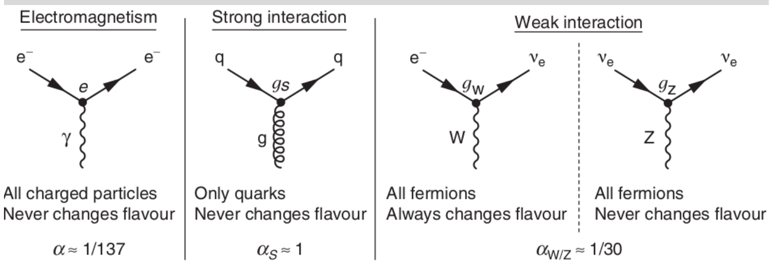
Chapter 1: Introduction

	Leptons				Quarks			
	Particle		Q	mass/GeV	Particle		Q	mass/GeV
First generation	electron	(e^-)	-1	0.0005	down	(d)	$-1/3$	0.003
	neutrino	(ν_e)	0	$< 10^{-9}$	up	(u)	$+2/3$	0.005
Second generation	muon	(μ^-)	-1	0.106	strange	(s)	$-1/3$	0.1
	neutrino	(ν_μ)	0	$< 10^{-9}$	charm	(c)	$+2/3$	1.3
Third generation	tau	(τ^-)	-1	1.78	bottom	(b)	$-1/3$	4.5
	neutrino	(ν_τ)	0	$< 10^{-9}$	top	(t)	$+2/3$	174

					strong	electromagnetic	weak
Quarks	down-type	d	s	b	✓	✓	✓
	up-type	u	c	t			
Leptons	charged	e^-	μ^-	τ^-		✓	✓
	neutrinos	ν_e	ν_μ	ν_τ			✓

Force	Strength	Boson		Spin	Mass/GeV
Strong	1	Gluon	g	1	0
Electromagnetism	10^{-3}	Photon	γ	1	0
Weak	10^{-8}	W boson	$W^\pm$	1	80.4
		Z boson	Z	1	91.2
Gravity	10^{-37}	Graviton?	G	2	0



Bethe-Bloch:

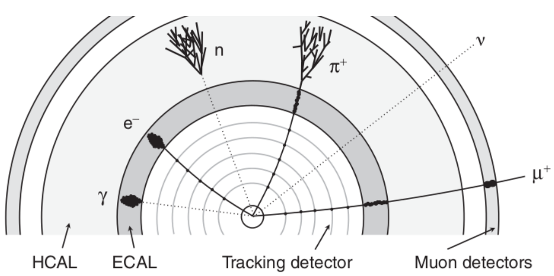
$$\frac{dE}{dx} \approx -4\pi\hbar^2 c^2 \alpha^2 \frac{nZ}{m_2 v^2}\left(\ln\left(\frac{2\beta^2\gamma^2 c^2 m_e}{I_e}\right) - \beta^2\right)$$

$v = \beta c, n = \text{num dens}, Z = \text{atom num}, I_e \approx 10Z\text{eV}, \alpha = 1/137$

Detectors: Mag field tracking: $p\cos(\lambda) = 0.3BR$

Cerenkov: $\cos(\theta) = \frac{1}{n\beta}$

After x rad. lengths $\langle E\rangle = \frac{E}{2^x}$. $x_{\max}$



B-tagging: look for secondary vertices

Number of events and cross sections: $N = \sigma \int \mathcal{L}(t)dt$

In 2 colliding bunches, $\mathcal{L} = f(\text{Hz})\frac{n_1 n_2}{4\pi\sigma_x\sigma_y}$

Chapter 2: Underlying Concepts

Quantity	[kg, m, s]	[$\hbar$, c, GeV]	$\hbar = c = 1$
Energy	kg m^2 s^{-2}	GeV	GeV
Momentum	kg m s^{-1}	GeV/c	GeV
Mass	kg	GeV/c^2	GeV
Time	s	(GeV/$\hbar$)$^{-1}$	GeV^{-1}
Length	m	(GeV/$\hbar c$)$^{-1}$	GeV^{-1}
Area	m^2	(GeV/$\hbar c$)$^{-2}$	GeV^{-2}

Sig. $(1, -1, -1, -1)$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$d' = d/\gamma$$
$$E^2 = \vec{p}^2 c^2 + m^2 c^4$$
$$x^\mu = (t, x, y, z)$$
$$x_\mu = (t, -x, -y, -z)$$
$$p = (E, -p_1, -p_2, -p_3)$$
$$p^2 = p^\mu p_\mu = E^2 - \vec{p}^2$$

$K = E - mc^2, E^2 = \vec{p}^2 + m^2$

Don't fuck up boosts:

$$x = \gamma(x^* - vt)$$
$$t = \gamma(t^* - vx^*)$$
$$x^* = \gamma(x + vt)$$
$$t^* = \gamma(t + vx)$$
$$p_x = \gamma(p_x^* - vE^*)$$
$$E = \gamma(E^* - vp_x^*)$$
$$p_x^* = \gamma(p_x + vE)$$
$$E^* = \gamma(E + vp_x)$$

General math stuff

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$
$$\Box = \partial^\mu \partial_\mu$$
$$p_1 \cdot p_2 = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2$$

Mandelstam: $1 + 2 \to 3 + 4$

$$s = (p_1 + p_2)^2 = E_{com}^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$

Spin algebra

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$
$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$S_- = S_x - iS_y, S_+ = S_x + iS_y$

Isospin, $\hat{T} = \frac{1}{2}\sigma$, same algebra, $T_+ d = u$

$T_3 u = \frac{1}{2}u, T_3 d = -\frac{1}{2}d$

$T_3 \bar{u} = -\frac{1}{2}\bar{u}, T\bar{d} = \frac{1}{2}\bar{d}$

$T_3 s = 0$

$T^2 = \frac{4}{3}I$

Chapter 3: Decay Rates & Cross Sections

Fermi's Golden Rule: $\Gamma_{fi} = 2\pi|T_{fi}|^2\rho(E_i)$

Γ_{fi} = number of transitions per unit time from i to f

Lifetime: $\tau = \frac{\hbar}{\Gamma}$

T_{fi} = Transition Matrix Element, H below is the perturbing Hamiltonian

$T_{fi} = \langle f|\, H\, |i\rangle + \sum_{j\neq i} \frac{\langle f|H|j\rangle\langle j|H|i\rangle}{E_i - E_j}$

$\rho(E_i)$ = density of final states (number of states per unit energy)

$\rho(E_i) = \left|\frac{dn}{dE}\right|_{E_1} = \int \frac{dn}{dE}\delta(E_i - E)dE$

Lor. Inv. T: M, $M_{fi}^{ab} = (2E_a 2E_b 2E_c 2E_d)^{\frac{1}{2}} T_{fi}^{ab}$

If we have an $a \to 1 + 2$ decay, $\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2}\int |M_{fi}|^2 d\Omega$ and

$p^* = \frac{1}{2m_a}\sqrt{[m_a^2 - (m_1 + m_2)^2]\,[m_a^2 - (m_1 - m_2)^2]}$

Cross Sections

For $1 + 2 \to 3 + 4$:

$\sigma = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}\int |M_{fi}|^2 d\Omega^*$ (CoM)

$\frac{d\sigma}{dt} = \frac{1}{64\pi^2|\vec{p}_i^*|^2}|M_{fi}|^2$ (all frames)

$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left[\frac{E_3}{ME_1}\right]^2 |M_{fi}|^2$ (lab, negl m of scattered particle)

where $|\vec{p}_i^*|^2 = \frac{1}{4s}\left[s - (m_1 + m_2)^2\right]\left[s - (m_1 - m_2)^2\right]$

For elastic $e^- p \to e^- p$ scattering: $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{E_3}{m_p E_1}\right)^2 |M_{fi}|^2$ $(E_3 = E_3(\theta))$

Chapter 4: The Dirac Equation

Relativistic QM

K-G: $(\partial^\mu\partial_\mu + m^2)\psi = 0$

Dirac: $(i\gamma^\mu\partial_\mu - m)\psi = 0$

$$\gamma^0 = \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$
$$\gamma^i = \beta\alpha_i = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$$

γ property: $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$

Adjoints:

$$\bar{\psi} = \psi^\dagger\gamma^0 = (\psi_0^*, \psi_1^*, -\psi_2^*, -\psi_3^*)$$

Dirac current:
$j^\mu = \psi^\dagger\gamma^0\gamma^\mu\psi = \bar{\psi}\gamma^\mu\psi$

Positive E solutions:

$$u_1(p) = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}$$
$$u_2(p) = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

Negative E: antiparticles

$$v_1(p) = \sqrt{E+m}\begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}$$
$$v_2(p) = \sqrt{E+m}\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix}$$

Helicity

$$h = \frac{\mathbf{S}\cdot\mathbf{p}}{p} = \begin{pmatrix} \sigma\cdot\hat{p} & 0 \\ 0 & \sigma\cdot\hat{p} \end{pmatrix}$$
$$s = \sin(\theta/2), c = \cos(\theta/2)$$
$$u_\uparrow = \sqrt{E+m}\begin{pmatrix} c \\ se^{i\phi} \\ \frac{p}{E+m}c \\ \frac{p}{E+m}se^{i\phi} \end{pmatrix}$$
$$u_\downarrow = \sqrt{E+m}\begin{pmatrix} -s \\ ce^{i\phi} \\ \frac{p}{E+m}s \\ -\frac{p}{E+m}ce^{i\phi} \end{pmatrix}$$

$$v_\uparrow = \sqrt{E+m}\begin{pmatrix} \frac{p}{E+m}s \\ -\frac{p}{E+m}ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix}$$
$$v_\downarrow = \sqrt{E+m}\begin{pmatrix} \frac{p}{E+m}c \\ \frac{p}{E+m}se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

Rel.Lim.: frac=1, $E+m=E$

In: $\theta=\phi=0$, out: $\theta=\phi=\pi$

Parity: $\hat{P}\psi = \gamma^0\psi$

Charge conj: $\hat{C}\psi = i\gamma^2\psi^*$

Parity Properties

$$\hat{P}u(m,0) = +u(m,0)$$
$$\hat{P}v(m,0) = -v(m,0)$$
$$\hat{P}u_\uparrow(p,\theta,\phi) = u_\downarrow(p,\pi-\theta,\pi+\phi)$$

(not conserved in weak interatcions)

Chapter 5: Interaction by Particle Exchange

General Feynman Tips

$-iM = \prod[\text{terms}]$

Put adjoints first

Put polarizations out front

Same index on all things involved in a vertex

QED

In lepton: $u(p)$

Out lepton: $\bar{u}(p)$

In anti-lepton: $\bar{v}(p)$

Out anti-lepton: $v(p)$

In photon: $\epsilon_\mu(p)$

Out photon: $\epsilon_\mu(p)^*$

Lepton prop: $\frac{i(\gamma^\mu q_\mu + m)}{q^2-m^2}$

Photon prop: $\frac{-ig_{\mu\nu}}{q^2}$

$\ell\to\ell\gamma$ vectex: $-iQ_\ell e\gamma^\mu$

$q\to q\gamma$ vectex: $-iQ_q e\gamma^\mu$

(Q = charge in e units, e.g. -1 for e^-)

Chapter 6: Electron-Positron Annihilation

Different processes $\mathcal{M}$ can add to give pos/neg interference.

Chirality: like helicity but LorInv.

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$
$$P_R = \frac{1}{2}(1+\gamma^5), P_R u_R = u_R$$
$$P_L = \frac{1}{2}(1-\gamma^5), P_L u_L = u_L$$
$$(\gamma^5)^2 = 1, \gamma^5\gamma^\mu = \gamma^\mu\gamma^5$$

$\gamma^5 u_R = +u_R, \gamma^5 u_L = -u_L, \gamma^5 v_R = -v_R, \gamma^5 v_L = +v_L$

In rel. limit (but not generally!),

$$u_R = u_\uparrow, u_L = u_\downarrow, v_R = v_\uparrow, v_L = v_\downarrow$$

I.e. $u_R = \lim_{rel} u_\uparrow$

For massless particles, helicity = chirality.

Calculated e^+e^- cross section using:

$\langle|M_{fi}|^2\rangle = \frac{1}{4}\sum_{\text{spins}}|M|^2 = \frac{1}{4}(|M_{LL\to LL}|^2 + |M_{LL\to LR}|^2 + \ldots)$

In particular, for $e^+e^-\to\mu^+\mu^-$ we had $\langle|M_{fi}|^2\rangle = 2e^4\left(\frac{t^2+u^2}{s^2}\right)$.

Chapter 7: e-p Elastic Scattering

As in ch 6, for $e^-q\to\mu^-q$: $\langle|M_{fi}|^2\rangle = 2Q_q^2e^4\left(\frac{s^2+u^2}{t^2}\right)$

Rutherford: non-rel. e^-, no proton recoil ($\lambda >> r_p$):

$\langle|M_{fi}|^2\rangle = \frac{m_p^2m_e^2e^4}{p^4\sin^4(\theta/2)}$, $\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E_K^2\sin^4(\theta/2)}$

Mott: rel. e^-, no proton recoil ($\lambda\sim r_p$)

$\langle|M_{fi}|^2\rangle = \frac{m_p^2e^4}{E^2\sin^4(\theta/2)}\cos^2(\theta/2)$, $\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4(\theta/2)}\cos^2(\theta/2)$

Form factors: p finite size, some recoil ($\lambda < r_p$)

$M_{fi} = M_{fi}^{pt}F(q^2), F(q^2) = \int\rho(r)e^{iq\cdot r}d^3r$

$\frac{d\sigma}{d\Omega}\to\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}|F(q^2)|^2$

$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4(\theta/2)}\frac{E_3}{E_1}\times\left(\frac{G_e^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2\tan^2\frac{\theta}{2}\right)$ (Rosenbluth)

G_E, G_M are no longer fourier transforms of moments, but in the low-Q^2 limit:

$G_E(Q^2)\approx G_E(q^2) = \int e^{iq\cdot r}\rho(r)d^3r, G_E(0) = 0$

$G_M(Q^2)\approx G_M(q^2) = \int e^{iq\cdot r}\mu(r)d^3r, G_M(0) = +2.79$

In the High-Q^2 limit, model starts to break: $\frac{d\sigma}{d\Omega}\propto\frac{1}{Q^6}\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$

Radius in terms of form factor: $\langle R^2\rangle = -6\left[\frac{dF(q^2)}{dq^2}\right]_{q^2=0}$

Chapter 8: Deep Inelastic Scattering

$Q^2 = -q^2$, q is the momentum transferred

Bjk x: frac of mom. transferred to hit quark, $x = \frac{Q^2}{2p_2\cdot q}$, $x\in[0,1], x=1$ is elastic

Elasticity (fraction of energy loss): $y = \frac{p_2\cdot q}{p_2\cdot p_1}$, $y\in[0,1]$

$v = \frac{p_2\cdot q}{m_p}$, if initial proton at rest $v = E_1 - E_3$ = energy lost

Any 2 of 4 variables are indep, x, y, v, Q^2

$y = \left[\frac{2m_p}{s-m_p^2}\right] v$

$Q^2 = (s - m_p^2)xy$

Deep inelastic scattering $Q^2 >> m_p^2 y^2$ ($\lambda << r_p$, quark sea):

$\frac{d^2\sigma}{dxdQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1 - y - \frac{M^2y^2}{Q^2})\frac{F_2(x,Q^2)}{x} + y^2 F_1(x, Q^2)\right]$

(replace $F_i(x, Q^2) \to f_i(Q^2)$ if elastic)

If $Q^2 >> M^2y^2$, $\frac{d^2\sigma}{dxdQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1 - y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x, Q^2)\right]$

F_2= EM Struct Func, F_1 = pure magnetic SF

In lab frame,

$\frac{d^2\sigma}{dE_3 d\Omega} \approx \frac{\alpha^2}{4E_1^2 \sin^4(\frac{\theta}{2})}\left[\frac{1}{v}F_2(x, Q^2)\cos^2(\frac{\theta}{2}) + \frac{2}{M}F_1(x, Q^2)\sin^2(\frac{\theta}{2})\right]$

$Q^2 = 4E_1E_3 \sin^2 \frac{\theta}{2}$

$x = \frac{Q^2}{2m_p(E_1-E_3)}$

$y = 1 - \frac{E_3}{E_1}$

F_i = structure functions.

Bjorken scaling: $F_i \approx$ flat over Q^2

Callan-Gross relation: $F_2(x) = 2xF_1(x)$.

Can rewrite σ in terms of q^2:

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right]$$

Symmetries and the Quark Model

Symmetries

Conservation of energy, momentum, charge, spin, baryon number, lepton number, colour charge, charge conjugation

Lepton flavour conserved at γ, Z but not W vertices.

Symmetries $\iff$ conserved quantities

- transform under symm, Taylor expand
- write as $\psi' = (1 + i\epsilon\hat{A})\psi$, $\implies$ $\hat{A}$ conserved
- e.g., translation $\leftrightarrow$ momentum
- Not applicable $\neq$ not conserved!

Strong interaction: tried to introduce isospin, with $u = (1, 0), d = (0, 1)$, corresponds to invariance under SU(2), $T_+u = 0, T_+d = u$.

Chapter 10: QCD

Local Gauge Invariance in QED

Local phase transformation of QED: U(1) trans. $\phi' = \phi e^{iq\chi(x)}$

To make QED invariant under that, must introduce photon field A_μ and modify Dirac eqn: $[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi = 0$.

$A'_\mu = A_\mu - \partial_\mu\chi(x)$

Now for QCD

Local phase transformation: SU(3) trans. $\phi' = \phi e^{ig\vec{\lambda}\cdot\vec{\theta}(x)}$

λ_i = the Gell-Mann matrices

$\theta_i(x)$ = 8 spin-1 bosons (gluons)

Wavefunctions now include colour, $\psi = \psi_{\text{flav}}\chi_{\text{spin}}\xi_{\text{col}}\eta_{\text{spc}}$

$$\lambda_1 = \begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix}, \lambda_2 = \begin{bmatrix}0&-i&0\\i&0&0\\0&0&0\end{bmatrix}$$

$$\lambda_3 = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&0\end{bmatrix}, \lambda_4 = \begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix}0&0&-i\\0&0&0\\i&0&0\end{bmatrix}, \lambda_6 = \begin{bmatrix}0&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix}0&0&0\\0&0&-i\\0&i&0\end{bmatrix}, \lambda_8 = \frac{1}{\sqrt{3}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix}$$

QCD Feynman Rules

Quarks: same as QED leptons

Gluons: same as QED photons

Gluon prop: $-\frac{ig_{\mu\nu}}{q^2}\delta^{ab}$

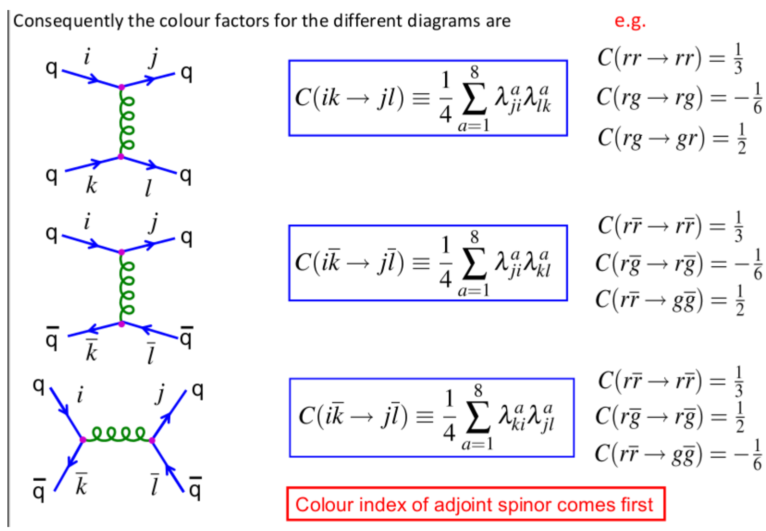
Vertices (qqg): $-ig_s\frac{1}{2}\lambda^a_{ji}\gamma^\mu$, i, j = (1,2,3) = (r, g, b) for initial, final quark colours

Other vertex terms exist but won't be tested.

Remember to have colour-neutral initial / final states

Colour factors $C(ik \to jl) = \frac{1}{4}\sum_{a=1}^8 \lambda^a_{ji}\lambda^a_{lk}$ (r=1, g=2, b=3). Remember colour conservation when applicable!

At each vertex, put the adjoint first, e.g. $u_i \to g + u_j$ gives λ^a_{ji}.



$$\langle |C|^2 \rangle = \frac{1}{9}\sum_{ijkl=1}^{3}|C(ij \to kl)|^2$$

$M_{QCD} = CM_{QED}$ for our purposes, also replace $e \to g_s^2$.

At low energies can't do QCD since $a_s \approx 1$, but at high energies can use perturbation theory just like QED = "asymptotic freedom."

Chapter 11: The Weak Interaction

All force carrying bosons have parity -1, e.g. $P(W^\pm) = -1$, not H though!

Parity conserved in QED (and therefore also high-energy QCD).

	Rank	Parity	Example
Scalar	0	+	Temperature, T
Pseudoscalar	0	−	Helicity, h
Vector	1	−	Momentum, $\mathbf{p}$
Axial vector	1	+	Angular momentum, $\mathbf{L}$

Parity not conserved in weak interaction. Discovered by Wu, who saw ${}^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$

Table 11.2 Lorentz-invariant bilinear covariant currents.

Type	Form	Components	Boson spin
Scalar	$\overline{\psi}\phi$	1	0
Pseudoscalar	$\overline{\psi}\gamma^5\phi$	1	0
Vector	$\overline{\psi}\gamma^\mu\phi$	4	1
Axial vector	$\overline{\psi}\gamma^\mu\gamma^5\phi$	4	1
Tensor	$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$	6	2

Experimentally we found the interaction is V-A, with a vertex term $\frac{-ig_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)$.

Weak Feynman Rules

Fermions: same as before, same vibe with neutrinos

W / Z: same as photons for incoming/outgoing

W prop: $-\frac{i[g_{\mu\nu}-q_\mu q_\nu/m_W^2]}{q^2-m_W^2}$

If q^2 small: $\frac{-ig_{\mu\nu}}{q^2-m_W^2}$

If q^2 really small: $\frac{ig_{\mu\nu}}{m_W^2}$

Vertex: $\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)$

For spin-0 kaons or pions, can replace $\bar{v}\gamma^\mu(1-\gamma^5)u$ with $f_\pi p_\pi^\mu$

If q^2 small, $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$

(Fermi: assuming no W boson propagator q^2 dependence)

Because we have $V-A$, only LH chiral particles and RH chiral anti-particles participate in weak CC interactions.

Remember $u/c/t \to W \to d/s/b$ only.

Chapter 12: Weak Interactions of Leptons

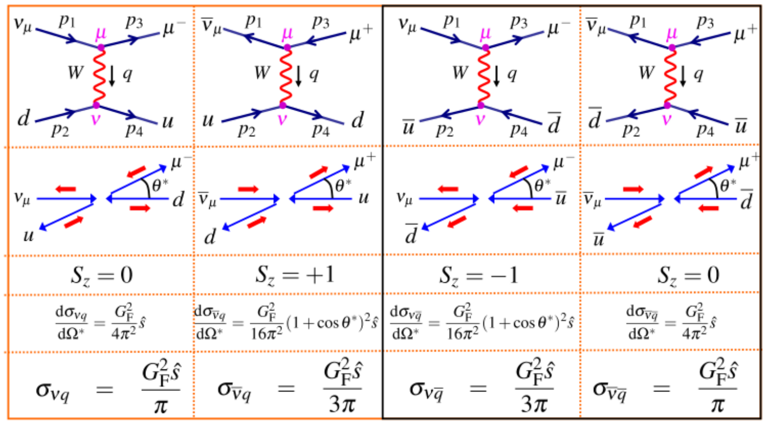
Lepton Universality: $G_F^{(e)} = G_F^{(\mu)} = G_F^{(\tau)}$ (within experimental error)

Neutrino-quark scattering:

$M_{fi} = \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\bar{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_1)\right]\left[\bar{u}(p_4)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_2)\right]$

$\implies \frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s} \implies \sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$

- Non-zero anti-quark component to the nucleon ⇨ also consider scattering from $\bar{q}$
- Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include LH particles and RH anti-particles



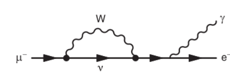
$F_2^{\nu p} = 2xF_1^{\nu p} = 2x[d(x) + \bar{u}(x)]$

$xF_3^{\nu p} = 2x[d(x) - \bar{u}(x)]$

$\sigma^{\nu N} = \frac{G_F^2 m_N E_\nu}{\pi}\left[f_q + \frac{1}{3}f_{\bar{q}}\right], \sigma^{\bar{\nu}N} = \frac{G_F^2 m_N E_{\bar{\nu}}}{\pi}\left[\frac{1}{3}f_q + f_{\bar{q}}\right]$

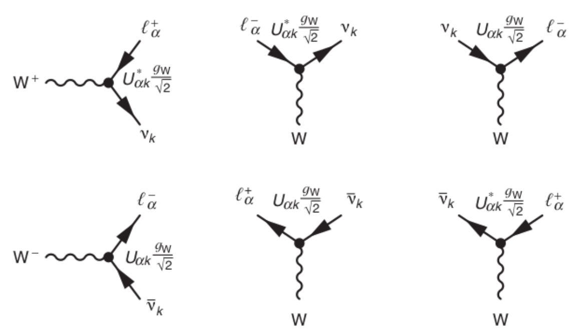
Neutrinos and Neutrino Scattering

Proof neutrinos have flavour: this doesn't happen



Solar neutrino problem: only 1/3 of neutrinos from the sun were electron ones, should be 100% but neutrino mixing happens.

PMNS matrix describes how neutrinos mix



The charged-current weak interaction vertices for charged lepton of flavour α = e, μ, τ and a neutrino of type k = 1, 2, 3.

$$-i\frac{g_W}{\sqrt{2}}\bar{e}\gamma^\mu\frac{1}{2}(1-\gamma^5)\nu_e,$$

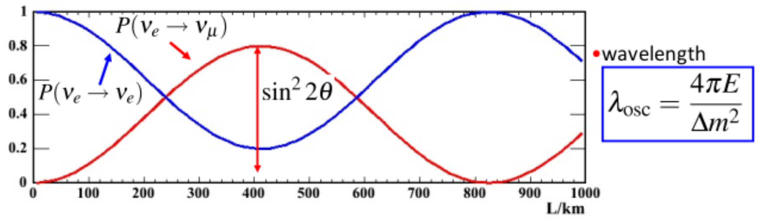
where here ν_e and $\bar{e}$ denote the electron neutrino spinor and the electron adjoint spinor. In terms of the neutrino mass eigenstates, the weak charged-current for a lepton of flavour α = e, μ, τ and a neutrino of type k = 1, 2, 3 takes the form

$$-i\frac{g_W}{\sqrt{2}}\bar{\ell}_\alpha\gamma^\mu\frac{1}{2}(1-\gamma^5)U_{\alpha k}\nu_k.$$

Studying 2-neutrino mixing can give the general idea:

Transition probability: $P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2(\frac{(m_1^2-m_2^2)L}{4E_\nu})$

Survival probability: $P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta)\sin^2(\frac{(m_1^2-m_2^2)L}{4E_\nu})$



With 3 flavours:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} \sim \begin{pmatrix} 0.85 & 0.50 & 0.17 \\ 0.35 & 0.60 & 0.70 \\ 0.35 & 0.60 & 0.70 \end{pmatrix}$$

An example of how to use that:

$$\begin{aligned} |\psi(\mathbf{x},t)\rangle = \; & (U_{e1}^* U_{e1} e^{-i\phi_1} + U_{e2}^* U_{e2} e^{-i\phi_2} + U_{e3}^* U_{e3} e^{-i\phi_3})|\nu_e\rangle \\ & (U_{e1}^* U_{\mu 1} e^{-i\phi_1} + U_{e2}^* U_{\mu 2} e^{-i\phi_2} + U_{e3}^* U_{\mu 3} e^{-i\phi_3})|\nu_\mu\rangle \\ & (U_{e1}^* U_{\tau 1} e^{-i\phi_1} + U_{e2}^* U_{\tau 2} e^{-i\phi_2} + U_{e3}^* U_{\tau 3} e^{-i\phi_3})|\nu_\tau\rangle. \end{aligned} \tag{13.21}$$

This can be expressed in the form $|\psi(\mathbf{x},t)\rangle = c_e|\nu_e\rangle + c_\mu|\nu_\mu\rangle + c_\tau|\nu_\tau\rangle$, from which the oscillation probabilities can be obtained, for example

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= |\langle \nu_\mu|\psi(\mathbf{x},t)\rangle|^2 = c_\mu c_\mu^* \\ &= |U_{e1}^* U_{\mu 1} e^{-i\phi_1} + U_{e2}^* U_{\mu 2} e^{-i\phi_2} + U_{e3}^* U_{\mu 3} e^{-i\phi_3}|^2. \end{aligned} \tag{13.22}$$

This allows us to get mass differences but not mass hierarchy.

CP violated in weak interaction only, CPT truly conserved (SM).

One possible parametrization:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$\Delta m_{21}^2 = (7.6 \pm 0.2) \times 10^{-5}\text{eV}^2$

$\sin^2(2\theta_{12}) = 0.87 \pm 0.04$

$\Delta m_{32}^2 = 2.3 \times 10^{-3}\,\text{eV}^2$

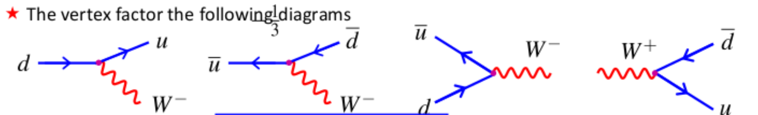
$\theta_{12} \approx 35°$, $\theta_{23} \approx 45°$, $\theta_{13} \approx 10°$

CP Violation and Weak Hadronic Interactions

Observations of kaons and pions decaying at different rates led Cabibbo to propose quark mixing.

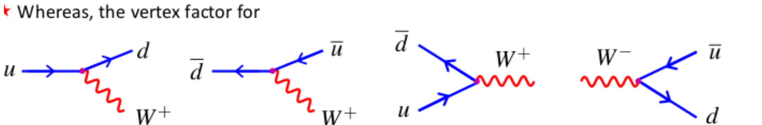
Quark mixing can happen at weak vertices through CKM matrix

★ The vertex factor the following diagrams



is $\boxed{-i\dfrac{g_W}{\sqrt{2}} V_{ud}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)}$

★ Whereas, the vertex factor for



is $\boxed{-i\dfrac{g_W}{\sqrt{2}} V_{ud}^*\gamma^\mu \tfrac{1}{2}(1-\gamma^5)}$

★ Assuming unitarity of CKM matrix, measure:

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \qquad \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

Near diagonal – very different from PMNS

CP violation gives things like K_S, K_L, B_H, B_L

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta'} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta'} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4).$$

CP violation… to understand in Kaon (same in B)

★ If CP is conserved in the Weak decays of neutral kaons, we expect decays to pions to occur from states of definite CP (i.e. the CP eigenstates K_1, K_2)

$$|K_1\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \quad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle \quad K_1 \to \pi\pi \quad \text{CP EVEN}$$

$$|K_2\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \quad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle \quad K_2 \to \pi\pi\pi \quad \text{CP ODD}$$

★ Expect lifetimes of CP eigenstates to be very different
- For two pion decay energy available: $m_K - 2m_\pi \approx 220\,\text{MeV}$
- For three pion decay energy available: $m_K - 3m_\pi \approx 80\,\text{MeV}$

★ Expect decays to two pions to be more rapid than decays to three pions due to increased phase space

★ We observe a short-lived state "K-short" which decays **mostly** two pions and a long-lived state "K-long" which decays **mostly** three pions
- But the long-lived Kaon we observe is *not* a pure K_2 (and hence not a pure CP eigenstate) but rather:

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_1\rangle + \varepsilon|K_2\rangle] \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]$$

K_1 is the 3-pion one, K_2 goes to 2 pions

★ Flavour eigenstates: $|K^0\rangle$ $|\overline{K}^0\rangle$
- ★ What is their quark flavour?

★ Mass eigenstates: $|K_S\rangle$ $|K_L\rangle$
- ★ What mass do you put in the wavefunction propagator part

★ CP eigenstates: $|K_1\rangle$ $|K_2\rangle$
- ★ Which decays to 2 pions, which to 3 pions

In practice, $K_S \approx K_1, K_L \approx K_2$

It's the same kind of thing with neutral B mesons,

$(B^0, \bar{B}^0)$ flav. eigs., (B_H, B_L) mass eigs

$$|B_L\rangle = \frac{1}{\sqrt{2}}\left(|B^0\rangle + e^{-i2\beta}|\overline{B}^0\rangle\right) \quad \text{and} \quad |B_H\rangle = \frac{1}{\sqrt{2}}\left(|B^0\rangle - e^{-i2\beta}|\overline{B}^0\rangle\right).$$

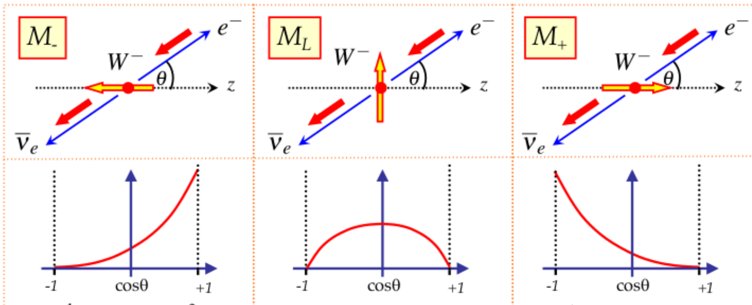
$\sin(2\beta) = 0.685 \pm 0.032$

Wolfenstein params: $\lambda = 0.2253, A = 0.811, \rho = 0.13, \eta = 0.345$

Oscillations arise because natural mesons are produced as flavour eigenstates and decay as either flavour or CP eigenstates, but propagate as physical mass eigenstates.

Electroweak Unification

- The angular distributions can be understood in terms of the spin of the particles



$\frac{1}{4}(1+\cos\theta)^2$ $\qquad$ $\frac{1}{2}\sin^2\theta$ $\qquad$ $\frac{1}{4}(1-\cos\theta)^2$

- The differential decay rate using Fermi's Golden Rule: $\dfrac{d\Gamma}{d\Omega} = \dfrac{|p^*|}{32\pi^2 m_W^2}|M|^2$

$p^* = \frac{m_W}{2}$

- Gives: $\boxed{\Gamma_- = \Gamma_L = \Gamma_+ = \dfrac{g_W^2 m_W}{48\pi}}$

- For a sample of unpolarized W-bosons, the decay is isotropic

$$\Gamma(W^- \to e^-\bar{\nu}_e) = \frac{g_W^2 m_W}{48\pi}$$

(multipliers below are relative to this)

★ The calculation for the other decay modes (neglecting final state particle masses) is same. For quarks need to account for colour and CKM matrix. No decays to top quark – the top mass (173 GeV) is greater than the W-boson mass (80.4 GeV)

$W^- \to e^-\overline{\nu}_e$	$W^- \to d\bar{u}$	$\times 3\|V_{ud}\|^2$	$W^- \to d\bar{c}$	$\times 3\|V_{cd}\|^2$
$W^- \to \mu^-\overline{\nu}_\mu$	$W^- \to s\bar{u}$	$\times 3\|V_{us}\|^2$	$W^- \to s\bar{c}$	$\times 3\|V_{cs}\|^2$
$W^- \to \tau^-\overline{\nu}_\tau$	$W^- \to b\bar{u}$	$\times 3\|V_{ub}\|^2$	$W^- \to b\bar{c}$	$\times 3\|V_{cb}\|^2$

★ Unitarity of CKM matrix gives, e.g. $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

★ Hence $BR(W \to qq') = 6BR(W \to e\nu)$

and thus the total decay rate : $\boxed{\Gamma_W = 9\Gamma_{W\to e\nu} = \dfrac{3g_W^2 m_W}{16\pi} = 2.07\,\text{GeV}}$

Local gauge transformation of QED: SU(2) trans. $\phi' = \phi e^{i\vec{\alpha}(x)\cdot\frac{\vec{\sigma}}{2}}$

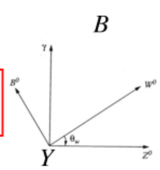
Generators: Pauli matrices, $\alpha_i(x)$ = local phases, give 3 gauge bosons, $W_1^\mu, W_2^\mu, W_3^\mu$.

$W^\pm = \frac{1}{\sqrt{2}}\left(W^1 \mp W^2\right)$

- W^1 and W^2 become the 2 charged W bosons
- Tempting to identify the W^3 as the Z boson
- However this is not the case (experimentally, NC is not pure V-A!)
- The physical bosons (the Z and photon field, A) are:

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

θ_W is the weak mixing angle



- The charge of this $U(1)_Y$ symmetry is called WEAK HYPERCHARGE

$\boxed{Y = 2Q - 2I_W^3}$ Q is the EM charge of a particle; I_W^3 is the third comp. of weak isospin

Overall EW Unification Idea:

$U(1)_{EM} \implies$ QED

$SU(2)_L \implies W_1, W_2 = W^\pm, W_3$

$SU(3)_{col} \implies$ QCD

If you introduce $U(1)_Y \implies B_\mu$ you can unify EM & weak.

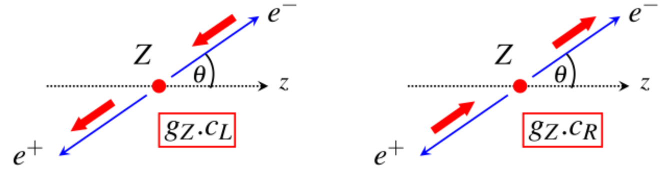
Once θ_W is known, properties of Z are determined.

Experimentally, roughly, $\sin^2(\theta_W) = 0.23$

Z couples to left- and right-handed chiral states but not equally.

I_W = weak isospin, neutrinos have 1/2 leptons have −1/2

- In terms of left and right-handed combinations need to calculate



- For unpolarised Z bosons (similar to W boson + RH current): $\langle|M_{fi}|^2\rangle = \frac{1}{3}g_W^2 m_W^2$

$$\langle|M_{fi}|^2\rangle = \tfrac{1}{3}[2c_L^2 g_Z^2 m_Z^2 + 2c_R^2 g_Z^2 m_Z^2] = \tfrac{2}{3}g_Z^2 m_Z^2(c_L^2 + c_R^2)$$

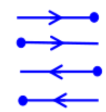
average over polarization

- Using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $\dfrac{d\Gamma}{d\Omega} = \dfrac{|p^*|}{32\pi^2 m_W^2}|M|^2$

$$\Rightarrow \boxed{\Gamma(Z \to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)}$$

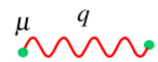
- External Lines

spin 1/2
- incoming fermion $u(p)$
- outgoing fermion $\bar{u}(p)$
- incoming anti- fermion $\bar{v}(p)$
- outgoing anti- fermion $v(p)$



- Internal Lines (propagators)

spin 1 Z $\boxed{\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}}$



- Neutral weak currents

$$\begin{aligned} j_\mu^Z &= g_Z(I_W^3 - Q\sin^2\theta_W)[\bar{e}_L\gamma_\mu e_L] - g_Z Q\sin^2\theta_W[e_R\gamma_\mu e_R] \\ &= g_Z c_L[\bar{e}_L\gamma_\mu e_L] + g_Z c_R[e_R\gamma_\mu e_R] \end{aligned}$$

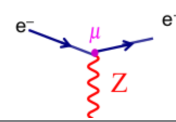


Table 15.1 The charge, $I_W^{(3)}$ and weak hypercharge assignments of the fundamental fermions and their couplings to the Z assuming $\sin^2\theta_W = 0.231\,46$.

fermion	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	−1	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	−1	$-\frac{1}{2}$	−1	−2	−0.27	+0.23	−0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{4}{3}$	+0.35	−0.15	+0.19	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{2}{3}$	−0.42	+0.08	−0.35	$-\frac{1}{2}$

Tests of the Standard Model

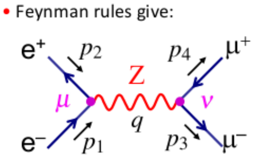
Can get a bunch of parameters using stuff we've learned by now:

$m_Z = 91.1875\,\text{GeV}, \sin^2(\theta_W) = 0.23146$

$m_W = 80.385\,\text{GeV}, m_t = 173.5\,\text{GeV}$

E.g.

- E.g. $e^+e^- \to Z \to \mu^+\mu^-$
  - Feynman rules give:



e⁺e⁻ vertex: $\bar{v}(p_2)\cdot -ig_Z\gamma^\mu\frac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

μ⁺μ⁻ vertex: $\bar{u}(p_3)\cdot -ig_Z\gamma^\nu\frac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)$

$$\boxed{M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2}g_{\mu\nu}[\bar{v}(p_2)\gamma^\mu\tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].[\bar{u}(p_3)\gamma^\nu\tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]}$$

- Convenient to work in terms of helicity states by explicitly using the Z coupling to LH and RH chiral states (ultra-relativistic limit so helicity = chirality)

$$\begin{aligned} \tfrac{1}{2}(c_V - c_A\gamma^5) &= \tfrac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) & c_V = c_L + c_R \\ &= c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5) & c_A = c_L - c_R \end{aligned}$$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}[(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2]}$$

The Higgs Boson

- In the previous example, the Higgs mechanism was used to generate masses for a single gauge boson of a U(1) local gauge symmetry.
- The Standard Model is based on the unified $SU(2)_L \times U(1)_Y$ electroweak theory
- This means, 3 Goldstone bosons are required to provide the longitudinal degrees of freedom of the W^+, W^- and Z bosons
- The minimal Higgs model that gives this consists of two complex scalar fields

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \boxed{= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}}$$

- With Lagrangian $\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi)$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \phi^\dagger \phi = \tfrac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}$$

- Has infinite set of minima
- This will result in massive W^+, W^- and Z bosons and one Higgs boson
- The photon will remain massless, therefore the minimum must correspond to a non-zero vacuum expectation value for the neutral field
- The electroweak theory successfully employs the Higgs mechanism
- It is described by just 4 parameters
  - The gauge couplings g_W and g'
  - The two free parameters of the Higgs potential: μ and λ
- These are related through the vev through

$$v^2 = \frac{-\mu^2}{\lambda} \quad \text{and} \quad m_H^2 = 2\lambda v^2.$$

- From the measured mass of the W boson, we can infer the vev

$$m_W = \tfrac{1}{2} g_W v \quad \rightarrow \quad \boxed{v = 246\ \mathrm{GeV}}$$

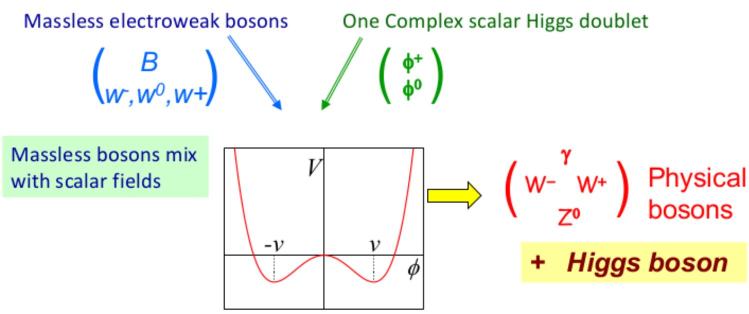
- The Higgs coupling to the physical weak gauge bosons can be obtained using:

$$W^{\pm} = \tfrac{1}{\sqrt{2}}\left(W^{(1)} \mp i W^{(2)}\right)$$

- And the relevant part of the Lagrangian becomes

$$\tfrac{1}{4} g_W^2 W_\mu^- W^{+\mu} (v+h)^2 = \tfrac{1}{4} g_W^2 v^2 W_\mu^- W^{+\mu} + \tfrac{1}{2} g_W^2 v W_\mu^- W^{+\mu} h + \tfrac{1}{4} g_W^2 W_\mu^- W^{+\mu} hh$$

- We have triple and quartic couplings to the Higgs boson



- So we have seen how the Higgs mechanism explains how the weak vector bosons acquire mass in a (local) gauge invariant way by 'eating' Goldstone bosons
- The mechanism also provides an explanation of how fermions acquire mass through Yukawa interaction with the Higgs field
- The price we had to pay is an additional massive Higgs boson
- The Higgs boson couples to all particles proportionally to their mass
- Next we need to use all of this to figure out out how the Higgs boson is produced, how it decays
  - That will explain how people were looking for the Higgs...
  - And how people found it...
  - And how people now measure its properties...

Higgs field permeates space, interacts with all massive particles, interaction strength $\propto$ mass.

$g_f = \sqrt{2}\frac{m_f}{v}$

Description	Free Parameters	Related Parameters
Lepton masses	m_e, m_μ, m_τ, $m_{\nu 1}$, $m_{\nu 2}$, $m_{\nu 3}$	Yukawa coupling to Higgs
Quark masses	m_u, m_c, m_t, m_d, m_s, m_b	Yukawa coupling to Higgs
CKM matrix	$\alpha, \beta, \gamma, i\delta$	CKM elements V_{ij}
PNS matrix	$\Theta_{12}, \Theta_{13}, \Theta_{23}, i\delta$	PNS elements V_{ij}
Coupling strength	g_e, g_w, g_s	$\sin\theta_W$
Higgs sector	m_H, m_W/m_Z	μ, λ, v

- What is the solution to the Hierarchy problem of a fundamental Higgs?
- Why three generations of fermions ?
- Unification of the Forces
- Matter/anti-matter asymmetry (CP violation)
- What is Dark Matter ?
- Why is the weak interaction V-A ?
- Why are neutrinos so light ?
- Ultimately need to include gravity