Cheat Sheet Comp
Miho Wakai 1

1 Calm down Miho

Dimensional Analysis?Virial Theorem is powerful.relativistic/non-relativistic?Force, energy, momentum?

2 General Terms

Terms

Z = atomic/proton A (mass number) = Z + N molar mass of air 28.97 g/mol

$1L = 0,001m^3$ $\rho_{H20} = 1000\frac{kg}{m^3}$ $\rho_{air} = 1.2754\frac{kg}{m^3}$ Interior T_{sun}: 10^6 °C Outer T_{sun}: $5778K$ fm: 10^{-15} peta: 10^{15} pico:10^{-12} atto: 10^{-18}

3 Math

$f(x) = \frac{1}{\sqrt{2\pi}}\int F(k)e^{ikx}dk$ $F(k) = \frac{1}{\sqrt{2\pi}}\int f(x)e^{ikx}dx$ Eigenvector: $\lambda\cdot$(matrix)$\cdot$(eigenvector) $= \lambda\cdot$eigenvector $f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ...$ $ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$ $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + ...$ $c^2 = a^2 + b^2 - 2abcos(\theta)$ $cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ $sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ ODEs: $b^2 - 4ac > 0 : y = C_1e^{r^1t} + C_2e^{r_2t}$ $b^2 - 4ac < 0 with r = \lambda \pm \mu i y = C_1e^{\lambda t}cos(\mu t) + C_2e^{\lambda t}sin(\mu t)$ $b^2 - 4ac = 0 : y = C_1e^{rt} + C_2te^{rt}$ $y'' + By + C = 0 \rightarrow y = C_1sin(\sqrt{B}t) + C_2cos(\sqrt{B}t) - \frac{C}{B}$ $\Sigma_i^n a_i = a\left(\frac{1-r^n}{1-r}\right)$ when |r|<1 $\frac{a}{1-r}$ $\int_0^\infty x^ne^{-\frac{x}{a}}dx = n!a^{n+1}$ $\int_0^\infty x^{2n}e^{-\frac{x^2}{a^2}}dx = \sqrt{\pi}\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$ $\int_0^\infty x^{2n+1}e^{-\frac{x^2}{a^2}}dx = \frac{n!}{2}a^{2n+2}$ $\int lnxdx = x(lnx - 1) + C$ $\int_{-\infty}^{\infty}e^{-a(x+b)^2}dx = \sqrt{\frac{\pi}{a}}$ $\int_{-\infty}^{\infty}e^{-ax^2+bx+c}dx = \sqrt{\frac{\pi}{a}}e^{\frac{b^2}{4a}+c}$

Trig Identities

$sin(x) = \frac{e^{ix}-e^{-ix}}{2i}$ $cos(x) = \frac{e^{ix}+e^{-ix}}{2}$ $tan(x) = \frac{e^{ix}-e^{-ix}}{i(e^{ix}+e^{-ix})}$ $csc(x) = (sin(x))^{-1}$ $sec(x) = (cos(x))^{-1}$ $sin(a\pm b) = sin(a)cos(b) \pm sin(b)cos(a)$ $cos(a\pm b) = cos(a)cos(b) \mp sin(a)sin(b)$ $tan(a\pm b) = \frac{tan(A)\pm tan(B)}{1\mp tan(A)tan(B)}$ $sin(2\theta) = 2sin(\theta)cos(\theta)$ $cos(2\theta) = cos^2\theta - sin^2\theta = 2cos^2(\theta) - 1 = 1 - 2sin^2(\theta)$ $\frac{d}{dx}tan(x) = sec^2x$

Cylindrical Coordinates

$dl = dr\hat{r} + rd\phi\hat{\phi} + dz\hat{z}$ $dV = rdrd\theta dz$ $\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial(A_\phi)}{\partial\phi} + \frac{\partial(A_z)}{\partial z}$ $\nabla\times\vec{A} = \hat{r}\left[\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right] + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right] + \hat{z}\frac{1}{r}\left[\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial\phi}\right]$ $\nabla f = \hat{r}\frac{\partial f}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial f}{\partial\phi} + \hat{z}\frac{\partial f}{\partial z}$ $\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$ $x = rcos\phi$ $y = rsin\phi$ $r = \sqrt{x^2+y^2}$ $\phi = \frac{y}{x}$ $\hat{x} = cos\phi\hat{r} - sin\phi\hat{\phi}$ $\hat{y} = sin\phi\hat{r} + cos\phi\hat{\phi}$ $\hat{r} = cos\phi\hat{x} + sin\phi\hat{y}$ $\hat{\phi} = -sin\phi\hat{x} + cos\phi\hat{y}$

Spherical Coordinates

$d\vec{r} = dr\hat{r} + rd\theta\hat{\theta} + rsin\theta d\phi\hat{\phi}$ $dV = r^2sin\theta drd\theta d\phi$ $\nabla f = \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial\theta} + \hat{\phi}\frac{1}{rsin\theta}\frac{\partial f}{\partial\phi}$ $\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{rsin\theta}\frac{\partial}{\partial\theta}(sin\theta A_\theta) + \frac{1}{rsin\theta}\frac{\partial}{\partial\phi}(A_\phi)$ $\nabla\times\vec{A} = \hat{r}\frac{1}{rsin\theta}\left[\frac{\partial(sinA_\phi sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right] + \hat{\theta}\frac{1}{r}\left[\frac{1}{sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right] + \hat{\phi}\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]$ $\nabla^2 f = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial f}{\partial\theta}) + \frac{1}{r^2\theta}\frac{\partial^2 f}{\partial\phi^2}$ $x = rsin(\theta)cos(\phi)$, $y = rsin(\theta)sin(\phi)$ $z = rcos(\theta)$ $\hat{x} = sin(\theta)cos(\phi)\hat{r} + cos(\theta)cos(\phi)\hat{\theta} - sin(\theta)\hat{(\phi)}$ $\hat{y} = sin(\theta)sin(\phi)\hat{r} + cos(\theta)sin(\phi)\hat{\theta} + cos(\phi)\hat{(\phi)}$ $\hat{z} = cos(\theta)\hat{r} - sin(\theta)\hat{\theta}$ $\hat{r} = sin(\theta)cos(\phi)\hat{x} + sin(\theta)sin(\phi)\hat{y} - cos(\theta)\hat{(z)}$ $\hat{\theta} = cos(\theta)cos(\phi)\hat{x} + cos(\theta)sin(\phi)\hat{y} - sin(\theta)\hat{(z)}$ $\hat{\phi} = -sin(\phi)\hat{x} + cos(\phi)\hat{y}$ $\theta = \left(\frac{\sqrt{x^2+y^2}}{z}\right)$ $\phi = \left(\frac{y}{x}\right)$

Stats

$x_{randwalk} = \sqrt{N}$ $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $z = cx^ay^b \rightarrow \frac{\delta z}{z} = \sqrt{\left(a\frac{\delta x}{x}\right)^2 + \left(b\frac{\delta y}{y}\right)^2}$

Vector Calculus

Poisson: $\nabla^2\Phi = -\frac{\rho}{\varepsilon_0}$ $\Phi(r) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r')}{|r-r'|}d^3r'$ Diffusion $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$ Solution $c(x,t) = \frac{C_{x=0}}{\sqrt{4\pi Dt}}e^{\frac{-x^2}{4Dt}}$ $\int_a^b = (\nabla f)dl = f(b) - f(a)$

Div Theorem $\oint_S F\cdot ds = \int_V(\nabla\cdot F)dV$

Stokes's Theorem $\oint_l F\cdot dl = \int_S(\nabla\times F)ds$

$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ $\nabla\times A = \begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}$

4 Mechanics

Rotation

$F = \frac{mv^2}{r}$ $\tau = Frsin(\theta) = I\alpha$ $L = I\omega$ $KE = \frac{1}{2}I\omega^2$ $L = mvr$ $\frac{dL}{dt} = 0$ $P = \tau\omega$ $R_{CM} = \frac{1}{M}\int xdm$ $I_1\dot\omega_1 + (I_3 - I_2)\omega_2\omega_3 = M_1$ $I_2\dot\omega_2 + (I_1 - I_3)\omega_3\omega_1 = M_2$ $I_3\dot\omega_3 + (I_2 - I_1)\omega_1\omega_2 = M_3$ $I = \int_0^V\rho r^2dV = \int r^2dm$ Solid cylinder/disc: $I = \frac{1}{2}MR^2$ Hoop: $I = MR^2$ Solid Sphere: $\frac{2}{5}MR^2$ Rod/solid cylinder/recta about center: $\frac{1}{12}MR^2$ Rod about edge: $\frac{1}{3}ML^2$ Thin spherical shell $\frac{2}{3}MR^2$ Rectangle: $\frac{bh^3}{12}$ Rectangle edge: $\frac{bh^3}{3}$ $I_{parr} = I_{CM} + Md^2$ roll: $v = \omega\times R$ $\alpha = \frac{R\times f(=\mu mg)}{I}$ $T_{load} = T_{hold}e^{\mu\phi}$ Rotat Liquid: $dA\left(\frac{\partial P}{\partial r}dr\right) = \omega^2r\rho dAdr$ $h(r) = \frac{\omega 2r^2}{2g}$ $V_{eff} = \frac{L^2}{2\mu r^2} + V(r)$ orbit stable when $\frac{\partial^2 V_{eff}}{\partial r^2} > 0$ $F = -\frac{\partial V(r)}{\partial r}$

Lagrangian

$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial\dot x} = 0$ $p_\theta = \frac{\partial L}{\partial\dot\theta}$ $H = \sum_i\dot q_i\frac{dL}{d\dot q_i} - L$ $x_1(t) = Ae^{i\omega t}$ $Mx'' = kx$ $det(k - \omega^2M) = 0$ $v = \sqrt{\frac{T}{\mu}}$ $\ddot s = \frac{T}{\rho}s''$ $s(x,t) = g(x)f(t)$ $\frac{c^2g''}{g} = \frac{\ddot f}{f} = -\omega^2$ $u = \frac{1}{2}\left(\rho\left(\frac{\partial s}{\partial t}\right)^2 + T\left(\frac{\partial s}{\partial x}\right)^2\right)$

Astro

$M\frac{dv}{dt} = -v_{exmass}\frac{dm}{dt}$ find mass of sun $F = \frac{GMm}{r^2} = \frac{mv^2}{r}$ $KE[T] = -\frac{1}{2}PE[V]$ GPE of sphere $U = -\int_0^R\frac{G\left(\frac{4\pi r^3}{3}\right)\rho 4\pi r^2\rho dr}{r} = \frac{3}{5}\frac{GM^2}{R}$ $\mu = \frac{m_1m_2}{m_1+m_2}$ $\frac{dP}{dr} = -g(r)\rho(r)$ BE $= (Zm_p + Nm_n - M)c^2$ $\frac{n_n}{n_p} = e^{-\frac{(m_n-m_p)c^2}{kT}}$

SHM

$k_{||} = k_1 + k_2$ $\frac{1}{k_{series}} = \frac{1}{k_1} + \frac{1}{k_2}$ $\omega_{pend} = \sqrt{g/l}$ $F = ma = -kx$ $x(t) = Acos(\omega t) + Bsin(\omega t)$ $\omega = \sqrt{\frac{k}{m}}$ $E_{max} = \frac{1}{2}kA^2$ $v_{max}\rightarrow\Delta x = 0$

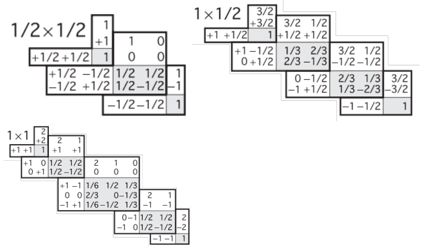
Waves

$v_g = \frac{\partial\omega}{\partial k}$, $v_p = \frac{\omega}{k}$ if $\omega = ak$ then $v_g = v_p$ deep water grav waves: $\omega = \sqrt{gk}$ $v_g = \frac{v_p}{2}$

Falling Chimney

$\tau = I\ddot\theta = F\cdot r$ Make sure $l = \frac{L}{2}$. Find τ_{orig} of chimney at angle which can be written in terms of $\ddot\theta$. Do the same for τ_p around a point p. $\tau(x) - \tau_p = I_p\ddot\theta$ take $\frac{d\tau}{dx}$

5 QM



NORMALIZE Φ

1. Write out Schroedinger
2. Solve second order $\rightarrow\Psi(x)$
3. Use boundary conditions
4. Solve E using the wavefunction

Finding Eigenvalues/vectors:

1. Apply operator on states to find eigenvalues then find eigenvector
2. If Hamiltonian matrix is found, find the eigenvalue/vector from the matrix

If a state is an eigenstate of an operator, the state should return to its original state when applying the operator $\alpha = \frac{e^2}{\hbar c}\approx\frac{1}{137}$ Ry $= \frac{me^4}{2\hbar^2}\approx 13.6eV$ $a_B = \frac{\hbar^2}{m_ee^2}$ $p = \frac{h}{\lambda} = \frac{hk}{2\pi} = \hbar k$ $\langle f|g\rangle = \int_a^b f(x)^*g(x)dx$ $[\hat A,\hat B] = \hat A\hat B - \hat B\hat A$ $\sigma^2 = \langle\Psi|x^2|\Psi\rangle - \langle\Psi|x|\Psi\rangle^2$ $\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = 1$ $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{\frac{ipx}{\hbar}}$ $[x_i,p_j] = i\hbar\delta_{ij}$ $\Psi(r,t) = \Psi(r)e^{-\frac{iEt}{\hbar}}$ orth. basis $\Psi(x,0) = \Sigma_{n=1}^{\infty}C_n\Psi_n(x)$

Heisenberg's Uncertainty Principle

$\Delta p\Delta x\geq\frac{\hbar}{2}$, $\Delta t\Delta E\geq\frac{\hbar}{2}$ $L = mvr = p\cdot r$ $\theta = \frac{x}{r}$ $\Delta L\Delta\theta\geq\frac{\hbar}{2}$ $\Delta x\sim N_e^{-\frac{1}{3}}$

Bohr Model

$\frac{m_ev^2}{r} = \frac{Zk_ee^2}{r^2}$ $L = mvr = n\hbar$ $E = -\frac{Zk_ee^2}{2r_n} = -\frac{Z^2(k_ee^2)^2m_e}{2\hbar^2n^2}$ Rydberg Energy $(R_E) = \frac{(k_ee^2)^2m_e}{2\hbar^2}$ $E_\gamma = R_E\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Schroedinger's Equation

$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$ Time Dependent: $\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = i\hbar\frac{\partial\chi}{\partial t} = H\chi$ $H_{rel} = \sqrt{m^2c^4 + p^2c^2} + V(x)$

Spin

$S_r = S\cdot\hat r = S_xsin(\theta)cos(\phi) + S_ysin(\theta)sin(\phi) + S_zcos(\theta)$ $S = \frac{\hbar}{2}\sigma$ $\sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}$ $\sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$ $\sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $S_+ = \hbar\begin{pmatrix}0&1\\0&0\end{pmatrix}$ $S_- = \hbar\begin{pmatrix}0&0\\1&0\end{pmatrix}$ $\chi_{+x} = \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}$ $\chi_{-x} = \frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix}$ $\chi_{+y} = \frac{1}{\sqrt2}\begin{pmatrix}1\\i\end{pmatrix}$ $\chi_{-y} = \frac{1}{\sqrt2}\begin{pmatrix}1\\-i\end{pmatrix}$ $\chi_{+z} = \frac{1}{\sqrt2}\begin{pmatrix}1\\0\end{pmatrix}$ $\chi_{-z} = \frac{1}{\sqrt2}\begin{pmatrix}0\\1\end{pmatrix}$ $\chi(0) = C_+\chi_+ + C_-\chi_-$ Use same basis as B field $\chi(t) = \alpha\chi_+e^{-\frac{iE_-t}{\hbar}} + \beta\chi_-e^{-\frac{iE_+t}{\hbar}}$ $\hat S^2 = \hat{S_x}^2 + \hat{S_y}^2 + \hat{S_z}^2 = \hat{S_z}^2 + \hat{S_+}\hat{S_-} - \hbar\hat{S_z}$ $\hat{S_+} = \hat{S_x} + i\hat{S_y}$ $\hat{S_-} = \hat{S_x} - i\hat{S_y}$

Magnetic Field

$E_+ = -\mu B$ $E_- = +\mu B$ $\mu = \gamma S$ $H = -\gamma B\cdot S$ Ex. $H = -\gamma B_0S_z\rightarrow\chi_+, E_+ = \frac{-\gamma B_0\hbar}{2}, \chi_-, E_- = \frac{+\gamma B_0\hbar}{2}$ using boundary conditions $\rightarrow\chi(t) = [state]$ Larmor Precess. $\tau = \frac{e}{2m_e}LBsin(\theta)$ $\omega = \frac{e}{2m_e}B_0$ Stern-Gerlach Exp: $F = \nabla(\mu\cdot B)\nabla(\gamma S\cdot B)$ Relate $e^{-\frac{iE_+T}{\hbar}} = e^{\frac{ipx}{\hbar}}$ to find momentum of the split beams

Operators

$L^2|\Psi\rangle = \hbar^2l(l+1)|\Psi\rangle$ $L_z|\Psi\rangle = \hbar m|\Psi\rangle$ $S^2|\Psi\rangle = \hbar^2s(s+1)|\Psi\rangle$ $S_z|\Psi\rangle = \hbar S_z|\Psi\rangle$ $J^2|\Psi\rangle = \hbar^2J(J+1)|\Psi\rangle$ $J_z|\Psi\rangle = \hbar J_z|\Psi\rangle$ $J_+|\Psi\rangle = \hbar\sqrt{J(J+1) - J_z(J_z+1)}|J_z+1\rangle$ $J_-|\Psi\rangle = \hbar\sqrt{J(J+1) - J_z(J_z-1)}|J_z-1\rangle$ $L_+|\Psi\rangle = \hbar\sqrt{l(l+1) - m(m+1)}|m+1\rangle$ $L_-|\Psi\rangle = \hbar\sqrt{l(l+1) - m(m-1)}|m-1\rangle$ $L_+ = L_x + iL_y$ $L_- = L_x - iL_y$ $I\omega = \hbar m$ $\hat R = e^{\frac{-i\phi L_z}{\hbar}}$ $\hat U_{time} = e^{\frac{-i\hat Ht}{\hbar}}$ $\hat T_{transl} = e^{\frac{-ipx}{\hbar}}$

Two Particle State

$S^2 = (S_1)^2 + (S_2)^2 + 2S_1\cdot S_2$ $S_1\cdot S_2(\uparrow\downarrow) = (S_{x1}\uparrow)(S_{x2}\downarrow) + (S_{y1}\uparrow)(S_{y2}\downarrow) + (S_{z1}\uparrow)(S_{z2}\downarrow) = \frac{\hbar^2}{4}(2\downarrow\uparrow - \uparrow\downarrow)$ $S_-(\uparrow\uparrow) = (S_{1-}\uparrow)\uparrow + \uparrow(S_{2-}\uparrow) = \hbar(\downarrow\uparrow + \uparrow\downarrow)$ Singlet $|S=0,S_z=0\rangle = \frac{(\uparrow\downarrow - \downarrow\uparrow)}{\sqrt2}$ Triplet $|11\rangle = \uparrow\uparrow$ $|10\rangle = \frac{(\uparrow\downarrow + \downarrow\uparrow)}{\sqrt2}$ $|1-1\rangle = \downarrow\downarrow$ $S_1\cdot S_2|10\rangle = \frac{\hbar^2}{4}|10\rangle$ $S_1\cdot S_2|00\rangle = -\frac{3\hbar^2}{4}|00\rangle$ $S^2|10\rangle = 2\hbar^2|10\rangle$ $S^2|00\rangle = 0$

Infinite Square Well (only bound state)

$\frac{d^2\Psi}{dx^2} = -k^2\Psi, k = \frac{\sqrt{2mE}}{\hbar}, E\geq 0, \Psi = Asin(kx) + Bcos(kx)$ $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$ $\psi_n(x) = \sqrt{\frac{2}{a}}sin\left(\frac{n\pi}{a}x\right)$ 3D: $\Psi(r)$

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$\sqrt{\frac{8}{V}} sin\left(\frac{n_x \pi x}{L_x}\right) sin\left(\frac{n_y \pi y}{L_y}\right) sin\left(\frac{n_z \pi z}{L_z}\right)$

$E = \frac{\hbar^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$

Finite Square Well

Bound State (E<0) $k = \frac{\sqrt{-2mE}}{\hbar}$ Three types of wavefunctions due to sections of well (within well: $\Psi(x) = Dcos(lx)l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$) $tan(z) = \sqrt{(Z_0/Z)^2 - 1}$ $z = la$ Wide deep well $\rightarrow E_n + V_0 \approx \frac{n^2\pi^2\hbar^2}{2m(2a)^2}$ Shallow, narrow well $\rightarrow a \rightarrow 0, V_0 \rightarrow \infty$ $E = \frac{-ma^2 V_0^2}{2\hbar^2}$ (δ well) for E>0 $E + V_0 = \frac{n^2\pi^2\hbar^2}{2m(2a)^2}$

Harmonic Oscillator (only bound state)

$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2\psi = E\psi$ $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 = \hbar\omega\left(a_+a_- + \frac{1}{2}\right)$ $a_+\Psi_n = \sqrt{n+1}\Psi_{n+1}$ $a_-\Psi_n = \sqrt{n}\Psi_{n-1}$ $E_n = (n+\frac{1}{2})\hbar\omega$ (sum for 2) $x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$ $p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$ 3D: $E = \left(n+\frac{3}{2}\right)\hbar\omega$ $g = \frac{1}{2}n(n+1)(n+2)$

Free Particle V = 0

$\psi_k(x,t) = Ae^{i(kx - \frac{\hbar k^2}{2m}t)}$ $k = \pm\frac{\sqrt{2mE}}{\hbar}$ k > 0 $\rightarrow$ right $v_{quantum} = \sqrt{\frac{E}{2m}} = \frac{1}{2}v_{classical} = v_{group} = v_{phase}$ $\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k)e^{i(kx - \frac{\hbar k^2}{2m}t)}dk$ $\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\Psi(x,0)e^{-ikx}dx$

Delta Function Well

Bound state with E < 0 take $k = \frac{\sqrt{-2mE}}{\hbar}$ use $-\frac{\hbar^2}{2m}\Delta\left(\frac{\partial\Psi}{\partial x}\right) + \int_{-\epsilon}^{\epsilon}V(x)\Psi(x)dx = 0$ $V = -\alpha\delta(x)$ $\psi(x) = \frac{\sqrt{m\alpha}}{\hbar}e^{\frac{-m\alpha|x|}{\hbar^2}}$ $E = \frac{-m\alpha^2}{2\hbar^2}$ Scat. state with E > 0 $R = \frac{\beta^2}{1+\beta^2}$ $T = \frac{1}{1+\beta^2}$ $\beta = \frac{m\alpha}{\hbar^2 k}$ $k = \frac{\sqrt{2mE}}{\hbar}$

Hydrogen Atom

e transitions $hf = \frac{Z^2me^4}{8h^2\epsilon_0^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Particle in a Ring

$-\frac{\hbar^2}{2mR^2}\frac{\partial\Psi}{\partial\phi} = E\Psi|$ $E = \frac{L_z^2}{2I}|\Psi(\phi) = Ce^{\pm in\phi}$

Perturbation Theory

$E_n^1 = \langle\Psi_0|H'|\Psi_0\rangle$

$\Psi_n^1 = \Sigma_{m\neq n}\frac{\langle\Psi_m^0|H'|\Psi_n^0\rangle}{(E_n^0 - E_m^0)}\Psi_m^0$

$E_n^2 = \Sigma_{m\neq n}\frac{|\langle\Psi_m^0|H'|\Psi_n^0\rangle|^2}{(E_n^0 - E_m^0)}$ $E_\pm^1 = \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{((W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\right]$

$P_{a\rightarrow b} = |c_b(t)|^2 \approx \frac{|V_{ab}^2|}{\hbar^2}\frac{sin^2[(\omega_0-\omega)\frac{t}{2})]}{(\omega_0-\omega)^2}$

$H' = -ezE_z = -ercos(\theta)E_z$ $V = qEd$

6 EM

Maxwell's Equations

$\nabla \cdot E = \frac{\rho}{\epsilon_0}$ $\nabla \times E = \frac{\partial B}{\partial t}$ $\nabla \cdot B = 0$

$\nabla \times B - \frac{1}{c^2}\frac{\partial E}{\partial t} = \mu_0 J$

Electrostatics

$\oint E \cdot dA = \frac{Q}{\epsilon_0}$ $E = -\vec{\nabla}\Phi(\vec{x}) = -\frac{dV}{dx}$ $F = qE$ $\Delta V = -\int_{-}^{+}E \cdot dl$ $J = \sigma E$ $\rho = \frac{1}{\sigma}$ $\frac{dq}{dt} = I = JA$

Magnetostatics

$\oint B \cdot dl = \mu_0 I = \int(\nabla \times B)ds$ $\vec{\nabla} \times \vec{B} = \mu_0\vec{J}$ (no $\frac{\partial E}{\partial t}$) $\vec{B} = \vec{\nabla} \times \vec{A}$ $\mu_0\vec{j} = \vec{\nabla}(\nabla \cdot \vec{A}) - \nabla^2\vec{A}$ $\Phi = \int B \cdot dA = BAcos(\theta)$ $F = \int I(dl \times B) = BILsin(\theta)$ Solenoid $B_0 = \frac{nI}{\epsilon_0 c^2}$ Biot Savart Law $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$ F per unit l between wires: $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$

Generating EMF

$F = q(\vec{E} + \vec{v} \times \vec{B})$ $\varepsilon = -\frac{\partial F}{\partial t} = \oint\vec{E} \cdot dl = -\oint\frac{\partial\vec{B}}{\partial t} \cdot \hat{n}da = -\frac{d\phi_B}{dt}$

Gauss Law stuff with Capacitance +Q -Q

Pill box $E = \frac{\sigma}{2\epsilon_0}$ Parallel Plates: $E = -\frac{Q}{\epsilon_0 A}\hat{z}C = \frac{\epsilon_0 A}{d}$ Concen. Spheres: $E = -\frac{Q}{4\pi r^2\epsilon_0}\hat{r}$ $C = 4\pi\epsilon_0\frac{ab}{b-a}$ Concen. Cylinders $E = -\frac{Q}{2\pi RL\epsilon_0}\hat{r}$ $C = 2\pi\epsilon_0 L ln\left(\frac{a}{b}\right)$

EM Waves

$E = cB$ $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$ $u(x,t) = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{\mu_0}B^2 = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$

S power per unit area $= \frac{1}{\mu_0}E \times B = \epsilon_0 cE^2$

g momentum density $= \frac{S}{c^2} = \frac{u}{c}$

Electric Dipoles

$U = -p \cdot E$ $\tau = p \times E$ $p = \int r\rho(r)d^3r$ $V_{dip}(r) = \frac{1}{4\pi\epsilon_0}\frac{p \cdot \hat{r}}{r^2}$ $p = qd$ $p = \alpha E$ $F \approx p \cdot \nabla V$ $E_{dip} = \frac{-p}{4\pi\epsilon_0 r^3}[2cos(\theta)\hat{r} + sin(\theta)\hat{\theta}]$

$E_{dip} = \frac{[3(p \cdot r)r - pr^2]}{4\pi\epsilon_0 r^5}$

Magnetic Dipoles

$m = I\int da = Ia$ $dm = dIA + IdA$ $dI = dq\frac{\omega}{2\pi}$ $m = \frac{1}{2}\int(r \times J)d\tau$ $A_{dip}(r) = \frac{\mu_0}{4\pi}\frac{m \times \hat{r}}{r^2}$ $B_{dip}(r) = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(m \cdot \hat{r})\hat{r} - m]$ $B_{\text{dip in z axis}}(r) = \frac{\mu_0 m}{4\pi}\frac{1}{r^3}[2cos(\theta)\hat{r} + sin(\theta)\hat{\theta}]$ Nuclear Magneton $\mu_N = \frac{e\hbar}{2m_p}$ $F = \nabla(m \cdot B)$ $\tau = m \times B$ $PE = -m \cdot B$

Magnetized Sphere

$m = \frac{4\pi R^3}{3}M$ $B_{\text{inside}} = \frac{2}{3}\mu_0 M$ vol. current: $J = \nabla \times M$ surf. current: $K = M \times \hat{n}$

Reflection Refraction Transmission

$n = \frac{c}{v}$ $n_1 sin(\theta_1) = n_2 sin(\theta_2)$ Brewster's angle perfectly polarized light is transmitted without reflection. Reflection of unpolarized light will be reflected as perfectly polarized. $\theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)$ $E_0 = E_0 e^{i(k_1 z - \omega t)}\hat{x}, B_0 = \frac{E_0}{v_1}e^{i(k_1 z - \omega t)}\hat{y}$ $E_R = E_R e^{i(-k_1 z - \omega t)}\hat{x}B_R = -\frac{E_R}{v_1}e^{i(-k_1 z - \omega t)}\hat{y}$ $E_T = E_T e^{i(k_2 z - \omega t)}\hat{x}, B_T = \frac{E_T}{v_2}e^{i(k_2 z - \omega t)}\hat{y}$ Boundary conditions must be satisfied at x = 0, x = d etc for both E field and B fields.

Diffraction

Single Slit: $tan(\theta) = \frac{y}{D}$ Condition for min.: $y \approx \frac{m\lambda D}{a}$ (a = width of slit) Resolvance of Grating: $R = \frac{\lambda}{\Delta\lambda} = mN$ (m = order of diffraction, N = total of slits) Double Slit: Condition for max.: $Dsin(\theta) = m\lambda \rightarrow y \approx \frac{m\lambda D}{d}$ (d = dist between slits) Circ. $\theta = 1.22\frac{\lambda}{D}$

Dielectrics

$P = \chi\epsilon_0 E$ $D = \epsilon_0 E + P$ $\sigma_{pol} = P \cdot n$ $E = \frac{\sigma_{free}}{\varepsilon}$ $\varepsilon = (1+\chi)\epsilon_0$ $C = \frac{\varepsilon A}{d}$

Circuits

$\varepsilon = V + IR$ $P = I^2R = VI = \frac{V^2}{R}$ $C[F] = \frac{Q}{V}$ $C_{par} = C_1 + C_2$ $\frac{1}{C_{ser}} = \frac{1}{C_1} + \frac{1}{C_2}$ $\tau = R \times C$ $L = \frac{\mu k N^2 S(\text{A of coil})}{L(\text{coil length})}$ $\varepsilon = L\frac{dI}{dt}$ $\tau = \frac{L}{R}$ $E = \frac{1}{2}LI_L^2$ $I = I_0(1 - e^{\frac{t}{\tau}})$ for inductors: $I\uparrow\varepsilon\downarrow$ Work done to charge capacitor $\rightarrow W = \int_0^Q\frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

Method of Images

Write potential with z+d and z-d summing both charges, Surface charge induced on conductor $= \sigma = -\varepsilon(\partial V/\partial n)$ Compute Q for total surface area. Find F with 2d, find E which gets halved.

Lenses

$Power = \frac{1}{f}$ $\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

$f_{\text{spher curved mirror}} = -\frac{R}{2}$

Relativity

$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$ for inverse $+\beta\gamma$ $tan(\theta) = \frac{y}{x} = \frac{1}{\gamma}\frac{sin(\theta')}{cos(\theta') + \frac{v}{c}}$ $\omega' = \gamma(1 - \beta cos\theta)\omega$ $f_r = \frac{f_s}{\gamma}$ $L = \frac{L_0}{\gamma}$ $T = \gamma T_0$ $u_{\text{on shore}} = \frac{v + u'_{\text{on boat}}}{1 + \left(\frac{vu'}{c^2}\right)}$ when v_x: $u = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x}$ $u = \frac{u'_y}{\gamma + \frac{v}{c^2}v'}$

$J^\mu = (c\rho, \vec{J}) = \rho_0(\gamma c, \gamma\vec{u}) = \rho_0 u^\mu$ $A^\mu = (\Phi, \vec{A})$ $Doppler_{resolution}: R = \frac{c}{\Delta v}$

7 Thermal Physics

$\rho_{water} = 997\frac{kg}{m^3}$ ice to water $= L = 334\frac{kJ}{kg}$ water to gas $= 2264.705\frac{kJ}{kg}$ $Cs = \frac{Q}{m\delta T}$ $L = \frac{Q}{m}$ Conduction $\frac{Q}{t} = -\frac{kA\Delta T}{d}$ $\frac{dQ}{dt} = -\frac{kAdT}{dx}$ $P = A\epsilon\sigma T^4$ Fusion=solid $\rightarrow$ liquid Isentropic: $\Delta S = 0$ Isobaric: $\Delta P = 0$ Isochoric: $\Delta P = 0$ Adiabatic: $Q = 0$ $dU = Tds - pdV = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$ Helmholtz: $F = E - TS$ Enthalpy: $H = U + PV$ Gibbs: $G = U - TS + PV = H - TS$ $df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$ $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ $\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$

Entropy

$S = kln(\Omega)$ when energy and of molecules are fixed $\Delta S = Nkln(\frac{V_f}{V_i})$ $\Delta S = \frac{\Delta Q}{T}(revsysonly, if not, U/T)$ Reversible $= \Delta S = 0$ $S_{\text{ideal}} = Nk\left[ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{\frac{3}{2}}\right) + \frac{5}{2}\right]$

Ideal Gas

$\Delta U = Q - W$ $\Delta T = 0 \rightarrow \Delta U = 0$ $C_v = \frac{gR}{2}$ $C_P = C_V + R$ Monoatomic $C_V = \frac{3}{2}R$ Diatomic $C_V = \frac{5}{2}R$ $C_v = \frac{dU}{dT}$ $U_{\text{mono}} = \frac{3}{2}nRT$ PDF of velocity $\rightarrow f(v)dv = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}4\pi v^2 e^{-\frac{mv^2}{2kT}}dv$ For reversible only: $P^{1-\gamma}T^\gamma = C$ $VT^{\frac{f}{2}} = C$ $TV^{\gamma-1} = C$ Adiabatic: $\Delta Q = 0$ $dS = 0$ $\Delta T \neq 0$ $(P_1, V_1 \rightarrow P_2, V_2): W = \frac{P_1V_1 - P_2V_2}{1-\gamma} = Nk\frac{1}{\gamma-1}(T_f - T_i)$

Adiabatic Relation

$ndT = \frac{dU}{C_v} = \frac{-PdV}{C_v} = \frac{PdV + VdP}{R}$ $PV = nRT$ $P = \frac{\rho RT}{M}$ $\gamma = \frac{C_P}{C_V}$ $PV^\gamma = const$ $P^{1-\gamma}T^\gamma = C$ $VT^{\frac{f}{2}} = C$ $TV^{\gamma-1} = C$ $P_1V_1 to P_2V_2 \rightarrow W = \frac{const\left(V_f^{1-\gamma} - V_i^{1-\gamma}\right)}{1-\gamma} = Nk\frac{1}{\gamma-1}(T_f - T_i)$ $PV^\gamma = (nRT)^\gamma$ Barometric Formula $\frac{dT}{dz} = -\frac{M}{R}g\left(\frac{\gamma-1}{\gamma}\right)$

Black Body Radiation

$B = \frac{2hv^3}{c^2}\frac{1}{e^{hv/kT} - 1}$ $\lambda_{peak} = \frac{2.8978 \times 10^3}{T}$ Power radiated across all ν : $j^\star = \varepsilon\sigma T^4$ $P_{\text{net}} = A\sigma\varepsilon\left(T^4 - T_0^4\right)$ Radiation Pressure Perfect Reflector: $P = 2\frac{I}{c}$ Perfect Absorber: $P = \frac{I}{c}$

8 Stat Mech

$Z_{\text{classical}} = \frac{1}{h^3}\int e^{\frac{-H}{k_BT}}d^3qd^3p$ $S = k_B ln(\Omega)$ no of microstates $\Omega = \sum_k w_k$ (coin $= 2^N$) ΩMultiplicity $= \frac{N!}{n!(N-n)!}$ $P_r = \frac{e^{\frac{-E_r}{k_BT}}}{Z}$ $Z = \Sigma_r e^{\frac{-E_r}{k_BT}}$ $E_{avg} = -\frac{1}{Z}\frac{dZ}{d\beta} = -\frac{\partial LnZ}{\partial\beta}$ non-int, distinct: $Z_{tot} = Z_1Z_2$ non-int, indistinct: $Z_{tot} = \frac{1}{N!}Z_1^N$ $P = \frac{1}{\beta}\frac{\partial lnZ}{\partial V}$ Stirling $ln(N!) = NlnN - N + \mathcal{O}(lnN)$ $N! \approx \sqrt{2\pi N}\left(\frac{N}{e}\right)^N$ $\beta = \frac{1}{k_BT}$ $n(E) = g(E)f(E)$ f(E): Maxwell Boltzman: $\frac{1}{e^{\beta(E-\mu)}}$ Fermi Dirac: $\frac{1}{e^{\beta(E-\mu)} + 1}$ Bose Einstein: $\frac{1}{e^{\beta(E-\mu)} - 1}$ $\lambda = \frac{h}{p}$ $p = \sqrt{2mkT}$ $F = -k_B ln(Z)$

Fermi Gas

$n = \frac{N}{V}$ $g(\epsilon) = \frac{\pi(8m)^{\frac{3}{2}}}{2h^3}V\sqrt{\epsilon} = \frac{3N}{2\epsilon_F^{3/2}}\sqrt{\epsilon}$ $E = \frac{h^2k^2}{8mL^2}$ $\epsilon_F = \mu(T=0) = \frac{h^2k_{max}}{8mL^2} = \frac{h^2}{8m}\left(\frac{3N_{max}}{\pi V}\right)^{2/3}$ $E_{tot} = \frac{\pi^3\hbar^2}{10mL^2}\left(\frac{3N}{\pi}\right)^{\frac{5}{3}}$ $P = \frac{\pi^3\hbar^2}{15m}\left(\frac{3n}{\pi}\right)^{\frac{5}{3}}$ $\epsilon_F \approx \frac{1}{mv^2}$ $N_{max} = 2\frac{1}{8}\frac{4\pi}{3}k_{max}^3$ $N = \frac{8\pi}{3}L^3\frac{(2mE)^{3/2}}{h^3}$ $\rho(E) = \frac{dn}{dE} = \frac{4\pi(2m)^{3/2}E^{1/2}}{h^3}$ if T = 0: $N = \int_0^{\epsilon_F}\rho(E)dE$ $T_F = \frac{\epsilon_F}{k_B}$ $P = \frac{2}{3}\frac{U}{V}$ $C_V = \frac{\pi^2Nk^2T}{2\epsilon_F}$ Strong Degeneracy $= K_BT << \epsilon_F$