Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 12, 2018
2:00PM to 4:00PM
General Physics (Part II)
Section 6.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 6 (General Physics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" × 11" paper (double-sided) you have prepared on General Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1. Suppose you are doing a spectroscopy experiment on a cloud of ^{84}Kr atoms at room temperature and negligible pressure. You want to tune a laser to probe a transition at 811.51 nm (with respect to an atom at rest) by passing the laser through the gas in one direction. To make your measurements, you would like to use an old interferometer you found hidden in the bottom of a drawer in your lab. It has a resolving power of 10^7. You may use the following information: the RMS velocity of ^{84}Kr atoms at room temperature is 300 ms^{-1} and the collision cross-section for ^{84}Kr atoms is $\approx 10^{-15}$ cm^2.

   (a) Estimate the dominant relative uncertainty on the measurement of the line. Suppose you could first cool the cloud to 1 K. Estimate the expected improvement in your results, if any.

   (b) Suppose that for the measurement above, the atoms are first put into a long-lived excited state (i.e. metastable state). The atoms must stay in this state as they freely expand from the excitation chamber to the detection chamber, 50 cm away. If the atoms collide with each other, however, they are likely to de-excite (a process called collisional quenching). Assume that collisions with the walls are negligible. Estimate the maximum pressure of gas that can be safely used in the region between chambers, both for a gas at 300 K and a gas at 1 K.

   (c) Once the atoms enter the detection chamber, they must each be probed for 0.5 s. What is the maximum pressure that can safely be maintained in this chamber, again for a gas at 300 K and at 1 K?

2.

(a) Using classical electricity and magnetism arguments (i.e. Coulombs potential), make an order of magnitude comparison of chemical energy to nuclear fission energy.

(b) Write a primary reaction for uranium fission. State two key features necessary for the production of nuclear energy.

(c) Shortly after detonation the fireball of a uranium fission bomb consists of a sphere of gas of radius 15 meters and temperature of 100 million K. Assuming that the expansion is adiabatic and that the fireball remains spherical, estimate the radius of the ball when the temperature is 3000 K.

(d) Thermonuclear weapons can be three orders of magnitude more powerful than fission bombs. What is the primary explanation for this enormous increase in energy?

3. The peaks of tall mountains are generally much colder than their bases. How much colder?

   Assume our lower atmosphere, heated from below by contact with the warmer earth surface, is slightly convective, because it is made slightly buoyant by that contact. Adiabatic convection results in a variation of temperature, T and pressure, P with altitude, h. Evaluate either $T(h)$ or $P(h)$ in terms of $T(0)$ or $P(0)$ using the following:

   The equation of state of air is $PV = RT$, where R is Boltzmann constant multiplied by the Avogadro number N_A. Air heat capacity c_V (per N_A molecules) is from rigid diatomic molecules of average mass $\bar{m}$ with classically calculated translational and rotational energies (but no excitable vibrational or electronic ones).

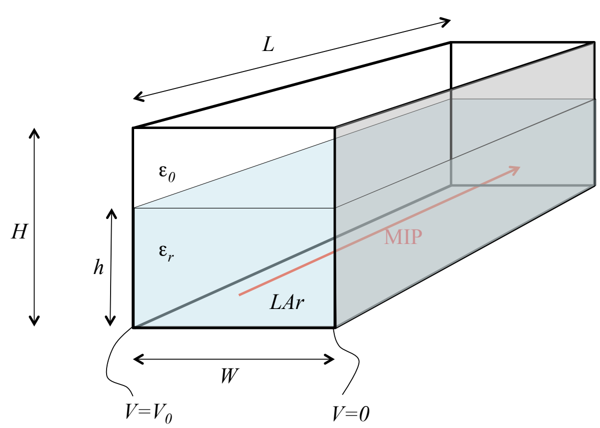
4. Neutron stars form when a solar mass is compressed to a radius $R \sim 10$ km. Estimate the maximum spin rate of a neutron star. Express your answer in revolutions per second. Recall that the radius of Earths orbit around the sun is $r_E \approx 1.5 \times 10^8$ km.

5. Typical dust particles in the Solar System have mass density ρ. They experience both gravitational and radiation pressure effects of the Sun (mass $M = 2 \times 10^{30}$ kg, emitted light power $P = 4 \times 10^{26}$ W).
   (a) Are the smaller or the larger dust particles more likely to be ejected from the Solar System?
   (b) Estimate, symbolically, the critical radius R of a particle that would not be ejected from the Solar System.
   (c) Provide a numerical estimate to part (b).

6. The MicroBooNE Liquid Argon Time Projection Chamber (LAr TPC) detector is a large rectangular volume with dimensions shown in the figure. It is nominally filled with liquid argon. Ionization electrons, liberated by a charged particle propagating through the liquid argon along the particle's path, drift under an applied uniform electric field toward a plane of anode wires that lie in the shaded plane in the figure. The wires are strung vertically from the top to the bottom of the TPC, and spaced 3 mm apart along the length (L) of the TPC. Answer the following questions:

   (a) During the filling of the TPC, the liquid level h changed with a constant rate dh/dt. The potential difference across the TPC was held fixed at V_0. The resulting change in effective capacitance in the TPC resulted in increasing noise levels on the TPC anode wires as a function of increasing liquid level. Calculate the rate of change of effective TPC capacitance, dC/dt.

   (b) A minimum ionizing particle (MIP) traverses the length of the TPC (see figure) near the bottom when the detector is half full. The particle loses energy due to ionization in the argon at a rate of 2 MeV/cm. The ionization energy of argon is 23.6 eV. You may assume that half of the ionization charge liberated is lost during the drift. The noise level on a single TPC wire for an empty detector, measured in Equivalent Noise Charge (ENC), is 400 electrons; when the detector is full, the ENC is 450 electrons. Calculate the signal-to-noise ratio, on a single TPC wire, under these conditions.



GENERAL QUALS 2018: EXPERIMENTAL TECHNIQUES

1. Spectroscopy

Suppose you are doing a spectroscopy experiment on a cloud of ^{84}Kr atoms at room temperature and negligible pressure. You want to tune a laser to probe a transition at 811.51 nm (with respect to an atom at rest) by passing the laser through the gas in one direction. To make your measurements, you would like to use an old interferometer you found hidden in the bottom of a drawer in your lab. It has a resolving power of 10^7.

Will this interferometer be good enough? Estimate the dominant relative uncertainty you would expect for this measurement. Suppose you could first cool the cloud to 1 K. Estimate the expected improvement in your results, if any.

2. Vacuum

Suppose that for the measurement above, the atoms are first put into a long-lived excited state (i.e. metastable state). The atoms must stay in this state as they freely expand from the excitation chamber to the detection chamber, 50 cm away. If the atoms collide with each other, however, they are likely to de-excite (a process called collisional quenching). Assume that collisions with the walls are negligible. Estimate the maximum pressure of gas that can be safely used in the region between chambers, both for a gas at 300 K and a gas at 1 K.

Once the atoms enter the detection chamber, they must each be probed for 0.5 s. What is the maximum pressure that can safely be maintained in this chamber, again for a gas at 300 K and at 1 K?

Recall that the collision cross section for ^{84}Kr atoms is $\approx 10^{-15}\text{cm}^2$.

Section 6 - Problem 1
Aprile

Spectroscopy Solution: The spectral peaks will be significantly broadened due to Doppler broadening, i.e. the thermal velocity of the gas cloud. The effect on the resolution can be approximated by considering the fractional Doppler shift.

First calculate the average speed at room temperature and at 1 K. At or below room temperature, the thermal energy is safely approximated as non-relativistic, so

$$\begin{aligned} 3/2k_BT &= 1/2mv^2 \\ \Rightarrow v &= \sqrt{3k_BT/m} \end{aligned}$$

Note: The above is actually the RMS velocity. For a Maxwell-Boltzmann distribution, the most probable speed is $\sqrt{2k_BT/m}$, but this is only small correction, and can be neglected in this approximation.

Recall that $k_B \approx 1.4 * 10^{-23}$ m^2 kg s^{-2} K^{-1}, and at room temperature, T$\approx$ 300K

$$\Rightarrow v_{T=300} \approx 300m/s$$
$$\Rightarrow v_{T=1} \approx 20m/s$$

Recall the Doppler shift formula

$$\omega' = \gamma\omega(1 - \beta Cos(\theta))$$

is well approximated at non-relativistic velocities as

$$\omega' = \omega(1 - \beta Cos(\theta)) = \omega - \vec{k} \cdot \vec{v}$$

It's magnitude is maximum for $\theta \in \{0, \pi\}$. So, for $\omega' = \omega_0 = 2\pi c/\lambda_0$, we find that the magnitude of the fractional Doppler broadening is approximated to first order by

$$\frac{\omega - \omega_0}{\omega_0} = \frac{v}{c}$$

To improve the approximation, the absorption probability would need to be taken into account, but this changes the result by less than a factor of 2.

We thus find that the fractional Doppler shift is $\approx 10^{-6}$ at room temperature, and $\approx 6.7 * 10^{-8}$ at 1 K. So, resolving powers of roughly 10^6 and $1.5 * 10^7$ would be necessary to differentiate between Doppler broadened peaks.

Hence, at room temperature, the relative uncertainty of our measurement will be dominated by the width of the Doppler peak to $\approx 10^{-6}$. In this case, our old interferometer is no problem, as it can resolve such wide peaks. But, at 1 K, the relative uncertainty due to the resolution of the interferometer will be on the same order as that from the Doppler broadening, at $\approx 10^{-7}$, and thus can not be neglected. So, a better interferometer may help at the lower temp, but not much. Regardless, a factor of 10 improvement is expected when decreasing the temperature from 300 K to 1 K.

Vacuum Solution: A good approximation of the maximum allowable pressure between chambers is that which gives a mean free path between collisions of 50 cm.

Recall that the mean free path is given by

$$\lambda_{mfp} = \frac{1}{\sqrt{2}n\sigma}$$

For an ideal gas, P = nk$_B$T, so we can rewrite as

$$P = \frac{k_BT}{\sqrt{2}\lambda_{mfp}\sigma}$$

Hughes

Problem 6-2

(a) Using classical electricity and magnetism arguments (i.e. Coulomb's potential), make an order of magnitude comparison of chemical energy to nuclear fission energy.

$V = \frac{Z^2e^2}{r}$

chemical reaction $\quad Z \sim 1 \quad$ and $\quad r \sim 10^{-10}m$

nuclear reaction (uranium) $\quad Z \sim 100$ and $\quad r \sim 10^{-14}m$

nuclear/chemical $= (\frac{\frac{Ze^2}{r}_{nuclear}}{\frac{Ze^2}{r}_{chemical}}) = 100/(\frac{10^{-10}m}{10^{-14}m}) \sim 10^6$

(b) Write a primary reaction for uranium fission. State two key features necessary for the production of nuclear energy.

$n + U^{235} \rightarrow Sr + Xe + n + n$

* chain reaction

*critical mass

(c) Shortly after detonation the fireball of a uranium fission bomb consists of a sphere of gas of radius 15 meters and temperature of 100 million K. Assuming that the expansion is adiabatic and that the fireball remains spherical, estimate the radius of the ball when the temperature is 3000 K.

$TV^{\gamma-1} =$ constant

$\mathrm{T}_{initial} = 10^8$ K

$\mathrm{T}_{final}=$ 3000 K

$\mathrm{V}_{initial} = 4/3\pi(15m)^3 \sim 13,500\ m$

For nitrogen, $\gamma \sim 7/5$

$\gamma - 1 = 2/5$

$V_{final} = \left(\frac{T_i}{T_f}\right)^{5/2} V_{initial} \sim \left(\frac{10^8}{3x10^3}\right)^{5/2} (13,500m^3) \sim 2$ x $10^{15}\ m^3$

$r \sim$ 100 km

(d) Thermonuclear weapons can be three orders of magnitude more powerful than fission bombs.

No critical mass limitation in fusion bomb.

Problem 2

A system with two nondegenerate energy levels, E_0 and E_1 ($E_1 > E_0 > 0$) is populated by N distinguishable particles at temperature T.

(a) What is the average energy per particle?
Express answer in terms of E_0, E_1 and $\Delta E = E_1 - E_0$.

(b) What is the average energy per particle as $T \to 0$?
Express answer in terms of E_1 and ΔE.

(c) What is the average energy per particle as $T \to \infty$?
Express answer in terms of $E_0 + E_1$ and ΔE.

(d) What is the specific heat at constant volume, c_V, of this system?
Express answer in terms of ΔE.

(e) Compute c_V in the limits $T \to 0$ and $T \to \infty$ and make a sketch of c_V versus $\Delta E/kT$.

Problem 3

For the infinite square well with walls located at $x = a$ and $x = -a$, the ground state energy is

$$E_1 = \frac{\pi^2 \hbar^2}{8ma^2}$$

and the ground state wavefunction is

$$\psi_1 = \frac{1}{\sqrt{a}} cos(\pi x/2a)$$

The position-momentum uncertainty relationship for this state is:

$$\Delta x \Delta p \geq \kappa \ \hbar/2$$

Find κ.

POSSIBLY USEFUL FORMULAS

$\int (\mathrm{sin\ ax})(\mathrm{cos^2\ ax})\mathrm{dx} = -\frac{\mathrm{cos^{m+1}\ ax}}{\mathrm{(m+1)a}}$

$\int (\mathrm{sin^m\ ax})(\mathrm{cos\ ax})\mathrm{dx} = \frac{\mathrm{sin^{m+a}\ ax}}{\mathrm{(m+a)a}}$

$\int (\mathrm{sin^3\ ax})\mathrm{dx} = -\frac{1}{3\mathrm{a}}(\mathrm{cos\ ax})(\mathrm{sin^2\ ax} + 2)$

$\int (\mathrm{cos^3\ ax})\mathrm{dx} = \frac{1}{3\mathrm{a}}(\mathrm{sin\ ax})(\mathrm{cos^2\ ax} + 2)$

$\int (\mathrm{sin^2\ ax})\mathrm{dx} = \frac{x}{2} - \frac{1}{4\mathrm{a}}(\mathrm{sin\ 2ax})$

$\int (\mathrm{cos^2\ ax})\mathrm{dx} = \frac{x}{2} + \frac{1}{4\mathrm{a}}(\mathrm{sin\ 2ax})$

$\int (\mathrm{xsin^2\ ax})\mathrm{dx} = \frac{\mathrm{x^2}}{4} - \frac{\mathrm{xsin\ 2ax}}{4\mathrm{a}} - \frac{\mathrm{cos\ 2ax}}{8\mathrm{a^2}}$

$\int (\mathrm{xcos^2\ ax})\mathrm{dx} = \frac{\mathrm{x^2}}{4} + \frac{\mathrm{xsin\ 2ax}}{4\mathrm{a}} + \frac{\mathrm{cos\ 2ax}}{8\mathrm{a^2}}$

$\int (\mathrm{x^2sin^2\ ax})\mathrm{dx} = \frac{\mathrm{x^3}}{6} - (\frac{\mathrm{x^2}}{4\mathrm{a}} - \frac{1}{8\mathrm{a^3}})\mathrm{sin\ 2ax} - \frac{\mathrm{xcos\ 2ax}}{4\mathrm{a^2}}$

$\int (\mathrm{x^2cos^2\ ax})\mathrm{dx} = \frac{\mathrm{x^3}}{6} + (\frac{\mathrm{x^2}}{4\mathrm{a}} - \frac{1}{8\mathrm{a^3}})\mathrm{sin\ 2ax} + \frac{\mathrm{xcos\ 2ax}}{4\mathrm{a^2}}$

Physical Phenomena (or thermodynamics)

Mal Ruderman

The peaks of tall mountains are generally much colder than their bases. How much colder?

Assume our lower atmosphere, heated from below by contact with the warmer earth surface, is slightly "convective" because it is made slightly buoyant by that contact. Large "blobs" slowly rise if they are slightly warmer than their environment (other blobs) and slightly cooler ones descend. Heat transfer between adjacent blobs is not significant. Local ~~there~~ temperature (T) and pressure (P) depend only on altitude (h).

What is T(h)?

or

What is T(P)?

In terms of g, T(0) or P(0), and whatever else is needed from below

Equation of state of air is

$PV = RT$ $\quad$ R = Boltzman Constant k_B $\times$ Avogadro number N_A

Air heat capacity C_v (per N_A molecules) is from rigid diatomic molecules of average mass $\bar{m}$ with classically considered translational and rotational energies (but no excitable vibrational or electronic ones).

Answer

$$DQ = TdS = 0 = dU + pdV = C_v dT + pdV$$

$$= C_v dT + RdT - VdP$$

Add $V = \frac{RT}{P}$

and $C_v = \frac{3}{2}R + \frac{2}{2}R = \frac{5}{2}R$

$$0 = C_p\, dT - RT\frac{dP}{P} \qquad (C_p \equiv C_v + R)$$

$$\boxed{T = T_0\left(\frac{P}{P_0}\right)^{2/7}}$$

or use

$$\frac{dP}{P} = -\frac{\bar{m} g n\, dh}{n k_B T}$$

$g \equiv$ acc. of gravity in line 2 above

$$\frac{dT}{~~T_0~~} = -\frac{2}{7}\frac{\bar{m} g\, dh}{k_B}$$

$$\boxed{k_B(T - T_0) = -\frac{2}{7}\bar{m} g h}$$

W.A. Zajc

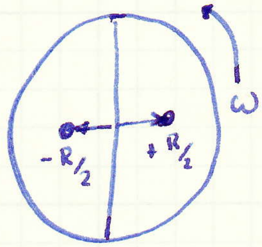
1 Problem: Maximum Neutron Star Spin Rate

Neutron stars form when a solar mass is compressed to a radius $R \sim 10$ km. Estimate the maximum spin rate of a neutron star. Express your answer in revolutions per second. Recall that the radius of Earth's orbit around the sun is $r_E \approx 1.5 \times 10^8$ km.

Section 6 - Problem 4
Zajc

NS SPIN RATE — ZAJC

At the maximum spin rate, the gravitational attraction just balances required centripetal force:



$$G\frac{\frac{M}{2}\cdot\frac{M}{2}}{R^2} = \frac{M}{2}\cdot\frac{R}{2}\cdot\omega_{max}^2$$

$$\Rightarrow \omega_{max}^2 = G\frac{M}{R^3} \qquad \text{(i.e, is set by density)}$$

To find $GM_\odot$, use data about Earth's orbit:

$$G\frac{m_E M_\odot}{r_E^2} = m_E r_E \omega_E^2$$

$$\Rightarrow GM_\odot = \omega_E^2 r_E^3 \Rightarrow \omega_{max}^2 = \omega_E^2\left(\frac{r_E}{R}\right)^3$$

It's convenient to express $\omega_E = \frac{1 \text{ rev}}{1 \text{ yr}} \simeq \frac{1 \text{ rev}}{\pi\cdot 10^7 \text{ s}}$

(students of course can work out 1 yr in seconds if they don't know this.)

$$\text{So} \quad \omega_{max} = \omega_E\left(\frac{r_E}{R}\right)^{3/2}$$

$$= \frac{1 \text{ rev}}{\pi\cdot 10^7 \text{s}}\cdot\left(\frac{1.5\cdot 10^8 \text{ km}}{10 \text{ km}}\right)^{3/2}$$

$$= \frac{1 \text{ rev}}{\pi\cdot 10^7 \text{s}}\left(15\cdot 10^6\right)^{3/2}$$

$$\cong \frac{1 \text{ rev}}{\pi\cdot 10^7 \text{s}}\cdot\left(4\cdot 10^3\right)^3$$

$$\simeq \frac{64}{\pi}\cdot\frac{10^9}{10^7}\frac{\text{rev}}{\text{s}} \sim \underline{\underline{2000\ \frac{\text{rev}}{\text{s}}}}$$

The fastest observed pulsar is 716 rev/s.
Note that for our Newtonian estimate, surface rotational velocity is $\sim \frac{1}{2}c$.

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Space Dust.

Typical dust particles in the Solar System have mass density ρ. They experience both gravitational and radiation pressure effects of the Sun (mass $M = 2 \times 10^{30}$ kg, emitted light power $P = 4 \times 10^{26}$ W).
(a) Are the smaller or the larger dust particles more likely to be ejected from the Solar System?
(b) Estimate (symbolically) the critical radius R of a particle that would not be ejected from the Solar System.
(c) Provide a numerical estimate to part (b).

QUALIFYING EXAM, 2017 - 2018

Space Dust. SOLUTION.

(a)

Radiation pressure repels particles from the Sun, while the gravitational force attracts them. The former scales with R^2 while the latter scales with R^3. Therefore, the smaller particles are more likely to be ejected from the Solar System.

(b)

The gravitational force is

$$F_g = \frac{GMm}{d^2} \approx \frac{4GMR^3\rho}{d^2}, \tag{1}$$

where G is the gravitational constant and d is the distance between the particle and the Sun.

The radiation pressure force is

$$F_r = \frac{IA}{c} \approx \frac{PR^2}{4cd^2}, \tag{2}$$

where I is the light intensity at the particle's location, A is the cross-sectional area of the particle, and c is the speed of light.

Balancing the forces yields

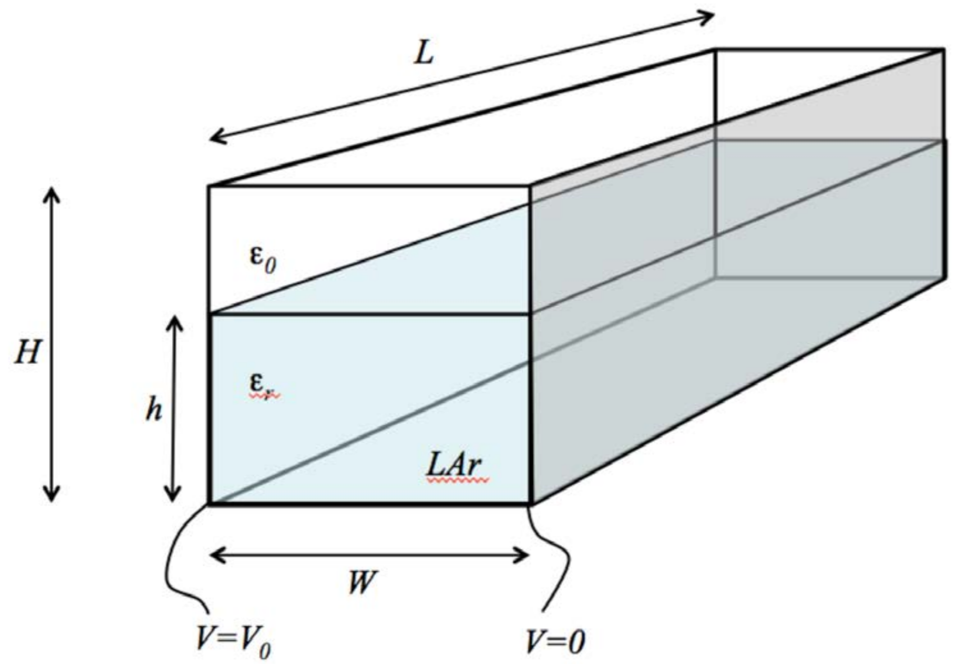
$$R \approx \frac{P}{16GMc\rho}. \tag{3}$$

(c)

Using the result of part (b) and assuming $\rho \sim 10^3$ kg/m^3, we find $R \sim 10^{-6}$ m.

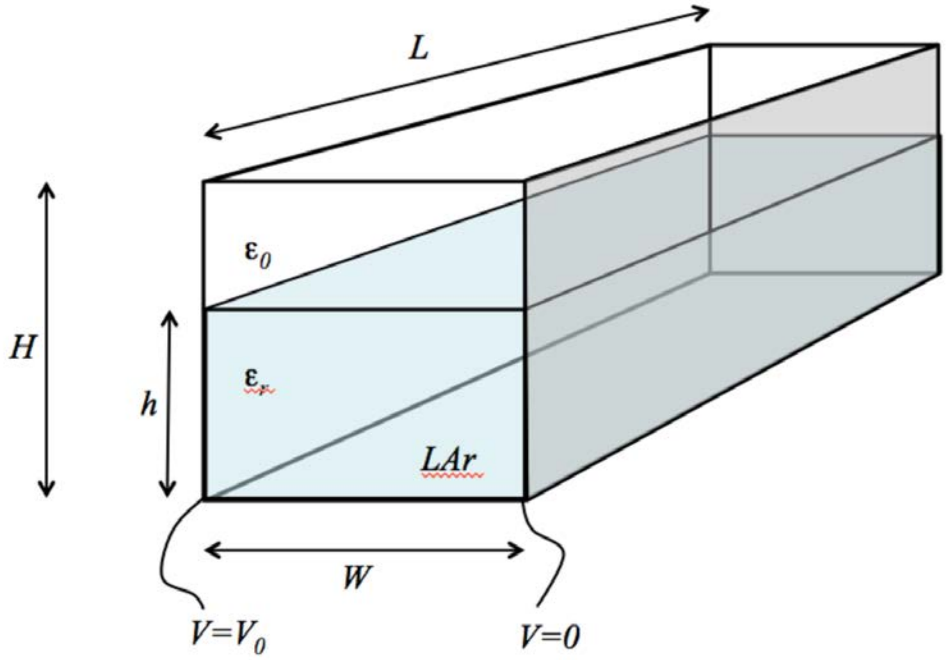
(a) During the filling of the MicroBooNE Liquid Argon Time Projection Chamber (LAr TPC) detector, the rectangular TPC was filled with liquid argon at a rate such that the liquid level h changed with a constant rate dh/dt. The potential difference across the TPC was held fixed at V_0. The resulting change in effective capacitance in the TPC resulted in increasing noise levels on the TPC wires as a function of increasing liquid level. Calculate the rate of change of effective TPC capacitance, dC/dt.

(b) A minimum ionizing particle (MIP) traverses the detector horizontally, along the bottom of the TPC, and parallel to the TPC wires, when the detector is half full. Calculate what would be the observable MIP signal-to-noise ratio, on a single TPC wire, if the detector were operated during that instance. Note: The TPC wires are vertically aligned along the grey-shaded plane in the figure, and the wire spacing is 3 mm. The noise level on a single TPC wire for an empty detector, measured in Equivalent Noise Charge (ENC), is 400 electrons; when the detector is full, the ENC is 450 electrons. The ionization energy of argon is 23.6 eV. You may assume that half of the ionization charge is lost while it drifts through the liquid argon volume toward the TPC wires. MIP energy loss due to ionization in argon is 2 MeV/cm.



(a) Two (parallel plate) capacitors in parallel:

$C = C_{air} + C_{LAr}$
$dC/dt \;= dC_{air}/dt + dC_{LAr}/dt$
$\quad = d(\varepsilon_0(H\text{-}h)L/W)/dt + d(\varepsilon_r hL/W)/dt$
$\quad = L/W\,(\varepsilon_r - \varepsilon_0)\,dh/dt$



(b) MIP signal per wire S $= ((2E6\ \text{eV/cm}) / (23.6\ \text{eV/electron})) (50\%) (0.3\ \text{cm})$
$= 12.5E3$ electrons per wire

Noise signal per wire, N, is proportional to capacitance per wire, C_{tot}.
When h=0, $N_0 = 400$ electrons per wire
When h=H, $N_H = 450$ electrons per wire
For any h, $C'_{tot} = C_{tot} + \kappa h$, where κ is a constant (see part (a))
So, when h=½H, N = 425 electrons.

→ S/N = 12.5E3 / 425 ≈ 30 when the detector is half full