(Unofficial) UBC PHAS Comprehensive 2022 Solutions

April 29, 2023

S P H E R E

2. A closed sphere contains an ideal gas, which has a pressure of 10^5 Pa at a temperature of 300 K. At what temperature will the radiation pressure pushing outward on the walls of the sphere be equal to the pressure from the gas? Assume that the gas is in thermal equlibrium with the walls, and that the walls (inside and outside) emit as blackbodies.

Let's use little p for pressure. $p = 1 \times 10^5\,\mathrm{Pa}$ at $T = 300\,\mathrm{K}$.
Ideal gas, so initially:

$$pV = NkT$$
$$\frac{p_0}{T_0} = \frac{Nk}{V}$$

And after changing temperature:

$$p = \frac{NkT}{V}$$

Black-body power radiated: $P = A\sigma T^4$, where $\sigma = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$ (on the formula sheet).
Here, $A = 4\pi R^2$ (surface area of sphere).
$P = 4\pi R^2 \sigma T^4$.

Radiation pressure:

Honestly there might be an extra factor of two somewhere here, I forget?

$$
\begin{aligned}
p &= \frac{I}{c} \\
&= \frac{\left(\frac{P}{A}\right)}{c} \\
&= \frac{\left(\frac{P}{4\pi R^2}\right)}{c} \\
&= \frac{\left(\frac{4\pi R^2 \sigma T^4}{4\pi R^2}\right)}{c} \\
&= \frac{\sigma T^4}{c}
\end{aligned}
$$

When will this be equal to the pressure from the gas? Set our expressions equal:

$$
\begin{aligned}
\frac{NkT}{V} &= \frac{\sigma T^4}{c} \\
\frac{Nk}{V}\frac{c}{\sigma} &= T^3 \\
\sqrt[3]{\frac{p_0}{T_0}\frac{c}{\sigma}} &= T = 1.2 \times 10^6\,\mathrm{K}
\end{aligned}
$$

Key take-aways:
Note: had to look up the radiation pressure / blackbody stuff
Ideal gas: $pV = NkT$
Blackbody total power radiated: $P = A\sigma T^4$
Radiation pressure: $p = \frac{I}{c}$