2016

1. 2nm 2m

$\lambda =$

3D random walk, step size 50nm

A. Number of steps $N = \frac{2}{50 \times 10^{-9}} = 4 \times 10^7$

$$P(r) = \left(\frac{a}{\sqrt{\pi}}\right)^3 \exp(-a^2 r^2) \quad \text{where } a = \left(\frac{3}{2N\lambda^2}\right)^{1/2}$$

What to use for ball diameter?
mean, RMS or most probable?

I'm going to use most probable which is $\frac{1}{a}$.

$$D = \frac{1}{a} = \left(\frac{2N\lambda^2}{3}\right)^{\frac{1}{2}} = 2.58 \times 10^{-4} \text{ m.}$$

$$= 258 \mu m$$

Way too big to fit in cell nucleus. ($\sim 10 \mu m$

If I hadn't had all of these handy formulae, wou
have used dimensional analysis & knowledge that

$$D \propto \sqrt{N} \approx \sqrt{N}\lambda$$

B. $$\Delta S = k_B \ln\left(\frac{V_{coiled}}{V_{balled}}\right) = k_B \ln\left(\frac{\frac{4}{3}\pi (5\mu m)^3}{\frac{4}{3}\pi (130\mu m)^3}\right)$$

$$= k_B \ln\left(\frac{5^3}{130^3}\right) = -1.34 \times 10^{-22}$$

human body temp
$= 37°C$

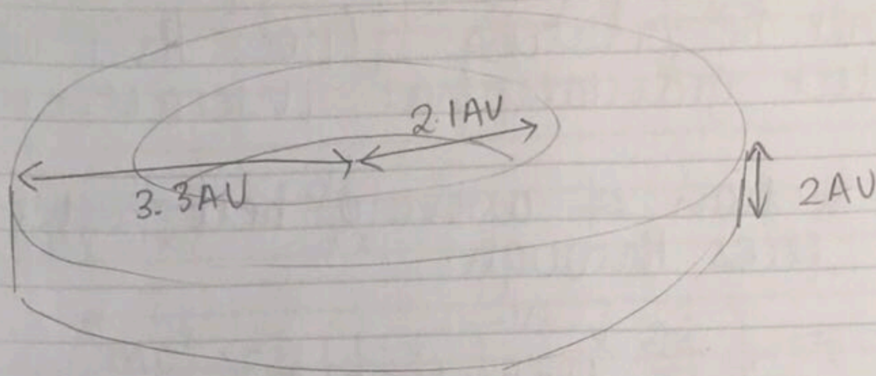
And then $\Delta E = T\Delta S \approx 310 \times 1.34 \times 10^{-22}$

$$= 4.18 \times 10^{-20} \text{ J}$$

$$= \underline{0.26 \text{ eV}}$$

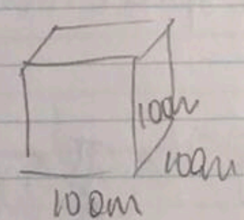
Seems maybe reasonable cell energy.

2.



Earth volume $= \frac{4}{3}\pi r^3 = 1 \times 10^{21} m^3$

Divided into



cubes

$V = 1 \times 10^6 m^3$

$N_{asteroids} = 1 \times 10^{15}$

$1 AU = 1.5 \times 10^{11} m$

Distributed evenly through volume of belt:

$$V_{belt} = \left(\pi (3.3 \times 1.5 \times 10^{11})^2 - \pi (2.1 \times 1.5 \times 10^{11})^2\right) \times 2 \times 1.5 \times 10^{11}$$

$$= 1.37 \times 10^{35} m^3$$

So as a percentage, volume of space taken up by asteroids

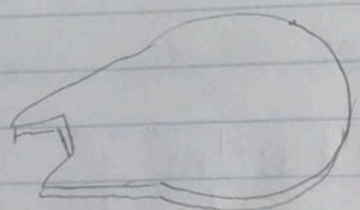
$$= \frac{10^{21}}{1.4 \times 10^{35}} = 1 \times 10^{-14}$$

$$= 1 \times 10^{-12} \%$$

Flying at near light speed so will have length contraction of the belt from the spaceship's point of view. But this should not affect answer

Asteroids will be orbiting. (Does this come under 'neglect gravitational interactions'?)

Think of section of asteroid belt which spaceship flies through.

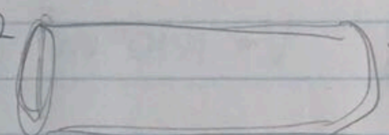


diameter = 30 m

not sure which diameter?

Assume spherical spaceship.

$A = \pi \times \left(\frac{d_f + d_a}{2}\right)^2$



1.2 AU

$V = \pi \times \left(\frac{30+100}{2}\right)^2 \times 1.2 \times 1.5 \times 10^{11}$

$= 2.39 \times 10^{15}\ m^3$

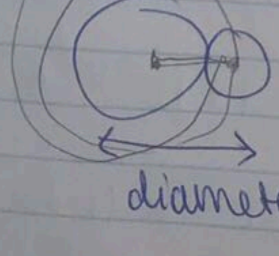
So probability of asteroid being in this volume

$P(\text{asteroid}) = N \frac{v}{V} = 10^{15} \times \frac{2.39 \times 10^{15/14}}{1.37 \times 10^{35}}$

$\approx 10^{-5}$

$\Rightarrow$ Probability of hitting asteroid $\approx 10^{-5} : 1$

This doesn't take into account size of asteroid



diameter = falcon + asteroid

$-\left(1-\frac{\dot{x}^2}{c^2}\right)^{\frac{1}{2}}$ $\qquad\qquad$ $\gamma^2(1-\beta^2) = 1$

3. $L = mc^2\left(1-\sqrt{1-\frac{\dot{x}^2}{c^2}}\right) - \frac{1}{2}kx^2$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

$$\frac{\partial}{\partial t}\left(mc^2 \times -\frac{\cancel{1}}{\cancel{2}}\left(1-\frac{\dot{x}^2}{c^2}\right)^{-1/2} \times \frac{\cancel{-2}\dot{x}}{c^2}\right) = -kx$$

$$\frac{\partial}{\partial t}\left(m\dot{x}\,\frac{1}{\left(1-\frac{\dot{x}^2}{c^2}\right)^{1/2}}\right) = -kx.$$

$$\gamma m\ddot{x} + \cancel{-}\frac{\cancel{1}}{\cancel{2}} m\dot{x}\,\frac{1}{\left(1-\frac{\dot{x}^2}{c^2}\right)^{3/2}} \times \frac{\cancel{-2}\dot{x}}{c^2}\,\ddot{x} = -kx.$$

$$\gamma m\ddot{x} + \gamma^3\beta^2 m\ddot{x} = -kx$$

$$\ddot{x} = \frac{-k}{\gamma m(\underbrace{1+\gamma^2\beta^2}_{=\gamma^2})}\,x$$

But definitely can't equate this to any kind of frequency as β is a function of x

Find conjugate momentum

$$p_x = \frac{\partial L}{\partial \dot{x}} = \gamma m\beta c \qquad \text{as expected.}$$

A. $H = p_x\dot{x} - L = \gamma m\beta^2c^2 - mc^2\left(1-\frac{1}{\gamma}\right) + \frac{1}{2}kx^2$

$$= mc^2\left(\gamma\beta^2 - 1 + \frac{1}{\gamma}\right) + \frac{1}{2}kx^2$$

If L has no time dep then $H = E$ = conserved

$$\gamma^2(1-\beta^2)=1 \quad \rightarrow \gamma - \gamma\beta^2 = \frac{1}{\gamma} \qquad \gamma - \frac{1}{\gamma} = \gamma\beta^2$$

$$H = mc^2\left(\gamma - \cancel{\frac{1}{\gamma}} - 1 + \cancel{\frac{1}{\gamma}}\right) + \frac{1}{2}kx^2$$

$$= \underline{(\gamma-1)mc^2 + \frac{1}{2}kx^2} \qquad = E$$

B. At max extension $\quad x = A \quad \dot{x} = 0 \rightarrow \gamma = 1$

$$E = \frac{1}{2}kA^2$$

At max velocity $\quad x = 0 \quad \dot{x} = \text{max}$

$$(\gamma-1)mc^2 = \frac{1}{2}kA^2$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{kA^2 + 2m^2c^4}{2mc^2}$$

$$1 - \frac{v^2}{c^2} = \frac{4m^2c^4}{(kA^2 + 2m^2c^4)^2}$$

$$v = \frac{(kA^2 + 2m^2c^4)^2 - 4m^2c^4}{(kA^2 + 2m^2c^4)^2} \quad c^2$$

$$= \frac{(k^2A^2 + 4m^4c^8)c^2}{(kA^2 + 2m^2c^4)^2}$$

C. At $x = \frac{A}{2}$ $\qquad E = (\gamma-1)mc^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA^2$

$$\gamma^2 - \gamma^2\beta^2 = 1$$

$$(\gamma - 1)mc^2 = \frac{3}{8}kA^2$$

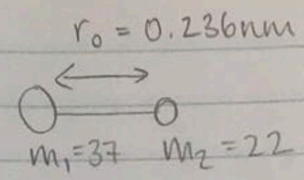
$$\gamma = \frac{3}{8}\frac{kA^2}{mc^2} + 1$$

Equation of motion from before:

$$\ddot{x} = \frac{-k}{\gamma^3 m} x$$

$$= \underbrace{\frac{-k}{\left(\frac{3}{8}\frac{kA^2}{mc^2} + 1\right)^3 m} \frac{A}{2}}$$

4.



$$V(r) = \frac{-e^2}{4\pi\varepsilon_0 r} + \frac{A}{r^k}$$

Model as SHM with $\mu\ddot{r} = -kr$ $\qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$

$= 13.8 \text{ g/mol}$

$= 2.29 \text{ kg/molecule}$

$$\frac{\partial V(r)}{\partial r} = \frac{e^2}{4\pi\varepsilon_0 r^2} - \frac{kA}{r^{k-1}}$$

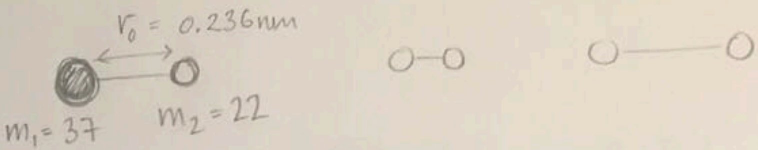
$$\frac{\partial^2 V(r)}{\partial r^2} = -\frac{e^2}{2\pi\varepsilon_0 r^3} + \frac{k(k-1)A}{r^{k-2}}$$

At V_0, r_0 (equilibrium point

$$V_0 = \frac{-e^2}{4\pi\varepsilon_0 r_0} + \frac{A}{r_0^k}$$

$$\frac{\partial V(r_0)}{\partial r} = \frac{e^2}{4\pi\varepsilon_0 r_0^2} - \frac{kA}{r_0^{k-1}} = 0$$

∂



4. $$V(r) = \frac{-e^2}{4\pi \varepsilon_0 r} + \frac{A}{r^k}$$

Want to model as SHM

$$\mu \ddot{x} = -kx \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$x = A\cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{k}{\mu}}$$

In the case $V = \frac{-1}{2} k_s x^2$

$$\frac{\partial V}{\partial r} = \frac{+e^2}{4\pi \varepsilon_0 r_0^2} - \frac{kA}{r_0^{k+1}} = 0$$

Find A and k from given info:

(A) $$0 = \frac{e^2}{4\pi \varepsilon_0 r_0^2} - \frac{kA}{r_0^{k+1}}$$

(B) $$V_0 = \frac{-e^2}{4\pi \varepsilon_0 r_0} + \frac{A}{r_0^k}$$

<u>do</u> want to do this

Then Taylor expand. Find ω.

$$\frac{A}{r_0^k} - V_0 = \frac{e^2}{4\pi \varepsilon_0 r_0} \qquad \frac{kA}{r_0^{k+1}} = \frac{e^2}{4\pi \varepsilon_0 r_0^2}$$

$$\frac{\cancel{r_0^{k+1}}(A - V_0 r_0^k)}{\cancel{r_0^k}(kA)} = r_0^2 \qquad A\cancel{r_0} - V_0 r_0^{k+1} - r_0^2 kA = 0.$$

$$A = \frac{V_0 r_0^k}{(1 - r_0 k)}$$

Put back into B

$$V_0 = \frac{-e^2}{4\pi\varepsilon_0 r_0} + \frac{V_0}{(1-r_0 k)}$$

$$1 - r_0 k = \frac{V_0}{V_0 + \frac{e^2}{4\pi\varepsilon_0 r_0}}$$

$$r_0 k = \frac{\cancel{V_0} + e^2/4\pi\varepsilon_0 r_0 - \cancel{V_0}}{V_0 + e^2/4\pi\varepsilon_0 r_0}$$

Put into A

$$0 = \frac{e^2}{4\pi\varepsilon_0 r_0^2} \quad \frac{-k V_0 r_0^{\cancel{k}} r_0}{\cancel{r_0^{k+1}}(1-r_0^2 k)}$$

$$\frac{e^2}{4\pi\varepsilon_0 r_0^2} = \frac{k V_0 r_0}{1-r_0^2 k}$$

$$\frac{e^2}{4\pi\varepsilon_0 r_0^2} - \frac{e^2}{4\pi\varepsilon_0} k = k V_0 r_0$$

$$k = \frac{e^2}{4\pi\varepsilon_0 r_0^2 \left(V_0 r_0 + \frac{e^2}{4\pi\varepsilon_0}\right)}$$

$r_0 = A$
$V_0 = B$

$$k = \frac{1}{r_0 + \frac{V_0 4\pi\varepsilon_0 r_0^2}{e^2}} = \frac{e^2}{r_0 (e^2 + V_0 4\pi\varepsilon_0 r_0)}$$

$r_0 = 0.236\text{nm}$
$V_0 = -4.26\text{eV}$

Put in numbers: $k = 1.\cancel{3}40 \times 10^{10}$ oh no not good

My calculator can't handle an exponent this big?

Anyway, would Taylor expand potential

$$V(r) = \frac{-e^2 r^k + A 4\pi\varepsilon_0 r}{4\pi\varepsilon_0 r^{k+1}}$$

$$V(r) = V(r_0) + \underbrace{V'(r_0)}_{=0}(r-r_0) + V''(r_0)\frac{(r-r_0)^2}{2} + \dots$$

$$\frac{\partial V(r)}{\partial r} = F = m\ddot{r} \qquad r = r_0 + \xi$$

$$V''(r_0)(r-r_0) = m\ddot{r}$$

$$\ddot{\xi} = \frac{V''(r_0)}{m}\xi$$

$$\frac{\partial^2 V(r_0)}{\partial r^2} = \frac{-e^2}{2\pi\varepsilon_0 r_0^3} + \overbrace{\frac{kA(k+1)}{r_0^{k+2}}}^{I}$$

$$\omega^2 = \frac{-V''(r_0)}{\mu}$$

Don't want to have k or A in there - replace using $\frac{\partial V(r_0)}{\partial r} = 0$ $\quad V(r_0) = V_0$

$$A = \frac{e^2 r_0^{k+1}}{4\pi\varepsilon_0 r_0^2 k}$$

$$I = \frac{e^2 r_0^{k-1}}{4\pi\varepsilon_0 \cancel{k}} \frac{\cancel{k}(k+1)}{r_0^{k+2}}$$

$$= \frac{e^2}{4\pi\varepsilon_0 r_0^3}(k+1)$$

$$\Rightarrow \omega^2 = \left(\frac{e^2}{2\pi\varepsilon_0 r_0^3} - \frac{(k+1)e^2}{4\pi\varepsilon_0 r_0^3}\right) \Big/ \mu$$

$$= \frac{e^2}{4\pi\varepsilon_0 r_0^3 \mu}(1-k)$$

$$\mu = \frac{22 \times 37}{37+22}$$
$= 13.8 g/mol$
$= 2.29 \times 10^{-26} kg/molecule$

$$= \frac{e^2 (1 - 1.4\times10^{10})}{4\pi\varepsilon_0 (0.236nm)^3 \times 2.29\times10^{-26}}$$

$$= 1.07\times10^{37}$$

$\omega = 3.28\times10^{18}$ rad/s

$f = 5.21\times10^{17}$ Hz $\quad = 521$ PHz ??

Hilroy

5. $M_{mars} = 6.4 \times 10^{23}$ kg $\qquad R_{mars} = 3.4 \times 10^{6}$ m

$P = 100$ kPa $\qquad T = 283$ K

as will be N_2 and O_2

Averaged mass $= (0.78 \times 28 + 0.22 \times 32) \times 2$
$= 58$ g/mol
$= 9.6 \times 10^{-26}$ kg/molecule

Escape velocity

$$\frac{1}{2} m v_{esc}^2 = \frac{GMm}{r}$$

$$v_{esc} = \sqrt{\frac{2GM}{r}} = 5010 \text{ m/s}$$

for particle at surface.

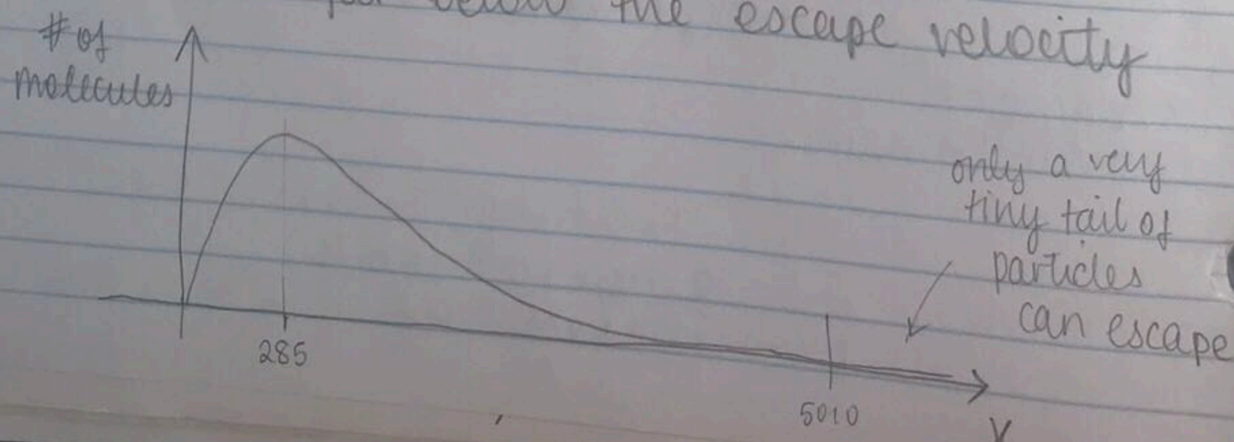
Find most likely speed for molecule from maxwell Boltzmann dist.

$$\frac{1}{2} m v^2 \approx kT \qquad \left(P(v) \propto v^2 e^{-\frac{mv^2}{2kT}} \right)$$

$$v \approx \sqrt{\frac{2kT}{m}}$$

$$= 285 \text{ m/s.}$$

So this is far below the escape velocity



How long would it take the atmosphere to escape into space?

$$P(5010 \to \infty \text{ m/s}) = \int_{5010}^{\infty} \overbrace{\left(\frac{m}{2\pi kT}\right)^{3/2}}^{A} 4\pi v^2 e^{\frac{-mv^2}{2kT}}\, dv$$

This will be a complicated integral to find

$$I = \int_{5010}^{\infty} v^2 e^{-\frac{mv^2}{2kT}}\, dv \qquad x^2 = \frac{mv^2}{2kT} \qquad dx = \sqrt{\frac{m}{2kT}}\, dv$$

$$= \left(\frac{2kT}{m}\right)^{3/2} \int_{5010}^{\infty} x^2 e^{-x^2}\, dx$$

$$u = x^2 \quad dv = e^{-x^2}$$
$$du = 2x \quad v = -2xe^{-x^2}$$

$$= \left(\frac{2kT}{m}\right)^{3/2} \left(\overbrace{\left[-2x^3 e^{-x^2}\right]_{5010}^{\infty}}^{\text{negligably small?}} + 4 \underbrace{\int_{5010}^{\infty} x^2 e^{-x^2}\, dx}_{\left(\frac{m}{2kT}\right)^{3/2} I} \right)$$

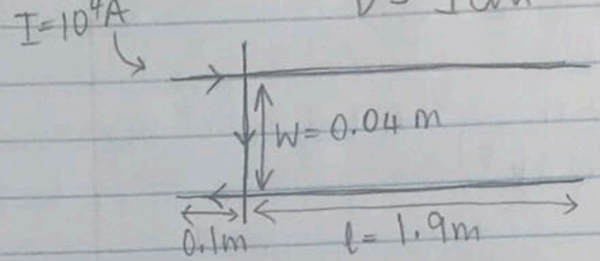
$$I = \left(\frac{2kT}{m}\right)^{3/2} 2(5010)^3 e^{-5010^2} + 4I$$

$$I \overset{?}{=} -\frac{2}{3} \left(\frac{2kT}{m}\right)^{3/2} 2(5010)^3 e^{-5010^2}$$

$$P \approx \frac{2}{3} \frac{1}{(\pi)^{3/2}} 4\pi \, 2 \, (5010)^3 e^{-5010^2}$$

6. Rail gun

$D = 1\,cm$ $\rho_m = 9000\,kg/m^3$ $\rho_r = 2 \times 10^{-8}\,\Omega m$



Current through loop

$$F = I\,\vec{W} \times \vec{B}$$

Two B forces acting $\qquad B = \dfrac{\mu_0 I}{2\pi s}$

$$F = 2\int_r^W \frac{\mu_0 I^2}{2\pi s}\,ds$$

$$= \frac{\mu_0 I^2}{\pi} \ln\left(\frac{W}{r}\right)$$

Force remains constant along rails

$$v^2 = v_0^2 + 2as$$

$$v^2 = 0 + 2\frac{\mu_0 I^2}{m\pi} \ln\left(\frac{W}{r}\right) l$$

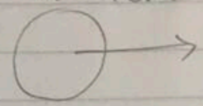
$$m = \rho \times 6\,cm \times \pi \times (0.5\,cm)^2$$

$\mu = 0.05$

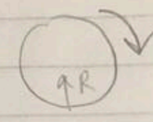
$M = 7kg$

$R = 11cm$

7.



$v_0 = 7m/s$



$R = Mg$

$$I = \frac{2}{5} M R^2$$

$$F = \mu R \qquad \ddot{x} = -\mu g$$

$\boxed{\text{Ball will roll when } \dot{x} = R\dot{\theta}}$ key point

$$\dot{x} = v_0 - \mu g t$$

$$\tau = R \times F$$
$$= I\ddot{\theta}$$

$$\mu \cancel{M} g \cancel{R} = \frac{2}{5} \cancel{M} R^{\cancel{2}} \ddot{\theta}$$

$$\ddot{\theta} = \frac{5}{2} \frac{\mu g}{R} \Rightarrow \dot{\theta} = \frac{5}{2} \frac{\mu g}{R} t$$

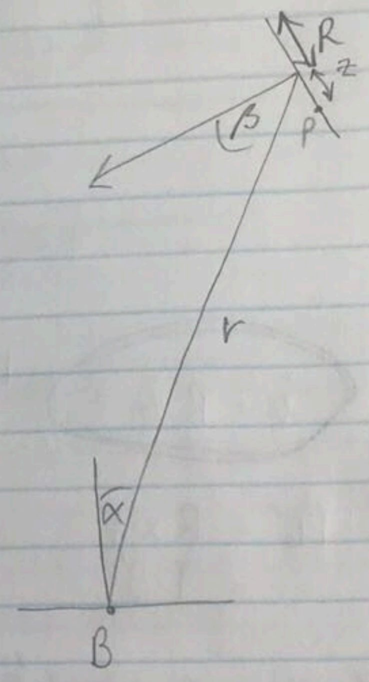
Time when $\dot{x} = R\dot{\theta}$

$$v_0 - \mu g t = \frac{5}{2} \mu g t$$

$$\boxed{t = \frac{2 v_0}{7 \mu g}}$$

Or $t = 4.08$ seconds (doesn't depend on M or R)

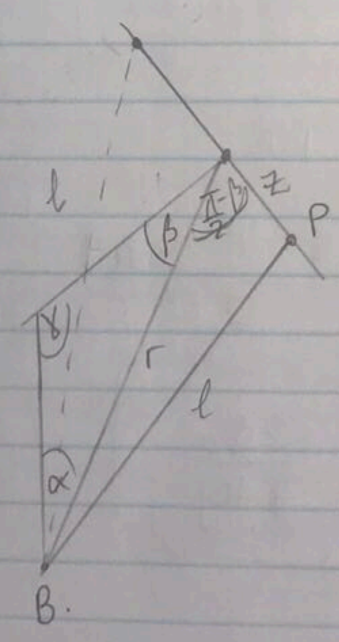
8.



$R << r$

Point on surface P, intensity I

B is a point detector. (?)



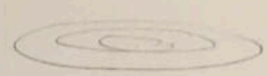
$\alpha + \beta + \gamma = \pi$

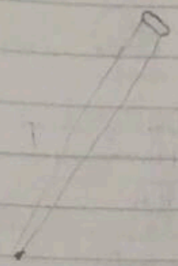
$\cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta.$

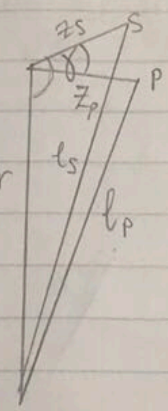
$l_p^2 = r^2 + z_p^2 - 2rz\cos\left(\frac{\pi}{2} - \beta\right)$

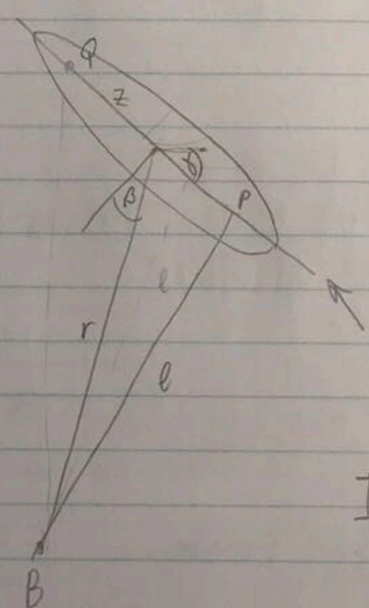
I_B (due to P) $= \underbrace{\frac{I_p}{2\pi\, l_p^2}}$

divide by area of half sphere surface at l_p.









$$= -\sin\beta$$

$$l_Q^2 = r^2 + z_Q^2 - 2rz_Q\cos\left(\frac{\pi}{2}+\beta\right)$$

contributions from points along this line:

$$I_B(\text{line}) = \frac{I_P}{2\pi}\int_0^R \frac{1}{z^2 - 2rz\sin\beta + r^2}\,dz$$

$$+ \frac{I_P}{2\pi}\int_0^R \frac{1}{z^2 + 2rz\sin\beta + r^2}\,dz$$

$$= \frac{I_P}{2\pi}\int_0^R \frac{2z^2+2r^2}{(z^2-2rz\sin\beta+r^2)(z^2+2rz\sin\beta+r^2)}\,dz$$

$$= \frac{I_P}{2\pi}\int_0^R \frac{2(z^2+r^2)}{\left(z^4 + \cancel{2rz^3\sin\beta} + z^2r^2 - \cancel{2rz^3\sin\beta} - 4r^2z^2\sin^2\beta - \cancel{2r^3z\sin\beta} + z^2r^2 + \cancel{2r^3z\sin\beta} + r^4\right)}\,dz$$

$$= \frac{I_P}{2\pi}\int_0^R \frac{2(z^2+r^2)\,dz}{\left(z^4 + 2z^2r^2\underbrace{(1-2\sin^2\beta)}_{\cos 2\beta} + r^4\right)}$$

$\cos^2 - \sin^2 = \cos 2$

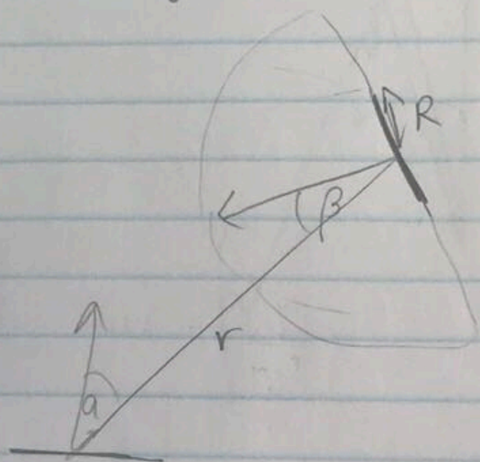
$1 - 2\sin^2$

$(z^2+r^2)(z^2 \times r^2$

$z^4 \times 2z^2r^2 \times r^4$

$$I_B\,(\text{line}) = \frac{I_p}{2\pi}\int_0^R \frac{2\,(z^2+r^2)}{z^4 + 2z^2r^2\cos 2\beta + r^4}\,dz$$

Simpler, Berry's

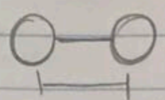


Think of emitting plate as covering solid angle $d\Omega$ on $2\pi r^2$ surface.

$$d\Omega = \frac{A}{r^2} = \frac{\pi R^2}{r^2}$$

$$\text{Flux} = \frac{\pi R^2 \cos\beta}{r^2}\cos\alpha$$

9.



L = spacing $\approx 2a_0$

$$V(r) = \frac{e^2}{4\pi\varepsilon_0 r} \qquad \frac{\partial^2 V(r)}{\partial r^2} = \frac{-e^2}{2\pi\varepsilon_0 r^3}$$

$$E = \frac{\text{Force}}{\text{Area}}\frac{L}{\Delta L} = \frac{\text{Force}}{L\,\Delta L}$$

(Area $= L^2$)

$$F = -\frac{\partial V}{\partial r} = \frac{+e^2}{4\pi\varepsilon_0 r^2}$$

$$V(r) = V(L) + \underbrace{\frac{\partial V(L)}{\partial r}}_{=0}(r-L) + \frac{\partial^2 V(L)}{\partial r^2}\frac{(r-L)^2}{2} + \ldots$$

$$\frac{\partial V(r)}{\partial r} \approx \frac{\partial^2 V(L)}{\partial r^2}(r-L)$$

$$\therefore \quad F = \frac{-e^2}{2\pi\varepsilon_0 L^3}\,\Delta L$$

$$\Longrightarrow \quad E = \frac{e^2\,\cancel{\Delta L}}{L\,\cancel{\Delta L}\,2\pi\varepsilon_0 L^3}$$

very high! ✓

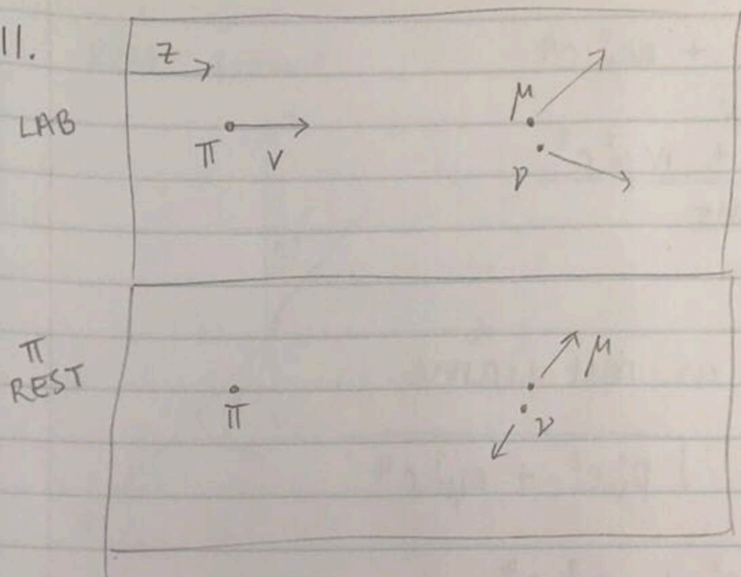
$$= \frac{e^2}{2\pi\varepsilon_0 L^4} \sim 6 \times 10^{13} \text{ Pa}$$

check this units: $\dfrac{C^2\,Nm^2}{C^2\,m^4} = \dfrac{N}{m^2}$ ✓

Hilroy

$m_\pi = 140 \frac{MeV}{c^2}$ $\qquad$ $m_\mu = 106 \frac{MeV}{c^2}$

11.



$$E^2 = p^2c^2 + m_0^2c^4$$

A. π rest frame:

Cons of 4-mom:

$$\begin{pmatrix} m_\pi c^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} E_\mu/c \\ p_\mu \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} p_\nu c \\ p_\nu \\ 0 \\ 0 \end{pmatrix} \qquad p_\mu = -p_\nu$$

$$m_\pi c^2 = \sqrt{p_\nu^2 c^2} + \underbrace{\sqrt{p_\mu^2 c^2 + m_\mu^2 c^4}}_{E_\mu}$$

$$E_\mu = m_\pi c^2 - p_\nu c$$

$$= m_\pi c^2 - p_\mu c \implies p_\mu^2 c^2 = (m_\pi c^2 - E_\mu)^2$$

$$E_\mu^2 = p_\mu^2 c^2 + m_\mu^2 c^4$$

$$= (m_\pi c^2 - E_\mu)^2 + m_\mu^2 c^4$$

$$= m_\pi^2 c^4 - 2m_\pi c^2 E_\mu + E_\mu^2 + m_\mu^2 c^4$$

$$2m_\pi c^2 E_\mu = m_\pi^2 c^4 + m_\mu^2 c^4$$

$$E_\mu = \frac{m_\pi^2 c^2 + m_\mu^2 c^2}{2m_\pi}$$

B. Velocity of muon in rest frame:

$$m_\pi c^2 = p_\nu c + \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4}$$

$$(m_\pi c^2 - p_\mu c)^2 = p_\mu^2 c^2 + m_\mu^2 c^4$$

$$m_\pi^2 c^4 - 2m_\pi c^3 p_\mu + \cancel{p_\mu^2 c^2} = \cancel{p_\mu^2 c^2} + m_\mu^2 c^4$$

$$p_\mu = \frac{m_\pi^2 c^2 - m_\mu^2 c^2}{2m_\pi c}$$

$$\gamma_u = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$= \gamma_u m_\mu \underbrace{u_\mu'}_{}$$

velocity of muon in pion rest frame

Velocity transformations

in lab frame (pion moved in x direction)

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} \qquad u_y = \frac{u_y'}{\gamma_v\left(1 + \frac{u_y' v}{c^2}\right)} \qquad u_z = \frac{u_z'}{\gamma_v\left(1 + \frac{u_z' v}{c^2}\right)}$$

↑ changed to x for simplicity

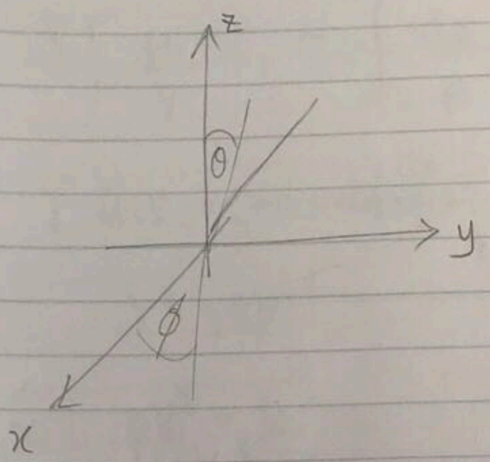
Because isotropic $\langle u_x' \rangle = \langle u_y' \rangle = \langle u_z' \rangle = 0$

$$u_x'^2 + u_y'^2 + u_z'^2 = u_\mu'^2$$

$$\langle u_x \rangle = \left\langle \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} \right\rangle$$

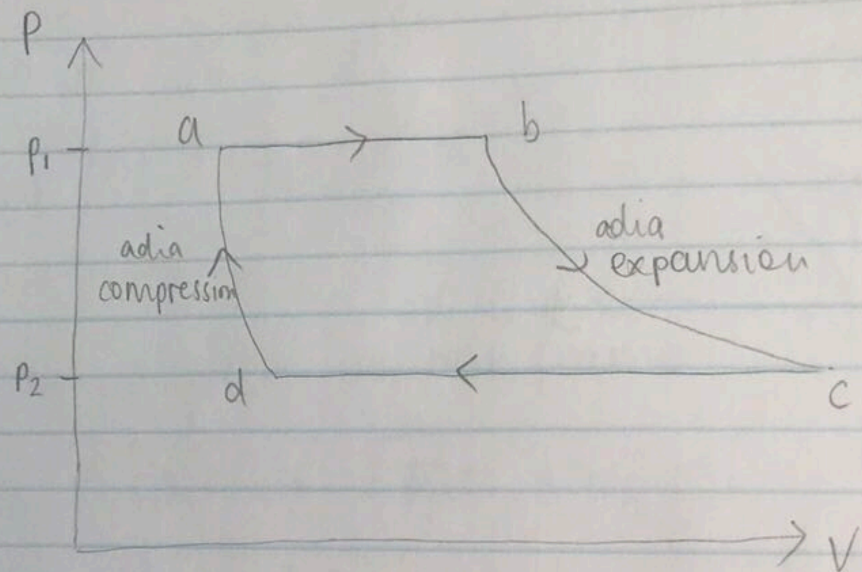
$$p^\mu = \left(\frac{E}{c}, \vec{p}\right) = \left(\frac{\gamma mc^2}{c}, \gamma m\vec{v}\right)$$

Rest frame



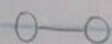
$$\vec{u} = u_\mu \begin{pmatrix} \sin\phi \sin\theta \\ \cos\phi \sin\theta \\ \cos\theta \end{pmatrix}$$

12.

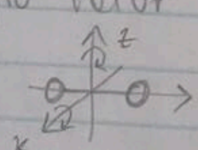


A. monoatomic gas: $g = 3$ degrees of freedom (translation in 3 directions)

diatomic gas: $g = 5$ (3 translational + 2 rotational)



rigid rotor $\therefore$ no vibrational dof



Cannot have rotation around y axis as too high E physically

B. Adiabatic $\therefore$ $dQ = 0$

$dU = -pdV = C_v dT$ (infinitesimal change in V)

$dT = \frac{1}{R} d(pV) = \frac{1}{nR}(pdV + Vdp)$

$\frac{C_v}{R}(pdV + Vdp) = -pdV$

$\underbrace{(C_v + nR)}_{C_p} pdV + C_v Vdp = 0$

$$\gamma = \frac{C_p}{C_v}$$

can also do with

$U = \frac{g}{2} k_B T N$ from the beginning

$$\gamma p\,dV = -V\,dp$$

$$\gamma \int \frac{dV}{V} = -\int \frac{dp}{p}$$

$$\gamma \ln V = -\ln p + A$$

$$V^{\gamma} = kp^{-1}$$

$$\underline{\underline{pV^{\gamma} = k}}$$

$$U = \underline{g \times \frac{1}{2} k_B T \times N}$$

$$C_v = \left.\frac{dU}{dT}\right|_V = \frac{N g k_B}{2}$$

$$C_p = C_v + N k_B \qquad N k_B = nR$$

$$\gamma = \frac{C_p}{C_v} = \frac{\cancel{Nk_B}\left(1 + \frac{g}{2}\right)}{\cancel{Nk_B}\frac{g}{2}}$$

$$\boxed{\gamma = \frac{2+g}{g}}$$

Check for monoatomic gas

$g = 3 \qquad \gamma = \frac{5}{3} \qquad C_v = \frac{3}{2}R \qquad C_p = \frac{5}{2}R$ ✓

derive with $pV^\gamma = \text{const}$.

$\Delta T = \frac{P \Delta V}{N k_B}$ & $pV^\gamma = \text{const}$

to prove.

bit of a guess

C. $pV = nRT$

$\Rightarrow$ d has lowest T, b has highest.

D. $a \to b$: $\uparrow V - P$, T must be increasing $\therefore$ entropy $\uparrow$ $\Delta S = +ve$ ✓

$b \to c$: adiabat, $\Delta S = 0$ ✓ (no heat exchange)

$c \to d$: $\downarrow V - P$, T must be decreasing $\therefore$ entropy $\downarrow$ $\Delta S = -ve$ ✓
(also need $\Delta S = 0$ around loop)

$d \to a$: adiabat, $\Delta S = 0$ ✓

E. $a \to b$: WD by gas $= \int_{V_a}^{V_b} p\, dV = p_1 (V_b - V_a)$

$= \underline{+ve}$ ✓

from looking at comparative values of V_a, V_b, V_c, V_d

$b \to c$: $pV^\gamma = c$ $\quad$ $WD = \int_{V_b}^{V_c} \frac{c}{V^\gamma} dV = \frac{c}{1-\gamma}\left(V_c^{1-\gamma} - V_b^{1-\gamma}\right)$

$= \underline{+ve}$ ✓

$c \to d$: $WD = \int_{V_c}^{V_d} p\, dV = p_2 (V_d - V_c)$

$= \underline{-ve}$ ✓

$d \to a$: $WD = \frac{c}{1-\gamma}\left(V_a^{1-\gamma} - V_d^{1-\gamma}\right) = \underline{+ve}$ ✓

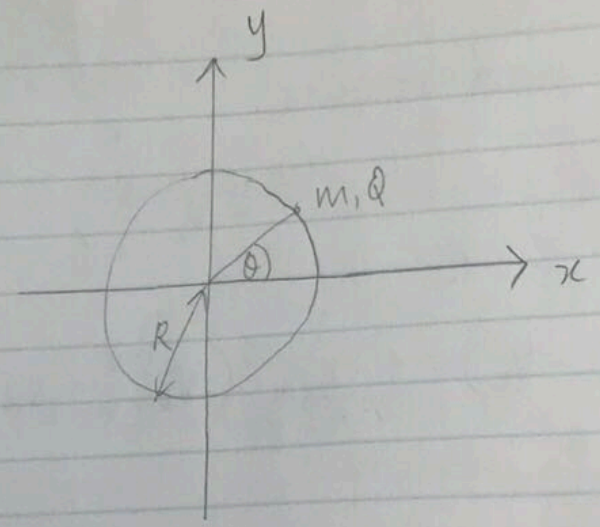
F. System acts as a heat engine.

$$\text{Total WD} = p_1(V_b - V_a) + \frac{c}{1-\gamma}\left(V_c^{1-\gamma} - V_b^{1-\gamma}\right)$$

$$+ p_2(V_d - V_c) + \frac{c}{1-\gamma}\left(V_a^{1-\gamma} - V_d^{1-\gamma}\right)$$

Work done at high T.
Clockwise = positive work out.
Look at area under each line

14.



Essentially moves in 1D

$$\psi(\theta) = \psi(\theta + 2\pi)$$

(periodic)

A. $$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi$$

Working in polar coordinates.

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}$$

$$\frac{\partial\psi}{\partial r} = 0$$ (not varying)

$$\nabla^2\psi = \frac{1}{R^2}\frac{\partial^2\psi}{\partial\theta^2}$$

$$\frac{\partial^2\psi}{\partial\theta^2} = \frac{-2mER^2}{\hbar^2}\psi$$

Solution like SHM.

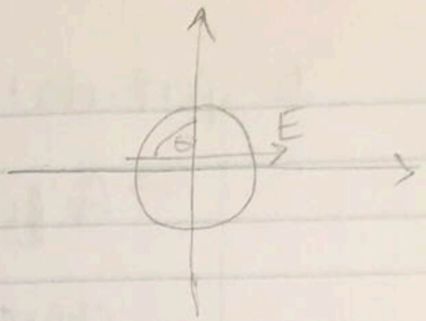
$$\omega^2 = \frac{2mER^2}{\hbar^2}$$

$$\psi_\pm = Ae^{\pm i\omega\theta}$$

Normalise $$\int_0^{2\pi} A^2\,d\theta = 1$$

$$A = \frac{1}{\sqrt{2\pi}}$$

$$\psi_\pm = \frac{1}{\sqrt{2\pi}} e^{\pm i w \theta}$$



Require $\psi_\pm(\theta) = \psi_\pm(\theta + 2\pi)$

$$\Rightarrow e^{\pm i w \theta} = e^{\pm i w(\theta + 2\pi)}$$

$$\Rightarrow e^{\pm i w 2\pi} = 1 = e^{\pm i 2\pi n}.$$

i.e. w can be any integer value n

$$n^2 = \frac{2mER^2}{\hbar^2}$$

$$\Rightarrow E = \frac{n^2\hbar^2}{2mR^2} \qquad n = 0, \pm 1, \pm 2, \pm 3 \ldots$$

So $E = 0$ state has degeneracy 1, but all states with $n > 0$ have degeneracy 2.

B. Ground state without E field $\psi(\theta) = \frac{1}{\sqrt{2\pi}}$ (constant all around circle)

$$\vec{E} = F\hat{x} \qquad V = -Fx.$$

$$\frac{-\hbar^2}{2mR^2}\frac{\partial^2 \psi}{\partial \theta^2} - FR\cos\theta\,\psi = E\psi$$

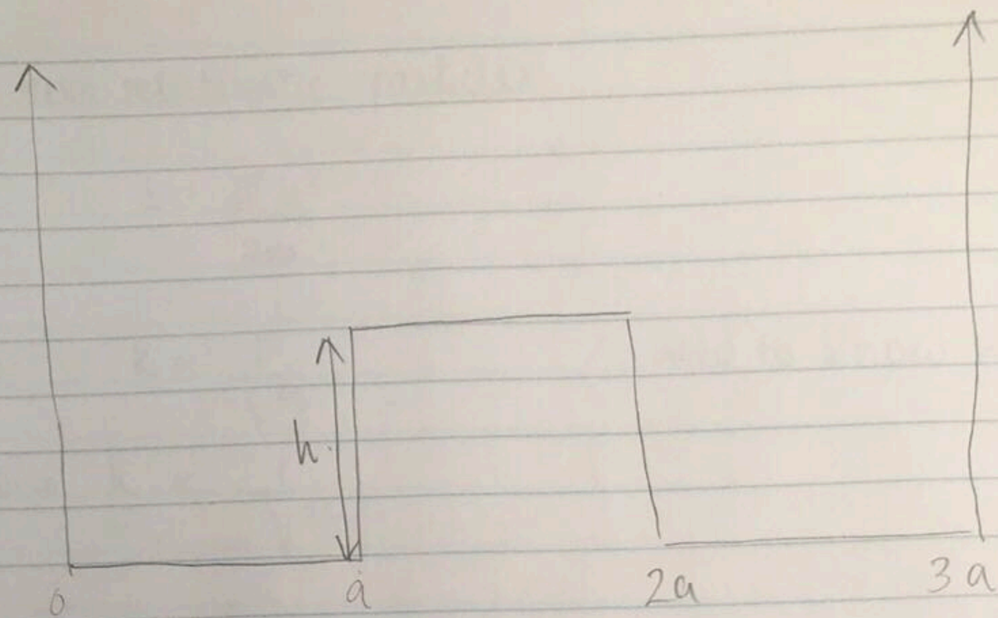
Don't know how to solve this?

Do perturbation theory.

→ Griffith's derivation

charge density $= Q|\psi^2|$

15.



← do we know this?

Ground state in L well

$$\psi_a = \sqrt{\frac{8}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

need to see my solution to QS3 Q3 from Quantum

Use WKB approximation:

$$\psi_b = \sqrt{\frac{8}{b}} \sin\left(\frac{n\pi x}{b}\right)$$

$$\psi_+ = \frac{\psi_a + \psi_b}{\sqrt{2}} \qquad \psi_- = \frac{\psi_a - \psi_b}{\sqrt{2}}$$

$$\psi_+(t) = \psi_+ e^{-\frac{iE_+t}{\hbar}} \qquad \psi_-(t) = \psi_- e^{-iE_-t/\hbar}$$

$$\psi(t=0s) = \psi_+ + \psi_-$$

E_+

16. For non relativistic particles

$$E = \frac{\hbar^2 k^2}{2m}$$

Bosons: $k \propto \frac{1}{V^{1/3}}$

Fermions $k \propto \frac{1}{V^{2/3}}$

need to know why!

$$dn = \frac{d^3k}{\left(\frac{2\pi}{L}\right)^3}$$

$$= \frac{V\, 2\pi k^2\, dk}{(2\pi)^3}$$

$$\frac{dE}{dk}$$

$$E = \int n(\varepsilon)\, f(\varepsilon)\, \varepsilon\, d\varepsilon.$$

$$\frac{N}{V}$$

$$E = \frac{p^2}{2m}$$