QUANTUM

H.O. $V(x) = \frac{m\omega^2}{2} \rightarrow \psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right)$, $a_+\psi_n = \sqrt{n+1}\,\psi_n$, $a_-\psi_n = \sqrt{n}\,\psi_n$, $a_\pm = \frac{\mp i\hat{p} + m\omega x}{\sqrt{2\hbar m\omega}}$

$E_n = \left(n+\frac{1}{2}\right)\hbar\omega$

I.S.W. $\psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$, $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

FOURIER $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(k)e^{ikx}\,dk$

$F(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ikx}\,dx$

SPIN $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\hat{S}_i = \frac{\hbar}{2}\sigma_i$, $S_\pm = S_x \pm iS_y$

PERTURB $E_n^1 = \langle\psi_n^0|H'|\psi_n^0\rangle$, $\psi_n^1 = \sum_{m\neq n}\frac{\langle\psi_m^0|H'|\psi_n^0\rangle}{(E_n^0 - E_m^0)}\psi_m^0$

OPS $\hat{p} = -i\hbar\frac{\partial}{\partial x_i}$ $\hat{T} = \frac{\hat{p}^2}{2m} = \frac{\hat{L}^2}{2I}$

ANG. MOM. $S_\pm|s,m\rangle = \hbar\sqrt{s(s+1) - m(m\pm1)}\,|s,m\pm1\rangle$

$S^2|s,m\rangle = s(s+1)\hbar^2|s,m\rangle$

HYDROGEN $E_n = \frac{-13.6\,eV}{n^2}$, $\psi_{100} = \frac{e^{-r/a}}{\sqrt{\pi a^3}}$

E+M

MAXWELL'S EQNS $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$, $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$, $\vec{\nabla}\times\vec{B} = \mu_0\left[\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right]$ $\vec{\nabla}\cdot\vec{B} = 0$

integrals: $\iint\vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, $\oint\vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint\vec{B}\cdot d\vec{A}$, $\oint\vec{B}\cdot d\vec{l} = \mu_0 I + \varepsilon_0\frac{\partial}{\partial t}\iint\vec{E}\cdot d\vec{A}$

POTENTIALS $\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{-\rho}{\varepsilon_0}$, $\varphi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\frac{\rho(\vec{r}', t - |\vec{r}-\vec{r}'|/c)}{|\vec{r}-\vec{r}'|}$

$A = \left(\frac{\varphi}{c}, \vec{A}\right)$ $\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J}$, $\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int d^3r'\,\frac{\vec{J}(\vec{r}', t - |\vec{r}-\vec{r}'|/c)}{|\vec{r}-\vec{r}'|}$

MATCHING

$\hat{n}\times(\vec{E}_2 - \vec{E}_1) = 0$

$\hat{n}\cdot(\vec{D}_2 - \vec{D}_1) = \sigma$

$\hat{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{J}$

$\hat{n}\cdot(\vec{B}_2 - \vec{B}_1) = 0$

FIELDS $\vec{E} = -\vec{\nabla}\varphi - \frac{\partial\vec{A}}{\partial t} = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\frac{\rho(\vec{r}', t - |\vec{r}-\vec{r}'|/c)\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$, $\vec{D} = \varepsilon\vec{E}$

$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{\mu_0}{4\pi}\int d^3r'\,\frac{\vec{J}(\vec{r}', t - |\vec{r}-\vec{r}'|/c)\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$, $\vec{H} = \frac{\vec{B}}{\mu}$

DIPOLES $\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E} = \vec{\nabla}(\vec{m}\cdot\vec{B})$

$\vec{\tau} = \vec{p}\times\vec{E} = \vec{m}\times\vec{B}$

ENERGY $u = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$, $\vec{S} = (\vec{E}\times\vec{H}) = \frac{(\vec{E}\times\vec{B})}{\mu_0}$, $P = \frac{\langle S\rangle}{c}$

WAVE $\vec{E} = c\vec{B}\times\hat{k}$

MECH

LAGRANGIAN $\mathcal{L} = T - U$, $\frac{d}{dt}\left[\frac{\partial\mathcal{L}}{\partial\dot{q}}\right] - \frac{\partial\mathcal{L}}{\partial q} = 0$

ROT MOMENTS sphere $\frac{2}{5}MR^2$ disc $\frac{1}{2}MR^2$

rod abt end $\frac{1}{3}ML^2$ box $\frac{1}{12}M(l^2+w^2)$

EXTRA hydrostat. equil. $\frac{dP}{dr} = -\frac{GM\rho}{r^2}$

THERMAL

1st LAW: $dU = \delta Q - dW$, W done by gas, $dW = PdV$ for P-V work @ constant P

2nd LAW: irreversibility! $\delta Q = TdS$ for reversible processes

equipartition theorem: each DoF gets $\frac{1}{2}k_BT$ of energy per quanta

VELOCITIES $P(v) = 4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2\exp\left(\frac{-mv^2}{2k_BT}\right)$ $v_{mpv} = \sqrt{\frac{3k_BT}{m}}$

IDEAL GAS

$S = Nk_B\log\left[\frac{V}{N}\left(\frac{4\pi mU}{3h^2N}\right)^{3/2}\right] + \frac{5}{2}$

REL. MECH.

DOPPLER $\frac{\lambda_r}{\lambda_s} = \frac{f_s}{f_r} = \sqrt{\frac{1+\beta}{1-\beta}}$ if parallel, or $\nu' = \gamma\nu(1-\beta\cos\theta)$ in general $\frac{\Delta\lambda}{\lambda} = \frac{-\Delta f}{f} \approx \frac{a\Delta x}{c^2}$

$\beta > 0$ indicates movement away

LORENTZ $x_0' = \gamma(x_0 - \beta x_1)$ $x = (ct, \vec{x})$ $p = mu = (E/c, \vec{p})$ $p^2 = m^2c^2 = E^2/c^2 - \vec{p}\cdot\vec{p}$

$x_1' = \gamma(x_1 - \beta x_0)$ $u = \gamma(c, \vec{u})$ $k = (\omega/c, \vec{k})$ $F = \gamma(P/c, \vec{F})$

SPACETIME $d\tau = dt' = \frac{dt}{\gamma}$, $\Delta x' = \frac{\Delta x}{\gamma}$ if $\Delta t' = 0$, $c\Delta t' = \gamma(c\Delta t - \beta\Delta x)$, $\Delta s^2 = c^2\Delta t^2 - \Delta x^2$

simultaneity

Ryan Quinn

CIRCUITS

	Resistor	Capacitor	Inductor
Impedance	R	$\frac{-i}{\omega C}$	$i\omega L$
I-V	$V = IR$	$I = C\frac{dV}{dt}$	$V = L\frac{dI}{dt}$
Energy	$P = IV$	$\frac{1}{2}CV^2$	$\frac{1}{2}LI^2$

$k_B = 8.6\text{E-}5$ eV K^{-1}
$\hbar = 6.6\text{E-}16$ eV s
$\hbar c = 0.2$ eV μm

AST
$T_\odot \sim 6\text{E}3$ K $\quad L = \sigma T^4$
$D_\odot \sim 150\text{E}9$ km $\quad \lambda_{max} T = \text{const.}$

TAYLOR EXPANSIONS $\quad \frac{1}{1-x} \approx 1+x \quad \cos(x) \approx 1 \quad e^x \approx 1+x \quad (1+x)^n \approx 1+nx$
$\sin(x) \approx x \quad \log(1+x) \approx x \quad \log(n!) = n\log n - n$

SUMMATIONS $\quad \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$, $r<1$; $\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$, $|x|<1$

TRIG $\quad \cos^2\theta = \frac{1+\cos(2\theta)}{2} \quad \sin(2\theta) = 2\sin\theta\cos\theta$

Random Walk $\sqrt{N}$ $\quad$ Gaussian $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

Maxwell $Z = \frac{1}{e^{\beta(E-\mu)}}$ $\quad$ Fermi $Z = \frac{1}{e^{\beta(E-\mu)}+1}$ $\quad$ Bose $Z = \frac{1}{e^{\beta(E-\mu)}-1}$, $\quad P = \frac{1}{\beta}\frac{\partial \log Z}{\partial V}$

Energy $\langle E \rangle = \frac{1}{Z}\frac{\partial Z}{d\beta} = -\frac{\partial \log Z}{\partial \beta}$ $\quad$ non. int. distinct: $Z = Z_1 Z_2$ $\quad$ non. int. identical. $Z = \frac{Z_1^N}{N!}$

$\vec{E}_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}$, $\quad \vec{B}_{dip} = \frac{\mu_0}{4\pi}\frac{3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}}{r^3}$, $\quad V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$, $\quad \vec{A}_{dip} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$

gyromag $\gamma_i = \frac{|q|}{2m} g_i$ $\quad g_i$(classical) $= 1$ $\quad g_i$(electron) ≈ 2 $\quad \gamma_N = g_n \frac{\mu_N}{\hbar}$ $\quad \mu_N = \frac{e\hbar}{2m_p}$

diffraction $\tan\theta = \frac{y}{D}$ $\quad$ minimum @ $y \approx \frac{m\lambda D}{a}$ a - width of slit $\quad$ max @ $D\sin\theta = m\lambda \rightarrow y \approx \frac{m\lambda D}{d}$ d - dist. bw slits