Online Appendix: Supply Chain Constraints and Inflation

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1 Quantitative Model

In this appendix, we discuss the quantitative version of the general model. We start by presenting the log-linear approximation of the model equilibrium conditions and the stochastic processes for exogenous variables. We then present the calibration and model estimation procedure. We provide supplemental results on parameter estimates, model fit, and robustness checks in later sections.

1.1 Log-Linearization of the Model Equilibrium Conditions

To construct the piece-wise linear solution to the model, we log-linearize the model equilibrium conditions for both the unconstrained and constrained equilibria around the steady state.

We normalize Home prices relative to the domestic price level, and we denote "real" prices with the letter r attached to the price. Further, lower case variables with hats denote log deviations from steady state. For example, the log deviation in the real wage from steady state is given by $\widehat{rw}_t = \hat{w}_t - \hat{p}_t$, while the real price of home output in sector s is $\widehat{rp}_{Ht}(s) = \hat{p}_{Ht}(s) - \hat{p}_t$, and so on.[1] Foreign currency prices (denoted by stars) are normalized relative to the foreign price level; for example, foreign real marginal costs are $\widehat{rmc}^*_t(s) = \widehat{mc}^*_t - \hat{p}^*_t$. We also define deviations in the value of constraints from steady state: $\hat{\bar{y}}_t(1) = \ln \bar{Y}_t(1) - \ln \bar{Y}_0(1)$ and $\hat{\bar{y}}^*_t(1) = \ln \bar{Y}^*_t(1) - \ln \bar{Y}^*_0(1)$. Finally, to reduce the number of potential foreign shocks, we assume that foreign export demand is given by $X^*_t(s) = \varpi(s)\left(\frac{P^*_t}{P^*_t(s)}\right)^{-\sigma(s)} C^*_t$, where we treat $\frac{P^*_t}{P^*_t(s)}$ and $\varpi(s)$ as constants, so $\hat{x}^*_t(s) = \hat{c}^*_t$.

We present the log-linear equilibrium conditions in Tables 1 and 2. Table 1 contains equilibrium conditions that hold in both unconstrained and constrained equilibria. Table 2 collects equilibrium conditions that differ across equilibria, depending on which constraints are slack or binding.

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[1] For completeness, $\widehat{rp}_t(s) = \hat{p}_t(s) - \hat{p}_t$, $\widehat{rp}_{Ft}(s) = \hat{p}_{Ft}(s) - \hat{p}_t$, $\widehat{rmc}_t(s) = \widehat{mc}_t(s) - \hat{p}_t$, $\widehat{rp}_{Mt}(s) = \hat{p}_{Mt}(s) - \hat{p}_t$, $\widehat{rp}_{Mt}(s', s) = \hat{p}_{Mt}(s', s) - \hat{p}_t$.

Table 1: Common Equilibrium Conditions for Unconstrained and Constrained Equilibria

Labor Supply	$-\rho\hat{c}_t+\widehat{rw}_t=\psi\hat{l}_t$
Consumption Allocation	$\hat{c}_t(s)=\hat{\zeta}_t(s)-\vartheta\widehat{rp}_t(s)+\hat{c}_t$ with $\sum_s\zeta_0(s)\hat{\zeta}_t(s)=0$ $\hat{c}_{Ht}(s)=-\varepsilon(s)\left(\widehat{rp}_{Ht}(s)-\widehat{rp}_t(s)\right)+\hat{c}_t(s)$ $\hat{c}_{Ft}(s)=-\varepsilon(s)\left(\widehat{rp}_{Ft}(s)-\widehat{rp}_t(s)\right)+\hat{c}_t(s)$
Euler Equation	$0=\mathbf{E}_t\hat{\Theta}_{t+1}-\hat{\Theta}_t-\rho\left(\mathbf{E}_t\hat{c}_{t+1}-\hat{c}_t\right)+i_t-\mathbf{E}_t\pi_{t+1}$
Consumer Prices	$0=\sum_s\left[\zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\right]\left[\widehat{rp}_t(s)+\frac{1}{1-\vartheta}\hat{\zeta}_t(s)\right]$ $\widehat{rp}_t(s)=\gamma(s)\left(\frac{P_{H0}(s)}{P_0(s)}\right)^{1-\varepsilon(s)}\widehat{rp}_{Ht}(s)+(1-\gamma(s))\left(\frac{P_{CF0}(s)}{P_0(s)}\right)^{1-\varepsilon(s)}\widehat{rp}_{Ft}(s)$
Labor Demand	$\widehat{rw}_t+\hat{l}_t(s)=\widehat{rmc}_t(s)+\hat{y}_t(s)$
Input Demand	$\widehat{rp}_{Mt}(s)+\hat{m}_t(s)=\widehat{rmc}_t(s)+\hat{y}_t(s)$ $\hat{m}_t(s',s)=-\kappa\left(\widehat{rp}_{Mt}(s',s)-\widehat{rp}_{Mt}(s)\right)+\hat{m}_t(s)$ $\hat{m}_{Ht}(s',s)=-\eta(s')\left(\widehat{rp}_{Ht}(s')-\widehat{rp}_{Mt}(s',s)\right)+\hat{m}_t(s',s)$ $\hat{m}_{Ft}(s',s)=-\eta(s')\left(\widehat{rp}_{FMt}(s')-\widehat{rp}_{Mt}(s',s)\right)+\hat{m}_t(s',s)$
Marginal Cost	$\widehat{rmc}_t(s)=-\hat{z}_t(s)+(1-\alpha(s))\widehat{rw}_t(s)+\alpha(s)\widehat{rp}_{Mt}(s)$
Input Prices	$\widehat{rp}_{Mt}(s)=\sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)\left(\frac{P_0(s',s)}{P_{M0}(s)}\right)^{1-\kappa}\widehat{rp}_{Mt}(s',s)$ $\widehat{rp}_{Mt}(s',s)=$ $\xi(s',s)\left(\frac{P_{H0}(s')}{P_0(s',s)}\right)^{1-\eta(s')}\widehat{rp}_{Ht}(s')+(1-\xi(s',s))\left(\frac{P_{MFt}(s')}{P_0(s',s)}\right)^{1-\eta(s')}\widehat{rp}_{FMt}(s')$
Consumption Import Pricing	$\pi_{Ft}(s)=\frac{\varepsilon-1}{\phi(s)}\left(\widehat{rmc}^*_t(s)+\hat{q}_t-\widehat{rp}_{Ft}(s)\right)+\beta\mathbf{E}_t\pi_{Ft+1}(s)$
Domestic Pricing for Services	$\pi_{Ht}(2)=\frac{\varepsilon-1}{\phi(2)}\left(\widehat{rmc}_t(2)-\widehat{rp}_{Ht}(2)\right)+\beta\mathbf{E}_t\pi_{Ht+1}(2)$
Input Import Pricing for Services	$\pi_{MFt}(2)=\frac{\varepsilon-1}{\phi(2)}\left(\widehat{rmc}^*_t(2)+\hat{q}_t-\widehat{rp}_{FMt}(2)\right)+\beta\mathbf{E}_t\pi_{FMt+1}(2)$
Market Clearing	$\hat{y}_t(s)=\left(\frac{C_{H0}(s)}{Y_0(s)}\right)\hat{c}_{Ht}(s)+\sum_{s'}\left(\frac{M_{H0}(s,s')}{Y_0(s)}\right)\hat{m}_{Ht}(s,s')+\left(\frac{X_0(s)}{Y_0(s)}\right)\hat{x}_t(s)$ $\hat{x}_t(s)=-\sigma(s)\left(\widehat{rp}_{Ht}(s)-\hat{q}_t\right)+\hat{c}^*_t$ $\hat{y}^*_{Ct}(s)=\hat{c}_{Ft}(s)$ $\hat{y}^*_{Mt}(s)=\sum_{s'}\left(\frac{M_{F0}(s,s')}{Y^*_{M0}(s)}\right)\hat{m}_{Ft}(s,s')$ $\hat{\Theta}_t-\rho\left(\hat{c}_t-\hat{c}^*_t\right)+\hat{q}_t=0$ $\sum_s\left(\frac{L_0(s)}{L_0}\right)\hat{l}_t(s)=\hat{l}_t$
Monetary Policy Rule	$i_t=\rho_i i_{t-1}+\omega(1-\rho_i)\hat{\bar{\pi}}_t+(1-\rho_i)\rho_y\hat{y}_t+\hat{\Psi}_t$ with $\hat{y}_t=\sum_s\left(\frac{P_0(s)Y_0(s)}{Y_0}\right)\hat{y}_t(s)$
Auxiliary Inflation Definitions	$\pi_{Ht}(s)=\widehat{rp}_{Ht}(s)-\widehat{rp}_{Ht-1}(s)+\pi_t$ $\pi_{Ft}(s)=\widehat{rp}_{Ft}(s)-\widehat{rp}_{Ft-1}(s)+\pi_t$ $\pi_{FMt}(s)=\widehat{rp}_{FMt}(s)-\widehat{rp}_{FMt-1}(s)+\pi_t$ $\bar{\pi}_t=\pi_t+\sum_s\zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\left(\widehat{rp}_t(s)-\widehat{rp}_{t-1}(s)\right)$

Table 2: Equilibrium Conditions with Binding Constraints for Goods

Panel A: Only Domestic Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\bar{\mu}}_t(1) + \beta \mathbf{E}_t \pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)\right) + \beta \mathbf{E}_t \pi_{FMt+1}(1)$
Domestic Constraint	$\hat{y}_t(1) = \hat{\bar{y}}_t(1) + \ln(\bar{Y}_0(1)/Y_0(1))$
Panel B: Only Foreign Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)\right) + \beta \mathbf{E}_t \pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\bar{\mu}}^*_t(1) + \beta \mathbf{E}_t \pi_{MFt+1}(1)$
Import Constraint	$\hat{y}^*_t(1) = \hat{\bar{y}}^*_t(1) + \ln(\bar{Y}^*_0(1)/Y^*_0(1))$
Panel C: Both Constraints Bind	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\bar{\mu}}_t(1) + \beta \mathbf{E}_t \pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\bar{\mu}}^*_t(1) + \beta \mathbf{E}_t \pi_{MFt+1}(1)$
Domestic Constraint	$\hat{y}_t(1) = \hat{\bar{y}}_t(1) + \ln(\bar{Y}_0(1)/Y_0(1))$
Import Constraint	$\hat{y}^*_t(1) = \hat{\bar{y}}^*_t(1) + \ln(\bar{Y}^*_0(1)/Y^*_0(1))$

Table 3: Calibration

Parameter	Value	Reference/Target
ψ	2	Labor supply elasticity of 0.5
ρ	2	Intertemporal elasticity of substitution of 0.5
β	.995	Annual risk-free real rate of 2%
ϑ	0.5	Elasticity of substitution across sectors in consumption
ε	4	Elasticity of substitution between varieties
κ	0.3	Elasticity of substitution for inputs across sectors
$\sigma(s)$	1.5	Export demand elasticity
ϕ	35.468	To yield first order equivalence to Calvo pricing, with average price duration of 4 quarters.

1.2 Stochastic Processes

We collect log deviations in exogenous domestic and foreign variables – $\hat{\Theta}_t$, $\hat{\zeta}_t(1)$, $\hat{c}_t^*$, and $\left\{\hat{z}_t(s), \widehat{rmc}_t^*\right\}_s$ – into vector $\hat{F}_t$, and we assume that $\hat{F}_t$ is a first-order vector autoregressive process, as in $\hat{F}_t = \Lambda \hat{F}_{t-1} + \varepsilon_t$, where Λ is a diagonal matrix of autoregressive coefficients (denoted λ_x for variable x) and ε_t is a vector of shocks.[2] We assume the vector of shocks has a multivariate normal distribution, with $var(\varepsilon_t) = \Sigma$ having diagonal elements σ_x^2 for each variable x and zeros off diagonal, and $cov(\varepsilon_t, \varepsilon_{t+s}) = 0$ at all leads and lags ($s \neq 0$).

We assume that the constraint for imports of consumption goods is not binding in all periods. Similarly, we assume that constraints are not binding for services. This leaves $\bar{Y}_t(1)$ and $\bar{Y}_{Mt}^*(1)$ as the remaining constraints.[3] We assume they follow autoregressive processes:

$$\hat{\bar{y}}_t(1) = \rho_{\bar{y}} \hat{\bar{y}}_{t-1}(1) + \varepsilon_{\bar{y}t}(1) \tag{1}$$

$$\hat{\bar{y}}_t^*(1) = \rho_{\bar{y}^*} \hat{\bar{y}}_{t-1}^*(1) + \varepsilon_{\bar{y}^*t}(1), \tag{2}$$

where $\gamma \in (0,1)$ and $\varepsilon_{\bar{y}t}(1)$ and $\varepsilon_{\bar{y}^*t}(1)$ denote capacity shocks. We assume the capacity shocks are independent, mean zero normal random variables, with variances $var(\varepsilon_{\bar{y}t}(1)) = \sigma_{\bar{y}}^2$ and $var(\varepsilon_{\bar{y}^*t}(1)) = \sigma_{\bar{y}^*}^2$, and $cov(\varepsilon_{\bar{y}t}(1), \varepsilon_{\bar{y},t+s}(1)) = cov(\varepsilon_{\bar{y}^*t}(1), \varepsilon_{\bar{y}^*,t+s}(1)) = 0$ at all leads and lags ($s \neq 0$).

1.3 Calibration

We set values for a subset of the structural parameters based on standard values in the literature, which we collect in Table 3. We use input-output data compiled by the US Bureau of Economic Analysis to pin down values for steady-state expenditure shares. We

[2] We assume that foreign real marginal costs are equal for goods and services: $\widehat{rmc}_t^*(s) = \widehat{rmc}_t^*$. Because the services sector is relatively closed, this restriction is unimportant.

[3] Reliable data on capacity at high frequencies is generally not available, so we cannot include capacity among the observable variables. Existing data on capacity, such as the series used by the Federal Reserve Board to produce its G.17 series, are not well suited to our exercise. One reason is that the Federal Reserve's survey data is collected at an annual frequency, while we are interested in capacity dynamics at higher frequencies. Further, the capacity estimates are nearly time invariant at medium term (multi-year) frequencies, which means that capacity utilization mainly reflects the dynamics of industrial production. A second problem concerns how capacity survey questions are posed to firms. Specifically, the survey instrument asks firms to report how much they could produce if they had access to all the labor and materials they need to produce. This question fails to capture key aspects of production that effectively limit true capacity. For example, firms make predetermined choices about essential labor, material inputs, and other aspects of the production process that limit their ability to produce today, but this would be not be picked up by the survey.

Table 4: Steady State Shares

Model and Data	Description
$\begin{bmatrix} \zeta_0(1)\left(\frac{P_0(1)}{P_0}\right)^{1-\vartheta} \\ \zeta_0(2)\left(\frac{P_0(2)}{P_0}\right)^{1-\vartheta} \end{bmatrix} = \begin{bmatrix} 0.26 \\ 0.74 \end{bmatrix}$	Sector shares in consumption expenditure
$\begin{bmatrix} \gamma(1)\left(\frac{P_{H0}(1)}{P_0(1)}\right)^{1-\varepsilon} \\ \gamma(2)\left(\frac{P_{H0}(2)}{P_0(2)}\right)^{1-\varepsilon} \end{bmatrix} = \begin{bmatrix} 0.80 \\ 0.995 \end{bmatrix}$	Home shares in consumption expenditure by sector
$\begin{bmatrix} \alpha(1) \\ \alpha(2) \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$	Input expenditure share of gross output
$\begin{bmatrix} \left(\frac{\alpha(1,1)}{\alpha(1)}\right)\left(\frac{P_0(1,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(1,2)}{\alpha(2)}\right)\left(\frac{P_0(1,2)}{P_{M0}(2)}\right)^{1-\kappa} \\ \left(\frac{\alpha(2,1)}{\alpha(1)}\right)\left(\frac{P_0(2,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(2,2)}{\alpha(2)}\right)\left(\frac{P_0(2,2)}{P_{M0}(2)}\right)^{1-\kappa} \end{bmatrix} = \begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix}$	Sector shares in input expenditure
$\begin{bmatrix} \xi(1,1)\left(\frac{P_{H0}(1)}{P_0(1,1)}\right)^{1-\eta} & \xi(1,2)\left(\frac{P_{H0}(1)}{P_0(1,2)}\right)^{1-\eta} \\ \xi(2,1)\left(\frac{P_{H0}(2)}{P_0(2,1)}\right)^{1-\eta} & \xi(2,2)\left(\frac{P_{H0}(2)}{P_0(2,2)}\right)^{1-\eta} \end{bmatrix} = \begin{bmatrix} 0.77 & 0.84 \\ 0.99 & 0.98 \end{bmatrix}$	Home shares in input expenditure
$\begin{bmatrix} \frac{C_{H0}(1)}{Y_0(1)} & \frac{M_{H0}(1,1)}{Y_0(1)} & \frac{M_{H0}(1,2)}{Y_0(1)} & \frac{X_0(1)}{Y_0(1)} \\ \frac{C_{H0}(2)}{Y_0(2)} & \frac{M_{H0}(2,1)}{Y_0(2)} & \frac{M_{H0}(2,2)}{Y_0(2)} & \frac{X_0(2)}{Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.41 & 0.32 & 0.16 & 0.11 \\ 0.61 & 0.07 & 0.29 & 0.03 \end{bmatrix}$	Domestic output allocation
$\begin{bmatrix} \frac{M_{F0}(1,1)}{Y^*_{M0}(1)} & \frac{M_{F0}(1,2)}{Y^*_{M0}(1)} \\ \frac{M_{F0}(2,1)}{Y^*_{M0}(2)} & \frac{M_{F0}(2,2)}{Y^*_{M0}(2)} \end{bmatrix} = \begin{bmatrix} 0.76 & 0.24 \\ 0.08 & 0.92 \end{bmatrix}$	Foreign output allocation for inputs
$\begin{bmatrix} \frac{P_{H0}(1)Y_0(1)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \\ \frac{P_{H0}(2)Y_0(2)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.29 \\ 0.71 \end{bmatrix}$	Sector shares in gross output

report these shares, which reflect mean values over the 1997-2018 period, in Table 4, along with their corresponding definitions in the model.

1.4 Estimation Procedure

Building on Kulish, Morley and Robinson (2017) and Kulish and Pagan (2017), we treat the duration of the potentially binding constraints as an estimable parameter. To explain the method, we first discuss how to solve the model for given constraint durations, and then we describe the estimation procedure.

1.4.1 Solving the Model for Given Durations

To construct a piecewise linear solution to the model, we take linear approximations of the model equilibrium for four regimes: the unconstrained regime, a second regime in which only domestic constraints bind, a third regime in which foreign constraints bind, and a fourth regime in which both constraints bind. Further, the linear approximations are all taken around the non-stochastic (unconstrained) steady state of the model. The solution procedure combines these local approximations to solve for the policy function.

The linear approximation to the unconstrained system can be written as:

$$\mathbf{A}X_t = \mathbf{C} + \mathbf{B}X_{t-1} + \mathbf{D}\mathbb{E}_t X_{t+1} + \mathbf{F}\varepsilon_t,$$

where x_t is an $n \times 1$ vector of model variables, ε_t is an $l \times 1$ vector of structural shocks, and $\mathbf{A}$, $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{F}$ are conformable matrices determined by the structural equations. If agents expect the economy to remain unconstrained from date t forward, then standard rational expectations solution procedures imply that the reduced-form solution is given by: $X_t = \mathbf{J} + \mathbf{Q}X_{t-1} + \mathbf{G}\varepsilon_t$, where $\mathbf{J}$, $\mathbf{Q}$, and $\mathbf{G}$ describe the policy function and model dynamics.

There are three regimes in which one or both constraints bind, and let us index these by $r \in \{1,2,3\}$. Then we can express the linear approximation to the model equilibrium in each case as:

$$\bar{\mathbf{A}}_r X_t = \bar{\mathbf{C}}_r + \bar{\mathbf{B}}_r X_{t-1} + \bar{\mathbf{D}}_r \mathbb{E}_t X_{t+1} + \bar{\mathbf{F}}_r \varepsilon_t,$$

where $\bar{\mathbf{A}}_r$, $\bar{\mathbf{C}}_r$, $\bar{\mathbf{B}}_r$, $\bar{\mathbf{D}}_r$, and $\bar{\mathbf{F}}_r$ are conformable matrices that correspond to the structural equations for each.

We summarize the expected evolution of regimes from a given date t forward by the durations that the individual constraints are expected to bind, as in $\mathbf{d}_t = [d_t, d_t^*]$. To fix ideas, suppose that $d_t = 1$, which means that the domestic constraint binds today, and then is expected to be slack in the future. Further, suppose that $d_t^* = 0$, so the foreign constraint is slack today and in the future. This implies that the constrained system governs model responses in period t and then the unconstrained system applies thereafter. Working backwards from the unconstrained solution, then $\mathbb{E}_t X_{t+1} = \mathbf{J} + \mathbf{Q}X_t$, so then $\bar{\mathbf{A}}_1 X_t = \bar{\mathbf{C}}_1 + \bar{\mathbf{B}}_1 X_{t-1} + \bar{\mathbf{D}}_1 (\mathbf{J} + \mathbf{Q}X_t) + \bar{\mathbf{F}}_1 \varepsilon_t$, where $r = 1$ is the system that applies when the domestic constraint binds and the foreign constraint is slack. Solving this linear equation yields the reduced form solution for X_t.

Generalizing this idea, the system will evolve according to:

$$\mathbf{A}_t X_t = \mathbf{C}_t + \mathbf{B}_t X_{t-1} + \mathbf{D}_t \mathbb{E}_t X_{t+1} + \mathbf{F}_t \varepsilon_t, \tag{3}$$

where $\mathbf{A}_t$, $\mathbf{C}_t$, $\mathbf{B}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$ are the structural matrices that apply at date t. Then the piecewise linear solution is given by:

$$X_t = \mathbf{J}_t + \mathbf{Q}_t X_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{4}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are determined via the following backward recursion, which is initialized as starting from the unconstrained solution:

$$\begin{aligned}
\mathbf{Q}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} \mathbf{B}_t \\
\mathbf{J}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} (\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}) \\
\mathbf{G}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} \mathbf{F}_t.
\end{aligned} \tag{5}$$

At this point, it is useful to note that this recursive solution coincides with the recursion employed by the Dynare OccBin toolkit to obtain policy functions for a given guess about the sequence of regimes. The Occbin toolkit then proceeds to verify whether the guess about the sequence of regimes is consistent with model equilibrium, given the current value of the shocks. Put differently, it solves for endogenous constraint durations. While we do not discuss this second step here, we do solve for endogenous durations (using Occbin) when we analyze counterfactuals in the model. We also take the dependence of $\mathbf{d}_t$ on ε_t into account in the estimation procedure, with details below.

While Equations 4 and 5 present the model solution for a given anticipated sequence of regimes, note that the anticipated sequence changes as durations evolve over time. The duration $\mathbf{d}_t$ implies a particular sequence of regimes anticipated at dates $t+1$, $t+2$, etc. Given this sequence and the maintained assumption that agents do not anticipate future shocks, one then uses the recursion above to solve for the associated policy matrices: $\mathbf{J}(\mathbf{d}_t, \theta), \mathbf{Q}(\mathbf{d}_t, \theta)$, and $\mathbf{G}(\mathbf{d}_t, \theta)$, where the notation captures the dependence of these matrices on $\mathbf{d}_t$. At date $t+1$, a new value for durations ($\mathbf{d}_{t+1}$) will be realized, and one then solves the recursion anew to obtain $\mathbf{J}(\mathbf{d}_{t+1}, \theta), \mathbf{Q}(\mathbf{d}_{t+1}, \theta)$, and $\mathbf{G}(\mathbf{d}_{t+1}, \theta)$. And so on. The state (transition) equation of the model then features time-varying coefficients:

$$X_t = \mathbf{J}(\mathbf{d}_t, \theta) + \mathbf{Q}(\mathbf{d}_t, \theta) X_{t-1} + \mathbf{G}(\mathbf{d}_t, \theta)\, \varepsilon_t. \tag{6}$$

When $\mathbf{d}_t = 0$, the unconstrained solution applies, so $\mathbf{J}(\mathbf{d}_t, \theta) = \mathbf{J}(\theta)$, $\mathbf{Q}(\mathbf{d}_t, \theta) = \mathbf{Q}(\theta)$, and $\mathbf{G}(\mathbf{d}_t, \theta) = \mathbf{G}(\theta)$ are time invariant.

1.4.2 Joint Estimation of Durations and Structural Parameters

We assume that a vector of observables (S_t) are linked to underlying model states via the measurement equation: $S_t = \mathbf{H}_t X_t + \nu_t$, where ν_t is an i.i.d. vector of normally distributed measurement errors and $\mathbf{H}_t$ is a conformable (potentially time-varying) matrix linking states to observables. Using this state space representation of the model, we can apply the Kalman filter to construct the Likelihood function $\mathscr{L}(\theta, \mathbf{d} | \{S_t\}_{t=1}^{T})$, where $\mathbf{d} = \{\mathbf{d}\}_{t=1}^{T}$ is the sequence of durations.

We put priors over structural parameters and independent priors over durations to construct the posterior, and then estimate the model via Bayesian Maximum Likelihood. We construct draws from the joint posterior distribution $p\left(\theta, \mathbf{d} | \{S_t\}_{t=1}^{T}\right)$ using a Metropolis-Hastings algorithm with two blocks – one for the structural parameters, which are continuous, and a second for the discrete duration parameters – as in Kulish, Morley and Robinson (2017). We use a uniform proposal density for the durations, between 0 (unconstrained) and a sufficiently large maximum duration. We discuss the priors in Section 1.4.4 below.

In evaluating proposed parameter and durations draws, we recognize that it is desirable for posterior estimates of constraint durations to be consistent with agents' forecasts about how long constraints will endogenously bind given shocks. To this end, we constrain admissible draws to enforce this constraint, in an approximate sense. For a given proposed joint parameter (θ^i) and duration draw ($\mathbf{d}^i$), we construct the piecewise linear solution for the model and use the Kalman filter to obtain smoothed structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^{T}$ and equilibrium variables $\{\tilde{X}_t^i\}_{t=1}^{T}$ given the data. At each sample period $\tau \in [1, \ldots, T]$, we then use the piecewise linear solution to project model outcomes forward given the state and current shock – $\left(\tilde{X}_{\tau-1}^i, \tilde{\varepsilon}_\tau^i\right)$, assuming that there are no anticipated future shocks.[4] We then check for violations of the output capacity constraints. If projected home or foreign output violates the constraints, then we reject the proposed parameter draw as inconsistent with model equilibrium. Otherwise, we accept the parameter draw, evaluate the likelihood, and proceed through the estimation algorithm. Under this procedure, we accept about 25% of the proposed parameter/duration draws, so the estimation proceeds at reasonable computa-

[4] Recall that in the absence of future shocks, agents anticipate that the model will return to the unconstrained state over time, where the duration of binding constraints ticks down toward zero in each passing period. We project model outcomes forward using this expected path for durations.

tional pace.

In this procedure, we reject the proposed draw when it implies that constraints will be violated in expectation. In turn, we accept draws for which constraints are satisfied. Strictly speaking, we do not explicitly check whether the duration $\mathbf{d}_\tau$ is equal to the endogenous equilibrium duration consistent with $\left(\tilde{X}^i_{\tau-1}, \tilde{\varepsilon}^i_\tau\right)$ in the model. Nonetheless, our approach provides a good approximation to model outcomes with endogenously binding constraints. To demonstrate this, we turn to simulation evidence.

1.4.3 Validating the Estimation Procedure

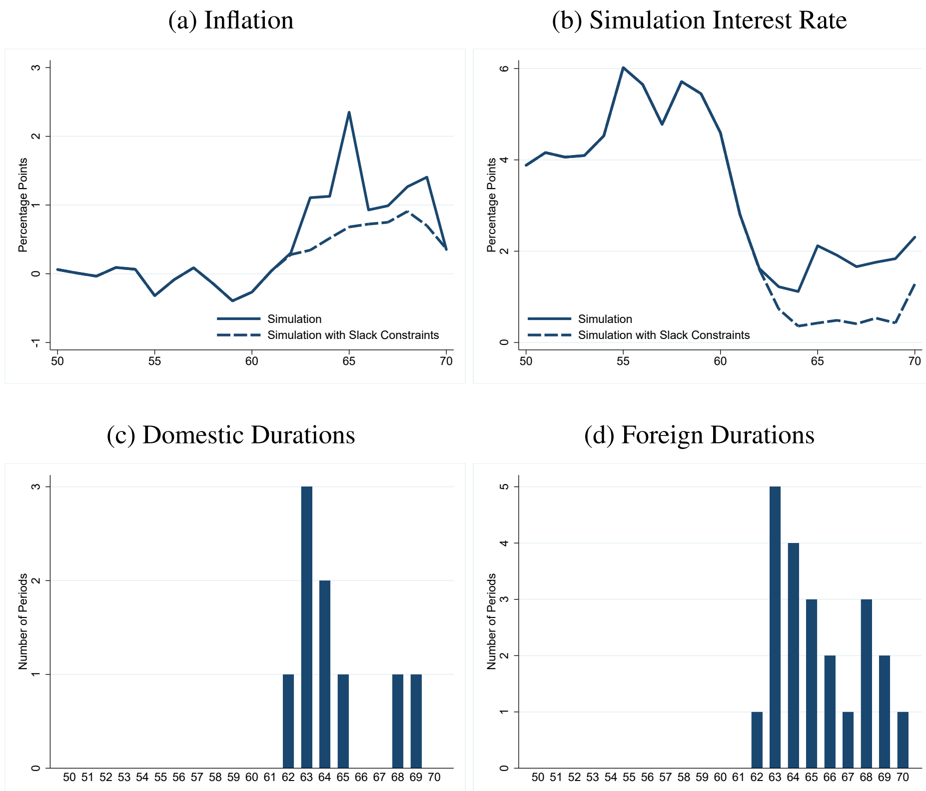
We provide results for two exercises to evaluate the accuracy of the estimation procedure. First, using simulated data, we demonstrate that the procedure is capable of identifying latent durations. Moreover, we show that it accurately recovers the corresponding multipliers on the constraints. Second, using results from the full estimation of the model with real world data, we compare smoothed inflation to simulated model results.

Estimation using Simulated Data The first step is to generate simulated data from the model, for given parameters.[5] Specifically, we draw a set of i.i.d. shocks for all variables over 70 quarters, and then impose a sequence of large, expansionary monetary policy shocks for quarters 61 to 69 (the size of the monetary policy shocks is set to three standard deviations). These shocks are large enough to trigger the capacity constraints. Since we can identify when the constraints are anticipated to bind in the simulation, we know the true sequence of durations.

We plot several simulated data series in Figure 1 to illustrate the set up, under both the maintained assumption that constraints are potentially binding and the counterfactual assumption that constraints are slack in all periods. The top two panels contain simulated inflation and the policy interest rate, while the implied durations for domestic and foreign constraints are recorded in the bottom two panels. The expansionary policy shocks evidently cause the policy rate to be low in periods 62 through 70, where inflation more than doubles at its peak relative to a simulation without capacity constraints.

[5]In contrast to the main quantitative model, we assume there is zero measurement error, so observable variables are equal to corresponding objects in the simulated data. Further, we set steady-state capacity levels so that there is 4% excess capacity for both home and foreign goods firms, so that we can trigger binding constraints with demand shocks alone (i.e., without negative capacity shocks). Remaining parameters are set to the mode of our baseline estimates.

Figure 1: Simulation



Note: Inflation is reported at a quarterly rate in percentage points. The interest rate is in percentage points.

Treating the simulated series as observable data, we illustrate that our empirical model is capable of identifying the true durations by directly examining model likelihood functions. Setting all parameters in the state and observation equations (other than durations) to their true values used to generate the simulated data, we compute the likelihood of the model for different values of the domestic and foreign durations, at given points in time. For example, setting the duration of the foreign constraint to its true value in a given period, we then trace out the likelihood over alternative values of the duration of the domestic constraint. And vice versa. We present the results from period 60, before the constraints become binding, through period 70, when the domestic capacity constraint stops binding and the foreign capacity constraint binds for one more quarter.

Figure 2 plots the inverse of the likelihood value across durations of the domestic constraint, where each panel corresponds to a period and the vertical line identifies the true duration. Figure 3 plots the corresponding results for the foreign constraint. As both figures illustrate, the inverse likelihood is minimized at the true values in every quarter, which confirms that the likelihood procedure we implement is able to discriminate between durations of different length. Importantly, for periods when the constraint does not bind, the likelihood is maximized at a duration value of zero.

Turning to estimation of the multipliers, we conduct a full estimation of the model using the simulated data, in which we estimate both the structural parameters and durations, as in the main analysis.[6]Here we focus on the estimated (smoothed) multipliers on the capacity constraints, as these play a key role in the framework. In Figure 4, we plot the true paths for the multipliers in the simulation, along with smoothed multipliers recovered via estimation. As is evident, the smoothed values of the multipliers match the exact simulation values closely, meaning the procedure does a good job at pinning down the reduced-form impact of constraints on inflation.

Smoothed vs. Simulated Inflation Drawing on results presented below and in the main text, we briefly compare smoothed inflation outcomes obtained via our estimation procedure with outcomes from the full structural model with endogenously binding constraints. This comparison serves to check that the empirical model with estimated durations replicates the outcomes of the structural model with endogenously binding constraints. Specifically, suppose we feed the structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^T$ obtained from our estimation procedure through the model, where structural parameters are set to their modal values and we use the OccBin procedure to solve for the endogenous duration of binding constraints in each period following the realization of shocks. We then plot this simulated inflation series to the smoothed inflation series from our estimation in Figure 5. As is evident, the two series track each other closely, so we conclude that our approach to capturing endogenously binding constraints in the estimation routine performs well.

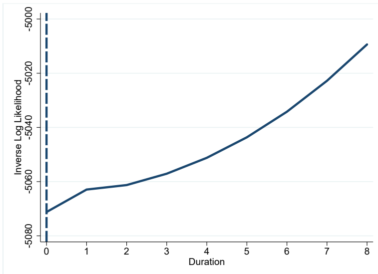
1.4.4 Priors

The full set of priors for structural parameters is included in Table 5. We use standard priors on autoregressive persistence of exogenous variables, parameters in the monetary policy rule, elasticities, and the standard deviations of most structural shocks. We set priors on the persistences of the exogenous capacity shocks that are wider than the priors on the other exogenous variables, as well as wide (uniform) priors on the standard deviations of the capacity shocks, since these are nonstandard parameters.

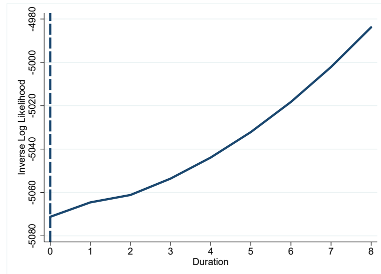
[6]In this estimation, we allow constraints to potentially bind for two quarters before the first period in which they actually bind in the simulated data. Further, we use the same priors here as in the baseline estimation.

Figure 2: Likelihood Over Domestic Durations

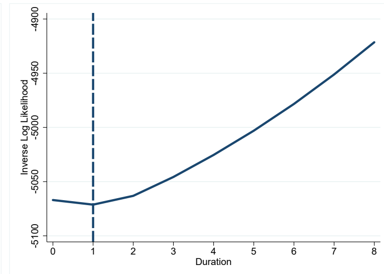
(a) Period 60



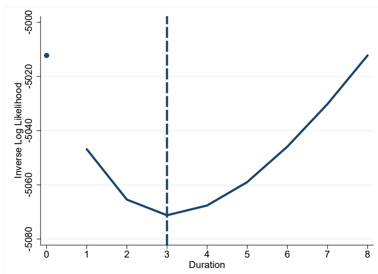
(b) Period 61



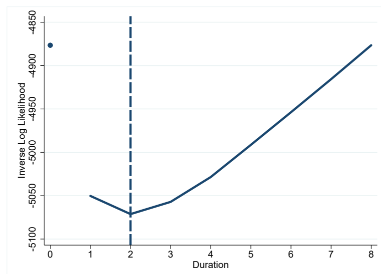
(c) Period 62



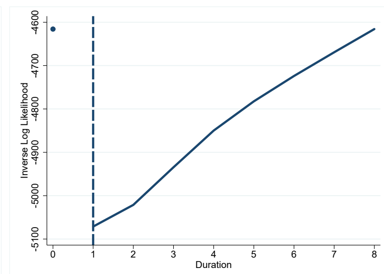
(d) Period 63



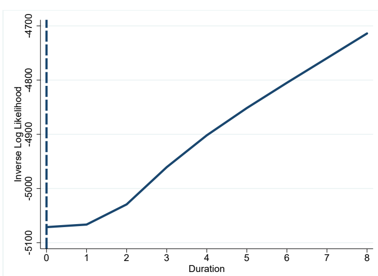
(e) Period 64



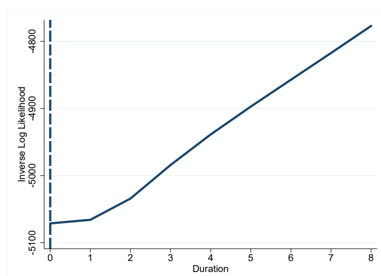
(f) Period 65



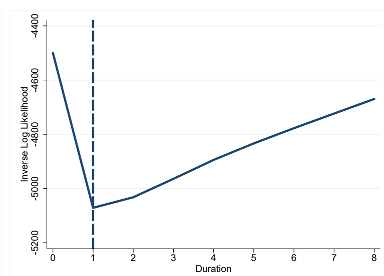
(g) Period 66



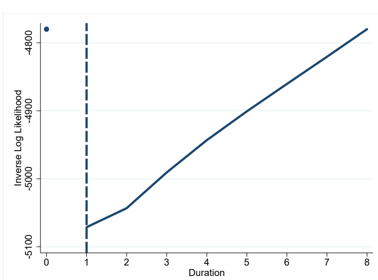
(h) Period 67



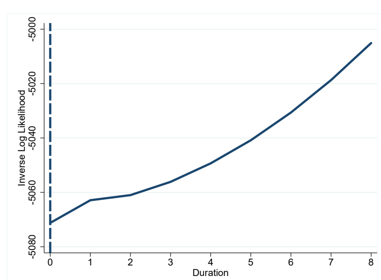
(i) Period 68



(j) Period 69



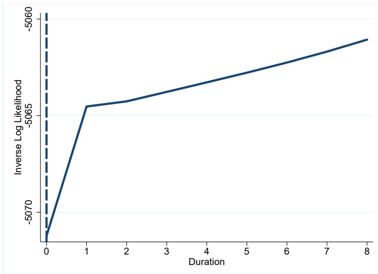
(k) Period 70



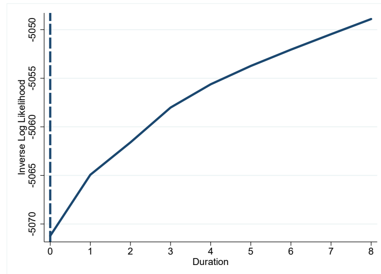
Note: The vertical dashed line marks the true duration of the constraint in the simulation for each period. In some figures, the dot denotes a value of the inverse likelihood that is substantially higher than the other values plotted in the figure; the dot is located at the maximal value depicted in the figure for visual reference.

Figure 3: Likelihood Over Foreign Durations

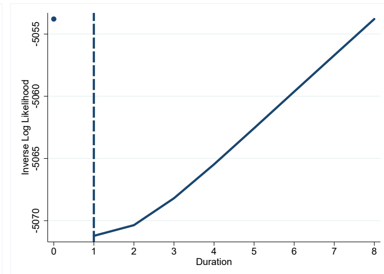
(a) Period 60



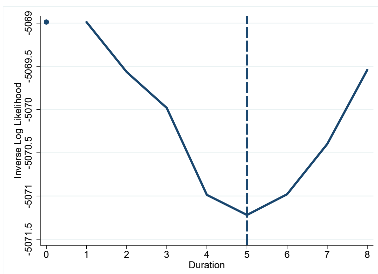
(b) Period 61



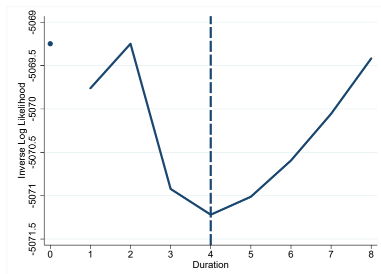
(c) Period 62



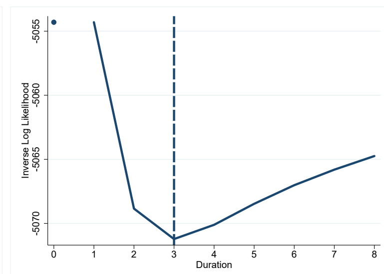
(d) Period 63



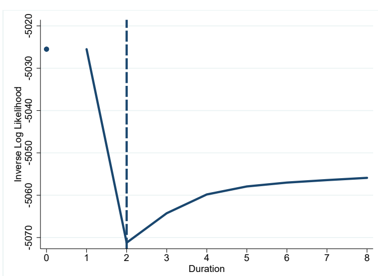
(e) Period 64



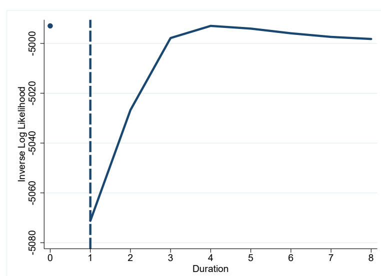
(f) Period 65



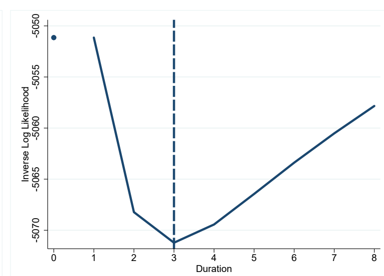
(g) Period 66



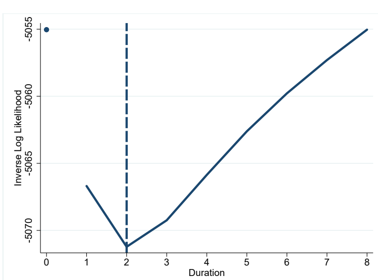
(h) Period 67



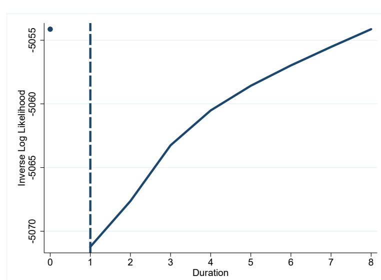
(i) Period 68



(j) Period 69



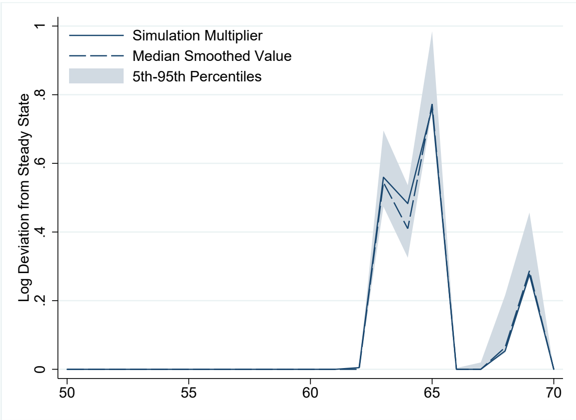
(k) Period 70



Note: The vertical dashed line marks the true duration of the constraint in the simulation for each period. In some figures, the dot denotes a value of the inverse likelihood that is substantially higher than the other values plotted in the figure; the dot is located at the maximal value depicted in the figure for visual reference.

Figure 4: Multipliers Capacity Constraints: Simulation vs. Estimation

(a) Multiplier on the Domestic Constraint



(b) Multiplier on the Foreign Constraint

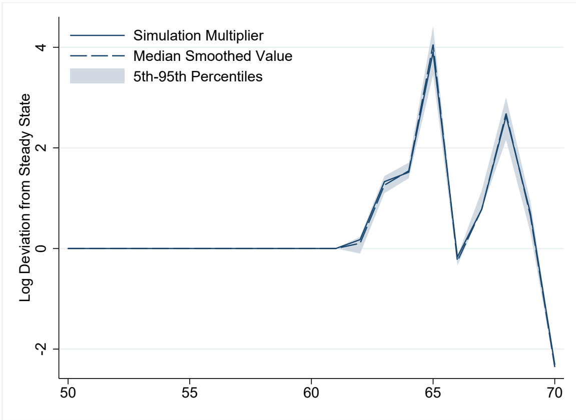
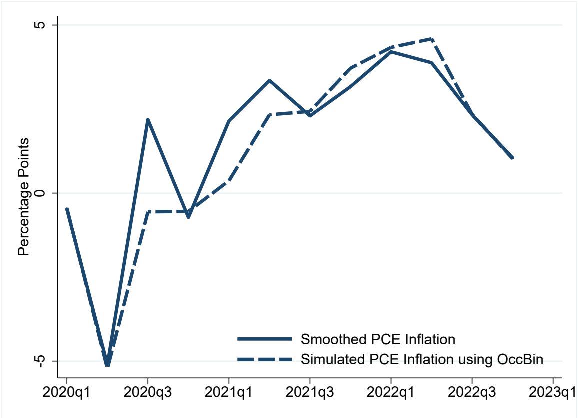


Figure 5: Comparison Between Smoothed Inflation and OccBin Simulated Inflation



Note: Smoothed PCE Inflation is Kalman-smoothed consumer price inflation, where the filter is parameterized using the modal values of structural parameters and durations from the empirical estimation. Simulated PCE Inflation using OccBin is obtained by simulating model responses to smoothed shocks (see the text for further description).

We set uniform priors on measurement errors associated with three inflation series – consumer price inflation for goods, imported input price inflation, and imported consumption goods price inflation. Further, looking forward, we will report below that the posterior estimates are pushed toward the boundary of the allowed parameter space for these parameters. The logic for constraining the measurement error parameters in this way is twofold. First, because our focus is on inflation outcomes and the role of constraints in driving them, we want to lean heavily on the realized data here. Second, we estimate the model using both pre-2020 and post-2020 data. As is evident in raw data series, the post-2020 COVID period features extreme variability in outcomes relative to the pre-2020 data. One way for the model to make sense of this is to assign very high measurement errors to the data. This is unpalatable from our perspective, as we wish to parse the actual data for this period. Thus, we effectively constrain the model to treat the post-2020 inflation data as an accurate representation of latent unobserved model variables. We envision experimenting with alternatives to this approach (e.g., allowing for different measurement error or shock processes before and after 2020), as thinking about how to model the COVID period evolves.

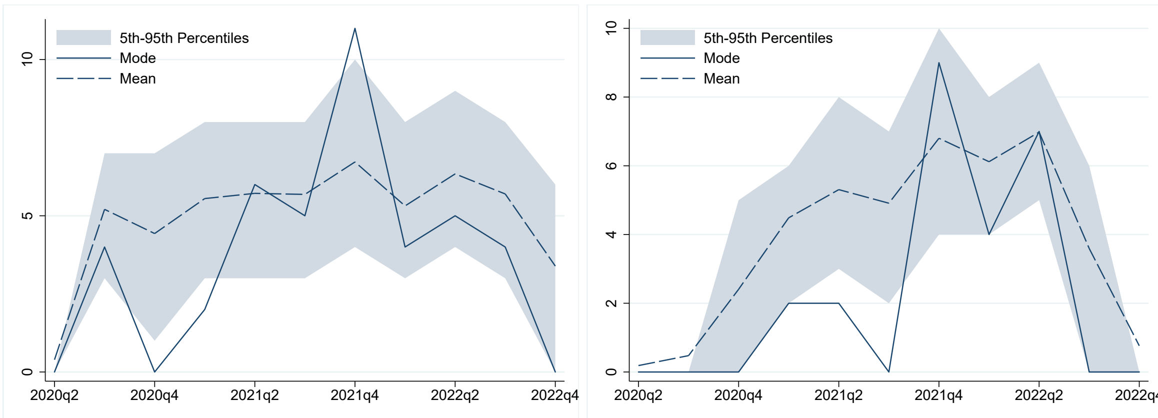
As noted in the main text, we allow constraints to potentially bind only starting in the second quarter of 2020. That is, we put zero mass on positive durations at all dates at/before 2020:Q1, which can be thought of as a dogmatic prior that constraints were not substantively important prior to the pandemic. Thereafter in each period, we place equal mass on durations of 0 to 4 quarters, summing to 60% total (12% on each discrete duration). We place 30% mass on durations of 5, 6, 7, and 8 quarters, again equally spread (7.5% each). The remaining 10% mass is spread equally over durations 9 through 12, and we place zero mass on durations longer than 12 quarters.

1.5 Estimation Results

In Table 5, we provide the mode, mean, and 5th-95th percentiles for the posterior distributions of the structural parameters. As noted in the text, we find that domestic and foreign goods inputs are complements on the production side, while domestic and foreign goods are substitutes in consumption. The Taylor rule coefficient on inflation is near 1.5, which is standard. Interest rates also depend positively on deviations of output from steady state, and the policy rule features a significant degree of inertia. The stochastic processes for shocks generally feature persistence, with auto-regressive coefficients generally between 0.7 and 0.9. Building on the discussion of measurement error above, we note that posterior

Figure 6: Posterior Distributions for Constraint Durations

(a) Domestic Constraint Durations (b) Foreign Constraint Durations



Note: At each date, there is a posterior distribution for constraint durations. Each figure presents the mean, mode, and interquartile range for this posterior distribution.

estimates for measurement errors on consumer goods price inflation and import price inflation are pushed toward the boundary of their prior distributions, reflecting tension in the model between fitting data before and during the COVID period. For all the other parameters, posterior distributions are generally well behaved, with single peaks well inside the allowable parameter space and reasonably tight distributions.

Turning to duration estimates, we plot statistics for the posterior distributions of domestic and foreign constraint durations in Figure 6. Due to skewness in the distributions, modal values for the duration (our preferred approach to summarizing the posterior distribution) are below the mean value in most periods. The time path for the duration estimates mimics the path of estimated multipliers on the constraints, as reported in the main text.

1.6 Model Fit

In the main text, we presented results on model fit for core inflation series. To evaluate model fit more broadly, we present data and smoothed values for the remaining observable variables in Figure 7.[7] For legibility in the figures, we focus on the 2017-2022 period – the key period leading up to and through our analysis. The model fits most series well, even capturing the whiplash dynamics of the data in 2020. The model struggles to replicate data on US labor productivity, particularly in 2020 for services. Through the lens of the model,

[7] We assume the interest rate is measured without error, so it is omitted here.

Table 5: Prior and Posterior Distributions for Structural Parameters

Panel A: Elasticity and Taylor Rule Parameters

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Consumption Armington Elasticity: ι	G	1.5	0.25	1.500	1.469	1.124	1.825
Input Armington Elasticity: η	G	0.5	0.15	0.549	0.563	0.362	0.796
Taylor Rule Inflation: ω	N	1.5	0.12	1.553	1.545	1.354	1.745
Taylor Rule Inertia:α_i	B	0.75	0.1	0.877	0.872	0.850	0.892
Taylor Rule Output: α_y	G	0.12	0.05	0.249	0.246	0.173	0.331

Panel B: Stochastic Processes

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Preference for Goods: σ_ζ	IG	1	2	0.314	0.403	0.192	0.642
Discount Rate: σ_Θ	IG	1	2	3.373	3.487	3.120	3.913
Foreign Costs: σ_{rmc*}	IG	1	2	2.194	2.297	1.955	2.717
Goods Productivity: $\sigma_{z(1)}$	IG	1	2	0.192	0.196	0.128	0.273
Services Productivity: $\sigma_{z(2)}$	IG	1	2	0.205	0.206	0.139	0.282
Foreign Constraint: $\sigma_{\bar{y}*}$	U	1	0.58	0.079	0.086	0.041	0.143
Domestic Constraint: $\sigma_{\bar{y}}$	U	1	0.58	0.019	0.025	0.012	0.050
Monetary Policy Shock: σ_i	IG	1	2	0.153	0.154	0.134	0.176
Preference for Goods: ρ_ζ	B	0.5	0.15	0.825	0.715	0.424	0.906
Discount Rate: ρ_Θ	B	0.5	0.15	0.727	0.727	0.670	0.780
Foreign Costs: ρ_{rmc*}	B	0.5	0.15	0.922	0.917	0.875	0.952
Goods Productivity: $\rho_{z(1)}$	B	0.5	0.10	0.542	0.538	0.371	0.697
Services Productivity: $\rho_{z(2)}$	B	0.5	0.15	0.911	0.813	0.434	0.946
Foreign Constraint: $\rho_{\bar{y}*}$	B	0.5	0.20	0.715	0.683	0.454	0.882
Domestic Constraint: $\rho_{\bar{y}}$	B	0.5	0.20	0.914	0.863	0.698	0.957

Panel C: Measurement Error

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Goods PCE: $\sigma^{\mathrm{me}}_{\mathrm{pceg}}$	IG	1	2	1.014	1.032	0.879	1.194
Services PCE: $\sigma^{\mathrm{me}}_{\mathrm{pces}}$	IG	1	2	0.664	0.663	0.558	0.773
Goods PCE Inflation: $\sigma^{\mathrm{me}}_{\pi(1)}$	U	0.25	.14	0.499	0.497	0.492	0.500
Services PCE Inflation: $\sigma^{\mathrm{me}}_{\pi(2)}$	IG	1	2	0.155	0.163	0.124	0.213
Imp. Input Goods Expenditure: $\sigma^{\mathrm{me}}_{\mathrm{inp}}$	IG	1	2	3.205	3.253	2.929	3.629
Imp. Consumption Goods Expenditure: $\sigma^{\mathrm{me}}_{\mathrm{fimp}}$	IG	1	2	2.868	2.948	2.659	3.287
Imp. Input Goods Inflation: $\sigma^{\mathrm{me}}_{\mathrm{inpp}}$	U	0.75	.43	1.498	1.487	1.463	1.499
Imp. Consumption Goods Inflation: $\sigma^{\mathrm{me}}_{\mathrm{fimp}}$	U	0.075	0.043	0.145	0.112	0.033	0.148
Goods Productivity: $\sigma^{\mathrm{me}}_{\mathrm{prod1}}$	IG	1	2	1.151	1.163	1.012	1.324
Services Productivity: $\sigma^{\mathrm{me}}_{\mathrm{prod2}}$	IG	1	2	1.053	1.059	0.930	1.203
Industrial Production: $\sigma^{\mathrm{me}}_{\mathrm{ip}}$	IG	1	2	0.930	0.961	0.836	1.100
Aggregate Nominal GDP: $\sigma^{\mathrm{me}}_{\mathrm{nva}}$	IG	1	2	0.476	0.474	0.397	0.556

Note: G denotes the gamma distribution, IG denotes the inverse gamma distribution, U denotes the uniform distribution, B denotes the beta distribution, and N denotes the normal distribution.

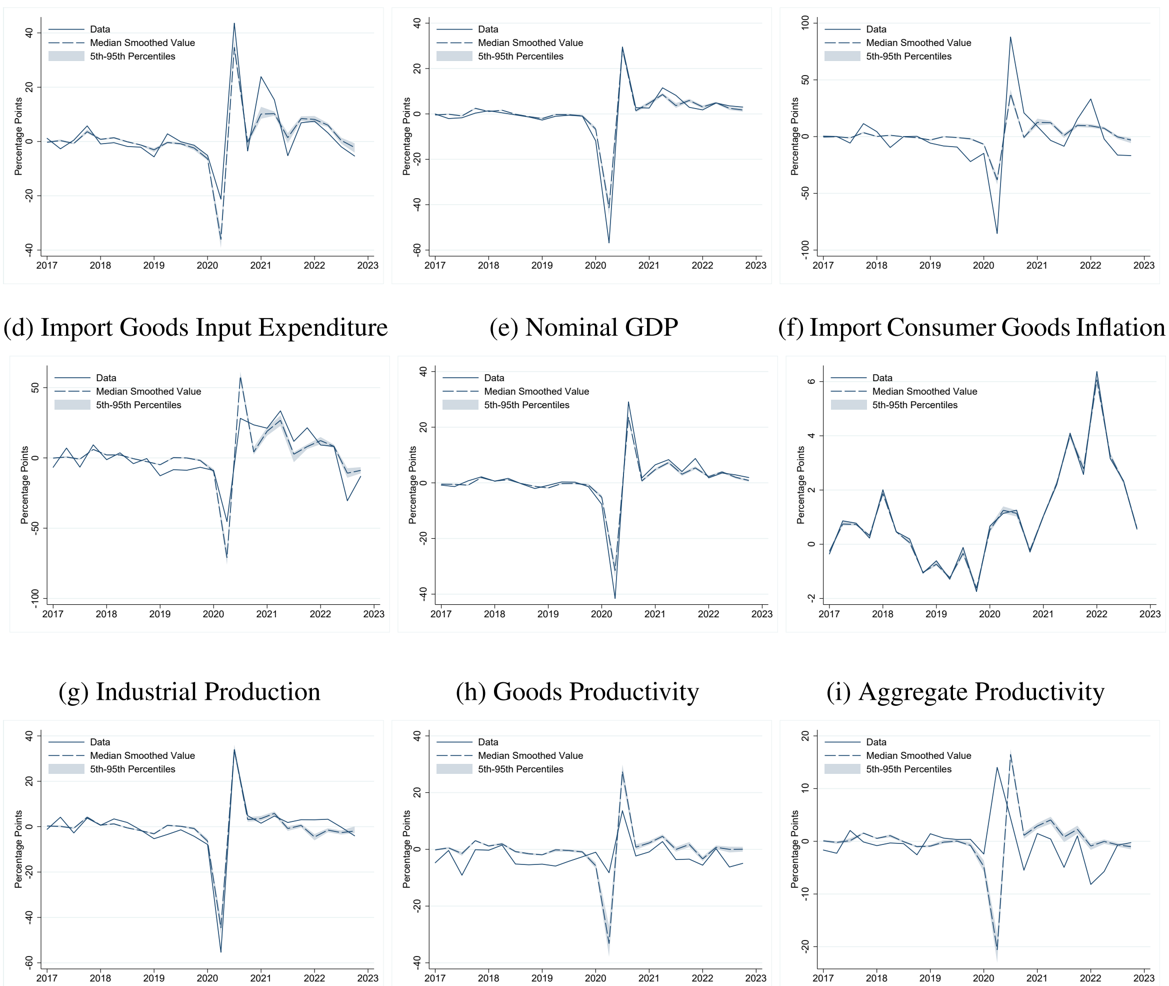
this implies that the data contains substantial measurement error during the pandemic period, which seems plausible to us. More broadly, a more sensitive treatment of the impact of lockdowns on the services sector would likely be needed to match data in the middle quarters of 2020. Nonetheless, referring back to the main text, the model is able to capture the dynamics of services inflation well overall, particularly in 2021-2022 when inflation escalates.

Turning to "non-targeted data," we now compare smoothed values for multipliers attached to the constraints to an external measure of supply chain disruptions. Specifically, we use the Global Supply Chain Pressure Index (GSCPI), developed by the New York Federal Reserve [Benigno et al. (2022)], which combines data on transportation costs (sea and air freight rates) with elements of Purchasing Managers' Index surveys pertaining to supply chain management from major industrial countries (China, the Eurozone, Japan, United States, etc.). To be clear, this data is not tightly related to the theoretical construct that we recover from the data; it also is not scaled in way that is directly comparable to our estimates.[8] Further, it is a proxy for global conditions, which doesn't distinguish between US-based and foreign supply chain constraints, so we compare it to a weighted mean of the median multipliers on the domestic and foreign constraints. With all these caveats, we plot the GSCPI and the weighted mean multiplier in Figure 8. As is evident, both the composite multiplier and the GSCPI index rise and fall in tandem.

Lastly, in the text, we noted that fluctuations in the reduced-form markup shocks in the Phillips Curves implied by binding constraints do not behave like standard markup shocks estimated from historical data. To illustrate this, we introduce an exogenous markup shock into the domestic and foreign price Phillips Curves of the baseline model, and we assume the markup shocks follow an AR1 stochastic process. We then re-estimate the model including exogenous markup shocks using only data from 1990:Q1-2019:Q4, under the assumption that constraints are slack throughout this period. We then filter the data to recover smoothed values for the markup shocks. In Figure 9, we plot the median smoothed values for the exogenous markup shocks over the period 1990-2020. We then also plot the median smoothed values for the reduced-form markup shocks implied by binding constraints

[8]The raw GSCPI index is reported as deviations from its mean value, in units of the standard deviation of the series. The NY Fed does not report either the mean or standard deviation, so we cannot compute log changes in the underlying index. Further, there is no obvious relationship between units attached to the multipliers – which summarize impacts of constraints on inflation – and units on the GSCPI. Because the GSCPI is reported at the monthly frequency, we take simple means across three month intervals to form quarterly values.

Figure 7: Data and Smoothed Model Observables



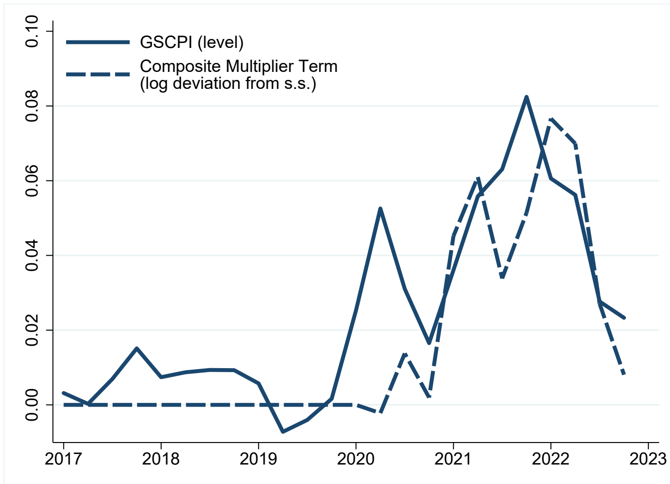
(a) Goods Cons. Expenditure (b) Services Cons. Expenditure (c) Import Cons. Goods Expenditure

(d) Import Goods Input Expenditure (e) Nominal GDP (f) Import Consumer Goods Inflation

(g) Industrial Production (h) Goods Productivity (i) Aggregate Productivity

Note: All data and simulated series are annualized values for de-meaned quarterly growth rates in percentage points. Data is raw data. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the dashed line. We shade the area covering the the 5% to 95% percentile for smoothed values.

Figure 8: Comparing the NY Fed GSCPI to the Weighted Mean of Multipliers on Domestic and Foreign Constraints



Note: To make the scale of the GSCPI index comparable to the multiplier, we plot the raw level of the GSCPI index divided by 50. The Composite Multiplier is computed as $0.75\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\hat{\mu}}_t(s)+0.25\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\hat{\mu}}^*_{ut}(s)$. The weight on the domestic term is 0.75 and the weight on the foreign term is 0.25, which roughly correspond to shares of total spending allocated to domestic and foreign goods.

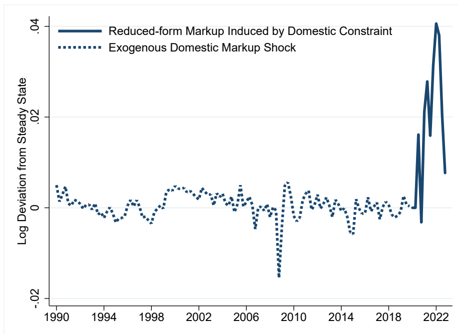
during the 2020:Q2-2022:Q4 period, obtained from the estimation above. As is evident, constraints induce markups shocks that are substantially larger than those that are consistent with historical data; further, the reduced-form markup shocks are also less persistent than the historical exogenous markup process.

1.7 Estimated Capacity Levels

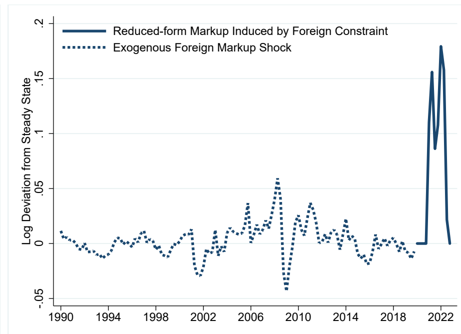
In the preceding (main) model, we calibrated the levels of domestic and foreign goods capacity in steady state. However, we could instead estimate those levels, with an important caveat. The caveat is that we allow constraints to bind only after 2020 in the estimation. The "steady-state" capacity level is the level to which capacity reverts in the long run, in the absence of shocks. We are able to estimate this level conditional on the data in periods in which constraints are potentially binding. Thus, if we estimate capacity levels, we are attempting to infer the capacity level only using post-2020 data. Naturally, since constraints were binding for much of this period, plausibly due to negative shocks that pushed realized capacity down, using only this data will tend to lead us to estimate a relatively low level for steady-state capacity. And in fact, this is that we find when we treat capacity levels as pa-

Figure 9: Exogenous and Reduced-Form (Binding Constraint) Markup Shocks

(a) Domestic Markup Shocks



(b) Foreign Markup Shocks



Note: The solid lines depict reduced-form markup shocks induced by binding constraints. The dashed lines are exogenous markup shocks obtained by estimating the model with exogenous markup shocks and slack constraints using only data for 1990:Q1-2019:Q4. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed values.

rameters to be estimated: steady state goods capacity is roughly 1% above the steady state level of goods output, which is lower than the calibrated value we have used previously. Nonetheless, this difference in the level of steady-state capacity has little import for our quantitative assessment, as we noted in the main text.

To demonstrate this, we provide supplemental figures illustrating results from a version of the model in which capacity is estimated in Figure 10. In Figure 10a, we replicate the counterfactual in which we relax both the domestic and import goods constraints. In Figure 10b, we replicate the simulated impact of individual shocks on inflation. To interpret this figure, we note that these counterfactuals are comparable to those in which we feed individual shocks into the model together with capacity shocks. The reason is that capacity shocks essentially lower the average capacity level to near the estimated steady-state capacity level recovered using only post-2020 data. The results are both qualitatively and quantitatively similar to prior results, which further demonstrates that the core counterfactual results are largely robust to the level of steady-state capacity.

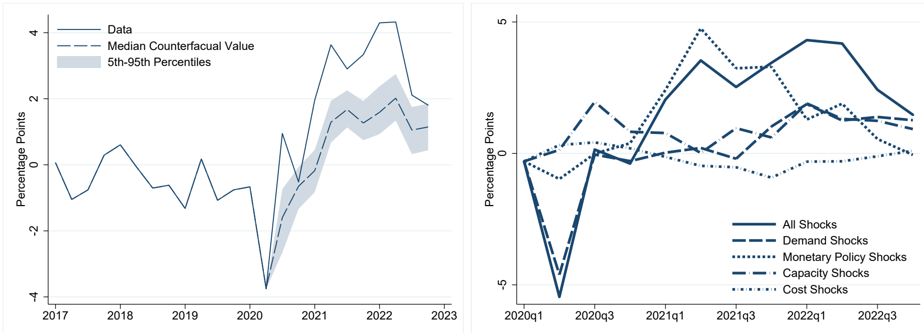
2 Labor Market Extension

This section provides details about how we extend the baseline model to incorporate sticky wages, potentially binding labor market constraints, and shocks to the disutility of labor. Conveniently, all three extensions can be formalized by re-writing the consumer-side of the

Figure 10: Counterfactual Inflation in Model with Estimated Steady-State Capacity Levels

(a) Aggregate Consumer Price Inflation

(b) Counterfactual Inflation for Individual Shocks



Note: In Figure 10a, we take 1000 draws from the posterior distribution of model parameters (re-estimated for this application including estimated capacity levels), compute the Kalman-smoothed values for model variables for each draw, add measurement error to the observables, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values. In Figure 10b, each series represents the simulated path of consumer price inflation (quarterly value, annualized) for the indicated subset of smoothed shocks during 2020-2022. See text for definition of the counterfactuals.

model as follows.

We now assume there is a unit continuum of consumers, indexed by $j \in (0,1)$. Consumers are identical, with one exception: each is the monopolistic supplier of its own differentiated labor services to the market. Further, the amount of labor that each consumer is able to supply in a given period is bound above by $\overline{L}_t$, which is exogenous and time varying. Differentiated labor services supplied by consumers are costlessly aggregated into a composite bundle by competitive intermediaries and sold to firms. The labor aggregation technology is given by $L_t = \left(\int_0^1 L_t(j)^{(\varepsilon_L-1)/\varepsilon_L} dj\right)^{\varepsilon_L/(\varepsilon_L-1)}$, where $\varepsilon_L > 1$ is the elasticity of substitution between differentiated labor services and the price index for the labor composite is $W_t = \left(\int_0^1 W_t(j)^{1-\varepsilon_L} dj\right)^{1/(1-\varepsilon_L)}$. Finally, each consumer faces pays Rotemberg-type adjustment costs to modify the nominal wage at which it supplies labor, as in Born and Pfeifer (2020).

Consumer j chooses its consumption, wage, and asset holdings to maximize utility, subject to its budget constraint, the demand curve for its labor, and the labor supply constraint:

$$\max_{\{C_t(j),W_t(j),B_{t+1}(j)\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[\frac{(C_t(j))^{1-\rho}}{1-\rho} - \Lambda_t \frac{L_t(j)^{1+\psi}}{1+\psi} \right] \tag{7}$$

$$\text{s.t.} \quad P_t C_t(j) + E_t \left[S_{t,t+1} B_{t+1}(j) \right] \leq B_t(j) + W_t(j) L_t(j) - \frac{\phi_W}{2} \left(\frac{W_t(j)}{W_{t-1}(j)} - 1 \right)^2 W_t L_t, \tag{8}$$

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_L} L_t, \quad \text{and} \quad L_t(j) \leq \overline{L}_t, \tag{9}$$

where ϕ_W is a parameter governing wage adjustment costs and Λ_t governs the time-varying disutility of labor supply. In a symmetric equilibrium, the first order condition for the optimal wage is:

$$1 - \varepsilon_L \left(1 - \frac{MRS_t + (\mu_{Lt}/C_t{}^{-\rho})}{\left(\frac{W_t}{P_t}\right)} \right) - \phi_W \left(\Pi_{Wt} - 1 \right) \Pi_{Wt} + E_t \left[\beta \frac{\Theta_{t+1}}{\Theta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{1}{\Pi_{t+1}} \phi_W \left(\Pi_{Wt+1} - 1 \right) \Pi_{Wt+1}^2 \frac{L_{t+1}}{L_t} \right] = 0, \tag{10}$$

where μ_{Lt} is the multiplier on the labor constraint, $\Pi_{Wt} \equiv \frac{W_t}{W_{t-1}}$, and $MRS_t = \frac{\Lambda_t L_t^{\psi}}{C_t^{-\rho}}$ is the marginal rate of substitution between consumption and labor supply in preferences. Further, the complementary slackness condition applies: $\left(L_t - \overline{L}_t \right) \mu_{Lt} = 0$, with $\mu_{Lt} \geq 0$.

Taking a log linear approximation for this equation, we arrive at the wage Phillips Curve presented in the main text:

$$\pi_{Wt} = \left(\frac{\varepsilon_L - 1}{\phi_W} \right) \left[\widehat{mrs}_t - \widehat{rw}_t \right] + \left(\frac{\varepsilon_L}{\phi_W} \frac{P_0}{W_0} \right) \hat{\tilde{\mu}}_{Lt} + \beta E_t \left(\pi_{Wt+1} \right), \tag{11}$$

where $\pi_{Wt} \equiv \hat{w}_t - \hat{w}_{t-1} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$ is nominal wage inflation, $\widehat{rw}_t \equiv \hat{w} - \hat{p}_t$, $\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$ with $\hat{\lambda}_t \equiv \ln \Lambda_t - \ln \Lambda_0$, and $\hat{\tilde{\mu}}_{Lt} \equiv \ln \tilde{\mu}_{Lt} - \ln \tilde{\mu}_{L0}$ where $\tilde{\mu}_{Lt} \equiv 1 + (\mu_{Lt}/C_t{}^{-\rho})$ is a function of the multiplier on the labor constraint.

To define equilibrium in this model, we modify the equilibrium conditions from Tables 1 and 2 as follows. First, we drop the "labor supply" condition from the baseline model, as labor supply is no longer determined by equating the marginal rate of substitution to the real wage. Second, we add the equilibrium conditions in Table 6, where Panel A corresponds to an equilibrium when labor constraints are slack, and Panel B corresponds to

Table 6: Equilibrium Conditions with Binding Constraints for Labor

Panel A: Labor Constraint is Slack	
Wage Setting	$\pi_{Wt} = \left(\frac{\varepsilon_L - 1}{\phi_W}\right)\left[\widehat{mrs}_t - \widehat{rw}_t\right] + \beta E_t\left(\pi_{Wt+1}\right)$
Marginal Rate of Substitution	$\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$
Auxiliary Inflation Definition	$\pi_{Wt} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$
Panel B: Labor Constraint Binds	
Wage Setting	$\pi_{Wt} = \left(\frac{\varepsilon_L - 1}{\phi_W}\right)\left[\widehat{mrs}_t - \widehat{rw}_t\right] + \left(\frac{\varepsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\mu}_{Lt} + \beta E_t\left(\pi_{Wt+1}\right)$
Marginal Rate of Substitution	$\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$
Auxiliary Inflation Definition	$\pi_{Wt} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$
Labor Market Constraint	$\hat{l}_t = \hat{\bar{l}}_t + \ln\left(\bar{L}_0 / L_0\right)$

the case when they are binding. The new endogenous variables in the equilibrium system are: $\{\pi_{Wt}, \widehat{mrs}_t\}$ when the labor constraint is slack, and $\left\{\pi_{Wt}, \widehat{mrs}_t, \hat{\mu}_{Lt}\right\}$ when the labor constraint binds. Combined with the goods constraints, this defines eight model regimes with different combinations of binding and slack constraints.

Turning to quantitative implementation of this model, we start by describing new calibrated parameters. We set $\varepsilon_L = 21$, following Christiano, Eichenbaum and Evans (2005). We then choose ϕ_W so that the slope of the wage Phillips Curve is equivalent to a Calvo model with wage adjustment parameter 0.4, when $\varepsilon_L = 21$. This Calvo wage adjustment target is taken from Fitzgerald et al. (forthcoming), who estimate it based on state-level data. The implied slope of the wage Phillips Curve is then about 0.02, which is relatively flat. We calibrate the level of the labor constraint ($\bar{L}_0$) to be 1% higher than steady state labor supply. Because the actual level of the constraint at a given point in time is a realization of a stochastic process, results are not sensitive to this value.

We assume the disutility of labor evolves according to $\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \varepsilon_{\lambda t}$, where $var\left(\varepsilon_{\lambda t}\right) = \sigma_\lambda^2$ and $cov\left(\varepsilon_{\lambda t}, \varepsilon_{\lambda t+s}\right) = 0$ for $s \neq 0$, and we estimate ρ_λ and σ_λ. Further, we assume that the labor constraint is subject to shocks, such that $\ln \bar{L}_t - \ln \bar{L}_0 \equiv \hat{\bar{l}}_t = \varepsilon_{\bar{l}t}$ with $var\left(\varepsilon_{\bar{l}t}\right) = \sigma_{\bar{l}}^2$ and $cov\left(\varepsilon_{\bar{l}t}, \varepsilon_{\bar{l},t+s}\right) = 0$ for $s \neq 0$, and we estimate $\sigma_{\bar{l}}$.

We assume observables (aggregate hours worked and real wage growth) are measured with error and estimate the variance of the measurement errors. We also re-estimate all the same structural parameters and stochastic processes using this version of the model. To do so, we assemble data on aggregate hours worked and real wage growth from raw data provided by the US Bureau of Labor Statistics.[9] To construct real wage growth, we

[9]We retrieve these data from the FRED database, maintained by the Federal Reserve Bank of St. Louis:

use hourly compensation data for the non-farm business sector to proxy for nominal wage growth (FRED series id: COMPNFB), taking log growth rates of that quarterly index. We then deflate this nominal wage growth using the aggregate PCE price index, used in prior sections. To build an aggregate hours series, we combine several series. We use average weekly hours of production and nonsupervisory works in the private sector (FRED series id: AWHNONAG) to proxy hours per worker. We then compute the ratio of employment (FRED series id: CE16OV) to population (FRED series id: CNP16OV), were we smooth population estimates by taking means within two-year moving windows in order to eliminate jumps due to data revisions. We then multiply average weekly hours by the employment to population ratio, take logs of that index, and compute deviations from the sample mean of the index over the 1992:Q2 to 2019:Q4 (the pre-COVID sample).

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https://fred.stlouisfed.org/. So, we provide FRED series identifies here.