The Inflation Accelerator

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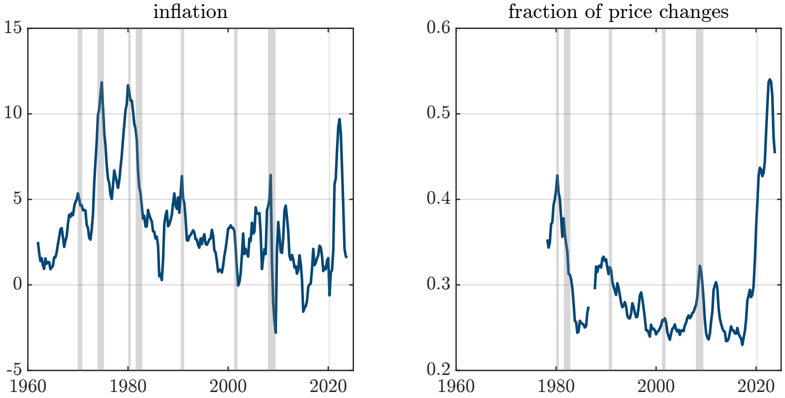
[1] The views expressed herein are those of the authors and not necessarily those of the Board of Governors of the Federal Reserve System, the Federal Reserve Bank of Atlanta or the Federal Reserve System.

Motivation

- Data: fraction of price changes increases with inflation
  - Gagnon (2009), Alvarez et al. (2018), Blanco et al. (2024)

Evidence from the U.S.



- Source: Nakamura et al. (2018), Montag and Villar (2023). Fraction quarterly.
- Inflation computed using CPI without shelter (year-to-year changes).

▸ extensive margin decomposition

Motivation

- Data: fraction of price changes increases with inflation
  - Gagnon (2009), Alvarez et al. (2018), Blanco et al. (2024)

- Existing sticky price models inconsistent with this evidence

  1. Time-dependent models
     - fraction constant by assumption

  2. State-dependent (menu cost) models
     - fraction nearly constant provided match micro-price data
     - less tractable because state includes distribution of prices

Our Paper

- We develop a tractable model with endogenously varying fraction
  - multi-product firms choose *how many*, but not *which*, prices to change
  - exact aggregation: reduces to one-equation extension of Calvo

- Model predicts highly non-linear Phillips curve
  - slope fluctuates from 0.02 in 1990s to 0.12 in 1970s and 1980s

- Mostly due to feedback loop between fraction and inflation
  - *inflation accelerator*
  - $\uparrow$ inflation $\rightarrow \uparrow$ fraction $\rightarrow \uparrow$ inflation $\rightarrow \ldots$

- Absent feedback loop slope would increase to only 0.04 in 1970s and 1980s

Outline

1. Model setup
2. Steady state as a function of trend inflation
3. Phillips curve in the time-series
4. Robustness to parameters and Taylor rule

Model

Model Overview

- Multi-product firms: each sells continuum of goods
  - decreasing returns labor-only technology
  - cost of changing prices
- Monetary policy targets nominal spending M_t
  - only source of aggregate uncertainty
- Golosov-Lucas assumptions on preferences
  - nominal wages proportional to M_t

Consumers

- Life-time utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log c_t - h_t\right)$$

- Budget constraint

$$P_t c_t + \frac{1}{1+i_t} B_{t+1} = W_t h_t + D_t + B_t$$

- Monetary policy targets nominal spending $M_t \equiv P_t c_t$

$$\log M_{t+1}/M_t = \mu + \varepsilon_{t+1}$$

- Log-linear preferences imply $W_t = M_t$

Technology

- Final good used for consumption, produced using CES aggregator

$$c_t = y_t = \left(\int_0^1 \int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

  – y_{ikt} output of good k produced by firm i, sold at price P_{ikt}

- Demand for individual product

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t} \right)^{-\theta} y_t, \qquad \text{where} \qquad P_t = \left(\int_0^1 \int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k \mathrm{d}i \right)^{\frac{1}{1-\theta}}$$

- Individual goods produced with decreasing returns technology

$$y_{ikt} = (l_{ikt})^{\eta}$$

Firm Objective

- PV of profits of firm i

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\beta^t}{P_t c_t} \left(\int_0^1 \left(P_{ikt} y_{ikt} - \tau W_t l_{ikt} \right) \mathrm{d}k - W_t F\left(n_{it}\right) \right)$$

  – $\tau = 1 - 1/\theta$ subsidy to eliminate markup distortion

- Price adjustment cost

$$F\left(n_{it}\right) = \frac{\xi}{2} \left(n_{it} - \bar{n}\right)^2 \quad \text{if} \quad n_{it} > \bar{n}$$

- Firm chooses fraction of prices to change n_{it} and reset price P_{it}^*

  – but not which prices to change (similar to Greenwald 2018)

  – if adjust $P_{ikt} = P_{it}^*$, otherwise $P_{ikt} = P_{ikt-1}$

Firm Objective

- Period t profits depend on two moments of price distribution

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\frac{P_{it}}{P_t} \right)^{1-\theta} - \tau \left(\frac{X_{it}}{P_t} \right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}} - F(n_{it}) \right)$$

  - firm price index P_{it} and losses from misallocation X_{it}

$$P_{it} = \left(\int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k \right)^{\frac{1}{1-\theta}} \quad \text{and} \quad X_{it} = \left(\int_0^1 (P_{ikt})^{-\frac{\theta}{\eta}} \, \mathrm{d}k \right)^{-\frac{\eta}{\theta}}$$

- History encoded in two state variables, P_{it-1} and X_{it-1}

  - exact aggregation because adjustment hazard does not depend on P_{ikt-1}

Relationship to Menu Cost Models

- In general, adjustment hazard in menu cost models non-linear $n_{it}(P)$

- So need to record distribution of old prices $g_{it}(P)$ and characterize l.o.m

  – for example, price index

$$P_{it} = \left(\left(P_{it}^{*}\right)^{1-\theta} \int n_{it}(P) \mathrm{d}g_{it}(P) + \int P^{1-\theta} \big(1 - n_{it}(P)\big) \mathrm{d}g_{it}(P) \right)^{\frac{1}{1-\theta}}$$

  – either use Krusell-Smith or study one-time shocks

- In our model adjustment $n_{it}(P) = n_{it}$, so, e.g.

$$P_{it} = \left(n_{it} \left(P_{it}^{*}\right)^{1-\theta} + \left(1 - n_{it}\right) \left(P_{it-1}\right)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

Optimal Reset Price

- Optimal reset price depends on PV of future marginal costs

$$\frac{P_{it}^*}{P_t} = \left(\frac{1}{\eta}\frac{b_{2it}}{b_{1it}}\right)^{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}$$

  – weighted by probability that price survives to that period

$$b_{1it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t}\right)^{\theta-1}$$

$$b_{2it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t}\right)^{\frac{\theta}{\eta}} (y_{t+s})^{\frac{1}{\eta}}$$

- Similar to Calvo, except n_{it} time-varying

Optimal Fraction of Price Changes

- Equate marginal cost to marginal benefit

$$\xi\left(n_{it}-\bar{n}\right)=b_{1it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{1-\theta}-\left(\frac{P_{it-1}}{P_{t}}\right)^{1-\theta}\right)-\tau b_{2it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{-\frac{\theta}{\eta}}-\left(\frac{X_{it-1}}{P_{t}}\right)^{-\frac{\theta}{\eta}}\right)$$

- Marginal benefit: higher n_{it}
  - changes firm price index
  - and reduces misallocation
  - weighted by the same terms b_{1it} and b_{2it} that determine P_{it}^{*}

Symmetric Equilibrium

- Since firms identical, in equilibrium $P_{it}^* = P_t^*$, $n_{it} = n_t$, …

- Let $p_t^* = P_t^*/P_t$, $x_t = X_t/P_t$, $\pi_t = P_t/P_{t-1}$

- Optimal choices

  – reset price

$$p_t^* = \left(\frac{1}{\eta}\frac{b_{2t}}{b_{1t}}\right)^{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}$$

  – fraction of price changes

$$\xi\left(n_t - \bar{n}\right) = b_{1t}\left(\left(p_t^*\right)^{1-\theta} - \left(\pi_t\right)^{\theta-1}\right) - \tau b_{2t}\left(\left(p_t^*\right)^{-\frac{\theta}{\eta}} - \left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\left(\pi_t\right)^{\frac{\theta}{\eta}}\right)$$

Symmetric Equilibrium

- Inflation pinned down by definition of price index

$$1 = n_t \left(p_t^*\right)^{1-\theta} + \left(1 - n_t\right) \left(\pi_t\right)^{\theta-1}$$

- Losses from misallocation

$$\left(x_t\right)^{-\frac{\theta}{\eta}} = n_t \left(p_t^*\right)^{-\frac{\theta}{\eta}} + \left(1 - n_t\right) \left(x_{t-1}\right)^{-\frac{\theta}{\eta}} \left(\pi_t\right)^{\frac{\theta}{\eta}}$$

- Model reduces to one-equation extension of Calvo
  - as $\xi \to \infty$, $n_t = \bar{n}$ so our model nests Calvo
- Unlike Calvo, important non-linearities, so use projection methods
  - third-order perturbation also accurate

Parameterization

Calibration Strategy

- Assigned parameters
  - period 1 quarter so $\beta = 0.99$
  - demand elasticity $\theta = 6$ and span of control $\eta = 2/3$

- Calibrated parameters
  - mean and standard deviation of nominal spending growth μ and σ
  - fraction of free price changes $\bar{n}$ and price adjustment cost ξ

- Calibration targets
  - mean and standard deviation of inflation
  - mean fraction of price changes
  - slope of fraction of price changes on absolute value of inflation

Calibrated Parameters

Targeted Moments

	Data	Our model	Calvo
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $\|\pi_t\|$	0.016	0.016	–

Calibrated Parameters

		Our model	Calvo
μ	mean spending growth rate	0.035	0.035
σ	s.d. monetary shocks	0.022	0.022
$\bar{n}$	fraction free price changes	0.241	0.297
ξ	adjustment cost	1.767	–

- Price adjustment costs account for 0.65% of all labor costs

Steady State

Overview

- Show how steady-state outcomes vary with trend inflation
- Responses to monetary shocks
- Derive Phillips curve

Fraction of Price Changes

- Let $\pi = \exp(\mu)$ denote level of trend inflation
  - variable without t subscript is value in non-stochastic steady state

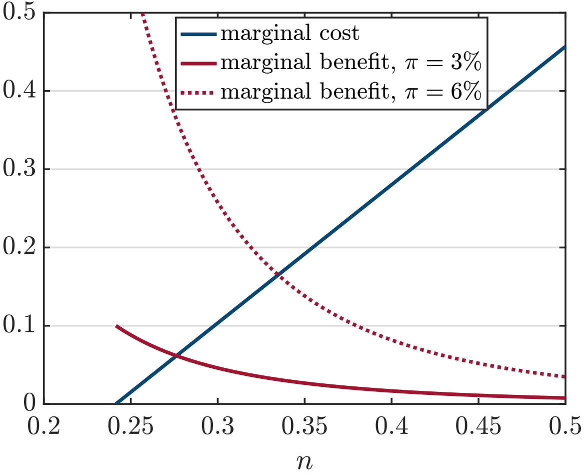
- Steady state fraction of price changes n

$$\xi (n - \bar{n}) = \frac{1}{1 - \beta (1 - n) \pi^{\theta - 1}} \frac{1}{n} \left(1 - \pi^{\theta - 1} - \tau \eta \frac{1 - (1 - n) \pi^{\theta - 1}}{1 - (1 - n) \pi^{\frac{\theta}{\eta}}} \left(1 - \pi^{\frac{\theta}{\eta}} \right) \right)$$

- Marginal cost linearly increasing in n

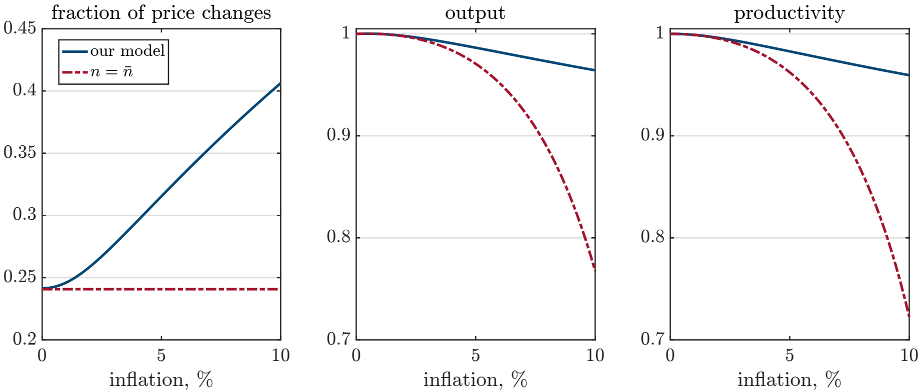
- Marginal benefit
  - absent trend inflation (i.e. $\pi = 1$), marginal benefit is zero and $n = \bar{n}$
  - when $\pi > 1$, decreasing in n

Fraction of Price Changes



Fraction of price changes increases with inflation

Output and Productivity



- Inflation less distortionary in our model

  equations

  - because more frequent price changes, as in menu cost models

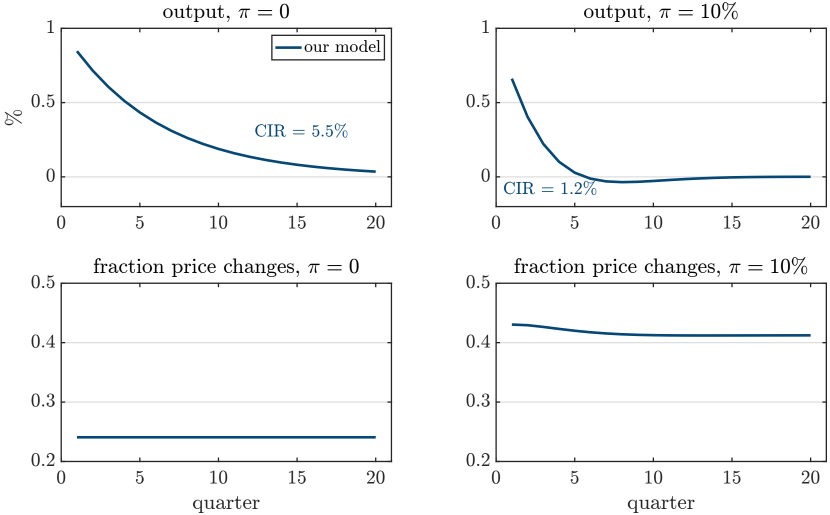
Real Effects of Monetary Shocks

- Responses to small monetary shocks
  - in economies with 0 and 10% trend inflation
  - compare to $\xi = \infty$ economy, frequency as in steady state of our model
  - use first-order approximation to build intuition

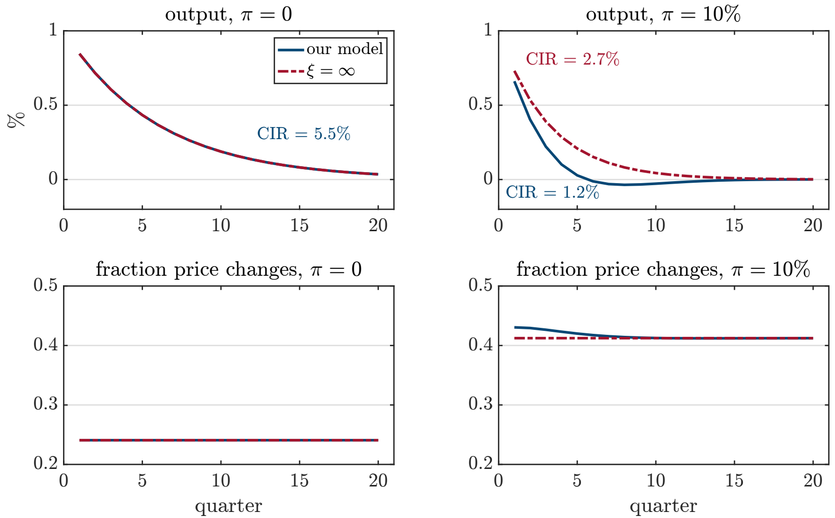
- Responses to large monetary shocks
  - use non-linear solution

- Focus on output response
  - $\Delta M_t = \Delta P_t + \Delta y_t$, so output response depends on how sticky prices are

Response to 1% Shock: Our Model



Response to 1% Shock: Calvo Model



Response to 1% Money Shock

- Absent trend inflation, our model responses identical to Calvo

- Output responds less in economy with 10% trend inflation
  - cumulative response: 1.2% vs. 5.5%

- Partly due to higher steady-state frequency: in $\xi = \infty$ economy
  - cumulative response: 2.7%

- Remaining difference due to increase in fraction of price changes
  - 0.41 to 0.43 on impact

Intuition

- Small jump in frequency (0.41 to 0.43) has large effect

- To see why, log-linearize expression for aggregate price index

$$\hat{\pi}_t = \underbrace{\frac{1-(1-n)\,\pi^{\theta-1}}{(1-n)\,\pi^{\theta-1}}}_{\mathcal{N}}\,\hat{p}_t^* + \underbrace{\frac{1}{(1-n)\,\pi^{\theta-1}}\,\frac{\pi^{\theta-1}-1}{\theta-1}}_{\mathcal{M}}\,\hat{n}_t$$

- Elasticity $\mathcal{N}$ to reset price changes: identical to Calvo
  - increases with n, decreases with π (lower weight on new prices)

- Elasticity $\mathcal{M}$ to frequency: zero if $\pi = 1$, increases with inflation

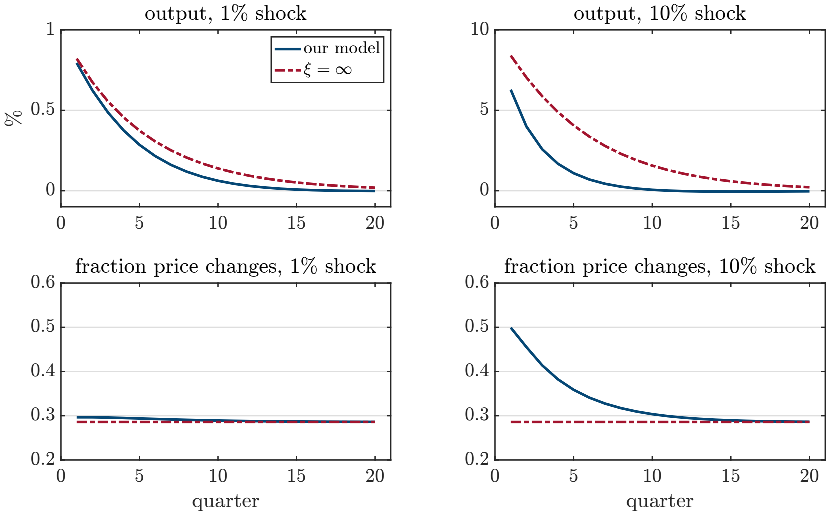
Intuition

- Why is price level more responsive to changes in n at high inflation?

$$\mathcal{M} = \frac{1}{(1-n)\,\pi^{\theta-1}} \frac{\pi^{\theta-1}-1}{\theta-1}$$

- Inflation $\approx$ average price change $\times$ fraction of price changes
  - $\pi = 1$: average price change $= 0$ so fraction inconsequential
  - π is high: average price change is large
    - so Δn increases inflation considerably
    - mechanism in Caplin and Spulber (1986) menu cost model
- To a first-order, changes in n only matter because of trend inflation
  - prices even more responsive to large shocks using non-linear solution

Response to Large Shock, Baseline Model



Strong non-linearities, as in menu cost model of Blanco et al. (2024)

Towards the Slope of the Phillips Curve

- Log-linearize optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}\hat{\pi}_t + \mathcal{B}\hat{p}_t^* - \mathcal{C}\hat{x}_{t-1} + \frac{n-\bar{n}}{n}\hat{b}_{1t}$$

- Elasticities $\mathcal{A}$ and $\mathcal{B}$ increase with trend inflation

$$\mathcal{A} = \frac{\theta-1}{\xi n}\frac{1}{1-\beta(1-n)\pi^{\theta-1}}\frac{\pi^{\frac{\theta}{\eta}}-\pi^{\theta-1}}{1-(1-n)\pi^{\frac{\theta}{\eta}}}$$

$$\mathcal{B} = (1-\tau\eta)\frac{\theta-1}{\xi n}\frac{1-(1-n)\pi^{\theta-1}}{1-\beta(1-n)\pi^{\theta-1}}\frac{1}{n}\frac{\pi^{\frac{\theta}{\eta}}-1}{1-(1-n)\pi^{\frac{\theta}{\eta}}}$$

  - zero when $\pi = 1$, so our model identical to Calvo up to a first-order

Inflation Accelerator

- Recall expression for price index

$$\hat{\pi}_t = \mathcal{N}\hat{p}_t^* + \mathcal{M}\hat{n}_t$$

- Optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}\hat{\pi}_t + \mathcal{B}\hat{p}_t^* - \mathcal{C}\hat{x}_{t-1} + \frac{n - \bar{n}}{n}\hat{b}_{1t}$$

- Feedback loop amplifies inflation response to changes in reset price

$$\hat{\pi}_t = \frac{\mathcal{N} + \mathcal{M}\mathcal{B}}{1 - \mathcal{M}\mathcal{A}}\hat{p}_t^* - \frac{\mathcal{M}\mathcal{C}}{1 - \mathcal{M}\mathcal{A}}\hat{x}_{t-1} + \frac{\mathcal{M}}{1 - \mathcal{M}\mathcal{A}}\frac{n - \bar{n}}{n}\hat{b}_{1t}$$

Phillips Curve

- Let $\widehat{mc}_t = \frac{1}{\eta}\hat{y}_t$ aggregate real marginal cost

- Phillips curve

$$
\begin{aligned}
\hat{\pi}_t \quad = \quad & \mathcal{K}\widehat{mc}_t + \beta(1-n)\left(\frac{\frac{\theta}{\eta}\pi^{\frac{\theta}{\eta}} - (\theta-1)\pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{N}+\mathcal{M}\mathcal{B}}{1-\mathcal{M}\mathcal{A}} + \pi^{\frac{\theta}{\eta}}\right)\mathbb{E}_t\hat{\pi}_{t+1} \\
+ \quad & \beta(1-n)\left(\frac{\pi^{\frac{\theta}{\eta}} - \pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{N}+\mathcal{M}\mathcal{B}}{1-\mathcal{M}\mathcal{A}} - \pi^{\frac{\theta}{\eta}}\frac{\mathcal{M}}{1-\mathcal{M}\mathcal{A}}\frac{n-\bar{n}}{n}\right)\mathbb{E}_t\hat{b}_{1t+1} \\
- \quad & \beta n\frac{\pi^{\frac{\theta}{\eta}} - \pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{N}+\mathcal{M}\mathcal{B}}{1-\mathcal{M}\mathcal{A}}\mathbb{E}_t\hat{n}_{t+1} \\
+ \quad & \beta(1-n)\pi^{\frac{\theta}{\eta}}\frac{\mathcal{M}\mathcal{C}}{1-\mathcal{M}\mathcal{A}}\hat{x}_t - \frac{\mathcal{M}\mathcal{C}}{1-\mathcal{M}\mathcal{A}}\hat{x}_{t-1} + \frac{\mathcal{M}}{1-\mathcal{M}\mathcal{A}}\frac{n-\bar{n}}{n}\hat{b}_{1t}
\end{aligned}
$$

Slope of the Phillips Curve

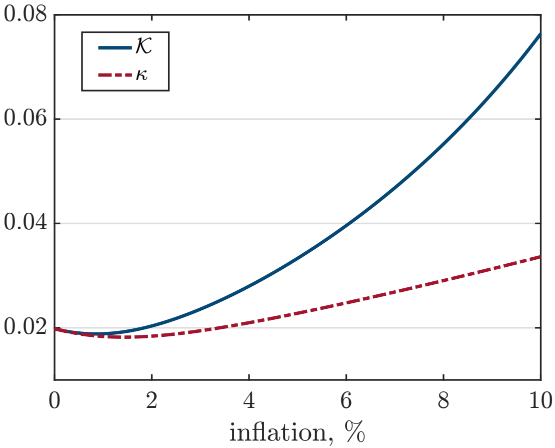
$$\mathcal{K} = \underbrace{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}_{\text{complementarities}} \quad \times \quad \underbrace{\left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)}_{\text{horizon effect}} \quad \times \quad \underbrace{\frac{\mathcal{N}+\mathcal{MB}}{1-\mathcal{MA}}}_{\text{reset price}}$$

- If $\xi = \infty$, reduces to slope in Calvo

$$\kappa = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)\frac{1-\left(1-n\right)\pi^{\theta-1}}{\left(1-n\right)\pi^{\theta-1}}$$

- Difference between $\mathcal{K}$ and κ captures inflation accelerator

Slope of the Phillips Curve



Much steeper at high inflation, mostly due to inflation accelerator

Phillips Curve in the Time-Series

Approach

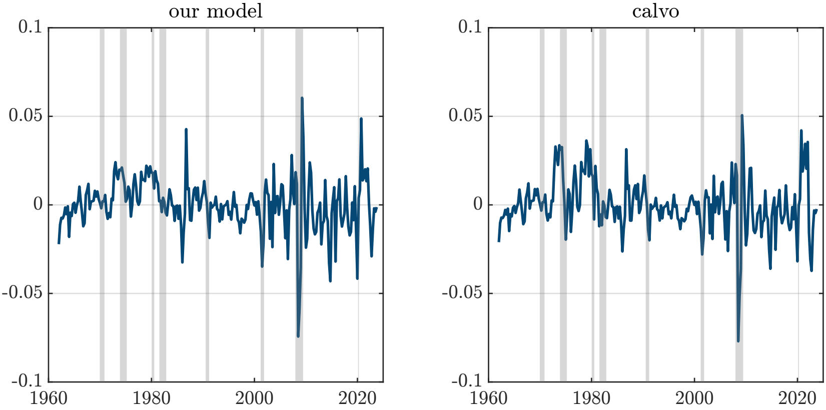
- Use non-linear solution to back out shocks that match U.S. inflation series

$$\pi_t = \pi\left(\frac{p_{t-1}}{\exp\left(\mu + \varepsilon_t\right)}, x_{t-1}\right)$$

  – initialize 1962 in stochastic steady state

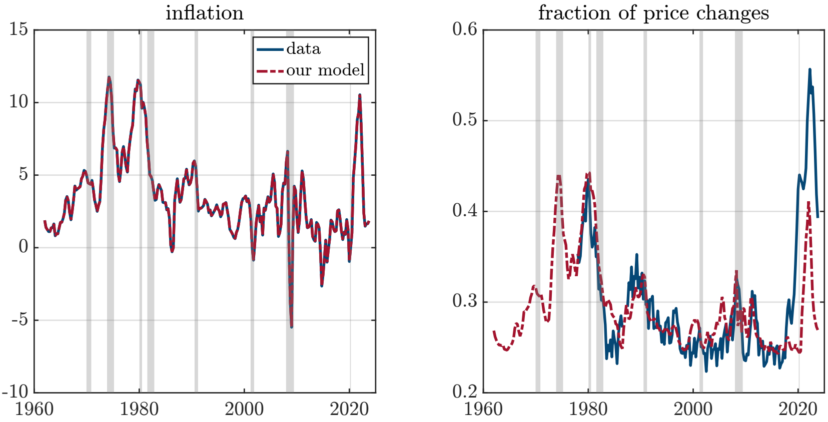
- Derive Phillips curve by perturbing equilibrium conditions at each date

Monetary Policy Shocks



Our model requires smaller monetary shocks to explain 1970s and 1980s

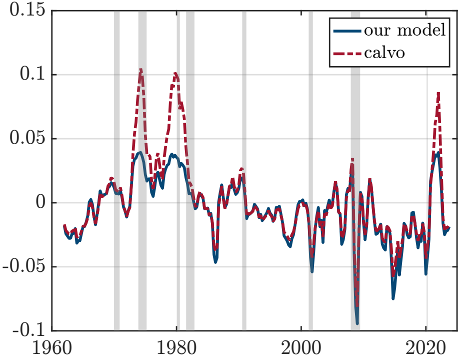
Fraction of Price Changes



- Reproduces fraction well, except post-Covid
  - many more price decreases, presumably due to sectoral shocks

▸ extensive margin model

Output Gap



Our model: smaller increase in output gap in periods of high inflation

Slope of the Phillips Curve

- First-order perturbation around equilibrium point at each date t

$$\hat{\pi}_t = \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}\,\hat{p}_t^* + \underbrace{\frac{1}{(1-n_t)\,\pi_t^{\theta-1}}\,\frac{\pi_t^{\theta-1}-1}{\theta-1}}_{\mathcal{M}_t}\,\hat{n}_t$$

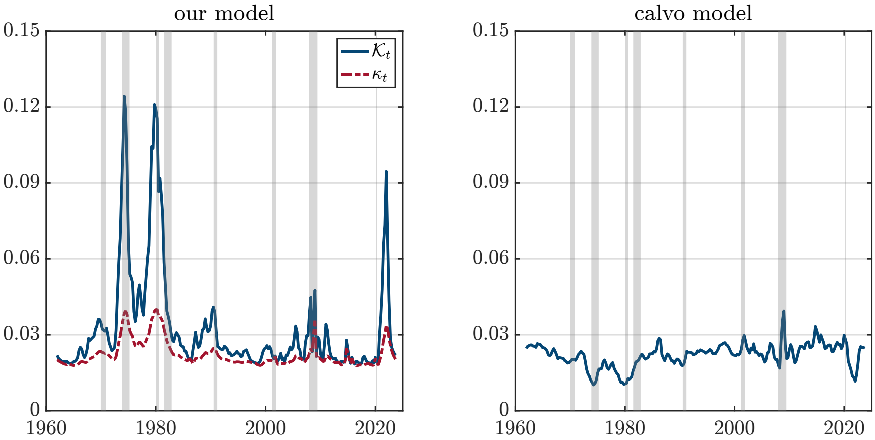
- Slope of the Phillips curve

$$\mathcal{K}_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\,\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}\,\frac{\mathcal{N}_t+\mathcal{M}_t\mathcal{B}_t}{1-\mathcal{M}_t\mathcal{A}_t}$$

- Absent endogenous frequency response

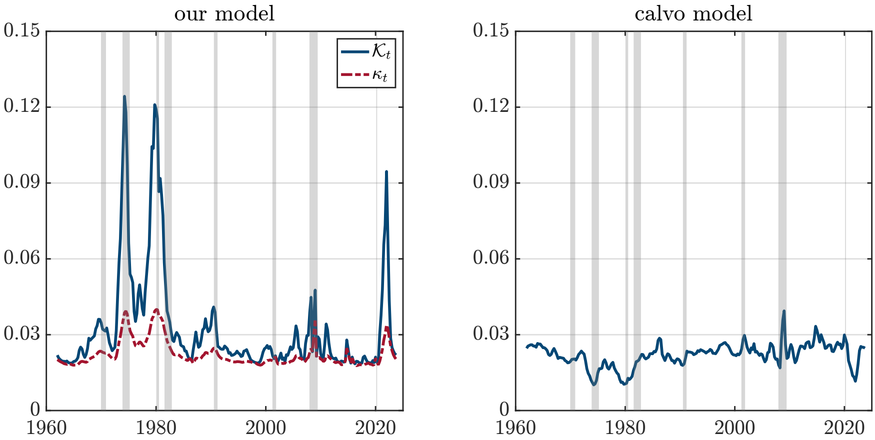
$$\kappa_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\,\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}\,\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}$$

Time-Varying Slope of the Phillips Curve



Ranges from 0.02 to 0.12, mostly due to inflation accelerator

Time-Varying Slope of the Phillips Curve



In Calvo model slope falls in periods of high inflation

Time-Varying Responses to Monetary Shocks

- Build intuition by computing log-linear approximation

$$\mathbf{A}_t \mathbf{z}_t = \mathbf{B}_t \mathbf{z}_{t-1} + \mathbf{C}_t \mathbf{z}_{t+1}$$

  – $\mathbf{z}_t$ log-deviations from initial equilibrium point

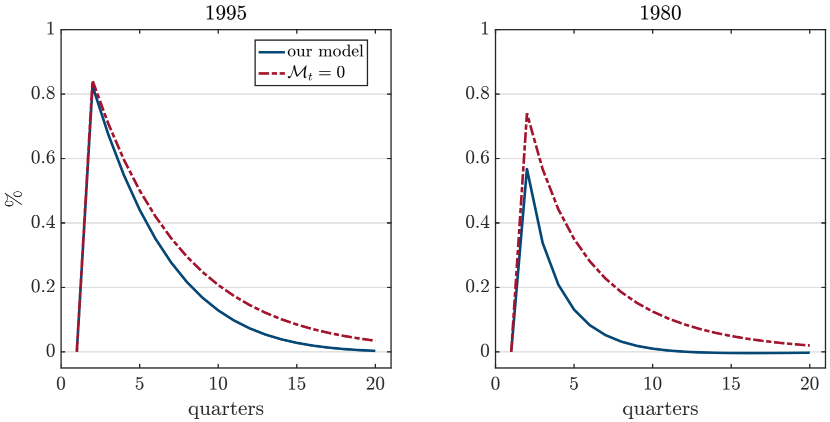
  – $\mathbf{A}_t$ to $\mathbf{C}_t$ collect time-varying elasticities, including $\mathcal{M}_t$

  – compute using ε_t that match U.S. inflation up to that date, zero after

- Solution $\mathbf{z}_t = \mathbf{Q}_t \mathbf{z}_{t-1}$, where $\mathbf{Q}_t = \left(\mathbf{A}_t - \mathbf{C}_t \mathbf{Q}_{t+1}\right)^{-1} \mathbf{B}_t$

- Repeat setting $\mathcal{M}_t = 0$ to isolate inflation accelerator

Time-varying IRF to 1% Money Shock



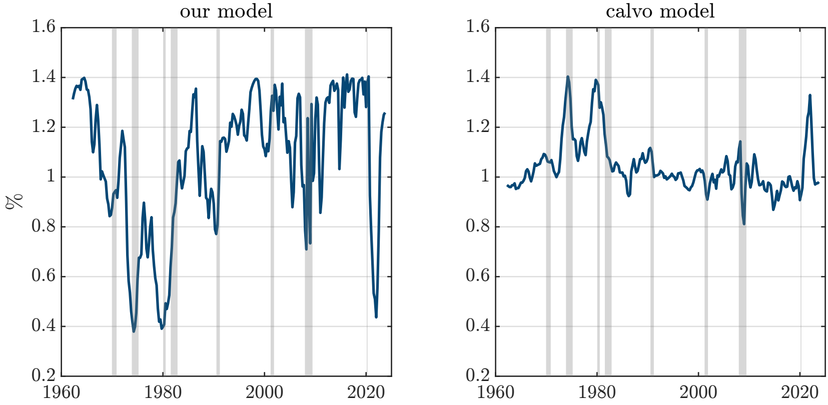
CIR: 4.0% in 1995 vs. 1.4% in 1980, mostly due to accelerator

Sacrifice Ratio

- Time-varying slope: reducing inflation less costly when inflation is high

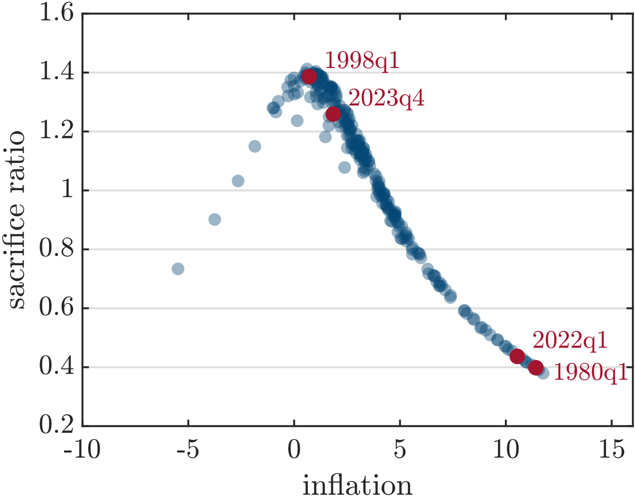
- Calculate average drop in output needed to reduce π by 1% over a year
  - use non-linear solution of the model
  - report average decline in output over 4 quarters of that year

Sacrifice Ratio



Ranges from 0.4% to 1.4%, opposite of Calvo

Inflation and the Sacrifice Ratio



Robustness

Eliminate Strategic Complementarities

- Set $\eta = 1$, recalibrate model

Targeted Moments

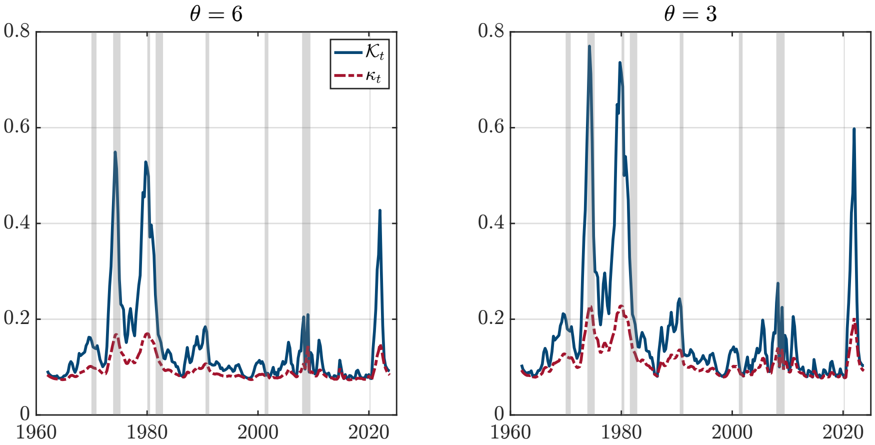
	Data	$\theta = 6$	$\theta = 3$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $\lvert\pi_t\rvert$	0.016	0.016	0.016

Calibrated Parameters

		$\theta = 6$	$\theta = 3$
μ	mean spending growth rate	0.035	0.035
σ	s.d. monetary shocks	0.019	0.018
$\bar{n}$	fraction free price changes	0.232	0.227
ξ	adjustment cost	0.365	0.109

- Smaller price adjustment costs because less curvature in profit function

Slope of the Phillips Curve



Larger absent complementarities, but fluctuates as much

Taylor Rule

- Replace nominal spending target with Taylor rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_i} \left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y}\right)^{1-\phi_i} u_t$$

- Two versions
  - u_t shocks iid
  - serially correlated with persistence ρ to match autocorrelation inflation
- Use Justiniano and Primiceri (2008) estimates
  - $\phi_i = 0.65$, $\phi_\pi = 2.35$, $\phi_y = 0.51$

Calibration of Economy with a Taylor Rule

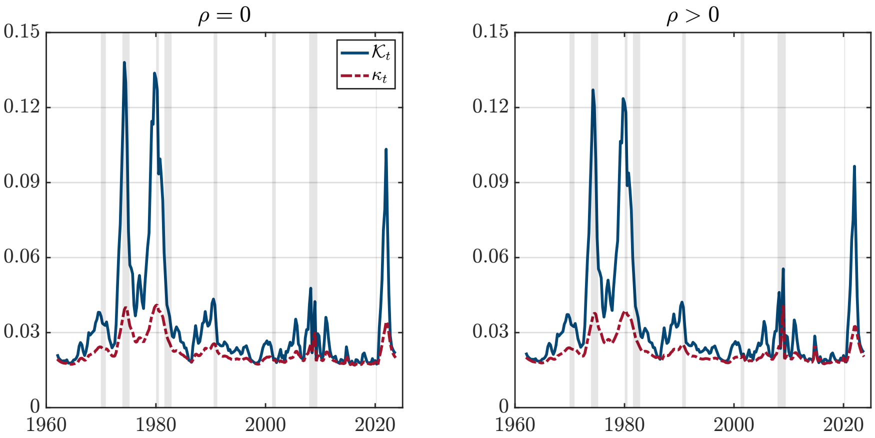
Targeted Moments

	Data	$\rho = 0$	$\rho > 0$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $\lvert\pi_t\rvert$	0.016	0.016	0.016
autocorr. inflation	0.942	*0.913*	0.942

Calibrated Parameters

		$\rho = 0$	$\rho > 0$
$\log \pi$	inflation target	0.040	0.037
σ	s.d. monetary shocks $\times 100$	2.626	0.551
ρ	persistence money shocks	–	0.685
$\bar{n}$	fraction free price changes	0.241	0.241
ξ	adjustment cost	1.671	1.688

Slope of the Phillips Curve



Our results robust to assuming a Taylor rule

Conclusions

- Data: fraction of price changes increases with inflation

- Developed tractable model consistent with this evidence
  - firms choose how many, but not which prices to change
  - reduces to one-equation extension of Calvo

- Implies slope of Phillips curve increases considerably with inflation
  - partly because more frequent price changes
  - primarily due to endogenous frequency response – *inflation accelerator*

Firm Problem

- Real flow profits of firm i

$$\int_0^1 \left(\left(\frac{P_{ikt}}{P_t} \right)^{1-\theta} y_t - \tau \frac{W_t}{P_t} \left(\frac{P_{ikt}}{P_t} \right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}} \right) \mathrm{d}k$$

- Log-linear preferences and montary policy rule imply $c_t = \frac{W_t}{P_t} = y_t$

- So the value of the firm is

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\int \left[\left(\frac{P_{ikt+s}}{P_{t+s}} \right)^{1-\theta} - \tau \left(\frac{P_{ikt+s}}{P_{t+s}} \right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}} \right] \mathrm{d}k - \frac{\xi}{2} \left(n_{it+s} - \bar{n} \right)^2 \right)$$

- Or, using the definition of P_{it} and X_{it}

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\left(\frac{P_{it+s}}{P_{t+s}} \right)^{1-\theta} - \tau \left(\frac{X_{it+s}}{P_{t+s}} \right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}} - \frac{\xi}{2} \left(n_{it+s} - \bar{n} \right)^2 \right]$$

back

Losses from Misallocation

$$
\begin{aligned}
(X_{it+s})^{-\frac{\theta}{\eta}} = {} & n_{it+s} \left(P^*_{it+s}\right)^{-\frac{\theta}{\eta}} + (1 - n_{it+s})\, n_{it+s-1} \left(P^*_{it+s-1}\right)^{-\frac{\theta}{\eta}} + \cdots \\
& + \prod_{j=1}^{s} (1 - n_{it+j})\, n_{it} \left(P^*_{it}\right)^{-\frac{\theta}{\eta}} + \prod_{j=1}^{s} (1 - n_{it+j})\,(1 - n_{it})\,(X_{it-1})^{-\frac{\theta}{\eta}}
\end{aligned}
$$

back

Optimal Reset Price

- Optimal reset price depends on present value of future marginal costs

$$P_{it}^{*} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \omega_{it+s} MC_{it+s}$$

- Marginal cost in $t+s$ of producing a good with price last reset in t

$$MC_{it+s} = \frac{1}{\eta} W_{t+s} \left(y_{it+s} \right)^{\frac{1}{\eta}-1} = \frac{1}{\eta} W_{t+s} \left(\frac{P_{it}^{*}}{P_{t+s}} \right)^{-\theta\left(\frac{1}{\eta}-1\right)} y_{t+s}^{\frac{1}{\eta}-1}$$

- Weight on $t+s$ reflects probability that a price is still in effect then

$$\omega_{it+s} = \frac{\beta^s \left(P_{t+s}\right)^{\theta-1} \prod_{j=1}^{s} \left(1-n_{it+j}\right)}{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(P_{t+s}\right)^{\theta-1} \prod_{j=1}^{s} \left(1-n_{it+j}\right)}$$

back

Output and Productivity

- Steady-state output

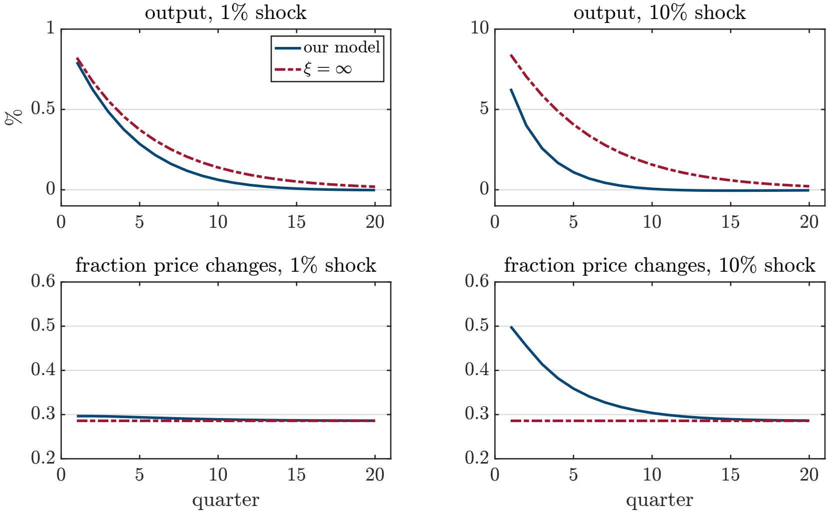
$$y^{\frac{1}{\eta}} = \eta \frac{1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}}{1-\beta\left(1-n\right)\pi^{\theta-1}}\left(\frac{n}{1-\left(1-n\right)\pi^{\theta-1}}\right)^{\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{\theta-1}}$$

- Steady-state productivity

$$x^{\theta} = \left(\frac{1-\left(1-n\right)\pi^{\frac{\theta}{\eta}}}{n}\right)^{\eta}\left(\frac{1-\left(1-n\right)\pi^{\theta-1}}{n}\right)^{-\frac{\theta}{\theta-1}}$$

back

Response to Large Shock, Baseline Model



Strong non-linearities, as in menu cost model of Blanco et al. (2024)

back

Elasticities $\mathcal{A}$ and $\mathcal{B}$

- Log-linearize optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}\hat{\pi}_t + \mathcal{B}\left(\hat{p}_t^* - \hat{p}_t\right) - \mathcal{C}\hat{x}_{t-1} + \frac{n - \bar{n}}{n}\hat{b}_{1t}$$

- Elasticities $\mathcal{A}$ and $\mathcal{B}$ increase with trend inflation

$$\mathcal{A} = \frac{\theta - 1}{\xi n}\frac{1}{1 - \beta\left(1 - n\right)\pi^{\theta - 1}}\frac{\pi^{\frac{\theta}{\eta}} - \pi^{\theta - 1}}{1 - \left(1 - n\right)\pi^{\frac{\theta}{\eta}}}$$

$$\mathcal{B} = \left(1 - \tau\eta\right)\frac{\theta - 1}{\xi n}\frac{1 - \left(1 - n\right)\pi^{\theta - 1}}{1 - \beta\left(1 - n\right)\pi^{\theta - 1}}\frac{1}{n}\frac{\pi^{\frac{\theta}{\eta}} - 1}{1 - \left(1 - n\right)\pi^{\frac{\theta}{\eta}}}$$

  - zero when $\pi = 1$, so our model identical to Calvo up to a first-order

back

Phillips Curve

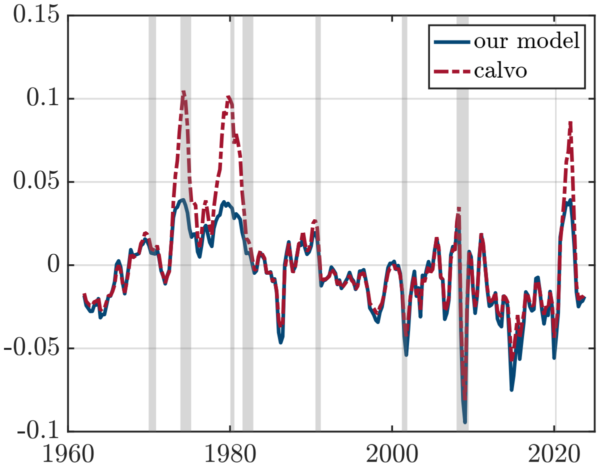
- Let $\widehat{mc}_t = \frac{1}{\eta}\hat{y}_t$ aggregate real marginal cost

- Phillips curve

$$
\begin{aligned}
\hat{\pi}_t \quad = \quad & \mathcal{K}\widehat{mc}_t + \beta(1-n)\left(\frac{\frac{\theta}{\eta}\pi^{\frac{\theta}{\eta}} - (\theta-1)\pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}} + \pi^{\frac{\theta}{\eta}}\right)\mathbb{E}_t\hat{\pi}_{t+1} \\
+ \quad & \beta(1-n)\left(\frac{\pi^{\frac{\theta}{\eta}} - \pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}} - \pi^{\frac{\theta}{\eta}}\frac{\mathcal{M}}{1-\mathcal{MA}}\frac{n-\bar{n}}{n}\right)\mathbb{E}_t\hat{b}_{1t+1} \\
- \quad & \beta n\frac{\pi^{\frac{\theta}{\eta}} - \pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}}\mathbb{E}_t\hat{n}_{t+1} \\
+ \quad & \beta(1-n)\pi^{\frac{\theta}{\eta}}\frac{\mathcal{MC}}{1-\mathcal{MA}}\hat{x}_t - \frac{\mathcal{MC}}{1-\mathcal{MA}}\hat{x}_{t-1} + \frac{\mathcal{M}}{1-\mathcal{MA}}\frac{n-\bar{n}}{n}\hat{b}_{1t}
\end{aligned}
$$

back

Output Gap



Our model: smaller output gap in periods of high inflation

back

Time-Varying Responses to Monetary Shocks

- Build intuition by computing log-linear approximation

$$\mathbf{A}_t \mathbf{z}_t = \mathbf{B}_t \mathbf{z}_{t-1} + \mathbf{C}_t \mathbf{z}_{t+1}$$

  - $\mathbf{z}_t$ log-deviations from initial equilibrium point
  - $\mathbf{A}_t$ to $\mathbf{C}_t$ collect time-varying elasticities, including $\mathcal{M}_t$
  - compute using ε_t that match U.S. inflation up to that date, zero after

- Solution $\mathbf{z}_t = \mathbf{Q}_t \mathbf{z}_{t-1}$, where $\mathbf{Q}_t = \left(\mathbf{A}_t - \mathbf{C}_t \mathbf{Q}_{t+1}\right)^{-1} \mathbf{B}_t$

- Repeat setting $\mathcal{M}_t = 0$ to isolate inflation accelerator

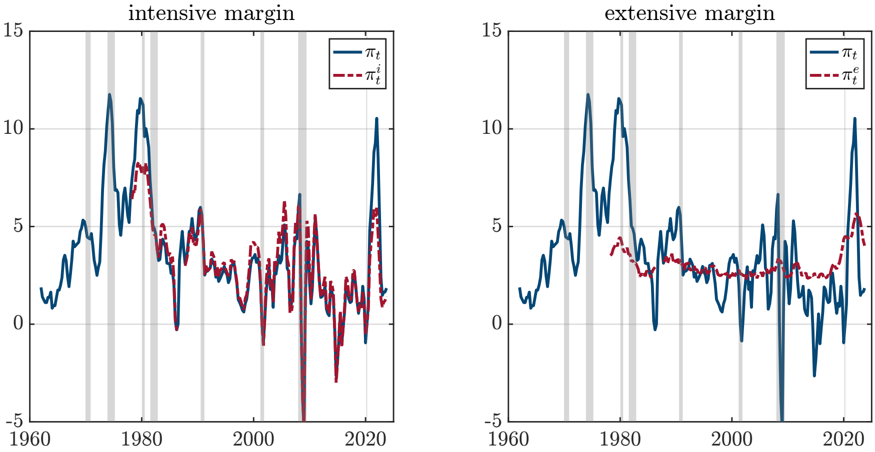
back

Role of Extensive Margin

- Decompose $\pi_t = \Delta_t n_t$ into two components
  - Δ_t : average price change conditional on adjustment
  - n_t : fraction of price changes

- Isolate role of each using Klenow and Kryvtsov (2008) decomposition
  - intensive margin: $\pi_t^i = \Delta_t \bar{n}$
    - $\bar{n}$: mean fraction of price changes
  - extensive margin: $\pi_t^e = \bar{\Delta} n_t$
    - $\bar{\Delta}$: mean average price change

Role of Extensive Margin: Data



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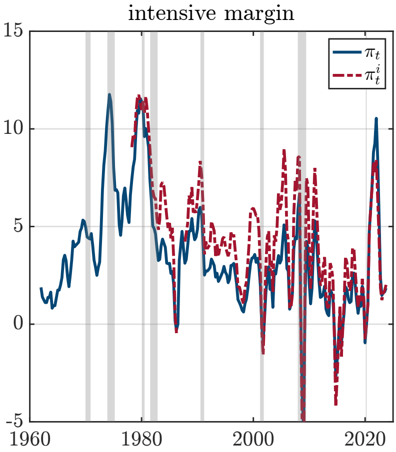
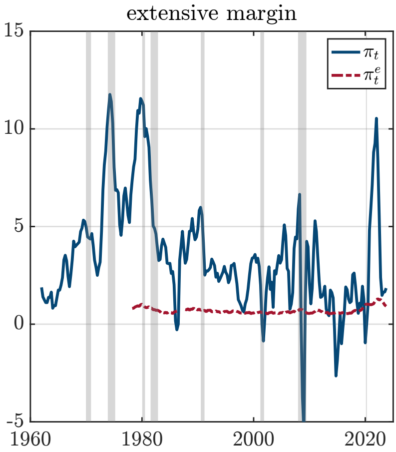
Montag and Villar (2024)

- Argue that extensive margin plays no role post Covid
- Same decomposition but set $\bar{n}$ and $\bar{\Delta}$ equal to January 2020 values
  - due to seasonality, unusually large n and low Δ
- Illustrate fixing $\bar{n}$ and $\bar{\Delta}$ at January 2020 values

Role of Extensive Margin using January 2020





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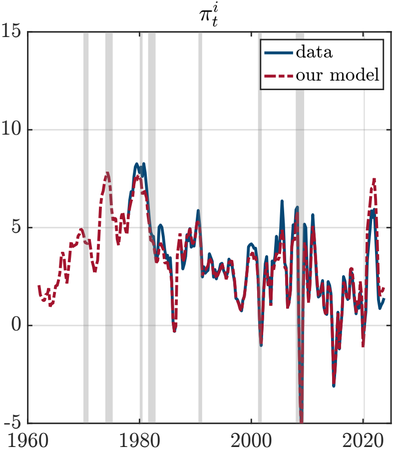
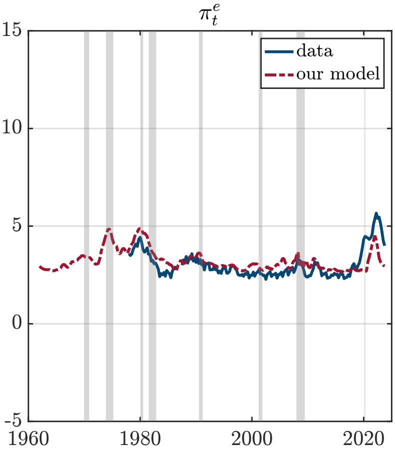
Role of Extensive Margin: Our Model





back

Eliminate Strategic Complementarities

- Set $\eta = 1$, recalibrate model

back

Targeted Moments

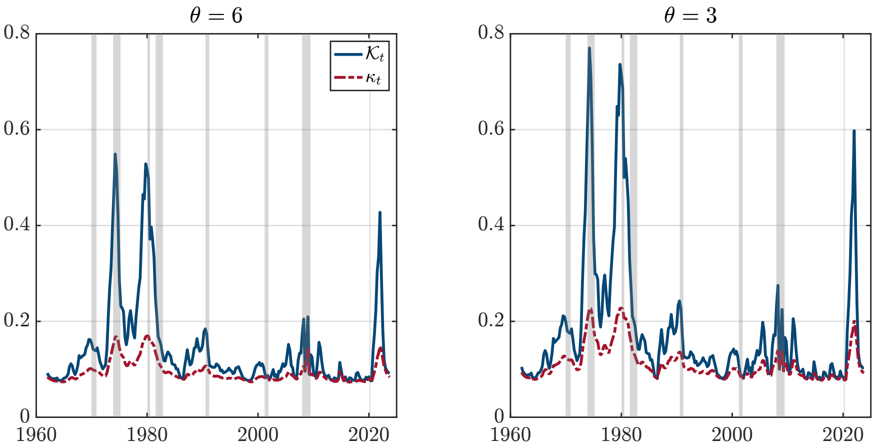
	Data	$\theta = 6$	$\theta = 3$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean frequency	0.297	0.297	0.297
slope of n_t on $\lvert\pi_t\rvert$	0.016	0.016	0.016

Calibrated Parameters

		$\theta = 6$	$\theta = 3$
μ	mean spending growth rate	0.035	0.035
σ	s.d. monetary shocks	0.019	0.018
$\bar{n}$	fraction free price changes	0.232	0.227
ξ	adjustment cost	0.365	0.109

- Smaller price adjustment costs because less curvature in profit function

Slope of the Phillips Curve



Larger absent complementarities, but fluctuates as much

back

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- Two versions
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back

Calibration of Economy with a Taylor Rule

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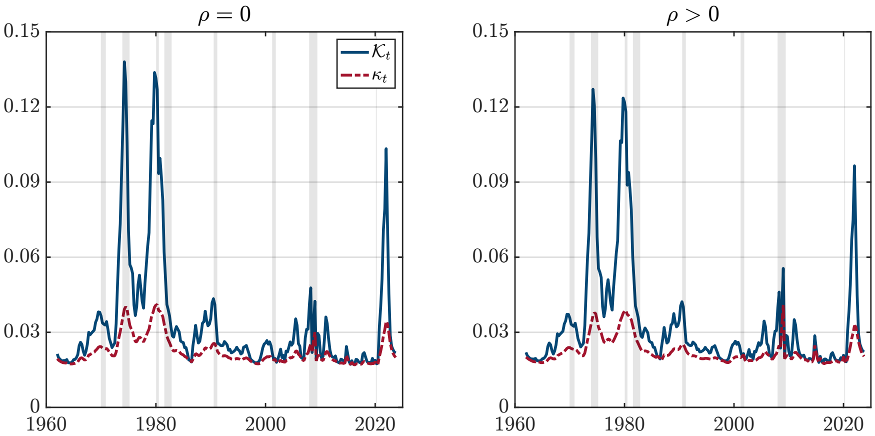

Our results are robust to assuming a Taylor rule

back