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Revisiting the accuracy of standard VaR methods for risk assessment: Using the Copula–EVT multidimensional approach for stock markets in the MENA region

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ABSTRACT

The aim of this study is twofold. First, it aims to show how to overcome some of the shortcomings of the standard risk measurement methods using value-at-risk (VaR) in the context of extreme events and propose an alternative empirical method. Second, it aims to fill the research gap in analysis of risk in the Middle East and North Africa (MENA) region. In this regard, for six daily stock indices in the MENA region, from January 3, 2005 to December 31, 2014, we employ a vine copula-based generalized autoregressive conditional heteroskedastic (GARCH) method and the extreme value theory (EVT) to model the dependence between the marginal distributions of returns and forecast the VaR. Based on backtesting, we assess the efficiency of the standard risk measurement models from the following families—the exponentially weighted moving average, the historical simulation, and the GARCH. By implementing the GARCH-EVT-C-vine method in the MENA region, we find that, empirically, standard methods overestimate (underestimate) the violation ratio, implying an underestimation (overestimation) of the risk, and therefore a misallocation of the capital covering the risk. The method also provides better VaR estimates for the MENA stock markets than that of the standard methods. We also verify that the greater the openness of the capital accounts and the flexibility of the exchange rate regimes, the greater will be the conditional dependence of the MENA countries on the developed markets. Finally, the empirical method we propose has an important implication for the MENA countries in that it can be adopted by these countries to forecast VaR effectively.

1. Introduction and overview

In the 1990s, the Middle East and North Africa (MENA) region initiated a wave of financial liberalization strategies focused on financial integration (FI). There are two types of arguments favouring the implementation FI strategies. The first type is based on the theoretical considerations. Specifically, through a better adjustment of the rates of returns, FI promotes a better allocation of resources, and thereby mitigates the efficiency losses induced by financial repression. FI also increases the capital flows to developing countries and enables investors to diversify their portfolios and achieve higher risk-adjusted returns (Stulz, 1999). The resulting increase in the mobility and flow of capital accelerates the convergence of the income of the less rich countries to its stationary state at a faster pace than that of a scenario characterized by less available and mobile capital (Barr, Gregory Mankiw, & Sala-i-Martin, 1995). Finally, in the face of random shocks, an access to international financial markets allows countries to borrow in order to smoothen consumption and seize the potential growth gains from international risk sharing (Obstfeld, 1994).

The second type of argument is empirical in nature. It is based on the positive effect of high private capital inflows since the early 1980s on several Southeast Asian countries. These effects prompted policymakers in the MENA region to implement FI and accelerate the inflow of foreign capital inflows after 1990 (Agénor, 2003); however, there has been a notable decline in the net inflows in the last decade.[1]

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[1] Over the period 2009–2018, the ratio of net inflows of FDI/GDP decreased on an annual average basis by 9, 0.3, 11, 8.8, 4.9, 12.5, 26.8, and 162.4 percentage points (pp) in Tunisia, Egypt, Lebanon, Algeria, Morocco, Jordan, Saudi Arabia, and Kuwait,

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In regard to the successful implementation of FI in the MENA region, it must be noted that institutional prerequisites, such as control and transparency (Collins & Abrahamson, 2006; Harvey, 1995), play a key role in reducing the cost of capital and increasing investment. Compared to more financially integrated emerging markets (East Asian and South American countries), these conditions are less noticeable in the MENA region (Guyot, Lagoarde-Segot, & Neaime, 2014). Concerning the level of FI, different degrees of FI implementation exert a different influence on these markets. Gradual FI exposes the MENA countries to macro-financial vulnerabilities during financial crises (Lagoarde-Segot and Lucey, 2006). These volatilities are independent of whether the MENA countries are net oil exporters or importers and whether they operate in open financial markets (e.g. Lebanon and Jordan) or under partial financial pressure (e.g. Tunisia and Algeria). This is notable in the case of the Gulf Cooperation Council (GCC) economies, which are characterized by a low degree of diversification. In these economies, a fall in the oil prices (as in 2015 and 2020) widens fiscal deficits, decreases growth and employment, and raises the risks of deflation (as in 2016). Similarly, the external indebtedness of non-oil-exporting MENA countries have made them vulnerable to the negative effects of external shocks, which include high risk premiums, interest rates, and the exchange rate risk. These effects have hindered the access of these countries to international capital markets. In the last decade, the political and social unrest in these countries (e.g. Egypt, Tunisia, Lebanon, and Algeria) has reduced their foreign investment appeal and increased their cost of capital (Belkhir, Boubakri, & Grira, 2017). In the context of progressive global FI, the real estate bubble formation can increase their vulnerabilities and lead to systemic risks (Neaime, 2012). These specific vulnerabilities should be considered when conducting a risk assessment of the MENA countries. After the 2007 global financial crisis, the early warning systems were adopted by some MENA countries and banks (main financial intermediaries) in the region, others failed to implement these systems (Calice, 2014). Hence, the aforementioned assessment will contribute towards identifying the vulnerabilities in the latter group of countries and banks. This assessment is also crucial because the emerging MENA financial markets are smaller, less liquid, and more volatile than the global financial markets (Domowitz, Glen, & Madhavan, 1998). It is crucial for most of the MENA countries to reform and modernize their financial systems in order to achieve resilience and better manage risks (Pietro, 2014).[2]

Concerning the FI and the cost of capital at a firm level, although empirical arguments show that financial integration contributes towards a decline in the cost of capital in the emerging market, these arguments are inadequate as they are based on studies conducted outside of the crisis events (Guyot et al., 2014). For example, based on a firm-level analysis of the impact of capital market liberalization in 18 emerging markets, Patro and Wald (2005) revealed that the firm-level cost of capital declined after liberalization. These models must assume that the correlation coefficient of the returns on domestic assets with returns on global assets must be less than the ratio of the domestic/global market volatilities (Stulz, 1999). If this condition does not hold, then an increase in the volatility of domestic assets could offset the diversification effect of liberalization and lead to a higher risk premium and a high cost of capital. Given this assumption, it must be noted that the MENA region is continually threatened by additional risks, such as structural macro-financial vulnerabilities (as shown by Gaysset et al., 2019)), and corporate governance issues, as per a report by the Organisation for Economic Co-operation and Development (OECD, 2012). Hence, this study aims to conduct an appropriate risk assessment for this region.

Overall, the MENA region is of interest for the following reasons. First, in this region, gradual FI has been undertaken to achieve efficiency and sustainable economic growth, though these goals are expected to materialize over a period. Second, the region's economies have intrinsic economic and institutional characteristics, which make them vulnerable to international financial shocks. Third, these sources of vulnerability can increase the volatility of the domestic assets more than that of the global assets, and therefore offset the expected diversification effects allowed by the FI. Fourth, MENA oil-exporting countries have a higher financing potential than those of the emerging markets, such as the East Asia. Specifically, in institutional and information terms, MENA financial markets are more volatile, less liquid, smaller, and less developed than the global financial markets. Fifth, MENA countries must undertake structural reform of their financial systems in order to build a capacity to manage financial crises. Sixth, studies have used conventional value-at-risk (VaR) measurement methods for the MENA region; however, these methods suffer from shortcomings, which affect the accuracy of these measurements. Hence, we focus on the financial vulnerability in the MENA region and develop a mechanism to better measure and manage these vulnerabilities. We also examine their dependence on the developed markets, given that some empirical works, like Ghorbel and Attafi (2014), show the intensity of dependence between the financial markets in the region increases during high volatility periods.

Specifically, we aim to provide a more appropriate framework than the studies using the conventional methods to model the extreme financial events in the MENA region. The financial crisis of 2007 provides an appropriate context for the aforementioned examination. This crisis triggered widespread empirical research on various aspects. It remains relevant to analyse the effects of this extreme event on the financial market conditions of the MENA region. In addition, modelling and forecasting the volatility in the context of dependence between markets is a key to managing financial risk and pricing instruments. From this perspective, and given the context described above, there are at least two additional reasons to question the effects of the financial crisis on MENA markets when modelling financial risk and market interdependence.

First, most empirical work has not paid sufficient attention to the assessment of risk measurement in MENA markets, which are witnessing heightened FI. Despite their vulnerability to extreme events, several MENA countries have been slow to adopt the necessary reforms and appropriate assessment methods (Neaime, 2016). Second, owing to the shortcomings of extreme event studies using VaR as a common risk measure, it is essential to review the methods and results of these studies.[3]

We highlight the limitations of the classical approaches—parametric, non-parametric, or semi-parametric—in using VaR for measuring the risk in the MENA market, in the context of extreme events. In this regard, it must be noted that the omission of dependence could affect the accuracy of the VaR estimate. The restrictive assumptions of normality and the independent and identical distribution (*i.i.d.*) as well as the classical use of the bivariate processes would increase the violation rate and underestimate the realized return and the risk incurred. Therefore, it is necessary to consider an alternative approach to the standard methods, based on the extreme value theory (EVT) and copulas.

respectively. However, this ratio has increased in United Arab Emirates by 24 pp. (Source: WB Development Indicators database, 2020).

[2] To address the financial vulnerability in the MENA region, in 2012, the World Bank designed a risk indicator for the financial institutions in the region; however, it has not been officially adopted by these countries.

[3] See our literature survey in the next section.

The EVT provides a flexible framework for analysing extreme events, by relaxing the restrictive assumption of the normality of return distributions and allowing a more accurate modelling of fat tails and high volatilities of the structural conditions in the MENA markets. Concerning copulas, they help in modelling the dependence between the markets, independent of the nature of the correlation between the tails (whether linear or not).

The aim of this study is twofold. First, this study aims to overcome some of the shortcomings of the standard risk measurement methods for the MENA region, in the context of extreme events. To this end, the study uses the EVT and copulas techniques that allow the modelling of marginal distributions by constructing a joint multivariate distribution. In this manner, the study empirically elaborates a flexible multidimensional dependence structure for the market risk measurement. Following Jorion (2000), we consider VaR as the maximum loss due to market fluctuations, that is, equity prices, for a given level of confidence and a fixed time horizon.[4]

Second, this study aims to evaluate the impact of market dependence on risk measurement and analyse the impact of the global financial crisis on MENA markets. We compare the considered copulas and select the most appropriate one based on statistical tests. Using daily data for sample of developing MENA and developed countries, we conduct our empirical investigations from January 3, 2005 to December 31, 2014. Our sample comprises six countries—two countries with developed financial markets (e.g. France and the United States of America) and four countries with emerging markets (e.g. Tunisia, Morocco, the Kingdom of Saudi Arabia (KSA), and the United Arab Emirates (UAE)). The *Cotation Assistée en Continu* (CAC) 40 and the Standard and Poor's (S&P) 500 indices were introduced into the sample to take into account the contagion effects of the global financial crisis when interpreting dependency structures. The sampled developed markets represent the sources of global volatility during the crisis. The Tunisian and Moroccan stock markets are the net oil importing countries of the MENA region; although they occupy small sizes in the sample, they are important in terms of the financial repression. The KSA and the UAE are net oil exporters and have the largest stock markets in the region. Given these financial markets, we obtain the dependency structure between any couple of countries, irrespective of its location (within or outside the MENA region), by using the pair copula construction (PCC).

In order to conduct the investigation, we adopt the following three steps. First, we conduct a synthesis of the literature to deduce the main shortcomings of the standard approaches. Second, we present the copula approach with the EVT in order to construct a flexible multidimensional dependence structure for the risk measurement. Third, we conduct empirical investigations on the MENA region by comparing the US and the French financial markets.

Specifically, we introduce multidimensional modelling of dependency structures using the C-vine copula as a possible alternative to the standard approaches. To the best of our knowledge, this is the first study to apply this new model to the context of the MENA stock markets. In doing so, we empirically prove that the standard methods overestimate (underestimate) the violation ratio, implying an underestimation (over-estimation) of the risk, and therefore a misallocation of the capital covering it. We also demonstrate the dominance of the vine-copula over the C-vine approach. The results of the backtesting show that the VaR measures, according to the cases of independence (the exponentially weighted moving average (EWMA), the historical simulation (HS), and the generalized autoregressive conditional heteroskedastic (GARCH)), do not meet the assumptions of the unconditional coverage and independence, and hence are invalid, whereas the measure of the GARCH-EVT-C-vine method (case of dependence) is empirically valid. The article also verifies the robustness of this measure by applying various model specifications and comparing their fit. Finally, in MENA countries, we verify that the greater the openness of the capital accounts and the flexibility of the exchange rate regimes, the greater will be the conditional dependence of the MENA countries on the developed markets. These results have important implications for policymakers and practitioners in that they provide an improved measure of VaR, and thus facilitate better forecasts.

The remainder of this paper is structured as follows. Section 2 presents the literature review with a focus on the MENA stock markets. Section 3 presents the methodology and its technical modelling aspects. Section 4 describes the data and the results of the VaR forecast as well as interpretations under the alternative hypothesis of the dependent and independent series. Section 5 presents the concluding remarks and policy recommendations.

2. Literature review

Since the 1990s, the empirical literature on VaR has evolved by offering a wide range of methods for its estimation in order to overcome the limitations of its standard version, which have received several criticisms. Artzner, Delbaen, Eber, and Heath (1997) have shown that VaR estimates do not take into account the extent of extreme losses outside their quantile.[5] For Barone-Adesi, Bourgoin, and Giannopoulos (1998) show that the VaR is too conservative, especially during unusual market events, while Artzner et al. (1997) , Artzner, Delbaen, Eber, & Heath (1999) argued that VaR is not a coherent measure of risk as it does not meet the under-addition requirement for all return distributions.[6] Following Mandelbrot (1963, 1971) and Black (1976), Barone-Adesi et al. (1998) concluded that the large returns cluster over time, resulting in the persistence of the amplitudes of price changes. Hence, the fluctuations in daily volatility make the confidence levels unreliable when calculating VaR (Boudoukh, Richardson, & Whitelaw, 1995). Although these criticisms have used various estimation methods to enrich the empirical literature on the treatment of VaR, we will show that the main work on the MENA region is marred by shortcomings especially in relation to the financial crises.

In this vein, it must be noted that parametric, non-parametric, and semi- parametric methods are most widely used for estimating VaR. The parametric method is an analytical treatment of VaR, and it is based on the GARCH, risk metrics (RM), and vector autoregression (VAR). Proposed by Engle (1982) and developed by Bollerslev (1986), the popular family of ARCH/GARCH models estimates the latent volatility and provides a measure of risk. Riskmetrics (1996), based on the moments of returns' distributions, also proposes an estimate of VaR. Like the delta–gamma approximations described by Britten- Jones and Schaefer (1999), Rouvinez (1997), and Wilson (1999), Riskmetrics (1996) has popularized the 'variance–covariance' solution. Both methods consider that the value of the portfolio changes quadratically or linearly with changes in the market risk factors (Glasserman, Heidelberger, & Shahabuddin, 2001).

[4] For more details on the VaR developments, see Duffie and Pan (1997) and Dowd (1998).

[5] See also, Embrechts, Kluppelberg, and Mikosch (1997), Longin (1997), and Embrechts, Resnick, and Samorodnitsky (1998).

[6] A 'coherent risk measure' is a function satisfying the properties of monotony, translational invariance, sub-additivity, and homogeneity. In this regard, Kostas and Tunaru (2005) have proposed a 'coherent risk measure', under the filtered HS, as an alternative to VaR.

The studies on emerging countries have used the parametric models for assessing the VaR; however, its use has been relatively less for the MENA countries. In a sample of three developed and seven emerging markets in the MENA region, Neaime (2012) identified relationships between time- varying volatility and expected returns[7] ; drawing on a variety of ARCH/GARCH models family as well as VAR analysis, the study also identified trade as one of the main crisis transmission channels. In addition, he identified the common factors partially explaining the volatility of the markets in the region. Guot et al., (2014) used a panel vector autoregressive model to show that the cost of equity transmits financial shocks in the MENA region. The negative effect of the global crisis on the net actualized investment project value in the MENA region may partially explain the rise in the cost of equity during the crisis and the resulting decline in the overall investment in the region in the aftermath of the global crisis. In an analysis of the stock markets of some MENA countries, Assaf (2014) compared the power ARCH models, from which he deduced the filtered residuals to calculate the VaR and the asymmetric power ARCH by relaxing the hypothesis of normality.[8] Subsequently, he concluded that the measures of VaR based on the normal distribution fail to model the large positive and negative returns accurately, and hence the tail behaviour of these markets should be taken into account.

These types of parametric estimation may fully characterize the distributions of returns because of the normality assumption of the standardized residuals. However, we show that the aforementioned models underestimate the VaR, while the conditional value-at-risk (CVaR) models tend to overestimate the risk in most of the cases when the backtesting technique is applied (Dargiri, Shamsabadi, Thim, Rasiah, & Sayedy, 2013). The parametric approach of VAR models allows for the estimation of the convergent parameters. In order to estimate the dependence of the tails of returns' distributions based on an extension of the seminal works of Koenker and Bassett (1978) and Koenker (2005), White, Kim, and Manganelli (2015) analytically treat VaR through a simple version of the model using market the equity returns data. However, VAR models rely on the controversial normality assumptions of the financial returns distributions and the *i.i.d*. as shown by (Patton, 2012)

The non-parametric method uses the Monte Carlo simulation, HS, and the variance-covariance method. The Monte Carlo method (Hurd, 1985) describes the process of price formation according to a stochastic process generating data. By referring to Lo and Mackinlay (1988) and Fama and French (1988); Hakim and Neaime (2013) and Neaime (2015) verified the existence of a mean reversion in some MENA countries through Monte Carlo simulations.[9] They concluded that, in this region, the volatility of the equity returns increased with a decline in the reversion speed, which contributed towards an increase in the stock prices. In the case of fat-tailed return distributions, by using the Monte Carlo simulations and the high frequency foreign exchange and incorporating the estimate of the second order term of the tail expansion, Danielsson and De Vries (1997) improved the conventional tail index and the quantile estimator. The Monte Carlo method is flexible and yields manageable empirical distributions. However, concerning this study, the Monte Carlo method suffers from some shortcomings. First, it tends to underestimate the probability of extreme events such as a financial crisis (Longin, 2000). In fact, the Gaussian density function adopted in Monte Carlo simulations imposes particularly thin tails. Hence, it may underestimate the probabilities associated with extreme events such as the financial crisis. Assaf (2009) empirically shows this shortcoming for some countries of the MENA region. Second, the fat tails may not be accurately reflected in the simulation outcomes when the entered distributions are poorly fitted.

Concerning the HS method initiated by Boudoukh, Richardson, and Whitelaw (1998), Barone-Adesi et al. (1998), Barone-Adesi, Giannopoulos, and Vosper (1999), it does not make assumptions about the distribution of returns and is based on observed historical data. It simulates or constructs the cumulative distribution function (CDF) of asset returns over a period to predict VaR. For a sample of MENA countries (Egypt, Jordan, Morocco, and Turkey), Assaf (2009) [10] highlighted two stylized facts on the distribution of returns, subject to a large consensus in the empirical literature, as synthetized by Pagan (1996). Questioning the normality of the conditional distribution of returns, the first fact is about fat tails of the distribution; the second fact about the time- varying returns implies that there can be no observation corresponding to certain quantiles of the underlying distribution. In Barone-Adesi et al. (1998), the HS method suffers from the discretization of extreme returns and very few observations in tails. These issues make the VaR measurements volatile and erratic. In this regard, some extensions to the basic version of the HS estimate the current level of risk more accurately; these extensions have led to the adoption of the semi-parametric method (Danielsson and De Vries, 2000). Barone-Adesi et al. (1999) initiated a filtered HS (FHS) method; it models innovations using a mixture of the parametric modelling of the volatility of risk factors, the non-parametric bootstrap procedure, and the GARCH process. Therefore, this non-parametric model does not take into account the current volatility of the market (Alexander, 2008). Since the FHS overcomes most of the basic issues in the HS, the former is considered more accurate than the latter for estimating VaR. For instance, Hull and White (1998), for a large sample of derivative securities portfolios, considered the stochastic volatility and exchange rates. Using the FHS, they found that correlations are implicitly preserved; they also found that, at the long horizon, VaRs are too high for swap portfolios, while, at short horizon, they are too low for options and futures portfolios. However, FHS retains a restrictive assumption in that the returns' vector is *i.i.d*., which implies the constancy of its correlation matrix. On this basis, we conclude that the HS method should be improved to estimate VaR accurately in MENA countries, especially in a crisis context where dealing with tails of the return distribution is crucial.[11]

Faced with the inadequacies in the standard VaR methods, the empirical literature switched to the EVT, which only considers the distribution of extreme returns, instead of the distribution of all returns. In this line, Alexander and Rüdiger (2000) and Gençay, Selçuk, and Ulugülyağci (2003) adopted the EVT to overcome the problem of under/overestimation of the VaR. Indeed, extreme conditions should be taken into account at the level of risk measurement in order to obtain a more effective risk measure. In this regard, Alexander and Rüdiger (2000), Bali and Neftci (2003), and

[7] The sample is for the period from January 1, 2007 to December 31, 2010. The author considered three developed market Indices—S&P's 500 (the United States of America), the Financial Times Stock Exchange (FTSE) 100 (the United Kingdom), and CAC 40 (France). The author also considered the seven emerging market indices of the MENA region—the Egyptian Exchange (EGX) 30 (Egypt), Amman Stock Exchange (Jordan), MADEX (Morocco), Tunindex (Tunisia), Kuwait Stock Market Index (Kuwait), Tadawul All Share Index (the KSA), and the General Dubai Financial Market Index (DFMGI).

[8] In this empirical work, daily data cover the period from May 1, 2005 to April 26, 2012, for Morocco, Jordan, and Egypt.

[9] Hakim and Neaime (2003) verify the mean reversion for Jordan, Morocco, Egypt, and Turkey, while Neaime (2015) verifies it for Saudi Arabia, Jordan, and Bahrain.

[10] The work is done on daily time series of stock market indexes covering the period April 1, 1997 to April 26, 2002

[11] Bali (2003); McNeil and Frey (2000), and Longin (2000) show that traditional parametric VaR models with normal density do not accurately estimate losses during the financial crises.

Marimoutou, Raggad, and Trabelsi (2009) used the EVT to estimate the tail of distributions. The authors applied the conditional EVT (CEVT) to measure risk. First, they used the GARCH models to estimate the volatility of the returns; subsequently, they used the EVT to capture the tail of the standardized residual distribution of the GARCH models. Bali and Neftci (2003), Byström (2005), Fernandez (2005) , and Fong and Gray (2006) compared the accuracy of the GARCH-EVT model with that of the parametric family models. The authors argued that a more accurate estimate of VaR is provided by the GARCH-EVT than the parametric family models.

There are at least two main reasons for preferring EVT to parametric normal density volatility models. In the MENA countries, in most financial time series, the distribution of returns is not only fat but also skewed during high volatility periods. This makes normal distribution inappropriate. Furthermore, extreme returns and high volatility could be caused by structural mechanisms specific to MENA countries such as structural financial fragilities, inefficiencies in information, and market governance. These MENA-specific market conditions require the use of flexible econometric models to capture the extreme behaviour of financial variables, which the EVT allows. Hence, under these conditions, the modelling of the distribution of extremities should be separated from that of the rest of financial series' distribution (Neftci, 2000). By examining the unconditional distribution tails of the financial market returns in the MENA region, Assaf (2006) empirically shows that these series have significantly fat tails than those of the normal distribution, and therefore suggests the use of the EVT. In line with Al-Zoubi and Maghyereh (2006); Silva and Mendes (2003), Assaf (2009) shows that the distribution of market returns has significantly larger tails than the normal distribution and justifies the relevance of the extreme value approach in VaR estimation. This is consistent with the finding of several studies on developing countries, such as Gençay and Selçuk (2004) and Mendes (2000).

Concerning the dependence of the financial markets, research on the extremes in the multivariate case suffers from ordering the pairs of random variables when treating the extreme dependence. The copula modelling is a more general method that enables us to study the *structure of dependence* of a set of random variables.

The copula theory was introduced in the seminal study of Sklar (1959), and Embrechts, Resnick, and Samorodnitsky (1999) were among the first to use it in financial applications. Meanwhile, Joe (1997) and Nelsen (2006) provided the fundamental properties of this theory. The copula is a multivariate CDF whose marginal distributions are uniform on the interval [0, 1]. A unique copula represents the dependency structure of every multivariate distribution with continuous marginals (Nelsen, 2006).

There are two reasons copulas detect the dependency structure in MENA financial markets better than the other correlation-based methods (Forbes and Rigobon, 2000).[12] First, copulas allow variables to be modelled independent of the distributions they follow, even if they are not normal, as demonstrated by Cherubini and Luciano (2001). This avoids the problems of adopting normal distributions of market returns in the MENA stock markets when modelling their tails. Second, copulas make it possible to separate the dependency structure from the marginal distributions followed by the variables.

Since 2000s, several experiments have been conducted to examine the different applications of the copula in finance. Cherubini and Luciano (2001) applied copulas to two stock market indices; they obtained the marginal probability distributions from the estimated copulas and, subsequently, deduced the joint probabilities of extreme losses. Jondeau and Rockinger (2006) used a copula-GARCH model to study the dependency between four stock indices—the CAC 40, the Deutscher Aktien Index (DAX), the FTSE, and the S&P 500—and extract dependency. Wu, Chen, Ye, and Miao (2006) also applied the copula-GARCH model to analyse portfolio risk in the Chinese stock market, while Palaro and Hotta (2006) used a copula model to explain the dependence between the NASDAQ and the S&P 500 stock indices. Using an EVT-copula approach, Hsu, Huang, and Chiou (2012) estimated the VaR for the Asian financial markets for the period 2000–2007. The results of backtesting showed that the Clayton copula associated with the EVT provides a better measurement of risk than the copulas with conventionally used empirical distributions. The aforementioned and recent empirical studies (Cortese, 2019) conclude that copulas are relevant for tail distribution modelling as well as dependency.

However, most empirical studies on portfolio risk management using copula models have focused on capturing the bivariate interdependence structure (Razak and Ismail, 2016). Copulas, such as the Gaussian and Student-t copulas, present some restrictions as they do not allow for tail dependence, while the asymmetric Gumbel copula does not take into account the negative dependence. Therefore, to overcome this shortcoming, it is necessary to implement higher-dimensional copulas that consider different dependency structures between the pairs of variables. The PCC provides an alternative and more flexible way to build multivariate distributions.

Bedford and Cooke (2001), Bedford & Cooke, 2002 introduced the PCC of multivariate distribution based on the vine structure. Aas, Czado, Frigessi, and Bakken (2009) argued that it could be used with arbitrary pair-copulas, referred to as the illustration's graphical structure of D- and C-vines. Vine copulas allow for separately modelling multivariate distributions as well as marginal and dependencies; they also offer the flexibility of bivariate copulas. Thus, this type of modelling coupled with the EVT contributes towards the construction of a flexible multidimensional dependency structure.

3. Methodology

We discuss the modelling procedure to select the appropriate model for risk measurement and adopt the following three-step approach. First, we model the marginal distribution and tails using the EVT after extracting the filtered residuals from each series of returns. Second, we select the appropriate copula by testing the fit quality of each copula, based on the tests of Vuong (1989) and Clarke (2007). Finally, we perform risk analysis according to the main methods of the empirical literature; we also conduct backtesting for comparison purposes. We also consider Monte Carlo simulations to generate the series of dependent returns for estimating the VaR. This three-step method is discussed in the following subsections.[13]

3.1. *First step: modelling the marginal distributions and the tails using EVT*

The EVT appears to be an appropriate approach for modelling the behaviour of the fat tail because the real financial time series often have fat tails and volatility clustering that can be captured (Pfaff, 2013). Nevertheless, applying the EVT to the random return, $r_{i,t}$, of the i^{th} asset at time t does not yield accurate results when the return is not *i.i.d*. Modelling the tails of a distribution with a generalized Pareto distribution (GPD) requires observations to be

[12] This work is for the correaltion biaised methods in financial markets.

[13] The literature considers that the package of C and D Vine copulas allows for the diversity of bivariate copula families belonging to the two major classes of the elliptical and the Archimedean copulas (Joe, 1997; Nelsen, 2006). Hence, we focus on these two copula types.

approximately *i.i.d.* Therefore, we filter the stock returns with an $AR(1) - GARCH(1,\ 1)$model, and their innovations are fitted with GPD to capture the existence of fat tails. However, in the standard GARCH model, which is the main empirical framework for modelling the stock market volatility, a symmetrical effect of the positive and negative error terms on volatility is assumed; the asymmetric effects, such as the leverage effect, are not modelled.[14] Before the introduction of the non- linear extensions to the GARCH processes, this leverage effect was introduced by Black (1976) to deal with the asymmetrical effect on volatility.[15]

To filter the stock returns taking into account the possible asymmetrical effects, we consider the following $AR(s) - GJR - GARCH(p,\ q)$ model for daily returns:

$$\begin{cases} r_{i,t} = c_0 + \sum_{j=1}^{s} c_j r_{i,t-j} + e_{i,t} \\ h_{i,t} = \omega_{i,t} + \sum_{j=1}^{p} \left\{\alpha_j + \gamma_j I\left(e_{i,t-1} > 0\right)\right\} e_{i,t-j}^2 + \sum_{j=1}^{q} \beta_j h_{i,t-j} \\ e_{i,t} = h_{i,t}\varepsilon_{i,t}\,,\ \varepsilon_{i,t} \sim SKT\left(\nu, \lambda\right) \end{cases} \quad (1)$$

where $r_{i,t}$ are the returns of the ith asset at time t, for $i = 1, 2, 3, 4, 5, 6$, with $i = 1$, $i = 2$, $i = 3$, $i = 4$, $i = 5$, and $i = 6$ for the Moroccan index (MASI), the Tunisian index (TUNINDEX), the CAC 40, the S&P 500), the TASI, and the ADI, respectively.

The conditional variance of asset returns is $h_{i,t}$. The non-negative parameters are ω_j, α_j, and β_j , for $j = 1, .., s$, such that $\omega_j > 0$; $\alpha_j \geq 0$, $\beta_j \geq 0$, and $\alpha_j + \gamma_j \geq 0$, to ensure the positiveness of the conditional variance. The error term $\varepsilon_{i,t}$ is assumed to have a *skewed-t* distribution, which is used to describe the possibly asymmetric and heavy-tailed distribution. The parameters ν and λ are the degrees of freedom and asymmetry, respectively.

To model tails using EVT, let $\left\{X_i\right\}_{i=1}^{n}$ be a sequence of *i.i.d.* random variables with CDF F, and consider its behaviour such that

$$M_n \equiv max\left\{X_1, \cdots, X_n\right\}, \quad as\, n \to \infty \quad (2)$$

Generally, in order to specify the form of this distribution, we can take two approaches to EVT modelling. The first is based on the generalized extreme value distribution, known as the block-maxima-minima approach, developed by Fisher and Tippett (1928). However, modelling only the block's maximum data is inefficient if other data on extreme values are available. A more efficient approach towards modelling extreme events would be to focus not only on events at larger (maximum) magnitudes but also on all events that are larger than a pre-set high threshold. This represents the modelling of peaks on the threshold (POT) (Balkema & De Haan, 1974; Pickands, 1975). Thus, observations exceeding a certain threshold are referred to as extreme events. The distribution above the threshold, known as the 'conditional excess distribution function', is given by

$$F_u = P\left\{X - u \leq y\,/X > u\right\} = \frac{F\left(y+u\right) - F(u)}{1 - F(u)} \quad (3)$$

where F is a distribution function describing the time series X, y denotes the excesses of X over the threshold u, $y \equiv X - u$; $0 \leq y < X_F - u$,and $X_F \leq \infty$ is the right endpoint of F.

We are interested in estimating the extremes, namely F_u in Eq. (3). The POT method shows that the conditional excess distribution function , $F_{u,}$ is well-approximated by the GPD, which describes the limit distribution of scaled excesses over high thresholds:

$$F_u\left(y\right) \cong G_{\xi,\sigma}\, u \to \infty \quad (4)$$

This technique shows that, for a high threshold u, a GPD of extrema (maxima or minima) with a distribution function $\boldsymbol{G}_{\xi,\,u,\,\sigma}$ will be as follows:

$$\begin{cases} G_{\xi,\,u,\,\sigma}\left(y\right) = 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-1/\xi}; \text{ if } \xi \neq 0 \text{ for all } y \ \in R_+^* \\ G_{\xi,\,u,\sigma}\left(y\right) = 1 - exp\left(-\frac{y}{\sigma}\right); \text{ if } \xi = 0 \end{cases} \quad (5)$$

where the GPD is characterized by the parameters ξ and σ, referred to as the shape and scale parameters, respectively, which we estimate by the maximum likelihood method. The excess over the threshold, u, is expressed by $y_{i,t} = r_{i,t} - u$.

For $\xi = 0$, the distribution in Eq. (5) is closest to the Gumbel (1954) distribution, which is an extreme value distribution function. For $\xi > 0$, the distribution belongs to the heavy-tailed distribution. For < 0, the distribution is closest to the Weibull (1951) distribution (Omari, Mwita, & Gichuhi, 2018).

In this study, following Alexander and Rüdiger (2000), we adopt the mean-excess plot approach to choose the optimal threshold, where the mean-excess function, $e(u)$, for Xover threshold u is given by

$$e\left(u\right) = E\left(X - u/X > u\right) \quad (6)$$

The natural estimate of the *empirical* mean-excess function $e^{EMP}\left(\mu\right)$ is defined as the average of all observations exceeding the threshold u:

$$e^{EMP}\left(\mu\right) = \frac{\sum_{i=1}^{n}\left(X_i - \mu, 0\right)}{\sum_{i=1}^{n} I_{\left\{X_{i>\mu}\right\}}} \quad (7)$$

The empirical mean-excess function is linear in u,

$$e^{GPD_{\xi,\sigma}}\left(u\right) = \frac{\sigma}{1-\xi} + \frac{\xi}{1-\xi}u \quad (8)$$

where $0 < u \leq \infty$, if $0 \leq \xi < 1$, and $0 < u \leq -\frac{\sigma}{\xi}$, if $\xi < 0$

Eq. (8) is used to obtain u empirically

3.2. *Second stage: modelling the joint distributions through the Vine copula*

In line with the Sklar (1959) theorem, the univariate margins and the dependency structure can be separated, where the dependency structure is represented by the copula. Joe (1996), Bedford and Cooke (2001), Aas et al. (2009), and Czado (2010) introduced a pair copula in d dimensions, while the graphical structures or *regular copulas* were introduced by Bedford and Cooke (2001). Aas et al. (2009) implemented the two subclasses of the vine copula,[16] namely the C-vine (canonical vine) and D-vine (drawable vine) copulas, whose respective joint distribution expressions, $f\left(x_1, .., x_d\right)$,

[14] The hypothesis of the asymmetrical leverage effect is based on Black (1976) and later Christie (1982). Taking daily data, in the period 1964–1975, for a sample of 30 stocks, Black (1976) showed that each return reduction corresponds to higher volatility. Christie (1982) used the following equation to estimate and show the relationship between volatility, σ_t, and returns, r_{t-1}: $b\left(\frac{\sigma_t}{\sigma_{t-1}}\right) = \beta_0 + \theta_s r_{t-1} + u_t$, where b, β_0 and θ_s are parameters. His empirical result shows a positive relationship between the volatility and the leverage.

[15] The main extensions are EGARCH models (Nelson, 1991), threshold ARCH models (Zakoian, 1994; Koutmos, 1998), the Glosten–Jagannathan–Runkle generalized autoregressive (GJR) model (Glosten, Jagannathan, & Runkle, 1993), and the asymmetric power ARCH (APARCH) model (Ding, Granger, & Engle, 1993).

[16] See Appendix B for the general expression of the vine copulas with their decompositions.

are provided in Appendix A (Eq. (A.2)). A vine with $d$dimensions is represented by $(d-1)$ trees. The tree j has $(d+1-j)$ nodes and $(d-j)$ edges. The edges of tree j become the following nodes of the tree $(j+1)$, and the nodes represent random variables with inverse distribution functions. For our sample of six markets, these two copulas are presented as follows.

C-vine copula:

$$f(x_1,\ldots,x_6)=\begin{cases}=f_1f_2f_2f_3f_4f_5f_6\\ \times c_{12}(F_1,F_2)c_{13}(F_1,F_3)c_{14}(F_1,F_4)c_{15}(F_1,F_5)c_{16}(F_1,F_6)\\ \times c_{23/1}\left(F_{2/1},F_{3/1}\right)c_{24/1}(F_{2/1},F_{4/1})c_{25/1}(F_{2/1},F_{5/1})c_{26/1}(F_{2/1},F_{6/1})\\ \times c_{34/12}\left(F_{3/12},F_{4/12}\right)c_{35/12}(F_{3/12},F_{5/12})c_{36/12}(F_{3/12},F_{6/12})\\ \times c_{45/123}\left(F_{4/123},F_{5/123}\right)c_{46/123}\left(F_{4/123},F_{6/123}\right)\\ \times c_{56/1234}\left(F_{5/1234},F_{6/1234}\right)\end{cases} \tag{9}$$

Where the marginals are $f_1f_2f_2f_3f_4f_5f_6$; the unconditional pair is $c_{23/1}\left(F_{2/1},F_{3/1}\right)c_{24/1}(F_{2/1},F_{4/1})c_{25/1}(F_{2/1},F_{5/1})c_{26/1}(F_{2/1},F_{6/1})$; and the conditional pair is

$$c_{34/12}\left(F_{3/12},F_{4/12}\right)c_{35/12}(F_{3/12},F_{5/12})c_{36/12}(F_{3/12},F_{6/12})\times c_{45/123}\left(F_{4/123},F_{5/123}\right)c_{46/123}\left(F_{4/123},F_{6/123}\right)\times c_{56/1234}\left(F_{5/1234},F_{6/1234}\right)$$

The D-vine copula is expressed as

$$f(x_1,\ldots,x_6)=\begin{cases}=f_1f_2f_3f_4f_5f_6\\ \times c_{12}(F_1,F_2)c_{23}(F_2,F_3)c_{34}(F_3,F_4)c_{45}(F_4,F_5)c_{56}(F_5,F_6)\\ \times c_{13/2}\left(F_{1/2},F_{3/2}\right)c_{24/3}\left(F_{2/3},F_{4/3}\right)c_{35/4}\left(F_{3/4},F_{5/4}\right)c_{46/5}(F_{4/5},F_{6/5})\\ \times c_{14/23}\left(F_{1/23},F_{4/23}\right)c_{25/34}\left(F_{2/34},F_{5/34}\right)c_{36/45}\left(F_{3/45},F_{6/45}\right)\\ \times c_{15/234}\left(F_{1/234},F_{5/234}\right)c_{26/345}\left(F_{2/345},F_{6/345}\right)\\ \times c_{16/2345}\left(F_{1/2345},F_{6/2345}\right)\end{cases} \tag{10}$$

Where $f_k(.)$, for $k=1,d$ denotes the marginal densities, and $c_{i,j+i1,\cdots,j-1}$ is the bivariate copula density that conditionally connects the jth and $(j+i)th$ variables with those indexed by $1,\ldots,j-1$. Index j identifies the trees, and i runs over the edges in each tree. Only one node is connected with $(d-j)$ edges, and the other nodes are connected with one edge in the trees of C-vine and only as lines with the trees of D-vine.

The CDF $F(x/v)$ can be obtained for an $n-$ dimensional vector v. This can be performed in a pair-copula term in tree $n-1$ by using the pair-copulas of the previous trees $1,\cdots,n$, and then applying the following relationship:

$$h\left(x/v,\theta\right)=F\left(x/v\right)=\frac{\partial C_{xv_j/v_{-j}}/(F\left(x/v_{-j}\right),F\left(v_j/v_{-j}\right)/\theta)}{\partial F(v_j/v_{-j})} \tag{11}$$

where v_j is an arbitrary component of v, and v_{-j} denotes the $(n-1)$ -dimensional vector v, excluding v_j (Joe, 1996). The bivariate copula function is specified by $C_{xv_j/v_{-j}}$ with parameters θ specified in tree n.[17]

The two vine copula structures are different. Therefore, before estimating the coefficients of the retained copula, we should perform a goodness-of-fit test, based on Vuong (1989) and Clarke (2007), to identify the most appropriate copula in order to analyse the impact of the market dependence on risk measurement.

3.3. Third stage: Risk analysis and backtesting

VaR is defined as a statistical measure of potential losses over a given time horizon and level of confidence. It is the standard measure of risk used by financial institutions and a large body of empirical literature. Several studies on VaR estimation are based on the parametric and non-parametric models. Here, we present the models used to estimate the financial risk under the hypothesis of dependence and independence between stock market returns. Subsequently, we compare the performance of each model through backtesting.

Let X be a loss random variable and $F_X\left(F_X(u)=P\left\{X\le u\right\}\right)$ its distribution function, which models losses on a certain financial instrument over a given time horizon. Let $F_X^{-1}(v)$ be its quantile function $\left(F_X^{-1}(v)=inf\left\{u\varepsilon\mathbb{R}:F(x)\ge v\right\};0<v<1\right)$. For a fixed level, α, the VaR_α can be defined as the α -quantile of the distribution, F:

$$VaR_\alpha(X)=F^{-1}(\alpha) \tag{12}$$

To empirically show the limit of the main standard methods of $VaR_\alpha(X)$ measurement and forecasts for the case of the *independence* assumption, we apply them to our sample. These standard methods are RM, HS, and GARCH–EVT–VaR.

Concerning the main empirical method for estimating the dependence, we analyse the GARCH–EVT–copula–VaR. To determine the impact of dependence on risk measurement, based on Cherubini and Luciano (2001), we adopt the following four-step procedure in order to use the copula for measuring the portfolio risk:

Step 1: Fit the univariate $AR-GARCH$ model for daily returns series by obtaining the residuals and, subsequently, standardize them by the corresponding standard deviations.[18]

Step 2: Using the standardized residuals computed in step 1, fit the GPD. After choosing the threshold value u from Eq. (8) for the extremes of the distribution, denote the resulting distribution as $F_{evt,i}$. Subsequently, transform each standardized residual series into the uniform distribution over (0, 1) using probability-integral transformation.

[17] See Appendix B for the graphical illustration of the two copulas in our sample case.

[18] The error term $\varepsilon_{i,t,}$ in Equation (1), is an *i.i.d* sequence with zero mean, unit variance, and F_{it} distribution function (i.e. *skewed-t* distribution).

Step 3: From the transformed data of the previous step, fit a C -vine copula, using Eq. (9), which is proved to be better than the D-vine copula (Eq. (10)). Subsequently, estimate their parameters using the inference functions for margins (IFM) procedure of Joe and Xu (1996) . The joint density function (from Eqs. (9) and (10)) are decomposed into two parts, one for the marginal and the other for the copulas. This decomposition is applied to the log-likelihood function. Therefore, we first estimate the parameters of the marginal and, subsequently, we estimate the parameters of the copulas. The same procedure is carried out for each type of copula (C- and D-vines). The number of parameters depends on the type of copula used. While maximizing the log-likelihood of the bivariate copulas, the parameters are estimated using the algorithms of Aas et al. (2009) (refer to Appendix C.1 for the details).

Step 4: Use the Monte-Carlo simulation method to simulate 1000 standardized residuals from the estimated C -vine model of the previous step and compute the risk of the portfolio at various confidence levels.

Finally, we experiment with the VaR backtesting, by comparing the actual profits and losses for the VaR estimates. We calculate the number of times the observed loss is higher than that calculated by VaR. We consider an exception (or a violation) when the VaR is underestimated, that is, the portfolio experienced a loss greater than the estimated VaR.

The VaR model performance is measured by applying the conditional and unconditional coverage tests of Christoffersen (1998) and Kupiec (1995), respectively. Let $I_t(\alpha)$be a hit variable associated with a possible violation of the VaR at time t, and define the coverage rate α as

$$I_t(\alpha) = \begin{cases} 1\, if\; r_t \le -VaR_{t/t-1}(\alpha) \\ 0\, if\; r_t \; > -VaR_{t/t-1}(\alpha) \end{cases} \tag{13}$$

A VaR model is valid if the violation sequence $\left\{I_t\right\}_{t=1}^{T}$ satisfies the two assumptions of unconditional coverage and independence. The unconditional coverage hypothesis stipulates that the probability of a violation must equal the αcoverage rate, such that

$$Pr\left[I_t(\alpha) = 1\right] = \mathbb{E}\left[I_t(\alpha)\right] = \alpha \tag{14}$$

According to Kupiec (1995), the unconditional coverage test is defined by the following likelihood ratio:

$$LR_{UC} = -2\, ln\left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{T-I(\alpha)}\left(\frac{\hat{\alpha}}{\alpha}\right)^{I(\alpha)} \rightarrow \chi^2(1) \tag{15}$$

where $\hat{\alpha} = \frac{1}{T}I(\alpha)$, and $I(\alpha) = \sum_{t=1}^{T} I_t(\alpha)$.

The conditional coverage hypothesis implies that VaR violations observed on two different dates must be independently distributed (*IND*). The interval forecast and conditional test usually used for this purpose is that proposed by Christoffersen (1998), which is defined by the following:

$$LR_{IND} = -2\ln\left[\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0{}^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1{}^{n_{11}}}\right] \rightarrow \chi^2(1) \tag{16}$$

where $\pi_0 = \frac{n_{01}}{n_{00}+n_{01}}$, $\pi_1 = \frac{n_{11}}{n_{10}+n_{11}}$, and $\pi = \frac{n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}$

π_0 (resp π_1) is the probability of observing an exception condition on state 0 (resp 1) on the previous day; $n_{ij}$is the number of days when condition j occurred, assuming that condition i occurred on the previous day. Hence, if the model is accurate, then a VaR exception on day t should not depend on whether an exception occurred on day $t-1$.

4. Empirical evidence

4.1. Data analysis and dependence structure modelling

We use daily data for the closing spot prices of the CAC 40, the S&P 500, the Tunindex, the MASI, the TASI, and the ADI. We consider an equally weighted portfolio of the six assets. In order to take into account the 2007 crisis, we consider the period from January 3, 2005 to December 31, 2014; the sample comprises T = 2601 observations for each index.[19] To avoid missing data due to time differences and holidays, we replace all series using the same time standard, that is, coordinated universal time (UCT), and filter the series by using the previous day's observation. In addition, we remove all public holidays; to minimize bias, we adopt the approximation 'mean range medium' method.[20]

We use the daily logarithmic returns defined as $r_t = [Ln(P_t/P_{t-1}] \times 100$, where P_t is the closing price of the index at time t.

Table 1 shows the descriptive statistics of daily data, which reveal that all series have non-zero skewness and that the kurtosis of all series is greater than three. Indeed, a Jarque–Bera test rejects the null hypothesis that stock market returns are normally distributed. All the series have a fat tail and volatility clustering. Therefore, the EVT approach appears to be appropriate for modelling the fat-tail behaviour. Based on the literature review and the test results in Table 1, the GARCH effects should be filtered out from the data before envisaging the extreme value analysis. Moreover, the results of the *ARCH–LM* test given in Table 1 (row 8) prove the presence of ARCH effects in all the series. Then, we can use the $ARMA-GARCH$model as a filter providing serially independent innovations.

To identify the most suitable *AR–GARCH* model for each time series, we use Schwarz's Bayesian criterion (SBC). For the six return series, the SBC confirms that the $AR(1)-GJR-GARCH(1,\ 1)$ —fits better for modelling returns volatility.[21] To fit the series of returns, we run the model (Eq. (1)), assuming that the innovations follow an asymmetric distribution; this assumption is based on the fact that, of the six series, five have negative skewness. All the parameter estimates obtained by the quasi-maximum likelihood method are significant at the 5% level, except for the constant. The inequality $\alpha + \beta < 1$ means that the model is weak stationary.

In order to reduce autocorrelation, we experiment with data filtering through the model; the estimation results are presented in the following Table 2.

Given the absence of autocorrelation in the standardized residuals series, as shown in Table 2, the EVT method is applied. Based on the POT method, we model the standard innovation using the GDP for tail estimation (Eq. (5)), and the mean excess function (Eq. (7)) is used to determine the threshold. The obtained standard residuals are subject to $i.i.d.(0,\ 1)$uniform distribution and can well meet the requirements of the copula-GARCH modelling. Appendix C presents the parameter estimates of the $AR(1)GARCH-EVT$ model (Eqs. (1) and (5)).

In the Appendix, Table C gives the parameter estimates of the GPD fitted to the excesses over the optimal threshold for both the upper and lower tails. As mentioned in the Subsection 3.2, we use copulas to model the dependence between the returns of the indices. To this end, we use the converted uniform distribution to estimate the copula parameters. There are three ways to estimate copulas—the maximum likelihood, the IFM, and the semi- para-

[19] The data are taken from the Morgan Stanley Capital International Inc (http://www.msci.com).

[20] We obtain the statistical results using the programming language R.

[21] See Appendix D for these statistics.

Table 1
Descriptive statistics for daily stock market returns.

		CAC 40	S&P 500	MASI	TUNINDEX	TASI	ADI
1	**Mean**	0.0040	0.0207	0.0290	0.0515	8.5700e-004	0.0126
2	**Maximum**	10.5946	10.9572	4.5537	3.6133	16.4521	17.6123
3	**Minimum**	−9.4715	−9.4695	−5.0167	−5.0037	−15.2773	−14.5146
4	**Std Dev**	1.4364	1.2680	0.8248	0.5671	1.7209	2.0042
5	**Skewness**	0.0541	−0.2674	−0.3155	−0.6373	−0.9931	−0.1934
6	**Kurtosis**	9.8924	13.8213	8.1902	14.9280	17.7194	13.7724
7	**Jarque–Bera**	514.77 [a] (0.001)	1271.7 [a] (0.001)	296.14 [a] (0.001)	1558.9 [a] (0.001)	2389.9 [a] (0.001)	1258.8 [a] (0.001)
8	**ARCH–LM test (5)**	443.6291 (0.000)	676.4037 (0.000)	439.4107 (0.000)	651.0107 (0.000)	414.2051 (0.000)	324.9694 (0.000)
9	**Ljung–Box[b] (5)**	36.7827 (0.0066)	99.4045 (0.000)	173.8628 (0.000)	238.6984 (0.000)	45.5900 (0.000)	100.2665 (0.000)

Notes: (a) Significance at the 1% level; (b) The Ljung–Box test for autocorrelation of residuals series.

Table 2
Diagnostic statistics of standardized residuals obtained from the AR (1)-GJR-GARCH (1, 1) model.

	CAC 40	S&P 500	MASI	TUNINDEX	TASI	ADI
Ljung–Box	7.3661	4.5560	20.6067	20.2770	74.0244	83.7755
Probability	0.1948	0.4724	0.0010	0.0011	0.0849	0.1110
Jarque–Bera	178.6224	401.2289	658.5773	636.20	961.72	358.61
Probability	0.001	0.001	0.001	0.001	0.001	0.001

Table 3
Parameter estimates for six-dimensional C-vine copula decomposition.

Specific copula	Margin	ϑ_0	ϑ_1	Lower and Upper Tails
BB1	C_{12}	0.0391 (0.0255)	1.001 (0.0115)	2.035703e-08 0.0013
Rotated Clayton	C_{13}	0.0386 (0.0219)	–	0 1.589628e-08
Gumbel	C_{14}	1.0027 (0.0118)	–	0 0.0037
BB7	C_{15}	1.0174 (0.0126)	0.0017 (0.0200)	0 0.0235
Gaussian	C_{16}	−0.0002 (0.0203)	–	00
BB1	$C_{23/1}$	0.3708 (0.0424)	1.4680 (0.0331)	0.2798 0.3965
Rotated Joe	$C_{24/1}$	1.0001 (0.0182)	–	0.00013 0
Student	$C_{25/1}$	−0.0039 (0.0213)	15.6669 (5.1353)	0.00077 0.00077
Rotated Clayton	$C_{26/1}$	0.0431 (0.0228)	–	0 1.036443e-07
Student	$C_{34/12}$	0.0576 (0.0214)	13.8977 4.2392)	0.0024 0.0024
Rotated BB7	$C_{35/12}$	1.0632 (0.0180)	0.0103 0.0224)	0.0807 5.940005e-30
Joe	$C_{36/12}$	1.0040 (0.0144)	–	0 0.0055
BB7	$C_{45/123}$	1.215 (0.0266)	0.3501 0.0328)	0.1380 0.2308
Student	$C_{46/123}$	0.1443 (0.0204)	21.4732 9.3236)	0.0004 0.0004
BB7	$C_{56/1234}$	1.0090 (0.0145)	0.0954 (0.0244)	0.00069 0.0123

Notes: 1 = CAC 40, 2 = S&P 500, 3 = MASI, 4 = TASI, 5 = ADI, and 6 = Tunindex. Standard errors of the coefficient estimates are in parentheses. These indices (1–6) are indicated in the second column of the table.

metric methods. We use the IFM method proposed by Joe and Xu (1996) because it can solve the maximization problem in the case of high-dimensional distributions.

To model the dependency structure for our sample, we use the C- and D-vine copula parameters alternatively to approximate the upper and lower tail dependencies in various specific copulas. The empirical results are given in Tables 3 and 4.

Since the objective is to highlight the relevance of the adopted method, and not to analyse the crisis mechanisms exhaustively, we focus on the interpretation of the results in Table 1 on the most important conditional dependencies between the markets. The crisis effect was not identical for all the sampled countries. Between December 31, 2007 and December 31, 2008, in the United States of America and France, the S&P 500 and the CAC 40 indices fell 26 % and 43 %, respectively. In the KSA and the UAE, and the TASI and the DFI, the indices fell by 56 % and 72 %, respectively. However, the Tunindex increased to 11 %, while the MASI fell only by 13 %. In Table 1, the results of the estimated parameters, for the six-dimensional C- vine

Table 4
Parameter estimates for the decomposition of six-dimensional D-vine copula.

Copula Types	Margin	ϑ_0	ϑ_1	Lower and Upper Tail
Clayton	C_{12}	0.1054(0.0000)	–	0.001392935 0
Frank	c_{23}	0.2309(0.0000)	–	0 0
Frank	C_{34}	0.6799(0.0000)	–	0 0
Frank	C_{45}	0.3874(0.0000)	–	0 0
Gaussian	C_{56}	0.0818(0.0000)	–	0 0
Clayton	$C_{13/2}$	0.0364(0.0000)	–	5.621176e-09 0
Clayton	$C_{24/3}$	0.0278(0.0000)	–	1.513065e-11 0
rotated Gumbel	$C_{35/4}$	1.0365(0.0000)	–	0.04824502 0
Gumbel	$C_{46/5}$	1.0116(0.0000)	–	0 0.01586958
Clayton	$C_{14/23}$	0.0229(0.0000)	–	7.65338e-14 0
Student	$C_{25/34}$	0.2679(0.0000)	2.5757(0.0000)	0.232153 0.232153
Survival Joe	$C_{36/45}$	1.0548(0.0000)		0.07078288 0
Survival Joe	$C_{15/234}$	1.0094(0.0000)	–	0.01288376 0
Rotated Joe	$C_{26/345}$	1.0149(0.0000)	–	0 0
Student-t	$C_{16/2345}$	0.6186(0.0000)	10.078(0.0000)	0.13427870.1342787

Notes: 1 = CAC 40, 2 = S&P 500, 3 = MASI, 4 = TASI, 5 = ADI, and 6 = Tunindex. Standard errors of the coefficient estimates are in parentheses.

Table 5
Goodness-of-fit test based on the Vuong and Clarke tests.

Test	Statistic	*p-value*
Clarke	1186	8.407761e-06
Vuong	−2.806366	0.004010379

copula decomposition show the existence of conditional dependencies within the sampled financial markets.[22] These dependencies correspond to the capital account opening and the degree of openness of the economy. This is reflected through the dependencies between the US and the Moroccan markets ($C_{23/1}$) and the KSA and AD markets, ($C_{45/123}$). This dependence during extreme events, $C_{23/1}$, is explained by the free trade agreement between Morocco and the United States of America in 2006 and the progressive liberalization of the Moroccan capital account since 2007. These dependencies reflect the adoption of the measured of liberalisation, such as the relaxation of restrictions on foreign investment and the partial liberalization of Moroccan investments outside the country. Concerning the conditional dependence between the Saudi Arabian and Abu Dhabi markets, $C_{45/123}$, the result is expected owing to the economic integration between the two countries in the GCC, and their capital account openness with the same exchange regimes (the Saudi Riyal and the UAE Dirham are anchored to the US dollar). However, our results confirm the non-validation of conditional dependence for other pairs of countries whose intensity of bilateral trade is very low[23] or whose financial market is under financial repression, like the case of Tunisia (Chebbi, Louafi, & Hedhli, 2014). In fact, we could not verify the dependencies between the markets of Tunisia–Abu Dhabi and Tunisia–US. However, this does not mean that commercial protection and financial repression are ideal for preventing the effects of extreme events. The delayed effects of the global crisis in protected economies should be studied separately. In this regard, certain theoretical standpoints predict that, in the long-run, openness would absorb the effects of the crisis better than that of the protection (Butkiewicz & Yanıkkaya, 2008).

Table 4 shows the estimated parameters obtained using D-vine copulas according to their diverse types, where the highest conditional dependency is between the US and Abu Dhabi markets, that is, $C_{25/34}$. This result arises for the same reasons outlined above, that is, the opening of the capital account, the anchoring of the Emirati currency, as well as trade liberalization. However, the obtained statistical differences in the results between Tables 3 and 4 lead us to test the goodness-of-fit (Clarke, 2007; Vuong, 1989) to select the best copula, C-vine or D-vine, in order to deal with conditional dependencies. Table 5 presents the test results.

Thus, at a p-value of less than 0.05 chosen as the critical level for our hypothesis tests, and following Sriboonchitta, Liu, Kreinovich, and Nguyen (2014), our results suggest that, for our empirical data, the C-vine copula is better than that of the D-vine copula for the treatment of conditional dependencies.

4.2. Backtesting and interpretation

By comparing VaR measures under the alternative hypothesis of independence (EWMA, GARCH, and HS) and the dependence (C-vine copula model) of stock market returns in order to predict the losses that may occur, we apply the steps of the estimation procedure described in Subsection 2.2.3. We simulate the returns at time $t + 1$ based on the C-vine copula's dependency structure. With an equally weighted portfolio with six stock indices, we estimate the VaR of the portfolio validated by the violation ratio (VR) and the backtesting (the unconditional coverage test LR_{uc} and the conditional coverage test LR_{cc}).[24] The result of the estimated risk measures varies by method. Under the hypothesis of independence of the extreme returns (normal approach, HS, and GARCH), the *out-of-sample* performance of each method given by the violation ratio is shown in Table 6.

The results in Table 6 show that the GARCH-EVT-copula model gives the most accurate forecast of risk at the 99 %, 95 %, and 90 % levels, respectively, for the weighted portfolios. An accurate model is obtained when the expected violation ratio is $VR \in [0.8, 1.2]$. Furthermore, if $VR < 0.5$ or $VR > 1.5$, the model is imprecise.[25] The EWMA method is the only method that can be considered unacceptable, as its violation ratio exceeds 1.5 at the 95 % and 99 % levels. A high violation ratio implies an underestimation of the risk, which contributes towards an insufficient capital allocation to cover the risk. In this case, the model increases the risk exposure owing to the

[22] Following Manner (2007), Brechmann, Czado, and Aas (2012), and Dißmann, Brechmann, Czado, and Kurowicka (2013) , the selection of the copula family for each pair of variables is carried out on the basis of the minimum Akaike information criterion.

[23] For example, the annual trade volume during the period 2000−2010, between the Tunisian, the US, and the UAE markets are very low. They are about 1%, 0.9%, and 0.4%, respectively, of the total Tunisian trade volume (INS, Institut National de la Statistique). The exchange between Tunisia and France was 18% of the total Tunisian foreign trade, which is less than 4% of the French commercial volume.

[24] The main tools used in backtesting are violation ratios, in which the actual number of VaR violations are compared with the expected value. The violation ratio is $VR = \frac{\textit{Observed number of violations}}{\textit{Expected number of violations}}$. See Appendix C-2 for the details of the backtesting procedure.

[25] These parametric conditions are widely used in the empirical literature (see Dannielsson, 2011, chapter 8).

Table 6
Results of the violation ratio and Kupiec and Christoffersen tests' portfolio.

Model	Method	Violation ratios			LR_{UC}			LR_{Ind}		
Confidence level (%)		90	95	99	90	95	99	90	95	99
Conventional measures	**EWMA**	0.8121	1.0531	2.0526	9.3295	0.3271	19.2320	11.5640	6.4786	5.8636
	HS	0.9281	0.8924	0.9370	1.3135	1.4130	0.0915	50.2476	34.3825	11.5319
	GARCH	0.7362	0.8210	1.8741	18.8866	4.0114	13.7591	16.5061	7.9395	0.0549
GARCH-EVT-copula	**Vine copula**	0.9673	0.9727	0.9370	0.0838	0.0881	0.0915	0.0148	0.358268	0.3974

underestimation. An excessively lower violation ratio implies that the model signals a greater capital allocation than necessary. In this case, the portfolio holder allocates more liquidity than necessary and registers a return loss.

Besides a simple violation ratio analysis, we use a formal significance test of the violation ratio. The validity of the VaR model should be verified by both unconditional and independence coverage tests. To this end, we backtest our risk measurement methods using the Kupiec (1995) and Christoffersen (1998) tests for testing the unconditional and conditional coverages, respectively. Thus, the GARCH–EVT–C-vine approach performs better than all the other competing methods and offers the most acceptable VaR estimates, which neither overestimate nor underestimate risk at all quantiles. The results show that traditional VaR measures are least appropriate for risk measurement and far from a normally distributed series. This indicates that extreme dependencies between portfolio assets must be taken into account when conducting risk assessment and risk management. Our results confirm those of McMillan and Kambouroudis (2009) and Maugis and Guegan (2010), who concluded that the C-vine copula is better suited for VaR estimation than the traditional methods, such as the RM, GARCH, and EWMA methods. Finally, Zhang, Wei, Yu, Lai, and Peng (2014) applied C-vine, D-vine, and R-vine copula models to a 10-dimensional global portfolio to estimate the VaR of an international stock market portfolio forecast using the Monte Carlo method. The authors found that the vine copula models estimated VaR more accurately than that of the traditional methods, including HS, mean variance, and dynamic conditional correlation GARCH models.

5. Summary and conclusion

In this study, we investigate the methods for modelling and measuring market risk as well as determine the dependence between the financial markets of the MENA region (Morocco, Tunisia, Saudi Arabia, and Abu Dhabi) and those of the developed countries (the United States of America and France) for the period January 3, 2005 to December 31, 2014. By experimenting with multidimensional models of dependency structures using the C-vine copula, we adopt two different approaches for a comparative purpose. The first approach uses a measure based on the assumption of series independence, while the second retains the hypothesized series' dependence. We also use the Monte Carlo simulations to measure the risk of a portfolio comprising the daily returns of the country sample. To validate the VaR measurement, we undertook the unconditional coverage and independence tests.

The results indicate an interesting contribution of the dependence measure to the assessment of the market risk. According to the backtesting procedure, the validity of VaR forecasts rests on the verification of the unconditional coverage and independence assumptions. The backtesting results show that the VaR measurements, according to the first approach (independence case: EWMA, HS, and GARCH), do not meet the assumptions of unconditional coverage and independence, while the second approach verifies these assumptions (dependence case: GARCH–EVT–vine copula). Thus, the study significantly contributes through the introduction of the copula to the measure of risk. Concerning the MENA countries, this empirical method verifies that the greater the openness of the capital accounts and the flexibility of the exchange rate regimes, the greater will be the conditional dependence of the MENA countries on the developed markets. Thus, this multivariate approach for the empirical risk assessment makes it possible to go beyond the limits of the standard bivariate approaches (Halbert et al., 2015). It also verifies that trade and financial openness determine the conditional dependence of financial markets in the MENA region on those of the United States of America and France. There are a limited number of studies that use the multivariate approach and copula to analyse empirically the effects of the financial crisis on the MENA region. Moreover, the goodness-of-fit test allows us to verify statistically the dominance of the vine-copula over the C-vine approach. MENA countries can adopt the empirical method to assess VaR effectively. However, the results do not show the late effects of the extreme event once it has ended. This opens an interesting research direction for future work on the long-term effects of trade protection and financial repression on country risk because, in theory, these effects could be reversed over time under some conditions.

Declaration of Competing Interest

The authors report no declarations of interest.

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Appendix A.

Joint distribution expressions

Referring to Sklar (1959), let $X_1, \ldots\ldots, X_d$ be random variables with respective marginal distributions, $F_1, \ldots.., F_d$, and a joint distribution, F. A copula $C : [0, 1]^d \to [0, 1]$ exists such that

$$F(x_1, \ldots.., x_d) = C(F_1(x_1), \ldots\ldots, F_d(x_d)) \quad \text{(A.1)}$$

Conversely, if C is a copula and $F_1, \ldots.., F_d$ are distribution functions, then the function F defined in (A.1) is a joint distribution with margins $F_1, \ldots.., F_d$.

If F is continuous and strictly increasing, then the probability density function of x is defined as follows:

$$f(x_1, \cdots, x_d) \equiv f_d(x_d) f_{d-1/d}(x_{d-1}/x_d) \cdots f_{1/2\cdots d}\left(x_1/x_2, \cdots, x_d\right) \quad \text{(A.2)}$$

If all the marginal functions are continuous, then $C(F_1(x_1), \ldots\ldots, F_d(x_d))$exists and is *unique*. If not, then the copula C is not unique. Then, for every $u \in I^n$, we have

$$C(U_1, \cdots U_n) = F\left(F_1^{-1}(U_1), \cdots, F_n^{-1}(U_n)\right) \quad \text{(A.3)}$$

where $F_i^{-1}(t) = inf\left\{x \in R^1; F_i(x) \geq t\right\}$, for $i = 1, \ldots, d$: the inverse functions of $F_1, \cdots, F_d$

Appendix B.

General expression of the vine copulas with their decompositions

Vine copulas were proposed by Joe (1996) and developed by Bedford and Cooke (2001, 2002). Vines are a flexible graphical model that is used for describing multivariate copulas built up using a cascade of bivariate copulas.

The basic idea is to decompose a d-dimensional copula density C into a product of $d(d-1)/2$ called pair-copulas via conditioning. Let $X=(X_1, X_2, \cdots, X_d)$ with marginal distribution functions and corresponding densities $f_1, f_2, \cdots, f_d$. By recursive conditioning, we can write

$$f(x_1, \cdots, x_d) = f_d(x_d) f_{d-1/d}(x_{d-1}/x_d) \cdots f_{1/2\cdots d}\left(x_1/x_2, \cdots, x_d\right) \quad \text{(C1)}$$

By the Sklar theorem, we know that

$$f(x_1, x_2, \cdots, x_d) = \prod_{1\le i<j\le d} c_{i,ji+1,\, i+2,\cdots,j-2,j-1} \prod_{1\le k\le d} f_k(x_k) \quad \text{(C2)}$$

where

$c_{i,ji+1,\, i+2,\cdots,j-2,j-1} = c_{i,ji+1,\, i+2,\cdots,j-2,j-1}(F_i(x_i), F_j(x_j)/F_{i+1}(x_{i+1}), \cdots, F_{j-1}(x_{j-1})$ is the conditional copula of $F_i(X_i)$ and $F_j\left(x_j\right)$, given that $F_{i+1}(X_{i+1}) = F_{i+1}(x_{i+1}), \cdots, F_{i+1}(X_{i+1}) = F_{j-1}(x_{j-1})$

The representation of C in terms of pair-copulas is not unique. It is essential to organize the structure of a d-dimensional vine copula. Bedford and Cooke (2001) propose a sequence of linked trees. A d-dimensional vine tree structure is a sequence of d-1 trees. Tree $j$has $d+1-j$ nodes and $d-j$ edges. The edges of the tree j become the nodes of the tree $j+1$ and are joined by an edge if the corresponding edges in tree j share a node.

Graphical illustration of the D-vine and C-vine copulas

We graphically illustrate the C- and D-vine copulas in general. Fig. B1 illustrates the tree decomposition for canonical vines in an arbitrary case of six dimensions. It consists of five trees, Tj, for $j = 1, \ldots, 5$. Tree Tj has $6-j$ nodes and $d-j$ edges. Each tree has a unique node connected to all other nodes. The nodes in tree Tj are necessary for determining the labels of the next tree $Tj+1$. The node with $(d-1)$ edges in tree $T1$ is called the *root*. The edge labels show the indices of the corresponding pair-copula terms.

Fig. B2 shows a D-vine copula on six variables. The nodes' names appear in the circles, which indicate trees, and the edges' names appear under the edges of trees.

Appendix C.

Parameter estimates of the GPD fitted to the excesses over the optimal threshold for both upper and lower tails (Table C1).

Appendix C1

Vine copula parameters estimating the procedure on the basis of the IFM procedure of Joe and Xu (1996) and the Aas et al. (2009) algorithm.

This estimation takes place as follows:

1 We estimate the parameters of the copula in the first tree (T1) from the uniform marginal on [0, 1].
2 Using the parameters obtained in the first step, we compute the conditional distribution functions for the second tree.
3 Using the results obtained from the previous step, we estimate the parameters of the second tree (T2).
4 We repeat step (2) and (3) to estimate the parameters of the following trees: T3, T4, and T5. We compute the conditional distribution functions from the results of the previous step and then we estimate the parameters of the current tree.

Appendix C2

The back-testing procedure

Given a total of 2601 observations, we have chosen a moving window of width $w = 1000$ observations in order to obtain 'within the sample' forecasts' VaR. This corresponds to $(n-w)$ sub-samples to be processed. Subsequently, we simulated a $u_{1,t}^{(K)}, u_{j,t}^{(K)}$ sample with $t \in \{1, 2, \ldots, n-w\}$.

For the backtesting procedure in the case of six indices and a sample of observations, $\{1, \ldots, T\}$, we used the C-Vine copula in order to model the dependence that exists between the six indices. We implemented the following backtesting procedure:

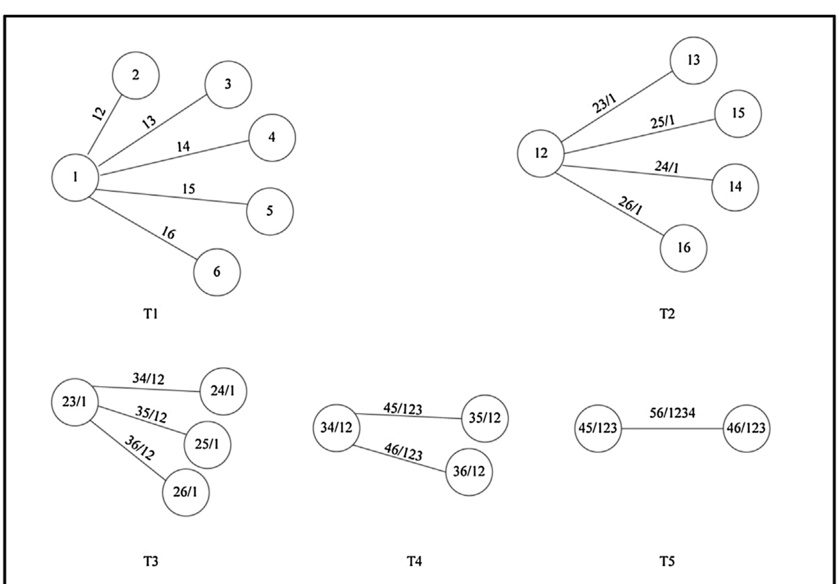

Fig. B1. Six-dimensional C-vine copula construction.

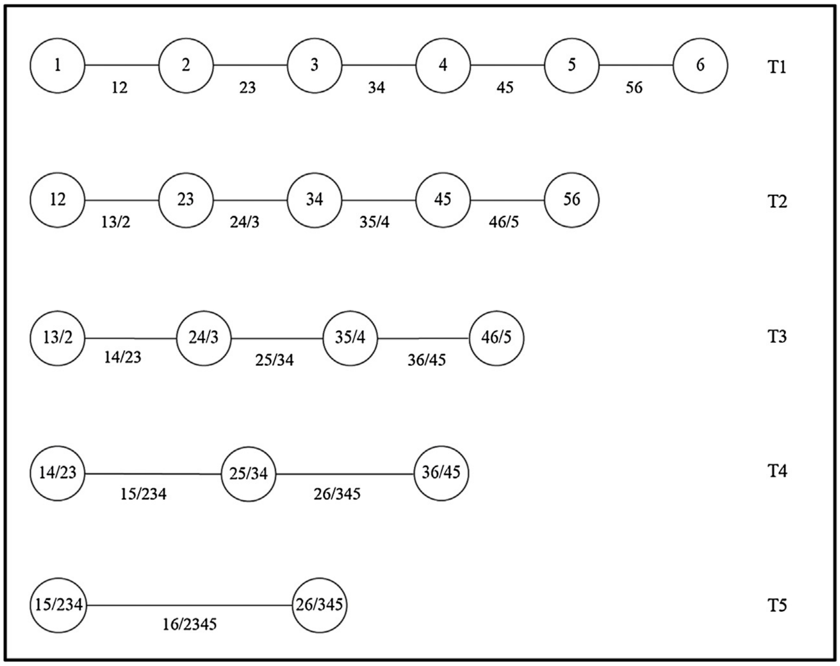

Fig. B2. Six-dimensional D-vine copula construction.

1 For $w = 1, \ldots, 1000$, from the estimated C-Vine copula, we simulate a sample $u_{1,t}^{(K)}, u_{j,t}^{(K)}, = 1, \ldots.., h$. For j = 1,. . ., 6, we transform $u_{j,t}^{(K)}$ into $\hat{\varepsilon}_{j,t}^{(K)}$, $t = 1, \ldots., h$ using the inverse of the Skewed-t function, $\hat{\varepsilon}_{j,t}^{(K)} = F_j^{-1} = u_{j,t}^{(K)}$. For j = 1,.. ., 6, we determine the returns forecasts as follows:

$$\hat{x}_{j,T+t}^{(t)} = \hat{\mu}_{j,T+t} + \sqrt{\hat{h}_{j,T+t}}\,.\hat{\varepsilon}_{j,t}^{(K)},\ \mathrm{t} = 1, \ldots, \mathrm{h}$$

where

$\hat{\mu}_{j,T+t}$ et $\hat{h}_{j,T+t}$ correspond to the one- day-horizon forecast of the mean and the conditional variance, respectively.

We calculate the portfolio forecast returns as follows:

$$\hat{x}_{P,T+1}^{(K)} = \sum_{j=1}^{6} \frac{\hat{x}_{j,T+t}^{(t)}}{6},\ \mathrm{t} = 1, \ldots.\mathrm{H}$$

2 For *confidence* levels,: $(1-\alpha)\varepsilon\{\{0.01,\ 0.05,\ 0.1\}$, we estimate the 1-day $VaR_{\alpha,1}$. This corresponds to the 100th percentile of $\hat{x}_{P,T+1}^{(K)}$, K = 1,. . .., 1000..

3 We *compare* the observed value of the portfolio for day T + 1, $x_{P,T+1}$ with the estimated value $V\hat{a}R_{\alpha,1}$ in order to record a violation.

Table C1
Parameter estimates for AR (1) GARCH-EVT models.

	CAC 40	S&P 500	MASI	TUNINDEX	TASI	ADI
Mean equation						
c_0	0.0191	0.0285	0.0076	0.0358	0.0658	0.0339
	(0.018)	(0.015)	(0.010)	(0.008)	(0.016)	(0.028)
c_1	−0.0706	−0.0800	0.1440	0.2191	−0.1320	−0.1543
	(0.020)	(0.018)	(0.024)	(0.022)	(0.019)	(0.023)
Variance equation						
Ω	0.0301	0.0162	0.0512	0.0499	0.0542	0.1508
	(0.009)	(0.004)	(0.017)	(0.012)	(0.022)	(0.117)
A	0.0000	0.0000	0.2832	0.3108	0.1567	0.2173
	(0.016)	(0.010)	(0.061)	(0.057)	(0.036)	(0.085)
B	0.8867	0.8933	0.6550	0.4742	0.8930	0.8557
	(0.022)	(0.016)	(0.072)	(0.086)	(0.026)	(0.050)
γ	0.1959	0.1868	0.0417	0.1179	0.0572	0.0336
	(0.033)	(0.029)	(0.060)	(0.060)	(0.065)	(0.069)
V	9.2852	6.5723	4.9882	4.7495	2.2223	2.4771
	(1.621)	(0.878)	(0.486)	(0.497)	(0.034)	(0.190)
Λ	−0.1143	−0.1600	−0.0419	0.0541	0.0173	0.0032
	(0.025)	(0.024)	(0.023)	(0.027)	(0.016)	(0.010)
EVT						
u_U	1.16601	0.96725	1.19687	1.22287	0.92699	1.0417
ξ_U	−0.08261	−0.11674	0.08663	0.05540	0.03763	0.06593
β_U	0.49557	0.48862	0.56916	0.54594	0.72214	0.57648
u_L	−1.26466	−1.18678	−1.1352	−1.07368	−0.93997	−0.98633
ξ_L	−0.03221	−0.00606	−0.01005	0.27367	0.02470	0.15859
β_L	0.64209	0.63460	0.66814	0.45765	0.97254	0.63467

Table D1
Fitted ARMA (s, v) + GARCH (p, q) model for the stock market returns.

Index	ARMA(s,v) + GARCH (p,q)	SBC
MASI	ARMA(1,0) + GARCH (1,1)	−2731.387
TUNINDEX	ARMA(1,0) + GARCH (1,1)	−1654.428
CAC40	ARMA(1,0) + GARCH (1,1)	−4143.777
S&P500	ARMA(1,0) + GARCH (1,1)	−3519.071
TASI	ARMA(1,0) + GARCH (1,1)	−4371.714
ADI	ARMA(1,0) + GARCH (1,1)	−5009.371

4 We assess the validity of the chosen method through validation tests, based on the occurrence of VaR violations.

Appendix D.

Fitting the marginal distribution

For each of the stock markets returns, we fit an ARMA (s, v) model. On these ARMA models, we fit a GARCH (p, q) model to the error terms (Table D1).

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