MA30287 Coursework

Analysis of a Latitude-Dependent Energy Balance Model

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Introduction

Throughout the forthcoming discussion with regards to the latitude-dependent energy balance model of the Earth, we aim to investigate the implications on the Earth's climate when differing parameter and albedo choices. We split this into 5 separate investigations, with each one looking into different variations and modifications to the model. Multiple sources will be used to aid our investigation; these are comprised of books, articles and research papers.

Investigation 1

Part (a)

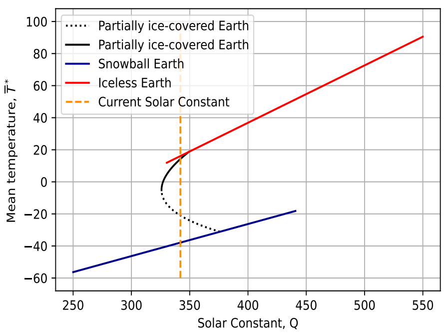

Figure 1: Investigating the mean temperature $\overline{T}^*$ as the solar constant Q varies

From Figure 1 we can see that the latitude dependent energy balance model exhibits hysteresis. Given that the Earth is in the current partially-iced state with the ice line $y_s \approx 0.939$ and all other parameters except Q fixed, it can be seen that as the solar constant Q decreases, the ice line y_s retreats away from the North Pole, until it reaches the tipping point at $y_s \approx 0.6$ (see Figure 1 in the coursework brief). This is where the the steady state $\overline{T}^*$ changes stability and becomes unstable. Upon solving $\frac{dQ}{d\hat{T}^*} = 0$ we discover that if the solar constant Q decreases past $\approx 325.84Wm^{-2}$, the Earth becomes fully glaciated. Thus, given that the current solar constant is $342Wm^{-2}$, an approximate decrease of at least $16.16Wm^{-2}$ in solar constant is required for this to happen.

If the Earth does become fully glaciated, then the solar constant has to increase past $Q \approx 441Wm^{-2}$ for the Earth to leave the glacial state. Thus an increase of approximately $115.2Wm^{-2}$ or more in solar constant is needed for the ice line to retreat from the equator.

Part (b)

We now explore how changing other parameter values affect the change in solar constant required for the Earth to change between the partially-iced, frozen and iceless states. For the forthcoming section, we shall use Q_1 to denote the change in solar constant required for the Earth to fall into the frozen state from it's current state, and Q_2 to represent the change in solar constant required to go from a frozen state to an iceless state.

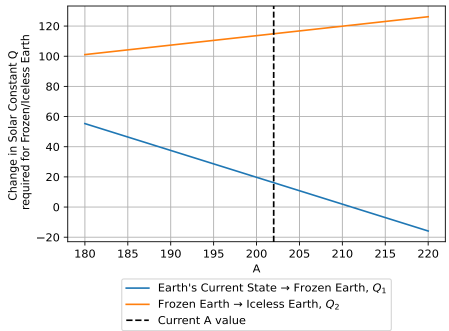

Figure 2: Investigating the critical values of the Earth's state as A varies

From Figure 2, we can observe linear relationships between A and our derived Q_1 & Q_2. As A is made larger, a smaller change in solar constant Q would be required for the Earth to transition from it's current state to that reminiscent of a frozen Earth. This opposes the change in solar constant needed to subsequently retreat the ice line from the equator, which increases as A is made bigger. Using our definitions for Q_1 and Q_2, this corresponds with Q_1 decreasing and Q_2 increasing with respect to an increasing A. Note the converse of this is all true for decreasing A from its current value. It can also be seen (in Figure 2) that the gradient of Q_1 (blue) is steeper than that of Q_2 (orange). Thus changes to A affect Q_1 more than Q_2.

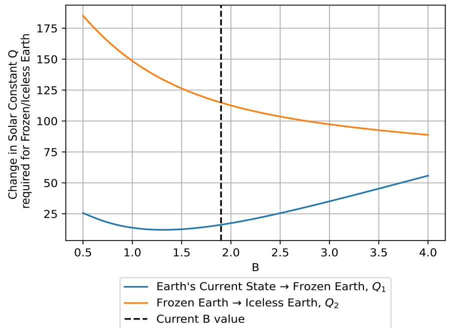

Figure 3: Investigating the critical values of the Earth's state as B varies

Figure 3 suggests increasing B decreases the value of Q_2. The opposite can be said for switching from Earth's current state to a frozen one, where Q_1 increases with B from a value of approximately $B \approx 1.3$. However from $B \approx 0.5$ to $B \approx 1.3$ Q_1 decreases with increasing B.

Upon observing how k affects the change in solar constant required (see Figure 4) it's seen that Q_2 increases with increasing k - the opposite to what happens in Figure 3. As k increases from $k \approx 2.5$ the Q_1 needed to switch from the Earth's current state to a frozen state increases. From $k \approx 0.5$ to $k \approx 2.5$ Q_1 decreases with increasing k - in fact the shape of the curve of Q_1 with respect to k matches fairly well with the same critical solar constant with respect to B. (Figure 3).

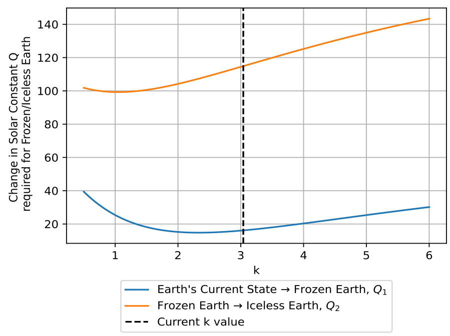

Figure 4: Investigating the critical values of the Earth's state as k varies

Next we shall see how these critical values of the solar constant vary with a change to the albedo formulation. We shall define δ to be a small change to the albedo function such that

$$a(y) = \begin{cases} a_i = 0.62 + \delta & y > y_s \\ a_w = 0.32 - \delta & y < y_s \\ 0.5(a_w + a_i) & y = y_s \end{cases}$$

(Note that $0.5(a_w + a_i) = 0.5(0.32 - \delta + 0.62 + \delta) = 0.5(0.94) = 0.47$ as before)

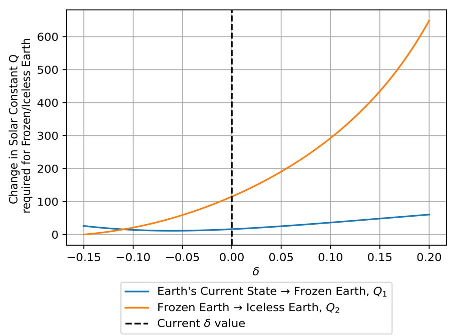

Figure 5: Investigating the critical values of the Earth's state as δ varies

Figure 5 shows that as δ increases from 0, the solar constant needed to change from the Earth's current state into a completely frozen state, Q_1, increases in a fairly steady linear fashion. This contrasts how the solar constant needed to convert a frozen Earth state into an iceless one, Q_2, increases exponentially with increasing δ; Q_2 is far more affected by changes in δ than Q_1.

Interestingly, when $a_i = a_w$ (ie. $\delta = -0.15$), Q_1 is a higher value than that of Q_2 (which has a value of 0). Q_1 is minimised at a value of $\delta \approx -0.06$, ie. when $a_i = 0.56$ and $a_w = 0.38$.

Investigation 2

Part (a)

Within the Energy Balance Model, we model the transportation of heat energy over the Earth's surface. The transport is modelled using Newton's Law of Cooling

$$E_{transport} = k(\overline{T} - T),$$

where k is the transport coefficient and $\overline{T}$ is the global mean temperature, which is given by

$$\overline{T} = \frac{1}{\text{surface area}} \iint TS = \int_0^1 T(y)dy.$$

Budyko used the transport coefficient $k = 3.91Wm^{-2\circ}C^{-1}$ while Warren and Schneider used the value $k = 3.74Wm^{-2\circ}C^{-1}$. The k given in our model is $k = 3.04$. To understand the sensitivity of the climate we have explored the temperature of the partially-iced Earth state - with respect to latitude - for different heat transport coefficients.

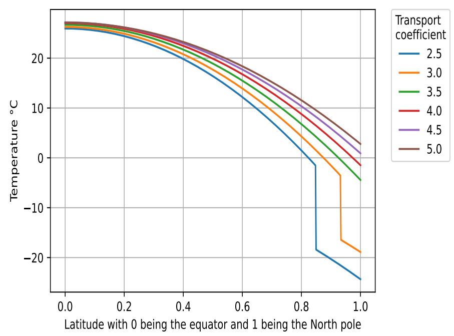

Figure 6: Investigating how climate changes with variation of the transport coefficient k

From Figure 6 we can see that smaller values, $k = 2.5Wm^{-2\circ}C^{-1}$, lead to a larger range of temperatures across the Earth and a larger drop in temperature when reaching the ice line. Whereas larger values, going towards $k = 5Wm^{-2\circ}C^{-1}$, cause the ice-line to disappear, resulting in an iceless Earth, and leading to a much smaller range in temperatures. In general, it is easy to see that larger transport coefficients lead to larger heat transfers from the equator and as such warmer poles. The model appears to be quite sensitive to the heat transport coefficient k, as small changes in the value of k used can cause fairly different climate predictions - most notably at the poles. Thus, the range in climate and position of the ice line is sensitive to the transport coefficient, k.

Part (b)

Upon viewing the Earth as being in it's current state, with the ice line at $y_s \approx 0.939$ and all other parameters fixed (excluding k) we explore how the climate across the globe changes depending on the transport coefficient k.

We begin by forming an asymptotic approximation for the temperature of the Earth when $k \ll 1$. From the steady state temperature equation,

$$T^*(y) = \frac{Q}{B+k}\left(s(y)(1 - a(y)) + \frac{k}{B}(1 - \overline{a})\right) - \frac{A}{B},$$

and using the expansion of the Earth's temperature as

$$T^*(y) = T_0^*(y) + kT_1^*(y) + k^2T_2^*(y) + \ldots,$$

we find that at leading order, we can approximate the Earth's temperature as

$$T^*(y) \sim T_0^*(y) = \frac{Q}{B} s(y)\Big(1 - a(y)\Big) - \frac{A}{B}.$$

Thus, when $k \ll 1$ the Earth's climate looks like below:

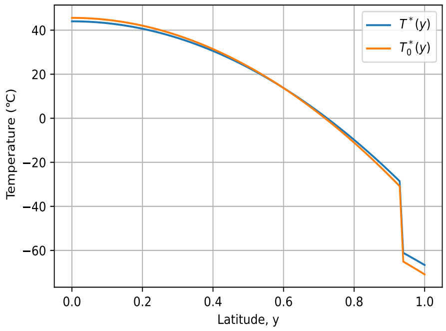

Figure 7: Investigating the Earth's climate when $k \ll 1$

Visually, we can see that when $k \ll 1$ there is massive difference in temperature between the equator and the north pole. This is to be expected as a low value for k implies that there is little heat transport between the equator and the north pole and as such, the heat from the equator does not disperse far, so the north pole will remain very cold.

On the contrary, we now explore when $k \gg 1$. We define $\epsilon = \frac{1}{k} \ll 1$ and find a two term asymptotic approximation (as the leading order term gives a constant temperature across the hemisphere). The two term approximation for $T^*(y) \sim T_0^*(y) + \epsilon T_1^*(y)$ is

$$T^*(y) \sim \frac{Q}{B}(1-\bar{a}) - \frac{A}{B} + \epsilon\Big(Qs(y)\big(1-a(y)\big) - Q\big(1-\bar{a}\big)\Big)$$

Thus, when $k \gg 1$ the Earth's climate looks as it does below:

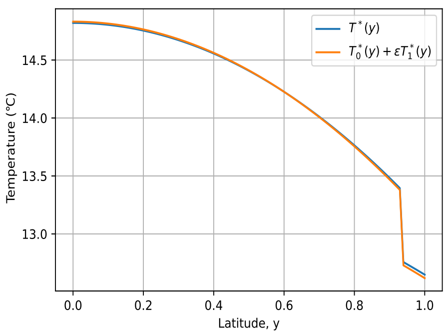

Figure 8: Investigating the Earth's climate when $k \gg 1$

As a result, when $k \gg 1$ - that is when there is a lot of heat energy being transported - the Earth has a less varied climate with similar mild temperatures near the equator and the north pole and no extreme cold or hot temperatures.

Now we shall investigate how sensitive these two cases' climates are to changes in the solar constant, Q, by deriving relationships between the change in temperature T, and the change in Q.

For the case of $k \ll 1$, we derive

$$\Delta T^* \sim \frac{\Delta Q}{B} s(y)\Big(1 - a(y)\Big)$$

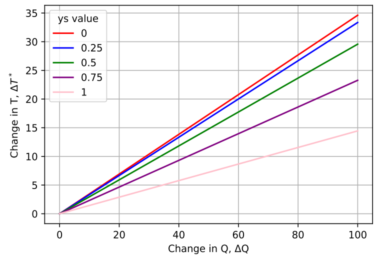

Figure 9: Change in T against change in Q, $k \ll 1$

So, dependent on the ice line value, a change in the solar constant by $20Wm^{-2}$ would approximately change the temperature between $3-7°C$. The climate with an ice line towards the north pole is less affected by change in solar constant than one with the ice line closer to the equator. This can perhaps be attributed to the low transfer of energy meaning changes in solar constant make less impact on colder areas of the planet.

When $k \gg 1$, we have

$$\Delta T^* \sim \frac{(1-\bar{a})}{B}\Delta Q + \epsilon\Big(s(y)\big(1-a(y)\big) - \big(1-\bar{a}\big)\Big)\Delta Q$$

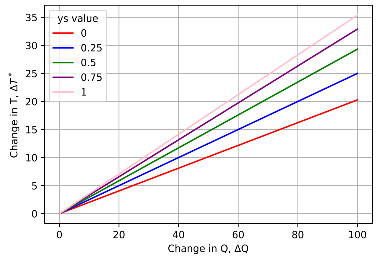

Figure 10: Change in T against change in Q, $k \gg 1$

This is similar to the previous graph in terms of a linear relationship between ΔT^* and ΔQ, however we see that an ice line at the north pole is far more affected by change in solar constant than having an ice line at the equator (the opposite of the $k \ll 1$ case).

Investigation 3

Part (a)

The critical temperature, T_c, is the temperature at which the surface of the Earth becomes ice-covered and therefore where the ice line, y_s, lies. In our original model, the critical temperature was $T_c = -10°C$. When modelling a more ice-covered Earth, i.e. an Earth with ice coverage on land over winter or seas covered with ice year-round, changing the critical temperature may be a way of modelling these alternate states of Earth.

Upon examining the climate with varied T_c values in Figure 11, unsurprisingly the lower the critical temperature the less ice covered the Earth is, as it must reach a much lower temperature for ice to form. This also means the albedo of our Earth in these cases will just be the water albedo $a_w = 0.32$. With increasing values of T_c we can see our Earth has a larger drop in temperature between land/sea and ice. When looking at the lines for $T_c \geq -6°C$ we notice that they don't actually converge and what is given is its sticking point, where no further progress is made. We can see this more in Figure 12, where it is evident these higher critical temperatures do not exist with the default solar radiation.

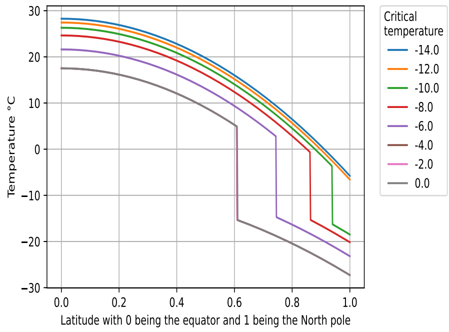

Figure 11: Investigating how the climate changes with variations in critical temperature

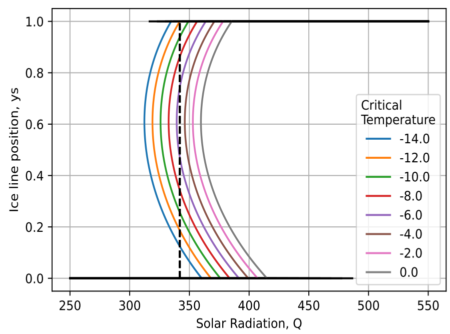

Figure 12: Investigation into how variations in Q along with changes in T_c affect ice line position

Figure 12 looks at how a change around the default value of solar radiation $Q = 342Wm^{-2}$ affects these new states. Plotting Q with the positioning of the ice line, y_s, we can see that our larger deviations from the default value, in the current climate, $y_s \approx 0.93$, do not exist with $Q = 342Wm^{-2}$. The higher critical values increase in similarity to our current climate with increases in Q and lower critical temperatures having ice lines in models with lower Q.

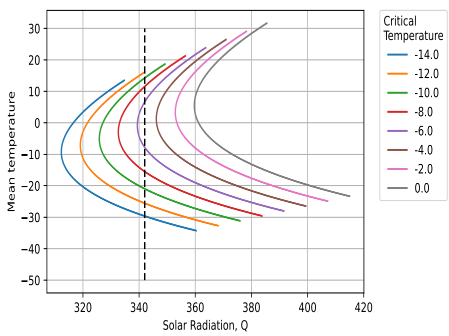

Figure 13: Investigation into how variations in Q along with changes in T_c affect global mean temperature

Then looking at Q vs the mean temperature, $\overline{T}$, we can see again that not all the variations of T_c exist in the current solar radiation. The larger values of T_c exist only in larger values of Q, needing solar radiation at $Q \approx 360Wm^{-2}$ to give a similar mean temperature to the current Earth, which is around $13.9°C$[1]. Overall, we can see with deviations from the default value of Q higher levels of solar radiation give more extreme global means whereas lower Q values give a less extreme deviation. Looking into more specific climates of earth we know that observations have shown fully ice-covered oceans at a mean temperature of $\overline{T} = -13°C$, we can see that $T_c \approx -7°C$ is most likely to give this mean in default Q. However, we can see that our ice line does not agree with this due to how our model is built.

Part (b)

We then return the critical temperature to its default value and look at how the ice albedo value affects the predicted climate and its sensitivity to changes in solar radiation, Q. Our original ice albedo value is $a_i = 0.62$, which is a mean value obtained by satellite,[2] but the value can range between 0.5 - 0.8 depending on many factors. For example, dry (non-melting) ice coverage in the central artic has an albedo of $a \approx 0.8$ according to observational data but melting ice in the summer has a dropped albedo of $a \approx 0.7$[3]. Cloud coverage, along with vegetation type can also change the albedo.

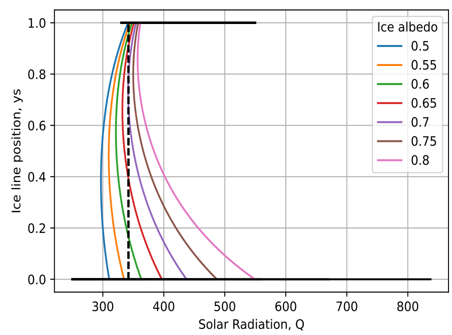

Figure 14: Investigation into how variations in Q along with changes in a_i affect ice line position

Looking at how the different ice albedo value models are affected by changes in solar radiation we see clearly that both extremes don't exist with ice lines in the default value of solar radiation, $Q = 342Wm^{-2}$. For the upper extreme $a_i = 0.8$ we need solar radiation upwards of $Q \approx 360Wm^{-2}$ for ice coverage to be present. Values between $a_i = 0.55$ and $a_i = 0.7$ imitate our original model with increasing values drawing the upper and lower ice line closer together towards $y_s = 0.7$ so around $63°N/S$. We can also see with increasing values of ice albedo the range in Q for which it is viable increases. The graph also displays the minimum value of Q for an ice-free Earth and the maximum Q for a glaciated Earth. Our minimum stays unchanged due to the fact it is based on having an Earth with only water albedo so our changes make no impact. However, with the maximum we see large increases with higher ice albedo values; going from $Q \approx 335Wm^{-2}$ for $a_i = 0.5$ to $Q \approx 837Wm^{-2}$ for $a_i = 0.8$.

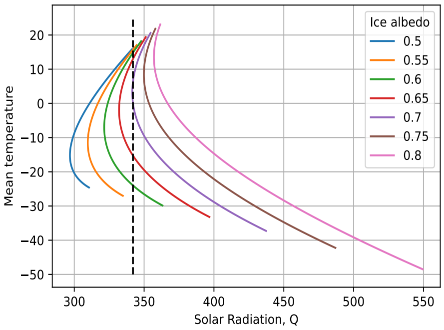

Figure 15: Investigation into how variations in Q along with changes in a_i affect global mean temperature

Then comparing the changes in solar radiation to the global mean temperature we see again higher ice albedo values give a much larger range of temperatures in a much larger range of solar radiation values. The albedo values between $a_i = 0.5$ and $a_i = 0.65$ give similar global mean temperatures to the actual in the warmer state; with further increases, we see a drop in the mean for the warmer state at our default Q and evidently, the higher values do not exist within the current solar radiation.

Investigation 4

Part (a)

There have been many suggestions for the values of the constants A and B determining the long-wave outgoing radiation from the planet and the effect of differing these values (while holding all others constant) on the climate can be seen in the plot of latitude vs temperature.

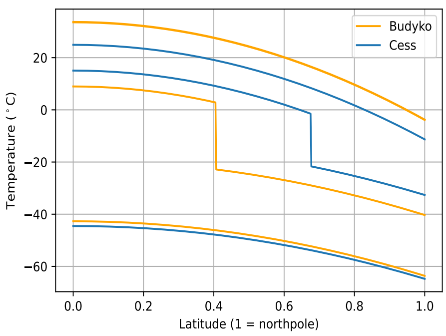

Figure 16: Effect of different outgoing radiation constants on the Earth's temperature

As shown in Figure 16 the Budyko and Cess values for A and B fundamentally change the possible steady state climate. Budyko's values suggest that there are 3 steady states and the only possible steady state with $y_s \in (0, 1)$ is around 0.4 which is much closer to the equator than are current ice-line at roughly 0.95. Hence, our current ice-line would retreat to the poles to the ice-free steady state. Likewise, the Cess values would suggest ice-line retreat to the poles, however, the stable steady state with ice line at roughly 0.7 is much closer to our actual ice-line latitude but not close enough to avoid tending to the ice-free state, without the influence of other factors.
However, a major limiting factor of the above model is using the same transport coefficient, $k = 3.04$. This is because B and k are closely linked by Meridional heat transportation. As ocean heat transportation influences ice extent as well as the width of tropical regions, which effects the moistness of sub-tropical regions where greenhouse gasses are trapped[4]. Therefore, simply changing the energy out coefficients without also changing the energy transport coefficient reduces the models capacity to capture the climate dynamics of the Earth hence reducing the accuracy of results.

Part (b)

Throughout the values given on page 2 of the coursework specification were used. In order to investigate the effect of changing B on the ice line, the formula for latitude dependent steady state temperature, $T^*(y)$, was rearrange to produce the quadratic equation,

$$B^2T^* + B[A + kT^* - Qs(1-a)] - k(Q(1-a) - A) = 0$$

The minimum ice-free state was calculated by setting $a = \hbar a = a_w$ for all $y \in [0, 1)$ and manipulating the same steady state temperature equation to give a quadratic equation.

$$B_{min} = \frac{Qs(1)(1-a_w)-kT_c-A+\sqrt{(kT_c+A-Qs(1)(1-a_w))^2+4T_c(Q(1-a_w)-Ak}}{2T_c}$$

The addition version of the quadratic formula was used to calculate the minimum ice free state to be -4.13. Therefore, under the current conditions the Earth can be ice free for all values of $B > 0$.
The maximum fully glaciated state was also calculated using the quadratic formula subtracting the square root term and replacing $s(1)$ with $s(0)$ and a_w with a_i. This value was calculated to be 7.49. The following bifurcation diagram was produced.

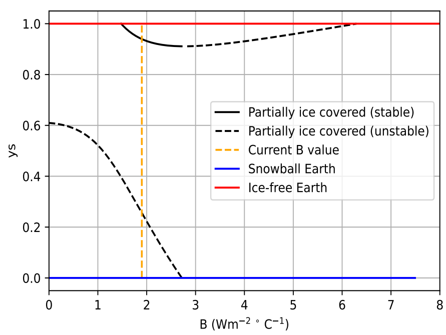

Figure 17: Investigating the ice line location as B varies

Next a bifurcation diagram was produced for B vs mean steady state temperature.

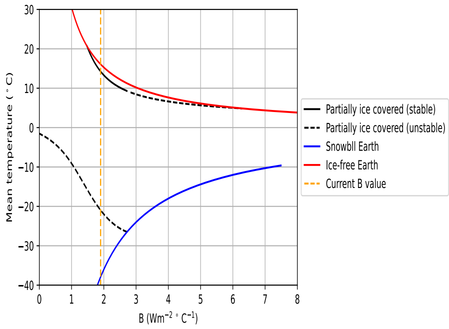

Figure 18: Investigating the mean steady state temperature as B varies

B is a measure of the temperature dependent feedback that occurs as a result of the Earth's climate. It amplifies or dampens cooling based on multiple factors and so a change in B could be the result of increased cloud droplet concentration which is a result of pollution. The effect of this would be an increase in planetary albedo as increased cloud cover would increase solar radiation reflection (Twomey 1977)[5]. However, an increase in cloud cover would also result in an increase in long-wave radiation absorption reflected from the Earth's surface which would have an isolating effect on the Earth. This can be seen in the ice-free steady state, which tends to an average temperature around $0°C$ when B increases. Additionally, a small increase in B from are current value would lead to a change in stability and tend to the ice-free steady state.

Investigation 5

Part (a)

There are several reasons why it seems advantageous that the Earth should be modelled in it's entirety, rather than via symmetry about the equator. One reason for this is due to the differing ice and land-sea distributions between the Northern and Southern Hemisphere, which will have varying reflective properties. It's understood that that around 90% of the Earth's ice is found in Antarctica [6] and also that the Northern Hemisphere is roughly 39.3% landmass and 60.7% water, whereas the Southern Hemisphere is 19.1% landmass and 80.9% water.[7].This leads to the forthcoming discussion regarding the possibility of non-symmetric steady states of the Earth's temperature about the equator.

Using data obtained from Earth Radiation Budget Experiment (ERBE)[8] we discover how the average albedo of the Earth differs at different latitude bands spanning the Southern and Northern Hemisphere.

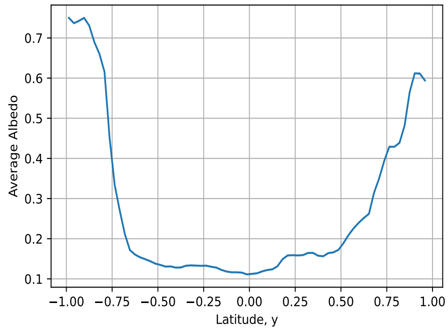

Figure 19: Average Albedo of the Earth's surface depending on the latitude

Upon observing the average albedo in Figure 19 we see that for the Southern Hemisphere - when $y \in [-1, 0]$ - there is a very sharp increase in the albedo after roughly $60°S$, which coincides with the beginning of Antarctica.

Interestingly, we equally notice that the Arctic Circle in the Northern Hemisphere has a lower average albedo than Antarctica. This is likely due to the geographical differences between the two poles. That is, the fact the Antarctica is a landmass covered in ice compared to the Arctic which is an ocean covered by a thin layer of perennial sea ice, surrounded by land.

To model the steady state temperature of the Earth as the latitude varies we use the equation

$$T^*(y) = \frac{Q}{k+B}\Big(s(y)\big(1-a(y)\big) + \frac{k}{B}(1-\overline{a})\Big) - \frac{A}{B}$$

Since we are using data to effectively implement a new albedo function, in order to approximate

$$\overline{a} = \int_{-1}^{1} s(y)a(y)\ dy,$$

the trapezoidal rule has been utilized at the discrete latitude points given by the data from the Earth Radiation Budget Experiment.[8]

With the given data, (and reflecting the Southern Hemisphere co-ordinates to $y \in [0, 1]$ for ease of comparison) the temperature of the Earth with respect to latitude appears as follows:

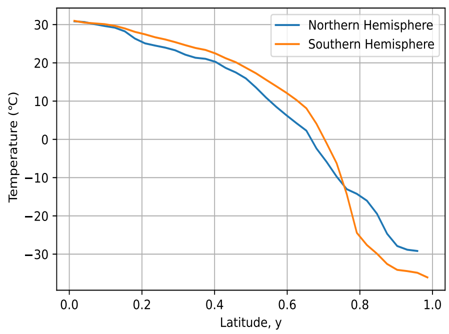

Figure 20: Temperature of the Earth depending on Latitude

From Figure 20, the average annual surface temperatures on Earth range from roughly $-40°C$ in Antarctica to $30°C$ near the equator which ties in with figures regarding atmospheric circulation.[9] It is equally found that the South Pole is colder than the North Pole as it appears so in reality.

However, there appear to be a couple of problems with this model. Firstly, the Southern Hemisphere appears warmer than the Northern Hemisphere for latitudes $y \in [0.2, 0.7]$ which is not accurate.[10] Secondly, the average annual surface temperatures within roughly $y \in [0.3, 0.7]$ appear slightly too high. As an example, at latitude approximately $52°S$ ($y \sim 0.58$), upon observing temperatures at the Falkland Islands around $1986-1989$ (since this is when the albedo data [8] was recorded for) we find that annual average temperature was roughly $7°C$[11] which is fairly lower than roughly $12°C$ observed in the model.

To improve upon this initial model, it's also worth noting that the radiation emitted from the Earth's surface will differ depending on the latitude in the Northern and Southern Hemisphere.[10] With regards to Cess' work[11], we formulate separate equations to model the radiation emitted in each hemisphere. In the Northern Hemisphere, we utilise $E_{out} = 211.5 + 1.575T(y,t)$ and for the Southern Hemisphere $E_{out} = 221.5 + 1.595T(y,t)$

Upon introducing these 2 specific equations for E_{out} in the Northern and Southern Hemisphere respectively, we observe that the temperature of the Earth with respect to latitude changes.

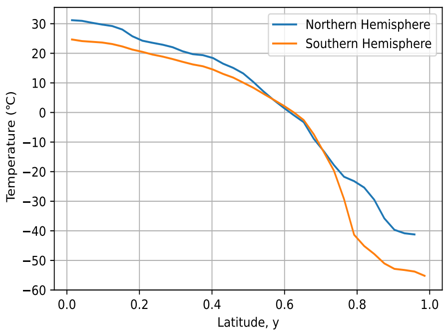

Figure 21: Temperature of the Earth depending on Latitude

It now appears that the Southern Hemisphere is colder than the Northern Hemisphere and we see that there appears to be lower temperature observations with respect to latitude in the region $y \in [0.2, 0.7]$ than there was in the previous model - most notably for the Southern Hemisphere. For example, the temperature modelled for the Falklands Islands now appears much more reasonable (roughly $5°C$). Equally, at latitude approximately $30°N (y \sim 0.33)$ Cairo in Egypt has an average annual temperature of $21.5°C$[12] which again seems fairly reasonable in Figure 21.

Obviously, this model is a massive simplification of the temperatures at different places on Earth, as there are clearly many other factors which will contribute. These range from altitude, seasonal variation and specific atmospheric circulation that occurs at certain latitudes. These are either not included or not modelled accurately enough in this model which could potentially causing invariance in temperatures from reality. An example of this is Quito, Ecuador which is situated upon the equator but due to it's high altitude is nowhere near as warm as the model predicts. [13] Notably, the climate at the North Pole in Figure 21 is too cold, hinting that important factors such as the heat transport in the Northern Hemisphere is perhaps not being modelled accurately enough as the circulation of warm air surrounding the Arctic Circle seems to not be impacting the temperature as much as it does in reality.

Part (b)

We shall investigate the use of a continuous albedo in place of the discontinuous one given. The continuous albedo we derive is

$$a(y) = \lambda tanh(\alpha(y - y_s)) + \mu,$$

where $\lambda = \frac{a_i - a_w}{2} = 0.15$, $\mu = \frac{a_i + a_w}{2} = 0.47$, and $\alpha \gg 1$. This albedo is pretty much constant at a_i and a_w other than when the function 'switches' at the ice line y_s. Setting higher values of α steepens the gradient of this switch, whilst lower values cause the switch to occur over a bigger area of latitude.

Investigating the non-trivial solutions when $Q = 342$ (using a discrete albedo) resulted in ice lines appearing at $y_s = 0.256$ and $y_s = 0.939$. To see how a continuous albedo affected these steady states, we plotted the value of α (used in our derived albedo) against y_s for both the low ice line, and the high one (Figure 22).

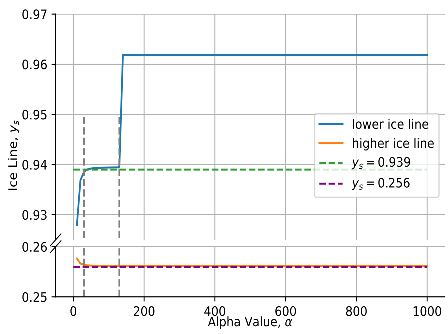

Figure 22: α against ice line y_s

This graph shows the unstable state at $y_s = 0.256$ is pretty much unaffected by the use of this continuous albedo (and values of α), other than when α is small ($\alpha \leq 10$) the lower ice line is a little higher. Given in our derivation of the continuous albedo we specified $\alpha \gg 1$, this small deviation can be ignored.

Worth more mention and investigating however is the stable state at $y_s = 0.939$, which does undergo changes as α increases. In Figure 22 we can see that between the grey dotted lines ($\alpha = 30$ and $\alpha = 130$), the continuous albedo obtains a higher ice line very close to the discrete albedo's $y_s = 0.939$. However at higher $\alpha(> 130)$ this ice line quickly changes/switches to a value of $y_s = 0.962$. The difference between this value and the previous value is approximately ≈ 0.023; the new higher ice line is at a value 2.45% bigger than before. Given the size of the Earth, this can be deemed a notable change in the stable steady state corresponding to the higher ice line.

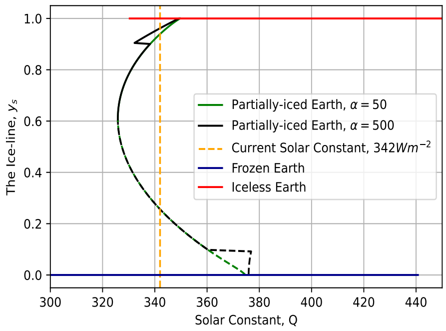

Figure 23: Investigating changes to the ice line due to variations in α

Looking at Figure 23 we can see why the higher ice line changed when increasing α. The line representing the partially-iced Earth state skews off its normal trajectory around $y_s \approx 0.9$, and then continues back to the iceless state. We couldn't find a particular reason why this happens for $\alpha > 130$, but this could perhaps be investigated in the future.

What can be said though, given the figures above, is that a continuous albedo can in fact slightly change the steady state corresponding to the higher ice line, with large values of α. Values of α ranging between $\alpha \approx 30$ and $\alpha \approx 130$ lead to similar solutions to that of the previous discontinuous albedo.

Part (c)

Given that the Earth has increased about $0.08°C$ per decade since 1880[1] and the oceans are getting hotter, one could suggest that as time evolves the heat transport coefficient on the Earth has been increasing. With this in mind, upon referring back to Figure 6 in Investigation 2 it can be observed that as time evolves and the heat transport coefficient increases the ice line moves closer and closer to the North Pole. In order to explore this further a finite difference method could be implemented where the temperature is discretised in 2 dimensions (latitude and time). A colour map could then be created by creating a colour bar in terms of temperature and upon varying k, it may be expected that upon increasing k the colourmap will become less varied in colour.

Conclusion

Upon our investigation of the solar constants required to change between the Earth's possible states, it was found that a decrease of at least $16.16Wm^{-2}$ from our current solar constant ($Q = 342$) was needed to fall into a frozen state from it's current state. From this frozen state it would then take an increase of $115.2Wm^{-2}$ for the ice to retreat from the equator. We then looked at how these newly found critical values of the solar constant changed when each parameter was varied; each one resulted in a slightly different graph (see Figures 2-5), but conclusively a variation in each parameter changed these critical values to some extent.

Budyko and Warren & Schneider used different values of transport coefficient k, resulting in slightly different climates. Keeping our other parameters the same and investigating these respective values of k, it was found the ice line disappeared. We have also investigated other suggested values of A and B (from Budyko and Cess), which were found to fundamentally change the possible steady state climate. Thus it can be concluded that when using a different value for a parameter, it is important to change the other parameters accordingly in order to retain a realistic model of the Earth's climate.

Changing the albedo was found to have various effects. For example, varying the ice albedo a_i from 0.5 to 0.8 changed the maximum solar constant from $Q \approx 335Wm^{-2}$ to $Q \approx 837Wm^{-2}$. Additionally, modifying the model to use a continuous albedo in place of the previous discontinuous one can result in a different value for the higher ice line, $y_s = 0.962$. This is dependent on the parameter α used in the continuous albedo we derived.

A separate modification to the model was made in order to investigate steady states across the whole world, and not just the Northern Hemisphere. It appears that the Southern Hemisphere is in fact colder than the Northern Hemisphere, thus implying that the steady states are non-symmetric about the equator.

Further analysis could be made into all of these topics, via additional changes to the model, varying parameters to a greater extent, or perhaps even introducing more variables. However given time and computational constraints, we shall leave that for others to explore.

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