Ohaaki Geothermal LPM

Model Formulation

The model will be composed of two ODEs:

The first pertains to the balance of mass within the control volume that represents the reservoir. Fradkin et al. (1981) defined a lumped parameter model for a reservoir that we will use to model changes in pressure within the Ohaaki geothermal field. The model is:

$$\frac{dP}{dt} = -aq - b(P - P_0) - c\frac{dq}{dt} \tag{1}$$

In this model, P represents the pressure in the reservoir, and P_0 is the pressure outside/surrounding the reservoir. q refers to the net mass flux in the reservoir, that is:

$$q = q_{out} - q_{in} \tag{2}$$

Where q_{out} is the mass flux out of the reservoir, and q_{in} is the mass flux into the reservoir. In this model, q_{out} will be Ohaaki geothermal field production, and q_{in} will be CO_2 injection (we have defined the model so that a net outflow of mass will yield a positive q value). Finally, a, b and c represent unknown lumped parameters dependent on the reservoir.

The second ODE pertains to the amount of CO_2 in the control volume/reservoir. For this, we will use the Conservation of Chemical Species (CO_2) in conjunction with Fick's Law. We will also assume that any CO_2 injected into the reservoir mixes perfectly with the hot water inside.

When CO_2 is injected, it is compressed, liquefied and injected into the reservoir at 100% concentration/weight percent. When hot water/steam is produced from the reservoir, it contains some percentage of CO_2, that can be modelled as an outflow of CO_2 from the system. Therefore, we can model the net change in CO_2 mass concentration with respect to time as follows:

$$\frac{d\varphi}{dt}V = q_{net} = y_c q_{out} - \ q_{in} \tag{3}$$

Where y_c is the weight percent of CO_2 in the hot water removed from the reservoir, φ is the mass concentration of CO_2 and V is total volume, which we assume to be a constant. Following the same pattern, the recharge flux from (1): $-b(P - P_0)$ will also contain dissolved CO_2. Therefore, the next term in the differential equation will be:

$$\frac{d\varphi}{dt}V_{recharge} = -by_c(P - P_0) \tag{4}$$

Furthermore, the slow drainage will also affect the CO_2 balance within the reservoir. Hence, we use q_{net} as defined in (3):

$$\frac{d\varphi}{dt}V = -aq_{net} - by_c(P - P_0) - c\frac{dq_{net}}{dt} \tag{5}$$

Where m_c is the mass of CO_2 in the reservoir. When CO_2 is added or removed from the system, this creates a concentration gradient with regards to CO_2, which will lead to the inflow/outflow of CO_2, as described by Fick's Law. This gives the fourth term of the ODE:

$$\frac{d\varphi}{dt} = D\frac{\partial^2\varphi}{\partial x^2} \tag{6}$$

Where D is diffusivity of CO_2 through the medium it travels through. x represents position, meaning that the derivative is with respect to distance/length. We have assumed the flow to be one dimensional as part of the reduced model. We have also assumed the mixture to be ideal, for simplicity. The final term of the model comes from the consumption of CO_2 as it reacts with the surrounding rock to produce carbonates. The reaction rate (and therefore rate of consumption of CO_2) is proportional to the over-supply of CO_2 in the reservoir, relative to the surroundings.

$$\frac{d\varphi}{dt} = -k(\varphi - \varphi_0) \tag{7}$$

Here, k represents an unknown constant of proportionality, and φ_0 is the concentration of CO_2 in the surrounding rock. Finally, we can put (5), (6) and (7) together to generate our final equation:

$$\frac{d\varphi}{dt}V = -aq_{net} - by_c(P - P_0) - c\frac{dq_{net}}{dt} + DV\frac{\partial^2\varphi}{\partial x^2} - kV(\varphi - \varphi_0) \tag{8}$$

We can divide through by V to lump it in with unknowns a, b and c, however, this will change their values. Therefore, we will arbitrarily assign three new lumped parameters f, g and h, to take their place:

$$\frac{d\varphi}{dt} = -fq_{net} - gy_c(P - P_0) - h\frac{dq_{net}}{dt} + D\frac{\partial^2\varphi}{\partial x^2} - k(\varphi - \varphi_0) \tag{9}$$