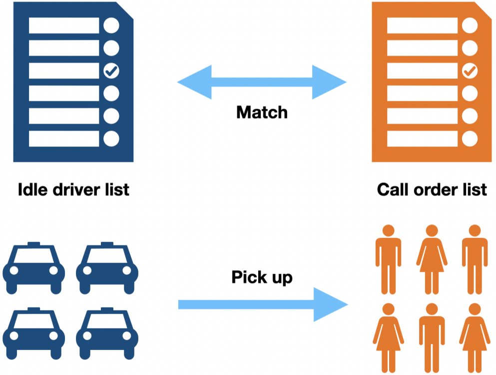
- Order dispatching



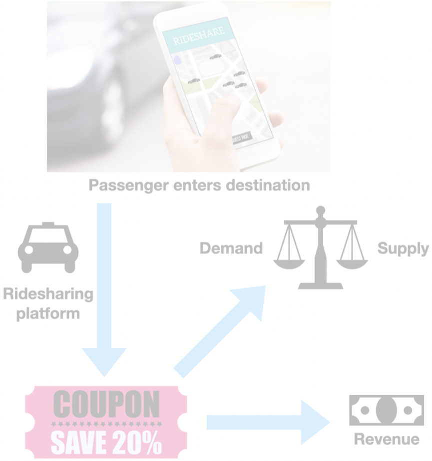
- Subsidizing

Policies of Interest

- **Order dispatching**



- Subsidizing



Time Series Data

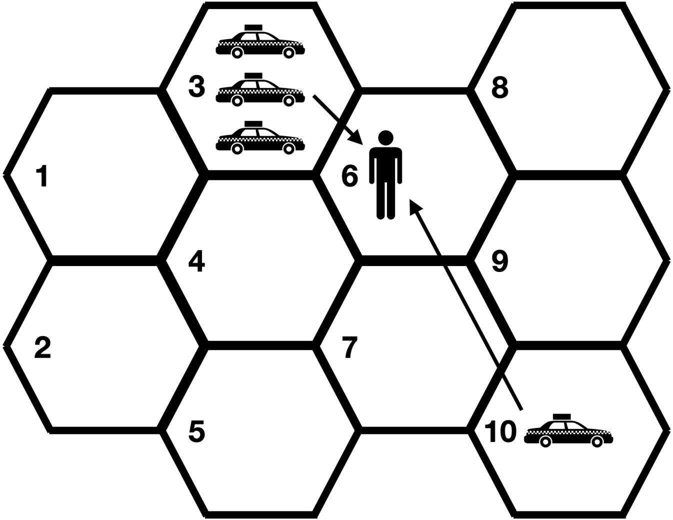
- Online experiment typically lasts for **two weeks**
- **30 minutes/1 hour** as one time unit
- Data forms a **time series** $\{(Y_t, U_t) : 1 \le t \le T\}$
- **Observations** $Y_t \in \mathbb{R}^3$:
  1. **Outcome**: drivers' income or no. of completed orders
  2. **Supply**: no. of idle drivers
  3. **Demand**: no. of call orders
- **Treatment** $U_t \in \{1, -1\}$:
  - **New** order dispatching policy B
  - **Old** order dispatching policy A

Challenges

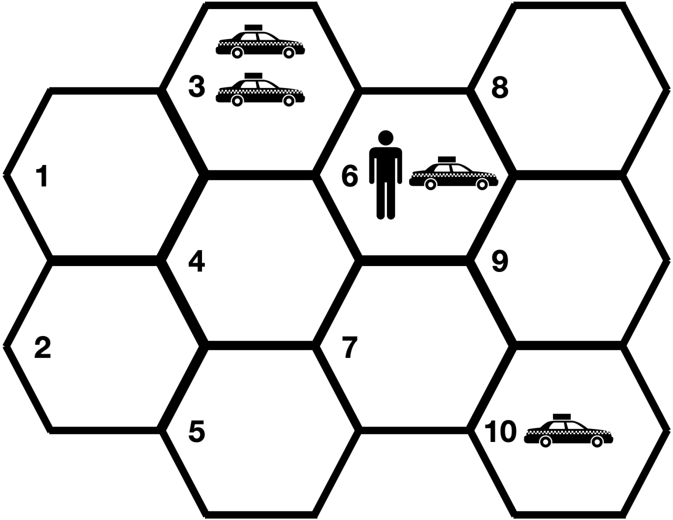
1. **Carryover Effects**:
   - Past treatments influence future observations [Li et al., 2024a, Figure 2] $\longrightarrow$
   - Invalidating many conventional A/B testing/causal inference methods [Shi et al., 2023].
2. **Partial Observability**:
   - The environmental state is not fully observable $\longrightarrow$
   - Leading to the violation of the Markov assumption.
3. **Small Sample Size**:
   - Online experiments typically last only two weeks [Xu et al., 2018] $\longrightarrow$
   - Increasing the variability of the average treatment effect (ATE) estimator.
4. **Weak Signal**:
   - Size of treatment effects ranges from 0.5% to 2% [Tang et al., 2019] $\longrightarrow$
   - Making it challenging to distinguish between new and old policies.

To our knowledge, **no** existing method has simultaneously addressed all four challenges.

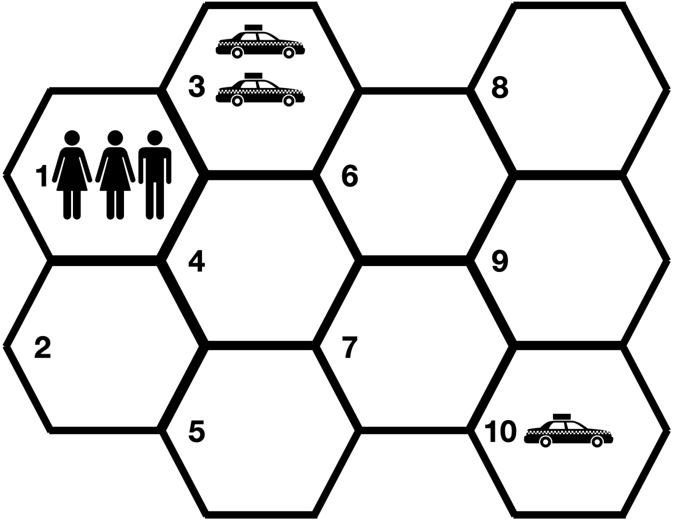
Challenge I: Carryover Effects



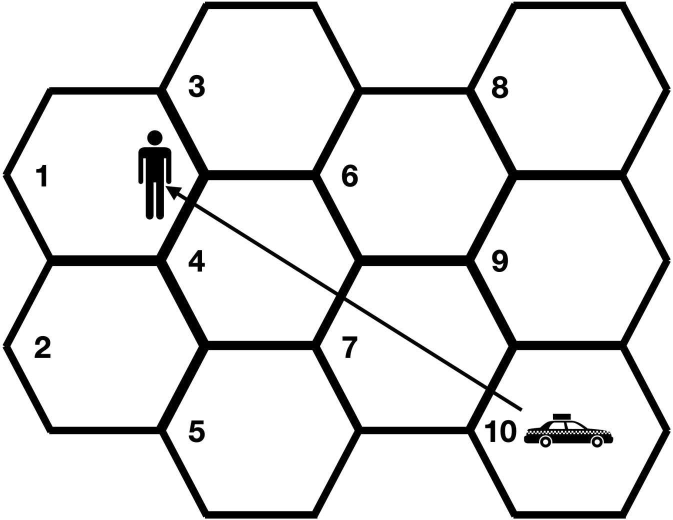
Adopting the Closest Driver Policy



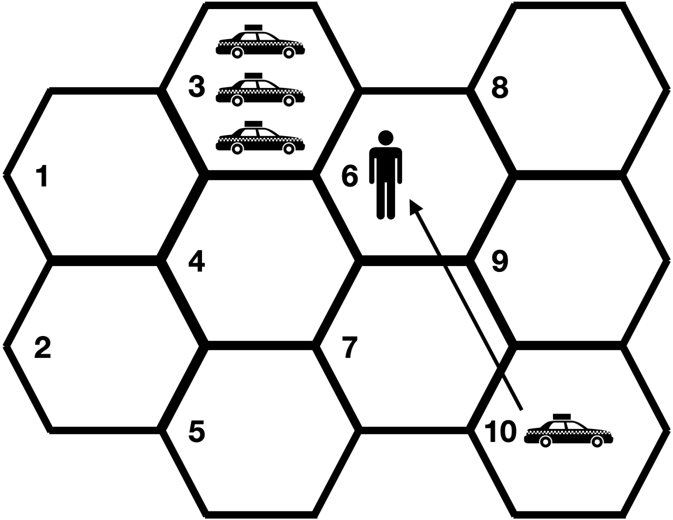
Some Time Later · · ·



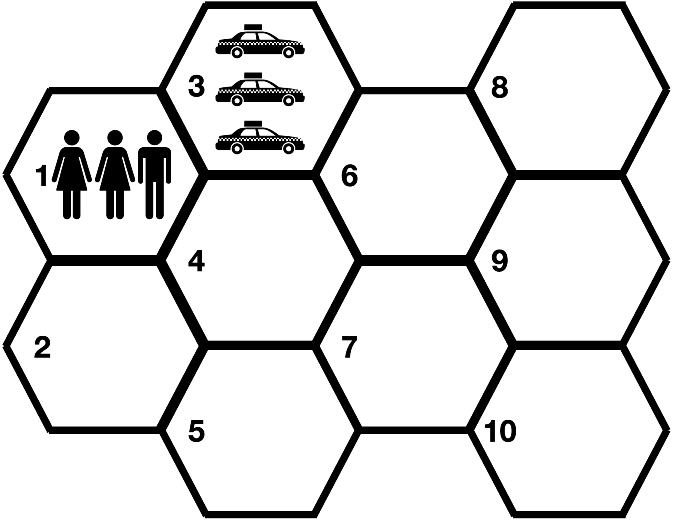
Miss One Order



Consider a Different Action



Able to Match All Orders

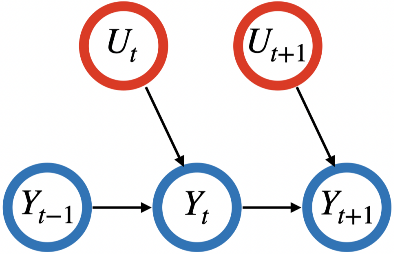


Challenge I: Carryover Effects (Cont'd)

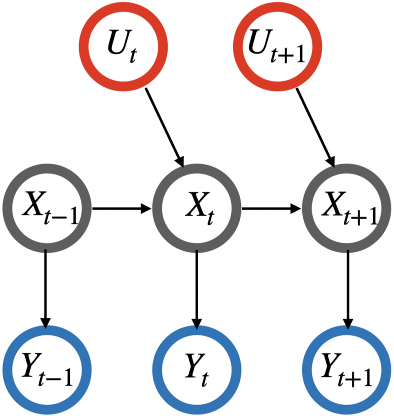
past treatments $\rightarrow$ distribution of drivers $\rightarrow$ future outcomes

Challenge II: Partial Observability

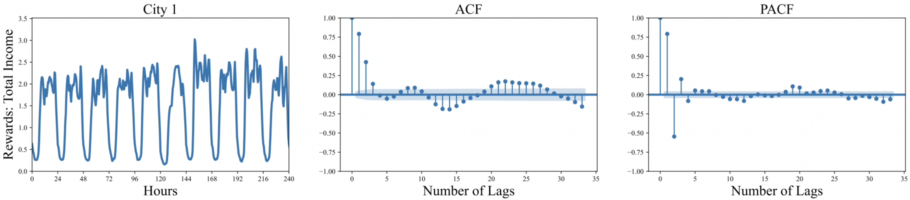
- **Fully Observable Markovian Environments**

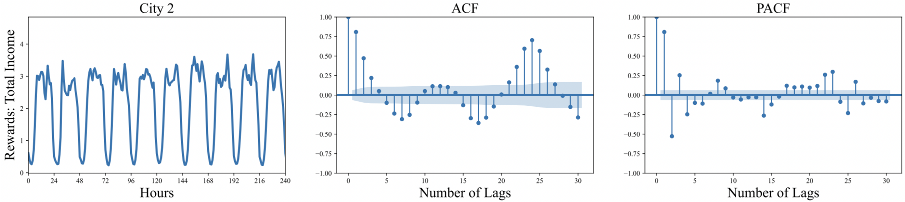


- **Partially Observable non-Markovian Environments**



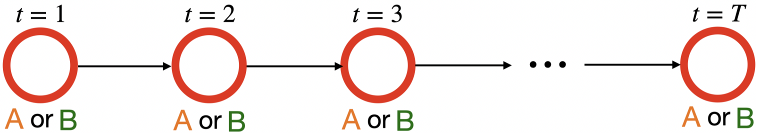
Challenge II: Partial Observability (Cont'd)



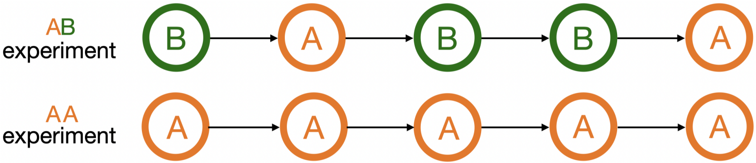


Challenge III & IV: Small sample & Weak Signal

- **Aim 1: Design**. Identify **optimal treatment allocation strategy** in online experiment that **minimizes MSE of ATE estimator**



- **Aim 2: Data Integration**. Combine **experimental data** (***A***/***B***) with **historical data** (***A***/***A***) to improve ATE estimation [Li et al., 2024b]



Optimal Treatment Allocation Strategies for A/B Testing in Partially Observable Environments

Joint work with Ke Sun, Linglong Kong & Hongtu Zhu

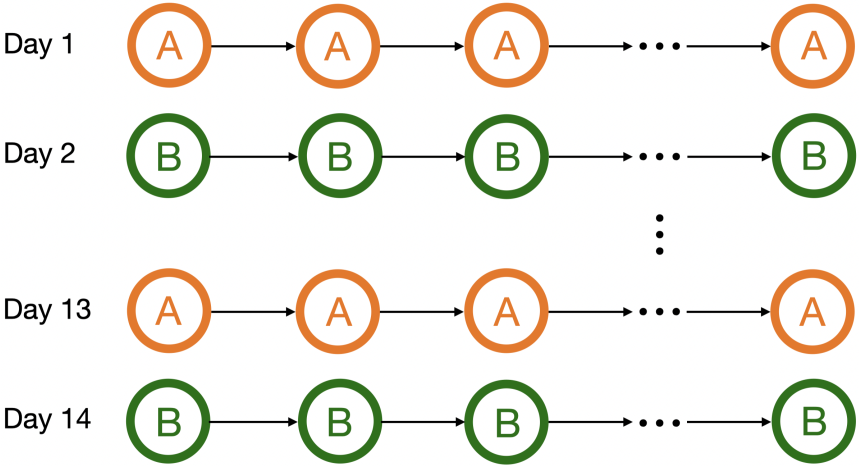
Average Treatment Effect

- Data summarized into a **time series** $\{(Y_t, U_t) : 1 \leq t \leq T\}$
- The first element of Y_t – denoted by R_t – represents the **outcome**
- **ATE** = **difference in average outcome** between the **new** and **old** policy

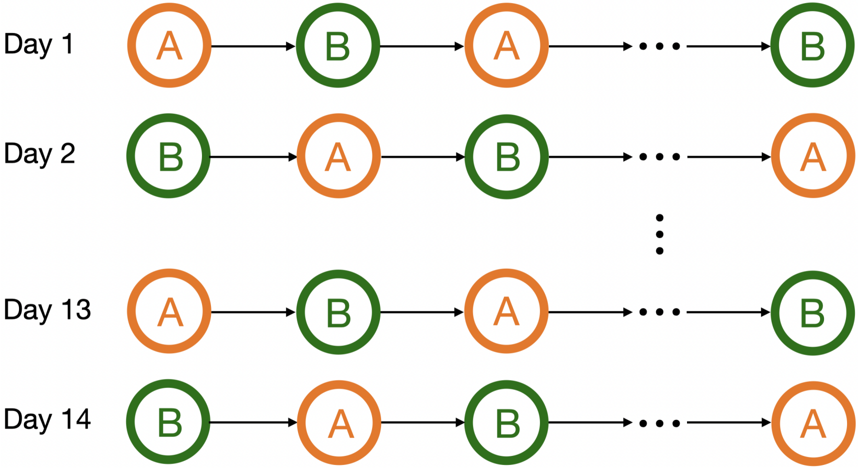
$$\lim_{T\to\infty}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}R_t\right] - \lim_{T\to\infty}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}R_t\right].$$

Letting $T \to \infty$ simplifies the analysis.

Alternating-day (AD) Design



Alternating-time (AT) Design



AD v.s. AT

Pros of **AD design**:

- Within each day, it is **on-policy** and avoids **distributional shift**, as opposed to **off-policy** designs (e.g., AT)
- On-policy designs are proven **optimal** in **fully observable Markovian** environments [Li et al., 2023].

Pros of **AT design**:

- Widely employed in ridesharing companies like Lyft and Didi [Chamandy, 2016, Luo et al., 2024]
- According to my industrial collaborator, AT yields **less variable ATE estimators** than AD

AD v.s. AT (Cont'd)

- **Q: Why can off-policy designs, such as AT, be more efficient than AD?**
- **A: Due to partial observability...**

A Thought Experiment

- A simple setting **without carryover effects**:

$$R_t = \beta_{-1}\mathbb{I}(U_t = -1) + \beta_1\mathbb{I}(U_t = 1) + \varepsilon_t$$

- ATE equals $\beta_1 - \beta_{-1}$ and can be estimated by

$$\widehat{\text{ATE}} = \frac{\sum_{t=1}^{T} R_t\mathbb{I}(U_t = 1)}{\sum_{t=1}^{T} \mathbb{I}(U_t = 1)} - \frac{\sum_{t=1}^{T} R_t\mathbb{I}(U_t = -1)}{\sum_{t=1}^{T} \mathbb{I}(U_t = -1)}$$

A Thought Experiment (Cont'd)

The ATE estimator's asymptotic MSE under AD and AT is proportional to

$$\lim_{t\to\infty} \frac{1}{t}\mathrm{Var}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \cdots + \varepsilon_t) \quad \text{and} \quad \lim_{t\to\infty} \frac{1}{t}\mathrm{Var}(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 + \cdots - \varepsilon_t)$$

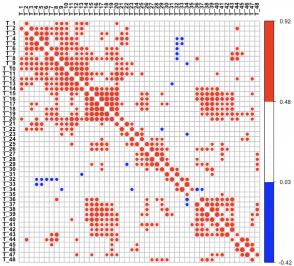
which depends on the residual correlation:

- With **uncorrelated residuals**, both designs yield **same** MSEs
- With **positively correlated residuals**:
  - **AD assigns the same treatment** within each day, under which ATE estimator's variance inflates due to **accumulation** of these residuals
  - **AT alternates treatments** for adjacent observations, effectively **negating** these residuals, leading to more efficient ATE estimation
- With **negatively correlated residuals**, AD generally outperforms AT

When Can AT Be More Efficient than AD

Key Condition: Residuals are positively correlated

- **Rule out full observablity** (Markovianity) where residuals are uncorrelated.
- Can only be met under **partial observability**.
- Suggest partial observability is more realistic, aligning with my collaborator's finding.
- **Often satisfied** in practice:



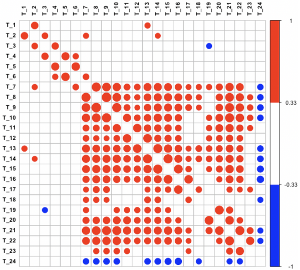


Figure: Estimated correlation coefficients between pairs of fitted outcome residuals from the two cities

Some Motivating Questions

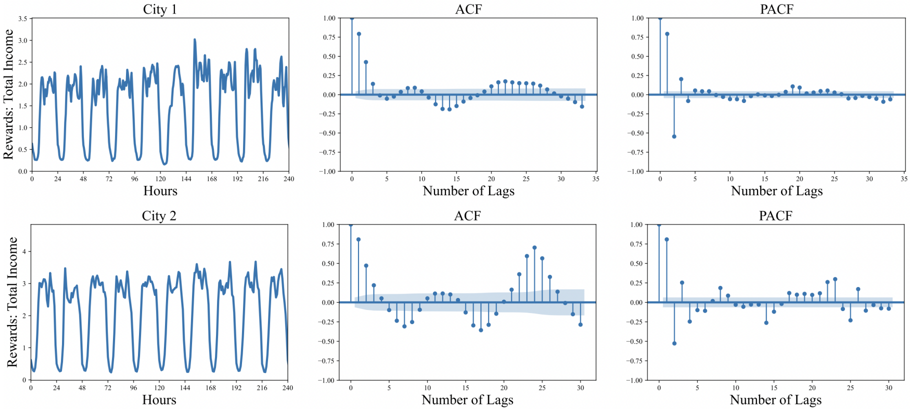
- **Q1: Previous analysis excludes carryover effects. Can we extend the results to accommodate carryover effects?**

- **Q2: Previous analysis focuses on AD and AT. Can we consider more general designs?**

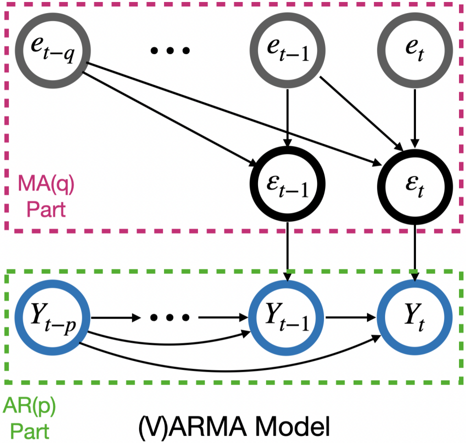
Our Contributions

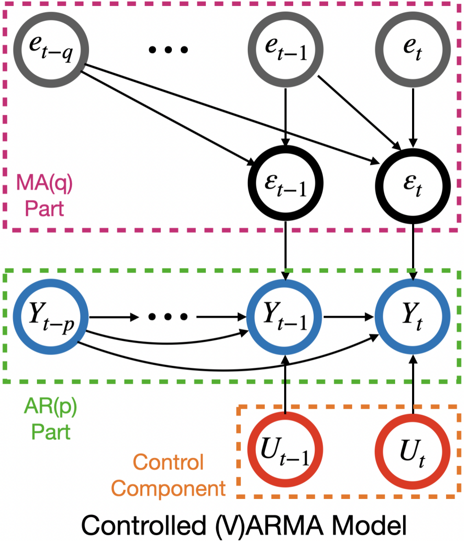
- **Methodologically**, we propose:
  1. A **controlled (V)ARMA** model $\rightarrow$ allow **carryover effects** & **partial observability**
  2. Two **efficiency indicators** $\rightarrow$ compare commonly used designs (AD, AT)
  3. A **reinforcement learning** (RL) algorithm $\rightarrow$ compute the **optimal design**
- **Theoretically**, we:
  1. Establish **asymptotic MSEs** of ATE estimators $\rightarrow$ compare different designs
  2. Introduce **weak signal condition** $\rightarrow$ simplify asymptotic analysis in sequential settings
  3. Prove the **optimal treatment allocation strategy** is $\boldsymbol{q}$-dependent $\rightarrow$ form the basis of our proposed RL algorithm
- **Empirically**, we demonstrate the advantages of our proposal using:
  1. A dispatch simulator (`https://github.com/callmespring/MDPOD`)
  2. Two real datasets from ridesharing companies.

Controlled VARMA Model: Motivation

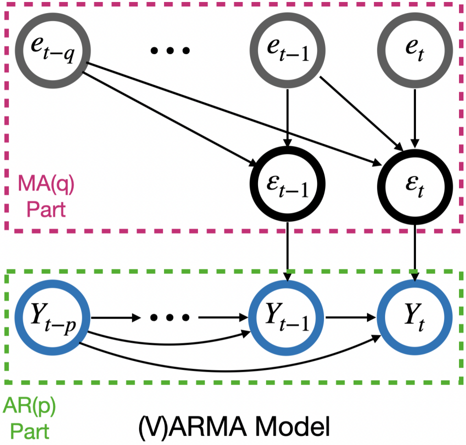


Controlled VARMA Model: Introduction

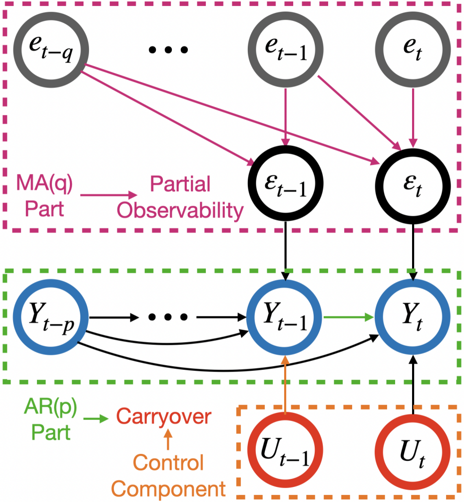




Controlled VARMA Model: Introduction



(V)ARMA Model



Controlled (V)ARMA Model

Controlled VARMA Model: Connection

- Closely related to **state space models** or **linear quadratic regulator** (LQR)
  - The latter being a rich sub-class of **partially observable MDPs**
  - Using VARMA as opposed to LQR allows to leverage asymptotic theories developed in time series to derive optimal designs
- Compared to **MDPs**
  - Both controlled VARMA and MDP accommodate **carryover effects**
  - MDPs require full observability whereas controlled VARMA allows **partial observability**

Controlled VARMA Model: Estimation

Consider a univariate controlled ARMA

$$Y_t = \mu + \underbrace{\sum_{j=1}^{p} a_j Y_{t-j}}_{\text{AR Part}} + \underbrace{bU_t}_{\text{Control}} + e_t + \underbrace{\sum_{j=1}^{q} \theta_j e_{t-j}}_{\text{MA Part}}$$

- **AR parameters** $\{a_j\}_j$ & **control parameter** b $\rightarrow$ **ATE**, equal to $2b/(1 - \sum_j a_j)$
    - Partial observability $\rightarrow$ standard OLS **fails** to consistently estimate b & $\{a_j\}_j$
    - Employ **Yule-Walker estimation** (method of moments) instead
    - Similar to **IV** estimation, utilize past observations as IVs
- **MA parameters** $\{\theta_j\}_j$ $\rightarrow$ **residual correlation** $\rightarrow$ **optimal design**

Theory: Weak Signal Condition

- **Asymptotic framework**: large sample $\boldsymbol{T} \to \infty$ & weak signal **ATE**$\to 0$
- **Empirical alignment**: size of ATE ranges from 0.5% to 2%
- **Theoretical simplification**: considerably simplifies the computation of ATE estimator's MSE in sequential settings. According to Taylor's expansion:

$$\widehat{\mathrm{ATE}} - \mathrm{ATE} = \frac{2\hat{b}}{1-\sum_j \hat{a}_j} - \frac{2b}{1-\sum_j a_j}$$

$$= \boxed{\frac{2(\hat{b} - b)}{1-\sum_j a_j}} + \boxed{\frac{2b}{(1-\sum_j a_j)^2}\sum_j (\hat{a}_j - a_j)} + \boxed{o_p\left(\frac{1}{\sqrt{T}}\right)}$$

Leading term. Easy to calculate its asymptotic variance under weak signal

Challenging to obtain the closed form of its asymptotic variance, but negligible under weak signal condition

High-order reminder

Theory: Asymptotic MSE

We focus on the class of **observation-agnostic** designs:

- U_1 is randomly assigned
- The distribution of U_t depends on $(U_1, \cdots, U_{t-1})$, independent of $(Y_1, \cdots, Y_{t-1})$

It covers three commonly used designs:

1. Uniform random (UR) design: $\{U_t\}_t$ are uniformly independently generated
2. AD: $U_1 = U_2 = \cdots = U_D = -U_{D+1} = \cdots = -U_{2D} = U_{2D+1} = \cdots$
3. AT: $U_1 = -U_2 = U_3 = -U_4 = \cdots = (-1)^{T-1} U_T$

Theorem (Asymptotic MSE)

Given an **observation-agnostic** *design, let $\xi = \lim_T \sum_{t=1}^T (\mathbb{E} U_t / T)$. Under the* **weak signal** *condition, its ATE estimator's asymptotic MSE (after normalization) equals*

$$\lim_T \frac{4}{(1-\sum_j a_j)^2 (1-\xi)^2 T} Var\Big[\sum_{t=1}^T (U_t - \xi)\varepsilon_t\Big].$$

Theory: Asymptotic MSE (Cont'd)

Corollary (Asymptotic MSE)

Under the **weak signal** *condition, the ATE estimator's asymptotic MSE (after normalization) under* **AD**, **UR** *and* **AT** *equals*

$$\begin{aligned}
\mathrm{MSE(AD)} &= \frac{4\sigma^2}{(1-\sum_j a_j)^2}\Big[\sum_{j=0}^{q}\theta_j^2 + \sum_{j_1\neq j_2}\theta_{j_1}\theta_{j_2}\Big] \\
\mathrm{MSE(UR)} &= \frac{4\sigma^2}{(1-\sum_j a_j)^2}\sum_{j=0}^{q}\theta_j^2 \\
\mathrm{MSE(AT)} &= \frac{4\sigma^2}{(1-\sum_j a_j)^2}\Big[\sum_{j=0}^{q}\theta_j^2 + \sum_{j_1\neq j_2}(-1)^{|j_2-j_1|}\theta_{j_1}\theta_{j_2}\Big],
\end{aligned}$$

where σ^2 denotes the variance of the white noise process.

Design: Efficiency Indicator

Define two efficiency indicators

$$\mathbf{EI}_1 = \sum_{j_1 \neq j_2} \theta_{j_1}\theta_{j_2} \qquad \text{and} \qquad \mathbf{EI}_2 = \sum_{j_1 \neq j_2} (-\mathbf{1})^{|j_2 - j_1|}\theta_{j_1}\theta_{j_2}.$$

They measure **residual correlations** and can be used to compare the three designs:

- If both $\mathbf{EI}_1$ and $\mathbf{EI}_2 > \mathbf{0}$, **UR** outperforms **AD** & **AT**
- If $\mathbf{EI}_2 < \mathbf{0}$ and $\mathbf{EI}_1 > \mathbf{EI}_2$, **AT** outperforms the rest
- If $\mathbf{EI}_1 < \mathbf{0}$ and $\mathbf{EI}_2 > \mathbf{EI}_1$, **AD** outperforms the rest

MA parameters can be estimated using historical data (even without treatment data).

Design: Optimality

Theorem (Optimal Design)

The optimal design must satisfy $\lim_T \sum_{t=1}^{T} (\mathbb{E} U_t / T) = 0$*. Additionally, it must minimize*

$$\sum_{k=1}^{q} \left[\lim_T \left(\frac{1}{T} \sum_{t=1}^{T} \mathbb{E} U_t U_{t+k} \right) \underbrace{\sum_{j=k}^{q} \theta_j \theta_{j-k}}_{c_k} \right]$$

Objective: learn the optimal observation-agnostic design that:

(i) **Minimizes** the above criterion

(ii) **Maintains** a zero mean asymptotically, i.e., $\lim_T \sum_{t=1}^{T} (\mathbb{E} U_t / T) = 0$

Design: An RL Approach

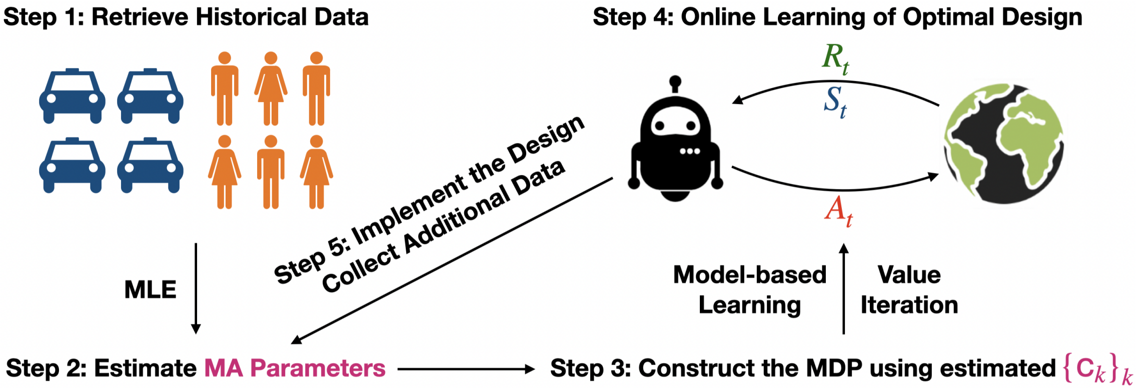
Solution: reformulate the minimization as an infinite-horizon average-reward RL problem

- **State S_t**: the collection of past q treatments $(U_{t-q}, U_{t-q+1}, \cdots, U_{t-1})$
- **Action A_t**: the current treatment $U_t \in \{-1, 1\}$
- **Reward R_t**: a deterministic function of state-action pair, $-\sum_{k=1}^{q} c_k (U_t U_{t-k})$

Easy to verify:

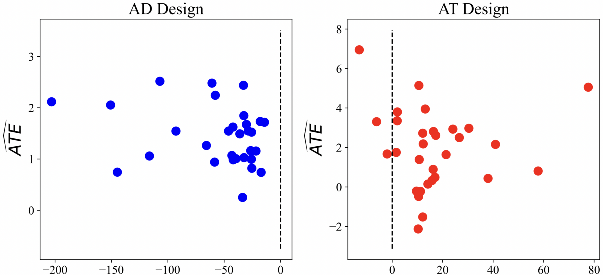
1. The minimization objective equals the negative average reward $\rightarrow$ equivalent to **maximizing the average reward**
2. The process is an **MDP** $\rightarrow$ there exists an optimal stationary policy maximizes the average reward $\rightarrow$ optimal design is q-**dependent**, i.e., U_t is a deterministic function of $(U_{t-q}, U_{t-q+1}, \cdots, U_{t-1})$ & this function is stationary in t
3. **Uniformly randomly** assign the first q treatments $\rightarrow$ the resulting design maintains a zero mean and is indeed optimal

Design: An RL Approach (Cont'd)



Empirical Study: Synthetic Environments

- A 9×9 dispatch simulator
- Available at `https://github.com/callmespring/MDPOD`
- Two efficiency indicators

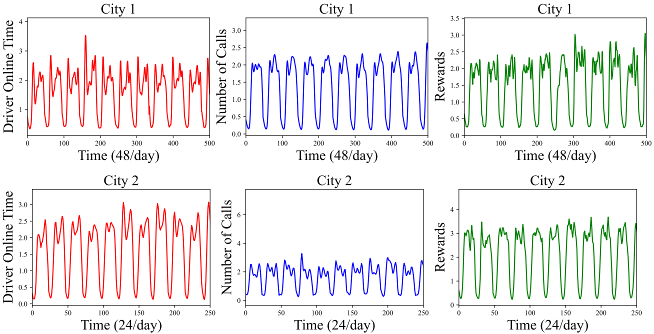


- ATE estimator's MSE under various designs

Design	AT	UR	Greedy	TMDP	NMDP	AD	Ours
MSE	8.33	2.23	1.10	0.56	0.42	**0.28**	**0.28**

Empirical Study: Real Datasets

- **Data**:



- We incorporate a **seasonal** term in our controlled VARMA model to account for seasonality. Below are MSEs of ATE estimators under different designs

City	EI_1	EI_2	AD	UR	AT	Ours
City 1	20.98	-21.11	11.98	11.63	9.72	**8.24**
City 2	-4.89	0.22	9.64	30.04	546.79	**8.38**

Thank You!

☺My RL short course

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