Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 17, 2012
3:10PM to 5:10PM
General Physics (Part II)
Section 6.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 6 (General Physics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on General Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1. The spectrum of a diatomic molecule includes transitions between electronic levels, between vibrational levels, and between rotational levels. Give an order of magnitude estimate of these transition energies in terms of the nuclear and electronic masses and other fundamental constants.

2. A dark nebula is a cold gas cloud in interstellar space that contains enough material to block out light from the stars behind it. A typical dark nebula is about 20 light-years in diameter and contains about 50 hydrogen atoms (i.e. monatomic hydrogen) per cubic centimeter at a temperature of about 20 K.

   (a) Estimate the mean free path for a hydrogen atom in a dark nebula. The radius of a hydrogen atom is 5.0×10^{-11}m.

   (b) Estimate the rms speed of a hydrogen atom and the mean free time between collisions. Based on this result, do you think that atomic collisions, such as those leading to H_2 molecule formation, are very important in determining the composition of the nebula?

   (c) Estimate the pressure inside a dark nebula.

   (d) Compare the rms speed of a hydrogen atom to the escape speed at the surface of the nebula (assumed spherical). If the space around the nebula were a vacuum, would such a cloud be stable or would it tend to evaporate?

   (e) The stability of dark nebulae is explained by the presence of the interstellar medium (ISM), an even thinner gas that permeates space and in which the dark nebulae are embedded. Show that for dark nebulae to be in equilibrium with the ISM, the numbers of atoms per volume and the temperatures of dark nebulae and the ISM must be related by:

$$\frac{(N/V)_{nebula}}{(N/V)_{ISM}} = \frac{T_{ISM}}{T_{nebula}}$$

   (f) In the vicinity of the Sun, the ISM contains about 1 hydrogen atom per 200 cm^3. Estimate the temperature of the ISM in the vicinity of the Sun. Compare to the temperature of the Sun's surface, about 5800 K. Would a spacecraft coasting through interstellar space burn up? Why or why not?

Notes: For atomic hydrogen, $M = 1.008 \times 10^{-3}$kg/mol, and 1 light-year is 9.46×10^{15}m. (Numerical factors in expressions for the mean free path, in part a., and the rms speed, in part b., may be ignored.)

3. An atom or ion with one electron has energy levels $E_n = -A/n^2$. Two neighboring lines in its spectrum at room temperature have wavelengths $\lambda_1 = 97.5$ nm and $\lambda_2 = 102.8$ nm. (Note: $hc = 1.240 \times 10^{-6}$ eV·m.)

   (a) What is the constant A?

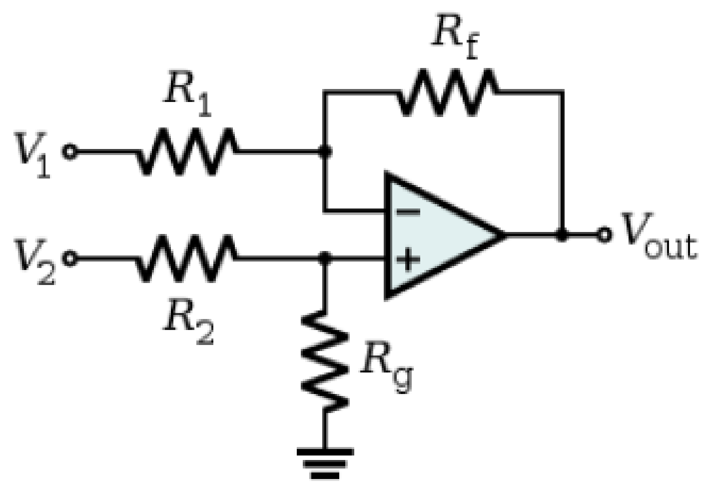
   (b) Identify the atom.

4. Find, by using dimensional analysis, the ratio of the muon and neutron lifetimes. The muon decays through $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ and the neutron through $d \to u e^- \bar{\nu}_e$. Assume the form of the weak Hamiltonian to be

$$H_{wk} = \frac{G_F}{\sqrt{2}} \bar{\psi}_1 O \psi_2 \bar{\psi}_3 O \psi_4,$$

with O some operator. $G_F \approx 10^{-5}/m_p^2$ with m_p the mass of the proton. The muon, electron, up and down quark masses are 105, 0.5, 2.3 and 4.8 MeV, respectively. You can neglect the neutrino masses.

5. Consider the electronic circuit shown below. Assume that the amplifier shown is an ideal operational amplifier, with infinite input impedance, zero output impedance, and infinite open-loop gain.



(a) Determine an expression for V_{out} in terms of V_1 and V_2.

(b) Considering the circuit's behavior, what might such a circuit be used for?

6. Fermi surface is one of the most fundamental notions in physics of metals. Suppose we have a single crystal of an unknown metal in front of us. Our goal is to characterize properties of the Fermi surface in this metal.

   The following measurements were performed:

   (a) Specific heat
   (b) Pauli susceptibility
   (c) DC-conductivity
   (d) Thermopower
   (e) Hall resistance
   (f) Plasmon frequency
   (g) Cyclotron resonance
   (h) Shubnikov-de Haas oscillations
   (i) Angular resolved photoemission spectroscopy

   Choose any four of the above listed measurements and describe briefly the idea of experiment and the properties of the Fermi surface which can be extracted from them.

Weinberg #2
solution

Let $m =$ electron mass
$M =$ atomic mass

$a = \frac{\hbar^2}{me^2}$ = Bohr radius

$$\Delta E_{elec} \sim \frac{e^2}{a} \sim \frac{me^4}{\hbar^2}$$

$$\Delta E_{vib} \sim \hbar\omega, \text{ with } \omega = \sqrt{\frac{V''(a)}{M}} \approx \sqrt{\frac{e^2}{Ma^3}}$$

$$\approx \frac{me^4}{\hbar^2}\sqrt{\frac{m}{M}}$$

$$\Delta E_{rot} \approx \frac{J^2}{I} \sim \frac{\hbar^2}{Ma^2} \approx \frac{me^4}{\hbar^2}\left(\frac{m}{M}\right)$$

2014 Quals General Question (Dodd)

A dark nebula is a cold gas cloud in interstellar space that contains enough material to block out light from the stars behind it. A typical dark nebula is about 20 light-years in diameter and contains about 50 hydrogen atoms (i.e. monatomic hydrogen) per cubic centimeter at a temperature of about 20 K.

a). Estimate the mean free path for a hydrogen atom in a dark nebula. The radius of a hydrogen atom is 5.0×10^{-11} m.

b). Estimate the rms speed of a hydrogen atom and the mean free time between collisions. Based on this result, do you think that atomic collisions, such as those leading to H_2 molecule formation, are very important in determining the composition of the nebula?

c). Estimate the pressure inside a dark nebula.

d). Compare the rms speed of a hydrogen atom to the escape speed at the surface of the nebula (assumed spherical). If the space around the nebula were a vacuum, would such a cloud be stable or would it tend to evaporate?

e). The stability of dark nebulae is explained by the presence of the interstellar medium (ISM), an even thinner gas that permeates space and in which the dark nebulae are embedded. Show that for dark nebulae to be in equilibrium with the ISM, the numbers of atoms per volume and the temperatures of dark nebulae and the ISM must be related by:

$$\frac{(N/V)_{nebula}}{(N/V)_{ISM}} = \frac{T_{ISM}}{T_{nebula}}$$

f). In the vicinity of the Sun, the ISM contains about 1 hydrogen atom per 200 cm^3. Estimate the temperature of the ISM in the vicinity of the Sun. Compare to the temperature of the Sun's surface, about 5800 K. Would a spacecraft coasting through interstellar space burn up? Why or why not?

Notes: for atomic hydrogen, $M = 1.008 \times 10^{-3}$ kg/mol. , and 1 light-year is 9.46×10^{15} m.

Solution:

a). First we need an expression for the mean free path λ

$$\lambda = (4\pi\sqrt{2}r^2(N/V))^{-1}$$

where r is the hydrogen radius. **(NOTE TO QUALS COMMITTEE:** students may not get the factors of 2,4 and π here – may want to give them some help, or ignore such factors in their answers.) So:

$$\lambda = (4\pi\sqrt{2}r^2(N/V))^{-1} = (4\pi\sqrt{2}(5.0\times10^{-11}\text{ m})^2(50\times10^6\text{ m}^{-3}))^{-1} = 4.5\times10^{11}\text{ m}.$$

b). The RMS speed is given by:

$$v_{\text{rms}} = \sqrt{3RT/M} = \sqrt{3(8.3145\text{ J/mol}\cdot\text{K})(20\text{ K})/(1.008\times10^{-3}\text{ kg/mol})} = 703\text{ m/s},$$

and the time between collisions is:

$$(4.5\times10^{11}\text{ m})/(703\text{ m/s}) = 6.4\times10^{8}\text{ s},$$

or about 20 years! So, collisions are not very important.

c). Assume ideal gas behavior, so we can determine the pressure:

$$p = (N/V)kT = (50/1.0\times10^{-6}\text{ m}^3)(1.381\times10^{-23}\text{ J/K})(20\text{ K}) = 1.4\times10^{-14}\text{ Pa}.$$

d). Assuming a spherical distribution, the escape speed is:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G(Nm/V)(4\pi R^3/3)}{R}} = \sqrt{(8\pi/3)G(N/V)mR^2}$$

$$v_{\text{escape}} = \sqrt{(8\pi/3)(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(50\times10^{6}\text{ m}^{-3})(1.67\times10^{-27}\text{ kg})(10\times9.46\times10^{15}\text{ m})^2}$$

$$v_{\text{escape}} = 650\text{ m/s}.$$

This is lower than v_{rms} and the cloud would tend to evaporate.

e). In equilibrium (clearly not *thermal* equilibrium), the pressures will be the same; from $pV = NkT$, $kT_{\text{ISM}}(N/V)_{\text{ISM}} = kT_{\text{nebula}}(N/V)_{\text{nebula}}$ and the result follows.

f). With the result of part e):

$$T_{\text{ISM}} = T_{\text{nebula}}\left(\frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}}\right) = (20\text{ K})\left(\frac{50\times10^{6}\text{ m}^3}{(200\times10^{-6}\text{ m}^3)^{-1}}\right) = 2\times10^{5}\text{ K},$$

i.e. more than three times the temperature of the sun. This indicates a high average kinetic energy, but the thinness of the ISM means that a ship would not burn up.

GENERAL PHYSICS

Atomic spectral lines. SOLUTION.

a) At room temperature, atoms are in the electronic ground state, so all transitions start from $n = 1$. Therefore, the wavelength ratio for the neighboring transitions is

$$\frac{\lambda_1}{\lambda_2} = 0.948 = \frac{1 - 1/n^2}{1 - 1/(n+1)^2}. \tag{1}$$

This ratio is 0.844 if $n = 2$ and 0.948 if $n = 3$.

Then

$$A = \frac{hc}{\lambda_2} \frac{1}{1 - 1/3^2} = \frac{1.240 \times 10^{-6}\ \mathrm{eV}}{102.8 \times 10^{-9}} \times 1.125 = 13.6\ \mathrm{eV}. \tag{2}$$

b) Since $A = 1$ Ry, the lines belong to the hydrogen spectrum.

2 Qualitative Problem

Find, by using dimensional analysis, the ratio of the muon and neutron lifetimes. The muon decays through $\mu^- \to \nu_\mu e^- \bar{\nu_e}$ and the neutron through $d \to u e^- \bar{\nu_e}$. Assume the form of the weak hamiltonian to be

$$H_{wk} = \frac{G_F}{\sqrt{2}} \bar{\psi_1} O \psi_2 \bar{\psi_3} O \psi_4, \tag{3}$$

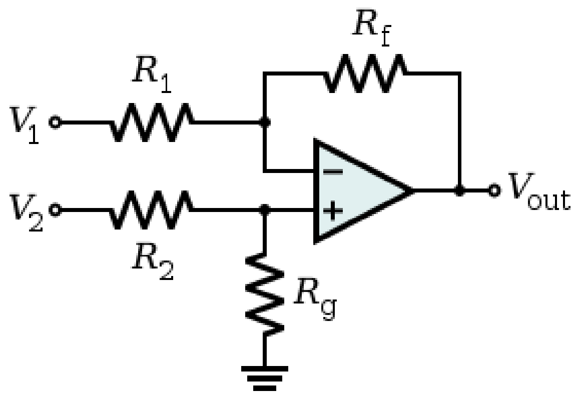
with O some operator. $G_F \approx 10^{-5}/m_p^2$ with m_p the mass of the proton. The muon, electron, up and down quark masses are 105, 0.5, 2.3 and 4.8 MeV, respectively. You can neglect the neutrino masses.

2.1 Solution

The decay width Γ will be proportional to G_F^2, but it also has dimension energy (or mass). So we must have $\Gamma \sim G_F^2 m^5$, where m is the characteristic mass of the process, i.e. the mass difference between the initial and final states. For muon decay, that is $\approx$ 105 MeV, and for neutron decay $\approx$ 2.5 MeV. The ratio of the muon and neutron lifetimes is thus $\sim \frac{(100)^5}{(2.5)^5} \approx 10^8$

General Physics

Question: Consider the electronic circuit shown below. Assume that the amplifier shown is an ideal operational amplifier, with infinite input impedance, zero output impedance, and infinite open-loop gain.



(a) Determine an expression for V_{out} in terms of V_1 and V_2.
(b) Considering the circuit's behavior, what might such a circuit be used for?

Soluion: (a) For an ideal opamp, no currents will flow into or out of the amplifier, and the inverting and non-inverting inputs will be held by the opamp at the same voltage (denoted V+ and/or V- hereafter, with V+ = V-).

The V2 part of the circuit can be viewed as a simple voltage divider, with
(1) V+ = Rg/(R2 + Rg) X V2

Therefore, (2) V- = V+ = Rg/(R2 + Rg) X V2

In the V1 part of the circuit, the current through R1 is
(3) I1 = (V1 – V-)/R1

and this same current flows through RF.

So, Vout = V1 – I1 (R1 + RF)
= V1 – [(V1 – V-)/R1] (R1 + RF) (substituting for I1 from eqn 3)
= V1 – [(V1 – Rg/(R2 + Rg) X V2)/R1] (R1 + RF) (substituting for V- from eqn 3)

Simplifying gives
Vout = V2 [Rg (R1 + RF)]/[R1 (R2 + Rg)] – V1 (RF/R1)

(b) The circuit is an example of a so-called difference amplifier, amplifying the difference between V1 and V2 (in this case, a weighted difference, with weights depending on the values chosen for the various resistors). It could be used, for example, in a differential system, where V1 and V2 are chosen to be opposite polarity versions of the same signal for transmission over long cables and providing cancellation of common mode noise.

Sec 6 - 6

Subject: Re: Reminder to please proofread Quals problem / Availability to grade?
From: "Yasutomo J. Uemura" <tomo@lorentz.phys.columbia.edu>
Date: 1/3/2014 5:49 PM
To: Randy <rt2255@columbia.edu>
CC: Randy Torres <rtorres@phys.columbia.edu>

Dear Randy:

Could you please remove
"and electron paramagnetic resonance" from (g),
and make (g) simply as Cytlotron resonance

I think having two items in the same column will be
confusing to the students.

Since this is a description problem, I would hesitate to
provide model answer. I will be happy to grade.
I will be in Japan until Jan. 17, and will become
availble at Pupin on Jan. 18. Is this OK for grading ?

Sincerely yours,

Tomo Uemura

On Thu, 2 Jan 2014, Randy wrote:

> Dear Tomo,
>
> A quick reminder to please proofread your Quals problem (Section 6, Question 6) and let us know if any modifications need to be made. Also, are you able to grade your problem? Would you also provide us with a solution to your question.
>
> Thanks,
>
> Randy
>
> --
> Randy Torres
> Senior Administrative Manager
> Department of Physics
> Columbia University
> 704 Pupin Hall
> Phone: 212-854-3366
> Fax: 212-854-3379