2013

1. Order of magnitude estimation

$$F_D = \frac{1}{2} \rho_{fluid} V^2 C_D A$$

$\rho_{fluid} \sim \frac{1kg}{m^3}$. (air at STP, although will be less dense at top of atmosphere, will ignore for now)

$$\frac{1}{2}\frac{GMm}{r^2} = \frac{mv_{orb}^2}{r}$$

$$v_{orb} = \frac{GM}{2r}$$

Don't have this so instead

$$\omega = \frac{2\pi}{365\ days}$$

$$V = r\omega$$
$$= \frac{1AU \times 2\pi}{365 days}$$

$$\approx 30km/s$$

At terminal vel:

$$mg = F_D$$

$$\Rightarrow v_{term} = \sqrt{\frac{2mg}{\rho_{air} A C_D}}$$

Assume that KE lost during slow down is given to heat meteor

$$\frac{1}{2} mv_{orb}^2 - \frac{1}{2} mv_{term}^2 = \Delta E$$

$A = \pi r^2$

$$m = \rho_{Fe} \frac{4}{3} \pi r^3$$

Energies given give energy required for state change, but also have to heat up iron to its melting point, which I don't know? Would also need heat capacity...

Going to assume energy required to vaporize meteor of mass $M = (250 + 6000) \times M$ (in g)

Although this doesn't seem right

meant to guess these?

$T_{melt,Fe} = 1538°C$ $\qquad$ $T_{boil,Fe} = 2862°C$

Huge amount of E produced will go into heating not just state change

$$C_{v,Fe} = 0.45 \frac{J}{g\,°C} = 450 \frac{J}{kg\,K}$$

Also have to worry about change in GPE

$$GPE = mg\Delta h$$

$$\Delta h = \frac{V_{term}^2 - V_{orb}^2}{2a}$$

$$a = F_D - mg$$

At ~~Earth's~~ surface (where $\Delta h = h = 10km$)

$\Delta GPE = mg\Delta h$

$\Delta KE = \frac{1}{2} m (v_{orb}^2 - v_{term}^2)$

Need $\Delta KE + \Delta GPE \geq C_v \Delta T m + (6000 + 250) m$.

Assume meteor in space is at $T \approx 0K$

$(v_{orb}^2 - v_{term}^2) + 2g\Delta h = 2C_v \Delta T + 2\times 6.25 \times 10^6$

$C_v = 450 \frac{J}{kg K}$ $\Delta T = 2862°C \approx 2500K$

$\Longrightarrow$ $v_{term}^2 = v_{orb}^2 + 2g\Delta h - 2C_v \Delta T - 2\times 6.25\times 10^6$

$A_{meteor} = \frac{2mg}{\rho_{air} C_D (v_{orb}^2 + 2g\Delta h - 2C_v\Delta T - 2\times 6.25\times 10^6)}$

$C_D \approx 1$

$m = \rho_{Fe} V$

$\frac{\pi r^2}{\frac{4}{3}\pi r^3}$

$\frac{3}{4 r_{met}} = \frac{2g\rho_{Fe}}{\rho_{air}(v_{orb}^2 + 2g\Delta h - 2C_v\Delta T - 2\times 6.25\times 10^6)}$

$r_{met} = \frac{3}{8}\frac{\rho_{air}(v_{orb}^2 + 2g\Delta h - 2C_v\Delta T - 2\times 6.25\times 10^6)}{g\rho_{Fe}}$

$= 4230 m$.

$= 4.23 km$ across Huge!

Hilroy

Check whether $C_v \Delta T$ term was negligable?

→ Seems not?

Did not consider the fact that, as outer layers vaporize, they will be lost so mass will be gradually decreasing.

Want to find a $\frac{dm}{dz}$ and see when $m = 0$ at ground

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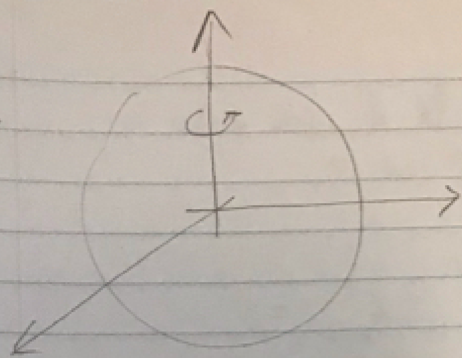
2.



$m = 0.1\,kg$ $\quad r = 5\,cm$

$\omega = \frac{10 \times 2\pi}{1} = 20\pi$ rad/s

$I = \frac{2}{5} m r^2$

A. Heisenberg's Uncertainty Principle

$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

Similarly $\quad \Delta L \Delta \theta \geq \frac{\hbar}{2}$

$$L = I\omega = \frac{2}{5} m r^2 \omega = 6.28 \times 10^{-3}$$

Limit $\quad \Delta\theta \geq \frac{\hbar}{2L}$

$$\geq 8.39 \times 10^{-33} \text{ rad}$$

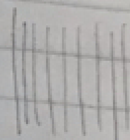
I think I can just put in L for ΔL

Check units $\quad \frac{J s^2}{kg\, m^2} = 0$ ✓

Very high accuracy (as large object)

$$\Delta L^2 = \langle L^2 \rangle - \langle L \rangle^2$$

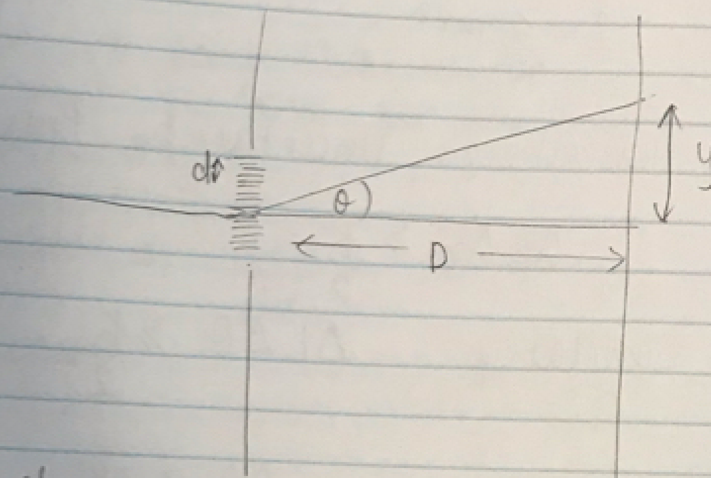
B.



$n\lambda = d\sin\theta$ for min

Diffraction in y direction

$y = D\tan\theta$



Make small angle approx

$\tan\theta \approx \sin\theta \rightarrow y \approx \frac{n\lambda D}{d}$

Energy time uncertainty

$$\Delta E \Delta t \geqslant \frac{\hbar}{2}$$

Energy resolution $\Delta E = \frac{hc}{\lambda^2} \Delta\lambda$

$E = hf$

$c = f\lambda$ $\quad E = \frac{hc}{\lambda} \quad \frac{dE}{d\lambda} = -\frac{hc}{\lambda^2}$

Resolving power of grating $\frac{\lambda}{\Delta\lambda} = N$

derivation not simple but on HyperPhysics

2012

$$\hbar = \frac{h}{2\pi}$$

$$\Delta t \approx \frac{\hbar \lambda^2}{2\, hc\, \Delta\lambda} = \frac{\lambda^2}{4\pi c\, \Delta\lambda}$$

$$= \frac{\lambda N}{4\pi c} = \frac{400\text{nm} \times 1\times10^4}{4\times\pi\times 3\times10^8}$$

$$= 1.06\times10^{-12}\ \text{s}.$$

Confused because asks for longest lifetime → surely shortest?

Time resolution = 1.06×10^{-12} s.

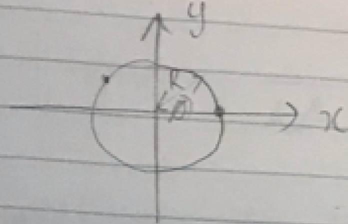
$$\int_0^{2\pi} |\psi^2| \partial\phi = 1$$

$$\int_0^{2\pi} A^2 \partial\phi = 1$$

$$A^2 2\pi = 1$$

(no potential)

4.



$$\frac{-\hbar^2}{2m} \frac{1}{R^2} \frac{\partial^2 \psi}{\partial \phi^2} = E\psi$$

$$\frac{\partial^2 \psi}{\partial \phi^2} + \frac{2mER^2}{\hbar^2}\psi = 0 \qquad w^2 = \frac{2mER^2}{\hbar^2}$$

$$\psi = e^{\pm iw\phi}$$

Check: $\frac{\partial^2\psi}{\partial\phi^2} = -w^2\psi$ ✓

Two eigenstates: $\psi_\pm = \frac{1}{\sqrt{2\pi}} e^{\pm iw\phi}$

Eigenvalue: $E_\pm = \frac{\hbar^2 w^2}{2mR^2}$

So these are degenerate

Boundary condition: $\psi(\phi) = \psi(\phi + 2\pi n)$

$$e^{\pm iw\phi} = e^{\pm iw(\phi + 2\pi n)}$$

$$\Rightarrow e^{\pm iw 2\pi} = 1$$

So can write $w = 0, \pm 1, \pm 2, \pm 3 \ldots$

With degeneracies = 1 for ground state, and two for all others

B. $\Delta E^1 = \langle \psi | H' | \psi \rangle$ $\qquad F = -\nabla V$

$\vec{E} = E_0 \hat{x}$ $\qquad V = -qE_0 x.$ $\qquad \psi_0 = 1$

$\qquad\qquad = -qE_0 R\cos\phi$

$$\Delta E^1 = qE_0 \int_{-\infty}^{\infty} x \, dx = 0.$$

$$\Delta E^2 = -\sum_{n=1}^{\infty} \frac{|\langle \psi_n | qE_0 x | \psi_0 \rangle|^2}{E_0 - E_n}$$

$E_0 = 0 \qquad \psi_0 = 1 \qquad \psi_n = e^{\pm in\phi}$

$$\Delta E^2 = \sum_{n=1}^{\infty} \frac{|qE_0 x e^{\pm in\phi}|^2}{E_n \, 2\pi}$$

modulus of this = 1

$$= \frac{q^2 E_0^2 R^2 \, 2mR^2}{4 \hbar^2 \, 2\pi} \sum_{n=1}^{\infty} \frac{|e^{in\phi}(e^{i\phi} + e^{-i\phi})|^2}{n^2} \qquad x = R\cos\phi$$

$$= \frac{E_0^2 \, mR^4 q^2}{2\hbar^2 \, 2\pi} \sum_{n=1}^{\infty} \frac{|e^{i(n+1)\phi} + e^{i(n-1)\phi}|^2}{n^2}$$

Want the first non-zero term, so set $n = 1, +1$

$$(1+e^{-i2\phi})(1+e^{+i2\phi})$$

$$\Delta E = \frac{E_0^2 mR^4 q^2}{2\hbar^2 \, 2\pi} \left[\left(1 + e^{-i2\phi}\right)^2 + \left(1 + e^{i2\phi}\right)^2 \right]$$

$$= \frac{E_0^2 mR^4 q^2}{4\pi \hbar^2} \left(1 + e^{i2\phi} + e^{-i2\phi} + 1 + 1 + e^{i2\phi} + e^{-i2\phi} + 1 \right)$$

Hilroy

$$\Delta E = \frac{E_0^2 m R^4 q^2}{4\pi\hbar^2}\left(4 + 2\cos 2\phi\right)$$

concerned about ϕ dependence

C. Polarizability of system

dipole moment $\vec{p} = q\langle x\rangle$

$$= \frac{\langle H'\rangle}{E_0} = \frac{E_0 m R^4 q^2}{4\pi\hbar^2}$$

$$\text{Polarizability} = \frac{|\vec{p}|}{E_0} = \frac{m R^4 q^2}{4\pi\hbar^2}$$

Hilroy

5. $\hat{H} = J\hat{S}_1 \cdot \hat{S}_2$

A. $(S_1 + S_2)^2 = S_1^2 + S_2^2 + 2S_1 \cdot S_2$

$$S_1 \cdot S_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2)$$

Time operator

$$U(t) = e^{-\frac{i\hat{H}t}{\hbar}}$$

$|1,0\rangle$

$$\hat{H}|\psi\rangle = J\hat{S}_1 \cdot \hat{S}_2 \frac{|\uparrow\downarrow + \downarrow\uparrow\rangle}{\sqrt{2}}$$

$$= \frac{\hbar^2 J}{2}\left(0 - \frac{1}{2} \times \frac{3}{2} + \frac{1}{2}\left(\frac{1}{2}\right) + 0 + \frac{1}{2}\left(\frac{1}{2}\right) - \right.$$

$$= \frac{\hbar^2 J}{4} \frac{|\uparrow\downarrow + \downarrow\uparrow\rangle}{\sqrt{2}}$$

$|0,0\rangle$

$$\hat{H}\frac{|\uparrow\downarrow - \downarrow\uparrow\rangle}{2} = -\frac{3\hbar^2 J}{4} \frac{|\uparrow\downarrow - \downarrow\uparrow\rangle}{\sqrt{2}}$$

Can write initial state

$$|\uparrow\downarrow\rangle = \frac{1}{2}\left(\frac{|\uparrow\downarrow + \downarrow\uparrow\rangle}{\sqrt{2}} e^{-\frac{i\hbar J}{4}t} + |\uparrow\downarrow - \downarrow\uparrow\rangle e^{\frac{+3i}{4}}\right.$$

$$|\langle\uparrow\uparrow|\uparrow\downarrow\rangle|^2 = 0.$$

$$|\langle\downarrow\downarrow|\uparrow\downarrow\rangle|^2 = 0$$

$$\hbar J t = s$$

$$|\langle\downarrow\uparrow|\uparrow\downarrow\rangle|^2 = \left|\frac{1}{2}\left(\frac{1}{\sqrt{2}} e^{-\frac{i\hbar J t}{4}} - \frac{1}{\sqrt{2}} e^{+\frac{3i\hbar J t}{4}}\right)\right|^2$$

$$= \frac{1}{8}\left(e^{-\frac{is}{4}} - e^{\frac{3is}{4}}\right)\left(e^{+\frac{is}{4}} - e^{-\frac{3is}{4}}\right)$$

$$= \frac{1}{8}\left(1 - e^{is} - e^{-is} + 1\right)$$

$$= \frac{1}{8}\left(2 - 2\cos s\right)$$

$$= \frac{1}{4}\left(1 - \cos(\hbar J t)\right)$$

B. Now assume $J(t) = J_0 \cos\omega t$

I think you would just plug in $J = J_0 \cos\omega t$ into what I just found

$$\Rightarrow P(\downarrow\uparrow) = \frac{1}{4}\left(1 - \cos(\hbar J_0 \cos\omega t \; t\right)$$

$$P(\uparrow\uparrow) = 0$$

$$P(\downarrow\downarrow) = 0$$

6. N independent spin-$\frac{1}{2}$ particles

A. $\hat{H} = -\vec{\mu}.\vec{B}$

$\vec{\mu} = \gamma \vec{S}$

$N - N_\uparrow \frac{1}{2}\left(N - \frac{E}{\mu B}\right)$

Total energy $E_T = \mu B(N_\downarrow - N_\uparrow) = \mu B(N - 2N_\uparrow)$

How many microstates for a given U?

$$\Omega(N_\uparrow) = \frac{N!}{N_\uparrow!(N-N_\uparrow)!} \times \underbrace{2s+1}$$

number of spin states for each level

$$= 2 \times \frac{N!}{N_\uparrow!(N-N_\uparrow)!}$$

can also write as

$$\Omega(E) = \frac{2N!}{\frac{1}{2}\left(N-\frac{E}{\mu B}\right)! \cdot \frac{1}{2}\left(N+\frac{E}{\mu B}\right)!}$$

B. $Z = \sum_{N_\uparrow=0}^{N} e^{-\beta\mu B(N-2N_\uparrow)}$

$$= e^{-\beta\mu BN} \sum_{N_\uparrow=0}^{N} e^{+2\beta\mu BN_\uparrow} = \frac{e^{-\alpha N}\left(1-e^{2\alpha(N+1)}\right)}{1-e^{2\alpha}}$$

$E = \mu B(N - 2N_E)$

$$P(E) = \frac{e^{-\beta E}}{Z}\Omega(E)$$

$\beta\mu B = \alpha$

$$= \frac{e^{+2\beta\mu BN_E}}{\sum_{N_\uparrow=0}^{N} e^{2\beta\mu BN_\uparrow}}\Omega(E)$$

$$= \frac{e^{2\alpha N_E}}{1 + e^{2\alpha} + e^{2\alpha\times 2} + e^{2\alpha\times 3} + \ldots}\Omega(E)$$

$$P(E) = \frac{e^{2\alpha N_E}(1 - e^{2\alpha})}{1 - (e^{2\alpha})^{N+1}} \Omega(E)$$

$$= \frac{e^{2\alpha N_E} - e^{2\alpha(N_E+1)}}{1 - e^{2\alpha(N+1)}} \Omega(E)$$

$$P(E) = \frac{e^{2\alpha N_E}(1 - e^{2\alpha})}{1 - e^{2\alpha(N+1)}} \frac{2N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)! \, \frac{1}{2}\left(\frac{E}{\mu B} - N\right)!}$$

C. Most probable E

$$\overline{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{Z}\mu B \frac{\partial Z}{\partial \alpha}$$

$$\alpha = \beta \mu B$$

$$d\alpha = \mu B \, d\beta$$

$$= -\frac{1}{Z}\mu B \left(\frac{}{(1-e^{2\alpha})^2} \right.$$

Try again

$$Z_1 = e^{-\frac{\mu B \hbar}{2}} + e^{\frac{\mu B \hbar}{2}}$$ call $\mu B \hbar = \alpha$.

$$= 2\cosh\alpha$$

$$Z_N = \frac{Z_1^N}{N!} = \frac{2\cosh^N\alpha}{N!}$$

~~need this unless distinct~~

$$P(E) = \frac{N! e^{-\beta E}}{2\cosh^N\alpha}$$

C. $$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial\beta} = -\frac{\mu B}{Z}\frac{\partial}{\partial\alpha}\left(\frac{2\cosh^N\alpha}{N!}\right)$$

most probable

$$= -\frac{\mu B}{Z_N}\frac{2N}{N!}\cosh^{N-1}\alpha\sinh\alpha.$$

Or is this only true for one particle? Nope seems to be true for many

$$= -\frac{\mu B}{Z_N} N \frac{2\cosh^N\alpha}{N!}\frac{\sinh\alpha}{\cosh\alpha}$$

$$= -\mu B N \tanh\alpha.$$

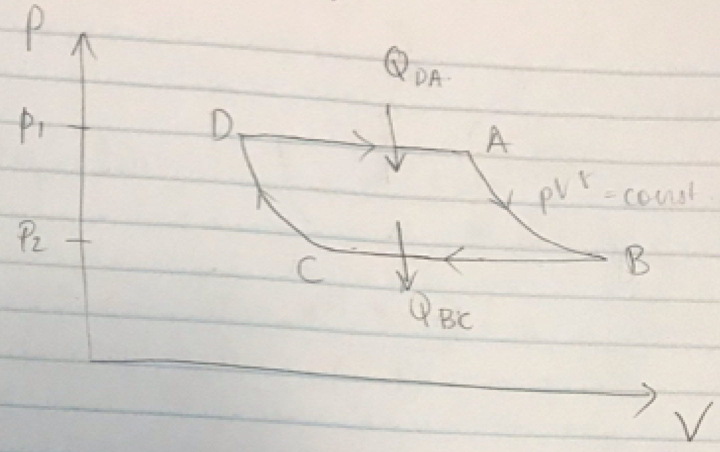
D. Distribution of E for large N.

$$P(E) = \frac{N!\, e^{-\beta E}}{(e^{\alpha} + e^{-\alpha})^N}$$

Hilroy

Reversible Brayton Engine

9.



$$\eta = \frac{W_{ABCD}}{Q_{DA}}$$

Calculate η in terms of C_p, C_v and $\frac{p_1}{p_2}$

Work:

A → B $\quad W = -C_v \Delta T$

$\quad = -C_v (T_B - T_A)$ ← step to remember

B → C $\quad W = \int_{V_B}^{V_C} p\,dV = p_2 (V_C - V_B)$

C → D $\quad W = -C_v (T_D - T_C)$

D → A $\quad W = p_1 (V_A - V_D)$

So total

$$W = p_1 (V_A - V_D) + p_2 (V_C - V_B) - C_v (T_B - T_A + T_D - T_C)$$

$$= nR (T_A - T_D + T_C - T_B) - C_v (T_B - T_A + T_D - T_C)$$

$$= C_v + nR (T_A - T_D + T_C - T_B)$$

$$C_v + nR = C_p.$$

$$W = C_p\left(T_A - T_D + T_C - T_B\right)$$

$$Q_{DA} = C_p\left(T_A - T_D\right)$$

$$\eta = \frac{W}{Q_{DA}} = \frac{T_A - T_D + T_C - T_B}{T_A - T_D}.$$

$pV^\gamma = \text{const}$ along $C \to D$ and $A \to B$ ☆ step to remember.

$$\Rightarrow T p^{\frac{1-\gamma}{\gamma}} = \text{const}$$

$$T_A p_1^{\frac{1-\gamma}{\gamma}} = T_B p_2^{\frac{1-\gamma}{\gamma}} \qquad \gamma = \frac{C_p}{C_v}$$

$$T_C p_2^{\frac{1-\gamma}{\gamma}} = T_D p_1^{\frac{1-\gamma}{\gamma}}$$

$$T_B = T_A\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}}$$

$$T_C = T_D\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}}$$

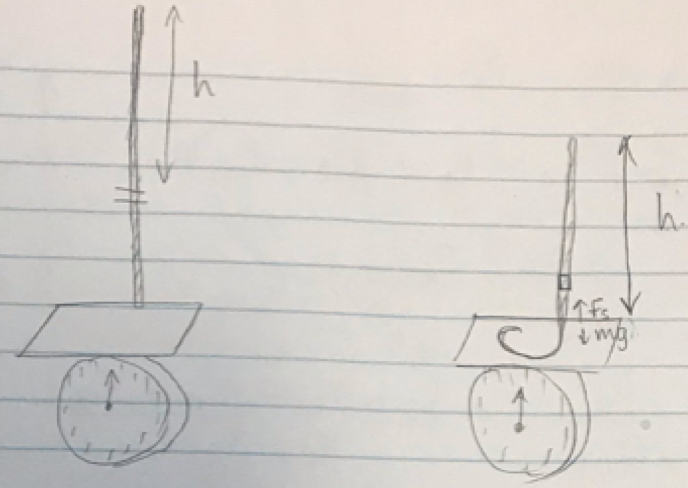
$$\eta = \frac{T_A - T_D - \left(T_A - T_D\right)\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}}}{T_A - T_D}$$

$$= 1 - \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} \quad ✓✓ \qquad \gamma = \frac{C_p}{C_v}$$

Hilroy

$\lambda = \frac{M}{L}$

10.



Force on scale will be combination of weight of rope resting & force from falling rope hitting scale

$$F_{resting} = \lambda(L-h)g$$

$$F_{hitting} = \frac{dp}{dt}$$

Momentum of small section Δm = $\Delta m v$

When $L-h$ is lying on scale already:

$$\cancel{\Delta m} g(L-h) = \frac{1}{2}\cancel{\Delta m} v^2$$ (saying GPE is zero at scale

$$v = \sqrt{2g(L-h)}$$

$$F_{hitting} = \frac{dm}{dt} v$$

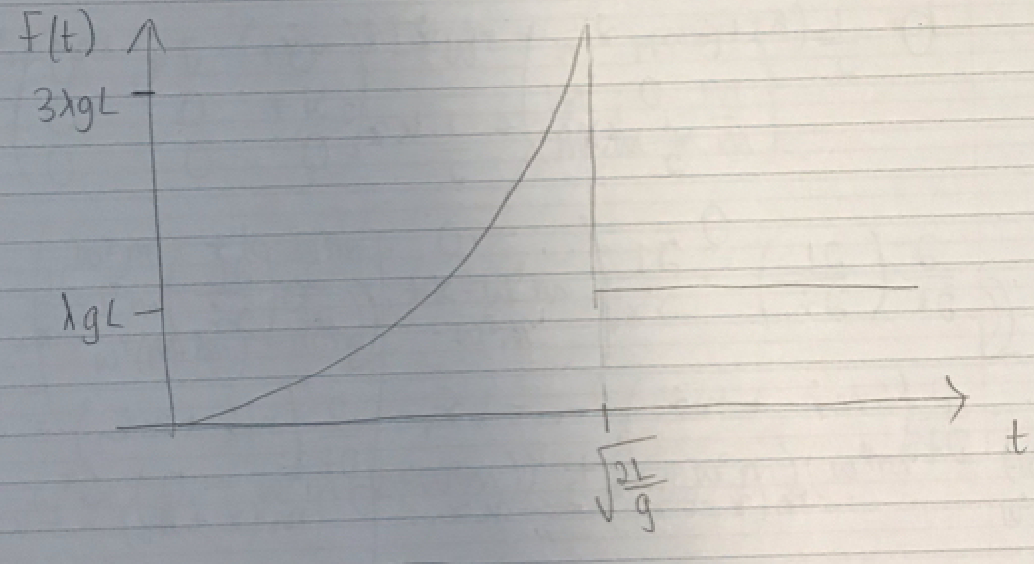
$$= \lambda \frac{dh}{dt} v$$

$$F_{hitting} = \lambda v^2$$
$$= 2\lambda g(L-h)$$

And so $F = 3\lambda g(L-h)$ adding these two together

$$h(t) = L - \frac{1}{2}gt^2$$

$$\Longrightarrow F = +\frac{3}{2}\lambda g^2 t^2$$

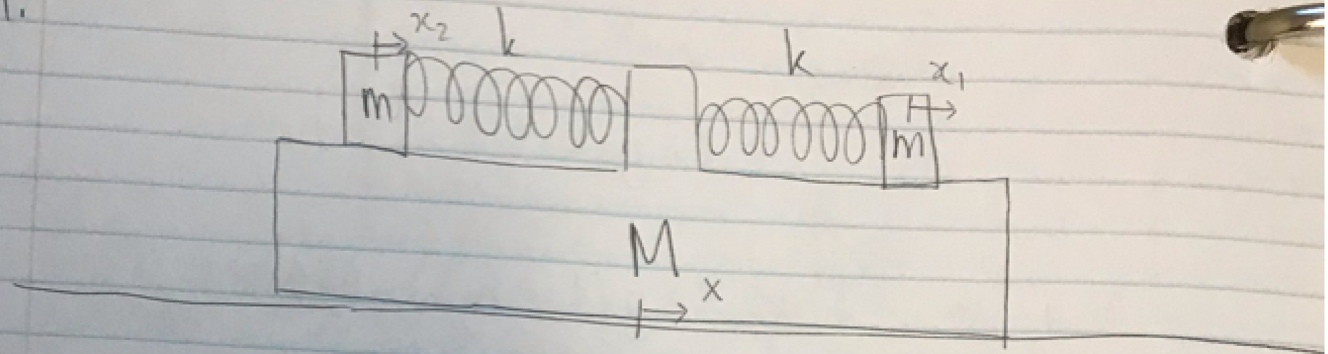


Time to fall $t = \sqrt{\frac{2L}{g}}$

Then $F = \frac{3}{\cancel{2}}\lambda g^{\cancel{2}}\frac{\cancel{2}L}{\cancel{g}} = 3\lambda g L$

After which it will sharply drop to $\lambda g L$ (Just the weight of the rope)

11.



$$T = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} m(\dot{x} + \dot{x}_1)^2 + \frac{1}{2} m(\dot{x} + \dot{x}_2)^2$$

$$V = \frac{1}{2} k x_1^2 + \frac{1}{2} k x_2^2$$

$$L = \frac{1}{2}(M + 2m)\dot{x}^2 + m\dot{x}(\dot{x}_1 + \dot{x}_2) + \frac{1}{2} m(\dot{x}_1^2 + \dot{x}_2^2)$$
$$- \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \frac{\partial L}{\partial x_1}$$

$$\frac{\partial}{\partial t}\left(m\dot{x} + m\dot{x}_1\right) = -kx_1$$

$$m(\ddot{x} + \ddot{x}_1) = -kx_1$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = \frac{\partial L}{\partial x_2}$$

$$\frac{\partial}{\partial t}\left(m\dot{x} + m\dot{x}_2\right) = -kx_2$$

$$m(\ddot{x} + \ddot{x}_2) = -kx_2.$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \Rightarrow \frac{\partial}{\partial t}\left((M + 2m)\dot{x} + m(\dot{x}_1 + \dot{x}_2)\right)$$
$$= 0$$

$$(M + 2m)\ddot{x} + m(\ddot{x}_1 + \ddot{x}_2) = 0.$$

2013

11.
$$\underbrace{\begin{pmatrix} m & m & 0 \\ m & 0 & m \\ M+2m & m & m \end{pmatrix}}_{\underline{\underline{M}}} \underbrace{\begin{pmatrix} \ddot{x} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix}}_{\underline{\ddot{x}}} = \underbrace{\begin{pmatrix} 0 & -k & 0 \\ 0 & 0 & -k \\ 0 & 0 & 0 \end{pmatrix}}_{-\underline{\underline{K}}} \underbrace{\begin{pmatrix} x \\ x_1 \\ x_2 \end{pmatrix}}_{\underline{x}}$$

$$-w^2 \underline{\underline{M}}\, \underline{x} = \underline{\underline{K}}\, \underline{x}$$

$$\left| \underline{\underline{K}} - w^2 \underline{\underline{M}} \right| = 0$$

$$\begin{pmatrix} 0 & +k & 0 \\ 0 & 0 & +k \\ 0 & 0 & 0 \end{pmatrix} - w^2 \begin{pmatrix} m & m & 0 \\ m & 0 & m \\ M+2m & m & m \end{pmatrix} = 0$$

$$\begin{pmatrix} -w^2 m & +k - w^2 m & 0 \\ -w^2 m & 0 & +k - w^2 m \\ -w^2(M+2m) & -w^2 m & -w^2 m \end{pmatrix} = 0$$

$$-w^2 m \left(0 - w^2 m(-k + w^2 m)\right) + (-k + w^2 m)\left(w^4 m^2 - (-k + w^2 m)\, w^2(M+2m)\right)$$

$$= 0$$

$$2w^{\cancel{4}\,2} m^2 (-k + w^2 m) - (-k + w^2 m)^2 \cancel{w^2} (M+2m) = 0.$$

$$\Rightarrow \quad w = 0 \qquad w^2 = \frac{+k}{m} \qquad w = \sqrt{\frac{k}{m}}$$

$$2w^2m^2 - (-k + w^2m)(M+2m) = 0$$

$$\cancel{2w^2m^2} - kM - 2km + w^2mM - \cancel{2w^2m^2} = 0$$

$$w = \sqrt{\frac{k(M+2m)}{mM}}$$

So normal modes with $w_1 = 0$, $w_2 = \sqrt{\frac{k}{m}}$ and $w_3 = \sqrt{\frac{k(M+2m)}{mM}}$.

Find motion:

$$\begin{pmatrix} -w^2m & k - w^2m & 0 \\ -w^2m & 0 & k - w^2m \\ -w^2(M+2m) & -w^2m & -w^2m \end{pmatrix} \begin{pmatrix} x \\ x_1 \\ x_2 \end{pmatrix} = 0$$

For w_1

$$\begin{pmatrix} 0 & k & 0 \\ 0 & 0 & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ x_1 \\ x_2 \end{pmatrix} = 0$$

$x_1 = x_2 = 0$

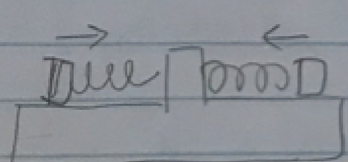
For w_2

$$\begin{pmatrix} -k & 0 & 0 \\ -k & 0 & 0 \\ -k\left(\frac{M+2m}{m}\right) & -k & -k \end{pmatrix} \begin{pmatrix} x \\ x_1 \\ x_2 \end{pmatrix} = 0$$

$-kx = 0$

$\Rightarrow x = 0$

$$-x\left(\frac{M+2m}{m}\right) = x_1 + x_2$$



$x_1 = -x_2$

Base is stationary and masses move in and out in opposition to each other.

2012

$$k - \frac{k(M+2m)}{M}$$

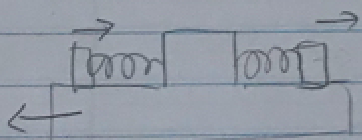
For ω_3

$$\begin{pmatrix} \frac{-k(M+2m)}{M} & \frac{-k2m}{M} & 0 \\ \frac{-k(M+2m)}{M} & 0 & -\frac{k2m}{M} \\ \frac{-k(M+2m)^2}{mM} & \frac{-k(M+2m)}{M} & \frac{-k(M+2m)}{M} \end{pmatrix} \begin{pmatrix} x \\ x_1 \\ x_2 \end{pmatrix} = 0$$

$$2m x_1 = (M+2m)x = 2m x_2$$

$$\left(\frac{M+2m}{m}\right)x + x_1 + x_2 = 0$$

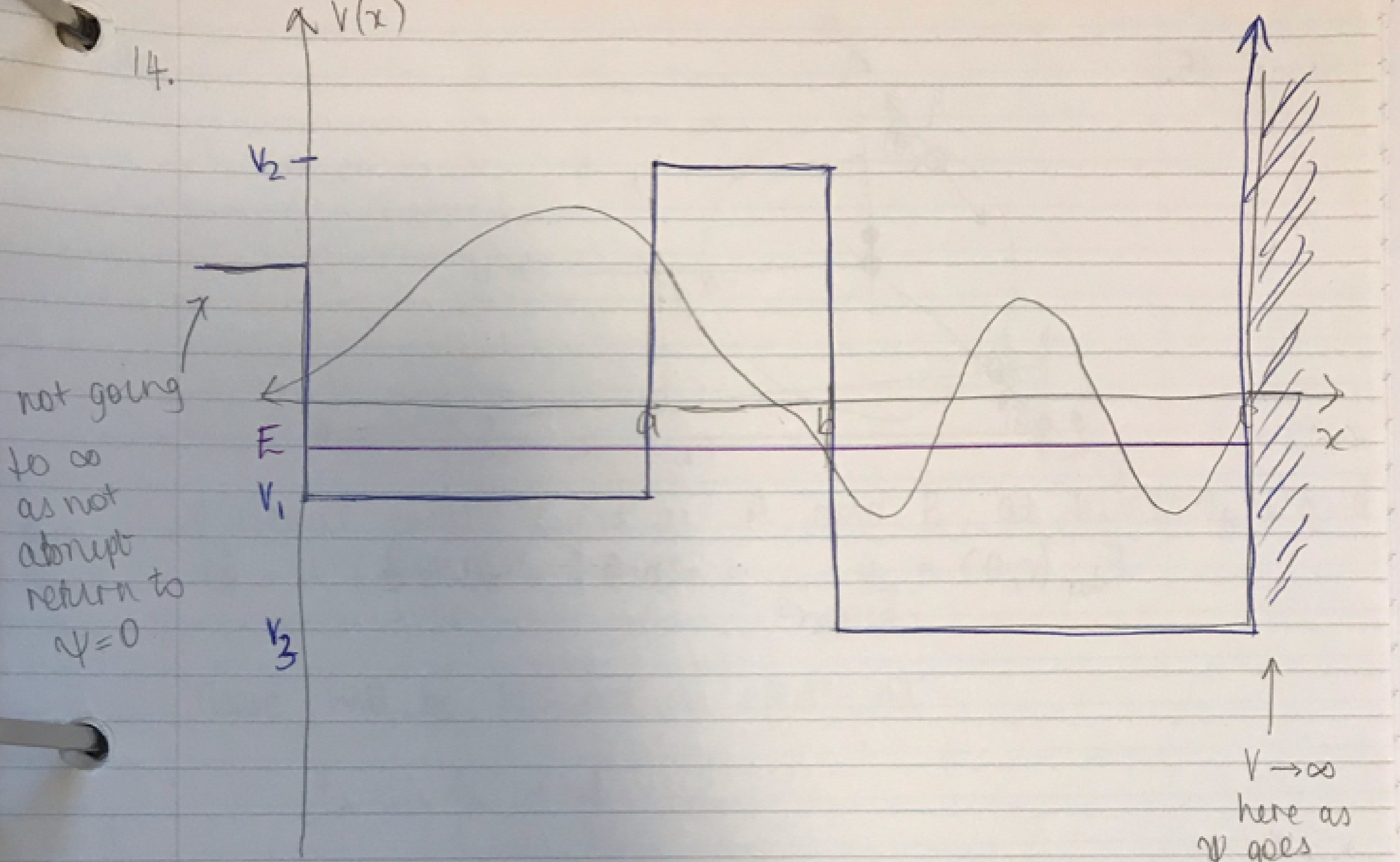
$x_1 = x_2$ $\qquad$ $x = \frac{-2m}{M+2m}$



Masses move both the same way, and base moves the opposite direction, a distance $\frac{2m}{M+2m}$.

2012

14.



$$k = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

$$k = 2\pi/\lambda$$

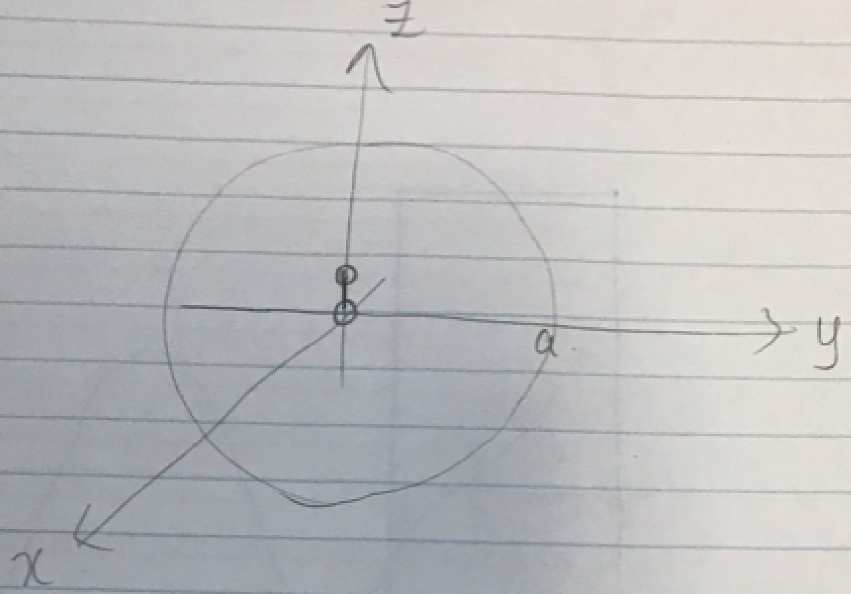
Can see longer λ on LHS, so smaller k, so V must be smaller (same E across whole state)

Can see exponential decay of wavefunction between points a and b $\Rightarrow$ $V > E$.

Lower potential V_3 as shorter wavelength.

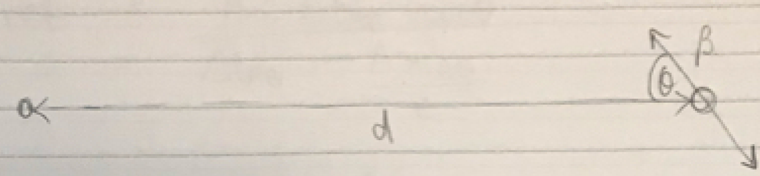
Seems that have bound state in both wells, so must have E higher than V_1 and V_3 but lower than $V(x<0)$ and V_2.

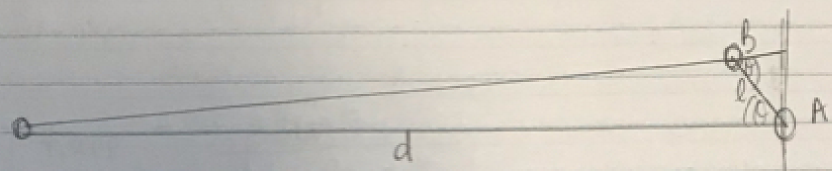
15.



$$E_{dip}(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{\underline{r}} + \sin\theta\,\hat{\underline{\theta}}\right)$$

16.





Material emits light at A and B, at times $t'_A = 0$ and t'_B in it's rest frame

These will be recieved on Earth at

$c = \frac{d}{t}$

$$t_A = t'_A + \frac{d}{c}$$

$$t_B = \frac{d - \Delta x'_{AB}}{c} + t'_B$$

$\beta = \frac{l}{\Delta t'_{AB} c}$

$$\Delta x'_{AB} = \beta c \Delta t'_{AB} \cos\theta$$

$$\Delta y'_{AB} = \beta c \Delta t'_{AB} \sin\theta$$

$$\Delta y_{AB} = \beta c \Delta t'_{AB} \sin\theta$$

$\Rightarrow$ Apparent velocity (transverse) $= \frac{\Delta y_{AB}}{\Delta t_{AB}}$

2012

$$\gamma^2 - \gamma^2\beta^2 = 1$$

$$\text{rel} = \frac{\beta c \Delta t'_{AB} \sin\theta}{\Delta t'_{AB} - \frac{\Delta x'_{AB}}{c}}$$

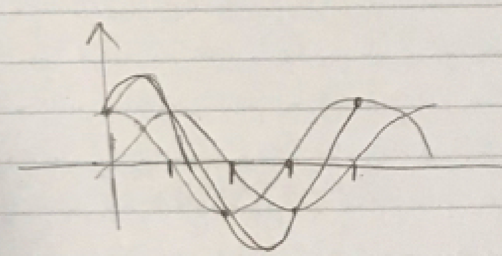
$$= \frac{\beta c \sin\theta}{1 - \beta\cos\theta}$$

$$\beta_{T,app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}$$

Find range where appears superluminal:

$$\frac{\beta\sin\theta}{1-\beta\cos\theta} > 1$$

$$\beta(\sin\theta + \cos\theta) > 1$$



Find turning points

$$\left(1 - \cos^2\theta + \cos\theta\right) - \frac{1}{\beta} = 0$$

$$\cos^2\theta - \cos\theta + \left(\frac{1}{\beta} - 1\right) = 0$$

$$\cos\theta = \frac{+1 \pm \sqrt{1 + 4 \times 1 \times \left(1 - \frac{1}{\beta}\right)}}{2}$$

$$= \frac{1 \pm \sqrt{1 + 4 - 4/\beta}}{2}$$

$$\cos\theta = \frac{1 \pm \sqrt{5 - 4/\beta}}{2}$$ ← within this range

Try for max $\beta = 1$

$$\rightarrow \cos\theta = \frac{1 \pm \sqrt{5-4}}{2}$$

$$= 0 \text{ or } 1$$

$$\theta = 0 \text{ or } \frac{\pi}{2}$$

Clear from sketch that for $\beta=1$,

$\theta = 0 \rightarrow \frac{\pi}{2}$ will look superluminal

Largest possible velocity

$$\frac{\partial \beta_{T,app}}{\partial \theta} = \frac{(1-\beta\cos\theta)\beta\cos\theta - \beta\sin\theta\,\beta\sin\theta}{(1-\beta\cos\theta)^2}$$

$$= \frac{\beta\cos\theta - \beta^2(\cos^2\theta + \sin^2\theta)}{(1-\beta\cos\theta)^2}$$

$$= 0$$

$$\beta\cos\theta - \beta^2 = 0$$

$$\beta = \cos\theta$$

$$\Rightarrow \beta_{T,app} = \frac{\cos\theta\sin\theta}{1-\cos^2\theta} = \cot\theta = \cot(\cos^{-1}\beta)$$

Hilroy

Or, more neatly

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\beta^2}$$

$$= \frac{1}{\gamma}$$

$$\Rightarrow \quad v_{T,app} = \frac{\beta/\gamma}{1-\beta^2} = \frac{\gamma^2\beta}{\gamma} = \underline{\underline{\gamma\beta}}$$

3. $F = (\vec{m} \cdot \vec{\nabla})\vec{B}$ 4 possible spin states

fraction of total beam

$\frac{1}{6}$ $|1,1\rangle$ $|\uparrow\uparrow\rangle$

$\frac{1}{6}$ $|1,-1\rangle$ $|\downarrow\downarrow\rangle$

$\frac{1}{6}$ $|1,0\rangle$ $\frac{|\uparrow\downarrow + \downarrow\uparrow\rangle}{\sqrt{2}}$

} orthopositronium

$\frac{1}{2}$ $|0,0\rangle$ $\frac{|\uparrow\downarrow - \downarrow\uparrow\rangle}{\sqrt{2}}$ - parapositronium

$\vec{m} = \gamma \vec{S}$

A. Equal number in ortho & para. Undeflected will be any $m_s = 0$ states

Fraction undeflected $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$\frac{2}{3}$ of original particles will pass through first beam stop.

Second B field in x direction.

State is $\frac{1}{2} : \frac{|\uparrow\downarrow - \downarrow\uparrow\rangle}{\sqrt{2}}$, $\frac{1}{6} : \frac{|\uparrow\downarrow + \downarrow\uparrow\rangle}{\sqrt{2}}$

Write these in x-basis: $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\chi_+^{(x)} + \chi_-^{(x)}\right)$

$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\chi_+^{(x)} - \chi_-^{(x)}\right)$

$$|\uparrow\downarrow\rangle = \frac{1}{2}(\chi_+^1\chi_+^2 - \chi_+^1\chi_-^2 + \chi_+^2\chi_-^1 - \chi_-^1\chi_-^2)$$
$$|\downarrow\uparrow\rangle = \frac{1}{2}(\chi_+^1\chi_+^2 + \chi_+^1\chi_-^2 - \chi_+^2\chi_-^1 - \chi_-^1\chi_-^2)$$

$$\frac{|\uparrow\downarrow - \downarrow\uparrow\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{1}{2}(2\chi_+^1\chi_+^2 - 2\chi_-^1\chi_-^2)$$

$$= \frac{1}{\sqrt{2}}(\chi_-\chi_+ - \chi_+\chi_-) \quad = |0,0\rangle_x$$

$$\frac{|\uparrow\downarrow + \downarrow\uparrow\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\chi_+\chi_+ - \chi_-\chi_-) \quad = \frac{1}{\sqrt{2}}(|1 1\rangle_x + |1 -1\rangle_x)$$

(a) $|1,-1\rangle_x = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

(b) $|0,0\rangle_x = \frac{1}{2}$

(c) $|1,1\rangle_x = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

total $= \frac{2}{3}$ of original beam.

B. Need to write back in z basis:

At (b) state is $\frac{|\uparrow\downarrow - \downarrow\uparrow\rangle}{\sqrt{2}}$

0% chance of same spin state

At (a) $|1,1\rangle_x = \chi_+\chi_+ = \frac{1}{2}(\uparrow + \downarrow)_1(\uparrow + \downarrow)_2$
$= \frac{1}{2}(|\uparrow\uparrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\downarrow\rangle)$

$\chi_\pm = \frac{1}{\sqrt{2}}(\uparrow \pm \downarrow)$

So 50% chance of same state $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$

At (c) $|1,-1\rangle_x = \chi_-\chi_- = \frac{1}{2}(\uparrow - \downarrow)_1(\uparrow - \downarrow)_2$
$= \frac{1}{2}(|\uparrow\uparrow\rangle - |\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle + |\downarrow\downarrow\rangle)$

So again 50% likelihood

Hilroy