A Structural Measure of the Shadow Federal Funds Rate*

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Abstract

We propose a shadow interest rate for structural macroeconomic models that measures the interest-rate-equivalent stance of monetary policy at the zero lower bound. The lower bound constraint, if expected to bind, is contractionary and increases the shadow rate compared to an unconstrained systematic policy response. By contrast, forward guidance that extends the expected duration of zero-interest-rate policy beyond the lower bound constraint is expansionary and decreases the shadow rate. Quantitative easing that shortens the expected duration of the binding constraint also decreases the shadow rate. We find that the estimated shadow federal funds rate from a workhorse structural model of the US economy better captures the stance of monetary policy than a shadow rate based only on the term structure of interest rates. Furthermore, both forward guidance and quantitative easing appear to be important drivers of our shadow federal funds rate.

Keywords: zero lower bound; forward guidance; quantitative easing; shadow rate; monetary policy

JEL classifications: E52; E58

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1 Introduction

In response to the Great Recession and the COVID-19 crisis, the Federal Reserve, like many other central banks, cut its policy interest rate close to zero. When this happens, the lower bound constraint on nominal interest rates makes it difficult to determine the overall stance of monetary policy for given economic conditions from the observed policy rate alone. In a highly influential paper, Wu and Xia (2016) use a term structure model to construct a 'shadow' policy rate intended to help quantify the interest-rate-equivalent stance of policy at the zero lower bound (ZLB). The basic idea is that the shadow rate reflects the effects of unconventional policies in terms of a hypothetical unconstrained short-term interest rate.[1]

Having a shadow rate that captures monetary policy at the ZLB is useful for at least two reasons. First, policymakers can gauge the scale of unconventional policy actions with a comparable measure to monetary policy conducted during conventional times. Second, researchers can easily extend their empirical analysis into periods in which the observed policy rate is at the ZLB, as has been done with the Wu-Xia shadow rate in studies such as Avdjiev et al. (2020) and Anderson et al. (2017).

Instead of basing the shadow rate on term structure models, such as in Ichiue and Ueno (2013), Krippner (2013), and Wu and Xia (2016), we propose a shadow rate for structural macroeconomic models.[2] The advantage of considering a structural model is it can better capture the stance of monetary policy for given economic conditions. This is because a structural model can help disentangle movements in the term structure due to unconventional policies from those due to other shocks.[3]

Term structure models can be used to extract market expectations about the duration

[1]This focus on an unobserved interest rate that leads to equivalent observed macroeconomic outcomes is distinct from the idea of a 'notional' shadow rate corresponding to hypothetical policy actions in the absence of a ZLB constraint that would have led to counterfactual macroeconomic outcomes. Examples of this 'notional' approach include De Michelis and Iacoviello (2016), Gust et al. (2017), Andrade et al. (2019), and Atkinson et al. (2020).

[2]Our approach also differs from a more statistical identification of an 'implied' interest rate based on a Tobit-type specification for the observed short-term interest rate in a VAR model, such as in Iwata and Wu (2006) and Nakajima (2011). It is closer to, but still different from, Aruoba et al. (2021a) and Mavroeidis (2021), who consider implied uncensored shadow rates in VAR models with occasionally-binding constraints, but allow observed macroeconomic outcomes to be due to a linear combination of the shadow rate and the actual constrained policy rate.

[3]Bauer and Rudebusch (2016) caution against using shadow rates from term structure models for an alternative reason that they can be highly sensitive to what would appear to be seemingly innocuous assumptions about model specification. Meanwhile, Johannsen and Mertens (2021) argue that it is important to augment term structure models with macroeconomic variables. However, their shadow rate is more of a notional rate that corresponds to a hypothetical policy rate in the absence of the ZLB rather than a measure of the stance of policy for given economic conditions.

of the ZLB, as in Ichiue and Ueno (2015). Yet, to capture the policy stance for given economic conditions, it is critical to uncover the underlying determinants of the duration. Specifically, is the policy rate expected to be zero because deteriorating economic conditions suggest the ZLB constraint is likely to bind for a long time? Or does the expected duration reflect unconventional 'lower-for-longer' zero-interest-rate policy beyond what the ZLB constraint would imply on its own?

Shadow rates from term structure models do not account for this distinction in terms of why the policy rate is expected to be zero. In the term structure approach, the observed interest rate follows $i_t = \max(i_t^*, 0)$, where i_t^* is the shadow policy rate. Therefore, it would be impossible to have a shadow rate greater than zero when $i_t = 0$. In our proposed structural approach, a persistent binding of the ZLB constraint is equivalent to a contractionary policy shock for an unconstrained system that increases the shadow rate relative to a level implied by the systematic policy response to economic conditions. Our structural measure of the shadow rate thus allows the possibility that $i_t^* > i_t = 0$, which would occur if, for example, the inability to cut the policy rate given large persistent negative economic shocks means that policy will remain much tighter than if the policy rate could be adjusted based on an unconstrained policy rule. Forward guidance that extends the expected duration of zero-interest-rate policy offsets the effects of the constraint, decreasing the shadow rate. Meanwhile, given a structural model that also incorporates quantitative easing (QE) in addition to forward guidance, different structural shadow rates can be constructed to separate out the interest-rate-equivalent effects of both types of unconventional policy.

The overall level of our structural shadow rate at any point in time reflects the net effects on observed macroeconomic outcomes of the ZLB constraint and unconventional policies. Depending on systematic policy responses and how much the unconventional policies offset a binding ZLB constraint, our shadow rate can be positive or negative when the actual policy rate is zero. In the term structure approach, the shadow rate is estimated from yield-curve data and a 3-month yield is typically used as the short-term rate. Consequently, it is technically feasible for a term structure measure to be greater than the federal funds rate, as in Figure 4 of Wu and Xia (2016) at the start of the ZLB in 2009 when the 3-month yield was still positive. But, unlike with our approach, the reason is not directly related to or reflective of the extent to which the ZLB constraint binds for the policy rate.

Section 2 describes how to construct our structural shadow rate, illustrating its link to the stance of monetary policy with a simple analytical example motivated by Sims et al. (2023). Section 3 presents the estimated shadow federal funds rate from the workhorse structural model of the US economy originally developed by Smets and Wouters (2007), with further analysis of a counterfactual shadow rate and macroeconomic outcomes in the absence of unconventional monetary policy and an extension to a model that accounts for QE. Section 4 considers three applications of our structural shadow federal funds rate to (i) illustrate its alignment with notable unconventional policy actions in the aftermath of the Great Recession, (ii) compare its performance with that of the Wu and Xia (2016) measure in a monetary VAR estimated using data from the ZLB, and (iii) determine what it implies about monetary policy in an extended sample period that includes the recent pandemic. Section 5 concludes.

2 Constructing the Structural Shadow Rate

In principle, the structural shadow rate can be constructed for any structural macroeconomic model that accounts for the ZLB. In the next section, we consider a particular estimated structural model that is widely regarded to capture the empirical transmission of monetary policy outside of the ZLB, but here we first describe how the structural shadow rate can be constructed in more general terms and provide a simple example using a stylized theoretical model to illustrate the approach analytically. Because our structural shadow rate is based on decomposing the expected duration of how long the ZLB will hold into its underlying sources, we begin with the details of this decomposition, as originally proposed in Jones et al. (2022).

2.1 Decomposition of ZLB Durations

Assume the observed policy rate, i_t, either follows an unconstrained Taylor-type policy rule or is fixed at a particular value. For expositional simplicity, we set the fixed level to zero, although it could be any feasible value, including an alternative effective lower bound. Thus, the policy rate can be described by

$$i_t = \begin{cases} \text{policy rule}, & \mathbb{I}_t = 0 \\ 0, & \mathbb{I}_t = 1, \end{cases} \tag{1}$$

where the indicator $\mathbb{I}_t$ keeps track of the policy-setting regime.

When $\mathbb{I}_t = 0$, the policy rate follows the unconstrained policy rule and linearized structural equations are given by

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}\varepsilon_t, \tag{2}$$

where x_t is an $n \times 1$ vector of model variables, ε_t is an $l \times 1$ vector of structural shocks that includes a monetary policy shock, which we will denote as $\varepsilon_{m,t}$, and $\mathbf{A}$, $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{F}$ are conformable matrices. The reduced-form solution when agents expect the regime to be $\mathbb{E}_t \mathbb{I}_{t+j} = 0$ for $\forall j \geq 0$ is then given by

$$x_t = \mathbf{J} + \mathbf{Q} x_{t-1} + \mathbf{G} \varepsilon_t, \tag{3}$$

where $\mathbf{J}$, $\mathbf{Q}$, and $\mathbf{G}$ are the standard linear rational expectations solution matrices.

When $\mathbb{I}_t = 1$, the structural equations are given by

$$\bar{\mathbf{A}} x_t = \bar{\mathbf{C}} + \bar{\mathbf{B}} x_{t-1} + \bar{\mathbf{D}} \mathbb{E}_t x_{t+1} + \bar{\mathbf{F}} \varepsilon_t, \tag{4}$$

where the parameters for the equation corresponding to the policy rate, which is now fixed at zero, are different from (2) and there is no longer a conventional monetary policy shock, i.e. $\varepsilon_{m,t} = 0$.[4] As discussed in Jones (2017) and Kulish et al. (2017), when the interest rate is fixed or expected to be fixed in the future such that $\mathbb{E}_t \mathbb{I}_{t+j} = 1$ for some $j \geq 0$, the solution, following Kulish and Pagan (2017), is a time-varying VAR of the form

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{5}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are time-varying matrices that depend on the expected timing of the fixed-interest-rate regime. If $\mathbb{I}_t = 1$ and the economy is expected to return to the unconstrained policy rule after d periods such that $\mathbb{E}_t \mathbb{I}_{t+j} = 0$ for $j \geq d$, we can summarize the timing of the fixed-interest-rate regime at time t by a fixed duration denoted $\mathbf{d}_t = d$. To keep track of the reduced-form that prevails at each point in time, we allow $\bar{T}$ to be an arbitrarily large upper-bound duration and find the sequences $\{\mathbf{J}_d\}_{d=1}^{\bar{T}}$, $\{\mathbf{Q}_d\}_{d=1}^{\bar{T}}$, and $\{\mathbf{G}_d\}_{d=1}^{\bar{T}}$ such that $\mathbf{J}_d$, $\mathbf{Q}_d$, and $\mathbf{G}_d$ are reduced-form matrices corresponding to a particular duration d.[5] The prevailing reduced-form in a given period can be relabeled as $\mathbf{J}_t = \mathbf{J}_{\mathbf{d}_t}$, $\mathbf{Q}_t = \mathbf{Q}_{\mathbf{d}_t}$, and $\mathbf{G}_t = \mathbf{G}_{\mathbf{d}_t}$ and, noting that $\mathbf{d}_t = 0$ in periods where $\mathbb{E}_t \mathbb{I}_{t+j} = 0$ for $\forall j \geq 0$ with $\mathbf{J}_0 = \mathbf{J}$, $\mathbf{Q}_0 = \mathbf{Q}$, and $\mathbf{G}_0 = \mathbf{G}$, the reduced-form solution over the full sample is also given by (5), providing the basis of estimation. At the estimated parameter values with the estimated structural shocks, (5) returns back the data.

[4] It is possible to allow parameters for other structural equations to also change in the fixed-interest-rate regime, such as would be the case if the propagation of certain shocks changes under the ZLB.

[5] Consistent with this notation for the reduced-form matrices that only depends on duration d, we find in our empirical applications that it is always the case that a fixed-rate regime is expected in the future only when the current regime is at the ZLB. Also, once the economy is expected to leave the fixed-rate regime, we always find that it is not expected to return to it. This reflects an inherent expected mean reversion following negative structural shocks that could cause the policy rate to hit the ZLB.

Following Jones et al. (2022), the duration of a fixed-interest-rate regime $\mathbf{d}_t$ corresponds to the actual duration expected by agents, which is not necessarily the same duration prescribed by the policy rule given the ZLB constraint. With the occasionally-binding-constraint solution of Guerrieri and Iacoviello (2015), we can find the duration in each period t prescribed by the policy rule – i.e. the expected duration implied by the estimated state x_{t-1}, the estimated non-policy structural shocks, and a given lower bound.[6] This duration corresponds to monetary policy following exactly $\max(\text{policy rule}, 0)$ in projecting when the policy rate lifts off from the ZLB. We denote this expected duration by $\mathbf{d}_t^{\text{lb}}$ and refer to it as the *lower-bound* duration. In the absence of future shocks, this duration is expected to fall by one period at a time as the effects of current shocks unwind.

Again following Jones et al. (2022), for each period of the fixed-interest-rate regime, we define the *forward-guidance* duration, $\mathbf{d}_t^{\text{fg}}$, as the difference between the actual duration $\mathbf{d}_t$ and the lower-bound duration $\mathbf{d}_t^{\text{lb}}$, so that the actual duration is decomposed as

$$\mathbf{d}_t = \mathbf{d}_t^{\text{lb}} + \mathbf{d}_t^{\text{fg}}. \tag{6}$$

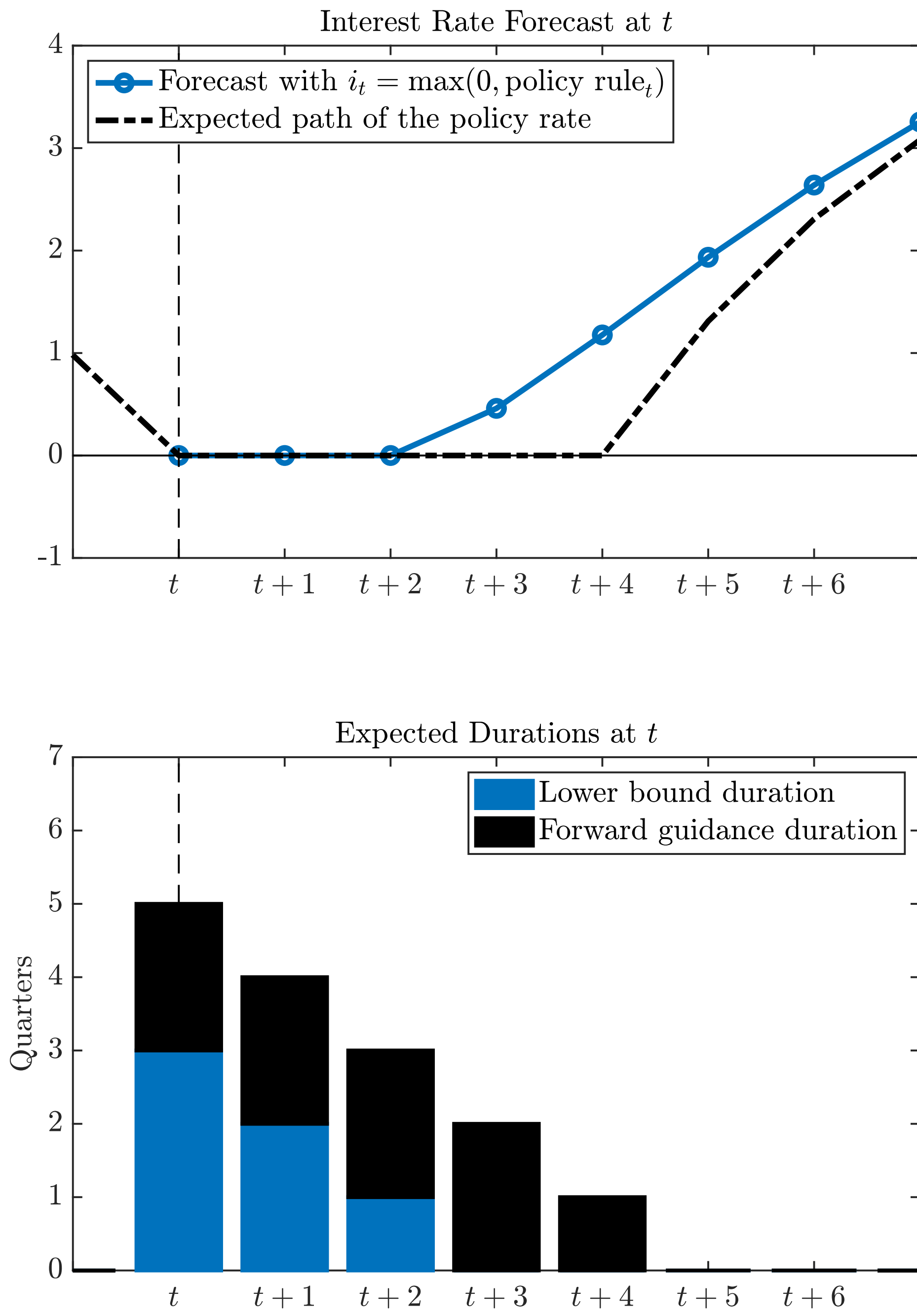
The forward-guidance duration captures announcements and other factors that change the actual duration beyond the lower-bound duration.[7] This decomposition is important because changes in $\mathbf{d}_t$ that originate from $\mathbf{d}_t^{\text{lb}}$ have different implications for the observed variables than those from $\mathbf{d}_t^{\text{fg}}$. For example, output and inflation might fall after a negative economic shock that makes the ZLB constraint bind for longer. But output and inflation might increase after a 'lower-for-longer' announcement extending the fixed-interest-rate regime beyond what the ZLB constraint implies.

Figure 1 illustrates the decomposition of the duration with a hypothetical example. It shows two forecasts for the policy rate associated with the decomposition. At time t, the actual duration is 5 quarters and the policy rate is expected to lift off in $t+5$. This duration is made up of 3 quarters of a lower bound duration and 2 quarters of a forward guidance duration. If monetary policy were to strictly follow $\max(\text{policy rule}, 0)$, the duration would be 3 quarters with its associated forecast for the policy rate path plotted in solid blue. In Figure 1, however, we assume the central bank has communicated that the

[6] Also see Jones (2017) and Aruoba et al. (2021b) on solving rational expectations models with occasionally-binding constraints.

[7] In addition to explicit central bank communications about durations, the forward-guidance duration could reflect other unconventional policies such as QE or public expectations that the central bank might deviate from its policy rule, such as by raising the policy rate sooner than implied by the ZLB constraint. For example, markets reassessed the likely timing of liftoff as the Fed was tapering the rate of bond purchases in 2013 and this can be thought of as having decreased the forward-guidance duration. We discuss the possible link to QE as part of our analytical example.

Figure 1: Hypothetical Example of a Duration Decomposition



interest rate will stay at zero 'lower for longer', extending the duration by an additional 2 quarters. So, in this example, $\mathbf{d}_t = 5$, with $\mathbf{d}_t^{\text{lb}} = 3$ and $\mathbf{d}_t^{\text{fg}} = 2$.

What drives the actual expected duration, as we demonstrate below, matters a lot for the stance of policy. If the duration were to be fully accounted for by the lower bound duration, that is by $\mathbf{d}_t = \mathbf{d}_t^{\text{lb}}$, then that duration would fully reflect persistent negative economic shocks that make the constraint bind. If, however, the actual duration were to be fully accounted for by the forward guidance duration, that is by $\mathbf{d}_t = \mathbf{d}_t^{\text{fg}}$, then that means the central bank is maintaining the policy interest rate at zero even though their policy rule calls for a positive rate, and so the central bank is effectively trying to provide stimulus by holding the rate at zero for longer than prescribed by their policy rule.

2.2 Mapping to a Shadow Economy

To construct the structural shadow rate, we find shocks under the unconstrained policy rule regime $\mathbb{I}_t = 0$ that would replicate outcomes in the data under the fixed-interest-rate regime $\mathbb{I}_t = 1$. Specifically, for the periods when $\mathbb{I}_t = 1$, we replace the monetary policy shock $\varepsilon_{m,t} = 0$ in the vector of structural shocks ε_t with a shadow rate shock, denoted $\varepsilon^*_{m,t}$, to obtain the vector of shocks in the shadow economy, denoted ε^*_t, where all other structural shocks are maintained at their realized values. The value of the shadow rate shock is determined such that outcomes in a shadow economy based on the structure in (3), given by

$$x^*_t = \mathbf{J} + \mathbf{Q}x^*_{t-1} + \mathbf{G}\varepsilon^*_t, \tag{7}$$

approximate the outcomes for at least some variables in x_t observed in the actual economy (5). Of course, when $\mathbb{I}_t = 0$, all of the shocks in the shadow economy ε^*_t will be the same as ε_t and the variables will be perfectly matched. The *structural shadow rate* is then defined as the policy interest rate, i^*_t, that prevails in the shadow economy (7) given all of the shocks including the shadow rate shock $\varepsilon^*_{m,t}$.

Formally, let ι denote a vector that selects which variables to target in matching the outcomes across systems (5) and (7), with $\Delta_t = \iota(x_t - x^*_t)$ denoting the difference between the observed targeted variables and the same variables as they evolve in the shadow economy. The shadow rate shock $\varepsilon^*_{m,t}$ is chosen each period to solve

$$\min_{\varepsilon^*_{m,t}} \Delta'_t \mathbf{W} \Delta_t, \tag{8}$$

where $\mathbf{W}$ is a diagonal weighting matrix that reflects the volatility of the targeted variables, with the diagonal elements being the inverse of the variance of each corresponding variable. This weighting scheme effectively standardizes the data and, as a result, implies equal weights in matching each of the targeted variables.[8] The shadow rate i_t^* is then constructed using the unconstrained policy rule with the shadow rate shock that minimizes the Euclidean distance for the standardized variables.

The next subsection presents a simple theoretical model to show analytically how changes in the expected duration of the ZLB constraint, forward guidance, and QE map into contractionary and expansionary shadow rate shocks and how the shadow rate reflects the stance of monetary policy.

2.3 Analytical Results from a Simple Model

In this section, we consider a number of analytical results using a simplified version of the New Keynesian (NK) model outlined in Sims et al. (2023). In this model, the central bank determines the value of the short-term interest rate subject to the zero lower bound and has an additional instrument, its balance sheet, through which it can implement QE. The equations of the model are

$$\hat{y}_t = -(1-z)\left(i_t - \bar{i} - \mathbb{E}_t\hat{\pi}_{t+1}\right) + \mathbb{E}_t\hat{y}_{t+1} + z\hat{\mathrm{qe}}_t + \varepsilon_{y,t} \tag{9}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa\hat{y}_t + \frac{z}{1-z}\kappa\hat{\mathrm{qe}}_t \tag{10}$$

$$i_t = \max(0, \bar{i} + \phi\hat{\pi}_t + \varepsilon_{m,t}), \tag{11}$$

where $\hat{y}_t$ is output in deviation from steady state, i_t is the nominal interest rate in levels with steady-state value $\bar{i} > 0$ and a ZLB constraint $i_t \geq 0$, $\hat{\pi}_t$ is inflation in deviation from steady state, $\hat{\mathrm{qe}}_t$ denotes the real market value of the central bank's long-term bond portfolio in deviations from steady-state, and $\varepsilon_{y,t}$ and $\varepsilon_{m,t}$ are mean-zero serially-uncorrelated IS and monetary policy shocks, respectively. We assume a positive discount factor, $\beta > 0$, positive slope of the Phillips curve, $\kappa > 0$, and a more than one-for-one systematic policy response to inflation, $\phi > 1$. In this model, the parameter z denotes the fraction of agents who do not supply labor and do not have an equity interest in firms; when $z = 0$, central bank asset purchases play no role, and the model collapses to the standard three-equation NK model.

[8] The choice of weighting scheme, including in terms of which variables to target, potentially matters because, in principle, shocks to the hypothetical policy rate in a linear system that replicate the dynamics of one variable need not be the same as those that replicate the dynamics of another. However, in our application to the Smets-Wouters model, we choose to match all of the observed variables other than interest rates and find that we can closely match all of the targeted variables simultaneously. Thus, the exact weighting scheme appears not to be so important in practice.

2.3.1 The Structural Shadow Rate for the Standard Three-Equation NK Model

To start, we consider analytical solutions for the shadow rate shock and the structural shadow rate without QE, i.e. the case for the model given in (9) through (11) when $z = 0$. In this case, conditional on $i_t > 0$, mean-zero shocks imply $\mathbb{E}_t \hat{\pi}_{t+1} = \mathbb{E}_t \hat{y}_{t+1} = 0$ and the solution is

$$\hat{y}_t = \frac{1}{1+\kappa\phi}(\varepsilon_{y,t} - \varepsilon_{m,t}) \tag{12}$$

$$\hat{\pi}_t = \frac{\kappa}{1+\kappa\phi}(\varepsilon_{y,t} - \varepsilon_{m,t}) \tag{13}$$

$$i_t = \bar{i} + \phi\frac{\kappa}{1+\kappa\phi}\varepsilon_{y,t} + \frac{1}{1+\kappa\phi}\varepsilon_{m,t}. \tag{14}$$

The Lower Bound. Suppose $\varepsilon_{m,t} = 0$ and the IS shock $\varepsilon_{y,t}$ is such that the ZLB constraint binds, which from (11) occurs whenever $\varepsilon_{y,t} \leq -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$. In this scenario, $i_t = 0$ and the solution is

$$\hat{y}_t^{\text{lb}} = \bar{i} + \varepsilon_{y,t} \tag{15}$$

$$\hat{\pi}_t^{\text{lb}} = \kappa(\bar{i} + \varepsilon_{y,t}). \tag{16}$$

A binding ZLB introduces a kink in the slope of the policy functions for output and inflation, as a comparison of (12) with (15) and (13) with (16) shows.

Our procedure for constructing the shadow rate is to find the hypothetical policy shock $\varepsilon_{m,t}^*$ in the no-ZLB solution (12) to (14) that replicates the outcomes $\hat{y}_t^{\text{lb}}$ and $\hat{\pi}_t^{\text{lb}}$. We can do so by choosing $\varepsilon_{m,t}^*$ to minimize $\Delta_t' \mathbf{W} \Delta_t$ from (8), which in this case is

$$w_y\left(\hat{y}_t^* - \hat{y}_t^{\text{lb}}\right)^2 + w_\pi\left(\hat{\pi}_t^* - \hat{\pi}_t^{\text{lb}}\right)^2,$$

where w_y and w_π are weights. Because $\hat{\pi}_t = \kappa\hat{y}_t$ and $\hat{\pi}_t^{\text{lb}} = \kappa\hat{y}_t^{\text{lb}}$, minimizing $\Delta_t' \mathbf{W} \Delta_t$ is equivalent to minimizing either $\left(\hat{y}_t^* - \hat{y}_t^{\text{lb}}\right)^2$ or $\left(\hat{\pi}_t^* - \hat{\pi}_t^{\text{lb}}\right)^2$. This can be done exactly with $\Delta_t = 0$ by setting $\hat{y}_t^* = \hat{y}_t^{\text{lb}}$, or, equivalently,

$$\frac{1}{1+\kappa\phi}(\varepsilon_{y,t} - \varepsilon_{m,t}^*) = \bar{i} + \varepsilon_{y,t},$$

which implies

$$\varepsilon_{m,t}^* = -\bar{i}(1+\kappa\phi) - \kappa\phi\varepsilon_{y,t} \geq 0. \tag{17}$$

The equality in (17) shows the contractionary shadow rate shock that is required to generate the same equilibrium outcomes in the shadow economy as those obtained under the binding ZLB. This shadow rate shock and (14) imply the structural shadow rate is $i_t^* = 0$, which is higher than the negative level for the interest rate implied by the systematic monetary policy response to economic conditions if the inequality $\varepsilon_{y,t} < -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$ is strict, i.e. $i_{t,\text{systematic}} = \bar{i}(1+\kappa\phi) + \kappa\phi\varepsilon_{y,t} < 0$, where $i_{t,\text{systematic}} \equiv \bar{i} + \phi\hat{\pi}_t$. Meanwhile, as illustrated next, more persistence in how long the constraint is expected to bind can actually generate a positive shadow rate at the ZLB.

Expected Negative IS Shock at the Lower Bound. Assume now that $\mathbb{E}_t\varepsilon_{y,t+1} = -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$, so that agents also expect a negative IS shock tomorrow will continue to cause the ZLB to bind beyond the negative IS shock $\varepsilon_{y,t} = -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$ today. $\mathbb{E}_t\hat{y}_{t+1}$ and $\mathbb{E}_t\hat{\pi}_{t+1}$ are determined from (15) and (16) iterated one period forward and, given $i_t = 0$,

$$\hat{y}_t^{\text{lb2}} = -\bar{i}\frac{(2+\kappa)}{\kappa\phi}$$
$$\hat{\pi}_t^{\text{lb2}} = -\bar{i}\frac{(2+\kappa+\beta)}{\phi}.$$

Unlike the previous scenario, $\hat{\pi}_t^{\text{lb2}} \neq \kappa\hat{y}_t^{\text{lb2}}$, so there is not necessarily a shadow rate shock that would lead to an exact match when targeting both output and inflation. Numerically, we could minimize

$$w_y\left(\hat{y}_t^* - \hat{y}_t^{\text{lb2}}\right)^2 + w_\pi\left(\hat{\pi}_t^* - \hat{\pi}_t^{\text{lb2}}\right)^2. \tag{18}$$

However, for analytical tractability, suppose we choose $\varepsilon_{m,t}^*$ to match inflation only (i.e. $w_y = 0$).[9] This match can be done exactly by setting $\hat{\pi}_t^* = \hat{\pi}_t^{\text{lb2}}$. In this case, the shadow rate shock is positive,

$$\varepsilon_{m,t}^* = \frac{\bar{i}}{\kappa\phi}(1+\kappa\phi)(1+\kappa+\beta) > 0,$$

and, from (14), the structural shadow rate is also positive,

$$i_t^* = \frac{\bar{i}}{\kappa\phi}(1+\kappa+\beta) > 0.$$

This scenario illustrates how persistent negative shocks generate contractionary expectations today that can push the shadow rate into positive territory, even though the actual

[9] If, instead, we were to match output only, we would get similar, but not identical, analytical expressions for the shadow rate shock and shadow rate, with the same implications in terms of their signs. However, the shadow rate would not be exactly equal to the sum of the implied systematic policy response and the shadow rate shock when matching output only. So, for consistency with the assumed policy rule, we choose to match inflation only.

interest rate is at zero, while the level implied by the systematic monetary policy response is again negative, i.e. $i_{t,\text{systematic}} = -\bar{i}(1+\kappa+\beta) < 0$. Thus, unlike the term structure approach in which the actual short rate is equal to $i_t = \max(i_t^*, 0)$, the shadow rate in our approach can be above zero when $i_t = 0$ because the ZLB constraint being expected to bind in the future acts equivalently to a contractionary monetary policy shock in the shadow economy given lower expected future inflation raising the current real interest rate.[10]

Forward Guidance. Suppose, instead, that the ZLB constraint binds in period t because of a negative IS shock $\varepsilon_{y,t} = -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$ today, but, in addition to setting $i_t = 0$, the central bank is able to credibly announce that it will continue to hold the interest rate at zero tomorrow such that $\mathbb{E}_t i_{t+1} = 0$. Then, $\mathbb{E}_t \hat{y}_{t+1} = \bar{i}$ and $\mathbb{E}_t \hat{\pi}_{t+1} = \kappa\bar{i}$. In this scenario,

$$\begin{aligned}
\hat{y}_t^{\text{fg}} &= \bar{i}\,(2+\kappa) + \varepsilon_{y,t} \\
\hat{\pi}_t^{\text{fg}} &= \kappa\bar{i}\,(2+\kappa+\beta) + \kappa\varepsilon_{y,t}.
\end{aligned}$$

The ‘lower-for-longer’ announcement boosts output and inflation today relative to the lower-bound solution (15) and (16). In constructing the structural shadow rate, similar to the case of an expected negative IS shock, it is analytically convenient to match inflation only, which again can be done exactly with a perfect match $\Delta_t = 0$ by setting $\hat{\pi}_t^* = \hat{\pi}_t^{\text{fg}}$. Rearranging to solve for the shadow rate shock gives

$$\varepsilon_{m,t}^* = -\bar{i}(1+\kappa\phi)(1+\kappa+\beta) < 0.$$

Thus, forward guidance maps into an expansionary shadow rate shock and, from (14), the shadow rate would clearly be less than the positive level implied by the systematic monetary policy response to economic conditions, i.e. $i_{t,\text{systematic}} = \kappa\phi\bar{i}(1+\kappa+\beta) > 0$, as it is strictly negative,

$$i_t^* = -\bar{i}(1+\kappa+\beta) < 0.$$

[10]We note that, if the current policy rate were positive, i.e. $i_t > 0$, but the ZLB was expected to bind in the future, our approach would imply an even more contractionary stance of monetary policy than for a zero rate. This is because the time-varying parameters in the general reduced-form solution (5) would imply an even more positive shadow rate for the corresponding shadow economy (7) than when the current policy rate is zero. However, this scenario is a theoretical curiosity only given that, as noted previously, we find in our empirical applications that the policy rate is always at zero whenever the ZLB is expected to bind in the future. Thus, in practice, when the observed policy rate is positive, we do not find a more positive shadow rate than the observed rate.

In this way, the structural shadow rate is able to reflect the more expansionary policy stance when there is forward guidance.[11]

2.3.2 The Structural Shadow Rate Allowing for QE

Up to this point, we have considered only one instrument of monetary policy, the policy interest rate, with this instrument subject to the ZLB constraint.

We now consider what happens when there is also QE, i.e. the case for the model given in equations (9) through (11) when $z > 0$. For simplicity, we assume qe_t purchases are exogenous, with $\mathrm{qe}_t = \varepsilon_{qe,t}$. As before, conditional on $i_t > 0$, mean-zero shocks imply $\mathbb{E}_t \hat{\pi}_{t+1} = \mathbb{E}_t \hat{y}_{t+1} = 0$ and, defining $\tilde{z} \equiv 1 - z$ for notational convenience, the solution is

$$
\begin{aligned}
\hat{y}_t &= \frac{1}{1+\kappa\tilde{z}\phi}(\varepsilon_{y,t} - \tilde{z}\varepsilon_{m,t}) + z\frac{(1-\phi\kappa)}{1+\kappa\tilde{z}\phi}\varepsilon_{qe,t} \\
\hat{\pi}_t &= \frac{\kappa}{1+\kappa\tilde{z}\phi}(\varepsilon_{y,t} - \tilde{z}\varepsilon_{m,t}) + \frac{\kappa z}{1+\kappa\tilde{z}\phi}\frac{(1+\tilde{z})}{\tilde{z}}\varepsilon_{qe,t} \\
i_t &= \bar{i} + \phi\frac{\kappa}{1+\kappa\tilde{z}\phi}\varepsilon_{y,t} + \phi\frac{\kappa z}{1+\kappa\tilde{z}\phi}\frac{(1+\tilde{z})}{\tilde{z}}\varepsilon_{qe,t} + \frac{1}{1+\kappa\tilde{z}\phi}\varepsilon_{m,t}.
\end{aligned}
$$

When $\phi < 1/\kappa$, all coefficients are positive, so a QE shock raises both inflation and output, as well as resulting in a positive endogenous response of the policy interest rate.

Again supposing $\varepsilon_{m,t} = 0$, the ZLB binds whenever the IS shock is negative enough such that

$$
\varepsilon_{y,t} \leq -\frac{\bar{i}(1+\kappa\tilde{z}\phi)}{\phi\kappa} - z\frac{(1+\tilde{z})}{\tilde{z}}\varepsilon_{qe,t}.
$$

That is, for a sufficiently negative IS shock $\varepsilon_{y,t}$ relative to any offsetting effects of the QE shock $\varepsilon_{qe,t}$, the ZLB binds and the solution is

$$
\begin{aligned}
\hat{y}_t^{\mathrm{lb}} &= \tilde{z}\bar{i} + \varepsilon_{y,t} + z\varepsilon_{qe,t} \\
\hat{\pi}_t^{\mathrm{lb}} &= \kappa(\tilde{z}\bar{i} + \varepsilon_{y,t}) + \kappa z\frac{(1+\tilde{z})}{\tilde{z}}\varepsilon_{qe,t}.
\end{aligned}
$$

As before, we can determine a shadow rate shock $\varepsilon^*_{m,t}$ that matches outcomes in the shadow economy to outcomes under the binding ZLB. In this case, suppose we match

[11] This contrasts with Hills and Nakata (2018), who define a shadow rate in an NK model as corresponding to the policy rate that would be set according to the policy rule in the absence of the ZLB constraint or monetary policy shocks, similar also to the measures reported in Kulish et al. (2017) based on the unconstrained systematic policy response. Specifically, this shadow rate is linked to the observed rate via $i_t = \max(i_t^*, 0)$, with a negative shadow rate $i_t^* < 0$ simply reflecting how much and for how long the constraint binds given the policy rule and other structural shocks, rather than providing a guide to the stance of monetary policy.

output only, so that we find the shadow rate shock that equates $\hat{y}_t^*$ to $\hat{y}_t^{\text{lb}}$. The resulting shadow rate shock is

$$\varepsilon_{m,t}^* = -(1 + \kappa \tilde{z}\phi)\bar{i} - \kappa\phi\varepsilon_{y,t} - z\kappa\phi\frac{(1+\tilde{z})}{\tilde{z}}\varepsilon_{qe,t}.$$

As in the standard NK model case, the more contractionary the IS shock $\varepsilon_{y,t}$, the greater the shadow rate shock needs to be to match macroeconomic outcomes. At the same time, expansionary QE offsets the impact of a contractionary IS shock and therefore lowers the size of the shadow rate shock needed to match observed outcomes. This logic is similar to the case where the central bank makes an announcement about the future value of the interest rate. Thus, both forward guidance ('lower for longer') and QE lower the shadow rate as they both can offset the impact of a contractionary IS shock.

In our empirical applications, we consider two structural shadow rates: one for the Smets-Wouters model in which the only explicit form of unconventional monetary policy is forward guidance and one for the model of Boehl et al. (2022) that also has quantitative easing. Thus, one may wonder how inferences about the shadow rate would differ in the two cases. To consider this, we can solve for shadow rate shocks in a shadow economy with no QE shocks so as to match the observed outcomes in the economy that reflect all shocks, including QE shocks, as well as the ZLB constraint. For example, suppose the data at the ZLB were driven in part by a QE shock, so that we observe

$$\hat{y}_t^{\text{lb}} = \tilde{z}\bar{i} + \varepsilon_{y,t} + z\varepsilon_{qe,t}.$$

Suppose then that we set the QE shock to zero in the shadow economy. Matching $\hat{y}_t^{\text{lb}}$ to the expression for output in the shadow economy–i.e. output given by (12)–we find the following shadow rate shock:

$$\varepsilon_{m,t}^* = -(1+\kappa\phi)\tilde{z}\bar{i} - \kappa\phi\varepsilon_{y,t} - z(1+\kappa\phi)\varepsilon_{qe,t},$$

which from (14) implies the associated shadow interest rate

$$i_t^* = z\bar{i} - z\varepsilon_{qe,t}.$$

Thus, setting the QE shock to zero in the shadow economy corresponds to a more negative shadow rate shock and a lower implied shadow interest rate to account for the data. This has two important implications. First, the shadow rate in the case of setting QE shocks to zero reflects the impact of all unconventional policies, including QE.[12] Second, the difference between this shadow rate for all unconventional policies and a shadow rate when QE shocks are not set to zero in the shadow economy can be used to quantify the interest-rate-equivalent effects of QE shocks.

3 Estimating the Structural Shadow Federal Funds Rate

3.1 Smets-Wouters Estimated Medium-Scale NK Model

To estimate the structural shadow federal funds rate, we consider the workhorse estimated structural model of the US economy originally developed by Smets and Wouters (2007), but allowing for the ZLB, as in Kulish et al. (2017). We are motivated to use this model as a baseline not just because of its widespread use in the literature, but primarily because it is thought to provide reasonable estimates of the empirical effects of conventional monetary policy and the responses of monetary policy to economic conditions. Also, this version of the model allowing for the ZLB following Kulish et al. (2017) includes a mechanism for unconventional policy to operate in the form of forward guidance and provides estimates of its empirical effects. Thus, the quantification of interest-rate-equivalent effects of unconventional policies in producing observed macroeconomic outcomes when measuring the structural shadow rate is informed by realistic empirical effects of conventional and unconventional monetary policies and estimates of systematic policy responses. Any structural model with similarly realistic estimated policy responses and effects of conventional and unconventional policies should produce a similar measure of the shadow rate in order to account for the same observed macroeconomic outcomes. Indeed, as discussed in more detail below, we find a similar shadow rate to our baseline measure from the Smets-Wouters model when we consider an extension to a model with QE and shut down all unconventional policy in the shadow economy.

The Smets-Wouters model allowing for the ZLB is estimated using US data over 1984Q1 to 2019Q4 and we make the other following changes to Smets and Wouters (2007): First, similar to Kulish et al. (2017), we expand the set of observables to include the 1-year and 5-year Treasury yields. Second, to capture the trend decline in interest rates over this period, we allow for a decline in trend growth. In particular, motivated by the evidence of structural break in trend growth in the 2000s documented in a number of studies includ-

[12] If the value of z is relatively small, and noting that Sims et al. (2023) calibrate it to $1/3$, the structural shadow rate with no QE shocks will be similar to the structural shadow rate for a model with no QE, which is exactly what we find in our empirical applications.

ing Fernald (2012), Antolin-Diaz et al. (2017), and Eo and Morley (2022), we allow for a one-time change in trend growth at a magnitude and date to be estimated. This captures the possibility that a decline in trend growth lowers the equilibrium level of the policy rate, which could cause the ZLB to be visited more frequently. Finally, we calibrate the inflation target to a 2% annualized rate to reflect the Fed's inflation objective.

We follow Smets and Wouters (2007) in all other respects, including the remaining observed variables and their construction, the set of estimated parameters, and priors. Motivated by the results in Kulish et al. (2017), we use modal reported values of durations from Blue Chip Financial Forecasts and the New York Fed Survey of Primary Dealers to measure expected durations of zero-interest-rate policy during the ZLB. Full details of the model and data are given in the appendix.

3.2 Baseline Estimates of the Structural Shadow Federal Funds Rate

The structural shadow rate for the Smets-Wouters model is constructed following the approach described in Section 2.2. We emphasize that this measure is designed to capture the stance of monetary policy in contrast to the measures reported in Kulish et al. (2017) that track the counterfactual unconstrained systematic policy responses. In constructing our shadow rate, we target all of the observed variables except the interest rates.[13]

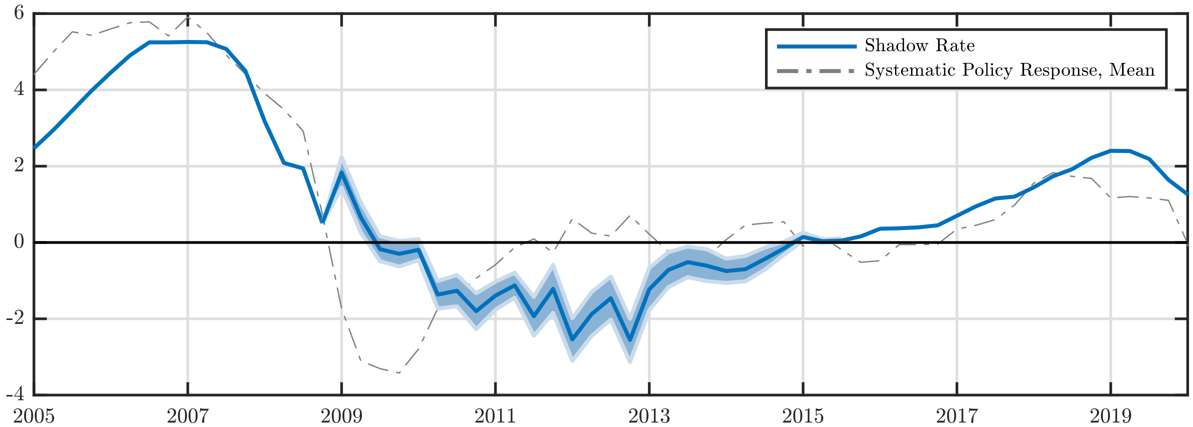
Figure 2 plots our baseline structural shadow federal funds rate and the corresponding shadow rate shocks. The posterior mean of the shadow rate reported in the top panel deviates from the observed federal funds rate when the latter hit the ZLB in 2009Q1, taking on a value of 1.6% with precise 90% posterior bands. This initial positive value for the shadow rate illustrates the contractionary effects of the ZLB constraint being expected to bind persistently given the large negative shocks that triggered the Great Recession. A contractionary stance of monetary policy due to the ZLB constraint is especially clear in comparison to the quick decline to values below -3% in the posterior mean of the systematic policy response implied by the policy rule and prevailing economic conditions.[14] However, from around the beginning of 2011, the estimated shadow rate implies rela-

[13]The appendix contains plots of the paths of targeted variables both in the data and in the shadow economy. The plots show that the variables in the shadow economy are very close to the actual data. The shadow rate shocks are thus able to capture, with great accuracy, the dynamics of all of the targeted variables. Furthermore, we find that our estimates are nearly identical if we also target the longer-term interest rates. These results confirm that there is considerable flexibility in practice with the exact weighting scheme.

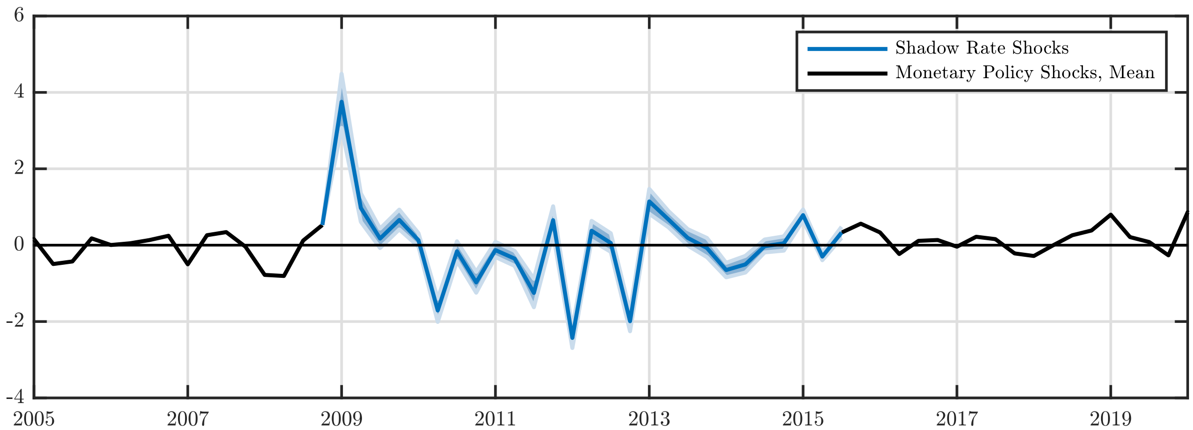
[14]The implied systematic policy response is calculated using the prevailing values of the target variables in the policy rule, but setting the monetary policy shocks and shadow rate shocks to zero and allowing for interest rate smoothing according to the estimated policy rule. The policy rule in the Smets-Wouters model assumes systematic responses to inflation, the output gap, and changes in the output gap.

Figure 2: Structural Shadow Federal Funds Rate from the Smets-Wouters Model

(a) Shadow Rate, i_t^*, and Systematic Policy Response, % Annualized



(b) Shadow Rate Shocks, $\varepsilon_{m,t}^*$



Notes: Panel (a) plots, in annualized percentage terms, the estimated shadow rate, along with the implied systematic policy response to prevailing economic conditions based on the policy rule without monetary policy or shadow rate shocks but allowing for interest rate smoothing, i.e. $i_{t,\text{systematic}} = \bar{i} + (1-\alpha_i)\alpha_p\hat{\pi}_t + (1-\alpha_i)\alpha_y\tilde{y}_t + \alpha_{\Delta y}\Delta\tilde{y}_t + \alpha_i(i_{t-1,\text{systematic}} - \bar{i})$, where the α's are the monetary policy response coefficients and $\tilde{y}_t$ is the output gap from the flexible price equilibrium for the Smets-Wouters model detailed in the appendix. Panel (b) plots, in annualized percentage point terms, the shadow rate shocks. The lines correspond to posterior means, while the bands show 90 percent equal-tailed posterior intervals for the shadow rate and shadow rate shocks during the ZLB period.

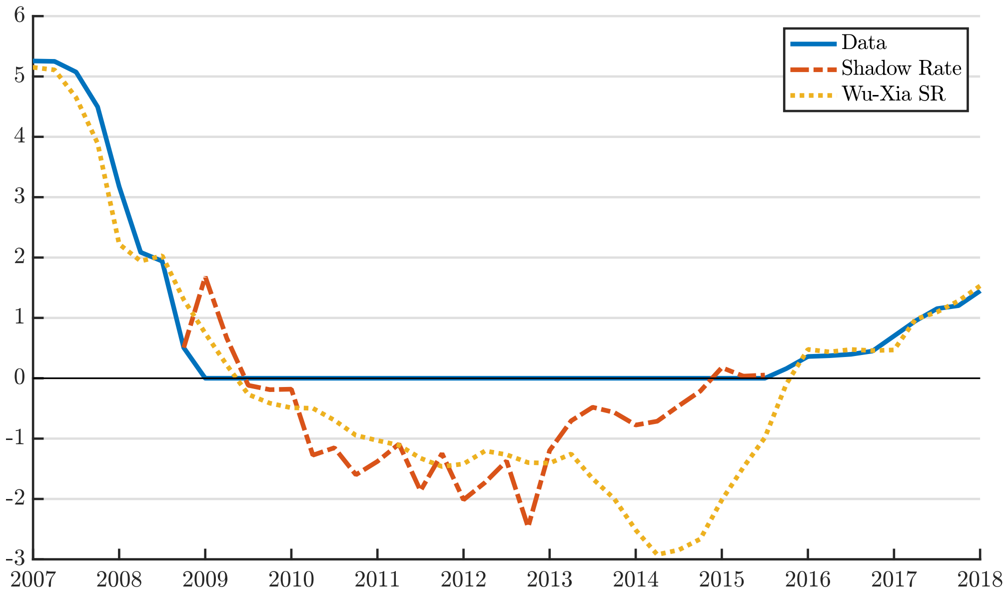
tively expansionary monetary policy compared to the systematic policy response, with a decline to about -2.4% in 2012Q4 given implementation of various unconventional policies including forward guidance that allowed the Fed to achieve the equivalent of an unconstrained negative rate in the shadow economy. After bottoming out in 2012, the estimated shadow rate increased back towards the level implied by systematic policy at around -0.5% in 2013Q2 and reaches zero in 2015Q1, just before liftoff. These shifts in the policy stance are reflected in the shadow rate shocks reported in the bottom panel. After the initial contractionary effects of the ZLB constraint being expected to bind for a number of quarters, the shadow rate shocks are typically estimated to be negative throughout the remaining ZLB period and quantify the interest-rate-equivalent effects of forward guidance and other unconventional policies.

There are important differences in the estimates of our shadow rate and other measures. In Figure 3, we compare our shadow rate to one constructed following Wu and Xia (2016). After being slightly more positive in 2009Q1, our shadow rate falls below the Wu-Xia measure in 2010 and has more pronounced fluctuations that appear closely related to forward guidance announcements, as discussed in more detail in Section 4.[15] At its trough in 2012Q4, just before the taper tantrum, our shadow rate is more than a percentage point below the Wu-Xia measure. A stark gap then opens up between our shadow rate and the Wu-Xia shadow rate over 2013 through 2015. Relative to a fairly flat implied systematic policy response to economic conditions at the time, our measure suggests that monetary policy was becoming relatively less accommodative in the lead up to liftoff from the ZLB, while the Wu-Xia measure fell by almost 2 percentage points to a low of -2.9% by 2014Q2 and seems to suggest that policy was becoming more expansionary at the time.[16] In Section 4, we compare the performance of both measures in a simple monetary VAR that controls for economic conditions in terms of inflation and output growth when identifying monetary policy shocks and find that our measure performs better due to the different signals it gives about the stance of policy during the ZLB.

[15]While both measures of the shadow rate initially start above zero in 2009, it is for completely different reasons. Our measure is above zero when the federal funds rate is at or close to zero because shocks interacting with the ZLB generate additional nonlinear contractionary effects that map into contractionary shadow rate shocks. By contrast, the Wu-Xia shadow rate is above zero when the federal funds rate is at zero because the measure of the short rate used in estimation – i.e. the 3-month forward rate – is positive.

[16]Sims and Wu (2020) show that the Wu-Xia measure is correlated with the Fed's balance sheet during this period, but acknowledge that the "shadow rate series is based on empirical term structure models that do not have an explicit mapping back into structural economic models or particular unconventional tools". In the appendix, we provide an example using simulated data from the Smets-Wouters model to show how the Wu-Xia measure provides a very different signal about the stance of monetary policy than our structural measure.

Figure 3: Comparison of Different Measures of the Shadow Federal Funds Rate



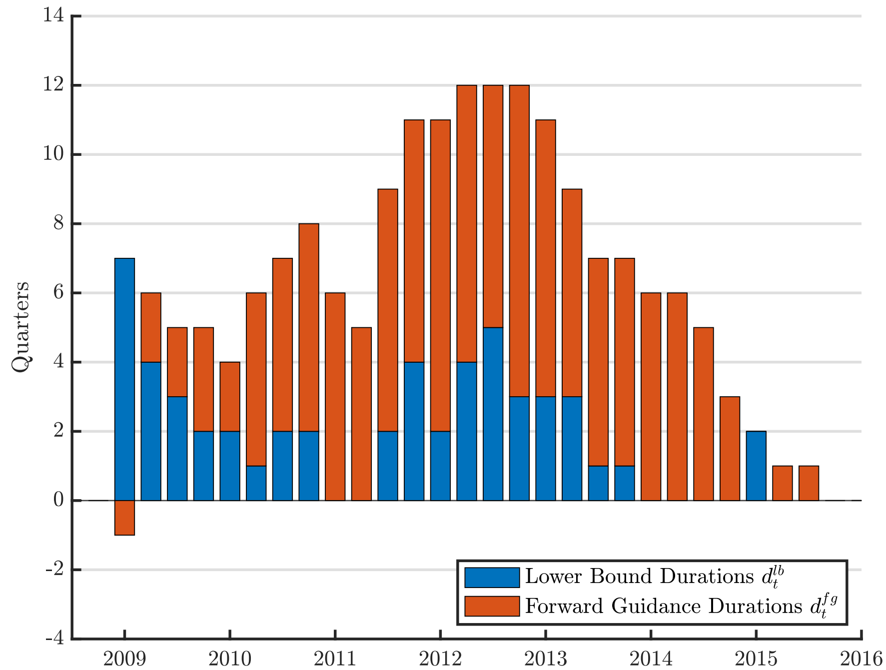
Note: Along with the data on the federal funds rate, the posterior mean of the structural shadow rate from the Smets-Wouters model and the Wu-Xia shadow rate are reported from 2007 to 2018. By construction, the structural shadow rate is the same as the data outside of the ZLB.

3.3 Durations and a No-Forward-Guidance Counterfactual

Figure 4 plots the durations of the ZLB decomposed into the lower-bound component, $\mathbf{d}_t^{\text{lb}}$, and the forward-guidance component, $\mathbf{d}_t^{\text{fg}}$. The sum of these two components gives the overall duration expected by agents in the economy, denoted by $\mathbf{d}_t$ in Section 2. The figure shows how the forward-guidance durations were initially quite short, but increased over 2011 and 2012. The forward-guidance duration was even briefly negative in 2009Q1, reflecting an estimated belief by economic agents that the Fed would deviate from the policy rule and begin raising rates one quarter before the ZLB constraint was expected to stop binding for the rule. This corresponds to the contractionary stance of policy in 2009Q1 evident in Figure 2. As unconventional policies were implemented, especially from 2011 onwards, the forward-guidance durations are longer than the lower-bound durations. From 2013 to 2015, as the federal funds rate moved closer to liftoff, the actual and forward-guidance durations fell back towards zero.

To help understand the quantitative effects of unconventional policies, we explore the counterfactual scenario of what would have happened in the aftermath of the Great Recession without forward guidance. We construct this counterfactual using the solution (5) implied by the overall expected durations $\mathbf{d}_t$ to obtain an estimate of the structural shocks ε_t and then feeding the estimated structural shocks through the occasionally-binding-

Figure 4: ZLB Durations at the Posterior Mode

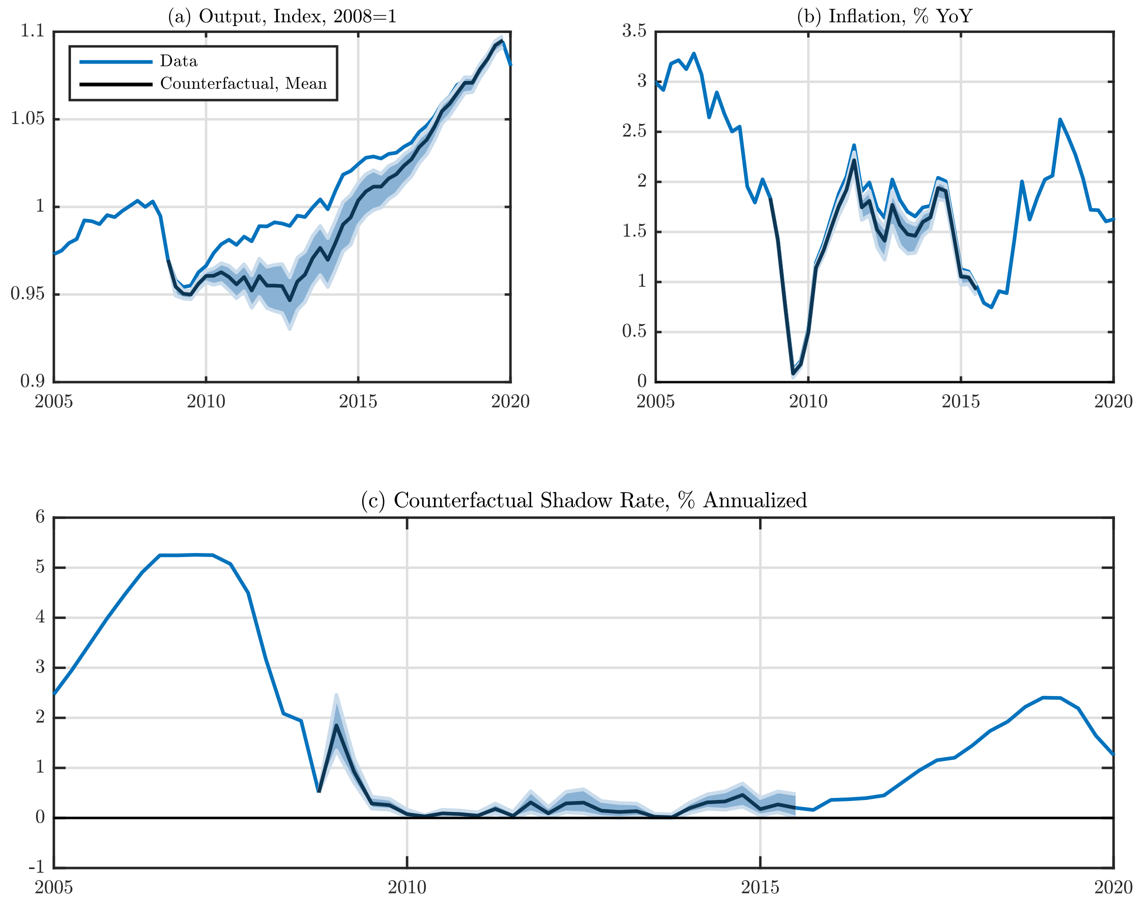


constraint solution to solve for the path of the economy if monetary policy simply followed the prescription of the policy rule constrained by the ZLB. In this case, the durations of the ZLB expected by agents are assumed to be only the lower-bound durations $\mathbf{d}_t^{\text{lb}}$.

Figure 5 plots the counterfactual paths of output, inflation, and a shadow rate removing forward guidance. These paths imply that unconventional policies extending expected durations raised the level of output by as much as 4 percent in 2012Q4, while inflation was less affected due to a strong degree of nominal rigidities according to the parameter estimates.[17] Using these variables together with the other non-interest-rate observables as targets in $\Delta_t = \iota(x_t - x_t^*)$, the counterfactual shadow interest rate is positive and not just in the early stages of the ZLB. Instead, it remains non-negative throughout the ZLB and rises somewhat above zero around 2012 when output would have been most depressed in the absence of unconventional policies. Again, the structural shadow rate captures the stance of monetary policy in this counterfactual scenario and its difference from our estimated shadow rate in Figure 2 provides an interest-rate-equivalent quantification of how unconventional policy altered the stance of monetary policy.

[17]The larger effects on real activity than inflation are in line with counterfactuals for the unemployment rate and inflation in Eberly et al. (2020) based on a structural VAR with external instruments. We note too that, as shown in the appendix, our shadow rate estimates are robust to an alternative calibration of Calvo parameters motivated by findings in Fitzgerald et al. (2020).

Figure 5: Counterfactual Removing Forward Guidance



Notes: For the data in solid blue and a counterfactual removing the effects of forward guidance in black, panel (a) plots an index of output normalized to 1 in the base year of 2008 and panel (b) plots, in annualized percentage terms, year-on-year inflation. Panel (c) plots, in annualized percentage terms, the shadow rate computed for this counterfactual path. The black lines for the counterfactuals correspond to posterior means, while the bands show 90 percent equal-tailed posterior intervals.

3.4 Accounting for QE

Our baseline estimated shadow federal funds rate is based on a structural model that abstracts from quantitative easing. In practice, monetary policy at the zero lower bound consists of two key tools: forward guidance about the path of the federal funds rate and QE via asset purchases or liquidity injections. To also consider the latter, we construct a shadow rate for a structural model that explicitly accounts for QE programs enacted by the Federal Reserve following the financial crisis. We quantify the extent to which quantitative easing stimulates the economy and substitutes for forward guidance. In doing so, we provide a mapping between the effects of QE policies and the structural shadow rate.

We use the model of Boehl et al. (2022), which integrates a banking sector as in Gertler and Karadi (2011) into a model that is otherwise similar to the Smets-Wouters model. In the model with QE, the full details of which are provided in the appendix of Boehl et al. (2022), the central bank can purchase treasury bonds or private capital assets, and can inject liquidity into banks. These features allow the model to account for various asset purchase programs enacted by the Federal Reserve in the aftermath of the financial crisis.

We set the parameters of the model with QE to the mean estimates reported by Boehl et al. (2022), and, as in our baseline results, maintain the durations at the modal reported values of durations from the Blue Chip Financial Forecasts and the New York Fed Survey of Primary Dealers. The set of observables used are also the same as those in our baseline estimation, though to be consistent with Boehl et al. (2022) we do not use longer-term interest rates but instead use four additional series that capture QE policies.[18] First, we use the Gilchrist and Zakrajsek (2012) measure of credit spreads. Second, to measure the Fed's government bond purchases, we use 10-year equivalents of the Fed's US Treasury holdings divided by nominal GDP. Third, to measure purchases of private capital securities, we use the sum of the current face value of mortgage-backed securities and federal agency debt securities held by the Fed, and express both as a fraction of nominal GDP. Finally, to measure the Fed's emergency liquidity injections, we combine central bank liquidity swaps, the net portfolio holdings of the Commercial Paper Funding Facility, the term auction credit and other loans held by the Fed, and express this combination as a fraction of nominal GDP.

We follow the same procedure to construct the shadow federal funds rate as in our baseline. Using the model's piecewise-linear solution, we first filter the observables for the model's structural shocks.[19] As before, we search for the shadow interest rate shocks that, in the linear shadow economy, approximate the paths of all observables except for the federal funds rate.[20] In constructing this shadow interest rate, the set of structural shocks that we use includes the QE policy shocks. The shadow interest rate from the QE model of Boehl et al. (2022), labelled 'Shadow Rate' is in the top panel of Figure 6.

To disentangle movements in the shadow rate due to QE from those related to forward guidance, we also calculate the shadow interest rate that would arise without the QE shocks. We do so by matching the non-QE observables as in our baseline. Thus, this shadow rate is what would be required to replicate the macroeconomic outcomes for

[18]In Figure E.4 of the appendix, we show that the implied macroeconomic effects of QE in the model are very similar to those reported by Boehl et al. (2022).

[19]Consistent with the piecewise linear solution we use, the monetary policy rule is not in operation during the ZLB. As a result, we also shut down the persistence of the monetary policy shock that is allowed for in Boehl et al. (2022).

[20]In the appendix, we plot the paths of observables together with the paths of the model variables that arise in the shadow economy. As in the baseline case, the plots show that the variables in the shadow economy are very close to the actual data.

output, consumption, investment, and other macroeconomic variables, if the QE shocks were set to zero. This shadow rate, labelled 'Shadow Rate, No QE Shocks', is also plotted in the top panel of Figure 6. Consistent with our analytical example, the shadow rate with no QE is lower than the one with QE. This is because in the absence of QE, monetary policy would require additional stimulus from forward guidance to achieve the same macroeconomic outcomes. The difference between the two shadow rates is shown in the middle panel of Figure 6 and provides a measure of the interest-rate-equivalent impact of QE. That is, this difference quantifies how much additional policy stimulus via a lower shadow rate would have been needed to generate the observed economic outcomes in the absence of QE. The difference widens to around 80 annual basis points in 2010 following the implementation of QE1. This result is consistent with those reported in Table 1 of Krishnamurthy and Vissing-Jorgensen (2011) that suggest QE1 lowered the 10-year and 5-year yields by 107 and 74 basis points respectively.

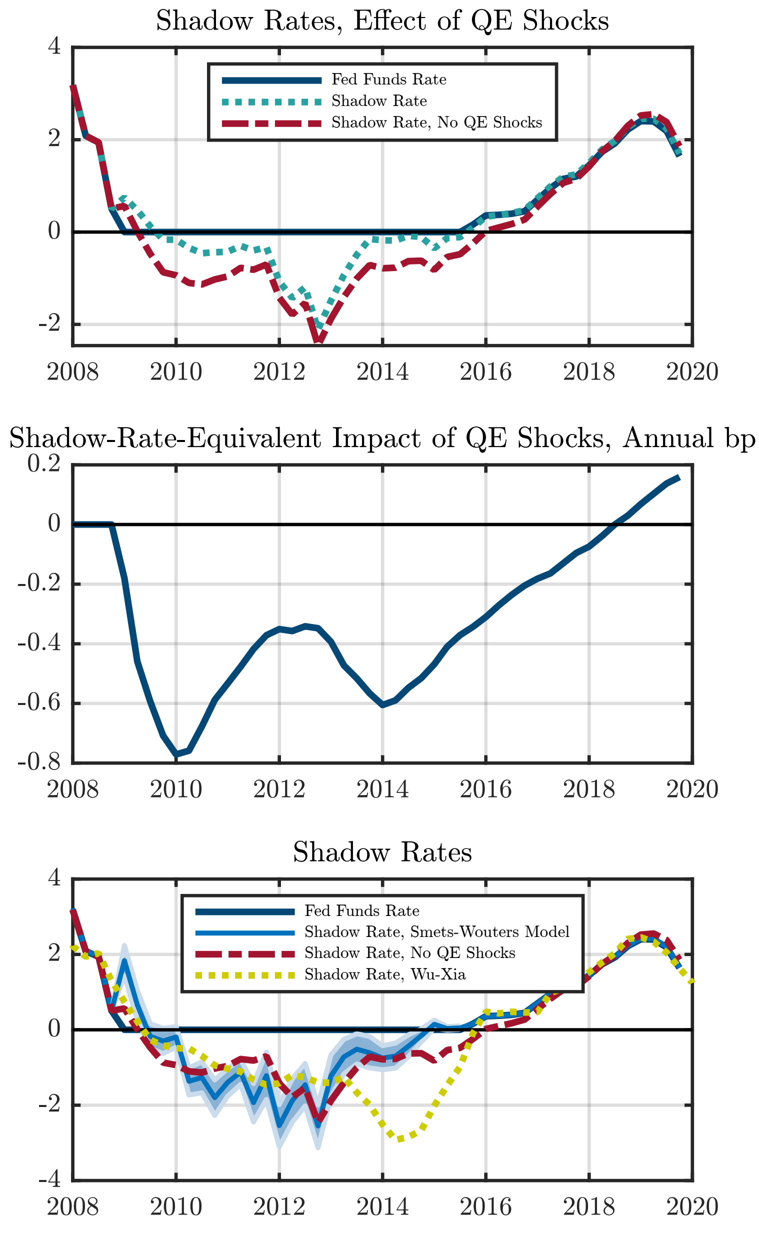
The bottom panel of Figure 6 compares the shadow rate from our baseline model to that from the model with QE but with no QE shocks and to the Wu-Xia measure. We can see that the two structural shadow rates are reasonably similar, reflecting the similar needed stimulatory policies in the respective shadow economies to match the same observed macroeconomic outcomes. The shadow rate from the model with QE but with no QE shocks is somewhat smoother than our baseline case, likely reflecting the different specification and estimates for interest rate smoothing in the Boehl et al. (2022) model than in the Smets-Wouters model.

The key point, however, is that the shadow rate from our baseline model manages to quantify the interest-rate-equivalent effects of all unconventional policies even though the model assumes forward guidance is the only channel for unconventional policy. This is because forward guidance can substitute for quantitative easing in matching observed outcomes. Meanwhile, the two structural shadow rates imply clear differences about the stance of monetary policy than the Wu and Xia (2016) rate, with these differences being relevant for the comparative performance of our baseline structural shadow rate and the Wu and Xia (2016) rate in a simple monetary VAR considered in the next section.

4 Applications

We explore three applications using our baseline structural shadow federal funds rate. The first two reflect the typical use of a policy rate in empirical analysis, while the third considers the implied stance of monetary policy during the recent pandemic.

Figure 6: Quantitative Easing and the Structural Shadow Federal Funds Rate



4.1 Shock Decomposition

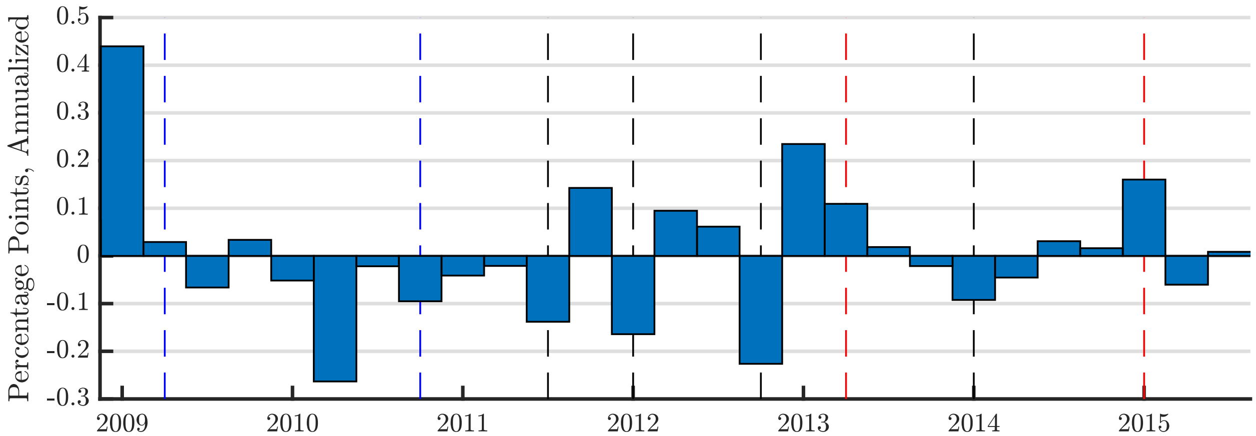
When the ZLB binds, the decomposition of observables into structural shocks is complicated by the nonlinearities introduced into the structural model. However, the structural shadow rate and its shocks can be used in the linear shadow economy to conduct such an historical decomposition more easily. Specifically, we obtain smoothed estimates of historical structural shocks, including the shadow rate shock, for the linear shadow economy. We then feed each shock one-by-one into the model to calculate the effect each shock has on the observed variables.

Figure 7 plots the path of the change in the annualized 5-year yield under the shadow rate shock alone, noting some key events related to unconventional policies. After initial contractionary effects from the ZLB constraint at the beginning of 2009, the shadow rate shocks largely act to reduce the long rate during the ZLB. Of particular note, the black dashed vertical lines correspond to quarters in which calendar-based forward guidance announcements were made in FOMC statements. The contribution of the shadow rate shocks to lowering the long rate closely aligns with the quarters of these announcements. For example, calendar-based forward guidance was initiated in 2011Q3 when the FOMC announced that the federal funds rate would be held at zero until "at least through mid-2013". In 2012Q1, this timeframe was extended to "at least through late-2014". In 2012Q4, the FOMC introduced threshold-based forward guidance, announcing that the federal funds rate would not be raised until certain values for unemployment and inflation were achieved. These three quarters – 2011Q3, 2012Q1, and 2012Q4 – saw three of the four largest contributions of the shadow rate shock to lowering the 5-year yield. Meanwhile, the red dashed vertical lines correspond to notable events when shadow rate shocks increased the 5-year yield. 2013Q2 covers the taper tantrum, when markets interpreted remarks by the Fed as a signal that it would slow asset purchases, while 2015Q1 covers the removal of references by the FOMC in its statement to maintaining the federal funds rate at the lower bound for a "considerable time" following the end of its asset purchase program. We note that some key movements in the 5-year yield attributed to the shadow rate shocks line up with announcements about large scale asset purchases, such as QE2 in 2011Q3, thus confirming that our shadow rate can reflect unconventional monetary policy more broadly than just forward guidance.

4.2 VAR Analysis

Another typical use of a shadow rate is in VAR analysis when the sample covers the ZLB. For this application, we consider our baseline structural shadow federal funds rate in

Figure 7: Contributions of Shadow Rate Shocks to Changes in the 5-Year Yield



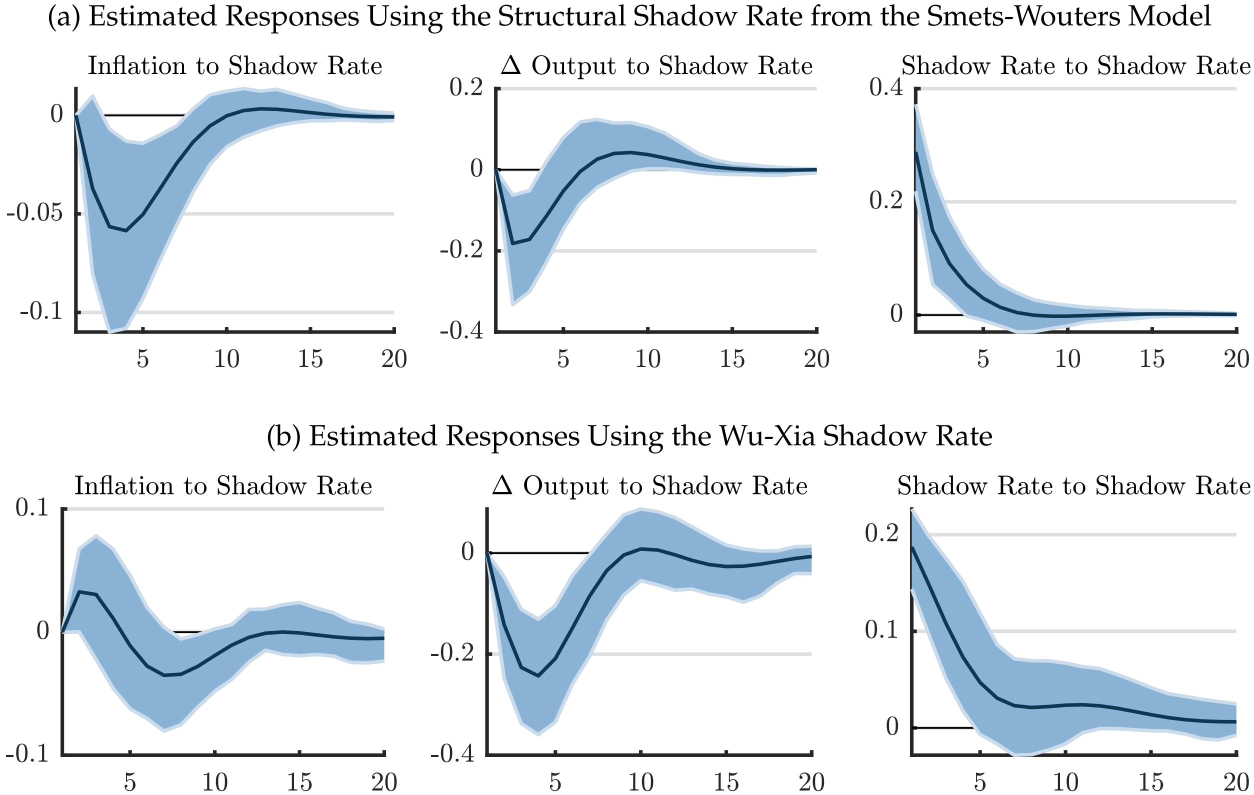
Notes: The bars are annualized percentage point contributions of shadow rate shocks to changes in the 5-year yield based on smoothed estimates. Dashed vertical lines represent the following dates: 2009Q2: QE1; 2010Q4: QE2; 2011Q3: calendar-based forward guidance "at least through mid-2013"; 2012Q1: calendar-based forward guidance "at least through late 2014"; 2012Q3: calendar-based forward guidance "at least through mid-2015"; 2012Q4: threshold-based forward guidance; 2013Q2: taper tantrum; 2014Q1: removal of threshold-based forward guidance; 2015Q1: removal of references to calendar-based forward guidance and to the maintenance of interest rates at the lower bound for a "considerable time" following the end of the asset purchasing program.

a simple three-variable monetary VAR that also includes quarterly inflation and output growth. Given that the reduced-form VAR might not be the same outside of the ZLB and for purposes of comparison across different shadow rate measures that are most different from each other during the ZLB, the VAR is estimated only over 2009Q1 to 2015Q3 and we include one lag based on diagnostics suggesting that the forecast errors are serially uncorrelated. For the comparison, we consider a VAR with the Wu-Xia shadow rate instead of our structural shadow rate.[21]

We employ a standard identification of monetary policy shocks by ordering the shadow rate last and using a Cholesky factorization of the forecast-error variance-covariance matrix to calculate impulse responses to a one-standard-deviation monetary policy shock under the assumption that the shadow rate can respond to contemporaneous information about inflation and output growth, but only affects these more sluggish variables with a lag. While acknowledging the potential limitations of this identification, we note that any limitations should apply equally regardless of which measure of the shadow rate we use.

[21]If we were to consider a VAR that also included variables related to QE such as those considered in the Boehl et al. (2022) structural model, we would want to include a shadow rate that only reflects interest-rate-equivalent effects of forward guidance because QE is already accounted for in the VAR. Specifically, we would want to consider the shadow rate when also allowing for QE shocks in the top panel of Figure 6. However, given our simple monetary VAR does not include QE variables and also for comparison to the Wu-Xia shadow rate that is intended to reflect all unconventional policies, we use our baseline shadow rate that was shown to be quite similar to the shadow rate from the QE model but with no QE shocks in the bottom panel of Figure 6.

Figure 8: Impulse Responses for a Three-Variable Monetary VAR



Notes: This figure displays responses of inflation and output growth in quarterly percentage terms and the shadow rate in annualized percentage terms to a one-standard deviation monetary policy shock in a three-variable VAR with quarterly inflation, output growth, and the shadow rate estimated over the ZLB period, 2009Q1 to 2015Q3. The shadow rate is ordered last for identification of monetary policy shocks using a Cholesky factorization of the forecast-error variance-covariance matrix. Panel (a) reports results for the baseline structural shadow rate from the Smets-Wouters model and panel (b) reports results for the shadow rate measure constructed following Wu and Xia (2016). The impulse responses are computed using a bootstrap procedure drawing residuals with replacement. The black lines correspond to mean responses and the bands show 90 percent equal-tailed bootstrap confidence intervals.

Figure 8 plots the impulse response functions for a monetary policy shock in the two cases of using our shadow rate or the Wu-Xia shadow rate. Given a contractionary shock, we find significant declines in both inflation and output growth within a one-year horizon when using our shadow rate to identify policy shocks. By contrast, there is a 'price puzzle' in the case of the Wu-Xia shadow rate in the form of initial positive (albeit insignificant) responses of inflation to a contractionary shock, although they do turn negative at longer horizons.

One interpretation of the price puzzle is that a VAR with Cholesky factorization does not cleanly identify monetary policy shocks, but mixes them with endogenous responses of monetary policy to other shocks with inflationary effects. The Wu-Xia shadow rate tends to decrease whenever there is a decline in long-term interest rates, regardless of the reason for the decline. If those declines reflect a deterioration in inflation expectations rather than more expansionary unconventional policies, then the Wu-Xia shadow rate will overstate how accommodative the policy stance has actually become. The identified policy shock in the VAR system will be negative when inflation expectations fall, thus leading to a positive correlation between the identified policy shock and inflation, i.e. the price puzzle. By contrast, our shadow rate should provide a more accurate reading of the policy stance relative to economic conditions and, therefore, can better avoid mixing policy shocks with endogenous responses to other shocks with inflationary effects. Our approach identifies whether a decline in long-term interest rates is due to a deterioration in inflation expectations increasing the expected duration of the ZLB because of the constraint or unconventional policy increasing the expected duration, with a corresponding increase or decrease in the shadow rate, respectively. Unlike with the Wu-Xia shadow rate, the identified policy shock in the VAR using our shadow rate would then be positive when inflation expectations fall, thus leading to a negative correlation between identified policy shock and inflation, i.e. avoiding the price puzzle.[22]

These VAR results support the idea that the structural shadow rate can better reflect the stance of policy than term structure measures, suggesting it can be easily employed in empirical analysis when the sample period covers the ZLB.[23] While an estimated structural macroeconomic model could be directly used to consider the empirical effects of monetary policy, as in Kulish et al. (2017), it may be useful to have a simple summary of the policy stance during the ZLB when conducting other empirical analysis. By providing a more accurate measure of monetary policy, our shadow rate can serve as a better control or indicator in such analysis. Meanwhile, we also note that using our structural shadow rate to estimate a linear version of the Smets-Wouters model leads to robust structural

[22] As highlighted by Krippner (2020) and Mavroeidis (2021), shadow rate estimates are generated regressors, implying that results from a VAR can be sensitive to alternative specifications of the underlying term structure model or structural macroeconomic model. Assessing the sensitivity of VAR results to model specification – as Krippner (2020) has done for term structure measures – is an exercise we leave for future research. However, it is notable that the price puzzle, which could reflect a generated regressor problem in the case of the widely-used Wu-Xia shadow rate, does not occur for our structural shadow rate. In the appendix, when considering simulated data from the Smets-Wouters model, we also find that the Wu-Xia shadow rate suffers from price and output puzzles with estimated impulse responses based on a VAR, while the structural shadow rate does not.

[23] Given the solution to the structural model during fixed-interest-rate regime is a time-varying VAR, one may wonder if it is appropriate to use the structural shadow rate in a constant-parameter VAR. However, as our measure is constructed from a linear VAR in a such a way so that the linear (shadow) system gets as close as possible to the nonlinear (actual) system, the structural shadow rate should be able to capture the nonlinearities reasonably well. Indeed, Carriero et al. (2021) suggest that, if monetary policy is effectively unconstrained by ZLB given unconventional policies, as argued, for example, by Swanson and Williams (2014), then a constant-parameter VAR including a shadow rate could even apply over a longer sample that includes observations both at the ZLB and away from it, although Aruoba et al. (2021a) and Ikeda et al. (2021) find that the ZLB is an empirically-relevant constraint in VAR analysis.

parameter and shock estimates compared to estimation based on the nonlinear version accounting for the ZLB, as in Kulish et al. (2017).[24] Thus, using the structural shadow rate has computational benefits even if one is considering a structural model for analysis rather than a VAR.

4.3 Monetary Policy during the Pandemic

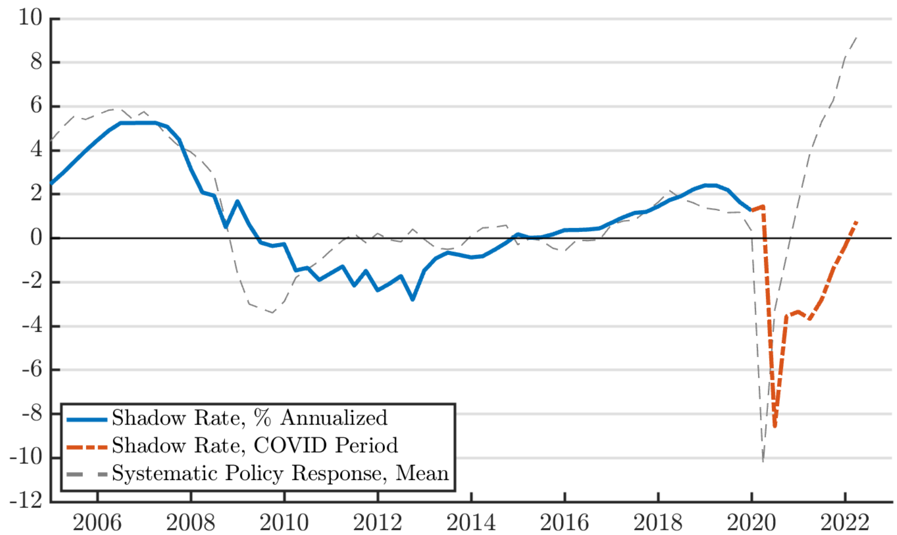
Our last application of the structural shadow rate extends the analysis to cover the large economic fluctuations associated with the recent pandemic. Given outliers in some variables, we do not re-estimate the model over an extended sample. Instead, we use parameter estimates for data from 1984Q1 to 2019Q4, but update the data to 2022Q2 to find structural shocks. During the second ZLB episode that starts in 2020Q2 and ends in 2022Q2, we again use expected durations based on the modal reported values from the New York Fed Survey of Primary Dealers.[25]

Figure 9 plots the estimated structural shadow federal funds rate given the updated data. The red dashed line highlights the shadow rate since 2020Q2. The estimated shadow rate remains positive in 2020Q2, reflecting the contractionary effects of a persistent expected ZLB constraint when the systematic policy response implied by the policy rule fell to below -10%. However, the estimated shadow rate quickly drops to more than -8% in 2020Q3, below the systematic policy response implied by the unconstrained policy rule. This decline is line with an immediate, albeit partial, recovery in economic conditions, along with the Fed's implementation of a number of extraordinary unconventional policies, including forward guidance related to a shift in the monetary policy framework to consider average inflation in August 2020, with a corresponding doubling of expected durations from 2 years in 2020Q2 to 4 years in 2020Q4 according to the New York Fed survey. The structural shadow rate implies that monetary policy remained highly accommodative in 2021, although the shadow rate increased by a similar amount to systematic policy response in 2021Q3 as the expected duration fell back down to under 2 years, with

[24] We report the parameter and shock estimates for the linear version of the model in the appendix. The robustness of the estimates is related to the validity of using the structural shadow rate in a linear VAR given that the reduced-form from the Smets-Wouters model for the shadow economy is a linear VAR.

[25] The survey implies durations of 10 quarters in 2020Q2, 14 quarters in 2020Q3, 16 quarters in 2020Q4, 12 quarters in 2021Q1, 9 quarters in 2021Q2, 7 quarters in 2021Q3, 2 quarters in 2021Q4, and 1 quarter in 2022Q1. Motivated by estimates for the euro area found in Haderer (2022), we allow for moving-average dynamics for the TFP innovations in 2020Q1 and 2020Q2 such that, unlike other innovations that are propagated by autoregressive dynamics, these particular two innovations have no persistent effects. By assuming the pandemic and effects of mitigation policies can be captured by a less persistent TFP shock process and that agents understood this at the time, we find the estimated lower-bound duration in 2020Q2 was similar to the overall survey duration of 8 quarters rather than much longer, which would imply a large positive spike in the shadow rate at the onset of the ZLB.

Figure 9: Updated Structural Shadow Federal Funds Rate including the COVID-19 Crisis



Notes: This figure plots, in annualized percentage terms, the estimated shadow rate up to 2021Q3, along with the implied systematic policy response. The red dashed line shows the estimated shadow rate over the second ZLB episode, starting from 2020Q2 and ending in 2022Q2. The implied systematic policy response to economic conditions is based on the policy rule without monetary policy or shadow rate shocks but allowing for interest rate smoothing, i.e. $i_{t,\text{systematic}} = \bar{i} + (1-\alpha_i)\alpha_p \hat{\pi}_t + (1-\alpha_i)\,\alpha_y \tilde{y}_t + \alpha_{\Delta y} \Delta \tilde{y}_t + \alpha_i (i_{t-1,\text{systematic}} - \bar{i})$, where the α's are the monetary policy response coefficients and $\tilde{y}_t$ is the output gap from the flexible price equilibrium for the Smets-Wouters model detailed in the appendix. The lines correspond to posterior means.

liftoff in 2022Q2. The rapid increases in the federal funds rate during 2022 are fully consistent with our estimates which show the shadow rate attempting to catch-up with the implied systematic policy response.

5 Conclusion

Identifying the stance of monetary policy at the ZLB requires a structural macroeconomic model that identifies underlying structural shocks and incorporates unconventional monetary policy. Term structure models can produce estimates of the expected duration of zero-interest-rate policy. But a structural macroeconomic model is crucial to uncover the drivers of the expected duration. Deteriorating economic conditions that make the ZLB constraint bind for longer are equivalent to tighter monetary policy, while unconventional policy that extends the duration corresponds to more expansionary policy.

In the term structure approach, the short rate follows $i_t = \max(i_t^*, 0)$, which constrains the behavior of that shadow rate to be non-positive when $i_t = 0$, that is when the short rate is at the ZLB. Given persistent large negative shocks that extend the ZLB constraint enough to make our structural shadow rate positive, a term structure measure would imply the stance of monetary policy is more expansionary than it actually is. By contrast, our structural shadow rate accurately reflects the interest-rate-equivalent stance of policy, as is evident in its strong coherence with announcements by the Fed related to unconventional policies, as well as its performance in VAR analysis when the sample period covers the ZLB.

The structural shadow rate is, by its nature, somewhat dependent on the structural model used in its estimation. This is no different than the term structure approach, which is sensitive to model specification, as highlighted in Bauer and Rudebusch (2016) and Krippner (2020). However, to the extent that expected durations are pinned down by survey data and the component of a duration related to the ZLB constraint is identified by reasonable estimates of structural shocks and the monetary policy rule, results should be fairly robust to related models, as we illustrate when we extend our analysis to a model that explicitly accounts for quantitative easing policies that the Federal Reserve pursued in the aftermath of the financial crisis.

Importantly, our approach can be applied to a wide range of structural models that involve nonlinear constraints. Thus, an interesting extension would be to consider other policy instruments subject to constraints, as would be the case for fixed exchange rate regimes, or fiscal policy subject to debt ceilings. These extensions are left for future research.

References

Anderson, Eric, Benjamin A Malin, Emi Nakamura, Duncan Simester, and Jón Steinsson, "Informational Rigidities and the Stickiness of Temporary Sales," *Journal of Monetary Economics*, 2017, *90*, 64–83.

Andrade, Philippe, Jordi Galí, Hervé Le Bihan, and Julien Matheron, "The Optimal Inflation Target and the Natural Rate of Interest," *Brookings Papers on Economic Activity*, 2019, *Fall*, 173–230.

Antolin-Diaz, Juan, Thomas Drechsel, and Ivan Petrella, "Tracking the Slowdown in Long-Run GDP Growth," *Review of Economics and Statistics*, 2017, *99*, 343–356.

Aruoba, S. Borağan, Marko Mlikota, Frank Schorfheide, and Sergio Villalvazo, "SVARs with Occasionally-Binding Constraints," *Journal of Econometrics*, 2021, *in press.*

_ , Pablo Cuba-Borda, Kenji Higa-Flores, Frank Schorfheide, and Sergio Villalvazo, "Piecewise-Linear Approximations and Filtering for DSGE Models with Occasionally-Binding Constraints," *Review of Economic Dynamics*, 2021, *41*, 96–120.

Atkinson, Tyler, Alexander W. Richter, and Nathaniel A. Throckmorton, "The Zero Lower Bound and Estimation Accuracy," *Journal of Monetary Economics*, 2020, *115*, 249–264.

Avdjiev, Stefan, Leonardo Gambacorta, Linda S. Goldberg, and Stefano Schiaffi, "The Shifting Drivers of Global Liquidity," *Journal of International Economics*, 2020, *125*, 103324.

Bauer, Michael D. and Glenn D. Rudebusch, "Monetary Policy Expectations at the Zero Lower Bound," *Journal of Money, Credit and Banking*, 2016, *48*, 1439–1465.

Boehl, Gregor, Gavin Goy, and Felix Strobel, "A Structural Investigation of Quantitative Easing," *The Review of Economics and Statistics*, 05 2022, pp. 1–45.

Carriero, Andrea, Todd E. Clark, Massimiliano Marcellino, and Elmar Mertens, "Forecasting with Shadow-Rate VARs," Working Paper 21-09, Federal Reserve Bank of Cleveland, 2021.

De Michelis, Andrea and Matteo Iacoviello, "Raising an inflation target: The Japanese experience with Abenomics," *European Economic Review*, 2016, *88*, 67–87.

Eberly, Janice C., James H. Stock, and Jonathan H. Wright, "The Federal Reserve's Current Framework for Monetary Policy: A Review and Assessment," *International Journal of Central Banking*, 2020, *16*, 5–71.

Eo, Yunjong and James Morley, "Why Has the U.S. Economy Stagnated since the Great Recession?," *The Review of Economics and Statistics*, 2022, *104*, 246–258.

Fernald, John, "Productivity and Potential Output before, during and after the Great Recession," Working Paper 18, Federal Reserve Bank of San Francisco, 2012.

Fitzgerald, Terry J., Callum Jones, Mariano Kulish, and Juan Pablo Nicolini, "Is There a Stable Relationship between Unemployment and Future Inflation?," Staff Report 614, Federal Reserve Bank of Minneapolis, 2020.

Gertler, Mark and Peter Karadi, "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 2011, *58*, 17–34.

Gilchrist, Simon and Egon Zakrajsek, "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, 2012, *102* (4), 1692–1720.

Guerrieri, Luca and Matteo Iacoviello, "OccBin: A Toolkit for Solving Dynamic Models With Occasionally Binding Constraints Easily," *Journal of Monetary Economics*, 2015, *70*, 23–38.

Gust, Christopher, Edward Herbst, David López-Salido, and Matthew E. Smith, "The Empirical Implications of the Interest-Rate Lower Bound," *American Economic Review*, 2017, *107*, 1971–2006.

Haderer, Michaela, "An Estimated DSGE Model of the Euro Area with Expectations about the Timing and Nature of Liftoff from the Lower Bound," mimeo University of Sydney, 2022.

Hills, Timothy S and Taisuke Nakata, "Fiscal Multipliers at the Zero Lower Bound: The Role of Policy Inertia," *Journal of Money, Credit and Banking*, 2018, *50*, 155–172.

Ichiue, Hibiki and Yoichi Ueno, "Estimating Term Premia at the Zero Bound: An Analysis of Japanese, US, and UK Yields," Working Paper 13-E-8, Bank of Japan, 2013.

_ **and** _ , "Monetary Policy and the Yield Curve at Zero Interest," *Journal of the Japanese and International Economies*, 2015, *38*, 1–12.

Ikeda, Daisuke, Shangshang Li, Sophocles Mavroeidis, and Francesco Zanetti, "Testing the Effectiveness of Unconventional Monetary Policy in Japan and the United States," Working Paper arXiv:2012.15158v2, 2021.

Iwata, Shigeru and Shu Wu, "Estimating Monetary Policy Effects when Interest Rates Are Close to Zero," *Journal of Monetary Economics*, 2006, *53*, 1395–1408.

Johannsen, Benjamin K and Elmar Mertens, "A Time-Series Model of Interest Rates with the Effective Lower Bound," *Journal of Money, Credit and Banking*, 2021, *forthcoming.*

Jones, Callum, "Unanticipated Shocks and Forward Guidance at the Zero Lower Bound," mimeo NYU, 2017.

_ , Mariano Kulish, and Daniel M. Rees, "International spillovers of forward guidance shocks," *Journal of Applied Econometrics*, 2022, *37*, 131–160.

Krippner, Leo, "Measuring the Stance of Monetary Policy in Zero Lower Bound Environments," *Economics Letters*, 2013, *118*, 135–138.

_ , "A Note of Caution on Shadow Rate Estimates," *Journal of Money, Credit and Banking*, 2020, *52*, 951–962.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," 2011. BPEA 2011.

Kulish, Mariano and Adrian Pagan, "Estimation and Solution of Models with Expectations and Structural Changes," *Journal of Applied Econometrics*, 2017, *32*, 255–274.

_ , James Morley, and Timothy Robinson, "Estimating DSGE Models with Zero Interest Rate Policy," *Journal of Monetary Economics*, 2017, *88*, 35–49.

Mavroeidis, Sophocles, "Identification at the Zero Lower Bound," *Econometrica*, 2021, *89*, 2855–2885.

Nakajima, Jouchi, "Monetary Policy Transmission under Zero Interest Rates: An Extended Time-Varying Parameter Vector Autoregression Approach," *The B.E. Journal of Macroeconomics*, 2011, *11*, 1–24.

Sims, Eric and Jing Cynthia Wu, "Are QE and Conventional Monetary Policy Substitutable?," *International Journal of Central Banking*, 2020, *16*, 195–230.

_ , _ , and Ji Zhang, "The four-equation new keynesian model," *Review of Economics and Statistics*, 2023, *105* (4), 931–947.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 2007, *97*, 586–606.

Swanson, Eric and John Williams, "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates," *American Economic Review*, 2014, *104*, 3154–3185.

Wu, Jing Cynthia and Fan Dora Xia, "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit, and Banking*, 2016, *48*, 253–291.

Appendix

For Online Publication

A Description of the Smets-Wouters Model

We provide here the linearized equations of the Smets-Wouters model. We use similar notation for variables and parameters as in Smets and Wouters (2007), but we substitute i for r when referring to the nominal interest rate and i^k for i when referring to investment. The model variables are presented in terms of deviations from steady state, but with hats suppressed for simplicity. A full description of the model is available in Smets and Wouters (2007) and its accompanying online appendix.

A.1 Sticky Price Economy

Factor prices:

$$mc_t = \alpha r_t^k + (1-\alpha) w_t - \varepsilon_{a,t}$$

$$r_t^k = w_t + l_t - k_t^s$$

$$z_t = \tfrac{1-\psi}{\psi} r_t^k$$

Investment:

$$i_t^k = \tfrac{1}{1+\beta\gamma} \left(i_{t-1}^k + \beta\gamma \mathbb{E}_t i_{t+1}^k + \tfrac{1}{\gamma^2 \phi} q_t \right) + \varepsilon_{i,t}$$

$$q_t = \tfrac{1-\delta}{1-\delta+\bar{R}^k} \mathbb{E}_t q_{t+1} + \tfrac{\bar{R}^k}{1-\delta+\bar{R}^k} \mathbb{E}_t r_{t+1}^k - i_t + \mathbb{E}_t \pi_{t+1} + \tfrac{\sigma_c (1+\lambda/\gamma)}{1-\lambda/\gamma} \varepsilon_{b,t}$$

Consumption:

$$c_t = \tfrac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \tfrac{1}{1+\lambda/\gamma} \mathbb{E}_t c_{t+1} + \tfrac{(\sigma_c - 1)\, W^* L^* / C^*}{\sigma_c (1+\lambda/\gamma)} \, (l_t - \mathbb{E}_t l_{t+1}) - \tfrac{1-\lambda/\gamma}{\sigma_c (1+\lambda/\gamma)} \, (i_t - \mathbb{E}_t \pi_{t+1}) + \varepsilon_{b,t}$$

Resource constraint:

$$y_t = c_t c_y + i_t^k i_y^k + z_t z_y + \varepsilon_{g,t}$$

Production function:

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha) \; l_t + \varepsilon_{a,t} \right)$$

$$k_t^s = z_t + k_{t-1}$$

Monetary policy rule:

$$i_t = (1-\alpha_i)\,\alpha_p \pi_t + (1-\alpha_i)\,\alpha_y \left(y_t - y_t^f\right) + \alpha_{\Delta y}\left(y_t - y_t^f - \left(y_{t-1} - y_{t-1}^f\right)\right) + \alpha_i i_{t-1} + \varepsilon_{m,t}$$

Longer term interest rates:

$$i_{4,t} = \varepsilon_{\eta,t} + \eta_{4,t}$$

$$i_{20,t} = \varepsilon_{\eta,t} + \eta_{20,t}$$

Evolution of capital:

$$k_t = \tfrac{(1-\delta)}{\gamma} k_{t-1} + \tfrac{(\gamma-1+\delta)}{\gamma} i_t^k + \tfrac{(\gamma-1+\delta)}{\gamma} \phi\, \gamma^2 \varepsilon_{i,t}$$

Price Phillips curve and wages:

$$\pi_t = \tfrac{1}{1+\beta\gamma\,\iota_p}\ \left(\beta\gamma\,\mathbb{E}_t \pi_{t+1} + \iota_p\,\pi_{t-1} + \tfrac{\left(1-\xi_p\right)\left(1-\beta\gamma\,\xi_p\right)}{\xi_p} \tfrac{1}{1+\left(\phi_p-1\right)\varepsilon_p} mc_t\right) + \varepsilon_{p,t}$$

$$\begin{aligned} w_t = w_1 w_{t-1} + (1-w_1)\,\mathbb{E}_t\left(w_{t+1} + \pi_{t+1}\right) - w_2\pi_t\ + w_3\pi_{t-1} - \\ w_4\left(\sigma_l l_t + \frac{1}{1-\lambda/\gamma} c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} - w_t\right) + \varepsilon_{w,t}, \end{aligned}$$

where $w_1 = \frac{1}{1+\beta\gamma}$, $w_2 = w_1\left(1+\beta\gamma\iota_w\right)$, $w_3 = w_1\iota_w$, and $w_4 = w_1 \frac{(1-\xi_w)\,(1-\beta\gamma\,\xi_w)}{\xi_w(1+(\phi_w-1)\,\varepsilon_w)}$.

A.2 Flexible Price Economy

The corresponding equations defining the flexible price economy are

$$\alpha r_t^{k,f} + (1-\alpha) w_t^f = \varepsilon_{a,t}$$

$$r_t^{k,f} = w_t^f + l_t^f - k_t^f$$

$$z_t^f = \tfrac{1-\psi}{\psi} r_t^{k,f}$$

$$k_t^f = z_t^f + kp_{t-1}^f$$

$$i_t^{k,f} = \tfrac{1}{1+\beta\gamma}\ \left(i_{t-1}^{k,f} + \beta\gamma\,\mathbb{E}_t i_{t+1}^{k,f} + \tfrac{1}{\gamma^2\,\phi}\, q_t^f\right) + \varepsilon_{i,t}$$

$$q_t^f = \tfrac{1-\delta}{1-\delta+\bar{R}^k}\,\mathbb{E}_t q_{t+1}^f + \tfrac{\bar{R}^k}{1-\delta+\bar{R}^k}\,\mathbb{E}_t r_{t+1}^{k,f} - i_t^f + \tfrac{\sigma_c(1+\lambda/\gamma)}{1-\lambda/\gamma}\,\varepsilon_{b,t}$$

$$c_t^f = \tfrac{\lambda/\gamma}{1+\lambda/\gamma}\, c_{t-1}^f + \tfrac{1}{1+\lambda/\gamma}\,\mathbb{E}_t c_{t+1}^f + \tfrac{(\sigma_c-1)\,W^* L^*/C^*}{\sigma_c(1+\lambda/\gamma)}\ \left(l_t^f - \mathbb{E}_t l_{t+1}^f\right) - \tfrac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)} i_t^f + \varepsilon_{b,t}$$

$$
\begin{aligned}
y_t^f &= c_t^f c_y + i_t^{k,f} i_y^{k,f} + z_t^f z_y + \varepsilon_{g,t} \\
y_t^f &= \phi_p \left(\alpha k_t^f + (1-\alpha)\, l_t^f + \varepsilon_{a,t} \right) \\
k_t^{p,f} &= \tfrac{(1-\delta)}{\gamma} k_{t-1}^{p,f} + \tfrac{(\gamma-1+\delta)}{\gamma} i_t^{k,f} + \tfrac{(\gamma-1+\delta)}{\gamma} \phi\, \gamma^2 \varepsilon_{i,t} \\
w_t^f &= \sigma_l l_t^f + \tfrac{1}{1-\lambda/\gamma} c_t^f - \tfrac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1}^f.
\end{aligned}
$$

A.3 Exogenous Processes

Letting ζ denote an i.i.d. standard normal innovation, the exogenous processes are

$$\varepsilon_{a,t} = \rho_a\, \varepsilon_{a,t-1} + \sigma_a\, \zeta_{a,t}$$

$$
\begin{aligned}
\varepsilon_{b,t} &= \rho_b\, \varepsilon_{b,t-1} + \sigma_b\, \zeta_{b,t} \\
\varepsilon_{g,t} &= \rho_g\, \varepsilon_{g,t-1} + \sigma_g\, \zeta_{g,t} + \rho_{ga} \sigma_a \zeta_{a,t} \\
\varepsilon_{i,t} &= \rho_i\, \varepsilon_{i,t-1} + \sigma_i\, \zeta_{i,t} \\
\varepsilon_{m,t} &= \rho_m\, \varepsilon_{m,t-1} + \sigma_i\, \zeta_{m,t} \\
\varepsilon_{p,t} &= \rho_p\, \varepsilon_{p,t-1} + \eta_{p,ma,t} - \mu_p\, \eta_{p,ma,t-1} \\
\eta_{p,ma,t} &= \sigma_p\, \zeta_{p,t} \\
\varepsilon_{w,t} &= \rho_w\, \varepsilon_{w,t-1} + \eta_{w,ma,t} - \mu_w\, \eta_{w,ma,t-1} \\
\eta_{w,ma,t} &= \sigma_w\, \zeta_{w,t} \\
\varepsilon_{\eta,t} &= \rho_\eta \varepsilon_{\eta,t-1} + \sigma_\eta\, \zeta_{\eta,t} \\
\eta_{4,t} &= \sigma_{i,4}\, \zeta_{4,t} \\
\eta_{20,t} &= \sigma_{i,20}\, \zeta_{20,t}.
\end{aligned}
$$

A.4 Measurement Equations

Finally, the measurement equations are

$$dy_t = \bar{\gamma} + y_t - y_{t-1}$$

$$
\begin{aligned}
dc_t &= \bar{\gamma} + c_t - c_{t-1} \\
di_t^k &= \bar{\gamma} + i_t^k - i_{t-1}^k
\end{aligned}
$$

$$
\begin{aligned}
dw_t &= \bar{\gamma} + w_t - w_{t-1} \\
\pi_t^{obs} &= \bar{\pi} + \pi_t \\
i_t^{obs} &= \bar{i} + i_t \\
i_{4,t}^{obs} &= \bar{i} + \bar{i}_4 + i_{4,t} \\
i_{20,t}^{obs} &= \bar{i} + \bar{i}_{20} + i_{20,t} \\
l_t^{obs} &= \bar{l} + l_t.
\end{aligned}
$$

B Data, Parameter Estimates, and Additional Results

B.1 Data Sources and Mapping to Model

We use the following data series and sources (with FRED mnemonics in parentheses):

- Real Gross Domestic Product (GDPC1)
- Fixed Private Investment (FPI)
- Personal Consumption Expenditures (PCEC)
- Inflation: Gross Domestic Product, Implicit Price Deflator (GDPDEF)
- Nonfarm Business Sector: Average Weekly Hours (PRS85006023)
- Nonfarm Business Sector: Compensation Per Hour (COMPNFB)
- Federal Funds Rate (FEDFUNDS)
- 1-Year Treasury Constant Maturity Rate (GS1)
- 5-Year Treasury Constant Maturity Rate (GS5)
- Population Level (CNP16OV)
- Employment Level (CE16OV)
- ZLB Durations: following Kulish et al. (2017), we use the ZLB durations extracted from the New York Fed Survey of Primary Dealers, conducted eight times a year

from 2011Q1 onwards and the Blue Chip Financial Forecasts survey before 2011.[26] For our measure of an expected duration, we take the mode of the distribution implied by these surveys.

We map these series to our observed variables in the following way:

$$\text{CNP16OV\_idx} = \frac{\text{CNP16OV}}{\text{CNP16OV}_{1992Q3}}$$

$$\text{CE16OV\_idx} = \frac{\text{CE16OV}}{\text{CE16OV}_{1992Q3}}$$

$$dc_t = 100 \times \Delta \frac{\text{PCEC}}{\text{GDPDEF}} \times \frac{1}{\text{CNP16OV\_idx}}$$

$$dy_t = 100 \times \Delta \frac{\text{GDPC1}}{\text{CNP16OV\_idx}}$$

$$di_t^k = 100 \times \Delta \frac{\text{FPI}}{\text{GDPDEF}} \times \frac{1}{\text{CNP16OV\_idx}}$$

$$dw_t = 100 \times \Delta \frac{\text{COMPNFB}}{\text{GDPDEF}}$$

$$l_t^{obs} = 100 \times \log\left(\text{PRS85006023} \times \frac{\text{CE16OV\_idx}}{\text{CNP16OV\_idx}}\right).$$

We demean l_t^{obs} over 1984Q1 to 2019Q4.

B.2 Parameter Estimates for the Baseline Model

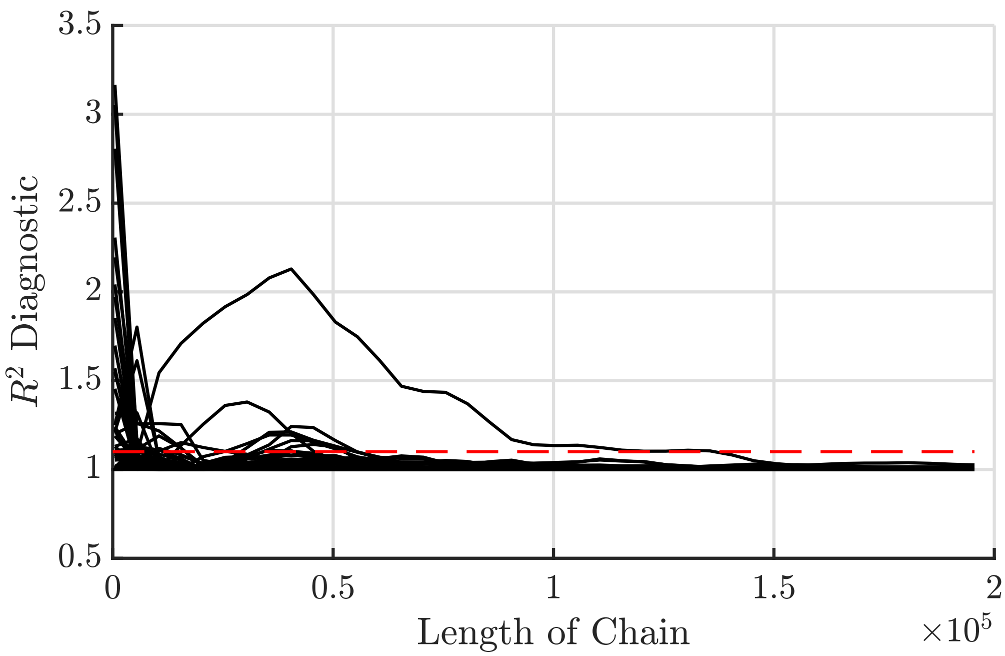
The prior and posterior distributions of the estimated parameters are given in Table B.1. The posterior for the Calvo price parameter is centered around a value of 0.93, indicating the aggregate data prefers strong nominal rigidities and a relatively flat Phillips curve, which helps to rationalize a relatively stable inflation rate with a large output and employment gap in the post-2009 sample. The posterior for the Calvo wage parameter is centered around a value of 0.36, in line with the estimates from Fitzgerald et al. (2020) that use relative US state-level data. In Appendix C below, we explore the robustness of our results to calibrating both the Calvo wage and price parameters to their estimates in Fitzgerald et al. (2020), namely a Calvo wage parameter of 0.4 and a Calvo price parameter of

[26] See the website https://www.newyorkfed.org/markets/primarydealer_survey_questions.html for more information on the Primary Dealers survey. For example, in the survey conducted on January 18 2011, one of the questions asked was: "Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase" (Question 2b). Responses were given in terms of a probability distribution across future quarters.

0.6. The results in terms of the shadow rate are highly robust. The posterior estimates of the response of the policy interest rate to inflation and output fluctuations are slightly lower than those reported in Smets and Wouters (2007), noting that their sample ends in 2004. The posterior estimate for trend growth is centered around 0.57% per quarter. We obtain a precisely estimated one-time decline in trend growth of −0.18% in 2003Q4, consistent with other studies noted above that find a decline in trend growth around that time. This decline translates into a difference in the annual rate of trend growth of 2.3% before 2003Q4 to 1.5% thereafter. Given our posterior estimate of the discount factor, the estimated decline in trend growth implies that the annualized steady-state nominal interest rate falls from 5.2% to 4.4%. In simulations, this decline in the steady-state nominal interest rate has the effect of raising the fraction of time spent at the ZLB from about 5% to about 15%.

Figure B.1 plots the convergence of the two chains along the chain using the Gelman R^2 diagnostic. The R^2 diagnostic lies below the value of 1.1 for all parameters by the end of the chain, indicating convergence, across chains, of the posterior distributions.

Figure B.1: Convergence of MCMC Chains



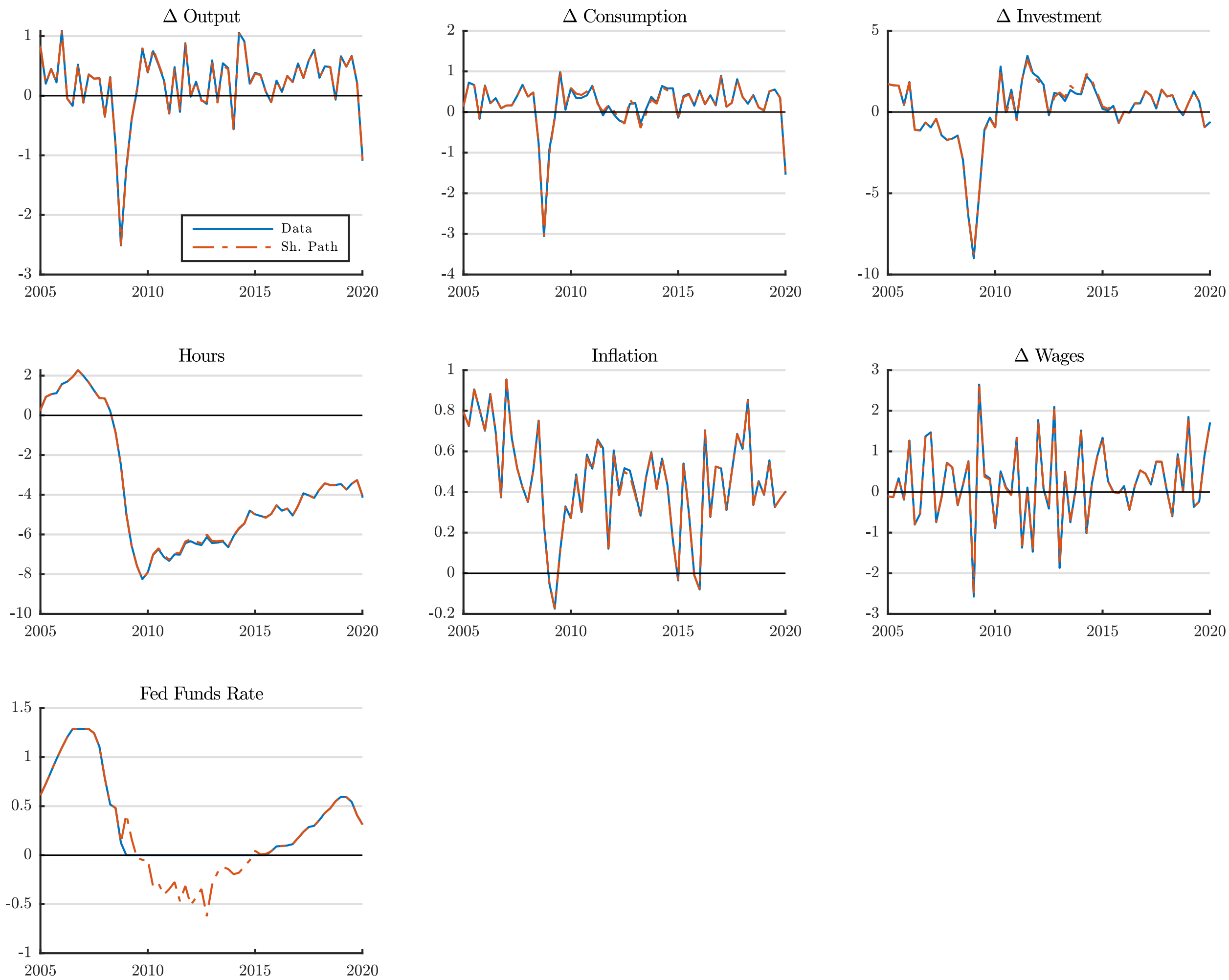
B.3 Additional Results for the Baseline Model

Figure B.2 plots the path of the variables targeted in the construction of the shadow interest rate in the first six panels, in the data (in blue), and in the shadow economy (in red)

Table B.1: Parameter Estimates for the Smets-Wouters Model

	Prior				Posterior			
Parameter	Type	Mean	5%	95%	Mode	Median	5%	95%
ϕ	N	4.0	1.5	6.5	6.98	6.79	5.05	8.74
σ_c	N	1.5	0.9	2.1	1.08	1.09	0.90	1.30
λ	B	0.7	0.5	0.9	0.27	0.26	0.19	0.35
ξ_w	B	0.5	0.3	0.7	0.33	0.36	0.25	0.50
σ_l	N	2.0	0.8	3.2	0.89	0.95	0.56	1.49
ξ_p	B	0.5	0.3	0.7	0.93	0.93	0.91	0.95
ι_w	B	0.5	0.3	0.7	0.37	0.40	0.18	0.65
ι_p	B	0.5	0.3	0.7	0.17	0.19	0.09	0.34
ψ	B	0.5	0.3	0.7	0.74	0.74	0.58	0.87
ϕ_p	N	1.2	1.0	1.5	1.50	1.50	1.37	1.64
α_p	N	1.5	1.3	1.7	1.64	1.65	1.50	1.80
α_i	B	0.8	0.6	0.9	0.79	0.76	0.57	0.90
α_y	N	0.1	0.0	0.2	0.06	0.06	0.05	0.08
$\alpha_{\Delta y}$	N	0.1	0.0	0.2	0.17	0.17	0.13	0.20
$100\left(\beta^{-1}-1\right)$	G	0.2	0.1	0.4	0.16	0.17	0.08	0.30
γ	N	0.4	0.2	0.6	0.58	0.57	0.53	0.62
α	N	0.3	0.2	0.4	0.15	0.15	0.13	0.18
$\bar{i}_4$	N	0.1	0.0	0.3	0.04	0.04	−0.00	0.08
$\bar{i}_{20}$	N	0.5	0.1	0.9	0.19	0.19	0.12	0.26
$\bar{l}$	N	0.0	−2.5	2.5	2.31	2.25	0.14	3.91
Persistence and Variances of Exogenous Processes								
ρ_a	B	0.5	0.2	0.8	0.93	0.93	0.91	0.95
ρ_b	B	0.5	0.2	0.8	0.98	0.98	0.97	0.99
ρ_g	B	0.5	0.2	0.8	0.99	0.99	0.98	1.00
ρ_i	B	0.5	0.2	0.8	0.82	0.81	0.73	0.88
ρ_p	B	0.5	0.2	0.8	0.80	0.79	0.69	0.87
ρ_w	B	0.5	0.2	0.8	0.99	0.99	0.99	1.00
μ_p	B	0.5	0.2	0.8	0.72	0.69	0.52	0.81
μ_w	B	0.5	0.2	0.8	0.74	0.73	0.56	0.86
ρ_{ga}	N	0.5	0.2	0.8	0.48	0.47	0.35	0.59
ρ_η	B	0.5	0.2	0.8	0.78	0.78	0.69	0.87
σ_a	IG	0.1	0.0	0.3	0.42	0.42	0.38	0.47
σ_b	IG	0.1	0.0	0.3	0.04	0.04	0.03	0.05
σ_g	IG	0.1	0.0	0.3	0.37	0.36	0.33	0.40
σ_i	IG	0.1	0.0	0.3	0.22	0.22	0.19	0.27
σ_m	IG	0.1	0.0	0.3	0.13	0.13	0.11	0.14
σ_p	IG	0.1	0.0	0.3	0.11	0.10	0.09	0.12
σ_w	IG	0.1	0.0	0.3	0.49	0.50	0.42	0.62
σ_η	IG	0.1	0.0	0.3	0.06	0.06	0.05	0.07
$\sigma_{i,4}$	IG	0.1	0.0	0.3	0.01	0.01	0.01	0.02
$\sigma_{i,20}$	IG	0.1	0.0	0.3	0.09	0.09	0.08	0.10
Change in Trend Growth								
$\Delta\bar{\gamma}$	N	0.0	−0.4	0.4	−0.18	−0.18	−0.23	−0.13
Date of $\Delta\bar{\gamma}$	U	2000Q1	1994Q3	2006Q3	2003Q4	2003Q4	2001Q1	2005Q2

Figure B.2: Paths of Targets in the Shadow Economy



given by the system (7). The plot shows that the paths of these variables under the shadow rate shocks are very close to the observed paths. The numerical matching procedure we use to find the shadow rate shocks is thus able to replicate the data. For completeness, the path of the policy rate and the shadow rate is also given in the last panel.

Figure B.3 plots the paths of all the variables in the data and in the counterfactual scenario where the forward-guidance durations are set to zero.

Figure B.3: Counterfactual Paths Removing Forward Guidance

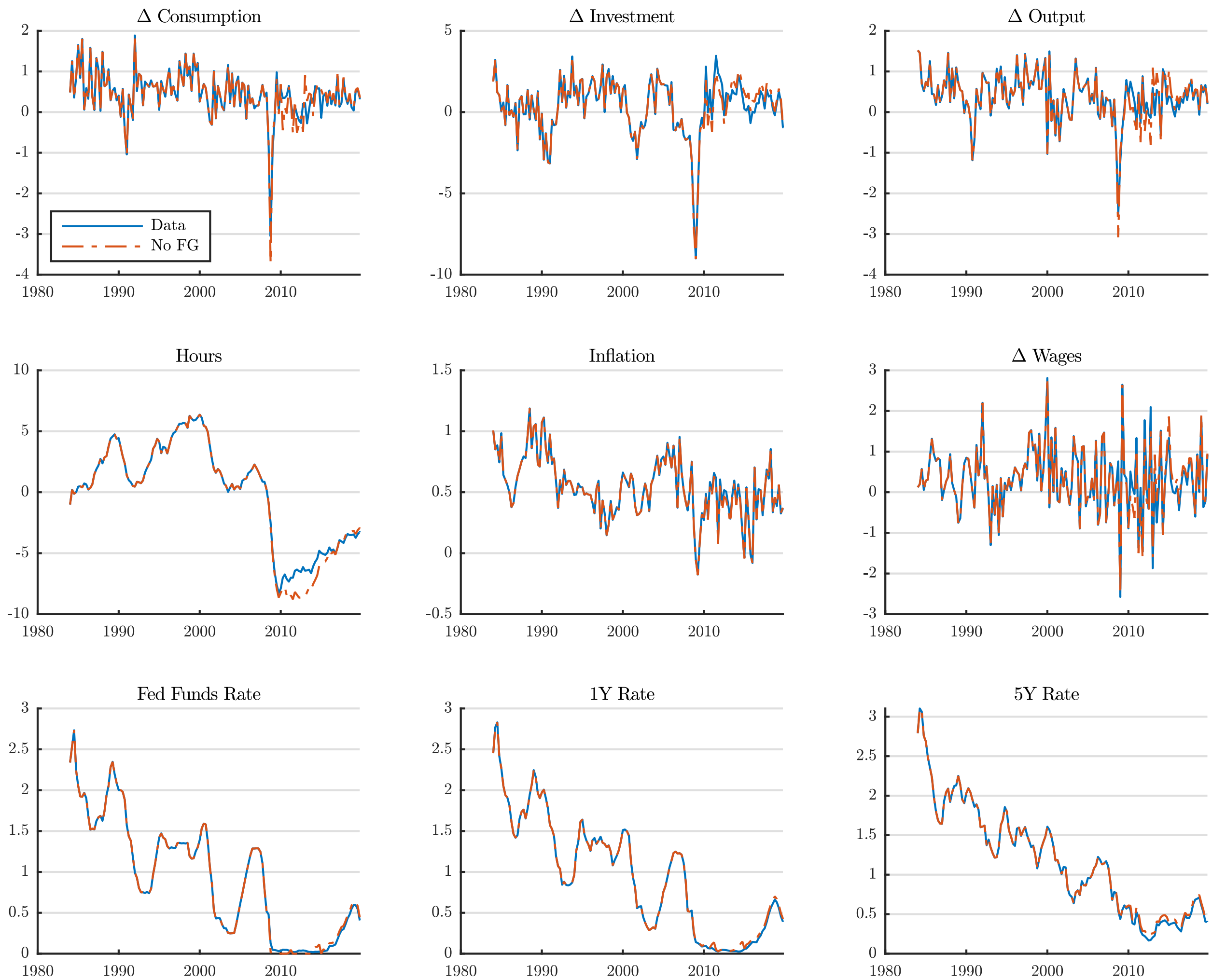
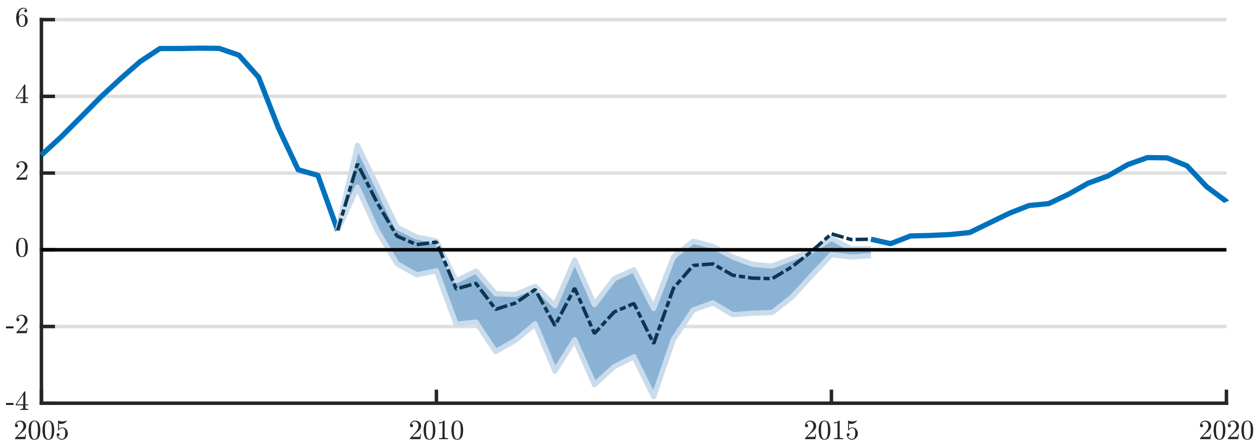


Figure C.1: Shadow Federal Funds Rate given Calibrated Calvo Parameters



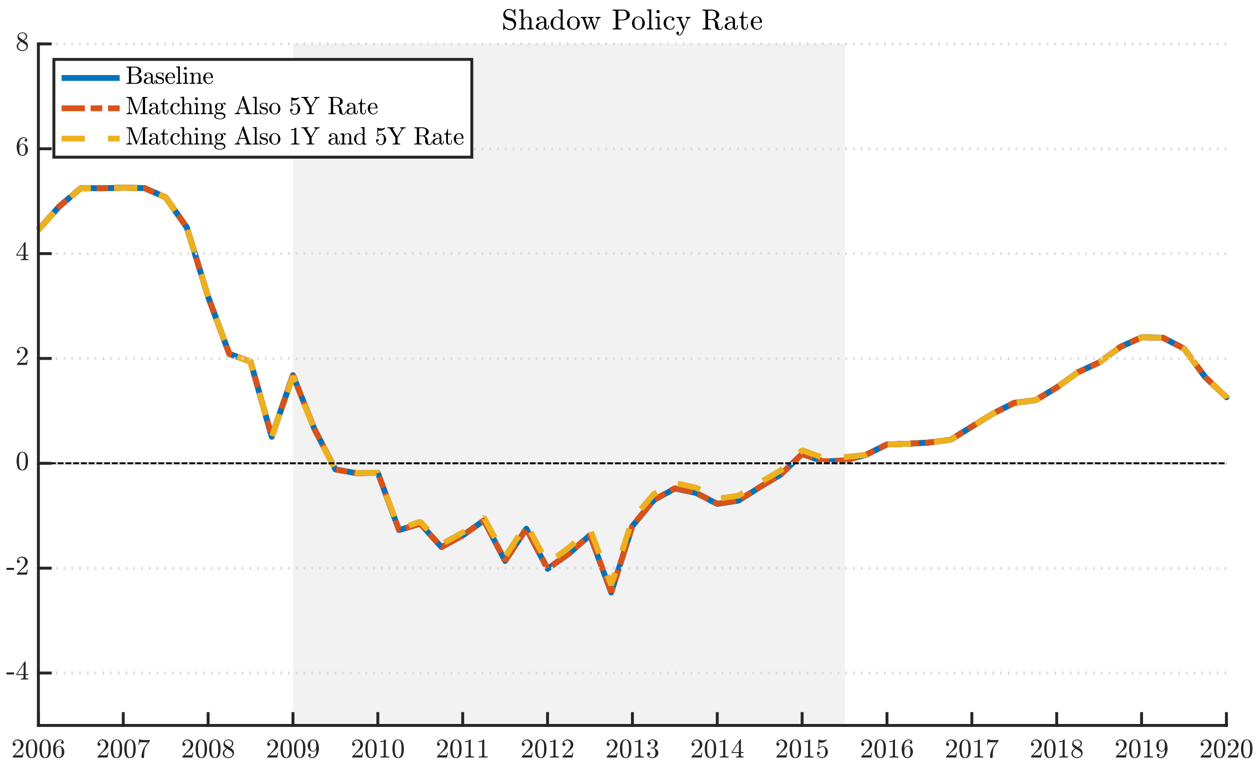
Notes: The figure plots, in annualized percentage terms, the estimated shadow rate from the Smets-Wouters model where the Calvo price and Calvo wage parameters are calibrated to $\xi_p = 0.6$ and $\xi_w = 0.4$, respectively. The dashed black lines correspond to posterior means, while the bands show 90 percent equal-tailed posterior intervals.

C Robustness

Figure C.1 plots the shadow rate estimates when the Calvo price and wage parameters are calibrated to $\xi_p = 0.6$ and $\xi_w = 0.4$, the values estimated in Fitzgerald et al. (2020) using state-level data, instead of estimated. The estimates are very similar to those reported in the main text.

Figure C.2 plots the shadow rate constructed when, in addition to the macroeconomic aggregates, we also target the 1-year and 5-year yields. The figure displays the shadow rate estimates including the pandemic and shows that there are almost no differences when compared to the baseline shadow rate we construct using macroeconomic aggregates alone. Figures C.3 and C.4 plot the paths of variables in the shadow economy given the alternative targets including the 1-year and 5-year yields and show that the fit of the macroeconomic aggregates is virtually unchanged to the baseline, while the interest rates, especially the 1-year yield, behave differently in the unconstrained shadow economy. This robustness exercise makes clear how our identification of the shadow rate is driven by the macroeconomic aggregates, not the term structure of interest rates, which is a key point of contrast to the existing literature on shadow policy rates.

Figure C.2: Shadow Federal Funds Rates Additionally Targeting Other Interest Rates



D Comparison with the Wu-Xia Approach

One illustrative way to contrast our approach with that of Wu and Xia (2016) is to simulate data from a structural model and use the two methods to construct the respective implied shadow interest rates. To do so, we estimate the Smets-Wouters model with additional yields up to 10 years to get our data generating process. Using this, we simulate a yield curve with the following interest rates – 1Q, 2Q, 1Y, 2Y, 5Y, 7Y, 10Y – that we can use to estimate a term structure model. We simulate a long sample of the model and choose a period from the simulation when the ZLB is a significant constraint. For the most contrast between methods, we abstract from any forward guidance.

The top two panels of Figure D.1 plot the simulated yield curve and the simulated output series. Also plotted in the output panel is the output series if the ZLB were removed as a constraint on monetary policy. Comparing this with output under the ZLB illustrates the contractionary forces that the ZLB can induce on the economy. The simulated federal funds rate is shown in the bottom two panels in blue – the ZLB binds on and off over the sample.

The shadow rate constructed using our approach is shown in the dashed red series in

Figure C.3: Paths of Targets in the Shadow Economy Additionally Targeting 1Y, 5Y Yields

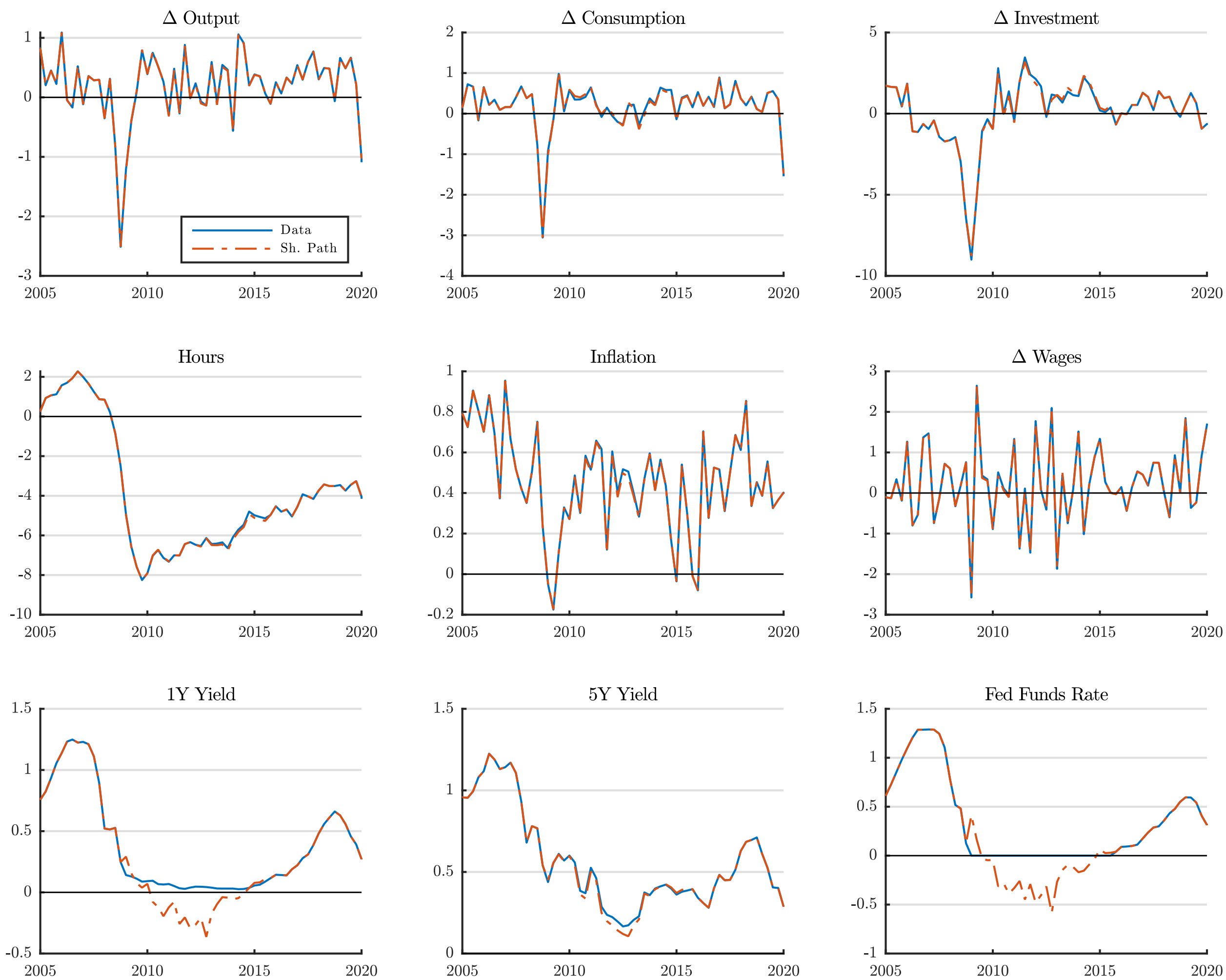


Figure C.4: Paths of Targets in the Shadow Economy Additionally Targeting 5Y Yield

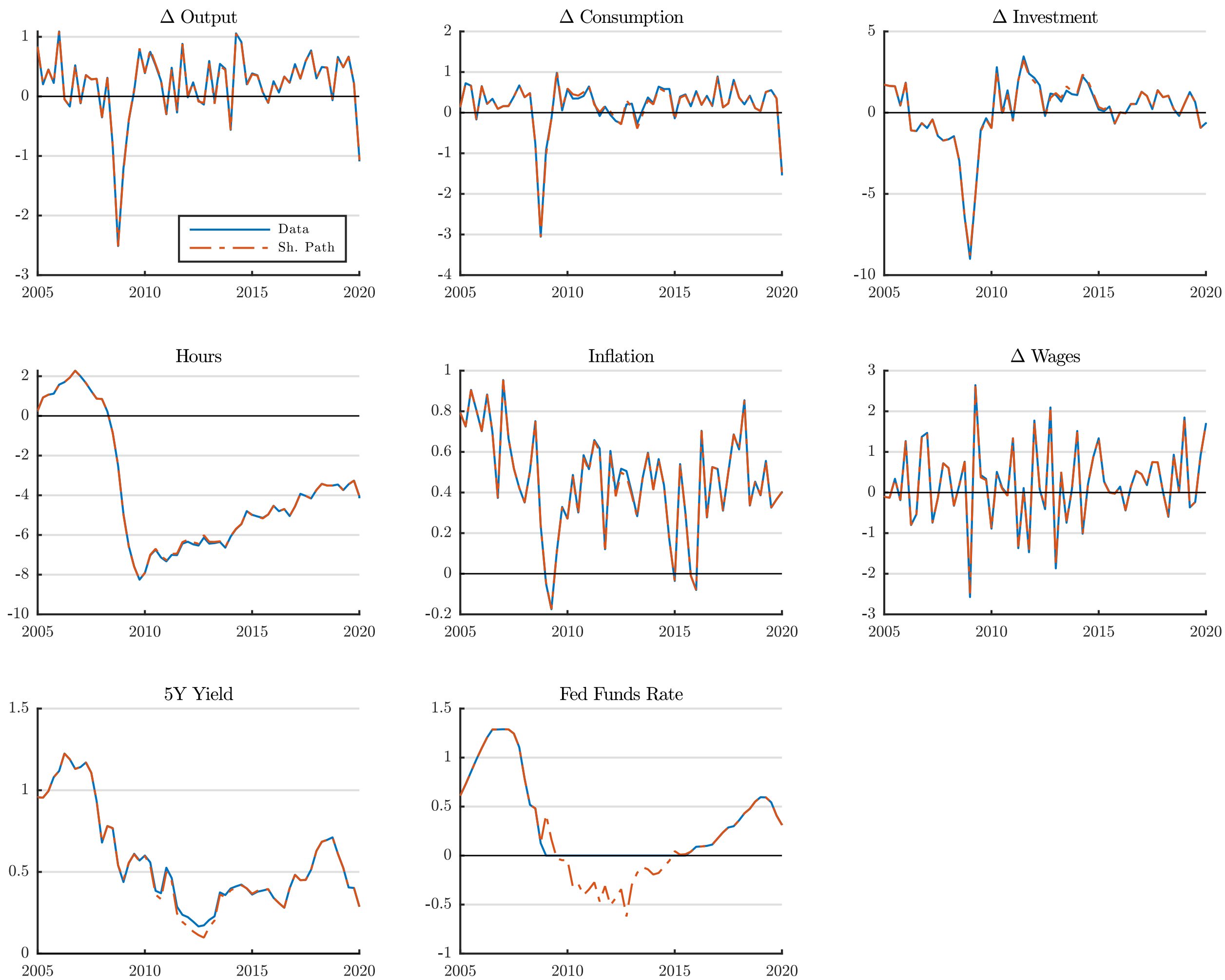
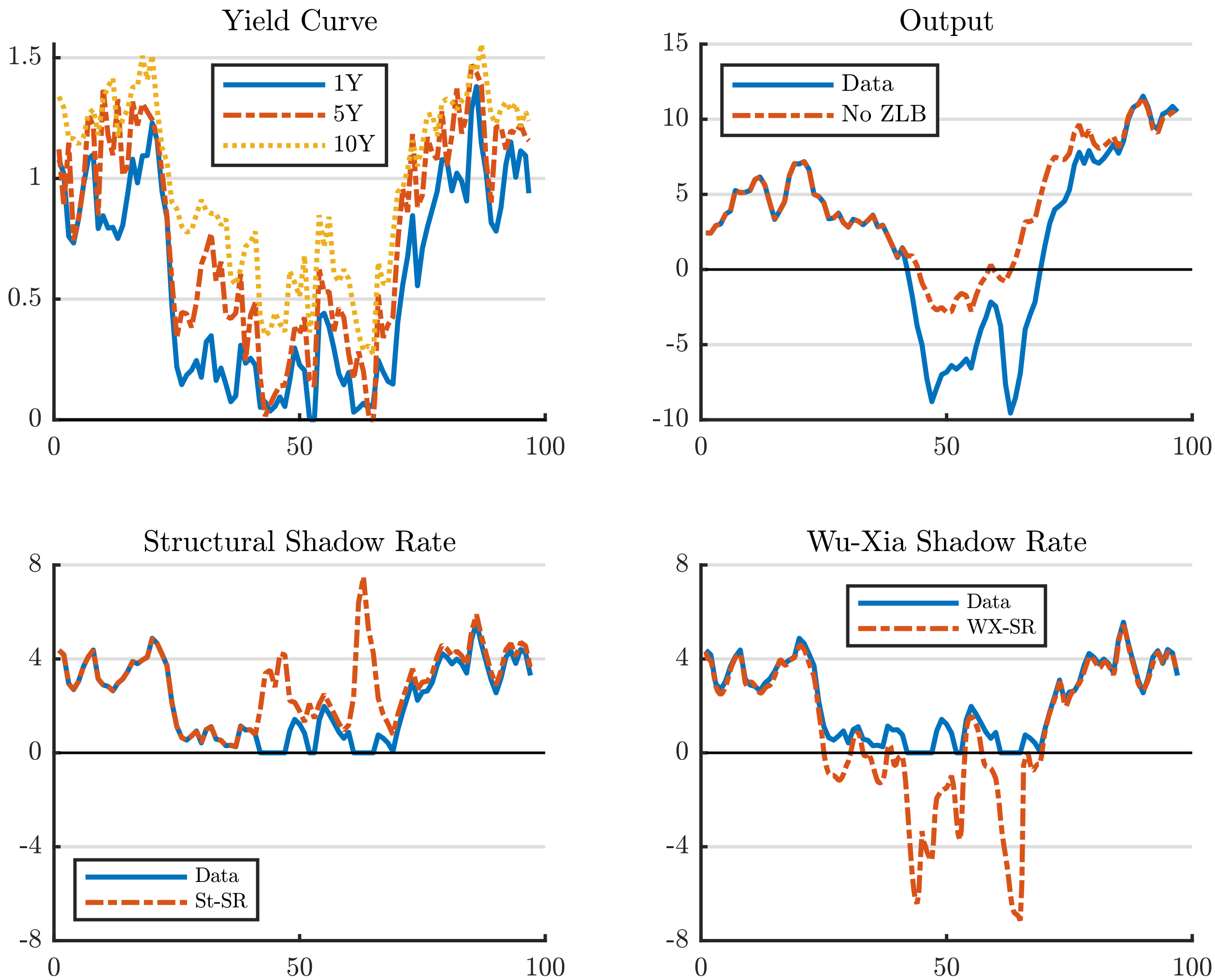


Figure D.1: Simulation to Compare Different Approaches to Measuring the Shadow Rate

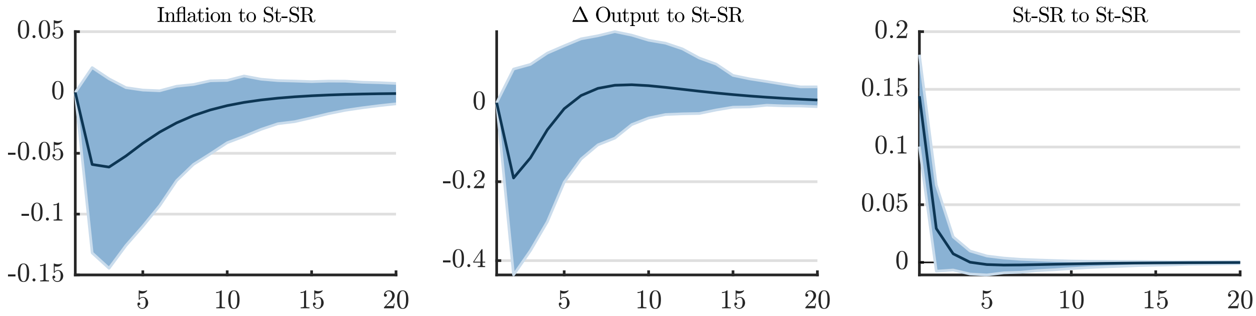


the bottom left panel. In order to replicate the contractionary effects of the ZLB, our procedure finds contractionary shadow rate shocks, which work to push the shadow interest rate well above zero in the periods that the ZLB binds.

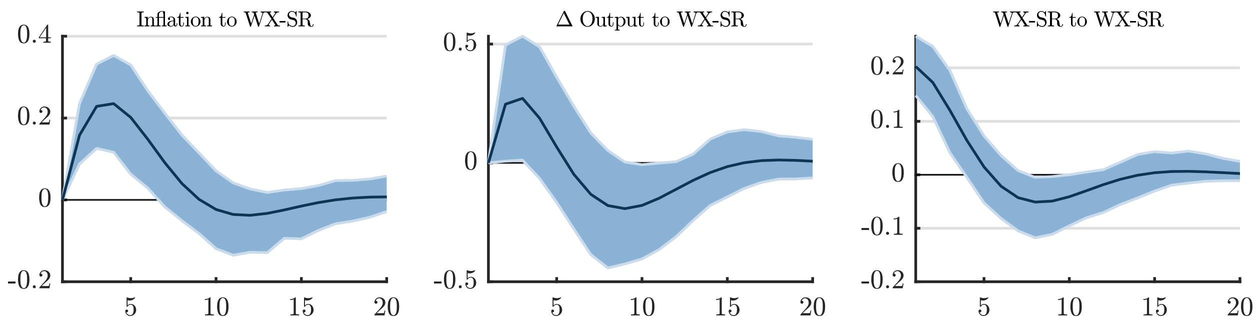
The constructed Wu-Xia shadow rate, presented in the final panel, provides a stark contrast, with the shadow rate measure falling well below zero and almost reaching -8%. Clearly, the inferences that would be made from this measure of the shadow rate would not line up with the conceptual basis of the shadow rate, which is that it reflects the policy stance of the central bank. In this simulation, the central bank does not react at all to shocks that occur at the ZLB. The Wu-Xia shadow rate instead simply reflects the behavior of the yield curve, but the yield curve can decline either because of negative shocks (which is the case here) or because of policy actions (which is not the case here). The Wu-Xia measure is not able to discriminate between the two sources of decline.

Figure D.2: Impulse Responses for Simulation with No Forward Guidance

(a) Estimated Responses Using the Structural Shadow Rate



(b) Estimated Responses Using the Wu-Xia Shadow Rate



Notes: This figure displays responses of inflation and output growth in quarterly percentage terms and the shadow rate in annualized percentage terms to a one-standard deviation monetary policy shock in a three-variable VAR with quarterly inflation, output growth, and the shadow rate estimated using simulated data with no forward guidance. The shadow rate is ordered last for identification of monetary policy shocks using a Cholsky factorization of the forecast-error variance-covariance matrix. Panel (a) reports results for the structural shadow rate and panel (b) reports results for the shadow rate measure constructed following Wu and Xia (2016). The impulse responses are computed using a bootstrap procedure drawing residuals with replacement. The black lines correspond to mean responses and the bands show 90 percent equal-tailed bootstrap confidence intervals.

The failure of the Wu-Xia shadow rate to capture the stance of policy in this simulated environment can be further illustrated by considering impulse response functions for an identified monetary policy shock in a VAR estimated using simulated data when the ZLB holds. The details of the VAR analysis are exactly as in Section 4 when applying to the actual data from the ZLB and finding a price puzzle for the Wu-Xia shadow rate. For the simulated data, the impulse responses reported in Figure D.2 when using the structural shadow rate given a contractionary shock suggest declines in inflation and output, although the responses are not significant. However, when using the Wu-Xia shadow rate, there is both a price and output puzzle, reflecting the fact that the Wu-Xia rate does not rise, but rather declines when there are negative economic shocks that cause the ZLB to bind for longer. In this case, a negative movement in the Wu-Xia shadow rate, even when controlling for inflation and output growth, will correspond to declines in inflation and output, with these declines being significant.

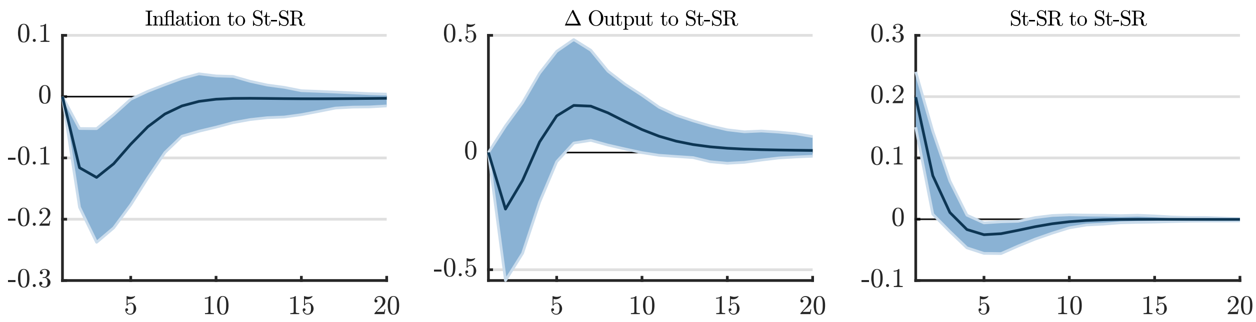
In practice, with the actual data from the ZLB, we find only a price puzzle, not an output puzzle, when using the Wu-Xia shadow rate. One reason is that there was some use of forward guidance in the actual data. To understand the effects of forward guidance in the simulated setting, we consider a simulation where there is forward guidance calibrated such that the expected ZLB duration is the maximum of 3 times the endogenous lower bound duration or the previous duration minus one. For example, if the lower bound duration is 2 quarters, the forward guidance duration would be 6 quarters. Then, in the next period, if the lower bound duration dropped to 0 quarters due to positive shocks, the forward guidance duration would reduce to 5 quarters. The impulse responses for this simulation with forward guidance are reported in Figure D.3. Both the structural shadow rate and the Wu-Xia shadow rate identify the negative effects of a contractionary shock. So the Wu-Xia approach is clearly useful when movements in durations largely reflect unconventional policies such as forward guidance. It is when there is a mix of changes in durations due to changes in the how long the ZLB will bind and forward guidance that the Wu-Xia approach will lead to price and possibly output puzzles in VAR analysis. By contrast, our structural shadow rate, by reflecting the separate identification of why durations have changed, captures the correct signs in terms of responses to identified monetary policy shocks when using VAR analysis with simulated data, just as it did with the actual data from the ZLB.

E Additional Results for the Model with QE

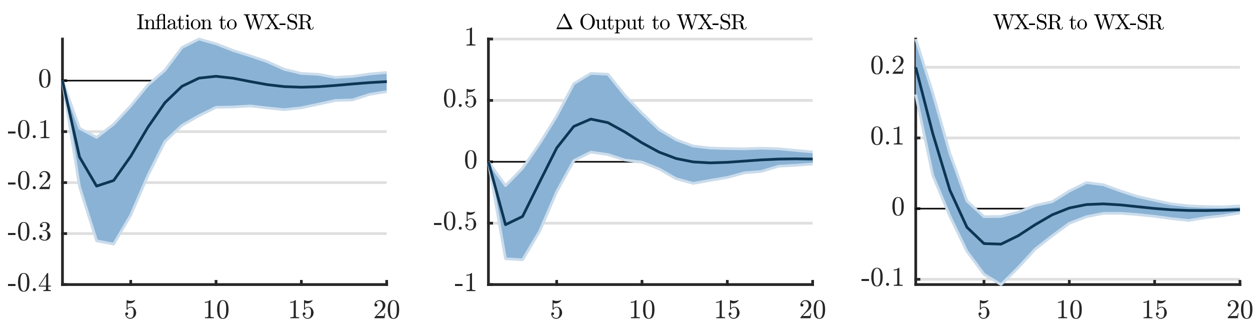
We present here some additional empirical results from the Boehl et al. (2022) model with QE. First, we validate that the macroeconomic effects of QE shocks are similar to what is presented in Boehl et al. (2022). Figure E.4 shows the change in the levels of output, consumption, and investment when QE shocks are set to zero from 2009:Q1 onwards. When QE shocks are turned off, output rises by almost 0.5 percent by 2015, with investment rising by at most almost 5 percent by the end of 2014.

Figure D.3: Impulse Responses for Simulation with Forward Guidance

(a) Estimated Responses Using the Structural Shadow Rate



(b) Estimated Responses Using the Wu-Xia Shadow Rate



Notes: This figure displays responses of inflation and output growth in quarterly percentage terms and the shadow rate in annualized percentage terms to a one-standard deviation monetary policy shock in a three-variable VAR with quarterly inflation, output growth, and the shadow rate estimated using simulated data with forward guidance. The shadow rate is ordered last for identification of monetary policy shocks using a Cholsky factorization of the forecast-error variance-covariance matrix. Panel (a) reports results for the structural shadow rate and panel (b) reports results for the shadow rate measure constructed following Wu and Xia (2016). The impulse responses are computed using a bootstrap procedure drawing residuals with replacement. The black lines correspond to mean responses and the bands show 90 percent equal-tailed bootstrap confidence intervals.

Figure E.4: Macroeconomic Effects of QE Shocks

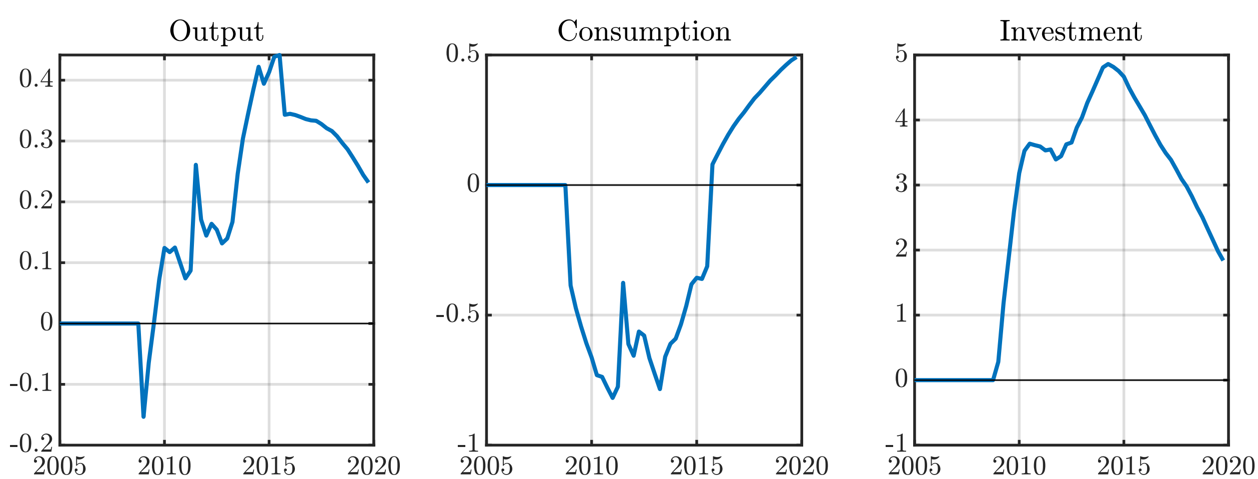
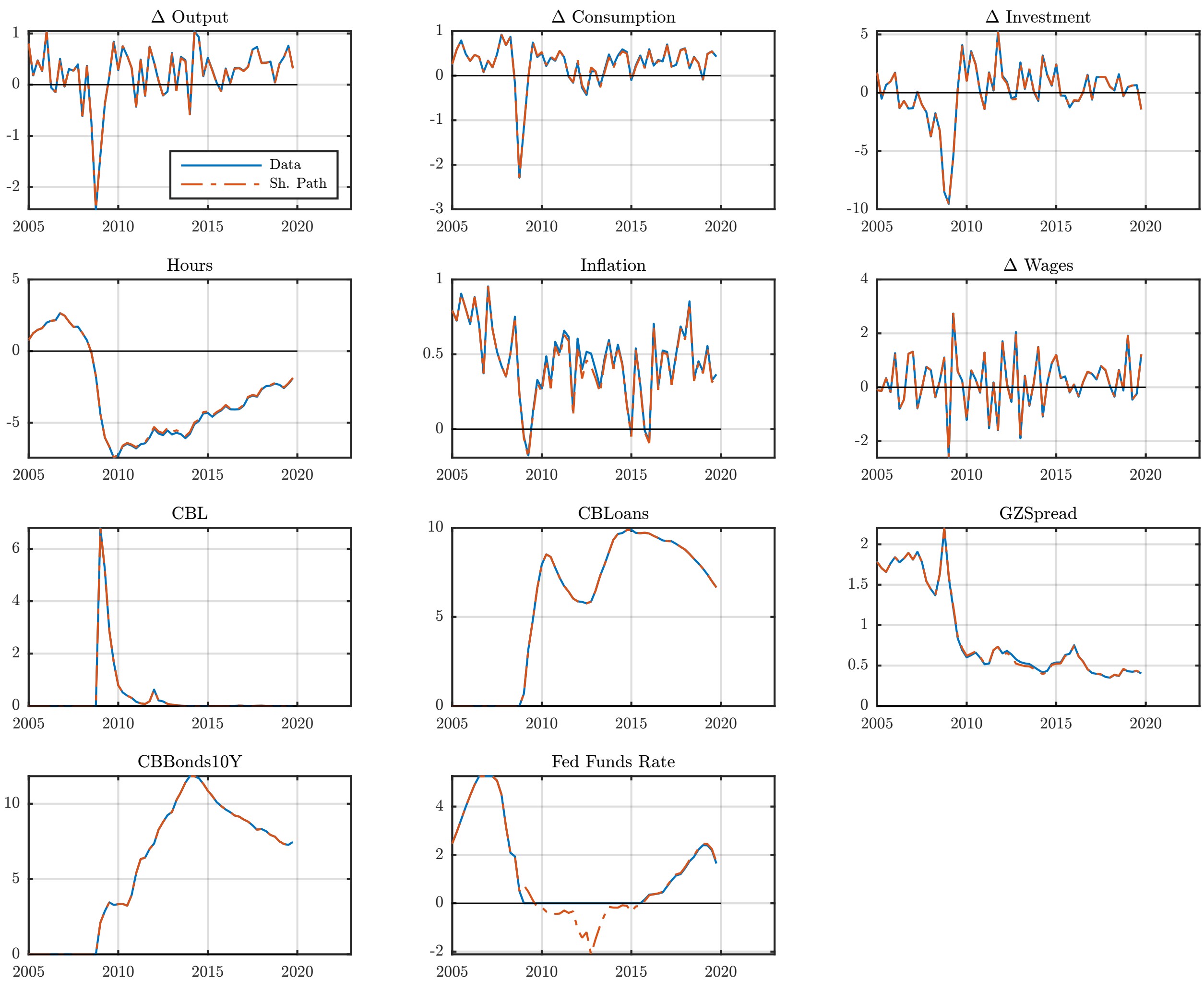


Figure E.5: Paths of Targets in the Shadow Economy with QE



F Model Estimation Using the Structural Shadow Rate

We report here the results for estimation of the Smets-Wouters model over 1984Q1 to 2019Q4, as in our baseline, but using the structural shadow federal funds rate as an observable in a linear setting, instead of explicitly accounting for the ZLB with nonlinear estimation, as in Kulish et al. (2017).

Posterior distributions of the parameters under our baseline and under the estimation with the shadow rate as an observable are given in Table F.1. The modal estimates are similar across the two estimations. Figure F.6 shows the filtered shocks at the modes for the two estimations. For the estimation with the shadow rate, we use the shadow rate as an observable when filtering for the shocks. The estimated shocks are similar, suggesting the structural shadow rate can be used as observed data when updating estimates of the Smets-Wouters model for a sample period that covers the ZLB.

Table F.1: Parameter Estimates for the Smets-Wouters Model Using the Shadow Rate

	Baseline Estimation with ZLB				Estimation with Shadow Rate			
Parameter	Mode	Median	5%	95%	Mode	Median	5%	95%
ϕ	6.98	6.79	5.05	8.74	4.54	5.47	3.81	7.99
σ_c	1.08	1.09	0.90	1.30	0.94	0.94	0.80	1.14
λ	0.27	0.26	0.19	0.35	0.34	0.34	0.25	0.42
ξ_w	0.33	0.36	0.25	0.50	0.37	0.33	0.23	0.44
σ_l	0.89	0.95	0.56	1.49	0.80	0.96	0.49	1.70
ξ_p	0.93	0.93	0.91	0.95	0.91	0.87	0.79	0.94
ι_w	0.37	0.40	0.18	0.65	0.51	0.50	0.26	0.74
ι_p	0.17	0.19	0.09	0.34	0.24	0.27	0.14	0.42
ψ	0.74	0.74	0.58	0.87	0.87	0.84	0.71	0.93
ϕ_p	1.50	1.50	1.37	1.64	1.31	1.32	1.19	1.47
α_p	1.64	1.65	1.50	1.80	1.72	1.75	1.59	1.90
α_i	0.79	0.76	0.57	0.90	0.77	0.76	0.57	0.90
α_y	0.06	0.06	0.05	0.08	0.07	0.07	0.05	0.09
$\alpha_{\Delta y}$	0.17	0.17	0.13	0.20	0.16	0.15	0.12	0.19
$100\left(\beta^{-1}-1\right)$	0.16	0.17	0.08	0.30	0.16	0.16	0.08	0.29
γ	0.58	0.57	0.53	0.62	0.52	0.52	0.47	0.58
α	0.15	0.15	0.13	0.18	0.17	0.16	0.13	0.19
$\bar{i}_4$	0.04	0.04	-0.00	0.08	0.08	0.08	0.02	0.15
$\bar{i}_{20}$	0.19	0.19	0.12	0.26	0.28	0.26	0.17	0.35
$\bar{l}$	2.31	2.25	0.14	3.91	2.69	2.50	1.30	3.64
	Persistence and Variances of Exogenous Processes							
ρ_a	0.93	0.93	0.91	0.95	0.94	0.94	0.91	0.96
ρ_b	0.98	0.98	0.97	0.99	0.98	0.98	0.97	0.98
ρ_g	0.99	0.99	0.98	1.00	0.99	0.98	0.97	0.99
ρ_i	0.82	0.81	0.73	0.88	0.83	0.81	0.73	0.88
ρ_p	0.80	0.79	0.69	0.87	0.90	0.87	0.75	0.93
ρ_w	0.99	0.99	0.99	1.00	0.99	0.99	0.98	1.00
μ_p	0.72	0.69	0.52	0.81	0.74	0.72	0.53	0.82
μ_w	0.74	0.73	0.56	0.86	0.66	0.65	0.48	0.80
ρ_{ga}	0.48	0.47	0.35	0.59	0.48	0.47	0.35	0.58
ρ_η	0.78	0.78	0.69	0.87	0.85	0.84	0.76	0.91
σ_a	0.42	0.42	0.38	0.47	0.41	0.42	0.38	0.47
σ_b	0.04	0.04	0.03	0.05	0.05	0.05	0.04	0.06
σ_g	0.37	0.36	0.33	0.40	0.35	0.35	0.32	0.39
σ_i	0.22	0.22	0.19	0.27	0.25	0.24	0.20	0.29
σ_m	0.13	0.13	0.11	0.14	0.16	0.16	0.14	0.18
σ_p	0.11	0.10	0.09	0.12	0.09	0.09	0.08	0.11
σ_w	0.49	0.50	0.42	0.62	0.51	0.52	0.44	0.66
σ_η	0.06	0.06	0.05	0.07	0.07	0.07	0.06	0.08
$\sigma_{i,4}$	0.01	0.01	0.01	0.02	0.02	0.02	0.01	0.04
$\sigma_{i,20}$	0.09	0.09	0.08	0.10	0.11	0.11	0.10	0.13
	Change in Trend Growth							
$\Delta\bar{\gamma}$	-0.18	-0.18	-0.23	-0.13	-0.17	-0.17	-0.24	-0.10
Date of $\Delta\bar{\gamma}$	2003Q4	2003Q4	2001Q1	2005Q2	2005Q1	2005Q1	2001Q1	2006Q4

Figure F.6: Filtered Shocks Accounting for the ZLB versus Using the Shadow Rate