1 `fakedata.out` contains 200 observations of three random variables: X, Y, and Z (each variable in its own column, listed in that order). Calculate the following for this data:

1. The mean values of X, Y, and Z.

   From the code, $\bar{X} = 49.85\bar{Y} = -1.56, \bar{Z} = -19.38$.

2. The standard deviations for all thre variables.

   From the code, $\sigma_X = 12.75\sigma_Y = 13.63, \sigma_Z = 11.06$.

3. The three correlation coefficients between the three variables.

   From the code, $C_{X,Y} = 0.30C_{X,Z} = 0.72, C_{Y,Z} = -0.30$.

4. The skew for X, Y, and Z.

   From the code, $\mathrm{Skew}(X) = -0.10\,\mathrm{Skew}(Y) = 0.05, \mathrm{Skew}(Z) = -0.31$.

2 Show that if (3.12), (3.13), and (3.14) are true, we have (3.15) and (3.16).

(3.12) and (3.13):

$$\delta_{ab}\psi_\sigma(\vec{x},t) = (x_a\nabla_b - x_b\nabla_a)\psi_\sigma(\vec{x},t)$$
$$\delta_{ab}\psi^{\sigma\dagger}(\vec{x},t) = (x_a\nabla_b - x_b\nabla_a)\psi^{\sigma\dagger}(\vec{x},t)$$

(3.14):

$$\begin{aligned}
\delta_{ab}\mathcal{L}(\vec{x},t) &= (x_a\nabla_b - x_b\nabla_a)\mathcal{L}(\vec{x},t) \\
\implies \delta_{ab}\mathcal{L}(\vec{x},t) &= \frac{\partial}{\partial t}(0) + (\nabla_b x_a\mathcal{L} - x_b\nabla_a\mathcal{L}) \\
&= \frac{\partial}{\partial t}(0) + \nabla\cdot(\hat{b}x_a\mathcal{L} - \hat{a}x_b\mathcal{L}) \\
\implies R = 0, J &= (\hat{b}x_a\mathcal{L} - \hat{a}x_b\mathcal{L})
\end{aligned}$$

(where $\hat{a},\hat{b}$ are the unit vectors in the ab plane.)
Start with general definition of $\mathcal{M}_{ab},\mathcal{M}_{cab}$ (Noether charge density and current density):

$$\begin{aligned}
\mathcal{M}_{ab} = \mathcal{R} &= \delta_{ab}\psi_\sigma\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi_\sigma} + \delta_{ab}\psi^{\sigma\dagger}\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi^{\sigma\dagger}} - R \\
&= (x_a\nabla_b - x_b\nabla_a)\psi_\sigma\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi_\sigma} + (x_a\nabla_b - x_b\nabla_a)\psi^{\sigma\dagger}\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi^{\sigma\dagger}} - 0 \\
&= x_a\nabla_b\psi_\sigma\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi_\sigma} - x_b\nabla_a\psi_\sigma\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi_\sigma} + x_a\nabla_b\psi^{\sigma\dagger}\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi^{\sigma\dagger}} - x_b\nabla_a\psi^{\sigma\dagger}\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi^{\sigma\dagger}} \\
&= x_a\left(\nabla_b\psi_\sigma\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi_\sigma} + \nabla_b\psi^{\sigma\dagger}\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi^{\sigma\dagger}}\right) - x_b\left(\nabla_a\psi_\sigma\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi_\sigma} + \nabla_a\psi^{\sigma\dagger}\frac{\partial\mathcal{L}}{\partial\frac{\partial}{\partial t}\psi^{\sigma\dagger}}\right) \\
&= x_aT_{tb} - x_bT_{ta}
\end{aligned}$$

Similarly (here it's pretty awkward to use δ as a variation and a Kronecker delta, but bear with me),

$$
\begin{aligned}
\mathcal{M}_{cab} = \mathcal{J}^c &= \delta_{ab}\psi_\sigma \frac{\partial \mathcal{L}}{\partial \nabla_c \psi_\sigma} + \delta_{ab}\psi^{\sigma\dagger}\frac{\partial \mathcal{L}}{\partial \nabla_c \psi^{\sigma\dagger}} - J^c \\
&= (x_a\nabla_b - x_b\nabla_a)\psi_\sigma \frac{\partial \mathcal{L}}{\partial \nabla_c \psi_\sigma} + (x_a\nabla_b - x_b\nabla_a)\psi^{\sigma\dagger}\frac{\partial \mathcal{L}}{\partial \nabla_c \psi^{\sigma\dagger}} - (\hat{b}x_a\mathcal{L} - \hat{a}x_b\mathcal{L})^c \\
&= x_a\nabla_b\psi_\sigma \frac{\partial \mathcal{L}}{\partial \nabla_c \psi_\sigma} - x_b\nabla_a\psi_\sigma \frac{\partial \mathcal{L}}{\partial \nabla_c \psi_\sigma} + x_a\nabla_b\psi^{\sigma\dagger}\frac{\partial \mathcal{L}}{\partial \nabla_c \psi^{\sigma\dagger}} - x_b\nabla_a\psi^{\sigma\dagger}\frac{\partial \mathcal{L}}{\partial \nabla_c \psi^{\sigma\dagger}} - (\hat{b}x_a\mathcal{L} - \hat{a}x_b\mathcal{L})^c \\
&= x_a\left(\nabla_b\psi_\sigma \frac{\partial \mathcal{L}}{\partial \nabla_c \psi_\sigma} + \nabla_b\psi^{\sigma\dagger}\frac{\partial \mathcal{L}}{\partial \nabla_c \psi^{\sigma\dagger}} - \delta_{cb}\mathcal{L}\right) - x_b\left(\nabla_a\psi_\sigma \frac{\partial \mathcal{L}}{\partial \nabla_c \psi_\sigma} + \nabla_a\psi^{\sigma\dagger}\frac{\partial \mathcal{L}}{\partial \nabla_c \psi^{\sigma\dagger}} - \delta_{ca}\mathcal{L}\right) \\
&= x_a T_{cb} - x_b T_{ca}
\end{aligned}
$$

So we have shown (3.15) and (3.16):

$$
\begin{aligned}
\mathcal{M}_{ab} &= x_a T_{tb} - x_b T_{ta} \\
\mathcal{M}_{cab} &= x_a T_{cb} - x_b T_{ca}
\end{aligned}
$$

3 Use (3.15), (3.16) to show $\frac{\partial}{\partial t}\mathcal{M}_{ab}(\vec{x},t)+\nabla^c\mathcal{M}_{cab}(\vec{x},t)=0$ is obeyed when $\frac{\partial}{\partial t}T_{tt}(\vec{x},t)+\nabla^b T_{bt}(\vec{x},t)=0$, $\frac{\partial}{\partial t}T_{ta}(\vec{x},t)+\nabla^b T_{ba}(\vec{x},t)=0$, and $T_{ab}(\vec{x},t)=T_{ba}(\vec{x},t)$.

(3.15) and (3.16):

$$\begin{aligned}\mathcal{M}_{ab}(\vec{x},t) &= x_a T_{tb}(\vec{x},t) - x_b T_{ta}(\vec{x},t)\\ \mathcal{M}_{cab}(\vec{x},t) &= x_a T_{cb}(\vec{x},t) - x_b T_{ca}(\vec{x},t)\end{aligned}$$

Take $\frac{\partial}{\partial t}$ of (3.15):

$$\begin{aligned}\frac{\partial}{\partial t}\mathcal{M}_{ab}(\vec{x},t) &= \frac{\partial}{\partial t}\left(x_a T_{tb}(\vec{x},t) - x_b T_{ta}(\vec{x},t)\right)\\ &= \frac{\partial}{\partial t}x_a T_{tb}(\vec{x},t) + x_a\frac{\partial}{\partial t}T_{tb}(\vec{x},t) - \frac{\partial}{\partial t}x_b T_{ta}(\vec{x},t) - x_b\frac{\partial}{\partial t}T_{ta}(\vec{x},t)\end{aligned}$$

Note that none of the x_i depend explicitly on t, so $\frac{\partial}{\partial t}x_i=0$,

$$\frac{\partial}{\partial t}\mathcal{M}_{ab}(\vec{x},t) = x_a\frac{\partial}{\partial t}T_{tb}(\vec{x},t) - x_b\frac{\partial}{\partial t}T_{ta}(\vec{x},t)$$

Use $\frac{\partial}{\partial t}T_{ta}(\vec{x},t)+\nabla^b T_{ba}(\vec{x},t)=0$

$$\frac{\partial}{\partial t}\mathcal{M}_{ab}(\vec{x},t) = -x_a\nabla^c T_{cb}(\vec{x},t) + x_b\nabla^c T_{ca}(\vec{x},t)$$

Now take ∇^c of (3.16):

$$\nabla^c\mathcal{M}_{cab}(\vec{x},t) = \delta^c_a T_{cb}(\vec{x},t) + x_a\nabla^c T_{cb}(\vec{x},t) - \delta^c_b T_{ca}(\vec{x},t) - x_b\nabla^c T_{ca}(\vec{x},t)$$

Now sum the delta terms over c and use $T_{ab}=T_{ba}$:

$$\begin{aligned}\nabla^c\mathcal{M}_{cab}(\vec{x},t) &= T_{ab}(\vec{x},t) + x_a\nabla^c T_{cb}(\vec{x},t) - T_{ba}(\vec{x},t) - x_b\nabla^c T_{ca}(\vec{x},t)\\ &= 0 + x_a\nabla^c T_{cb}(\vec{x},t) - x_b\nabla^c T_{ca}(\vec{x},t)\\ &= -\frac{\partial}{\partial t}\mathcal{M}_{ab}(\vec{x},t)\end{aligned}$$

$$\implies \frac{\partial}{\partial t}\mathcal{M}_{ab}(\vec{x},t) + \nabla^c\mathcal{M}_{cab}(\vec{x},t) = 0$$

4 Confirm that for an infinitesimal rotation in the $a-b$ plane,

$$-(x_a\nabla_b - x_b\nabla_a)\psi_\sigma(\vec{x},t) = \frac{1}{i\hbar}[\psi_\sigma(\vec{x},t), M_{ab}]$$

and

$$-(x_a\nabla_b - x_b\nabla_a)\psi^{\dagger\sigma}(\vec{x},t) = \frac{1}{i\hbar}[\psi^{\dagger\sigma}(\vec{x},t), M_{ab}]$$

where M_{ab} is given in (3.25).

(3.25)

$$\begin{aligned}M_{ab} = -\frac{i\hbar}{2}\int d^3x[&\psi^{\dagger\sigma}(\vec{x},t)(x_a\nabla_b - x_b\nabla_a)\psi_\sigma(\vec{x},t)\\ &- (x_a\nabla_b - x_b\nabla_a)\psi^{\dagger\sigma}(\vec{x},t)\psi_\sigma(\vec{x},t)]\end{aligned}$$

Start with the commutators:

$$\begin{aligned}\frac{1}{i\hbar}[\psi_\sigma(\vec{x},t), M_{ab}] &= \frac{1}{i\hbar}\psi_\sigma(\vec{x},t)M_{ab} - \frac{1}{i\hbar}M_{ab}\psi_\sigma(\vec{x},t)\\ &= -\frac{1}{2}\psi_\sigma(\vec{x},t)\int d^3y[\psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t)\\ &\quad - (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)]\\ &\quad + \frac{1}{2}\int d^3y[\psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t)\\ &\quad - (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)]\psi_\sigma(\vec{x},t)\\ &= \frac{1}{2}\int d^3y - \psi_\sigma(\vec{x},t)\psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t)\\ &\quad + \psi_\sigma(\vec{x},t)(y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)\\ &\quad + [\psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t)\\ &\quad - (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)]\psi_\sigma(\vec{x},t)\end{aligned}$$

Recall equal-time anti-commutators:

$$\begin{aligned}\psi_\sigma(\vec{x},t)\psi^{\rho\dagger}(\vec{y},t) + \psi^{\rho\dagger}(\vec{y},t)\psi_\sigma(\vec{x},t) &= \delta^\rho_\sigma\delta(\vec{x}-\vec{y})\\ \psi_\sigma(\vec{x},t)\psi_\rho(\vec{y},t) + \psi_\rho(\vec{y},t)\psi_\sigma(\vec{x},t) &= 0\end{aligned}$$

Also note that the ∇_is are w.r.t. y_i for now, so we can pass the $\vec{x}$ field operators through the $(y_a\nabla_b - y_b\nabla_a)$ terms:

$$
\begin{aligned}
\frac{1}{i\hbar}[\psi_\sigma(\vec{x},t), M_{ab}] = \frac{1}{2}\int d^3y &- [-\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{x},t) + \delta^\sigma_\sigma\delta(\vec{x}-\vec{y})](y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&+ (y_a\nabla_b - y_b\nabla_a)[-\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{x},t) + \delta^\sigma_\sigma\delta(\vec{x}-\vec{y})]\psi_\sigma(\vec{y},t) \\
&+ [\psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&- (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)]\psi_\sigma(\vec{x},t) \\
= \frac{1}{2}\int d^3y &\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{x},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&- \delta^\sigma_\sigma\delta(\vec{x}-\vec{y})(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&- (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{x},t) \\
&+ (y_a\nabla_b - y_b\nabla_a)\delta^\sigma_\sigma\delta(\vec{x}-\vec{y})\psi_\sigma(\vec{y},t) \\
&+ [\psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&- (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)]\psi_\sigma(\vec{x},t) \\
= \frac{1}{2}\int d^3y &- \psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t)\psi_\sigma(\vec{x},t) \\
&- \delta^\sigma_\sigma\delta(\vec{x}-\vec{y})(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&+ (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)\psi_\sigma(\vec{x},t) \\
&- \delta^\sigma_\sigma\delta(\vec{x}-\vec{y})(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&+ [\psi^{\dagger\sigma}(\vec{y},t)(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t) \\
&- (y_a\nabla_b - y_b\nabla_a)\psi^{\dagger\sigma}(\vec{y},t)\psi_\sigma(\vec{y},t)]\psi_\sigma(\vec{x},t) \\
= \int d^3y &0 + 0 - \delta^\sigma_\sigma\delta(\vec{x}-\vec{y})(y_a\nabla_b - y_b\nabla_a)\psi_\sigma(\vec{y},t)
\end{aligned}
$$

Note that σ is not summed, so $\delta^\sigma_\sigma = 1$. Integrate the $\delta(\vec{x}-\vec{y})$ and we get the desired result:

$$\frac{1}{i\hbar}[\psi_\sigma(\vec{x},t), M_{ab}] = -(x_a\nabla_b - x_b\nabla_a)\psi_\sigma(\vec{x},t)$$

(where now ∇_a is w.r.t. x_a)

5 Assume that

$$P_a |\mathcal{O}\rangle = 0, \langle\mathcal{O}| P_a = 0, M_{ab} |\mathcal{O}\rangle = 0, \langle\mathcal{O}| M_{ab} = 0.$$

Prove that $\langle\mathcal{O}| \psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle$ can only be a function of the coordinates $\vec{x}$ and $\vec{y}$ through the combination $|\vec{x} - \vec{y}| = \sqrt{(\vec{x} - \vec{y})^2}$.

Consider that expetation value of a commutator:

$$\begin{aligned}
&\frac{1}{i\hbar} \langle\mathcal{O}| \left[\psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t'), P_a\right] |\mathcal{O}\rangle \\
&= \frac{1}{i\hbar} \langle\mathcal{O}| \psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t')P_a |\mathcal{O}\rangle - \langle\mathcal{O}| P_a\psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle \\
&= \frac{1}{i\hbar} \langle\mathcal{O}| \psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t')(0) - (0)\psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle \qquad [\text{use } \langle\mathcal{O}| P_a = 0, P_a |\mathcal{O}\rangle = 0] \\
&= 0 - 0 \\
&= 0
\end{aligned}$$

Then consider $\left(\frac{\partial}{\partial x^a} + \frac{\partial}{\partial y^a}\right) \langle\mathcal{O}| \psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle$:

$$\begin{aligned}
&\left(\frac{\partial}{\partial x^a} + \frac{\partial}{\partial y^a}\right) \langle\mathcal{O}| \psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle \\
&= \langle\mathcal{O}| \frac{\partial}{\partial x^a}\psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle + \langle\mathcal{O}| \psi_\sigma(\vec{x}, t)\frac{\partial}{\partial y^a}\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle \\
&= \frac{1}{i\hbar} \langle\mathcal{O}| [\psi_\sigma(\vec{x}, t), \mathcal{Q}]\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle + \frac{1}{i\hbar} \langle\mathcal{O}| \psi_\sigma(\vec{x}, t)[\psi^{\dagger\rho}(\vec{y}, t'), \mathcal{Q}] |\mathcal{O}\rangle \\
&= \frac{1}{i\hbar} \langle\mathcal{O}| [\psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t')\mathcal{Q}] |\mathcal{O}\rangle
\end{aligned}$$

So in particular for P_a,

$$\begin{aligned}
0 &= \frac{1}{i\hbar} \langle\mathcal{O}| \left[\psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t'), P_a\right] |\mathcal{O}\rangle \\
&= \left(\frac{\partial}{\partial x^a} + \frac{\partial}{\partial y^a}\right) \langle\mathcal{O}| \psi_\sigma(\vec{x}, t)\psi^{\dagger\rho}(\vec{y}, t') |\mathcal{O}\rangle
\end{aligned}$$

And if we let $\vec{r} = \vec{x} + \vec{y}$,

$$\frac{\partial}{\partial r^a} = \frac{\partial}{\partial x^a} + \frac{\partial}{\partial y^a}$$

$$\implies \frac{\partial}{\partial r^a} \langle \mathcal{O} | \, \psi_\sigma(\vec{x}, t) \psi^{\dagger\rho}(\vec{y}, t') \, | \mathcal{O} \rangle = 0$$

This implies $\langle \mathcal{O} | \, \psi_\sigma(\vec{x}, t) \psi^{\dagger\rho}(\vec{y}, t') \, | \mathcal{O} \rangle$ depends on $\vec{x}$ and $\vec{y}$ through the combination $\vec{x} - \vec{y}$ rather than $\vec{x} + \vec{y}$.

Then, it is also true that

$$0 = \frac{1}{i\hbar} \langle \mathcal{O} | \left[\psi_\sigma(\vec{x}, t) \psi^{\dagger\rho}(\vec{y}, t'), M_{ab} \right] | \mathcal{O} \rangle$$

And finally to show this function only depends on the magnitude $|\vec{x} - \vec{y}|$, let's show that $(x_a \nabla_b - x_b \nabla_a) f(\vec{x}) = 0$ for every pair $(a, b) \implies f(\vec{x})$ depends only on $|\vec{x}|$.

Consider just x^1, x^2 for now.

$$(x^1 \nabla_2 - x^2 \nabla_1) f(x^1, x^2, \ldots) = 0 \implies f = f((x^1)^2 + (x^2)^2, \ldots)$$

Then x^1, x^2, x^3 (or equivalently for f, $(x^1)^2 + (x^2)^2, x^3$):

$$(x^2 \nabla_3 - x^3 \nabla_2) f((x^1)^2 + (x^2)^2, x^3, \ldots) = 0 \implies f = f((x^1)^2 + (x^2)^2 + (x^3)^2, \ldots)$$

And if we continue in the same way for however many coordinates we have, we find:

$$f = f(\sum_i (x^i)^2)$$

Or equivalently,

$$f = f(\sqrt{\sum_i (x^i)^2}) = f(|\vec{x}|)$$

Since we have a function of this type above, which satisfies $(x_a \nabla_b - x_b \nabla_a) f(\vec{x}) = 0$ for every pair (a, b), we can say $\langle \mathcal{O} | \, \psi_\sigma(\vec{x}, t) \psi^{\dagger\rho}(\vec{y}, t') \, | \mathcal{O} \rangle$ is only a function of $|\vec{x} - \vec{y}| = \sqrt{\sum_i (x_i - y_i)^2}$.

6 Show that the Galilean transformations defined above are a symetry of the theory defined by the action and Lagrangian density (3.5).

The Galilean transformations defined above:

$$\delta\psi_\sigma(\vec{x},t) = \left(-\vec{v}t\cdot\vec{\nabla} + \frac{i}{\hbar}m\vec{v}\cdot\vec{x}\right)\psi_\sigma(\vec{x},t)$$
$$\delta\psi^{\sigma\dagger}(\vec{x},t) = \left(-\vec{v}t\cdot\vec{\nabla} - \frac{i}{\hbar}m\vec{v}\cdot\vec{x}\right)\psi^{\sigma\dagger}(\vec{x},t)$$

To show this is a symmetry, we want to show $\delta\mathcal{L}(\vec{x},t) = \frac{\partial}{\partial t}R(\vec{x},t) + \vec{\nabla}\cdot\vec{J}(\vec{x},t)$ for some R, $\vec{J}$. The Lagrangian:

$$\mathcal{L} = \frac{i\hbar}{2}\psi^{\sigma\dagger}\frac{\partial}{\partial t}\psi_\sigma - \frac{i\hbar}{2}\frac{\partial}{\partial t}\psi^{\sigma\dagger}\psi_\sigma - \frac{\hbar^2}{2m}\nabla\psi^{\sigma\dagger}\cdot\nabla\psi_\sigma - \frac{\lambda}{2}\left(\psi^{\sigma\dagger}\psi_\sigma\right)^2$$

This is does not explicitly depend on $\vec{x}$ or t, so we have

$$\begin{aligned}
\delta\mathcal{L} &= \left(-\vec{v}t\cdot\vec{\nabla} + \frac{i}{\hbar}m\vec{v}\cdot\vec{x}\right)\mathcal{L} \\
&= \left(-v_a t\nabla_a\mathcal{L} + \frac{i}{\hbar}mv_a x_a\mathcal{L}\right) \\
&= 0 + \frac{\partial}{\partial t}\left(t\frac{i}{\hbar}mv_a x_a\mathcal{L}\right) \\
&= \frac{\partial}{\partial t}\left(t\frac{i}{\hbar}m\vec{v}\cdot\vec{x}\mathcal{L}\right) + \nabla\cdot\vec{0}
\end{aligned}$$

So these transformations are a symmetry, with $R = \frac{it}{\hbar}m\vec{v}\cdot\vec{x}\mathcal{L}$, $\vec{J} = 0$.