Volatility-of-volatility and tail risk hedging returns

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This paper reports that the volatility-of-volatility implied by VIX options has predictability for tail risk hedging returns. Specifically, an increase in the volatility-of-volatility as measured by the VVIX index raises current prices of tail risk hedging options, such as S&P 500 puts and VIX calls, and lowers their subsequent returns over the next three to four weeks. The results are robust to jump risk, skewness, kurtosis, option liquidity, variance risk premium, and limit of arbitrage. The predictability can be explained by either risk premiums for a time-varying crash risk factor or uncertainty premiums for a time-varying uncertain belief in volatility.

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1 Introduction

The recent financial crisis has provoked an interest in tail risk hedging strategies that are structured to generate positive payoffs in bad states of the world in which asset values plunge, market volatility soars, funding/market liquidity drops, and institutional investors are forced to deleverage their risk exposures because of higher margin and haircut rates. For example, equity portfolio risk can be hedged by buying stock index put options or volatility derivatives such as the VIX futures and options. Despite the growing interest in tail risk hedges, little is known about their expected returns. The financial media expresses skepticism about the efficacy of tail risk hedges because the increased demand following the 2007-2009 financial crisis has made them too expensive.[1] Against this backdrop, the objective of this paper is to propose a new measure of marketwide tail risk and examine its relation to expected returns on two popular forms of tail risk hedges: the out-of-the-money (OTM) S&P 500 (SPX) puts and the OTM VIX calls.

The market's perception of tail risk can be incorporated into the stock index dynamics through two stochastic channels: a jump process and a persistent volatility process with a leverage effect, which is a negative correlation between changes of prices and volatility. Most of the existing studies, such as Bollerslev and Todorov (2011) and Du and Kapadia (2012), identify tail risk through the lens of a jump process because it offers an easy-to-understand framework to conceptualize tail risk. However, the precision of the jump-based tail risk measures heavily relies on the existence and accuracy of deep OTM SPX puts. In fact, it is highly difficult to obtain precise estimates of jump-based tail risk due to the limited availability and poor liquidity of deep OTM SPX puts. This measurement-error issue becomes even worse during a crisis when a tail risk indicator would be most useful, because the pre-crisis OTM SPX puts often turn into in-the-money.

Unlike the jump-based tail risk literature, this paper takes the perspective that tail

[1] For example, Barclays' exchange-traded note on the first two front-month VIX futures lost more than one half of its value in the first half of 2012.

risk information may be impounded into volatility-of-volatility because even a small change in the variable has a critical influence on the tails of return distributions. Motivated by this insight, I suggest using the volatility-of-volatility implied by the VIX options as a tail risk indicator, in particular, the Chicago Board Options Exchange (CBOE) VVIX index. Calculated by applying the VIX methodology to a cross-section of the VIX options, the VVIX index represents a risk-neutral expectation of volatility of the 30-day forward VIX index.

A nice property of the VVIX index is that it is less prone to measurement errors than the extant tail risk measures for two reasons. First, the VIX options market has, on a per-contract basis, a larger trading volume, a lower bid-ask spread, and a lower Amihud (2002) illiquidity measure than the SPX options market. In short, the VIX options market has greater market liquidity than the SPX options market. Second, the VVIX index is the second moment of VIX return distributions, so its computation is more weighted toward slight and moderate OTM options than deep OTM ones. As such, the absence of deep OTM options is less of a concern for the computation of the VVIX index, while some tail risk measures, such as the Bollerslev and Todorov (2011) fear index, hinge on the existence of deep OTM puts.

A central hypothesis tested in this paper is the negative relation between tail risk and expected returns on tail risk hedging options. That is, a higher level of tail risk increases the current prices of tail risk hedges, lowering their subsequent returns over the next period. Consistent with the hypothesis, the VVIX index is predictive of tail risk hedging returns with a negative sign over the next three to four weeks, implying that the tail risk hedging options become more expensive when the VVIX index is high. A one standard deviation increase in the current VVIX index is associated with a 1.63% to 2.19% decrease in the next day's SPX put returns and a 0.68% to 0.87% decrease in the next day's VIX call returns. The results are robust to a wide range of control variables, including other jump and tail risk measures; skewness and kurtosis measures; option (il)liquidity measures; variance risk premiums; and limits of arbitrage.

As the VVIX index compounds both information on volatility-of-volatility risk and its associated risk premium, the true source of the predictability of the VVIX index is unclear. To uncover the true source I introduce an approach to separating the VVIX index into a physical measure of volatility-of-volatility ($RVVIX$) and a volatility-of-volatility risk premium ($VVRP$). $RVVIX$ is obtained by computing the realized variance of the five-minute front-month VIX future returns over the past one month. $VVRP$ is then defined as the difference between the squared VVIX index and the $RVVIX$ measure. By running predictive regressions of tail risk hedging returns against $RVVIX$ and $VVRP$, I find that they both significantly contribute to the forecasting power of the VVIX index, although the former is more statistically significant than the latter.

What can then explain the predictive relation of the VVIX index to future tail risk hedging returns? The primary explanation is risk-based. The volatility-of-volatility as measured by the VVX index contains information on the crash risk as perceived and priced by the options market, given that it is a critical determinant of the likelihood of a market crash. With that said, the volatility-of-volatility may be a priced crash risk factor, and the predictability is driven by a risk compensation for the time-varying crash risk factor. This interpretation is related to a large literature on rare disaster that tail risk has a crucial impact on asset returns (e.g., Rietz, 1988; Barro, 2006; Gabaix, 2008, 2012; Wachter, 2013).

An alternative uncertainty-based explanation is plausible. While return volatility is not observable, it is so crucial for pricing options that an ambiguity-averse agent may factor uncertainty over volatility into option pricing. In such a case, the VVIX index may be considered as a proxy for uncertainty over volatility, and an increase in the uncertainty measure will raise current prices of tail risk hedging options and lower their subsequent returns over the next period. Accordingly, it is the uncertainty premium for a time-varying uncertain belief in volatility—not the risk premium for a time-varying crash risk factor—that drives the predictability of the VVIX index. This interpretation is related to the equilibrium-based models that incorporate model uncertainty into option prices (e.g., Liu et al., 2005; Drechsler, 2013).

The rest of the paper is organized as follows. In Section 2, I propose using volatility-of-volatility as a tail risk indicator. The futures and options data are introduced in Section 3. In Section 4, I examine the predictability of the VVIX index for tail risk hedging returns. In Section 5, I offer two economic implications for the results. Finally, I conclude in Section 6.

2 Proposing volatility-of-volatility as a tail risk indicator

Measuring or predicting tail risk is challenging because a tail risk factor is latent and a tail risk event is rare by definition. The existing tail risk methodologies are based on different data sources. While Barro (2006) and Barro and Ursua (2008) compute the macroeconomic disaster risk from a cross-country data set of the consumptions and GDPs, others attempt to identify tail risk from financial market data. In particular, the options market offers a promising environment in which to measure tail risk, as options contain forward-looking information about future market returns. In light of this fact, Bollerslev and Todorov (2011) develop a new fear index from the deep OTM SPX puts, and Du and Kapadia (2012) devise an index of jump and tail risk from a cross-section of the SPX options. Alternatively, Kelly (2011) proposes a measure of tail risk from extreme cross-sectional stock returns. Researchers such as Barndorff-Nielsen and Shephard (2004) introduce a model-free estimate of jump risk by using high-frequency asset returns. Despite the substantive progress, a definitive measure of tail risk is still lacking, so this paper contributes to the literature by proposing volatility-of-volatility as a tail risk indicator.

2.1 How is volatility-of-volatility related to tail risk?

In this subsection, I explain how tail risk information may be impounded into volatility-of-volatility. To do this, I assume an affine stochastic volatility model, which allows for analytic solutions to skewness and kurtosis. Given a risk-neutral probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ and information filtration $\{\mathcal{F}_t\}$, the log stock index price, $s_t = \log(S_t)$, takes the following form under the risk-neutral measure:

$$\begin{aligned} ds_t &= (r_t^f - \frac{h_t}{2})dt + \sqrt{h_t}dW_{1,t} \\ dh_t &= \kappa(\overline{h} - h_t)dt + \sigma\sqrt{h_t}dW_{2,t}, \end{aligned} \tag{1}$$

where r_t^f is the risk-free rate, h_t is the volatility state, σ is the volatility-of-volatility, κ is the persistence of volatility, $\overline{h}$ is the long-run mean of volatility, and $W_{1,t}$ and $W_{2,t}$ refer to the standard Brownian motions under the risk-neutral measure, with a correlation of ρ.

I now show that volatility-of-volatility is a critical determinant of both skewness and kurtosis of return distributions and thus tails of return distributions. Specifically, under the affine framework, it can be shown that the negative skewness is proportional to the volatility-of-volatility (note that ρ is negative), and that the excess kurtosis is proportional to the squared volatility-of-volatility:

$$\begin{aligned} SKEW_t(\tau) &\propto \sigma\rho \\ KURT_t(\tau) &\propto \sigma^2(\rho^2 + constant), \end{aligned} \tag{2}$$

where $SKEW_t(\tau)$ and $KURT_t(\tau)$ denote the τ-horizon skewness and excess kurtosis at time t, respectively. The analytic forms of skewness and kurtosis are provided in Appendix A.

To further illustrate that σ has a significant impact on tails of return distributions, I compute volatility smirks for different levels of σ under the affine stochastic volatility framework. The relevant parameters, except for σ, are obtained by taking the averages of those reported by Christoffersen et al. (2009). That is, $\kappa = 2.5971$, $\rho = -0.6850$, and

$\overline{h} = 0.0531$. The low, mean, and high σ correspond to the lowest (0.3796), mean (0.6151), and highest (0.8516) values of their estimates of the volatility-of-volatility, respectively. The initial volatility state, h_t, is assumed to be equal to the long-run volatility. Lastly, r_t^f is assumed to be 2%.

The four panels of Figure 1 show the computed volatility smirks with 1, 3, 6, and 12 months to maturity where the moneyness is defined as strike prices divided by spot prices. OTM puts are more expensive in times of high volatility-of-volatility than those of low volatility-of-volatility, whereas OTM calls are more expensive in times of low volatility-of-volatility than those of high volatility-of-volatility. This result implies that a higher level of volatility-of-volatility makes the left tail thicker, while making the right tail thinner, resulting in more left-skewed return distributions.

2.2 Extracting volatility-of-volatility from the VIX options

Recently, the VIX option dollar volume had grown dramatically from 0.7 billion dollars in 2006 to 16 billion dollars in 2012. Moreover, the VIX calls are more actively traded than the VIX puts because the former are used as a tail risk hedging strategy as opposed to the latter.[2] What is special about the VIX options is that they contain information on the market's risk-neutral expectation of volatility-of-volatility. That is, the VIX options can be utilized to infer the volatility-of-volatility as perceived by the market.

While there are various model-free approaches to computing implied volatility, including Bakshi et al. (2003), Carr and Wu (2009), Jiang and Tian (2005), and Britten-Jones and Neuberger (2000), I use the CBOE VVIX index as a proxy for volatility-of-volatility because the data are publicly available from the CBOE website.[3] Computed by applying the VIX

[2] Researchers, such as Alexander and Korovilas (2012) and Engle and Figlewski (2014), discuss the risk management functioning of the VIX derivatives.

[3] At the time when the first draft of the paper was written, the VVIX data were not available so I computed the model-free, risk-neutral volatility-of-volatility based on the Bakshi et al. (2003) method. The empirical results that follow are robust to the implied volatility methods.

calculation method to a cross-section of OTM VIX options, the VVIX index represents a risk-neutral expectation of the volatility of the 30-day forward VIX index.[4]

A nice feature of the VVIX index is that it is less prone to the issue of measurement errors than other jump-induced tail risk measures for two reasons. First, the VIX options have greater market liquidity than the SPX options. Table 1 shows three kinds of (il)liquidity metrics for the SPX and VIX options across different moneyness levels, including average trading volumes ($VOLUME$), average relative bid-ask spreads ($SPREAD$), and average Amihud (2002) illiquidity ($ILLIQ$) measures. It is evident that the VIX options market has, on a per-contract basis, the larger trading volume, the lower relative bid-ask spread, and the lower $ILLIQ$ than the SPX options market.

Second, while some tail risk metrics such as the Bollerslev and Todorov (2011) fear index hinge on the existence and accuracy of deep OTM puts, the computation of the VVIX index is more weighted toward slight and moderate OTM VIX options than deep OTM ones. Table 1 shows that slight OTM options have greater market liquidity than deep OTM options. Especially, the relative bid-ask spread and the $ILLIQ$ both are far lower for slight OTM options than deep OTM options. More importantly, missing deep OTM options would be less problematic in the VVIX index because they make only a small contribution to its computation.

The VVIX data cover January 1, 2007 through January 31, 2013. To investigate the potential of the VVIX index as a tail risk indicator, I examine whether the spikes of the VVIX index correspond well with some of the recent financial crisis episodes. The top panel of Figure 2 presents the history of both the VVIX index and the VIX index. The VVIX index exhibits four pronounced crisis periods, which are associated with the Bear Stearns' two hedge funds suspension in July to August 2007, the Lehman Brothers bankruptcy in September 2008, the Greek debt crisis in May 2010, and the U.S. debt ceiling crisis in August

[4] See http://www.cboe.com/micro/VVIX/VVIXwhitepaper.aspx as a reference on the VVIX index.

2011, respectively.[5] Unlike the VIX index, the VVIX index shows a high level of fear risk for the New Century Financial Corporation bankruptcy in April 2007, which is one of the precursors to the subsequent financial crisis.

A market crash is usually accompanied by a stock market decline, a volatility increase, and a liquidity dry-up. To see whether the evolution of the VVIX index is consistent with those typical characteristics of a market crash, the SPX, VIX, $VVIX$, and liquidity changes are computed as follows:

$$\begin{aligned} \Delta SPX_t &= \log\left(\frac{S_t}{S_{t-1}}\right) \\ \Delta VIX_t &= \log\left(\frac{VIX_t}{VIX_{t-1}}\right) \\ \Delta VVIX_t &= \log\left(\frac{VVIX_t}{VVIX_{t-1}}\right) \\ \Delta LIQUID_t &= LIQUID_t - LIQUID_{t-1}, \end{aligned} \tag{3}$$

where $LIQUID$ is the stock market liquidity factor as measured by Pastor and Stambaugh (2003). $\Delta VVIX$ is negatively correlated with ΔSPX with a correlation of -0.52, positively correlated with ΔVIX with a correlation of 0.71, and negatively correlated with $\Delta LIQUID$ with a correlation of -0.18. These correlations indicate that when the VVIX index increases, the stock market index tends to decline; market volatility tends to climb; and market liquidity tends to drop.

2.3 Decomposing the VVIX index into $RVVIX$ and $VVRP$

As the VVIX index represents a risk-neutral expectation of the volatility of the 30-day forward VIX index, it contains both information on a physical expectation of the volatility of the 30-day forward VIX index and its associated risk premium. In this subsection, I

[5] The first spike occurred on August 16, 2007 when the Federal Reserve Board cut the discount rate by 50 bps down to 5.75%; the second spike took place on October 27, 2008, and two days later, the Federal Open Market Committee reduced its federal funds target rate by 50 bps down to 1.00%; and the fourth spike occurred on August 8, 2011, and one day later, the FOMC decided to keep the federal funds target rate at 0% to 1/4% as economic growth turned out to be considerably slower than expected.

propose an approach to separating the VVIX index into the $RVVIX$ and $VVRP$ measures.

First, I obtain $RVVIX$ by computing the annualized realized variance of the five-minute front-month VIX future returns over the past 22 trading days, based on Andersen et al., 2001; Andersen et al., 2003; Barndorff-Nielsen and Shephard, 2002:

$$RVVIX_t \equiv 12 \sum_{i=1}^{22} \sum_{j=1}^{1/\Delta} \left(f^V_{t-i+j\Delta} - f^V_{t-i+(j-1)\Delta} \right)^2, \tag{4}$$

where f^V_t denotes the log front-month VIX future price and Δ is the sampling interval for the intraday data. $RVVIX$ is used as a proxy for a physical expectation of the volatility of the 30-day forward VIX index.[6]

Following the literature on variance risk premiums, I now define the volatility-of-volatility risk premium:

$$VVRP_t \equiv VVIX_t^2 - RVVIX_t. \tag{5}$$

Figure 3 presents the time series plots of $RVVIX$ and $VVRP$. Similar to the VVIX index, $RVVIX$ also corresponds to the four recent crisis episodes mentioned above. $VVRP$ takes positive values most of the time, with an average of 0.41, implying that volatility-of-volatility is negatively priced. The negative pricing of volatility-of-volatility is not surprising because volatility-of-volatility tends to increase during market downturns, playing a hedging role against the stock market.

3 Futures and options data

The futures and options data set spans from January 1, 2007 through January 31, 2013. The high-frequency, front-month SPX futures data are used to compute the delta-neutral SPX option returns and high-frequency-based jump variations. In particular, I use small-

[6] I admit that $RVVIX$ is a biased measure of volatility-of-volatility risk because it measures the volatility-of-implied-volatility but not the volatility-of-physical-volatility. However, unfortunately, there is no such derivatives market where one can extract a physical expectation of the volatility-of-physical-volatility.

denomination (E-mini) SPX future prices because Hasbrouck (2003) finds that price information is discovered in the E-mini market as opposed to the regular market or the exchange-traded funds. The E-mini SPX futures data come from Thomson Reuters Tick History. Although the E-mini futures trade overnight in the electronic Globex venue, I include the five-minute returns only for 8:30 a.m. to 3:15 p.m. when the CBOE option market is open.

I also need the high-frequency, front-month VIX futures returns to compute $RVVIX$ and the delta-neutral VIX option returns. The high-frequency VIX futures data for 8:30 a.m. to 3:15 p.m. are obtained from Thomson Reuters Tick History.

The options data come from OptionMetrics. Option prices are taken from the bid-ask midpoint on each day's close of the options market. With respect to the SPX options, the CBOE option market closes 15 minutes later than the SPX spot market. To address the issue of nonsynchronous trading hours, I back out the spot price for each of the first three pairs of near-the-money SPX put and call options by using the put-call parity (e.g., Aït-Sahalia and Lo, 1998, 2000), and take an average of the three extracted spot prices. Then, the options that violate the usual lower bound constraints are eliminated using the following:

$$\begin{aligned} C_t^S(\tau, K) &\geq \max(0, S_t(\tau)\exp(-q_t\tau) - K\exp(-r_t^f\tau)) \\ P_t^S(\tau, K) &\geq \max(0, K\exp(-r_t^f\tau) - S_t(\tau)\exp(-q_t\tau)), \end{aligned} \tag{6}$$

where $C_t^S(\tau, K)$ and $P_t^S(\tau, K)$ are the time t prices of the SPX call and put options with time to maturity τ and strike price K, $S_t(\tau)$ is the time t implied SPX price with time to maturity τ, and q_t is the dividend payout rate. Treasury bill rates and dividend rates are also obtained from OptionMetrics.

In addition, I apply a set of filters to eliminate inaccurate or illiquid options. Specifically, I delete the SPX options for which the mid price is less than 0.5, there is no trading volume, the implied volatility is lower than 5% or higher than 100%, or the relative bid-ask spread is larger than 0.5, where the relative bid-ask spread is defined as $\frac{2(ask-bid)}{(ask+bid)}$. I also delete the VIX options for which the mid price is less than 0.2, there is no trading volume, the implied volatility is lower than 10% or higher than 150%, or the relative bid-ask spread is larger than 0.5.

With respect to maturities, I focus on relatively short-term options with 8 to 90 days to expiration.[7] With respect to moneyness, I define a triple of moneyness bins for each of the SPX puts and the VIX calls. Specifically, the SPX OTM puts are classified into the slight OTM ($0.95 < k < 1.00$), the medium OTM ($0.90 < k < 0.95$), and the deep OTM ($0.85 < k < 0.90$), where k denotes the moneyness defined as $k = \frac{K}{F_t(\tau)}$. I use wider ranges of moneyness for the VIX calls than the SPX puts because the volatility-of-volatility is more volatile than volatility. The VIX OTM calls are classified into the slight OTM ($1.0 < k < 1.1$), the medium OTM ($1.1 < k < 1.2$), and the deep OTM ($1.2 < k < 1.3$).

4 Predicting tail risk hedging option returns

4.1 Delta-neutral option returns

Analyzing risk premiums on options poses two challenges. First, options are levered investments with nonlinear payoffs so their return distributions are far from normality. For example, Broadie et al. (2009) argue that normality-based standard statistical tools can mislead a hypothesis test of interest, and find that the large returns to writing put options are not inconsistent with the Black–Scholes or the affine stochastic volatility models. Second, option returns are largely driven by innovations in the underlying asset price and volatility. It is therefore necessary to isolate the effect of a variable of interest—tail risk in this paper—from the effects of the two underlying risks.

To alleviate the difficulties, I first introduce the delta-neutral option returns that are immune to the underlying asset's price risk. The delta-neutral (excess) returns are computed

[7] The results are weaker for long-term options, although not reported in this paper.

as:

$$R^O_{t+1}(k,\tau) = \frac{O_{t+1}(k,\tau) - O_t(k,\tau) - \Delta_t(k,\tau)(F_{t+1}(\tau) - F_t(\tau))}{O_t(k,\tau)} - \frac{r^f_t N}{365}, \tag{7}$$

where, with abuse of notation, $O_t(k,\tau)$ means either the SPX option price or the VIX option price with an expiration of τ and a moneyness of k depending on the context of analysis; $F_t(\tau)$ indicates either the SPX future price or the VIX future price; and $\Delta_t(k,\tau)$ refers to the corresponding option delta that is directly obtained from OptionMetrics.[8]

Next, on each day I take an equal average of all delta-neutral option returns that belong to the i^{th} moneyness bin:

$$R^O_{i,t} = \frac{1}{N_{i,t}} \sum_{j=1}^{N_{i,t}} R^O_t(k_j,\tau_j), \tag{8}$$

where $N_{i,t}$ denotes the number of options that belong to the i^{th} moneyness bin on date t. Although I use the equal averaging scheme in the empirical analysis, the weighting rule does not affect the findings.

Table 2 shows the statistical characteristics of the delta-neutral option returns as in Equation (8) across different moneyness levels, including means, 99% confidence intervals of the bootstrapped means, minimums, maximums, standard deviations, skewness, kurtosis, one-day autocorrelations, and Sharpe ratios, where the confidence intervals are computed with 250,000 sample draws. The table shows that the deep and medium SPX puts lose 1.46% and 1.29% per day, respectively. Based on the bootstrapped confidence intervals, these means are significantly different from zero at 99% confidence levels. The slight SPX puts lose 0.40% a day, although the result is not statistically different from zero at a 99% confidence level. On the other hand, the VIX calls lose 0.65% to 0.74% per day depending on moneyness bins. The bootstrapped confidence intervals indicate that the delta-neutral VIX call returns are statistically different from zero at 99% confidence levels.

[8] Notice that the denominator of equation (7) is $O_t(k,\tau)$ but not $(O_t(k,\tau) - \Delta_t F_t(\tau))$, implying that I do not account for the required capital for investing in the futures. The reason is that the associated margin rate has been quite low historically; for example, the margin rate on the SPX futures has been around 6% on average. Frazzini and Pedersen (2011) also disregard the required capital for a stock position in computing delta-hedged option returns.

The negative delta-neutral returns indicate that some systematic risk factors may be priced in the tail risk hedging options.[9] Two well-known priced factors in the stock index options are stochastic variance and stochastic jumps. For example, Coval and Shumway (2001) and Bakshi and Kapadia (2003) show that stochastic variance is a significant priced factor in delta-neutral options, and there has been ample evidence on jump risk premiums, including Bates (2000), Pan (2002), and Broadie et al. (2007), among others.

Although the variance and jump risk premiums are important drivers of the delta-neutral returns, there may be other unknown risk factors in the tail risk hedging options. In particular, the tail risk factor as measured by the VVIX index may be priced in the tail risk hedging options beyond the variance and jump risk premiums. I attempt to answer this question by looking at the predictability of the VVIX index for tail risk hedging returns after controlling for the variance and jump risk measures.

4.2 Predictability of the VVIX index

To test the main hypothesis, I take the realized delta-neutral returns as a proxy for expected ones, following French et al. (1987) and Amihud (2002). I then run the predictive regressions of option returns onto the VVIX index for each moneyness bin:

$$\begin{aligned} R^O_{i,t} = {} & \beta_{i,0} + \beta_{i,1} VVIX_{t-2} + \beta_{i,2}\Delta SPX_t + \beta_{i,3}\Delta VIX_t \\ & + \beta_{i,4}\Delta SPX_t^2 + \beta_{i,5}\Delta VIX_t^2 + \beta_{i,6}\Delta SPX_t \Delta VIX_t + \varepsilon_{i,t}, \end{aligned} \tag{9}$$

where I skip one more day between the option returns and the VVIX index because option prices may be stale, although some illiquid options are filtered out. Note that there are five control variables: ΔSPX_t, ΔVIX_t, ΔSPX_t^2, ΔVIX_t^2, and $\Delta SPX_t \Delta VIX_t$.[10] The reason is

[9] Alternatively, the negative option returns can be explained by the behavioral explanation of Barberis and Huang (2008), which is based on the cumulative prospect theory of Tversky and Kahneman (1992). They argue that positively skewed assets can have negative returns as investors distort a subjective probability distribution, or decision weights, toward tails of a return distribution. This theory is consistent with the negative delta-neutral returns because they are severely positively skewed.

[10] In the case of VIX option returns, only ΔVIX_t and ΔVIX_t^2 are included among the control variables.

follows. Although delta hedging attenuates the price sensitivity in option returns, $R^O_{i,t}$ is not completely free of the price risk because the Black–Scholes delta used in this paper causes hedging errors and fails to account for the second-order (gamma) effect of price changes. More importantly, $R^O_{i,t}$ is largely driven by changes in asset volatility. By including these variables, I am able to study the effect of a tail risk factor on option returns in isolation from the price and volatility risks.

Table 3 presents the two-day-ahead predictive regression results for the SPX puts and the VIX calls. Each explanatory variable is divided by its standard deviation so each regression coefficient can be interpreted as the impact of a one standard deviation change in that variable, and statistical significance is computed by Newey and West (1987) robust t-statistics with an optimal lag.

Regression 1 is the baseline regression that includes only the VVIX index. Consistent with the hypothesis, the negative relation is statistically significant for the SPX puts, with an expected sign. Specifically, a one standard deviation increase in the VVIX index results in a 1.63% to 2.19% decrease in the next day's SPX put returns. The result is statistically significant at 99% confidence levels, with t-statistics of -3.85 to -4.80.

The negative relation is statistically significant for the VIX calls as well. Specifically, a one standard deviation increase in the VVIX index results in a 0.68% to 0.87% decrease in the next day's VIX call returns. The result is statistically significant at 99% confidence levels, with t-statistics of -3.01 to -3.57. Furthermore, the coefficients are more negative for the deep OTM VIX calls than the slight ones, as the former are more sensitive to tail risk than the latter.

In fact, it is the expected VVIX index that truly matters in predicting tail risk hedging returns. For this reason, I run the predictive regressions with the expected VVIX index that is obtained by assuming that the VVIX index follows an autoregressive process of order 1, and find that the expected VVIX index yields very similar empirical results to the current

VVIX index, although these results are not reported here.

In an unreported table, I also look at whether the VVIX index can have forecasting power for returns on SPX calls and VIX puts, which may be considered as tail risk taking assets that pay positive payoffs in good states of the world. However, I fail to find that the VVIX index has predictability for tail risk taking returns regardless of whether control variables are included.

4.3 Controlling for other factors

In this subsection, I look at whether the predictability of the VVIX index is robust to other pricing factors and whether they have incremental information for future tail risk hedging returns beyond the VVIX index. In particular, five distinct groups of the control variables are considered. The summary statistics and correlation matrix for the explanatory variables are provided in Table 4.

4.3.1 Jump/tail risk measures

I next examine whether the predictability of the VVIX index is robust to VIX, JV, and $TAIL$, among a variety of jump/tail risk measures that have been developed in the literature.[11] The computations of JV and $TAIL$ are provided in Appendix B. It is also prudent to control for the VIX index as it is commonly referred to as the Wall Street fear index, although it is intended to measure volatility risk rather than tail risk. The correlation matrix in Panel B of Table 4 shows that the VVIX index is positively correlated with both VIX and JV, and negatively correlated with $TAIL$.

Regression 2 in Table 3 shows the regression results with $VVIX$, VIX, JV, and $TAIL$. The predictability of the VVIX index is robust to all of the control variables. The coefficients

[11] I also consider the Du and Kapadia (2012) jump and tail index and the Bollerslev and Todorov (2011) fear index as control variables, and find that the result are robust to the two variables. However, I do not include them in the main analysis because the data have shorter sample periods than mine.

on $VVIX$ are statistically significant at 99% confidence levels with negative signs, regardless of moneyness levels. However, none of the control variables have incremental forecasting power for tail risk hedging returns beyond the VVIX index. The coefficients on $TAIL$ are only statistically significant at 95% confidence levels only for the medium and slight SPX puts.

The insignificance of JV appears to be at odds with the literature showing that return jumps are priced in the index options (e.g., Bates, 2000; Pan, 2002; Broadie et al., 2007). These contradictory results may be due to different empirical frameworks; I examine the predictability of option returns, whereas other researchers pin down jump risk premiums from option prices.

4.3.2 Skewness and kurtosis measures

The second control-variable group includes the realized skewness, the CBOE SKEW index, and the realized kurtosis. It should be noted that the SKEW index is constructed such that a high value of it is associated with more negative skewness (or a more left-skewed return distribution). The computation of the realized skewness and kurtosis is described in Appendix B.

Regression 3 in Table 3 shows the regression results with $VVIX$, $RSKEW$, $SKEW$, and $RKURT$. The predictability of the VVIX index is robust to all of the control variables. The coefficients of $VVIX$ are statistically significant at 99% confidence levels with negative signs, regardless of moneyness levels. Thus, the predictability of the VVIX index is not affected by the inclusion of the skewness and kurtosis measures.

Unexpectedly, there is no evidence that any of the control variables have incremental forecasting power for tail risk hedging returns beyond the VVIX index. This insignificant relation of higher moments to tail risk hedging returns is rather surprising because authors such as Harvey and Siddique (2000) have found higher moments to be important priced factors in the stock market. Perplexed by this result, I test some other measurement windows for the realized skewness and kurtosis but still fail to find any significant relation regardless of whether the VVIX index is included or not. Nonetheless, I do not interpret the result as implying that skewness and kurtosis are not priced in tail risk hedging options.

This result also seems to be at odds with the previous argument that the volatility-of-volatility is associated with skewness and kurtosis. Why is it that the VVIX index is priced in tail risk hedging options but the realized skewness and kurtosis are not? To understand this issue, note that the VVIX index and the realized moments are driven by completely different stochastic components. As pointed out by Du and Kapadia (2012), the VIX methodology is designed to capture the integrated variance rather than the quadratic variance, so the VVIX index is not driven by return jumps. In contrast, as pointed out by Amaya et al. (2013), the realized third and fourth moments separate out cubic and quartic variations due to return jumps from those due to a diffusive process with the leverage effect. Given such a difference, it is no longer surprising that the VVIX index and the realized higher moments have different results. Rather, the seemingly contradictory results clarify the argument that tail risk information can be impounded into the volatility of a diffusive volatility component.

4.3.3 Liquidity measures

As the third control-variable group, I consider three (il)liquidity measures: the trading volumes ($VOLUME$), the relative bid-ask spreads ($SPREAD$), and the Amihud (2002) illiquidity ($ILLIQ$) measures. Regression 4 in Table 3 shows the regression results with $VVIX$, $VOLUME$, $SPREAD$, and $ILLIQ$. The predictability of the VVIX index is robust to all of the control variables. The coefficients of $VVIX$ are statistically significant at 99% confidence levels with negative signs, regardless of moneyness levels. Thus, the VVIX index still has forecasting power even after the liquidity measures are controlled for.

The coefficients on $VOLUME$ in Table 3 are positive, implying that liquid SPX puts trade at lower prices than illiquid ones. The coefficients on $SPREAD$ are also positive, implying that liquid SPX puts trade at higher prices than illiquid ones. In short, $VOLUME$ implies liquidity discounts, whereas $SPREAD$ implies liquidity premiums. In fact, the literature has been reporting different liquidity effects in the derivatives markets. For example, Deuskar et al. (2011) find liquidity discounts in the over-the-counter interest rate caps and floors market, whereas Christoffersen et al. (2012) find liquidity premiums in the equity options market. Bongaerts et al. (2011) and Deuskar et al. (2011) argue that the pricing of liquidity depends on investors' characteristics such as risk aversion and nontraded risk exposure as well as market characteristics such as zero net supply.

4.3.4 Variance risk premiums

Bollerslev et al. (2009) introduce an equilibrium model in which the variance risk premium is determined only by the volatility-of-volatility. They find that the variance risk premium can forecast subsequent stock index returns. Motivated by their model implication, I now investigate whether the VVIX index is just picking up the variance risk premium.[12] Interestingly, the correlation between $VVIX$ and VRP is low at 0.33, although this number is statistically significant at a 99% confidence level (see Table 4).

Regression 5 in Table 3 shows the regression results with $VVIX$ and VRP. The negative relation is robust to VRP. The coefficients of $VVIX$ are still statistically significant and economically large for each option market, regardless of moneyness levels. In contrast, there is no evidence that VRP is associated with subsequent tail risk hedging option returns except for the deep SPX puts.

4.3.5 Limits of arbitrage

Bollen and Whaley (2004) and Gârleanu et al. (2009) find that demand pressure, which is

[12] I also use the forward-looking variance risk premium as in Bekaert and Hoerova (2014) and the results are virtually the same.

associated with limits of arbitrage, affects the pricing of the SPX puts. When arbitrage is highly implausible, tail risk hedging assets may become more costly, resulting in lower subsequent returns over the next period. I thus add the relative SPX futures margins ($MRGN$) and the TED spreads as proxies for limits of arbitrage to the regressions. The time series plots of $MRGN$ and TED are given in Figure 4.

Regression 6 in Table 3 shows the results of regressions of tail risk hedging returns on $VVIX$, $MRGN$, and TED. The predictability of the VVIX index is robust to all of the control variables. The coefficients of $VVIX$ are statistically significant at 99% confidence levels with negative signs, regardless of moneyness levels. However, except for the deep SPX puts, none of the control variables have incremental forecasting power for tail risk hedging option returns beyond the VVIX index.

4.4 Persistence of predictability

The regression results so far are based on two-day-ahead predictive regressions. In this subsection, I investigate how persistent the predictability is over multi-week horizons. Table 5 shows the multi-week-ahead predictive regression results onto the VVIX index and the three liquidity measures. As before, each explanatory variable is divided by its standard deviation, and Newey and West (1987) robust t-statistics with an optimal lag are shown in parentheses.

Regression 1 in Table 5 shows that one-week-ahead predictability is quite decisive at 99% confidence levels regardless of moneyness, with expected negative signs. Regressions 3 and 4 indicate that the predictability can persist over the next three to four weeks at 95% or 99% confidence levels. In fact, this long-run predictability does not come as a surprise given the fact that a future VVIX index is predictable over the next few weeks. Table 4 shows that the VVIX index has a weekly autocorrelation of 0.79, which corresponds to a half-life of 2.94 weeks. Simply put, the persistence of the VVIX index is being translated into its multi-week predictability for tail risk hedging returns.

4.5 $RVVIX$ versus $VVRP$

In Subsection 2.3, I introduce the decomposition of the VVIX index into the $RVVIX$ and $VVRP$ measures. The correlation between $VVIX$ and $RVVIX$ is positive, with $\rho = 0.63$, and so is the correlation between $VVIX$ and $VVRP$, with $\rho = 0.52$. To identify the true source of the predictability, I run the two-day-ahead predictive regressions on $RVVIX$, $VVRP$, and all control variables.

Table 6 presents the regression results, with Newey and West (1987) robust t-statistics with an optimal lag in parentheses. The coefficients on $RVVIX$ are all statistically significant at 99% confidence levels across all six different cases, with expected negative signs. The coefficients on $VVRP$ are also statistically significant at 99% confidence levels for the medium and slight SPX puts and 95% confidence levels for the deep SPX puts and all VIX calls. Taken together, $RVVIX$ and $VVRP$ both significantly contribute to the forecasting power of the VVIX index, although the former is more statistically significant than the latter.

4.6 Robustness to liquidity filters

In Subsection 4.3.3, I find that trading volume and relative bid-ask spreads are important determinants of tail risk hedging returns, so the predictability of the VVIX index may depend on a choice of liquidity filters. In the analysis so far, I apply the liquidity filter such that the trading volumes must be greater than zero and the relative bid-ask spreads must be less than 0.5. I now apply a tighter liquidity filter such that the trading volumes must be greater than 20 and the relative bid-ask spreads must be less than 0.25. This tighter liquidity constraint reduces the number of SPX puts from 110,545 to 72,639 and the number of VIX calls from 13,063 to 12,028.

Table 7 presents the regression results for the new set of options data. The coefficients on $VVIX$ are all statistically significant at 99% confidence levels across all six different cases, with negative signs. The positive coefficients of $VOLUME$ indicate liquidity discounts in the SPX puts, whereas the positive coefficients of $SPREAD$ imply liquidity premiums in both SPX puts and VIX calls.

4.7 Predicting the VIX returns

I next examine whether the VVIX index can be predictive of the VIX returns. Given that the VIX futures are another form of tail risk hedging, I expect that an increase in the current VVIX index may be associated with lower subsequent VIX returns. To test such a negative relation, I run the h-period-ahead predictive regressions of the form:

$$\begin{aligned}\frac{\sum_{i=1}^{h} r_{t+i}^{V}}{h} &= \beta_0 + \beta_1 VVIX_t + \beta_2 VRP_t + \beta_3 \log(P/E)_t + \beta_4 \log(P/D)_t \\ &\quad + \beta_5 DFSP_t + \beta_6 TMSP_t + \varepsilon_{t,t+h},\end{aligned} \tag{10}$$

where $r_t^V = \log(VIX_t/VIX_{t-1})$ is the monthly VIX return; (P/E) denotes the price-earning ratio; (P/D) denotes the price-dividend ratio; $DFSP$ the difference between Moody's BAA and AAA corporate yields; and $TMSP$ the difference between 10-year and 3-month Treasury yields. The control variables are included because they are known to have forecasting power for stock index returns.

Table 8 presents the regression results for the VIX index returns for one-, three-, and six-month horizons. Each explanatory variable is divided by its standard deviation so each regression coefficient can be interpreted as the impact of a one standard deviation change in that variable. The robust t statistics are computed based on the Newey and West (1987) method for a one-month horizon and the Hodrick (1992) method for longer-than-one-month horizons to control for overlapping data. Consistent with the expectation, the coefficient on $VVIX$ has a negative sign for every horizon, although the result is not statistically significant regardless of whether control variables are included.

Interestingly, VRP can be predictive of VIX returns with a negative sign. That is, higher levels of VPR are associated with lower subsequent returns on the VIX index. When the control variables are not included, the results are statistically significant at a 90% confidence level for the one-month horizon and a 95% confidence level for three- and six-month horizons. However, the statistical significance disappears to some extent as the control variables are included.

5 Economic explanations

In this section, I offer two economic explanations for the predictability of the VVIX index. The results will help shed light on the priced risk factors and the characteristic of investors' preferences.

5.1 Risk-based explanation

Some researchers extract the risk factors implied by the index options market as well as their risk premiums. They often assume that crash or tail risk is manifested through return jumps and thus take a measure of time-varying jumps as a proxy for crash risk (e.g., Bates, 2000, Santa-Clara and Yan, 2010; Bollerslev and Todorov, 2011; Du and Kapadia, 2012). Distinct from the existing studies, I argue that the market's consensus on crash risk can be transmitted into the stock index dynamics through stochastic volatility-of-volatility. This is motivated by the feature of standard stochastic volatility models that volatility-of-volatility is a critical determinant of the likelihood of a market crash; the higher the volatility-of-volatility, the higher the probability of a market crash.

If volatility-of-volatility was a priced crash factor, it should be able to forecast returns of assets that are susceptible to crash risk. Indeed, the VIX index is a strong and robust predictor of tail risk hedging returns, with a negative sign. The result implies that volatility-of-volatility is a crash risk factor priced in tail risk hedging options and that the predictability may be driven by a risk compensation for the time-varying crash risk.

This risk-based interpretation is related to the literature showing that tail risk has a crucial impact on asset returns. For example, Rietz (1988) and Barro (2006) show that introducing a tail risk factor to the standard asset pricing model can explain the equity premium puzzle with reasonable levels of risk aversion. Burnside et al. (2011) and Jurek (2014) argue that positive excess returns to carry trades are associated with the risk compensation for a rare disaster in currency rates. Collin-Dufresne et al. (2010) emphasize the importance of catastrophic risk in the pricing of super-senior collateralized debt obligation (CDO) tranches. Kelly (2011) and Kelly and Hao (2012) find that tail risk is a priced factor in cross sectional stock and hedge fund returns, respectively. Most closely related to this work, Gourio (2008), Gabaix (2008, 2012), and Wachter (2013) develop time-varying rare disaster models to resolve the well-known asset pricing puzzles such as return predictability and high stock market volatility.

5.2 Uncertainty-based explanation

The options markets contain information on not only risk factors but also investors' preferences through the pricing of the risk factors, so the finding may promote an understanding of the preferences that investors adopt in option pricing. In particular, I attempt to relate the empirical finding to model uncertainty introduced by Knight (1921). That is, the predictive relation that I find may be explained by an equilibrium model in which an uncertainty-averse agent factors time-varying uncertainty into options pricing.

Uncertainty, or interchangeably ambiguity, refers to a situation where an agent is unsure of the probability distribution that guides his/her decision. Several researchers examine the importance of uncertainty over beliefs in explaining the empirical features of option prices.

For example, Liu et al. (2005) argue that uncertainty over jumps in the aggregate endowment, which they call rare events, drives the shape of volatility smirk, using the robust control approach of Hansen and Sargent (2001). Based on the recursive multiple-priors model of Epstein and Schneider (2003), Drechsler (2013) offers an uncertainty-based description of asset prices, including option prices, variance premiums, and equity prices. Loosely related, Miao et al. (2012) provide an ambiguity-based interpretation for the large variance premium observed in the data using the recursive smooth ambiguity model with learning.

Considerable effort has been put into providing an ambiguity-based explanation of option prices. However, despite the theoretical advancement in this argument, there is not much empirical research. How can agents assess the extent of uncertainty in the options markets? How can one test whether a measure of uncertainty affects option prices and returns? the empirical analysis of this paper may be a good example to fill this gap.

Consider a situation where an agent, when pricing options, faces uncertainty over the return distribution of the underlying asset. Return volatility is so crucial for pricing options that the agent will incorporate the extent of uncertainty over volatility into option prices. In doing so, the agent might consider a measure of volatility-of-volatility as the degree of uncertainty over volatility.[13] If this is the case, an increase in the uncertainty proxy will raise the prices of tail risk hedging options, lowering their subsequent returns over the next period, which is exactly what the analysis of this paper shows. Thus, the results could imply that option traders have uncertainty-averse preferences and the predictability is driven by time-variation in uncertain volatility, which can be measured by volatility-of-volatility.

In fact, this is not the only paper that interprets volatility-of-volatility as a measure of uncertainty. Baltussen et al. (2012) take individual options' implied volatility-of-volatility as a proxy for uncertainty. They find that it is priced into the cross-section of stock returns. Brenner and Izhakian (2011) take the variance of daily variance as a proxy for uncertainty

[13] This measure of uncertainty should be distinguished from that of Anderson et al. (2009), which is the dispersion of analysts' forecasts. The former is a measure of uncertainty over expected volatility, whereas the latter is over expected mean.

and examine its forecasting power for stock index returns. Most closely related to this work, Agarwal et al. (2014) argue that volatility-of-volatility risk is a systematic risk factor for hedge fund returns.

6 Conclusion

The market's perception of tail risk may be impounded into volatility-of-volatility because it is a critical determinant of the likelihood of extremely low returns. In this paper, I suggest using a model-free, risk-neutral measure of the volatility-of-volatility implied by the VIX options as a tail risk indicator and tests whether the indicator can predict returns on tail risk hedging options such as the SPX puts and the VIX calls. In particular, I hypothesize that an increase in the volatility-of-volatility raises the prices of the tail risk hedging options and thus lowers their subsequent returns.

Consistent with the hypothesis, the volatility-of-volatility as measured by the VVIX index has a statistically significant impact on expected returns on both SPX puts and VIX calls. A positive shock to tail risk increases the current prices of tail risk hedges and lowers their subsequent returns over the next three to four weeks. The economic impact is large. A one standard deviation increase in the volatility-of-volatility results in a 1.63% to 2.19% decrease in the next day's SPX put returns and a 0.68% to 0.87% decrease in the next day's VIX call returns. These results are robust to various control variables, including tail and jump measures, skewness and kurtosis measures, option liquidity measures, variance risk premiums, and limits of arbitrage.

In fact, the true source of the predictability of the VVIX index is unclear because the VVIX index compounds both information on volatility-of-volatility risk and its associated risk premium. By separating the VVIX index into a physical measure of volatility-of-volatility and a volatility-of-volatility risk premium, I find that they both significantly contribute to the forecasting power of the VVIX index, although the former is more statistically significant than the latter.

Finally, I provide two economic explanations for the empirical finding. The first explanation is that the volatility-of-volatility as measured by the VVX index contains information on the crash risk priced by the options markets. Thus, the predictability may be driven by time-variation in the priced crash risk factor. This risk-based interpretation is related to the time-varying rare disaster models of Gourio (2008), Gabaix (2008, 2012), and Wachter (2013). The second uncertainty-based explanation is that the volatility-of-volatility may be a good proxy for uncertainty over volatility. Thus, the predictability may be explained by uncertainty premiums for a time-varying uncertain belief in volatility, consistent with the equilibrium model of Drechsler (2013).

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Appendix A. Skewness and kurtosis in the affine framework

Under the affine framework as given in Equation (1), Das and Sundaram (1999) provide closed-form expressions of τ-horizon skewness and kurtosis as follows:

$$\begin{aligned} SKEW_t(\tau) &= \frac{3\sigma\rho e^{\frac{\kappa\tau}{2}}}{\sqrt{\kappa}} \times \frac{\left[\overline{h}\left(2-2e^{\kappa\tau}+\kappa\tau+\kappa\tau e^{\kappa\tau}\right)-h_t\left(1+\kappa\tau-e^{\kappa\tau}\right)\right]}{\left[\overline{h}\left(1-e^{\kappa\tau}+\kappa\tau e^{\kappa\tau}\right)+h_t\left(e^{\kappa\tau}-1\right)\right]^{3/2}} \\ KURT_t(\tau) &= 3\left(1+\sigma^2 \times \frac{\overline{h}A_1-h_tA_2}{B}\right), \end{aligned} \tag{A.1}$$

where:

$$\begin{aligned} A_1 &= \left(1+4e^{\kappa\tau}-5e^{2\kappa\tau}+4\kappa\tau e^{\kappa\tau}+2\kappa\tau e^{2\kappa\tau}\right)+4\rho^2\left(6e^{\kappa\tau}-6e^{2\kappa\tau}+4\kappa\tau e^{\kappa\tau}+2\kappa\tau e^{2\kappa\tau}+\kappa^2\tau^2e^{\kappa\tau}\right) \\ A_2 &= 2\left(1-e^{2\kappa\tau}+2\kappa\tau e^{\kappa\tau}\right)+4\rho^2\left(2e^{\kappa\tau}-2e^{2\kappa\tau}+2\kappa\tau e^{\kappa\tau}+\kappa^2\tau^2e^{\kappa\tau}\right) \\ B &= 2\kappa\left[\overline{h}(1-e^{\kappa\tau}+\kappa\tau e^{\kappa\tau})+h_t(e^{\kappa\tau}-1)\right]^2. \end{aligned}$$

Appendix B. Construction of the control variables

B.1. High-frequency jump variation

The regression analysis uses a monthly measure of jump variation. To compute this variable, I first obtain daily realized volatility by summing the squared intraday returns over each day:

$$RV_t^{(d)} \equiv \sum_{j=1}^{1/\Delta}\left(f^S_{t-1+j\Delta}-f^S_{t-1+(j-1)\Delta}\right)^2, \tag{B.1}$$

where $f_t^S = \log(F_t^S)$ is the log SPX future price, and Δ is the sampling interval for the intraday data.

Next, I compute the daily bipower variation by using the approach of Barndorff-Nielsen and Shephard (2004):

$$BV_t^{(d)} \equiv \frac{\pi}{2} \sum_{j=2}^{1/\Delta} \left| f^S_{t-1+j\Delta} - f^S_{t-1+(j-1)\Delta} \right| \left| f^S_{t-1+(j-1)\Delta} - f^S_{t-1+(j-2)\Delta} \right|. \tag{B.2}$$

The daily jump variation is then defined by subtracting the daily bipower variation from the daily realized volatility, with a floor of zero, $JV_t^{(d)} \equiv \max(RV_t^{(d)} - BV_t^{(d)}, 0)$. Finally, monthly jump variation is obtained by summing daily jump variations over the past 22 trading days: $JV_t = \sum_{i=1}^{22} JV_{t-i+1}^{(d)}$.

B.2. $TAIL$ index

Following the approach of Kelly (2011) and Chollete and Lu (2012), stock returns exceeding a threshold level u_t are assumed to have the following tail distribution:

$$P_t(-r_{i,t+1} > r| - r_{i,t+1} > u_t, \mathcal{F}_t) \sim \left(\frac{r}{u_t} \right)^{-a_i \varsigma_t}, \tag{B.3}$$

where $\mathcal{F}_t$ means the time-t information filtration. The tail exponent $-a_i\varsigma_t$ is associated with tail risk, with a higher value of $-a_i\varsigma_t$ indicating fatter tails.

Let $r_{(1)} \leq r_{(2)} \leq ... \leq r_{(N_t)}$ be the sample order statistics of the stock returns over the past one month, where N_t indicates the number of stock returns at time t. $TAIL$ can be estimated by applying the Hill (1975) estimator to the order statistics on each date. The Hill (1975) estimator takes the form of

$$\frac{1}{\varsigma_t^{\text{Hill}}} = \frac{1}{K_t} \sum_{k=1}^{K_t} \left(\log |r_{(k)}| - \log |r_{(K_t+1)}| \right), \tag{B.4}$$

where K_t is the observation that corresponds to the fifth percentile of the sample order statistics.

As a higher level of $-\varsigma_t^{\text{Hill}}$ is associated with a higher level of tail risk, Kelly (2011) introduces the following standardized index, which is increasing in tail risk, via a sign change:

$$TAIL_t = -\frac{\varsigma_t^{\text{Hill}} - \text{mean}(\varsigma_t^{\text{Hill}})}{\text{std}(\varsigma_t^{\text{Hill}})}, \tag{B.5}$$

where "mean" and "std" refer to the mean and the standard deviation, respectively.

The returns data are obtained from the CRSP daily stock files, with share codes of 10 and 11 and exchange codes of 1, 2, and 3.

B.3. Realized skewness and kurtosis

Similar to Amaya et al. (2013) and Guo et al. (2013), I compute a monthly measure of the realized skewness and kurtosis by using the high-frequency data. To do this, I first obtain daily measures of the realized skewness and kurtosis, denoted by $RSKEW_t^{(d)}$ and $RKURT_t^{(d)}$, respectively:

$$\begin{aligned} RSKEW_t^{(d)} &\equiv \frac{\sum_{j=1}^{1/\Delta}\left(f_{t-1+j\Delta}^S - f_{t-1+(j-1)\Delta}^S\right)^3}{\sqrt{\Delta}\left[RV_t^{(d)}\right]^{3/2}} \\ RKURT_t^{(d)} &\equiv \frac{\sum_{j=1}^{1/\Delta}\left(f_{t-1+j\Delta}^S - f_{t-1+(j-1)\Delta}^S\right)^4}{\Delta\left[RV_t^{(d)}\right]^{2}}, \end{aligned} \tag{B.6}$$

where $f_t^S = \log(F_t^S)$ is the log SPX future price, and Δ is the sampling interval for the intraday data.

Next, monthly measures are obtained by averaging daily ones over the past 22 trading days:

$$\begin{aligned} RSKEW_t &= \frac{1}{22}\sum_{i=1}^{22} RSKEW_{t-i+1}^{(d)} \\ RKURT_t &= \frac{1}{22}\sum_{i=1}^{22} RKURT_{t-i+1}^{(d)}. \end{aligned} \tag{B.7}$$

B.4. Amihud (2002) $ILLIQ$ measure

I extend the Amihud (2002) illiquidity measure to the options markets, similar to Cao and Wei (2010). On each day I take an equal average of all $ILLIQ$ measures that belong to the i^{th} moneyness bin:

$$ILLIQ_{i,t} = \frac{1}{N_{i,t}} \sum_{j=1}^{N_{i,t}} \frac{|R_t^O(k_j, \tau_j)|}{VOLUME(k_j, \tau_j) O(k_j, \tau_j)}, \tag{B.8}$$

where $VOLUME(k_j, \tau_j)$ is the trading volume, $N_{i,t}$ is the number of options, and $R_t^O(k_j, \tau_j)$ is the delta-neutral option returns as defined in Equation (7).

Table 1: **Comparison of market liquidity between the SPX and VIX options**

$VOLUME$ denotes the average trading volume, $SPREAD$ denotes the average relative bid-ask spread, and $ILLIQ$ denotes the average Amihud (2002) illiquidity measure. The moneyness is defined as $k = \frac{K}{F_t(\tau)}$, where K is the strike price and $F_t(\tau)$ is the future price.

Moneyness	VOLUME	SPREAD	ILLIQ
Panel A: SPX options			
deep OTM put ($0.85 < k < 0.90$)	1,925	0.23	0.294
med. OTM put ($0.90 < k < 0.95$)	2,277	0.18	0.192
slight OTM put ($0.95 < k < 1.00$)	3,329	0.11	0.028
slight OTM call ($1.00 < k < 1.03$)	2,784	0.10	0.021
med. OTM call ($1.03 < k < 1.06$)	1,976	0.18	0.189
deep OTM call ($1.06 < k < 1.09$)	1,597	0.23	0.394
Panel B: VIX options			
deep OTM put ($0.85 < k < 0.90$)	4,374	0.13	0.090
med. OTM put ($0.90 < k < 0.95$)	4,504	0.10	0.069
slight OTM put ($0.95 < k < 1.0$)	4,360	0.08	0.041
slight OTM call ($1.0 < k < 1.1$)	4,576	0.07	0.025
med. OTM call ($1.1 < k < 1.2$)	4,774	0.09	0.036
deep OTM call ($1.2 < k < 1.3$)	4,855	0.11	0.075

Table 2: **Summary statistics for delta-neutral option returns**

The table presents the summary statistics for delta-neutral option returns. Panel A corresponds to the SPX put returns, while Panel B corresponds to the VIX call returns. The sample period is January 1, 2007 through January 31, 2013 on a daily basis. CI stands for 99% confidence intervals for the bootstrapped means with 250,000 sample draws. The moneyness is defined as $k = \frac{K}{F_t(\tau)}$, where K is the strike price and $F_t(\tau)$ is the future price.

	Mean(%)	CI(%)	Min.	Max.	Std.	Skew.	Kurt.	AR(1)	Sharpe
Panel A: SPX puts									
deep OTM put ($0.85 < k < 0.90$)	-1.46	[-2.04, -0.85]	-54.68	256.26	15.11	4.52	51.16	-0.02	-0.10
med. OTM put ($0.90 < k < 0.95$)	-1.29	[-1.92, -0.59]	-50.89	533.50	16.88	10.75	267.19	-0.02	-0.08
slight OTM put ($0.95 < k < 1.00$)	-0.40	[-0.95, 0.26]	-48.95	573.32	15.29	16.53	521.30	-0.04	-0.03
Panel B: VIX calls									
slight OTM call ($1.0 < k < 1.1$)	-0.65	[-1.17, -0.08]	-43.52	144.70	8.07	5.72	90.97	-0.15	-0.08
med. OTM call ($1.1 < k < 1.2$)	-0.74	[-1.29, -0.16]	-43.66	100.91	8.54	2.86	29.75	-0.07	-0.09
deep OTM call ($1.2 < k < 1.3$)	-0.71	[-1.33, -0.06]	-68.26	82.56	9.68	1.15	18.13	-0.07	-0.07

Table 3: **Predictive regressions of tail risk hedging option returns.**

The table presents the two-day-ahead predictive regression results of the delta-neutral option returns onto the VVIX index and control variables. The sample period is January 1, 2007 through January 31, 2013 on a daily basis. $VVIX_t$ denotes the CBOE VVIX index implied by the VIX options; VIX_t denotes the CBOE VIX index; JV_t denotes a monthly measure of the Barndorff-Nielsen and Shephard (2004) jump variation; $TAIL_t$ denotes the Kelly (2011) tail index; $RSKEW_t$ denotes a monthly measure of the realized skewness; $SKEW_t$ denotes the CBOE SKEW index; $RKURT_t$ denotes a monthly measure of the realized kurtosis; $VOLUME_{i,t}$ denotes the average trading volume for the i^{th} moneyness bin; $SPREAD_{i,t}$ denotes the average relative bid-ask spread for the i^{th} moneyness bin; $ILLIQ_{i,t}$ denotes the average Amihud (2002) illiquidity measure for the i^{th} moneyness bin; VRP_t denotes the variance risk premium as defined by Bollerslev et al. (2009); $MRGN_t$ denotes the SPX futures margins divided by the futures prices; and TED_t denotes the TED spread. Newey and West (1987) robust t-statistics with an optimal lag are shown in parentheses. *, **, and *** indicate statistical significance at 90%, 95%, and 99% confidence levels, respectively.

	SPX puts			VIX calls		
	deep	medium	slight	slight	medium	deep
Regression 1: Baseline regression						
constant	5.78***	8.20***	7.40***	2.04*	2.70	3.28*
	(2.65)	(2.98)	(2.90)	(1.74)	(1.54)	(1.95)
$VVIX_{t-2}$	-1.63***	-2.19***	-1.89***	-0.68***	-0.82***	-0.87***
	(-4.80)	(-4.34)	(-3.85)	(-3.57)	(-3.01)	(-3.36)
Adj. R^2	0.62	0.62	0.61	0.30	0.26	0.24
No. obs.	1520	1512	1514	1340	1383	1416
Regression 2: Controlling for tail risk measures						
constant	7.15***	11.55***	10.66***	2.61	3.93*	3.55*
	(2.86)	(3.24)	(2.98)	(1.58)	(1.78)	(1.76)
$VVIX_{t-2}$	-1.44***	-2.30***	-2.23***	-0.76***	-0.89***	-0.89***
	(-4.99)	(-3.96)	(-3.67)	(-3.36)	(-2.87)	(-3.20)
VIX_{t-2}	-1.06	-1.26	-0.52	-0.01	-0.39	0.05
	(-1.12)	(-1.45)	(-0.70)	(-0.02)	(-0.69)	(0.07)
JV_{t-2}	-0.79	-0.02	-0.14	0.03	0.15	-0.42
	(-0.95)	(-0.03)	(-0.17)	(0.08)	(0.22)	(-0.55)
$TAIL_{t-2}$	-0.40	-1.05**	-1.20**	-0.20	-0.37	-0.19
	(-1.02)	(-2.16)	(-2.45)	(-0.72)	(-1.18)	(-0.63)
Adj. R^2	0.62	0.62	0.61	0.30	0.26	0.24
No. obs.	1520	1512	1514	1340	1383	1416
Regression 3: Controlling for skewness and kurtosis						
constant	7.10	12.41	15.40**	5.51	5.07	0.12
	(1.21)	(1.53)	(2.04)	(1.55)	(1.24)	(0.03)
$VVIX_{t-2}$	-1.66***	-2.11***	-1.79***	-0.61***	-0.78***	-0.92***
	(-4.86)	(-4.71)	(-4.28)	(-3.26)	(-2.85)	(-3.32)
$RSKEW_{t-2}$	-0.12	0.05	-0.02	0.15	0.09	-0.03
	(-0.55)	(0.19)	(-0.09)	(1.01)	(0.52)	(-0.15)
$SKEW_{t-2}$	-0.02	-0.25	-0.40	-0.20	-0.12	0.18
	(-0.08)	(-0.68)	(-1.12)	(-1.08)	(-0.61)	(0.88)
$RKURT_{t-2}$	-0.10	0.06	-0.06	0.02	-0.03	-0.04
	(-0.33)	(0.18)	(-0.20)	(0.11)	(-0.16)	(-0.17)
Adj. R^2	0.62	0.62	0.61	0.30	0.26	0.23
No. obs.	1520	1512	1514	1340	1383	1416
Regression 4: Controlling for liquidity measures						
constant	-6.08	-0.95	-7.49	8.15***	9.85***	8.32***
	(-1.24)	(-0.12)	(-1.50)	(3.85)	(3.04)	(2.63)
$VVIX_{t-2}$	-1.64***	-2.21***	-1.84***	-0.91***	-1.25***	-1.43***
	(-4.76)	(-4.17)	(-3.73)	(-4.48)	(-3.76)	(-4.41)
$VOLUME_{i,t-2}$	3.75***	2.93***	2.02**	0.14	-0.28	-0.81**

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	SPX puts			VIX calls		
	deep	medium	slight	slight	medium	deep
	(4.93)	(3.34)	(2.27)	(0.55)	(-0.70)	(-2.03)
$SPREAD_{i,t-2}$	11.30***	10.50***	7.78***	4.63***	4.67***	5.02***
	(13.80)	(9.29)	(6.62)	(4.82)	(9.27)	(11.83)
$ILLIQ_{i,t-2}$	-1.59	5.32	6.71*	-0.44***	-0.30*	0.20
	(-1.13)	(1.33)	(1.70)	(-2.62)	(-1.69)	(0.98)
Adj. R^2	0.62	0.62	0.61	0.31	0.27	0.25
No. obs.	1520	1512	1514	1340	1383	1416
Regression 5: Controlling for variance risk premiums						
constant	5.02**	7.82***	7.51***	2.13*	2.61	3.00*
	(2.38)	(2.86)	(2.86)	(1.78)	(1.49)	(1.79)
$VVIX_{t-2}$	-1.39***	-2.07***	-1.93***	-0.71***	-0.79***	-0.79***
	(-4.30)	(-4.06)	(-3.70)	(-3.54)	(-2.87)	(-2.95)
VRP_{t-2}	-0.91**	-0.47	0.14	0.15	-0.11	-0.26
	(-2.07)	(-1.13)	(0.46)	(0.71)	(-0.48)	(-0.79)
Adj. R^2	0.62	0.62	0.61	0.30	0.26	0.24
No. obs.	1520	1512	1514	1340	1383	1416
Regression 6: Controlling for funding constraints						
constant	9.77***	10.24***	8.00***	1.52	2.84	3.13*
	(3.50)	(3.37)	(3.25)	(1.14)	(1.30)	(1.68)
$VVIX_{t-2}$	-1.67***	-2.22***	-1.90***	-0.67***	-0.81***	-0.84***
	(-5.10)	(-4.45)	(-3.93)	(-3.54)	(-3.03)	(-3.27)
$MRGN_{t-2}$	-1.06***	-0.60	-0.24	0.15	-0.03	0.16
	(-2.69)	(-1.44)	(-0.69)	(0.91)	(-0.12)	(0.72)
TED_{t-2}	-0.61*	-0.09	0.17	0.05	-0.06	-0.53
	(-1.71)	(-0.24)	(0.46)	(0.31)	(-0.27)	(-1.50)
Adj. R^2	0.62	0.62	0.61	0.30	0.26	0.24
No. obs.	1520	1512	1514	1340	1383	1416

Table 4: **Summary statistics and correlation matrix for explanatory variables**

The sample period is January 1, 2007 through January 31, 2013 on a daily basis. $VVIX_t$ denotes the CBOE VVIX index implied by the VIX options; VIX_t denotes the CBOE VIX index; $RVVIX_t$ denotes the realized volatility of the VIX futures returns; $VVRP_t$ denotes a volatility-of-volatility risk premium, which is defined as the difference between the squared $VVIX$ and $RVVIX$; JV_t denotes a monthly measure of the Barndorff-Nielsen and Shephard (2004) jump variation; $TAIL_t$ denotes the Kelly (2011) tail index; $RSKEW_t$ denotes a monthly measure of the realized skewness; $SKEW_t$ denotes the CBOE SKEW index; $RKURT_t$ denotes a monthly measure of the realized kurtosis; VRP_t denotes the variance risk premium as defined by Bollerslev et al. (2009); $MRGN_t$ denotes the SPX futures margins divided by the futures prices; and TED_t denotes the TED spread. ADF stands for the augmented Dickey-Fuller unit root test. *, **, and *** indicate statistical significance at 90%, 95%, and 99% confidence levels, respectively.

	$VVIX$	VIX	$RVVIX$	$VVRP$	JV	$TAIL$	$RSKEW$	$SKEW$	$RKURT$	VRP	$MRGN$	TED
Panel A: Summary statistics												
Mean	0.87	0.24	0.37	0.41	0.00	0.00	0.03	119.22	4.30	0.04	0.07	0.64
Median	0.86	0.21	0.31	0.41	0.00	0.09	0.03	119.32	4.23	0.03	0.06	0.38
Min.	0.60	0.10	0.11	-0.40	0.00	-3.45	-0.41	0.00	3.22	-0.13	0.04	0.09
Max.	1.45	0.81	1.36	1.32	0.03	1.89	0.65	139.25	6.25	0.37	0.15	4.58
Std.	0.13	0.11	0.22	0.19	0.00	1.00	0.14	5.82	0.62	0.04	0.02	0.64
Skew.	0.77	1.97	2.17	-0.12	4.06	-0.95	0.64	-5.44	0.75	3.33	0.80	2.26
Kurt.	3.87	7.79	8.31	4.31	22.13	4.09	4.98	117.05	3.20	20.81	3.29	9.73
AR(1)	0.94	0.98	0.99	0.90	0.99	0.98	0.93	0.65	0.97	0.83	1.00	0.99
ADF t-stat	−8.37***	−5.47***	0.08	−11.71***	−3.18**	−3.59***	−7.38***	−30.91***	−5.05***	−15.30***	−1.70	−1.34
Panel B: Correlation matrix												
$VVIX$	1.00***	0.34***	0.63***	0.52***	0.22***	-0.44***	-0.16***	0.23***	-0.14***	0.33***	0.01	0.11***
VIX		1.00***	0.61***	-0.24***	0.84***	-0.68***	-0.07***	-0.23***	-0.31***	0.82***	0.70***	0.57***
$RVVIX$			1.00***	-0.33***	0.58***	-0.66***	-0.11***	-0.03	-0.32***	0.34***	0.14***	0.48***
$VVRP$				1.00***	-0.35***	0.19***	-0.07***	0.31***	0.17***	0.05*	-0.13***	-0.39***
JV					1.00***	-0.47***	0.04*	-0.18***	-0.17***	0.55***	0.54***	0.68***
$TAIL$						1.00***	0.23***	0.13***	0.35***	-0.49***	-0.37***	-0.29***
$RSKEW$							1.00***	0.12***	0.35***	-0.08***	-0.03	-0.03
$SKEW$								1.00***	0.14***	-0.14***	-0.05*	-0.39***
$RKURT$									1.00***	-0.16***	-0.08***	-0.29***
VRP										1.00***	0.59***	0.36***
$MRGN$											1.00***	0.05**
TED												1.00***

Table 5: **Persistence of predictability**

The table presents the multi-week-ahead predictive regressions of the delta-neutral option returns. $VVIX_t$ denotes the CBOE VVIX index implied by the VIX options; $VOLUME_{i,t}$ denotes the average trading volume for the i^{th} moneyness bin; $SPREAD_{i,t}$ denotes the average relative bid-ask spread for the i^{th} moneyness bin; and $ILLIQ_{i,t}$ denotes the average Amihud (2002) illiquidity measure for the i^{th} moneyness bin. Newey and West (1987) robust t-statistics with an optimal lag are shown in parentheses. *, **, and *** indicate statistical significance at 90%, 95%, and 99% confidence levels, respectively.

	SPX puts			VIX calls		
	deep	medium	slight	slight	medium	deep
Regression 1: One-week ahead						
constant	-7.77*	-4.03	-10.70**	5.17***	6.80**	4.66*
	(-1.65)	(-0.57)	(-2.16)	(2.79)	(2.47)	(1.72)
$VVIX_{t-5}$	-1.55***	-1.82***	-1.38***	-0.61***	-0.94***	-1.03***
	(-4.30)	(-4.12)	(-3.38)	(-3.75)	(-3.23)	(-3.77)
$VOLUME_{i,t-5}$	3.86***	3.10***	2.24**	0.21	-0.21	-0.72*
	(5.10)	(3.45)	(2.43)	(0.81)	(-0.52)	(-1.84)
$SPREAD_{i,t-5}$	11.48***	10.77***	8.05***	4.53***	4.55***	4.88***
	(14.08)	(9.21)	(6.62)	(4.76)	(9.26)	(12.38)
$ILLIQ_{i,t-5}$	-1.55	5.33	6.75*	-0.40**	-0.25	0.25
	(-1.09)	(1.31)	(1.69)	(-2.39)	(-1.46)	(1.24)
Adj. R^2	0.62	0.62	0.61	0.30	0.27	0.24
No. obs.	1519	1511	1513	1339	1382	1415
Regression 2: Two-week ahead						
constant	-9.38*	-7.58	-15.07***	5.29***	6.79***	3.60
	(-1.76)	(-1.17)	(-3.08)	(2.95)	(2.60)	(1.32)
$VVIX_{t-10}$	-1.18***	-1.17***	-0.67**	-0.63***	-0.95***	-0.91***
	(-3.43)	(-3.14)	(-2.32)	(-3.58)	(-3.36)	(-3.43)
$VOLUME_{i,t-10}$	3.96***	3.21***	2.36**	0.23	-0.18	-0.68*
	(5.24)	(3.51)	(2.49)	(0.91)	(-0.46)	(-1.74)
$SPREAD_{i,t-10}$	11.64***	10.98***	8.23***	4.49***	4.50***	4.85***
	(14.48)	(9.20)	(6.59)	(4.75)	(9.25)	(12.73)
$ILLIQ_{i,t-10}$	-1.57	5.31	6.75*	-0.40**	-0.25	0.24
	(-1.09)	(1.29)	(1.67)	(-2.35)	(-1.45)	(1.17)
Adj. R^2	0.62	0.62	0.61	0.30	0.27	0.24
No. obs.	1514	1506	1508	1334	1377	1411
Regression 3: Three-week ahead						
constant	-8.87*	-6.85	-15.85***	4.47**	5.23**	1.03
	(-1.73)	(-1.05)	(-3.42)	(2.55)	(2.27)	(0.38)
$VVIX_{t-15}$	-1.30***	-1.26***	-0.59**	-0.56***	-0.80***	-0.61**
	(-3.45)	(-3.17)	(-2.24)	(-3.59)	(-3.01)	(-2.24)
$VOLUME_{i,t-15}$	3.96***	3.22***	2.38**	0.24	-0.17	-0.66*
	(5.25)	(3.54)	(2.50)	(0.95)	(-0.42)	(-1.67)
$SPREAD_{i,t-15}$	11.65***	10.98***	8.26***	4.47***	4.46***	4.78***
	(14.44)	(9.20)	(6.60)	(4.73)	(9.08)	(12.76)
$ILLIQ_{i,t-15}$	-1.47	5.40	6.80*	-0.40**	-0.23	0.28
	(-1.03)	(1.31)	(1.68)	(-2.35)	(-1.30)	(1.33)
Adj. R^2	0.62	0.62	0.61	0.30	0.26	0.23
No. obs.	1512	1504	1506	1332	1375	1409
Regression 4: Four-week ahead						
constant	-9.53*	-8.29	-16.13***	3.41**	3.54	0.08
	(-1.83)	(-1.28)	(-3.38)	(2.03)	(1.56)	(0.03)
$VVIX_{t-20}$	-1.13***	-1.03***	-0.53**	-0.43***	-0.60**	-0.47*
	(-3.67)	(-3.06)	(-2.32)	(-2.67)	(-2.27)	(-1.78)
$VOLUME_{i,t-20}$	4.01***	3.27***	2.41**	0.25	-0.15	-0.65*
	(5.30)	(3.58)	(2.52)	(1.01)	(-0.39)	(-1.66)
$SPREAD_{i,t-20}$	11.73***	11.07***	8.30***	4.45***	4.44***	4.79***
	(14.63)	(9.25)	(6.61)	(4.72)	(9.06)	(12.85)
$ILLIQ_{i,t-20}$	-1.47	5.40	6.80*	-0.38**	-0.20	0.26
	(-1.00)	(1.30)	(1.68)	(-2.23)	(-1.14)	(1.22)
Adj. R^2	0.62	0.62	0.61	0.30	0.26	0.23
No. obs.	1509	1501	1503	1329	1372	1406

Table 6: **Volatility of volatility risk versus its associated risk premium**

The table presents the predictive regression results of the delta-neutral option returns onto $RVVIX$ and $VVRP$. The sample period is January 1, 2007 through January 31, 2013. $RVVIX_t$ denotes the realized volatility of the VIX futures returns; $VVRP_t$ denotes a volatility-of-volatility risk premium, which is defined as the difference between the squared $VVIX$ and $RVVIX$; VIX_t denotes the CBOE VIX index; JV_t denotes a monthly measure of the Barndorff-Nielsen and Shephard (2004) jump variation; $TAIL_t$ denotes the Kelly (2011) tail index; $RSKEW_t$ denotes a monthly measure of the realized skewness; $SKEW_t$ denotes the CBOE SKEW index; $RKURT_t$ denotes a monthly measure of the realized kurtosis; VRP_t denotes the variance risk premium as defined by Bollerslev et al. (2009); $VOLUME_{i,t}$ denotes the average trading volume for the i^{th} moneyness bin; $SPREAD_{i,t}$ denotes the average relative bid-ask spread for the i^{th} moneyness bin; $ILLIQ_{i,t}$ denotes the average Amihud (2002) illiquidity measure for the i^{th} moneyness bin; $MRGN_t$ denotes the SPX futures margins divided by the futures prices; and TED_t denotes the TED spread. Newey and West (1987) robust t-statistics with an optimal lag are shown in parentheses. *, **, and *** indicate statistical significance at 90%, 95%, and 99% confidence levels, respectively.

	SPX puts			VIX calls		
	deep	medium	slight	slight	medium	deep
constant	1.48	1.70	-5.73	9.68*	9.88*	1.55
	(0.15)	(0.15)	(-0.66)	(1.70)	(1.71)	(0.33)
$RVVIX_{t-2}$	-1.73***	-2.40***	-2.26***	-0.93***	-1.69***	-1.43***
	(-3.96)	(-3.75)	(-3.61)	(-3.23)	(-4.08)	(-3.77)
$VVRP_{t-2}$	-0.76**	-1.46***	-1.65***	-0.55**	-0.69**	-0.80**
	(-2.50)	(-2.61)	(-3.07)	(-2.12)	(-2.01)	(-2.15)
VIX_{t-2}	-1.14	-0.83	-0.18	-0.93	-0.08	0.66
	(-0.56)	(-0.38)	(-0.11)	(-0.87)	(-0.07)	(0.48)
JV_{t-2}	-0.40	-0.08	-0.10	0.41	0.48	-0.23
	(-0.33)	(-0.07)	(-0.12)	(0.88)	(0.68)	(-0.33)
$TAIL_{t-2}$	-0.43	-1.09*	-1.21**	-0.48	-0.58	-0.11
	(-0.83)	(-1.67)	(-2.07)	(-1.43)	(-1.63)	(-0.34)
$RSKEW_{t-2}$	0.05	0.23	0.08	0.20	0.19	0.03
	(0.19)	(0.85)	(0.35)	(1.15)	(1.13)	(0.16)
$SKEW_{t-2}$	-0.36	-0.38	-0.25	-0.23	-0.12	0.03
	(-1.12)	(-1.03)	(-0.79)	(-0.97)	(-0.52)	(0.18)
$RKURT_{t-2}$	-0.47	0.04	0.12	-0.04	-0.15	-0.16
	(-1.32)	(0.08)	(0.24)	(-0.19)	(-0.78)	(-0.62)
VRP_{t-2}	0.04	-0.15	0.26	0.25	-0.29	-0.71
	(0.05)	(-0.19)	(0.45)	(0.44)	(-0.48)	(-0.83)
$MRGN_{t-2}$	-0.22	-0.19	-0.47	0.20	-0.35	0.17
	(-0.33)	(-0.27)	(-0.66)	(0.47)	(-0.58)	(0.33)
TED_{t-2}	-0.13	0.19	0.11	0.22	0.13	-0.08
	(-0.17)	(0.27)	(0.19)	(0.82)	(0.31)	(-0.13)
$VOLUME_{i,t-2}$	3.64***	2.90***	2.02**	0.13	-0.29	-0.80**
	(4.61)	(3.20)	(2.22)	(0.52)	(-0.72)	(-1.98)
$SPREAD_{i,t-2}$	11.10***	10.39***	7.81***	4.63***	4.66***	4.99***
	(13.23)	(8.66)	(6.41)	(4.78)	(9.25)	(11.74)
$ILLIQ_{i,t-2}$	-0.36	5.98	6.81*	-0.39**	-0.30*	0.21
	(-0.21)	(1.51)	(1.83)	(-2.37)	(-1.72)	(1.02)
Adj. R^2	0.63	0.62	0.61	0.31	0.27	0.24
No. obs.	1520	1512	1514	1340	1383	1416

Table 7: **Robustness to liquidity filters**

The table presents the predictive regression results for delta-neutral option returns with a stricter liquidity filter. For now, we require that the trading volumes be greater than 20 and the relative bid-ask spreads be less than 0.25. The sample period is January 1, 2007 through January 31, 2013. $VVIX_t$ denotes the CBOE VVIX index implied by the VIX options; $VOLUME_{i,t}$ denotes the average trading volume for the i^{th} moneyness bin; $SPREAD_{i,t}$ denotes the average relative bid-ask spread for the i^{th} moneyness bin; and $ILLIQ_{i,t}$ denotes the average Amihud (2002) illiquidity measure for the i^{th} moneyness bin. *, **, and *** indicate statistical significance at 90%, 95%, and 99% confidence levels, respectively. Newey and West (1987) robust t-statistics with an optimal lag are shown in parentheses.

	SPX puts			VIX calls		
	deep	medium	slight	slight	medium	deep
constant	-1.86	7.79	-6.28	9.40***	10.37***	7.61**
	(-0.51)	(0.68)	(-0.93)	(4.18)	(3.08)	(2.35)
$VVIX_{t-2}$	-1.12***	-3.26***	-1.70***	-0.98***	-1.31***	-1.42***
	(-4.40)	(-2.83)	(-4.10)	(-4.72)	(-3.76)	(-4.22)
$VOLUME_{i,t-2}$	3.18***	3.74**	1.78**	0.13	-0.27	-0.82**
	(5.25)	(2.54)	(2.42)	(0.49)	(-0.68)	(-2.14)
$SPREAD_{i,t-2}$	10.56***	12.16***	7.63***	4.71***	4.77***	4.39***
	(13.34)	(5.05)	(7.37)	(4.91)	(9.35)	(11.07)
$ILLIQ_{i,t-2}$	-0.49	17.72*	5.22	-0.49***	-0.29	0.24
	(-0.36)	(1.77)	(1.56)	(-2.62)	(-1.48)	(1.15)
Adj. R^2	0.63	0.61	0.62	0.31	0.27	0.20
No. obs.	1498	1510	1513	1335	1366	1398

Table 8: **Regressions of the VIX returns**

The table presents the predictive regression results of VIX returns across varying horizons. The sample period is January 1, 2007 through January 31, 2013. $VVIX_t$ denotes the CBOE VVIX index implied by the VIX options; VRP_t denotes the variance risk premium as defined by Bollerslev et al. (2009); P/E_t denotes the price-earning ratio; P/D_t denotes the price-dividend ratio; $DFSP_t$ denotes the default spread, which is the difference between Moody's BAA and AAA corporate yields; and $TMSP_t$ denotes the term spread, which is the difference between Treasury 10-year and 3-month yields. The robust-t statistics are computed based on the Newey and West (1987) method for one-month horizons and the Hodrick (1992) method for longer-than-one-month horizons to control for overlapping data. *, **, and *** indicate statistical significance at 90%, 95%, and 99% confidence levels, respectively.

const	$VVIX$	VRP	$\log(P/E)$	$\log(P/D)$	$DFSP$	$TMSP$	Adj. R^2
Regression 1: One-month horizon							
1.87	-0.06						-1.75
(0.13)	(-0.03)						
6.10**		-4.61*					5.36
(1.92)		(-1.69)					
-185.15	0.45	-3.70	4.01	8.30	2.23	-0.03	0.71
(-0.87)	(0.18)	(-1.40)	(1.15)	(0.88)	(0.31)	(-0.01)	
Regression 2: Three-month horizon							
17.54	-2.29						2.89
(1.23)	(-1.14)						
5.31*		-3.95**					12.90
(1.91)		(-2.65)					
17.32	-2.54	-3.15**	-0.63	0.64	-1.57	-0.56	17.49
(0.08)	(-1.19)	(-2.24)	(-0.15)	(0.07)	(-0.25)	(-0.26)	
Regression 3: Six-month horizon							
19.17	-2.54						10.61
(1.64)	(-1.60)						
4.05		-2.77**					12.42
(1.45)		(-2.45)					
-95.15	-3.09*	-1.58	0.48	5.76	2.24	-0.14	44.57
(-0.59)	(-1.72)	(-1.45)	(0.13)	(0.84)	(0.46)	(-0.07)	

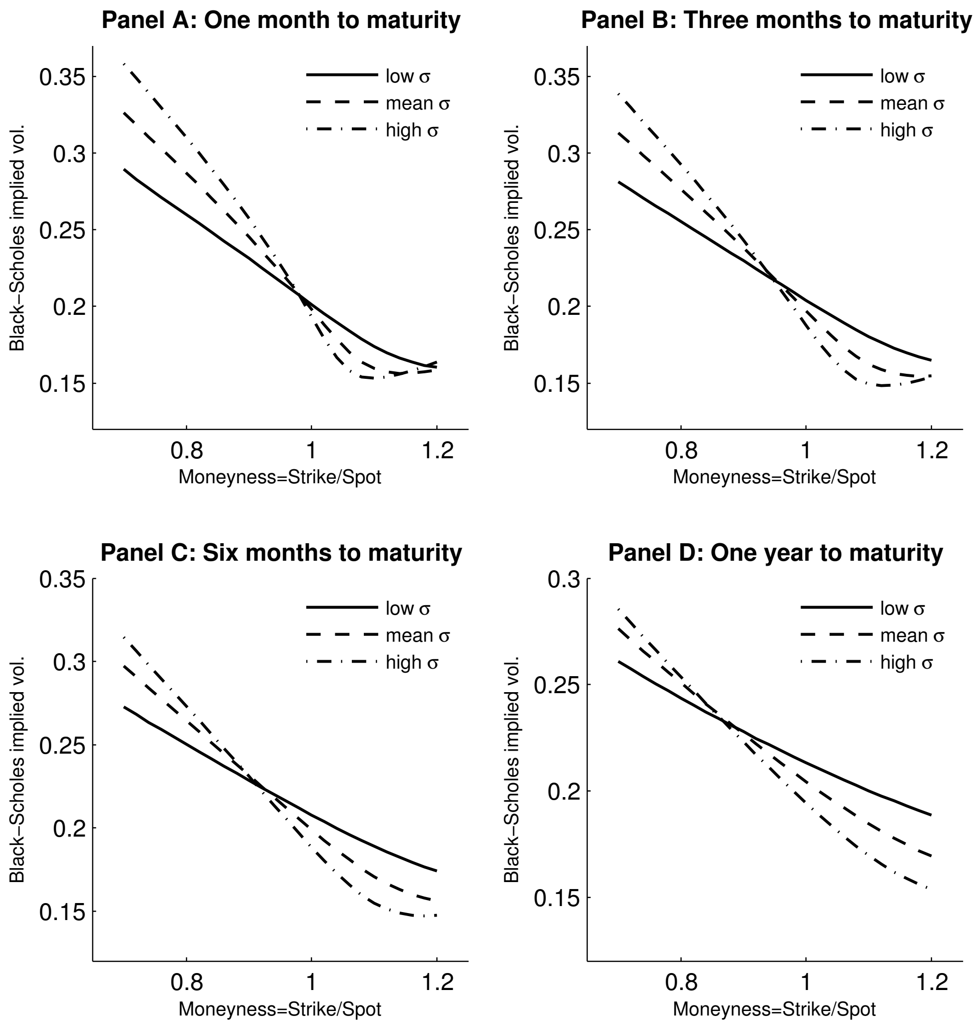


Figure 1: The impact of the volatility of volatility on volatility smirks.
This figure shows the shapes of volatility smirks with different levels of volatility of volatility under the affine stochastic volatility model. Four panels correspond to one, three, six, and twelve months to maturity, respectively. The x-axis represents moneyness defined as strike prices divided by spot prices, whereas the y-axis shows the Black-Scholes implied volatility. The Black-Scholes implied volatility is a decreasing function of moneyness with a hook at the right end, which is called the volatility smirk. The parameters are obtained by taking the averages of those reported by Christoffersen et al. (2009) except for the volatility of volatility. That is, the persistence, the leverage effect, and the long-run volatility are set at 2.5971, -0.6850, and 0.0531, respectively. The solid, dashed, and dashed-dotted lines of each panel correspond to the lowest (0.3796), mean (0.6151), and highest (0.8516) values of their estimates of the volatility of volatility, respectively. The initial volatility state and the interest rate are set at the long-run volatility and 2%, respectively.

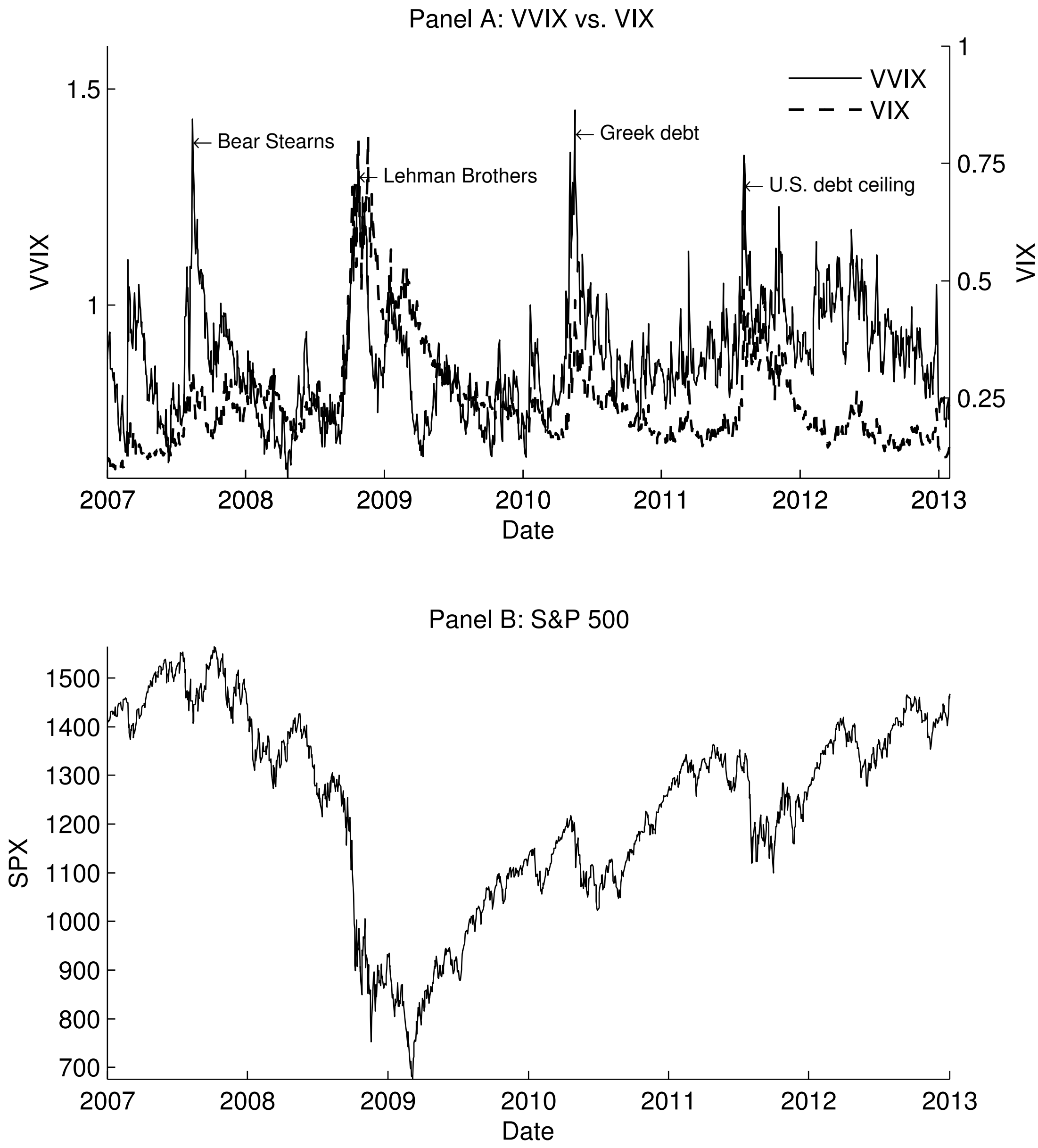
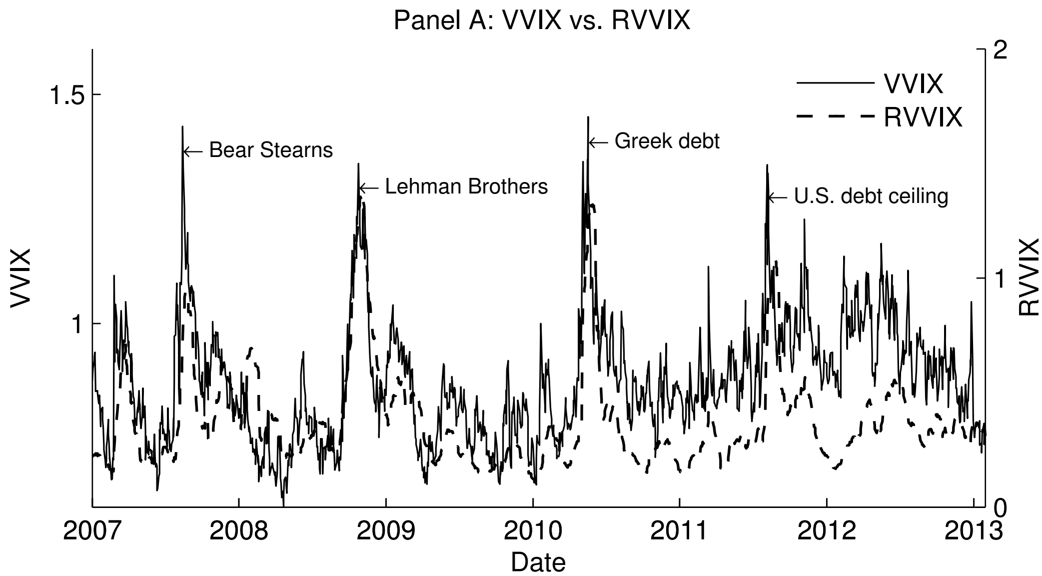


Figure 2: The VVIX index versus the VIX index (top) and the S&P 500 index (bottom). The upper panel shows the VVIX index in the y-axis and the VIX index in the z-axis, while the lower panel shows the time series plot of the S&P 500 index.



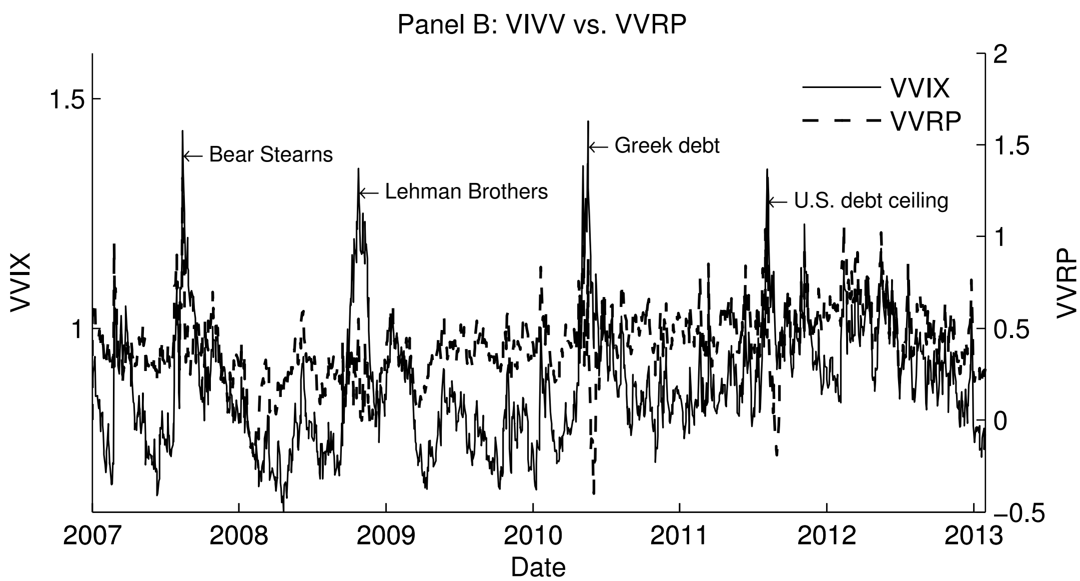
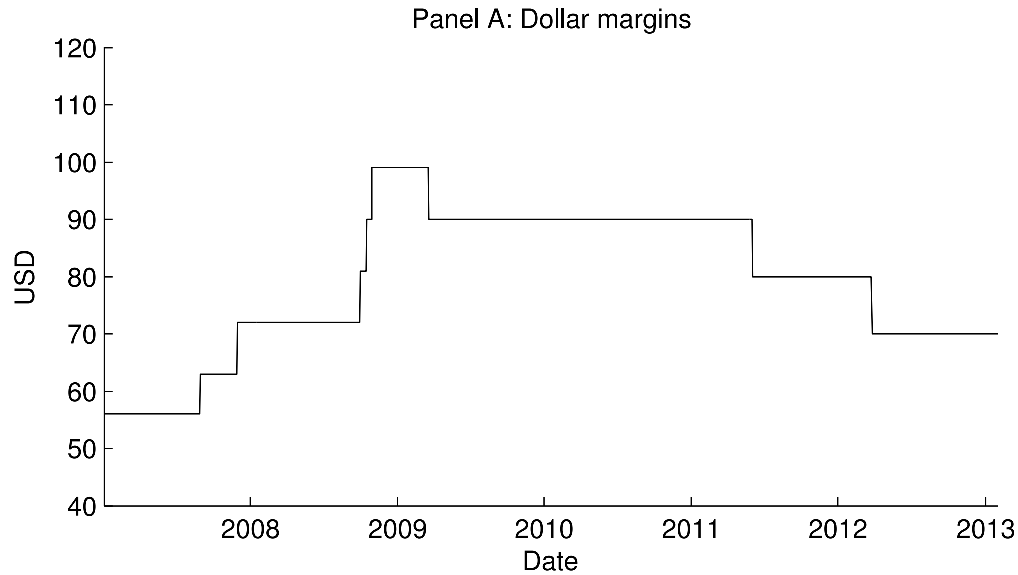


Figure 3: The VVIX index versus the realized volatility of the VIX futures returns (top) and versus the volatility of volatility risk premium (bottom).



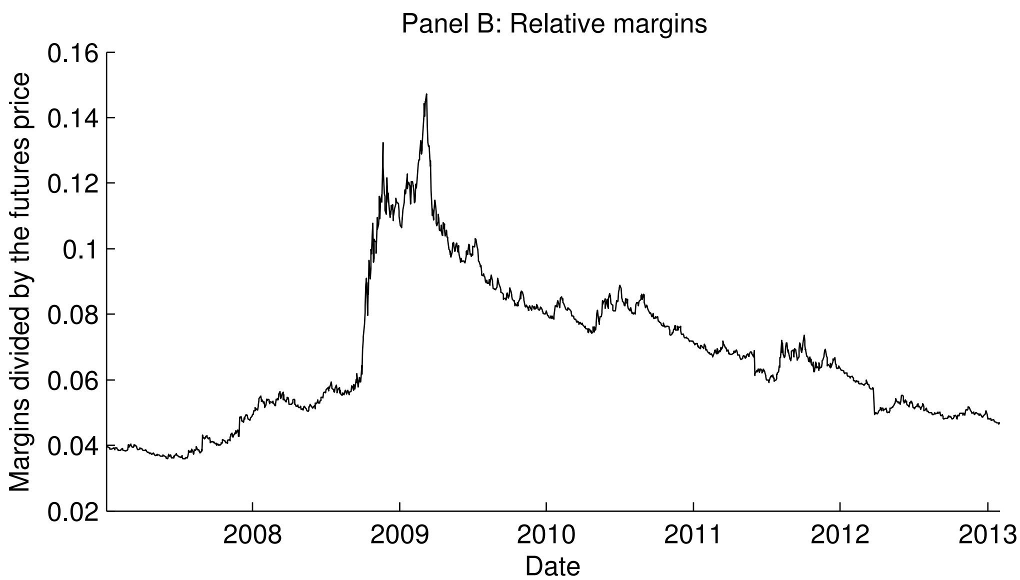


Figure 4: S&P 500 futures dollar margins (top) and relative margins (bottom).