

```
1. function evaluate_algorithm(expression)
    while there are tokens to be read do:
        if the token is a number then:
            push the number to the value stack.
        if the token is an open parenthesis '(' then:
            push the token onto the operator stack.
        if the token is a close parenthesis ')' then:
            if there is more than one token in the operator stack then:
                while the top element of the operator stack is not '(' do:
                    if the value stack has less than 2 elements then:
                        return NOTWELLFORMED.
                    else:
                        pop the top element of the operator stack and the top 2 elements of
                        the value stack
                        perform the operation on the popped values in the correct order.
                        push the result onto the value stack
                    pop the top element of the operator stack and discard it (open parenthesis)
            else:
                return NOTWELLFORMED
        if the token is an operator then:
            if there are no tokens in the operator stack then:
                if there are no values in the value stack then:
                    return NOTWELLFORMED
                else:
                    push the operator onto the operator stack
            else:
                if the top element of the operator stack is an open parenthesis '(' then:
                    push the current token onto the operator stack.
                else:
                    while the top element of the operator stack is higher precedence than the current
                    token do:
                        pop the top element of the operator stack and the top 2 elements of
                        the value stack
                        perform the operation on the popped values in the correct order
                        push the result onto the value stack
            else:
                return NOTWELLFORMED
        while there are tokens in the operator stack do:
            pop the top element of the operator stack and the top 2 elements of the value stack
            perform the operation on the popped values in the correct order
            push the result onto the value stack
    pop and return the top value of the value stack.
```

3. B)

The input stage of the program runs in  $O(n + m + 1)$  time. 'parkranger.c' works by first initializing an array of empty deques equal to the number of trees in the problem plus an extra deque to represent the starting position of the run, this process runs in  $O(n + 1)$  time. Next the program reads in the viable routes into the deques previously initialized, this process costs  $O(m)$  time.

The next stage of the program is the topological sorting stage, which is a dfs algorithm with a worst-case time complexity  $O(m)$ . This part of the algorithm explores possible paths from the start position 0, marking nodes as it backtracks into a topologically sorted array. This part of the program works by recursively calling 'dfs\_search' and keeping track of visited nodes so as not to explore them more than once.

The next stage involves checking to see if the tree trimmer can go from the top of the mountain to the bottom going from one node to the next in the topological ordering given by dfs\_sort. Since dfs\_sort only explores from node 0, we first check to see if all the nodes were visited in the sort - with costs  $O(n)$  time. If they weren't the program returns false. Next the program checks for each node in the topological sort, whether the previous node has an outgoing edge to the current one, this process has a nested for loop but only costs  $O(n + m)$  time since nodes are not revisited and also due to the design decision to use an adjacency list instead of matrix.

Finally, freeing the memory associated with running the program is bounded by  $O(n)$  time due to freeing each deque in our adjacency list.

Hence the program runs in linear time, since each of its constituent stages run in linear time leading to an overall time complexity of  $O(n + m)$  for large values of  $n$  and  $m$ .