The Inflation Accelerator

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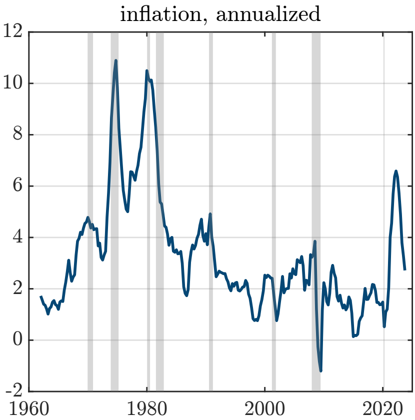
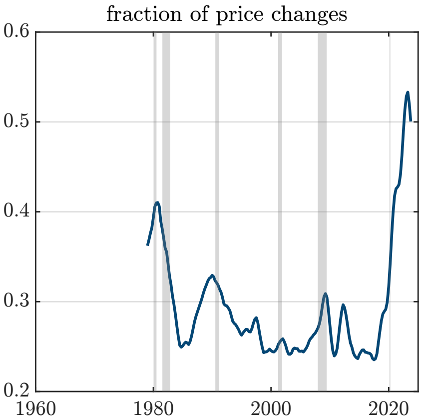
[1] The views expressed herein are those of the authors and not necessarily those of the Board of Governors of the Federal Reserve System, the Federal Reserve Bank of Atlanta or the Federal Reserve System.

Motivation

- Fraction of price changes increases with inflation
- Evidence from the US (Montag and Villar, 2023):





Consequences for Inflation Dynamics?

- Standard Calvo Phillips curve:

$$\pi_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$

  - κ increases with frequency of price changes

- Existing estimates: $\kappa \approx 0$: most inflation due to 'markup' shocks (u_t)

- But what if κ increases with size of shock?

  - Phillips curve non-linear

Our Paper

- Study a Calvo model in which fraction of price changes is endogenous
  - and time-varying
- Unlike menu cost models, no distribution of price changes to keep track of
  - so very tractable, one equation extension of Calvo
- Baseline Calvo: firms with older prices more willing to adjust them
  - so endogenizing frequency requires keeping track of price distribution
- Insight: multi-product firms choose
  - *how many* prices to change, but *not which*
  - no heterogeneity across firms, Calvo-like aggregation
- We find substantial variation in slope of Phillips curve in US data
  - varies from about 0.01 to 0.2

Model

Consumers

- The consumer has preferences over consumption and disutility from work

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_t - h_t\right)$$

subject to

$$P_t c_t + \frac{1}{1+i_t} B_{t+1} = W_t h_t + D_t + B_t,$$

- We assume a cash in advance constraint so

$$W_t = P_t c_t = M_t$$

- Money supply growth is iid

$$\log M_{t+1}/M_t = g_{m,t+1} = \bar{g}_m + \sigma_m \varepsilon_{m,t+1}$$

Technology

- Continuum of intermediate goods firms indexed by f
- They sell a continuum of products with technology

$$y_{it}\left(f\right) = l_{it}\left(f\right)^{\eta}$$

- y_{it} are aggregated in a CES fashion to produce a final composite good

$$y_t = \left(\int\int y_{it}\left(f\right)^{\frac{\theta-1}{\theta}}\,\mathrm{d}i\mathrm{d}f\right)^{\frac{\theta}{\theta-1}},$$

- This implies that the demand for an individual variety is

$$y_{it}\left(f\right) = \left(\frac{P_{it}\left(f\right)}{P_t}\right)^{-\theta} y_t,$$

- The final good y_t is used for consumption

$$y_t = c_t$$

Intermediate Goods Producers

- Per period nominal profits from producing a particular variety i are

$$P_{it}\left(f\right)y_{it}\left(f\right)-\tau W_t l_{it}\left(f\right)$$

- Firm composite output and total labor are:

$$y_t(f)=\left(\int y_{it}\left(f\right)^{\frac{\theta-1}{\theta}}\,\mathrm{d}i\right)^{\frac{\theta}{\theta-1}},\qquad l_t(f)=\int l_{it}(f)\mathrm{d}i$$

- Using this, the firm-level production function is

$$y_t\left(f\right)=\left(\frac{X_t\left(f\right)}{P_t\left(f\right)}\right)^{\theta}l_t\left(f\right)^{\eta},$$

where $P_t(f)=\left(\int P_{it}\left(f\right)^{1-\theta}\,\mathrm{d}i\right)^{\frac{1}{1-\theta}}$ denotes the price index of firm f and

$$X_t(f)=\left(\int P_{it}\left(f\right)^{-\frac{\theta}{\eta}}\,\mathrm{d}i\right)^{-\frac{\eta}{\theta}}$$

determines the extent of misallocation within the firm

Sticky Prices

- We assume that the firm has a convex cost of changing prices
  - increasing in the number of prices it changes, $n_t(f)$

$$\frac{\xi}{2}\left(n_t(f)-\bar{n}\right)^2, \quad \text{if } n_t(f)>\bar{n}$$

- Firms can chose how many prices change, but not which prices change

- The firm's objective is to maximize discounted real profits

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\left(\frac{P_{t+s}(f)}{P_{t+s}} \right)^{1-\theta} - \tau \left(\frac{X_{t+s}(f)}{P_{t+s}} \right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}} - \frac{\xi}{2}\left(n_{t+s}(f)-\bar{n}\right)^2 \right]$$

- Firm chooses the reset price $P_t^*(f)$ and fraction of prices to change $n_t(f)$

Choice of Reset Price

- $P_{it}(f) = P_t^*(f)$ for all products whose prices change
- $P_t^*(f)$ that maximizes the value of the firm satisfies

$$\left(\frac{P_t^*(f)}{P_t}\right)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta}\frac{b_{2t}(f)}{b_{1t}(f)}$$

where

$$b_{1t}(f) = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} \left(1 - n_{t+j}\left(f\right)\right) \left(\frac{P_{t+s}}{P_t}\right)^{\theta-1}$$

$$b_{2t}(f) = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} \left(1 - n_{t+j}\left(f\right)\right) \left(\frac{P_{t+s}}{P_t}\right)^{\frac{\theta}{\eta}} \left(y_{t+s}\right)^{\frac{1}{\eta}}$$

- This could also be written as,

$$P_t^*\left(f\right) = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \omega_{t,t+s}\left(f\right) MC_{t,t+s}\left(f\right),$$

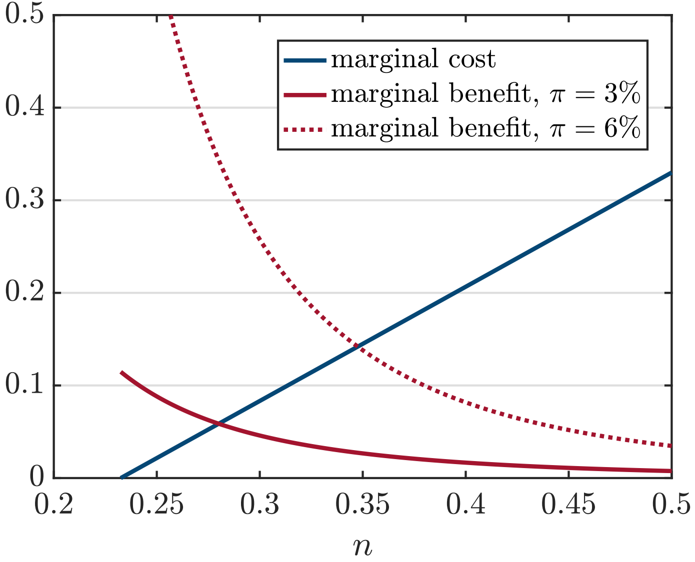
where $\omega_{t,t+s}\left(f\right) = \frac{\beta^s \prod_{j=1}^{s}(1-n_{t+j}(f))(P_{t+s})^{\theta-1}}{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s}(1-n_{t+j}(f))(P_{t+s})^{\theta-1}}$

Choice of Frequency

- The optimal choice of $n_t(f)$ satisfies

$$
\begin{aligned}
\underbrace{\xi\left(n_t(f)-\bar{n}\right)}_{\text{Marginal price adj. cost}} &= \underbrace{b_{1t}(f)\left(\left(\frac{P_t^*(f)}{P_t}\right)^{1-\theta}-\left(\frac{P_{t-1}(f)}{P_t}\right)^{1-\theta}\right)}_{\text{marginal benefit from changing prices}} \\
&\quad \underbrace{-\tau b_{2t}(f)\left(\left(\frac{P_t^*(f)}{P_t}\right)^{-\frac{\theta}{\eta}}-\left(\frac{X_{t-1}(f)}{P_t}\right)^{-\frac{\theta}{\eta}}\right)}_{\text{marginal benefit from reducing misallocation}}
\end{aligned}
$$

Steady-State Fraction of Price Changes



Symmetric Equilibrium

❶ definition of price index

$$1 = n_t \left(\frac{p_t^*}{p_t} \right)^{1-\theta} + (1 - n_t)\, \pi_t^{\theta-1}$$

❷ optimal reset price

$$\left(\frac{p_t^*}{p_t} \right)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta} \frac{b_{2t}}{b_{1t}}$$

❸ optimal choice of the fraction of price changes

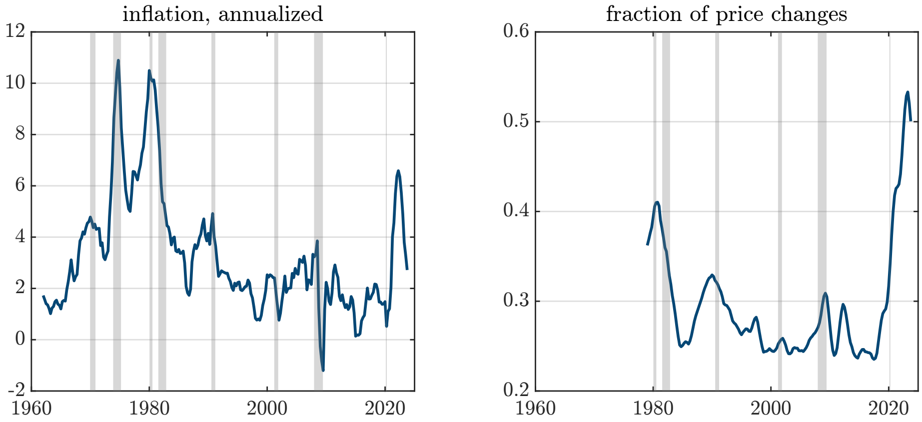
$$\xi\,(n_t - \bar{n}) = b_{1t} \left(\left(\frac{p_t^*}{p_t} \right)^{1-\theta} - \pi_t^{\theta-1} \right) - \tau b_{2t} \left(\left(\frac{p_t^*}{p_t} \right)^{-\frac{\theta}{\eta}} - x_{t-1}^{-\frac{\theta}{\eta}} \pi_t^{\frac{\theta}{\eta}} \right)$$

❹ losses from misallocation

$$x_t^{-\frac{\theta}{\eta}} = n_t \left(\frac{p_t^*}{p_t} \right)^{-\frac{\theta}{\eta}} + (1 - n_t)\, x_{t-1}^{-\frac{\theta}{\eta}} \pi_t^{\frac{\theta}{\eta}}$$

Parameterization

Data



inflation, annualized
12
10
8
6
4
2
0
-2
1960
1980
2000
2020
fraction of price changes
0.6
0.5
0.4
0.3
0.2
1960
1980
2000
2020

Assigned Parameters

β	0.99	discount factor
θ	6	demand elasticity
η	2/3	returns to scale

Calibrated Parameters

A. Targeted Moments

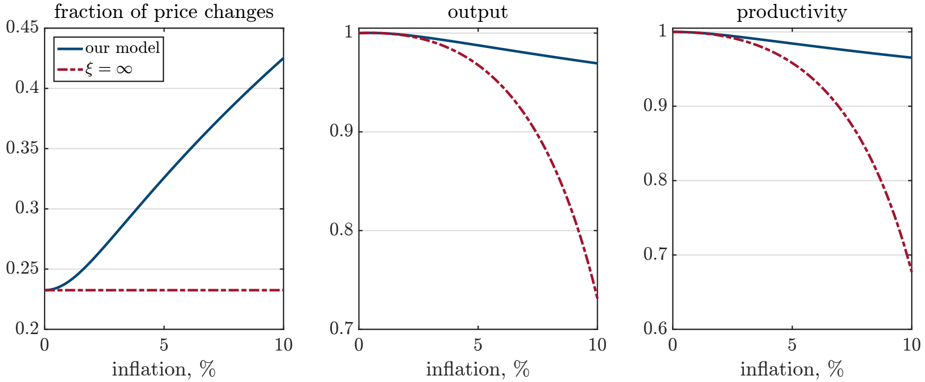
	Data	Our model	Calvo
mean inflation	3.291	3.291	3.291
s.d. inflation	2.313	2.313	2.313
mean frequency	0.294	0.294	0.294
slope of n_t on $\lvert\pi_t\rvert$	0.019	0.019	–

B. Calibrated Parameter Values

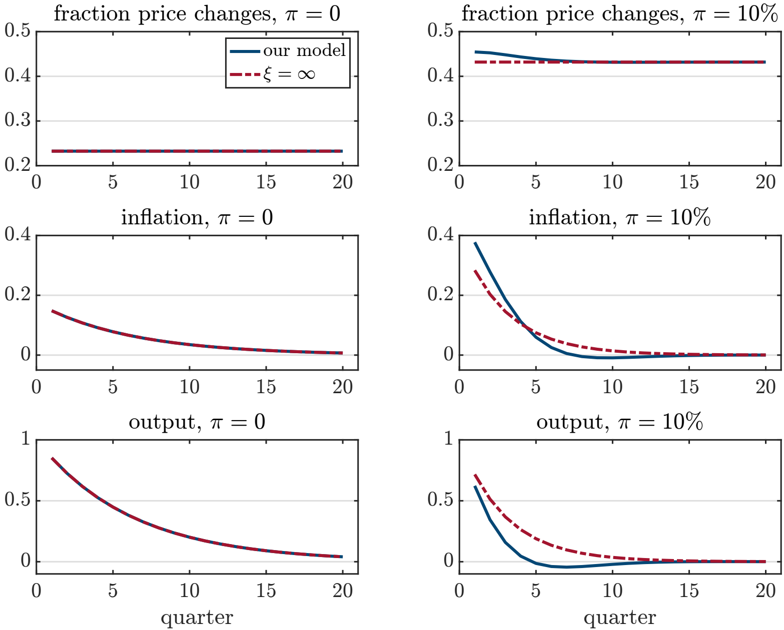
		Our model	Calvo
$\bar{g}_m$	mean spending growth rate	0.033	0.034
σ_m	s.d. idios. shocks	0.018	0.020
$\bar{n}$	fraction free price changes	0.232	0.294
ξ	adjustment cost	1.233	–

Note: The mean nominal spending growth rate is annualized.

Steady-state Outcomes



Impulse Response to 1% Money Shock



Log-linearization

- First-order expansion of the price index definition gives

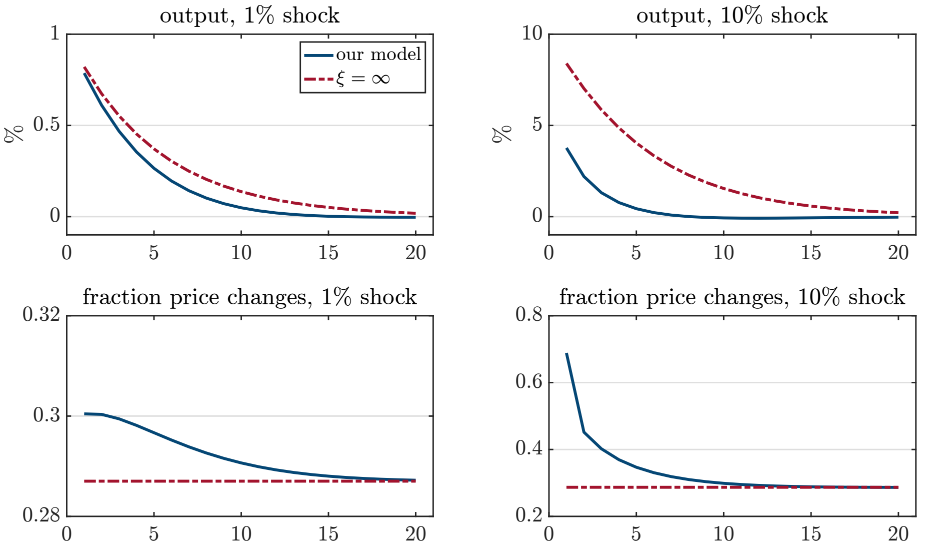
$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n)\,\pi^{\theta-1}}\frac{\pi^{\theta-1}-1}{\theta-1}}_{\mathcal{M}}\,\hat{n}_t + \underbrace{\frac{1-(1-n)\,\pi^{\theta-1}}{(1-n)\,\pi^{\theta-1}}}_{\mathcal{N}}\left(\hat{p}_t^* - \hat{p}_t\right).$$

- $\mathcal{N}$ reflects direct effect of higher reset price on inflation

- $\mathcal{M}$ reflects direct effect of higher frequency on inflation

- $\mathcal{M} = 0$ if $\pi = 1$ but increases with π
  - up to a first-order, $\log \pi_t = n_t \Delta_t$, where Δ_t is the average price change

Impulse Response, from $\pi = 3.3\%$ Steady-state



Inflation Accelerator

- The slope of the Phillips curve increases rapidly with trend inflation
  - due to feedback effect between inflation and the frequency of price changes
- An increase in the fraction of price changes increases inflation
  - more so in environments with higher trend inflation
- An increase in inflation increases firms' incentive to change prices
  - thus raising the frequency of price changes

Slope of the Phillips Curve

- Log-linearize the expression determining the optimal n_t

$$\hat{n}_t = \mathcal{A}\hat{\pi}_t + \mathcal{B}\left(\hat{p}_t^* - \hat{p}_t\right) - \mathcal{C}\hat{x}_{t-1} + \frac{n - \bar{n}}{n}\hat{b}_{1t},$$

- Combine this with the log-linearized expression of the price index:

$$\hat{\pi}_t = \frac{\mathcal{MB} + \mathcal{N}}{1 - \mathcal{MA}}\left(\hat{p}_t^* - \hat{p}_t\right) - \frac{\mathcal{MC}}{1 - \mathcal{MA}}\hat{x}_{t-1} + \frac{\mathcal{M}}{1 - \mathcal{MA}}\frac{n - \bar{n}}{n}\hat{b}_{1t}$$

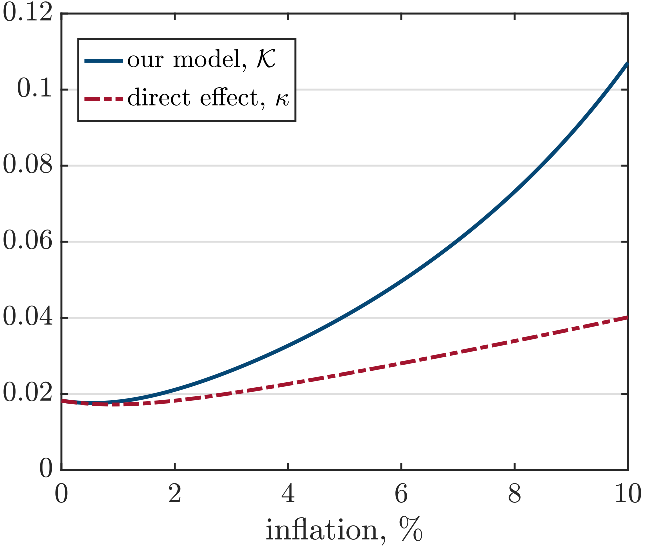
- Combine with log-linearized exp. for p_t^*, elast. of π_t to $mc_t = \frac{1}{\eta}\frac{W_t}{P_t}y_t^{\frac{1}{\eta}-1}$:

$$\mathcal{K} = \underbrace{\frac{1}{1 + \theta\left(\frac{1}{\eta} - 1\right)}}_{\text{strategic complementarities}} \underbrace{\left(1 - \beta\left(1 - n\right)\pi^{\frac{\theta}{\eta}}\right)}_{\text{horizon effect}} \underbrace{\frac{\mathcal{MB} + \mathcal{N}}{1 - \mathcal{MA}}}_{\text{impact of higher reset price}}$$

- Note: turning off feedback effect, $\mathcal{M} = 0$, gives the familiar slope

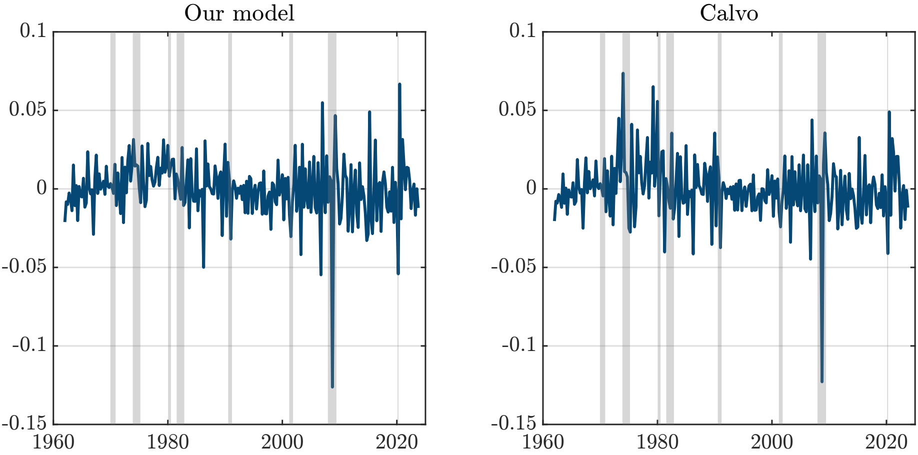
$$\mathcal{K} = \frac{1}{1 + \theta\left(\frac{1}{\eta} - 1\right)}\frac{\left(1 - \beta\left(1 - n\right)\pi^{\frac{\theta}{\eta}}\right)\left(1 - \left(1 - n\right)\pi^{\theta - 1}\right)}{\left(1 - n\right)\pi^{\theta - 1}}$$

Slope of the Phillips Curve

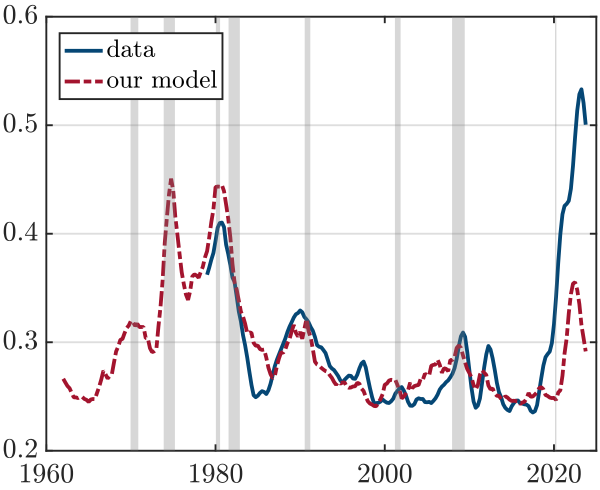


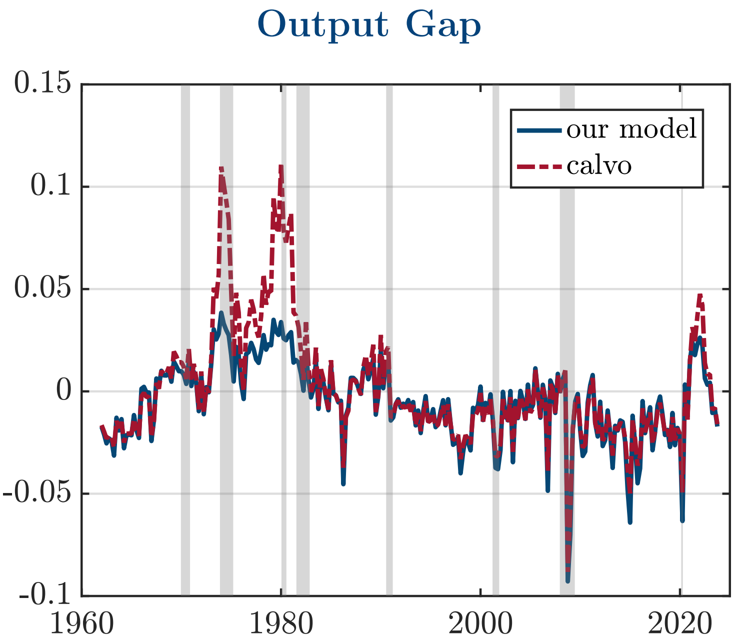
Time-series Analysis

Money Shocks to Match Inflation

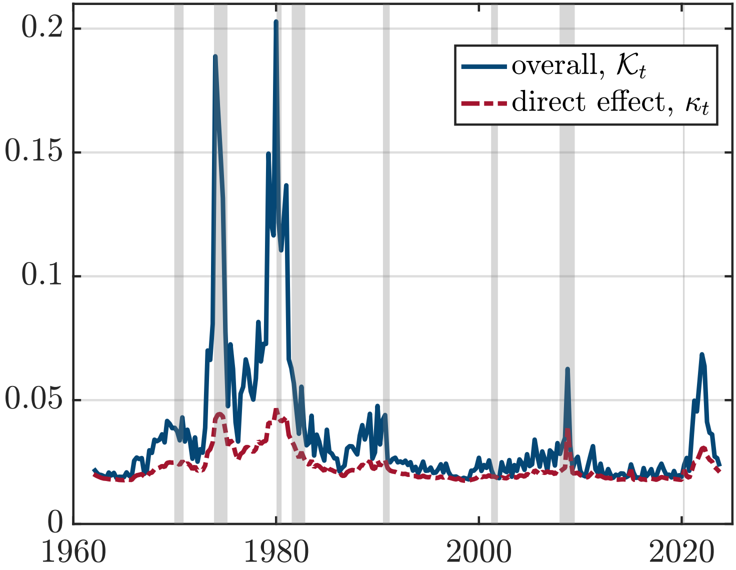


Model Fit of Frequency

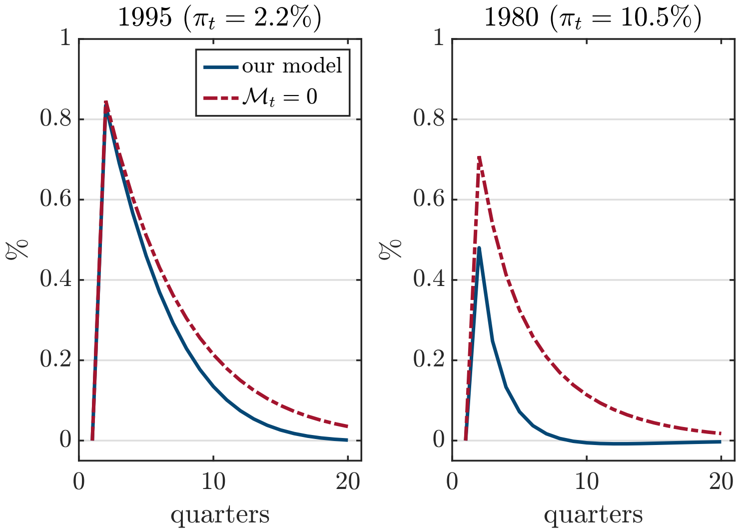



Output Gap
our model
calvo
0.15
0.1
0.05
0
-0.05
-0.1
1960
1980
2000
2020

Time-varying Slope of the Phillips Curve

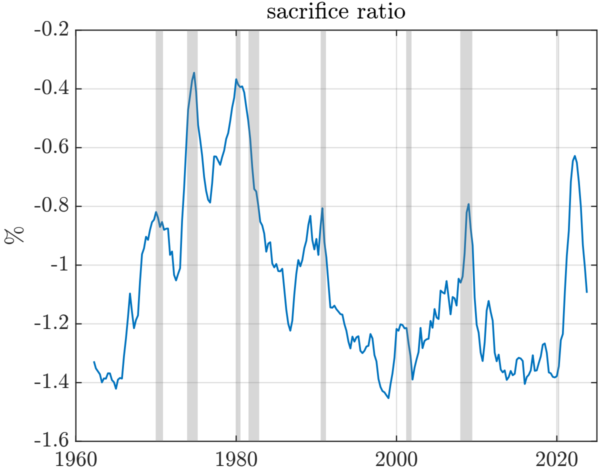


Time-varying IRF to 1% Money Shock



Sacrifice Ratio

Output cost of negative money shock that lowers infl by 1% after one year



Conclusions

- The frequency of price changes rises with inflation
- Tractable one-equation extension of the Calvo model
  - the frequency of price changes varies endogenously over time
- The slope of the Phillips curve fluctuates a great deal over time
- This is primarily due to the inflation accelerator:
  - the feedback effect between inflation and the frequency of price changes