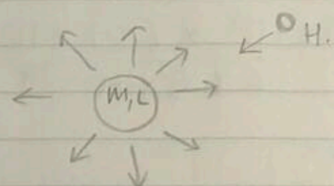
2017

1. EDDINGTON LUMINOSITY.

σ_T = effective area of proton for elastic scattering



Luminosity = power

$$\frac{L}{m^2} = \frac{L}{4\pi r^2}$$

$\uparrow$ power / area

Balance forces

$$\underbrace{\frac{G m_H M}{r^2}}_{\text{gravity}} = \text{rad pressure}$$

$E = Fd$
$P = Fv$

$$\text{Pressure} = \frac{F}{A} = \frac{L}{Ac}$$

So to get force from luminosity, have to divide by c (Speed that photons are hitting protons)

$$\frac{GMm_H}{r^2} = \underbrace{\frac{L_{Edd}\,\sigma_T}{4\pi r^2 c}}$$

check dimensions of this side

$$\frac{E \cancel{t} \cancel{\ell^2}}{\cancel{t} \cancel{\ell^2} \ell} = \frac{E}{\ell} = \frac{m}{\ell}\left(\frac{\ell}{t}\right)^2 = \frac{m\ell}{t^2} = F$$

dimensions of force as required

Rearrange: $L_{edd} = \dfrac{4\pi c\, G M m_H}{\sigma_T}$

Calculate for sun

$$L_{edd} = \frac{4\pi \times 3\times10^{8} \times 6.7\times10^{-11} \times 2\times10^{30} \times m_p}{6.65\times10^{-29}}$$

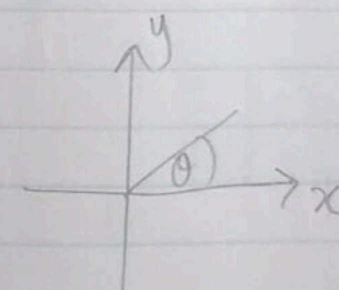
$$m_p = 938\ \frac{MeV}{c^2} = \frac{938\times10^{6} \times 1.6\times10^{-19}}{(3\times10^{8})^2}$$

$$= 1.67\times10^{-27}\ kg.$$

$$L_{edd} = 1.27\times10^{31}\ \frac{J}{s}$$

$$= 1.27\times10^{31}\ W$$

2. $V(r,\theta) = \frac{1}{2} k r^2 (1 + 3\sin^2\theta)$



A. $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$

$x = r\cos\theta$
$y = r\sin\theta$

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2}$$

$$V(x,y) = \frac{1}{2}k(x^2 + y^2 + 3y^2)$$

$$V(x,y) = \frac{1}{2}k(x^2+4y^2)$$

$$\psi = P_n(x)\,Q_n(y)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$-\frac{\hbar^2}{2m}\left(P''(x)Q(y) + P(x)Q''(y)\right) + \frac{1}{2}k(x^2+4y^2)P(x)Q(y)$$
$$= E\,P(x)\,Q(y)$$

parable

$$-\frac{\hbar^2}{2m}P''_{n_x}(x) + \frac{1}{2}kx^2P(x) = E_{n_x}P_{n_x}(x)$$

$$-\frac{\hbar^2}{2m}Q''_{n_y}(y) + 2ky^2Q(y) = E_{n_y}Q_{n_y}(y)$$

$$\cancel{\frac{1}{2}}k = \cancel{\frac{1}{2}}m\omega_x^2$$ ← relate to standard harmonic oscillator form.

$$2k = \frac{1}{2}m\omega_y^2$$

$$E = E_x + E_y = \left(n_x+\frac{1}{2}\right)\hbar\omega_x + \left(n_y+\frac{1}{2}\right)\hbar\omega_y$$
$$= \left(n_x+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{m}} + \left(n_y+\frac{1}{2}\right)\hbar\,2\sqrt{\frac{k}{m}}$$

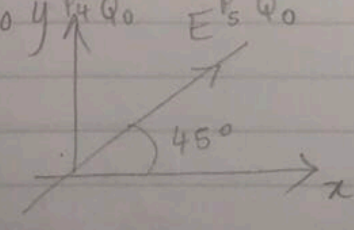
$$E = \hbar \underbrace{\sqrt{\frac{k}{m}}}_{\text{call this } \omega} \left(n_x + 2n_y + \frac{3}{2} \right)$$

n_x	n_y	N	E/ω	degeneracy
0	0	0	$\frac{3}{2}$	1
1	0	1	$\frac{5}{2}$	1
0	1	2	$\frac{7}{2}$	2
1	1	3	$\frac{9}{2}$	1
2	0	2	$\frac{7}{2}$	
0	2	4	$\frac{11}{2}$	2
2	1	4	$\frac{11}{2}$	
1	2	5	$\frac{13}{2}$	
2	2	6	$\frac{15}{2}$	
3	0	3	$\frac{9}{2}$	
0	3	6	$\frac{15}{2}$	

n_x	n_y	N	E/ω
3	1	5	
1	3	7	
3	2	7	
2	3	8	
3	3	9	
4	0	4	
0	4	8	
5	0	5	
0	5	10	

Energies:	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{11}{2}$	$\frac{13}{2}$
Degeneracies:	1	1	2	2	3	3
	P_0Q_0	P_1Q_0	P_0Q_1, P_2Q_0	P_1Q_1, P_3Q_0	P_0Q_2, P_2Q_1, P_4Q_0	P_1Q_2, P_3Q_1, P_5Q_0

B. Add $\vec{E} = E_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$



$$V = qE_0(x+y)$$

$$\vec{E} = -\vec{\nabla} V = -qE_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\omega = \sqrt{\frac{k}{m}}$$

So Schrödinger eq becomes:

$$-\frac{\hbar^2}{2m} P''_{n_x}(x) + \left(\frac{1}{2}kx^2 \mp qE_0 x\right)P(x) = E_x P(x)$$

$$-\frac{\hbar^2}{2m} Q''_{n_y}(y) + \left(2ky^2 \cancel{\mp} qE_0 y\right)Q(y) = E_y Q(y)$$

$$\frac{1}{2}k\left(x^2 + \frac{2qEx}{k}\right)$$

$$\left(x + \frac{qE}{k}\right)^2 = x^2 + \frac{2qE}{k}x + \left(\frac{qE}{k}\right)^2$$

$$\Rightarrow \frac{1}{2}k\left(x + \frac{qE}{k}\right)^2 - \left(\frac{qE}{k}\right)^2$$ ✓ good method.

Eigenstates of harmonic potential

$$P_0(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\Delta E_0 = \sqrt{\frac{m\omega}{\hbar\pi}}\int_{-\infty}^{\infty} qE_0 x\, e^{-\frac{m\omega}{\hbar}x^2}\, dx$$

$$= 0$$ (odd function over even interval)

$$\Rightarrow E_x = \hbar\omega\left(n_x + \frac{1}{2}\right) - \left(\frac{qE}{k}\right)^2$$

$$E_y = 2\hbar\omega\left(n_y + \frac{1}{2}\right) - \left(\frac{qE}{4k}\right)^2$$

should be + maybe?

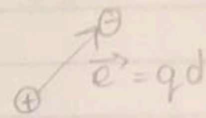
$$E = E_x + E_y$$

$$= \hbar\omega\left(n_x + 2n_y + \frac{3}{2}\right) - \frac{17}{16}\left(\frac{qE}{k}\right)^2$$

This would seem to lower energy of every state by a constant amount?

q6.5
p256

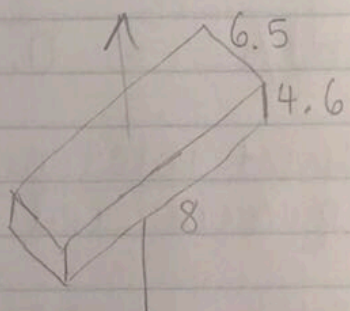
3. TUMOUR TREATING FIELDS.



$E_x = E_0 \sin wt$

$m = 110,000$ amu

$\vec{e} = 40e$ nm



Lowest freq rotation

$$I = \frac{1}{12} ml^2$$

If we approximate shape as rod rotating about centre

$$F = \nabla \vec{E} . \vec{p}$$

$$\tau = I\ddot{\theta}$$
$$= qE\sin\theta$$

Want to find natural rotation of molecule before field is applied.

Assume $\frac{1}{2}kT \approx \frac{1}{2}Iw^2$ from Equipartition Theorem

$$w^2 = \frac{kT}{I}$$
$$= \frac{12kT}{ml^2}$$

$$f = \frac{\omega}{2\pi}$$

Human body: $T = 37^\circ C = 310K$

$$\Longrightarrow \quad \omega = \sqrt{\frac{12 \times k \times 310}{1.1 \times 10^5 \times 1.66 \times 10^{-27} \times (8 \times 10^{-9})^2}}$$

$$= 2.09 \times 10^9 \text{ Hz}$$

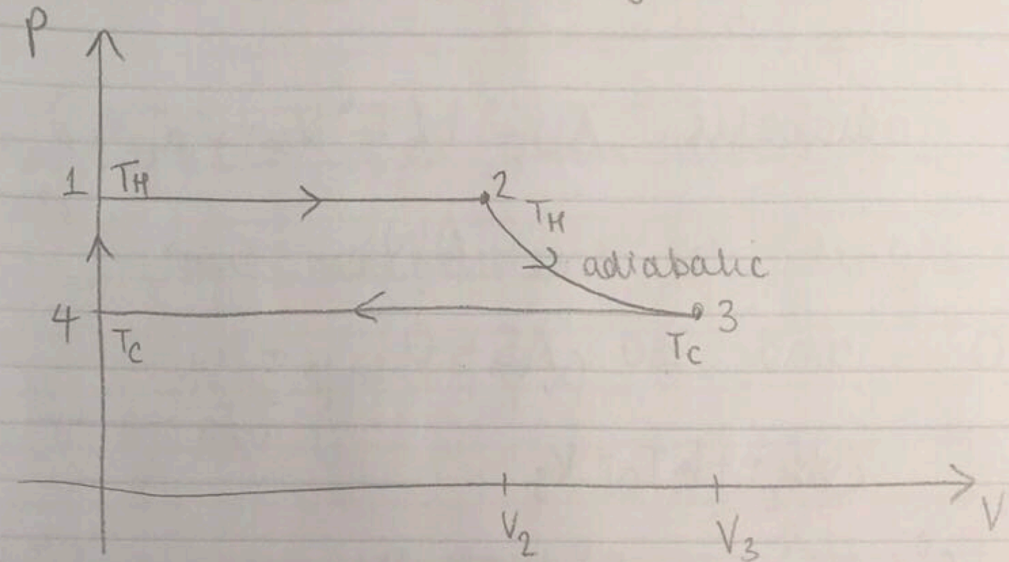
$$= 2.09 \text{ rad/s} = 0.33 \text{ GHz}$$

This will be the lowest rotation frequency as I about the long axis will be much smaller and

$$\omega^2 \propto \frac{1}{I}$$

So then require frequency of E field oscillation $\underline{\underline{\lesssim 330 \text{ MHz}}}$.

4. HEAT ENGINE - photon gas (bosons)



$U = bVT^4$ $\qquad$ $U = 3pV$

Combine these $\rightarrow$ $\quad 3pV = bVT^4$

$$p = \frac{b}{3}T^4$$

First law of thermodynamics: $\quad \Delta U = \underset{T\Delta S}{\Delta Q} + \underset{p\Delta V}{\Delta W}$

ΔW = work done <u>on</u> the system increasing internal energy.

Worried because $\quad dU = TdS - pdV$

gives $\frac{dU}{dV} = -p$ but we have $\frac{dU}{dV} = 3p$? Does that not apply to photon gas?

$$\Delta S = \frac{\Delta U - p\Delta V}{T}$$

<u>1 → 2</u> $\qquad \Delta U = bT_H^4 V_2$

$$\Delta W = \int_0^{V_2} p\,dV = \frac{b}{3}T_H^4 V_2$$

Work done <u>to</u> gas

$$\Rightarrow \Delta Q = \frac{2}{3}bT_H^4 V_2$$

And so $\Delta S = \frac{2}{3} b T_H^3 V_2$

Maybe use pV^γ and integrate along path

$\underline{2 \to 3}$ adiabatic $\Delta U = b(T_C^4 V_3 - T_H^4 V_2)$

$= \Delta W$

$\Delta Q = 0$ and so $\Delta S = 0$

$\underline{3 \to 4}$ $\Delta U = -b T_C^4 V_3$

$$\Delta W = \int_{V_3}^{0} p\, dV = \int_{V_3}^{0} \frac{b}{3} T_C^4\, dV = -\frac{b}{3} T_C^4 V_3$$

$$\Delta Q = \Delta U - \Delta W = -\frac{2}{3} T_C^4 V_3$$

$$\Delta S = -\frac{2}{3} T_C^3 V_3$$

$\underline{4 \to 1}$ $\Delta U = 0$

$\Delta W = 0$

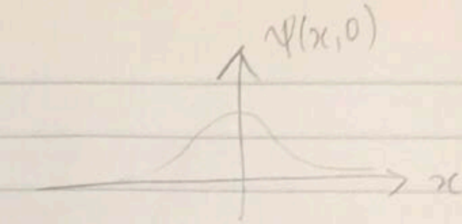
$\Delta Q = 0$ and so $\Delta S = 0$

↑ worried about this but I guess at zero volume changes in T do not change entropy?

Check that overall $\Delta U = 0$

$$b T_H^4 V_2 + b(T_C^4 V_3 - T_H^4 V_2) - b T_C^4 V_3 = 0 \checkmark$$

5. A. $\psi_0(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$



I am expecting it will spread out

Free particle with given initial state:

Time independent Schrödinger equation

$$\frac{\hbar^2 k^2}{2m}\psi(x) = E\psi(x)$$

$$k = \pm\frac{\sqrt{2mE}}{\hbar}$$

$$\psi(x,t) = Ae^{i\left(kx - \frac{\hbar k^2}{2m}t\right)}$$

standard stationary state time dependence.

Not a physical solution as not normalisable

General solution $\psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \phi(k)\, e^{i\left(kx - \frac{\hbar k^2}{2m}t\right)}\, dk$

Choose $\phi(k)$ to match initial condition given

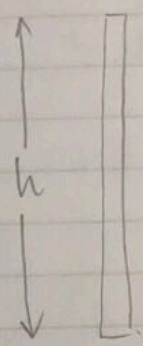
$$\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \psi(x,0)\, e^{-ikx}\, dx$$

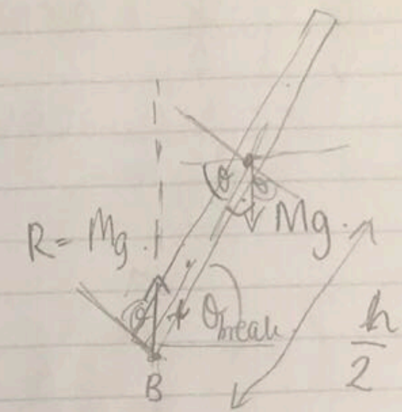
$$= \frac{1}{2\pi\sigma}\int_{-\infty}^{\infty} e^{-\left(\frac{x^2}{2\sigma^2} + ikx\right)}\, dx.$$

See Griffiths Q 2.22 Nasty algebra.

Hilroy

2017 Q6





$$\tau(x) = x\, R\cos\theta + \left(\frac{h}{2} - x\right) R\cos\theta$$

Incorrect because forces are <u>not</u> balanced → acceleration is occurring

$$\tau = I\ddot{\theta}$$

Calculate I from B.

$$I = \int_0^L r^2\, dm \qquad \frac{dm}{dr} = \lambda = \frac{M}{h}$$

$$= \int_0^L \lambda r^2\, dr$$

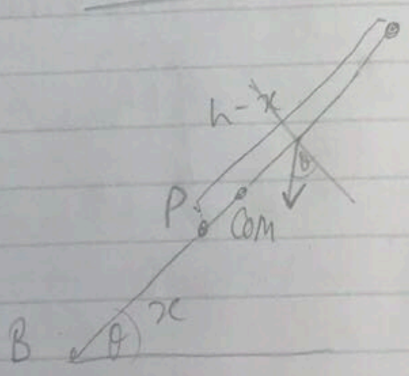
$$= \frac{\lambda h^3}{3} = \frac{M}{3} h^2$$

$$\tau = \frac{mh^2}{3}\ddot{\theta}$$

Torque from base: $\tau = -\frac{h}{2} Mg\cos\theta$

$$\ddot{\theta} = \frac{\cancel{h}}{2} \cancel{M} g \cos\theta \times \frac{3}{\cancel{M} h^{\cancel{2}}}$$

$$= -\frac{3}{2}\frac{g}{h} \cos\theta$$



$$m_{top} = \frac{h-x}{h} M$$

Torque at P from top part at point when it breaks.

$$\tau = \frac{h-x}{h} M g \times \frac{h-x}{2} \cos\theta$$

$$= \frac{(h-x)^2}{2h} Mg \cos\theta$$

$$= \frac{(h-x)^2}{\cancel{2h}} M\cancel{g} \frac{\cancel{2h}}{3\cancel{g}} \ddot{\theta}$$

$$= (h-x)^2 \frac{M}{3} \ddot{\theta}$$ instantaneously.

How are these two the same $\ddot{\theta}$?

Torque at P from com trying to keep rotating at same speed $= \tau(x)$

$$\tau(x) - (h-x)^2 \frac{M}{3} \ddot{\theta} = I\ddot{\theta}$$

$$\longrightarrow \quad = \frac{1}{3} M \frac{(h-x)}{h} (h-x)^2 \ddot{\theta}$$

from $I = \frac{1}{3} ML^2$

$$\tau(x) = \frac{M}{3}\ddot{\theta}\left(-(h-x)^2 + \frac{(h-x)^3}{h}\right) \qquad \frac{d\tau}{dx} = \frac{M\ddot{\theta}}{3}\left(-2(h-x) - 3\frac{(h-x)^2}{h}\right)$$

Solutions have +ve.

$$\tau(x) = \frac{M}{3}(h-x)^2\,\ddot{\theta}\left(1 + \frac{h-x}{h}\right)$$

Take $\frac{d\tau}{dx}$ and set $= 0$ to find max torque

$$\frac{d\tau}{dx} = -\frac{M}{3}\,2(h-x)\,\ddot{\theta}\left(2 - \frac{x}{h}\right)$$

$$- \frac{M}{3h}(h-x)^2\,\ddot{\theta}$$

$$\longrightarrow \quad 2(h-x)\left(2-\frac{x}{h}\right) + \frac{(h-x)^2}{h} = 0.$$

$$2(h-x)(2h-x) + (h-x)^2 = 0$$

Either $x = h$ (not possible) or

$$2(2h-x) = -h+x$$
$$4h - 2x = -h + x$$

$$x = -\frac{5h}{3} \qquad \text{oh no! algebra.}$$

$$-2h(h-x) + 3(h-x)^2 = 0$$

either $h = x$ or

$$2h = +3(h-x)$$
$$5h = 3x$$
$$x = \frac{5h}{3}.$$

9. $$\psi(r,\theta,\phi) = \frac{1}{\sqrt{4\pi}} \frac{2}{a_0^{3/2}} e^{-r/a_0}$$

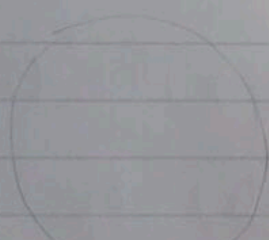
If a point particle:

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{e^2}{4\pi\varepsilon_0 r}\psi = E\psi$$

If a sphere of charge

$$V = \begin{cases} \frac{kQ}{2R}\left(\frac{r^2}{R^2} - 3\right) & r \leq R \\ \frac{kQ}{r} & r > R \end{cases}$$

$\sigma = \frac{Q}{\frac{4}{3}\pi R^3}$



$$\int E \cdot dS = \frac{Q_{encl.}}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{\frac{4}{3}\pi r^3 \sigma}{\varepsilon_0}$$

$$E = \frac{\sigma r}{3\varepsilon_0} = \frac{kQ}{R^3} r$$

$k = \frac{1}{4\pi\varepsilon_0}$

$$V = -\int_r^\infty E \cdot dr$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ outside.}$$

$$= +\int_R^\infty \frac{kQ}{r^2}\,dr + \int_r^R \frac{kQ}{R^3} r\,dr$$

$$= -\left[-\frac{kQ}{r}\right]_\infty^R - \left[\frac{kQ}{R^3}\frac{r^2}{2}\right]_R^r$$

$$V = 0 + \frac{kQ}{R} - \frac{kQ}{R^3}\frac{r^2}{2} + \frac{kQ}{2R}$$

$$= \frac{kQ}{2R}\left(-\frac{r^2}{R^2} + 3\right)$$

Use perturbation theory

$$H = H_0 + H'$$

$$= -\frac{\hbar^2}{2m}\nabla^2 - \frac{kq^2}{r} + \underbrace{\frac{kq^2}{r} - \frac{kq^2}{2R}\left(3 - \frac{r^2}{R^2}\right)}_{H'}$$

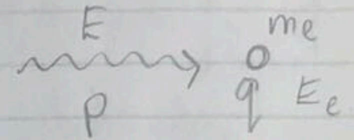
$$\Delta E = \langle\psi|H'|\psi\rangle$$

$$= \frac{kq^2}{\pi a_0^3}\int_{-\infty}^{\infty} e^{-2r/a_0}\left(\frac{1}{r} - \frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right)\right) 4\pi\, dr$$

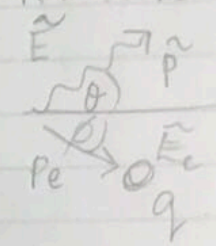
Can leave in integral form

12.

BEFORE



AFTER



$p_i = \dfrac{E_i}{c}$

Relativistic calculation so conserve 4-energy and 4-momentum.

$$\underline{p} = \underline{\tilde{p}} + \underline{p_e}$$

$$E + E_e = \tilde{E} + \tilde{E}_e$$

$$\tilde{E} = E + E_e - \tilde{E}_e$$

$$= pc + m_e c^2 - \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

Only need to get rid of p_e

$$\underline{p_e}^2 = (\underline{p} - \underline{\tilde{p}})^2$$

$$= p^2 - 2p\tilde{p}\cos\theta + \tilde{p}^2$$

$$\tilde{E} = E + m_e c^2 - \sqrt{E^2 - 2E\tilde{E}\cos\theta + \tilde{E}^2 + m_e^2 c^4}$$

$$(-\tilde{E} + E + m_e c^2)^2 = E^2 - 2E\tilde{E}\cos\theta + \tilde{E}^2 + m_e^2 c^4$$

$$\cancel{\tilde{E}^2} - 2E\tilde{E} - 2m_e c^2 E + \cancel{E^2} - 2\tilde{E} m_e c^2 + \cancel{m_e^2 c^4} = \cancel{E^2} - 2E\tilde{E}\cos\theta + \cancel{\tilde{E}^2} + \cancel{m_e^2 c^4}$$

$$E\tilde{E} + m_e c^2 (E + \tilde{E}) = E\tilde{E}\cos\theta$$

$$\tilde{E}\left(E\cos\theta - m_e c^2 - E\right) = E m_e c^2$$

$$\tilde{E} = \frac{E}{-1 + \frac{E}{m_e c^2}(\cos\theta - 1)}$$

signs are wrong.

Somehow need denominator x-1 but cant find mistake in.

$$\tilde{E} = \frac{E}{1 + \frac{E}{m_e c^2}(1-\cos\theta)}$$

14. $n + e^+ \rightarrow \bar{\nu}_e + p$

$n + \nu_e \longrightarrow p + e^-$

$t \ll 1s$ $n:p = 1:1$ then gradually more p

$T = 8\times10^9$ K reactions stop

t = few minutes 20% n decayed to p
rest $\rightarrow {}^4He$

Estimate H : ^{4}He nuclei.

Need to estimate proportion of n when T = 8×10

Number density $n(a)e^{U(a)/k_BT} = n(b)e^{U(b)/k_BT}$

$$\frac{n(p)}{n(n)} = \frac{e^{m_nc^2/k_BT}}{e^{m_pc^2/k_BT}}$$

$$= \frac{e^{940\times10^6\times1.6\times10^{-19}/1.38\times10^{-23}\times8\times10^9}}{e^{938\times10^6\times1.6\times10^{-19}/1.38\times10^{-23}\times8\times1}}$$

$$= \frac{e^{\frac{94000}{69}}}{e^{\frac{93800}{69}}}$$

$$= \left(\frac{e^{9.4}}{e^{9.38}}\right)^{\frac{10^4}{69}}$$

$$= (1.02020)^{144.9}$$

$$= 18.15$$

$$\frac{n(p)}{n(n)} = \exp\left(\frac{-\Delta mc^2}{k_BT}\right)$$

$$= \exp\left(\frac{-2\times10^6\times1.6\times10^{-19}}{1.38\times10^{-23}\times8\times10^9}\right)$$

= 18.15 as before.

should be 1:6 ← perhaps cooled quickly ∴ didn't reach equilibrium

So 18 times more protons than neutrons.

Seems approx reasonable?

Then 20% of these become protons

18:1 $\rightarrow$ 5.3% neutrons

−20% $\rightarrow$ 4% neutrons

96% protons.

^4He requires 2 neutrons, 2 protons.

$\rightarrow$ 8% of p and n form He

92 protons : 4 He nuclei

Ratio H : He = 1 : 0.0435

23 : 1

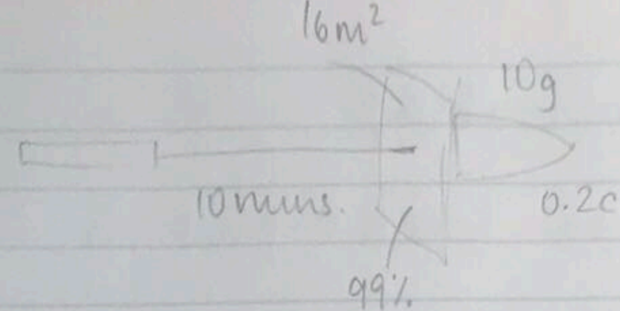
should be 12:1

By mass $\frac{75\%}{1} : \frac{25\%}{4}$

$\hookrightarrow$ 75 : 8

9 : 1

15.



Energy striking sail during acceleration

Start from E required to accelerate to 0.2c in 10 minutes. Then times $\frac{100}{99}$ (as 1% wa[s] absorbed)

Before

$E_r = m_0 c^2$

After

$E = \gamma m_0 c^2$

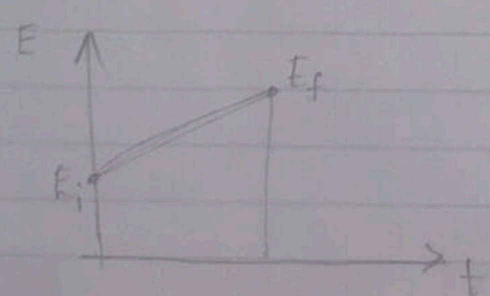
$= \frac{1}{\sqrt{1-\left(\frac{1}{5}\right)^2}}$

$= \frac{5\sqrt{6}}{12} m_0 c^2$

So need $\Delta E = \left(\frac{5\sqrt{6}}{12} - 1\right) m_0 c^2$ from photons hitting sail

$$E = \frac{100}{99}\left(\frac{5\sqrt{6}}{12} - 1\right) m_0 c^2 = 1.87 \times 10^{13}\,\text{J}$$

Or

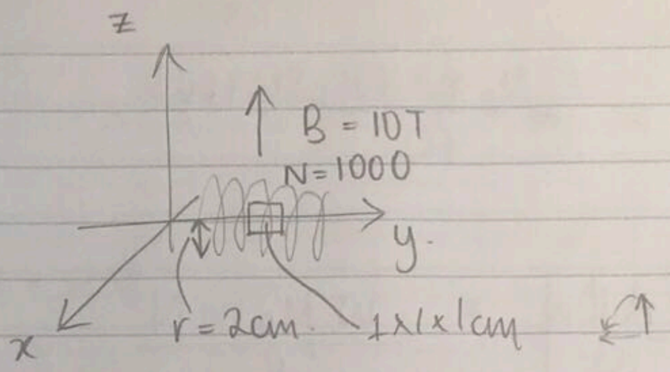


$$E = \int_0^{0.2c} \gamma m_0 c^2 \, dv$$

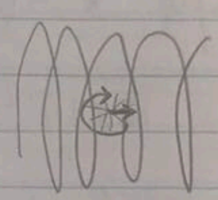
Calculate v in terms of acceleration and time. Integrate over acceleration E

$$B = -\frac{1}{\mu_0}\vec{m} = \frac{\vec{m}}{\mu_0 V}$$

8.



Amplitude of AC voltage induced in coil



$$\oint E \cdot dl = -\frac{d\Phi_B}{dt}$$

Need to calculate frequency of spin precession

$= $ larmor frequency $\quad \omega = \gamma B. = \frac{ge}{2m_p} B. = g\mu_N B$

$\uparrow \sim 2.8$

Don't think this is the right way as didn't know these things

$$\Phi_B = \int_0^R \underbrace{\frac{\cancel{2}\mu_0}{\cancel{4\pi}} \frac{m}{r^{\cancel{3}2}}}_{} 2\pi r \, dr. \quad N \cos\omega t$$

magnitude of B field integrated over area.

eek $\frac{1}{0}$?.

$$= \mu_0 m \left[-\frac{1}{r} \right]_0^R N \cos\omega t$$

$$= \frac{\mu_0 m}{R} N \cos\omega t$$

$$m_p = \frac{938\times10^6 \times 1.6\times10^{-19}}{(3\times10^8)^2} = 1.67\times10^{-29}$$

$$\frac{d\Phi_B}{dt} = -w\frac{\mu_0 m N}{R}\sin wt$$

$$|V| = -\left|\frac{d\Phi_B}{dt}\right| = \frac{w\mu_0 m N}{R}$$

$$= \frac{g\,\mu_N B\,\mu_0\, m_p\, N}{R}$$

$$= \frac{2.8\times 5\times10^{-27}\times 4\pi\times10^{-7}\times 1.67\times10^{-29}\times 100}{2\times10^{-2}}$$

$$= 1.5\times10^{-56}\ V$$

Very low! Wait, but this is one proton spin. Need to estimate how many there are in volume

$1\,cm^3$ of water $= 1g$ $\quad$ 1 mole of $H_2O = 18g$.

So have $\approx 2\times\frac{N_A}{18}$ Hydrogen nuclei

$|V| = 9.83\times10^{-34}\,V$ $\quad$ Still ridiculously small...

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

10. $$H = -J[S_+^1 S_-^2 + S_-^1 S_+^2] - h_1 S_z^1 - h_2 S_z^2$$

$$H\psi = E\psi$$

Spin $\frac{1}{2}$ particles so can be $\uparrow$ or $\downarrow$

Try $|\uparrow\uparrow\rangle$ $\quad S^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad S^2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$H|\uparrow\uparrow\rangle = -J[\,0\,] - h_1 \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{h_2}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= -\frac{h_1}{2}|\uparrow\uparrow\rangle - \frac{h_2}{2}|\uparrow\uparrow\rangle$$

$$= -\frac{(h_1 + h_2)}{2}|\uparrow\uparrow\rangle$$

$$H|\downarrow\downarrow\rangle = +\frac{(h_1+h_2)}{2}|\downarrow\downarrow\rangle$$ in a similar way

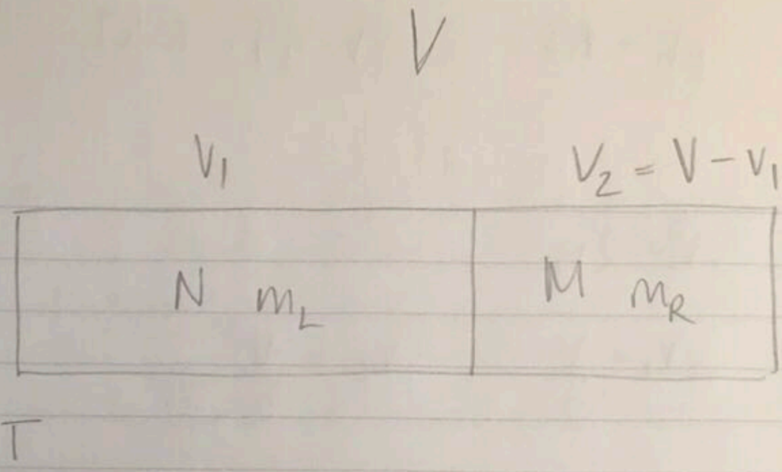
Try $|\psi_+\rangle = \dfrac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$

$$H|\psi_+\rangle = -\frac{J}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) - \frac{h_1}{\sqrt{2}}\left(\frac{1}{2}|\uparrow\downarrow\rangle - \frac{1}{2}|\downarrow\uparrow\rangle\right)$$
$$- \frac{h_2}{\sqrt{2}}\left(-\frac{1}{2}|\uparrow\downarrow\rangle + \frac{1}{2}|\downarrow\uparrow\rangle\right)$$

So this is <u>not</u> an eigenstate!

$\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$ would not work either

13.



A. Sackur-Tetrode equation

$$S = kN\left(\ln\left(\frac{V}{N}\left(\frac{4\pi m}{3h^2}\frac{U}{N}\right)^{3/2}\right) + \frac{5}{2}\right)$$

Internal energy $U = \frac{3}{2} N k_B T$

$$S_{tot} = S_1 + S_2$$

be careful M & N multiplier

$$= kN\left(\ln\left(\frac{v_1}{N}\left(\frac{\cancel{4}^{2}\pi m_L}{\cancel{3}h^2}\frac{\cancel{3}}{\cancel{2}}k_B T\right)^{3/2}\right) + \frac{5}{2}\right)$$

$$+ kM\left(\ln\left(\frac{v_2}{M}\left(\frac{\cancel{4}^{2}\pi m_R}{\cancel{3}h^2}\frac{\cancel{3}}{\cancel{2}}k_B T\right)^{3/2}\right) + \frac{5}{2}\right)$$

(For monoatomic ideal gas $C_V = \frac{3}{2}R$)

$\ln(v_1(V-v_1)) + \ln(MN) + \ln($ $)$

$$S_{tot} = \frac{5k}{2}(N+M) + \ln\left(\frac{v_1(V-v_1)}{MN}\left(\frac{2\pi}{h^2}k_B T\right)^3 (m_L m_R)^{3/2}\right)$$

Equilibrium v_1 value: Minimise S_{tot}

$$\frac{\partial S_{tot}}{\partial v_1} = \frac{1}{v_1(V-v_1)} \times (V - 2v_1)$$

$$= \frac{V - 2v_1}{v_1(V-v_1)} = 0$$

$$pV_1 = NkT \qquad p(V - V_1) = MkT$$

So $\quad V = 2V_1$

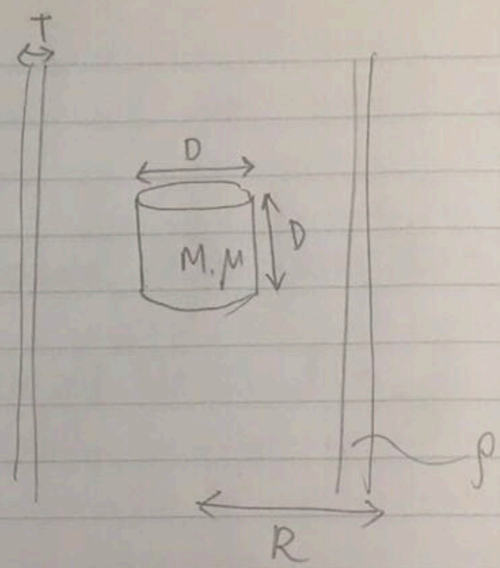
$$V_1 = \frac{V}{2} \qquad V_2 = \frac{V}{2}.$$

B. Total energy = E, can exchange heat between sides but not w outside

$$\frac{3}{2} N k_B T_1 + \frac{3}{2} M k_B T_2 = E$$

C. $\quad v_L^2 = \dfrac{3kT_1}{m_L} \qquad v_R^2 = \dfrac{3kT_2}{m_R}$

16.



$D \ll R \quad T \ll R$

E.m.f. induced $= -\dfrac{d\Phi_B}{dt}$