International Spillovers of Forward Guidance Shocks*

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Abstract

A two-country model of the US and Canada is estimated over the post 2009 sample to study the cross-country spillovers of forward guidance shocks. To do so, we provide a method to identify forward guidance shocks during the fixed interest rate regime. US forward guidance shocks have an impact which is on average twice as large as conventional monetary policy shocks. The effect of US forward guidance shocks on Canadian output, unlike conventional policy shocks, propagates non-linearly through expectations and depends on the state of the US risk premium shock. The estimated forward guidance shocks are externally validated with other measures of the stance of US monetary policy.

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1 Introduction

In response to the 2007-08 financial crisis, the Federal Reserve cut its policy interest rate to near zero and turned to unconventional monetary policies including forward guidance policy. The unprecedented monetary actions in the US provoked a debate about the effects of these policies on other open economies, which saw their currencies appreciate considerably against the US dollar. Rajan (2015), for instance, argues that the forward guidance policy of '*lower interest rates for longer*' triggered international capital flows that, in a 'search for yield', appreciated non-US currencies and shifted spending away from domestically produced goods. In contrast, Bernanke (2017) argues that growth in the US during the recent recovery has not been driven by exports and that the 'expenditure-augmenting' effects of expansionary US monetary policies were likely to have added to global demand and to have counteracted the 'expenditure-switching' forces that were the concern of Rajan (2015).

Structural small open economy models have both expenditure-augmenting and expenditure-switching forces. These two competing forces are highlighted in studies of optimal policy in open economies (Bodenstein et al., 2017; Fujiwara et al., 2013; Haberis and Lipinska, 2012). In theory, and as shown in these papers, the net effect of international monetary policy spillovers on domestic output is ambiguous and depends crucially on the trade elasticity of substitution between domestic and foreign goods. When domestic goods easily substitute for foreign goods, the exchange rate appreciation caused by an expansionary monetary policy shock abroad causes a decline in the demand for domestic goods that can outweigh any increase due to an expanding foreign economy – a 'beggar-thy-neighbor' effect. It is therefore an empirical issue to determine the relative strength of expenditure-augmenting and expenditure-switching forces and the net effect of monetary policy spillovers.

An important contribution of this paper, in our view, is to propose a method to identify forward guidance shocks in samples with fixed interest rate regimes in forward looking structural models. Our paper is related to Inoue and Rossi (2018) who propose a method to identify forward guidance shocks through exogenous shifts of the term structure in the context of a VAR model. As we discuss below, the identification of forward guidance shocks in forward looking structural models requires extending the estimation method of Kulish, Morley and Robinson (2017) with the occasionally binding constraint solution of Guerrieri and Iacoviello (2015) and Jones (2017).

We then quantify the spillovers of forward guidance shocks using our proposed method and an estimated structural open economy model. Structural evidence on the spillovers of forward guidance shocks has so far been absent in the literature. While there is some evidence on the international effects from the unconventional monetary policy period in the US, this evidence is mixed[1] and limited to the impact of quantitative easing policies on emerging markets using reduced form methods.[2] Although a considerable literature exploring the effects of interest rate shocks exists, these results do not carry over to the effect of forward guidance shocks because, as we discuss below, forward guidance shocks are changes to the expected path of the policy rate that propagate non-linearly through expectations. Unlike conventional shocks in linear models, the impact of forward guidance shocks is state-dependent.[3]

The model used to quantify the international spillovers of forward guidance by the Federal Reserve is an extension of the canonical two-country small open economy model of Galí and Monacelli (2005). We apply the model to the US and Canada and focus on the period from 2009 to 2014. This period includes the largest changes in the expected duration of the Federal Reserve zero interest rate policy. We choose Canada for two reasons. First, Canada is closely linked financially and through trade in goods and services with the US – around three quarters of Canada's merchandise exports go to the US, accounting for close to 20 per cent of its GDP. Second, the Bank of Canada closely followed US monetary policy after the crisis and responded with calendar-based forward guidance of its own in 2009 (see Murray, 2013). This provides an opportunity to quantify, in addition to the spillovers of US forward guidance, the impact of domestic forward guidance on a small open economy.[4]

As the theory emphasizes, the effect of foreign monetary policy on a small open economy is ambiguous. We first consider a simplified version of our estimated model in order to distill the transmission mechanism and identify the key parameters that govern the strength of monetary policy spillovers, namely the trade elasticity of substitution and the parameters governing the degree of price stickiness in both countries. In estimation, we put wide enough priors over these parameters to accommodate the perspectives of both Bernanke (2017) and Rajan (2015) and let the data speak through the lens of our model. Our estimates imply that expenditure-switching effects are stronger than expenditure-augmenting effects across the posterior range, so that expansionary US monetary policy is 'beggar-thy-neighbor' for Canada. But, as in Mundell-Fleming type models, we find using counterfactuals that although expansionary US monetary policy is 'beggar-thy-neighbor', in aggregate the US and Canada are both better off by jointly responding with expansionary forward guidance to the large contractionary shocks that took

[1] See Ammer et al. (2016), Dedola et al. (2017), Bernanke (2017), and Kim (2001).

[2] See Anaya et al. (2017), Chen et al. (2016) and Tillmann (2016) for evidence that relies on vector autoregressions, and Chen et al. (2014) for regression evidence using an event-study type identification for monetary policy shocks.

[3] See Bernanke and Reinhart (2004) and Eggertsson and Woodford (2003).

[4] Following the classification of Campbell et al. (2012), our focus is on calendar-based 'Odyssean' forward guidance policies.

place during the Great Recession.[5]

Regarding our estimates, we find that the expected durations of the fixed interest rate policy varied more in the US than they did in Canada. For the US, we find that the degree of expansionary forward guidance was strongest between 2011 and 2013, consistent with the explicit calendar-based announcements made by the Federal Reserve. We also find evidence for forward guidance stimulus in Canada whereby the expected duration of fixed interest rates is longer than the duration implied by the estimated structural shocks.

The estimated average forward guidance shock in the US is 2 quarters. The largest forward guidance shock in the US is estimated to be 4 quarters in duration in 2011Q2, the same quarter as the introduction of calendar-based forward guidance. As an additional assessment of the validity of our estimates, we compute the shadow policy rate which depends on the estimated forward guidance shocks. Our shadow rate is close to those of Krippner (2014) and Wu and Xia (2016) who rely on a different methodology. In Canada, the largest forward guidance shock is 5 quarters, which occurs when the policy interest rate first hits 0.25% in 2009Q2. This is in line with the calendar-based announcement made by the Bank of Canada that the policy would remain at 0.25% until 2010Q2.

An implication of our results is that, in contrast to conventional monetary policy shocks, the propagation of forward guidance shocks is state-dependent. In particular, international spillovers depend strongly on the state of the US risk premium shock: a large, negative state of the US risk premium shock diminishes both the expansionary consequences of an extension in the expected duration of fixed interest rate policy for the US as well as the implied contractionary impact on Canada. We show that an unexpected 2 quarter extension of the US expected duration in 2012Q2, when US demand was relatively weak, has an effect on US and Canadian output which is approximately that of a 2 standard deviation conventional monetary shock in normal times when interest rate constraints are not binding. Nevertheless, the effects of forward guidance are meaningful. In response to the 2 quarter forward guidance shock in 2012Q2, US output increases on impact by 0.3% and Canadian output declines by almost 0.2%. Following the US forward guidance shock, we estimate that the USD/CAD real exchange rate depreciates by almost 1%.

McKay, Nakamura and Steinsson (2016) argue that if agents face uninsurable income risk and borrowing constraints, their responses to changes in future interest rates are muted and so forward guidance would be less powerful. In our model, the power of forward guidance depends on the state of

[5] For earlier discussions of these mechanisms in Mundell-Fleming-type models see Mundell (1968), chapter 18, and Eichengreen and Sachs (1985).

the risk premium shock. Our findings are consistent with the view of McKay, Nakamura and Steinsson (2016), as borrowing constraints drive a wedge in the Euler consumption equation, as in Jones, Midrigan and Philippon (2018), in the same way as risk premium shocks in our model.

The rest of the paper is structured as follows. Section 2 outlines the model and discusses the transmission in a simplified version. Section 3 discusses the solution and estimation method and Section 4 discusses the identification of forward guidance shocks. Section 5 presents estimation results while Section 6 presents the cross-country effects of forward guidance in our estimated model and computes counterfactual paths of the US and Canadian economies absent forward guidance shocks. Section 7 concludes.

2 Model

We conduct our analysis using a small open economy model along the lines of Galí and Monacelli (2005). The model features two economies: a large (foreign) economy and a small (domestic) economy. Economic developments in the large economy affect the small economy, but the reverse is not true. As in De Paoli (2009a), the model is the limiting case of a two-country model where the relative size of one of the economies goes to zero. We extend this otherwise standard framework in four dimensions, including: (i) imperfect exchange rate pass-through; (ii) trend inflation; (iii) interest rates of longer maturities; and (iv) habits in the utility function.[6]

2.1 The Large Economy

This economy is populated by a large number of households who maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi_t^* \left(\log(C_t^* - hC_{t-1}^*) - \frac{N_t^{*\,1+\varphi}}{1+\varphi} \right) \right], \tag{1}$$

where N_t^* is labor supply and ξ_t^* denotes intertemporal preference shocks that follow an AR(1) process in logs. C_t^* is a composite consumption index given by $C_t^* = \left[\int_0^1 C_t^*(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}$, where ν is the elasticity of substitution between types of differentiated goods. Households face in every period t the flow budget constraint:

$$P_t^* C_t^* + \mathbb{E}_t \left\{ Q_{t,t+1}^* D_{t+1}^* \right\} \leq D_t^* + W_t^* N_t^* + T_t^*, \tag{2}$$

[6] The full set of log-linear equations used in estimation is given in the Appendix.

for all $t > 0$, where P_t^* is the large economy's CPI, W_t^* is the nominal wage rate and T_t^* denotes taxes and transfers. D_{t+1}^* is the nominal payoff in period $t+1$ of the portfolio held at the end of period t and $Q_{t,t+1}^*$ is the stochastic discount factor for one-period ahead nominal payoffs. Households have access to a complete set of internationally traded contingent claims.

Firms produce differentiated goods with the technology:

$$Y_t^*(i) = Z_t A_t^* N_t^*(i), \tag{3}$$

where $Y_t^*(i)$ is the production and $N_t^*(i)$ the labor input of firm i. Z_t is the trend component of productivity, which follows a deterministic process that grows at the rate μ, and A_t^* is the transitory component of productivity, which follows an AR(1) process in logs. Real marginal costs are common across firms:

$$MC_t^* = \frac{W_t^*}{P_t^* Z_t A_t^*}. \tag{4}$$

Firms face Calvo-style pricing frictions. Each quarter, a fraction of firms, $1-\theta^*$, sets prices optimally, while the remainder adjusts their prices by the steady-state inflation rate, Π^*. The pricing problem for a representative firm i is:

$$\max_{P_t^*(i)} \sum_{k=0}^{\infty} (\beta\theta^*)^k \mathbb{E}_t \left\{ \frac{\Lambda_{t+k}^* P_t^*}{\Lambda_t^*} \left[\frac{P_t^*(i)\,(\Pi^*)^k}{P_{t+k}^*} Y_{t+k}^*(i) - MC_{t+k}^* Y_{t+k}^*(i) \right] \right\}, \tag{5}$$

subject to the demand constraint $Y_{t+k}^*(i) = \left[\frac{P_t^*(i)(\Pi^*)^k}{P_{t+k}^*}\right]^{-\chi} Y_{t+k}^*$, where Y_{t+k}^* is aggregate output. In the expression above Λ_t^* is the shadow value of an additional unit of nominal income in period t. Goods market clearing in the large economy requires that all production is consumed, that is $Y_t^* = C_t^*$.

When the interest rate is not fixed, monetary policy follows an interest rate rate rule that responds to inflation, output growth and the deviation of the level of output from trend:

$$\frac{R_t^*}{R^*} = \left[\frac{R_{t-1}^*}{R^*}\right]^{\rho_R^*} \left[\left(\frac{\Pi_t^*}{\Pi^*}\right)^{\phi_\pi^*} \left(\frac{Y_t^*}{Y_{t-1}^*\mu}\right)^{\phi_g^*} \left(\frac{Y_t^*}{Z_t}\right)^{\phi_y^*} \right]^{1-\rho_R^*} \exp(\varepsilon_{R,t}^*) \tag{6}$$

where R_t^* is the policy rate in the large economy, $R^* = \mu\Pi^*/\beta$ is the steady-state policy rate and $\varepsilon_{R,t}^*$ is a monetary policy shock. After 2009Q1, the central bank fixes the level of the interest rate at $\bar{R}^*$, that is

$$\frac{R_t^*}{R^*} = \bar{R}^* \tag{7}$$

where $\bar{R}^*$ need not be the effective lower bound. For the US we set $\bar{R}^*$ to the mid point of the target range for the Federal Funds rate, to 0.125 %.

Longer-term interest rates are determined via the expectations hypothesis. We link model longer-term interest rates to observed longer-term interest rates following Graeve et al. (2009). For any maturity $m > 1$:

$$R_{m,t}^{*,\text{obs}} = R_{m,t}^* \exp\left(c_m^* \eta_t^* \varepsilon_{m,t}^*\right), \tag{8}$$

where $R_{m,t}^*$ is the interest rate on a bond that pays one unit of the large economy's currency in m quarters as determined by the expectations hypothesis, c_m^* is a constant risk premia on the m quarter interest rate, η_t^* is shock, common to all interest rates in the large economy, that follows exogenous autoregressive process and $\varepsilon_{m,t}^*$ is an idiosyncratic shock to the m quarter interest rate in the large economy. Because the expectations hypothesis holds, longer-term nominal interest rates in the model, $R_{m,t}$, are also subject to the lower bound of R_t^*.

2.2 The Small Economy

The structure of the small economy is similar to that of the large economy, except that households consume goods and services produced abroad and firms sell their output overseas as well as domestically.

Households The small economy is populated by a representative household that maximizes the expected present discounted value of lifetime utility, given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi_t \left(\log(C_t - hC_{t-1}) - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\right], \tag{9}$$

subject to the flow budget constraint in every period t:

$$P_t C_t + \mathbb{E}_t \left\{Q_{t,t+1} D_{t+1}\right\} \leq D_t + W_t N_t + T_t. \tag{10}$$

As in the large economy, households have access to a complete set of internationally traded contingent claims. Complete markets and household optimization implies a risk-sharing condition linking the marginal utilities of consumption in the large and small economies, $\Lambda_t = \frac{\Lambda_t^*}{Q_t}$ where Λ_t is the marginal value of an additional unit of domestic nominal income to households in the small economy, $Q_t = S_t P_t^* / P_t$ is the real exchange rate between the small and large economy and S_t is the nominal exchange rate between the small and large economy. The exchange rate is defined as the number of units of the small economy's currency required to purchase one unit of the large economy's currency. According to this definition, an increase in S_t corresponds to a depreciation of the domestic currency. To account for deviations from uncovered interest rate parity in estimation, we add a risk premium shock to the risk-sharing condition that follows an AR(1) process in logs.

Consumption Retailers The final consumption good is assembled by perfectly competitive retailers using the technology:

$$C_t = \left[(1-\alpha)^{\frac{1}{\tau}} (C_{H,t})^{\frac{\tau-1}{\tau}} + \alpha^{\frac{1}{\tau}} (C_{F,t})^{\frac{\tau-1}{\tau}}\right]^{\frac{\tau}{\tau-1}}, \tag{11}$$

where $C_{H,t}$ and $C_{F,t}$ are composite consumption indices of domestically- and foreign- produced final goods. The parameter τ is the elasticity of substitution between domestic- and foreign-produced goods. The price index corresponding to this bundle is:

$$P_t = \left[(1-\alpha)(P_{H,t})^{1-\tau} + \alpha (P_{F,t})^{1-\tau}\right]^{\frac{1}{1-\tau}}, \tag{12}$$

where $P_{H,t}$ is the price of the domestic composite good and $P_{F,t}$ is the price of the imported composite good, both expressed in domestic currency.

Domestic Final Goods Retailers The domestically-produced final good, $Y_{H,t}$ is assembled by a perfectly competitive final good retailer that combines domestically-produced intermediate goods using the technology:

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(i)^{\frac{\chi-1}{\chi}} di\right]^{\frac{\chi}{\chi-1}}, \tag{13}$$

where χ is the elasticity of substitution between varieties of domestic intermediate goods. The price of the domestic final good is:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\chi} di\right]^{\frac{1}{1-\chi}}. \tag{14}$$

It follows that the final goods firm's demand for each variety is given by:

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\chi} Y_{H,t}. \tag{15}$$

Domestic Intermediate Goods Producers Domestic intermediate goods producers manufacture heterogeneous goods using the technology:

$$Y_{H,t}(i) = Z_t A_t N_t(i), \tag{16}$$

where A_t is a stationary technology process that follows an AR(1) process in logs. Real marginal costs are equal across firms and given by:

$$MC_t = \frac{W_t}{P_t Z_t A_t}. \tag{17}$$

As for the US, firms in Canada face Calvo-style pricing frictions so that, each quarter, a fraction, $1-\theta$, of firms are able to adjust their prices freely. The remaining firms index their prices to the steady-state inflation rate, Π. The resulting pricing problem for firm i is

$$\max_{P_{H,t}(i)} \sum_{k=0}^{\infty} (\beta\theta)^k \, \mathbb{E}_t \left\{ \frac{\Lambda_{t+k} P_t}{\Lambda_t} \left[\frac{P_{H,t}(i) Y_{H,t+k}(i) \Pi^k}{P_{t+k}} - MC_{t+k} Y_{H,t+k}(i) \right] \right\}, \tag{18}$$

subject to the domestic final goods demand condition given above.

Exporters Exporters purchase the domestic final good at price $P_{H,t}$ and differentiate it through branding for sale in the foreign economy. The exporters are owned by domestic households. However, as in Burgess et al. (2013) and consistent with the prevalence of a dominant currency documented in Boz et al. (2018), all export contracts and prices are specified in the currency of the large economy. An export retailer bundles these goods before selling them overseas according to the technology:

$$X_t = \left[\int_0^1 X_t(i)^{\frac{\nu_X - 1}{\nu_X}} (i) \right]^{\frac{\nu_X}{\nu_X - 1}}, \tag{19}$$

where ν_X is the elasticity of substitution between different varieties for export. The corresponding price index, in foreign currency terms, is:

$$P^*_{X,t} = \left[\int_0^1 P^*_{X,t}(i)^{1-\nu_X} di \right]^{\frac{1}{1-\nu_X}}. \tag{20}$$

The export retailer faces the demand function:

$$X_t = \left(\frac{P^*_{X,t}}{P^*_t} \right)^{-\tau} Y^*_t, \tag{21}$$

As in Justiniano and Preston (2010) the elasticity of export demand, τ, is the same as the elasticity of substitution between domestic and foreign goods in the consumption basket. This implies that household's preferences in the small and large economies are the same, which for the US and Canada is a reasonable assumption. It follows that the demand for each exporter's goods are given by:

$$X_t(i) = \left(\frac{P^*_{X,t}(i)}{P^*_{X,t}} \right)^{-\nu_X} X_t. \tag{22}$$

Exporters face Calvo-style pricing frictions, with $1-\theta_x$ governing the share of firms that are able to adjust their prices each quarter. The resulting pricing problem for firm i is:

$$\max_{P^*_{X,t}(i)} \sum_{k=0}^{\infty} (\beta\theta_x)^k \, \mathbb{E}_t \left\{ \frac{\Lambda_{t+1} P_t}{\Lambda_t} \left[\frac{P^*_{X,t}(i)\Pi^k X_{t+k}(i)}{S_{t+k}P_{t+k}} - \frac{P_{H,t+k}X_{t+k}(i)}{P_{t+k}} \right] \right\}, \tag{23}$$

subject to the demand constraint given above.

Finally, market clearing for the domestic final good requires that all production is sold, either domestically or overseas. That is:

$$Y_{H,t} = C_{H,t} + X_t. \tag{24}$$

Importers Importers bring in homogeneous products from abroad at price $S_t P^*_t$ and differentiate them through branding. Importers then sell the differentiated goods to a retailer that combines them into the final imported good using the technology:

$$C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\nu_F - 1}{\nu_F}} di \right]^{\frac{\nu_F}{\nu_F - 1}}, \tag{25}$$

where $C_{F,t}(i)$ is the quantity of the imported good of variety i used in the production of the final imported good and ν_F is the elasticity of substitution between different imported good varieties. The price index corresponding to the imported final good is:

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\nu_F} di \right]^{\frac{1}{1-\nu_F}}. \tag{26}$$

Consequently, each importer faces the demand curve:

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\nu_F} C_{F,t}. \tag{27}$$

Importers face Calvo-style pricing frictions. Each quarter, a fraction, $1-\theta_m$ sets prices optimally, while the remainder adjusts their prices by the small economy's steady-state inflation rate, Π. The pricing problem for a representative firm i is:

$$\max_{P_{F,t}(i)} \sum_{k=0}^{\infty} (\beta\theta_m)^k \, \mathbb{E}_t \left\{ \frac{\Lambda_{t+1} P_t}{\Lambda_t} \left[\frac{P_{F,t}(i) C_{F,t}(i) \Pi^k}{P_{t+k}} - \frac{S_{t+k} P^*_{t+k} C_{F,t+k}(i)}{P_{t+k}} \right] \right\}, \tag{28}$$

subject to the demand constraint above.

Monetary Policy Before 2009Q2, when R_t is not fixed, the Canadian central bank follows the feedback rule:

$$\frac{R_t}{R} = \left[\frac{R_{t-1}}{R}\right]^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_{H,t}}{Y_{H,t-1}\mu}\right)^{\phi_g} \left(\frac{Y_{H,t}}{Z_t}\right)^{\phi_y} \right] e^{\varepsilon_{R,t}}. \tag{29}$$

As for the US, we allow for an arbitrary level of the fixed nominal interest rate. Canada fixed its policy rate at 0.25 % for a year and then at 1 % for about 5 years in our sample so, from 2009Q2, policy follows

$$\frac{R_t}{R} = \bar{R} \tag{30}$$

and from 2010Q3

$$\frac{R_t}{R} = \bar{\bar{R}} \tag{31}$$

The term structure of interest rates in the small economy is determined in a similar manner to the large economy. For any $m > 1$:

$$R^{\text{obs}}_{m,t} = R_{m,t} \exp\left(c_m \eta_t \varepsilon_{m,t}\right), \tag{32}$$

where $R_{m,t}$ is the interest rate on a bond that pays one unit of domestic currency in m quarters as determined by the expectations hypothesis, c_m is a constant risk premia on the m quarter interest rate, η_t is shock, common to all small economy interest rates, that follows an exogenous autoregressive process and $\varepsilon_{m,t}$ is an idiosyncratic shock to the m quarter interest rate in the small economy.

2.3 The Transmission of Foreign Monetary Policy

In the empirical implementation, we use the model that we describe above. But to understand the key forces that determine the strength of foreign monetary policy shocks in the larger model it is useful to consider the impact of foreign shocks in a simpler context. With three simplifying assumptions – no habits in consumption, complete exchange rate pass through at home and abroad and i.i.d. shock processes – our small economy model collapses to that of De Paoli (2009b) which can be characterized by the set of log-linear equations:

$$\begin{aligned}
\pi_t &= \kappa\left(c_t + \varphi y_{H,t} + \frac{\alpha}{1-\alpha} q_t\right) + \beta \mathbb{E}_t \pi_{t+1} \\
y_{H,t} &= (1-\alpha) c_t + \alpha y_t^* + \gamma q_t \\
c_t &= y_t^* + q_t \\
c_t &= -(r_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t c_{t+1}
\end{aligned}$$

and a monetary policy reaction function. In the equations above, variables represent percentage deviations from steady state, $\gamma = \frac{\tau\alpha(2-\alpha)}{(1-\alpha)}$ and $\kappa = \frac{(1-\theta\beta)(1-\theta)}{(1+\chi\varphi)\theta}$. The only exogenous shock is y_t^*. Although this system of equations does not feature foreign interest rates, it can nonetheless be used to examine foreign monetary policy spillovers following foreign monetary policy shocks, conditional on the response of foreign output, y_t^*.

First, assume that domestic monetary policy follows a rule of the form:

$$r_t = \phi_\pi \mathbb{E}_t \pi_{t+1} \tag{33}$$

where $\phi_\pi > 1$. Because the shock is i.i.d., this reaction function implies that the small economy's central bank stabilizes the *ex-ante* real interest rate. One can think of this exercise as illustrating the direct effects of a foreign monetary policy shock, holding the stance of domestic monetary policy fixed.

The monetary policy rule and Euler equation imply that the level of consumption in the small economy is stable at its steady state level. The risk sharing condition then ensures that the small economy's real exchange rate appreciates one-for-one with increases in foreign output. Substituting these results into the resource constraint, we can express output in the small economy as a function of foreign output:

$$y_{H,t} = \left[\alpha - \frac{\tau\alpha(2-\alpha)}{1-\alpha}\right] y_t^* \tag{34}$$

The expression in brackets summarizes the two forces that govern the response of domestic output. The first term, αy_t^*, is the income effect – the direct increase in exports from the small economy resulting from expansionary monetary policy abroad that increases demand in the large economy. This term is increasing in the openness of the domestic economy, reflected in the parameter α. The second term,

$\frac{\tau\alpha(2-\alpha)}{1-\alpha}y_t^*$, is the substitution effect – the reduction in demand for goods produced in the small economy resulting from expansionary monetary policy abroad that triggers an appreciation of the small economy's real exchange rate. The strength of this term is increasing in the openness of the domestic economy, as well as in the substitutability between domestic and foreign products, reflected in the parameter τ. In the case where the domestic central bank perfectly stabilizes the real interest rate, foreign monetary policy is beggar-thy-neighbour if $\tau > \frac{1-\alpha}{2-\alpha}$.

The mechanism described above also summarizes the effect of foreign forward guidance shocks on the domestic economy. With a constant *ex-ante* real interest rate, Equation (34) describes the relationship between domestic and foreign output in each period. An announcement about the future path of interest rates in the foreign economy triggers a sequence of foreign output realizations. These realizations will expand or contract domestic output over time according to the path of foreign output and the relative strength of the income and substitution effects.

If the domestic central bank follows a reaction function that responds to current realizations of domestic inflation:

$$r_t = \phi_\pi \pi_t \tag{35}$$

then the real interest rate is no longer constant and the relationship between foreign and domestic output depends also on the flexibility of domestic prices, on the response of domestic marginal costs to demand conditions, and on the response of monetary policy to inflation, summarized by the following equation:[7]

$$y_{H,t} = \left[\alpha - (1-\alpha+\gamma)\phi_\pi b - \frac{\tau\alpha(2-\alpha)}{1-\alpha}\right] y_t^* \tag{36}$$

Although neat analytical expressions are not available in this case, Equation (36) reveals which additional parameters – in particular, the elasticity of labor supply, the slope of the Phillips curve and the strength of the monetary policy response to inflation – also matter in determining the extent to which foreign monetary policy is beggar-thy-neighbour.

A lower wage elasticity of labor supply (a lower value of φ) implies that wages, and hence marginal costs, are more responsive to changes in labor demand. Hours worked, in contrast, are less responsive. Foreign monetary policy is less beggar-thy-neighbour when the elasticity of labor supply is low.

When prices are more flexible (a larger value of κ) a given change in marginal costs translates more into changes in prices and less into changes in quantities. In the case where expansionary foreign

[7]Where $b = \frac{\varphi(\alpha-\gamma)-\frac{\alpha}{1-\alpha}}{\frac{1}{\kappa}+\varphi(1+\varphi(1-\alpha)+\varphi\gamma+\frac{\alpha}{1-\alpha})}$ and $\gamma = \frac{\tau\alpha(2-\alpha)}{1-\alpha}$.

monetary policy triggers a fall in domestic output, more flexible prices dampen the expenditure switching channel. This is because the fall in labor demand and marginal costs translates into a larger reduction in the prices of domestically-produced goods. This cushions the fall in the consumption of domestic goods.

A central bank that responds to inflation will cut interest rates following an expansionary foreign monetary policy shock that lowers domestic production, wages and inflation. In doing so, the central bank reduces the interest rate differential between the domestic and foreign economies and diminishes the initial exchange rate appreciation of the domestic currency. This too will reduce the extent to which expansionary foreign monetary policy is beggar-thy-neighbour.

The mechanisms described above are also at work in our larger model. In estimation, we place priors over the parameters that govern the response of domestic output to foreign monetary policy disturbances, in particular the trade elasticity, τ, that are wide enough to allow the data to explain whether expansionary foreign monetary policy is also expansionary for the domestic economy, or beggar-thy-neighbour.

3 Solution and Estimation Methods

In our case, policy interest rates in the US and Canada can be fixed at different periods, or at the same time. This section describes how we solve and estimate the model with fixed interest rate regimes in the two-country setup. We use the solution methods proposed in Cagliarini and Kulish (2013), Kulish and Pagan (2017) and Jones (2017), which are related to the methods in Eggertsson and Woodford (2003) and Guerrieri and Iacoviello (2015), and the estimation method of Kulish, Morley and Robinson (2017). The Appendix provides additional details.[8]

3.1 Solution with Fixed Interest Rates

The economy can be in one of the following four possible regimes at a given point in our sample: (i) interest rates follow feedback rules, (ii) only the interest rate of the large economy is fixed, (iii) only the interest rate of the small economy is fixed, and (iv) both interest rates are fixed. Figure 1 illustrates one possibility, in which in an initial sub-sample conventional policy applies to both economies, then there is a period of time for which the interest rate is fixed only in the large economy. After that, interest rates are fixed in both economies, and eventually there is a return to conventional policy which takes

[8]These methods have more general application than the context we are considering, and can be applied to a range of problems in which one requires to solve a model subject to occasionally binding constraints or anticipated changes in the structural parameters efficiently.

place out-of-sample.

We first linearize the model around the steady state for which policy rates follow feedback rules, and write the resulting system of equations in matrix form as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}\varepsilon_t. \tag{37}$$

where x_t is the state vector and ε_t is the vector of structural shocks, which we take to be i.i.d. without loss of generality. If it exists and is unique, the standard rational expectations solution to (37) is $x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t$.

When only the *foreign* interest rate is fixed the structural equations are given by:

$$\mathbf{A}^\star x_t = \mathbf{C}^\star + \mathbf{B}^\star x_{t-1} + \mathbf{D}^\star\mathbb{E}_t x_{t+1} + \mathbf{F}^\star\varepsilon_t. \tag{38}$$

where the only equation that has changed in the starred system relative to (37) is the equation defining the *foreign* policy interest rate rule, which is now specified such that the nominal interest rate is fixed.[9]

When only the *domestic* interest rate is fixed the structural equations are given by:

$$\bar{\mathbf{A}}x_t = \bar{\mathbf{C}} + \bar{\mathbf{B}}x_{t-1} + \bar{\mathbf{D}}\mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}\varepsilon_t. \tag{39}$$

where the only equation that has changed relative to (37) is the equation defining the *domestic* policy interest rate rule, which is now specified such that the nominal interest rate is fixed.

And when both *foreign* and *domestic* interest rates are fixed the structural equations are given by:

$$\bar{\mathbf{A}}^\star x_t = \bar{\mathbf{C}}^\star + \bar{\mathbf{B}}^\star x_{t-1} + \bar{\mathbf{D}}^\star\mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}^\star\varepsilon_t. \tag{40}$$

Because at least one nominal interest rate is fixed in regimes (38), (39) or (40), if any of these regimes were expected to prevail indefinitely, they would be found to be inconsistent with a unique rational expectations solution.[10] However, provided both nominal interest rates are expected to be governed by policy rules consistent with determinacy, there is a unique path under temporarily fixed interest rates (Cagliarini and Kulish, 2013).

[9] The notation accommodates additional structural changes which have to be accounted for if the expansion point of the approximation changes. In our application we work around the intended steady state.

[10] This is the case in our model where monetary policy is 'active' in the terminology of Leeper (1991). As emphasized recently by Cochrane (2014), a fixed interest rate need not imply indeterminacy in models with 'active' fiscal policy.

In general, the structural equations in-sample may be written:

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \mathbb{E}_t x_{t+1} + \mathbf{F}_t \varepsilon_t. \tag{41}$$

where $\{\mathbf{A}_t, \mathbf{C}_t, \mathbf{B}_t, \mathbf{D}_t, \mathbf{F}_t\}_{t=1}^{T}$ is a sequence of structural matrices over the sample. For example, if interest rates follow feedback rules at time t, then $\mathbf{A}_t = \mathbf{A}$, $\mathbf{C}_t = \mathbf{C}$, $\mathbf{B}_t = \mathbf{B}$, and so. If both domestic and foreign policy interest rates are fixed then $\mathbf{A}_t = \bar{\mathbf{A}}^\star$, $\mathbf{C}_t = \bar{\mathbf{C}}^\star$, $\mathbf{B}_t = \bar{\mathbf{B}}^\star$, and so on.

The solution accomodates different durations in the two economies but suppose, for simplicity, that at period t' interest rates are fixed in both economies, so Equation (40) holds, and that feedback rules, Equation (37), are expected to govern monetary policy from some future period T' onwards. Thus the expected durations of the fixed interest rate regimes for both economies at t' is $\mathbf{d}_{t'} = T' - t'$. Agents foresee that the structural equations will evolve as follows $\{\mathbf{A}_t, \mathbf{C}_t, \mathbf{B}_t, \mathbf{D}_t, \mathbf{F}_t\}_{t=t'}^{T'}$ where in this case $\mathbf{A}_t = \bar{\mathbf{A}}^\star$, $\mathbf{C}_t = \bar{\mathbf{C}}^\star$, $\mathbf{B}_t = \bar{\mathbf{B}}^\star$ for $t', \dots, T'-1$ and $\mathbf{A}_t = \mathbf{A}$, $\mathbf{C}_t = \mathbf{C}$, $\mathbf{B}_t = \mathbf{B}$ in T' . Following Kulish and Pagan (2017), the solution is a time-varying VAR of the form:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{42}$$

which implies that $\mathbb{E}_t x_{t+1} = \mathbf{J}_{t+1} + \mathbf{Q}_{t+1} x_t$. Substituting $\mathbb{E}_t x_{t+1}$ into (41) implies that

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \left(\mathbf{J}_{t+1} + \mathbf{Q}_{t+1} x_t\right) + \mathbf{F}_t \varepsilon_t. \tag{43}$$

which implies by undetermined coefficients the following recursions:

$$\mathbf{J}_t = \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}\right) \tag{44}$$

$$\mathbf{Q}_t = \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t \tag{45}$$

$$\mathbf{G}_t = \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{F}_t. \tag{46}$$

The backward recursion for the sequence of time-varying reduced form matrices, $\{\mathbf{Q}_t\}_{t=t'}^{T'-1}$ starts from the terminal condition $\mathbf{Q}_{T'} = \mathbf{Q}$ and works its way back to period t', yielding the system of time-varying reduced form matrices corresponding to (42). With the sequence for $\{\mathbf{Q}_t\}_{t=t'}^{T'-1}$ in hand, the sequence for $\{\mathbf{G}_t\}_{t=t'}^{T'-1}$ may be computed as well as the sequence for $\{\mathbf{J}_t\}_{t=t'}^{T'-1}$ given the terminal condition $\mathbf{J}_{T'} = \mathbf{J}$. Terminal conditions, $\mathbf{J}$ and $\mathbf{Q}$, correspond to the solution matrices to the model for which feedback rules for policy interest rates apply, consistent with the assumption that agents do not expect interest rates to be fixed from period T' onwards.

The sequence of reduced-form matrices $\{\mathbf{Q}_{t'}, \mathbf{Q}_{t'+1}, \ldots, \mathbf{Q}_{T'-1}\}$ has an associated sequence of expected durations of the fixed interest rate regime given by $\{(T'-t'), (T'-t')-1, \ldots, 1\}$, so that the expected duration falls by one period in every period.

Now assume more generally that agents re-evaluate the timing of the return to conventional policy in each period of the fixed interest rate regime so the expected duration at t is $\mathbf{d}_t$. So given sequences $\{\mathbf{J}_k\}_{k=1}^{\bar{d}}$, $\{\mathbf{Q}_k\}_{k=1}^{\bar{d}}$ and $\{\mathbf{G}_k\}_{k=1}^{\bar{d}}$ computed for an arbitrary large upper bound $\bar{d}$, the reduced-form matrices that prevail when the expected duration is $\mathbf{d}_t$ in a given period t will be $\mathbf{J}_t = \mathbf{J}_{\bar{d}-\mathbf{d}_t+1}$, $\mathbf{Q}_t = \mathbf{Q}_{\bar{d}-\mathbf{d}_t+1}$ and $\mathbf{G}_t = \mathbf{G}_{\bar{d}-\mathbf{d}_t+1}$.[11] So when the interest rate is fixed, implicit in the evolution of the reduced form matrices are sequences of fixed interest rate durations.

3.2 The Likelihood Function

Since (42) is a linear system, the Kalman filter can be used to construct the likelihood. The likelihood is a function both of the structural parameters and the sequences of expected durations. Following Kulish, Morley and Robinson (2017) we estimate jointly the structural parameters, ϑ, and sequences of expected durations of the fixed interest rate regimes in the large and small economies. We denote the *estimated* durations of the small economy by $\{\mathbf{d}_t\}_{t=1}^{T}$ and of the large economy by $\{\mathbf{d}_t^*\}_{t=1}^{T}$.[12]

4 Identification of Forward Guidance Shocks

The very nature of an expected duration of the fixed interest rate regime, $\mathbf{d}_t$, is that, in the absence of future shocks, it is expected to shrink by one period in every period. That is,

$$\mathbb{E}_t \mathbf{d}_{t+1} = \mathbf{d}_t - 1 \;\text{ for }\; \mathbf{d}_t > 0 \tag{47}$$

We do not restrict the estimated expected durations, $\{\mathbf{d}_t\}_{t=1}^{T}$ and $\{\mathbf{d}_t^*\}_{t=1}^{T}$, to be the same as the duration implied by the structural shocks. Thus, the estimation allows for the possibility that central banks adopted the optimal policy prescription of Eggertsson and Woodford (2003) by extending the duration of the fixed interest rate policy beyond the horizon implied by the lower bound constraint itself.

[11]See Kulish et al. (2017).

[12]Details of the Kalman filter and Kalman smoother are relegated to the Appendix.

We next discuss how we identify the forward guidance component of the estimated duration and in turn the forward guidance shocks.

4.1 Lower Bound Durations

With the occassionally binding constraint algorithm of Jones (2017) we find at each period t, the expected duration which is implied by the estimated state x_{t-1}, the estimated structural shocks ε_t and a given lower bound.[13] We denote this expected duration at t by $\mathbf{d}_t^{\text{lb}}$ and refer to it as the *lower bound* duration. In the absence of future shocks, as the effect of current shocks ε_t unwind, the lower bound duration is expected to fall by one period in every period. In other words,

$$\mathbb{E}_t \mathbf{d}_{t+1}^{\text{lb}} = \mathbf{d}_t^{\text{lb}} - 1 \text{ for } \mathbf{d}_t^{\text{lb}} > 0 \tag{48}$$

As shocks unfold in $t+1$, these shocks may make the constraint bind for a longer or for a shorter period of time. With the estimated structural shocks and state of the economy in hand, we compute the lower bound durations that prevail in each quarter of the fixed interest rate regime for the small and large economies, that is $\{\mathbf{d}_t^{\text{lb}}\}_{t=1}^T$ and $\{\mathbf{d}_t^{lb^*}\}_{t=1}^T$.[14]

4.2 Forward Guidance Shocks

For each period t of the fixed interest rate, we define the *forward guidance* duration, $\mathbf{d}_t^{\text{fg}}$, as the difference between the *estimated* duration, $\mathbf{d}_t$ and the *lower bound* duration, $\mathbf{d}_t^{\text{lb}}$, that is

$$\mathbf{d}_t^{\text{fg}} \equiv \mathbf{d}_t - \mathbf{d}_t^{\text{lb}} \tag{49}$$

The *forward guidance* duration, $\mathbf{d}_t^{\text{fg}}$, captures announcements or other factors that can change the expected duration of the fixed interest rate regime beyond what the structural shocks and the constraint itself implies, that is beyond $\mathbf{d}_t^{\text{lb}}$.[15] This decomposition of the estimated duration characterizes monetary policy when the policy rate is fixed and is useful since the level of the nominal interest rate itself is an insufficient statistic of the stance of monetary policy.

[13] For each country we take the level of the fixed interest rate as a lower bound constraint.

[14] Jones (2017) shows that occasionally binding constraint solution constitutes a good approximation to the non-linear economy. Details of the occasionally binding constraint solution are given in full in the Appendix.

[15] Other factors beyond explicit central bank communication can influence the duration. As is well known, markets reassessed the time of lift-off as the Federal Reserve was tapering the rate of bond purchases.

Forward guidance shocks are unexpected changes of the forward guidance duration, that is

$$\varepsilon_t^{\text{fg}} = \mathbf{d}_t^{\text{fg}} - \mathbb{E}_{t-1}\mathbf{d}_t^{\text{fg}} \tag{50}$$

The expectation of the forward guidance duration, $\mathbb{E}_{t-1}\mathbf{d}_t^{\text{fg}}$, however, depends on whether the lower bound binds or not. Equations (47) and (48) imply that

$$\mathbb{E}_{t-1}\mathbf{d}_t^{\text{fg}} = \begin{cases} \mathbf{d}_{t-1}^{\text{fg}} - 1, & \text{if } \mathbf{d}_t^{\text{lb}} = 0 \\ \mathbf{d}_{t-1}^{\text{fg}}, & \text{otherwise} \end{cases} \tag{51}$$

If the lower bound binds, so that $\mathbf{d}_t^{\text{lb}} > 0$, the forward guidance shock is the change in the forward guidance duration, that is $\Delta\mathbf{d}_t^{\text{fg}}$. But when the lower bound constraint does not bind, $\mathbf{d}_t^{\text{lb}} = 0$, so that $\mathbf{d}_t = \mathbf{d}_t^{\text{fg}}$, the forward guidance duration is expected to fall by one in every quarter as dictated by Equation (47). Therefore, the expected one quarter reduction in $\mathbf{d}_t^{\text{fg}}$ is not a forward guidance shock but a continuation of policy. Forward guidance shocks in this case are defined as increases or decreases in the expected duration of the fixed interest rate policy beyond this expected one quarter change. Formally, the *forward guidance shock* in period t is given by:

$$\varepsilon_t^{\text{fg}} = \begin{cases} \mathbf{d}_t^{\text{fg}} - \mathbf{d}_{t-1}^{\text{fg}}, & \text{if } \mathbf{d}_t^{\text{lb}} \geq 1 \\ \mathbf{d}_t^{\text{fg}} - \mathbf{d}_{t-1}^{\text{fg}} + 1, & \text{if } \mathbf{d}_t^{\text{lb}} = 0 \end{cases} \tag{52}$$

Figure 2 illustrates a forward guidance shock at t with an example. At period $t-1$ the estimated duration, $\mathbf{d}_{t-1}$ of the lower bound is 5 quarters: the interest rate is expected to be fixed from period $t-1$ to period $t+3$. In the absence of structural and forward guidance shocks after $t-1$, in the forecast horizon, the expected duration falls by one quarter every period. In Figure 2, $\mathbb{E}_{t-1}\{\mathbf{d}_{t-1}, \mathbf{d}_t, ..., \mathbf{d}_{t+3}\} = \{5, 4, 3, 2, 1\}$ and its associated interest rate path is labelled the "*forecast path at* $t-1$".

At period $t-1$, the structural shocks, ε_{t-1}, imply an evolution of the lower bound durations. In this case, $\mathbb{E}_{t-1}\{\mathbf{d}_{t-1}^{\text{lb}}, \mathbf{d}_t^{\text{lb}}, \ldots, \mathbf{d}_{t+3}^{\text{lb}}\} = \{3, 2, 1, 0, 0\}$. The associated path for the interest rate is shown in the top panel of Figure 2 and is labelled the "*lower bound forecast*". These lower bound durations are represented by the blue bars in the middle panel of Figure 2. The difference between the fixed interest rate duration and the lower bound duration, illustrated by the black bars in the middle panel of Figure 2, is the forward guidance duration. At $t-1$, the forward guidance duration is expected to evolve as follows: $\mathbb{E}_{t-1}\{\mathbf{d}^{\text{fg}}_{t-1}, \mathbf{d}^{\text{fg}}_{t}, \ldots, \mathbf{d}^{\text{fg}}_{t+3}\} = \{2, 2, 2, 2, 1\}$.

The bottom panel of Figure 2 illustrates a one-quarter expansionary forward guidance shock in period t. We continue to assume that there are no structural shocks after $t-1$. Following the forward guidance shock at t, the lower bound and the forward guidance durations are expected to evolve in the forecast at t as shown by the bottom panel of Figure 2. At $t-1$, the estimated duration was expected to fall to 4 quarters by t, that is $\mathbb{E}_{t-1}\mathbf{d}_t = 4$. However, a one quarter forward guidance shock increases $\mathbf{d}_t$ to 5 quarters. Although the total duration is the same in $t-1$ as it is in t, that is $\mathbf{d}_t = \mathbf{d}_{t-1}$, the decomposition of the duration is different because of the forward guidance shock. This example illustrates that the identification of forward guidance shocks requires both a method to estimate the expected fixed interest rate duration and an occasionally binding constraint solution to decompose it.

5 Estimation

5.1 Data

For estimation, we use US and Canadian nominal interest rates, output growth, and inflation, and changes in the US/Canadian nominal exchange rate. These seven quarterly data series are plotted in Figure 3. We also use 2-year nominal interest rates for both the US and Canada in helping identify the fixed interest rate durations. The series start in 1984Q1 and end in 2014Q4. All details of the data are provided in the Appendix.

5.2 Assigned and Calibrated Parameters

We calibrate a number of structural parameters in line with values from the literature. The quarterly discount factor β, common to both countries, is set at 0.9978. The consumption habit formation parameter h is set at 0.71, the posterior mode of the estimated value in Smets and Wouters (2007), while the inverse of the Frisch elasticity of labor supply ψ, is calibrated to 1.5 in both countries, close to the values estimated in Smets and Wouters (2007) and Justiniano and Preston (2010). And the import share of the Canadian consumption basket, α, is set to 0.29 to match the average share of exports and imports to Canadian GDP.

5.3 Fixed Interest Rate Regimes

We measure expected durations in the US from the time at which the Federal Funds rate reached 0.125% in 2009Q1 until the end of our sample 2014Q4. Similarly, we measure these durations in Canada when the policy rate was at 0.25% between 2009Q2 and 2010Q2, and then when the rate was at 1% between 2010Q3 and the end of the sample. This modeling choice is motivated by the clear communication of the Bank of Canada providing forward guidance of its fixed interest rate policy at these values over the sample period.[16]

5.4 Estimation Results

Structural Parameters The moments of the prior and posterior distribution for each parameter that we estimate are reported in Table 1.[17] We use the same prior distributions for parameters common to both countries. The prior of our key parameter, τ, the trade elasticity of substitution is centered at 1, and allows for a wide range of possible outcomes for the sign and magitude of spillovers of US monetary policy to Canada. We also use loose priors over the slopes of the price equations, which translates into a wide range of possible degrees of price stickiness in both the US and Canada. Our priors over the annual trend growth rate and the targeted annual inflation rates in both the US and Canada are centered at 2%, while the 10th and 90th percentiles of the prior distributions over these parameters are 1.2% and 2.8% respectively.

The monetary policy feedback rules respond to the growth rate of GDP and to the detrended level of output, given the Fed's emphasis on the subdued labor market outcomes over the 2009-2015 period. We put loose priors over the coefficients on these variables. The estimates of the monetary policy rule will be important for accurately estimating anticipated forward guidance durations, as it is the monetary policy regime that the economy reverts to following a period of time at a fixed interest rate which is important for governing expectations about the path of the interest rate, and therefore in determining the stimulatory effect of forward guidance. Finally, we use wide priors for the standard deviations of the autoregressive shocks for both the US and Canada.

The estimated parameters of the monetary policy rules are similar for the two economies. We estimate the slope of the log-linearized Phillips curves, which we denote by κ^*, κ, κ_x and κ_m, rather than

[16] As an example, in the May 2013 press release, the Bank of Canada said that '*With continued slack in the Canadian economy, the muted outlook for inflation, and the constructive evolution of imbalances in the household sector, the considerable monetary policy stimulus currently in place will likely remain appropriate for a period of time, after which some modest withdrawal will likely be required, consistent with achieving the 2 per cent inflation target.*'

[17] We plot in the Appendix prior and posterior distributions for the structural parameters.

the Calvo parameters directly.[18] The slopes of the domestic and foreign Phillips curves are estimated to be quite small, and therefore imply quite sticky prices, in line with the estimates of del Negro et al. (2015), and others. The slopes of the Phillips curves for import and export prices are steeper than those of domestic prices, as in Justiniano and Preston (2010).

The posterior estimate of the trade elasticity, τ, is centered around 1.27, which is above the estimate for Canada obtained by Justiniano and Preston (2010) of about 0.8, but similar to those reported in other estimated small open economy models.[19] As discussed above, this parameter governs the response of domestic output to foreign monetary policy disturbances. We thus describe in more detail how it is identified in estimation.

The trade elasticity governs how much domestic output responds to changes in relative prices between the domestic and foreign economy. A lower trade elasticity makes domestic output less responsive to changes in relative prices. Less responsive output implies smaller changes in labor demand which in turn imply smaller changes in marginal cost and inflation. Through the policy rule this means the nominal interest rate is also less responsive. Following an expansionary monetary policy abroad, the interest rate differential between foreign and domestic interest rates would be larger the lower the trade elasticity. Through uncovered interest rate parity, this implies larger changes of the nominal exchange rate.

The estimation procedure chooses a value of τ that helps the model to jointly match the behaviour of domestic output and the nominal exchange rate. While a lower value of τ would imply that expansionary monetary policy in the US is also expansionary for Canada, a lower value of τ also reduces the variance of domestic output and increases the variance of the nominal exchange rate. In fact, we estimated a specification in which τ is constrained to $1/3$ and found that larger shocks were required to explain the nominal exchange rate.

In Figure 4, we compare the responses *on impact* of Canadian output to an expansionary US monetary policy shock at different values of τ, the trade elasticity. We draw parameters from the prior and posterior distributions and solve the model for each draw. Figure 4 illustrates that our priors are wide enough to allow an expansionary US monetary policy shock to be expansionary or contractionary for Canadian output. For the entire posterior distribution, however, we find that the output responses are negative.

Turning to the estimates of the shock processes, relative to the priors, the data prefer persistent demand shocks in the US, and relatively less persistent TFP shocks in Canada, consistent with the

[18] For example, $\kappa^* = (1-\theta^*)(1-\beta\theta^*)/\theta^*$.

[19] For example for Sweden, Adolfson et al. (2013) report an estimate of 1.41.

relative differences in persistence found in Kulish and Rees (2011). For both economies, demand shocks are more persistent and the standard deviation of demand shocks are estimated to be roughly four times as large than technology shocks, in line with the relative sizes of comparable shocks found in Ireland (2004) and Kulish and Rees (2011). To understand the implications of these parameter estimates, we report in Table 2 the forecast error variance decomposition of both US and Canadian variables into the structural shocks of the model.[20] US preference shocks account for about 90% of the variation in detrended US output in the long run, and 75% of the variation in the Fed Funds rate. Consistent with the findings of Smets and Wouters (2007), US monetary policy shocks drive a small percentage of US output and inflation. US preference shocks account for about 5% of the forecast error of Canadian output, while US monetary policy shocks account for little of the long-run variation in Canadian output. Instead, about 70% of the variation in Canadian output is driven by Canadian preference shocks. Risk premia shocks are important for Canadian business cycles, accounting for about 23% of the forecast error of output.

Expected Durations For the sequence of US expected durations, we use an informative prior as in Kulish, Morley and Robinson (2017), using survey data from Blue Chip from 2009 to 2010, and survey data from the NY Fed from 2011 to 2015. For Canada we use an uninformative prior as equivalent survey measures are not available.[21] The posterior distributions for the anticipated durations of the US Fed Funds rate at 0.125% between 2009Q1 and 2014Q4, and the Bank of Canada's Bank Rate at 0.25% between 2009Q2 and 2010Q2 and 1% between 2010Q3 and 2014Q4 are in Table 3.

The posteriors show that the expected durations are well identified. For the US, the central value of each posterior density lies between 3 and 10 quarters. The posterior densities noticeably shift towards longer durations in 2011Q2, with the mean of the posteriors increasing from 5 quarters in 2011Q1 to 8.5 quarters in 2011Q2, and staying around 8 quarters until the start of 2014Q1, after which the mean of the posteriors declines to around 6 quarters. These values are consistent with the results of Swanson and Williams (2014), who find expected durations of around 7 quarters for each quarter from 2011, and aligns with the explicit adoption of forward guidance from 2011Q3, when the Federal Reserve announced that it would maintain the interest rate at 0.125% until mid-2013. Subsequently, the Fed repeatedly extended the explicit liftoff date back, which is consistent with our estimated posterior distributions

[20] The shocks to the 2Y yields capture only deviations of the 2Y interest rates from the expectations hypothesis, and so do not affect the model's other variables.

[21] The priors and posterior distributions for the sequence of expected durations are plotted in the Appendix.

remaining centered around 6 to 7 quarters.

For Canada, the estimated posterior distributions are centered around lower fixed interest rate durations, with mean durations of between 3 and 7 quarters over the estimation sample. The mass of the posterior distribution of the fixed interest rate regimes shrinks towards smaller durations for the first three quarters that the Canadian Bank Rate is fixed at 0.25% in 2009, which is consistent with the Bank of Canada raising its policy interest rate to 1% in third quarter of 2010, and keeping it fixed thereafter.

6 Spillovers of US Monetary Policy

We next use the estimated model to study the spillovers of US monetary policy shocks on Canada. We first use our solution methods to measure forward guidance shocks in each country. We then compare the size and magnitude of the spillovers of conventional US monetary policy shocks when interest rates in the US and Canada follow a reaction function, to the spillovers that arise in response to a typical US forward guidance shock. Next, we show that the extent of these spillovers depends on the state of the US economy. Finally, we compute counterfactuals and use the model to quantify the spillovers from US forward guidance shocks between 2009 and 2014.

6.1 Identified Forward Guidance in the US and Canada

We first report our estimated measure of forward guidance in the US and Canada. To measure forward guidance, we repeatedly draw from the posterior distribution of parameters and durations. For each draw, we use the observables and the Kalman smoother to estimate the model's structural shocks. We then follow the methods discussed in Section 4 and use the structural shocks to identify the fixed interest rate durations which are consistent with the estimated structural shocks, which we define as the sequence of lower bound durations for each country.

Figure 6 plots the mean of the set of draws of fixed interest rate durations and lower bound durations for the US and Canada. For the US, the mean across durations is initially around 4 quarters to 6 quarters, from 2009 to early 2011 (see also Table 3), of which the mean across lower bound durations is roughly half the total duration, between 2 quarters and 3 quarters. We estimate therefore that US forward guidance was responsible for about half of the expected duration of fixed interest rate policy between 2009 and 2011, or between 2 quarters and 3 quarters in duration.

From early to mid-2011, to late 2013, we find that forward guidance shocks in the US expanded markedly. Following the Fed's explicit calendar-based commitment to holding the Fed Funds rate fixed from early to mid-2011, we estimate that the lower bound durations stayed roughly constant, between 2 quarters and 4 quarters, so that the forward guidance announcements were about 4 quarters to 5 quarters. As discussed below in the counterfactual simulations, we find that these announcements were stimulatory for the US and, owing to the estimated elastic demand by the US for Canadian exports, were contractionary in Canada.

In Panel B of Figure 6, we plot the corresponding decomposition of the estimated total expected durations of fixed interest rate policy in Canada.[22] The estimated lower bound duration is roughly constant over the 2009Q2 to 2014Q4 period, mostly being between 2 quarters and 4 quarters in duration.

Next, we assess the external validity of the estimated US forward guidance shocks by perfoming two checks. The first compares the estimated forward guidance shocks with the historical record of FOMC announcements. We examine two salient quarters: 2011Q2 when the Federal Reserve introduced explicit calendar-based forward guidance in its statement and 2013Q2 when the Federal Reserve announced a reduction in the pace of government bond purchases, which was interpreted by markets as corresponding to an earlier lift-off, prompting the so called 'taper tantrum'.[23] Figure 7 plots the posterior distributions of the forward guidance shocks in the US for 2011Q2, in Panel A, and 2013Q2, in Panel B. For 2011Q2, the forward guidance shock is 4 quarters at the mean with the bulk of the posterior distribution on positive durations. For 2013Q2, however, when the Federal Reserve announced the tapering of its asset purchases, the forward guidance shock is -2 quarters at the mean with the bulk of the posterior on negative durations.

Krippner (2014) and Wu and Xia (2016) use affine term structure models – a different methodology to ours – to construct estimates of the shadow interest rate, which they argue summarizes the stance of monetary policy (including the effects of unconventional policies) when the actual policy rate is fixed at its lower bound. We use the structural model to construct a measure of the shadow rate as follows. First, given the estimated structural shocks, we compute

$$\frac{\widetilde{R}_t^*}{R^*} = \left[\frac{\widetilde{R}_{t-1}^*}{R^*}\right]^{\rho_R^*} \left[\left(\frac{\Pi_t^*}{\Pi^*}\right)^{\phi_\pi^*} \left(\frac{Y_t^*}{Y_{t-1}^*\mu}\right)^{\phi_g^*} \left(\frac{Y_t^*}{Z_t}\right)^{\phi_y^*}\right]^{1-\rho_R^*} \exp(\varepsilon_{R,t}^*) \tag{53}$$

[22] Recall, these durations are for fixed interest rate policy at 0.25% between 2009Q2 and 2010Q1, and then at 1% from 2010Q2 to 2014Q4, the end of the sample.

[23] See http://www.federalreserve.gov/newsevents/press/monetary/20110809a.htm

without imposing a lower bound and under the assumption that $\widetilde{R}_t$ feeds back to the rest of the economy. Note that when the policy rate is at the lower bound, the estimated structural shocks $\varepsilon^*_{R,t}$ equal zero. Then, to compute a measure of the shadow rate, we construct a counterfactual policy rate, $\breve{R}^*_t$, using Equation (53), where we do not allow $\breve{R}^*_t$ to feedback into other variables in the model. This gives a measure of the policy rate implied by economic fundamentals. We then compute an alternative measure of $\breve{R}^*_t$, where instead of the actual values of inflation, Π^*_t, and output, Y^*_t, we use the values from the counterfactual of no forward guidance shocks, again assuming no feedback from $\breve{R}^*_t$ to the rest of the economy. This gives us a measure of what the policy rate implied by economic fundamentals would have been in the absence of forward guidance shocks. We can then use the difference between these two counterfactual policy rates to compute the implied monetary policy shocks to the shadow rate at the lower bound. Using these monetary policy shocks, we calculate the model's shadow rate – the implied policy rate that would have delivered the economic outcomes that we actually observed.

Figure 8 compares our structural measure of the shadow rate with those Krippner (2014) and Wu and Xia (2016). The correlation is 0.95 with the measure of Wu and Xia (2016) and 0.87 with Krippner (2014) which suggests that the identified forward guidance shocks line up with variations in unconventional policy as detected with different methodologies.

6.2 Conventional and Unconventional Shocks

The non-linearity of the solution during the period of fixed interest rates, as we showed above, implies a time-varying VAR of the form $x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t$. A forward guidance shock is a change to $\mathbf{Q}_t$ and so its impact depends on the state of the economy x_{t-1}. We will explore the state dependence in the next section, but to get a sense of the possible magnitudes and dynamics involved, we first compare impulse responses to a conventional shock with those to a forward guidance shock. We do so at the mode of the estimated values of the parameters and for illustration, we consider a 2 quarter forward guidance shock in 2012Q1. The forward guidance shock amounts to an extension of 2 quarters to the estimated duration, which at the median draw is 7 quarters. We choose 2 quarters because it is the mean across the sample for forward guidance shocks in the US.

Figure 9 compares impulse responses of a one standard deviation conventional expansionary monetary policy (about 50 annual basis points) with the impulse response to a 2 quarter forward guidance shock. For the forward guidance shock we use generalized impulse responses following Koop et al. (1996). These are computed as the difference between the forecast paths of variables with and without the forward guidance shock. This explains why an expansionary forward guidance shock implies a fall of the interest rate by the time of liftoff under the estimated duration, because the interest rate under the forward guidance shock remains lower under the fixed interest rate extension. A forward guidance shock therefore only affects the expect path of the Federal Funds rate but has no impact on its contemporaneous value.

A forward guidance shock implies the same qualitative responses as a conventional shock. An expansionary forward guidance shock increases output and inflation in the US and appreciates the Canadian real exchange rate which leads to a decrease in domestic output and inflation in Canada. At the modal draw notice that a forward guidance shock implies responses which are between 2 or 3 times larger than a conventional shock and, in the case of output responses, are more persistent.

6.3 State Dependence of Forward Guidance Shocks

The state of the US preference shock, ξ_t^*, can amplify or diminish the impact of a forward guidance shock. To see this, take the demand equation for the US, which is the only equation apart from the foreign interest rate rule where r_t^* enters, and set for simplicity the habits parameter to zero:

$$y_t^* = \mathbb{E}_t y_{t+1}^* - \left(r_t^* - \mathbb{E}_t \pi_{t+1}^* - (1-\rho_\xi^*)\xi_t^*\right) \tag{54}$$

Iterating this equation forward reveals that it is the expected path of the last term on the right hand side, $r_t^* - \mathbb{E}_t \pi_{t+1}^* - (1-\rho_\xi^*)\xi_t^*$, that matters for output. The expression makes clear why the impact of the expected duration depends on the nature of the risk premium shock. A large, negative and persistent ξ_t^* offsets any potentially expansionary impact of the expected duration. Conversely, a small, positive and persistent ξ_t^* can in fact amplify the expansionary impact of the expected duration. This also explains why one may find large estimates of the expected duration which do not give rise to implausibly large responses of aggregate variables, counter to the 'forward guidance puzzle' phenomenon identified by Carlstrom et al. (2015) and del Negro et al. (2012). Thus, an extension of the expected duration that lowers the expected path of r_t^* will have a smaller impact on output, the more negative and persistent is the foreign preference shock, ξ_t^*.

Figure 10 plots the impulse responses for the US and Canada to an extension of the forward guidance component of the duration of 2 quarters for two sets of different draws. In one case, we take draws for which the standard deviation of the US preference shock, ξ_t^*, is smaller than 0.35. And in the other case, we take draws for which the standard deviation of the US preference shock, ξ_t^*, is higher than

0.55.[24] We further condition on draws for which the duration in 2012Q1 is 8 quarters.

For each draw we obtained smoothed estimates of the shocks and states and use these to compute generalized impulse responses for output, inflation and the real exchange rate. A 2 quarter forward guidance shock in the US leads to mild spillovers on Canadian output (-0.06 % at mean) when the state of the US preference shock is very negative. On the other hand, when the state of the US preference shock is benign, the spillovers from US forward guidance can be up to four times as large (-0.2 % at the mean). We plot the mean of the impulse responses for all draws together with 50 other draws chosen at random, to illustrate the dispersion of the spillovers of forward guidance.

6.4 Counterfactual Scenarios

Next, we use the estimated model to construct a counterfactual in which we remove forward guidance in both the US and Canada, and a counterfactual in which we remove forward guidance in the US alone. In Figure 11, we plot the change in output in the counterfactual from that observed, for the US and Canada. We find that, absent forward guidance in the US and Canada, the cumulative decline in output would be about 27 %, suggesting that forward guidance was stimulatory, particularly between 2011 and 2014. Forward guidance in Canada was also stimulatory. We find that removing forward guidance in Canada and in the US would reduce Canadian output by almost 15 % in cumulative terms, with the deepest counterfactual declines occurring between 2010 and 2012.

Our second counterfactual explores the consequences of removing US forward guidance only, but keeping Canadian forward guidance. To construct this counterfactual, we do not allow the US to announce an extension of the fixed interest rate policy beyond the durations implied by a lower bound constraint. In response, we keep the expected fixed interest rate durations in Canada at their posterior medians. We plot the change in output in Figure 12. First, as one would expect, the decline in US output in Panel A is the same as the decline in the counterfactual removing forward guidance in both countries. However, the experience of Canada is different. Instead, between 2011 and 2014, output in Canada is higher, with output being, cumulatively, almost 4% higher when US forward guidance shocks are removed. This is a direct measure of the spillovers of US forward guidance, and shows that forward guidance announcements by the Bank of Canada helped to mitigate the contractionary effects of expansionary US monetary policy.

[24]The mode of the standard deviation of ξ_t^* is 0.42.

7 Conclusions

In this paper, we examine empirically how forward guidance of the US Fed Funds rate at its lower bound affects monetary policy and economic dynamics in Canada, a small open economy. To this end, we estimate a two-country small open economy model on US and Canadian data accounting for fixed interest rates from 2009Q1 onwards. In estimating these durations we allow, but do not require, that the durations be those that arise if we were to treat the level of the fixed interest rate as an occasionally binding lower bound constraint. This allows us to determine the extent to which central banks use forward guidance to extend the expected duration of the lower bound regimes and identify the effects of forward guidance shocks.

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Table 1: Estimated Parameters

Parameter	Prior				Posterior			
	Dist	Median	10%	90%	Mode	Median	10%	90%
A. Structural Parameters								
τ	G	1.0	0.5	3.3	1.27	1.31	1.12	1.52
$100 \times \kappa^*$	G	5.0	3.0	10.0	0.10	0.12	0.07	0.18
$100 \times \kappa$	G	5.0	3.0	10.0	0.31	0.32	0.24	0.43
$100 \times \kappa_x$	G	10.0	3.0	20.1	1.56	1.80	1.16	2.73
κ_m	N	1.0	0.8	1.2	0.77	0.78	0.56	1.00
$100 \times (\mu - 1)$	N	0.5	0.3	0.7	0.47	0.47	0.44	0.50
$100 \times (\pi^* - 1)$	N	0.5	0.3	0.7	0.50	0.49	0.39	0.58
$100 \times (\pi - 1)$	N	0.5	0.3	0.7	0.63	0.63	0.55	0.70
ρ_r^*	B	0.7	0.6	0.9	0.88	0.88	0.85	0.90
ϕ_π^*	N	2.0	1.8	2.2	1.98	1.96	1.78	2.14
ϕ_g^*	N	0.2	0.1	0.3	0.30	0.30	0.19	0.41
ϕ_y^*	G	0.2	0.1	0.3	0.11	0.12	0.07	0.17
ρ_r	B	0.7	0.6	0.9	0.85	0.85	0.82	0.87
ϕ_π	N	2.0	1.8	2.2	1.95	1.94	1.76	2.14
ϕ_g	N	0.2	0.1	0.3	0.29	0.28	0.16	0.39
ϕ_y	G	0.2	0.1	0.4	0.12	0.13	0.10	0.16
B. Shock Processes								
ρ_ξ^*	B	0.5	0.2	0.8	0.93	0.93	0.90	0.94
ρ_a^*	B	0.5	0.2	0.8	0.80	0.79	0.72	0.85
$10 \times \sigma_\xi^*$	IG	0.6	0.3	1.8	0.42	0.43	0.37	0.49
$10 \times \sigma_a^*$	IG	0.6	0.3	1.8	1.04	1.20	0.73	2.07
$100 \times \sigma_r^*$	IG	0.6	0.3	1.8	0.12	0.12	0.11	0.14
ρ_ξ	B	0.5	0.2	0.8	0.91	0.90	0.88	0.92
ρ_a	B	0.5	0.2	0.8	0.38	0.40	0.28	0.53
ρ_p	B	0.5	0.2	0.8	0.97	0.97	0.96	0.98
$10 \times \sigma_\xi$	IG	0.6	0.3	1.8	0.59	0.60	0.52	0.69
$10 \times \sigma_a$	IG	0.6	0.3	1.8	1.74	2.00	1.48	2.58
$100 \times \sigma_r$	IG	0.6	0.3	1.8	0.19	0.20	0.18	0.22
$10 \times \sigma_p$	IG	1.5	0.7	3.3	0.52	0.52	0.46	0.58
$100 \times \sigma_8$	IG	0.1	0.0	0.2	0.04	0.06	0.03	0.12
$100 \times c_8$	N	0.0	−0.2	0.2	−0.03	−0.01	−0.25	0.25
$100 \times \sigma_8^*$	IG	0.1	0.0	0.2	0.04	0.06	0.02	0.14
$100 \times c_8^*$	N	0.0	−0.2	0.2	0.03	0.03	−0.22	0.30

Table 2: Variance Decomposition Due to Shocks, %

Variable \ Shock	US Shocks: Preference	US Shocks: Productivity	US Shocks: Policy	Canadian Shocks: Preference	Canadian Shocks: Productivity	Canadian Shocks: Policy	Canadian Shocks: Risk Premia
A. US Variables							
Fed Funds Rate	76.4	10.9	12.6	–	–	–	–
Inflation	18.5	81.1	0.5	–	–	–	–
Output	89.5	7.3	3.2	–	–	–	–
B. Canadian Variables							
Bank Rate	5.7	0.1	0.0	71.6	1.9	17.4	3.2
Inflation	6.2	0.3	0.3	33.2	37.3	3.2	19.5
Output	5.3	0.1	0.1	69.1	0.9	2.0	22.6
Consumption	7.2	0.1	0.0	43.4	0.0	0.3	48.9
Real Exch Rate	10.4	0.3	0.5	18.2	0.1	0.7	69.7

Table 3: Posterior Distributions of ZLB Durations

	US				Canada			
Date	Mean	Median	10%	90%	Mean	Median	10%	90%
2009Q1	4.8	4	1	10	-	-	-	-
2009Q2	4.4	4	1	9	7.9	6	2	16
2009Q3	4.1	4	2	7	5.2	5	2	9
2009Q4	4.9	4	2	10	4.5	4	2	8
2010Q1	3.5	3	2	7	4.5	3	1	9
2010Q2	5.1	4	2	10	3.4	3	1	7
2010Q3	4.3	4	2	7	5.3	5	1	11
2010Q4	5.0	4	1	10	6.6	6	1	12
2011Q1	5.0	5	2	8	5.6	5	1	11
2011Q2	8.5	8	4	13	6.5	6	2	13
2011Q3	6.7	7	2	11	6.9	7	1	13
2011Q4	8.4	9	4	12	6.7	6	1	13
2012Q1	7.2	7	2	12	5.7	5	1	11
2012Q2	7.7	8	3	12	6.3	5	1	13
2012Q3	9.3	10	3	14	5.8	5	1	12
2012Q4	8.3	9	3	13	6.2	5	2	12
2013Q1	8.3	9	3	13	6.6	5	1	14
2013Q2	6.7	7	2	11	5.8	5	1	11
2013Q3	8.0	8	3	13	6.1	5	1	13
2013Q4	8.8	9	3	14	5.5	4	1	11
2014Q1	6.3	6	3	9	7.2	6	1	14
2014Q2	6.5	6	3	12	6.6	6	1	13
2014Q3	5.7	4	2	12	6.0	5	1	12
2014Q4	4.0	3	2	7	6.3	6	1	12

Figure 1: Timing and Four Possible Regimes in the Model

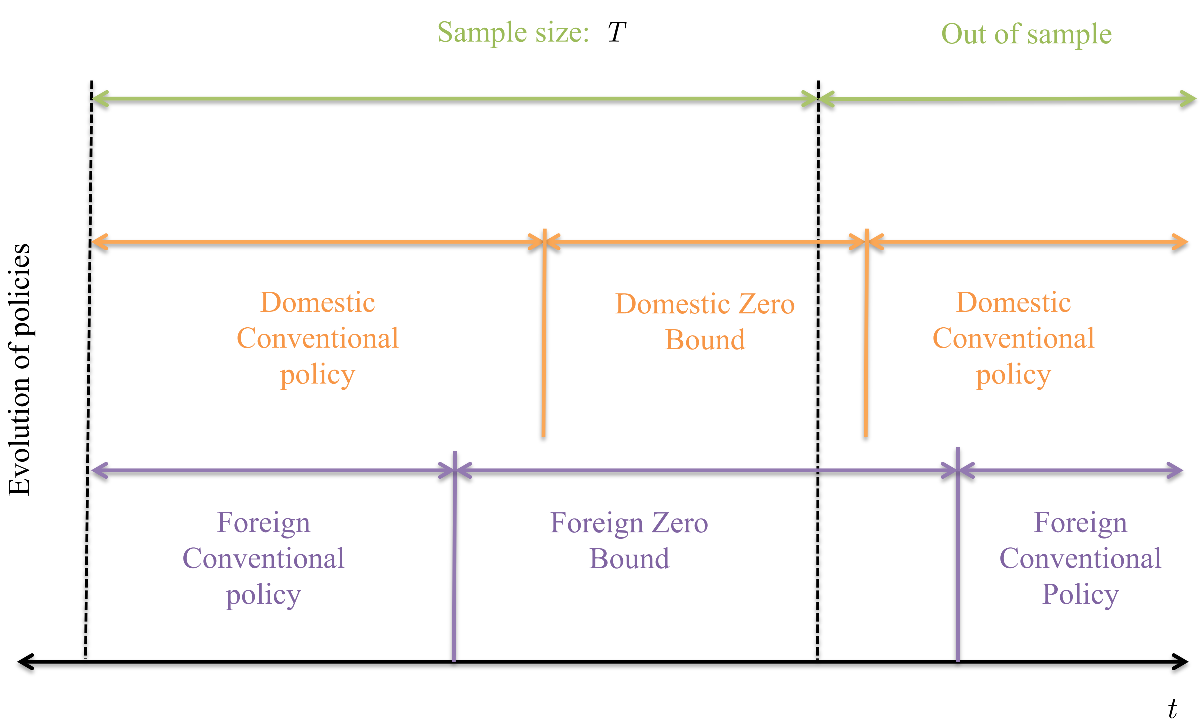

Figure 2: Example of a Forward Guidance Shock at t

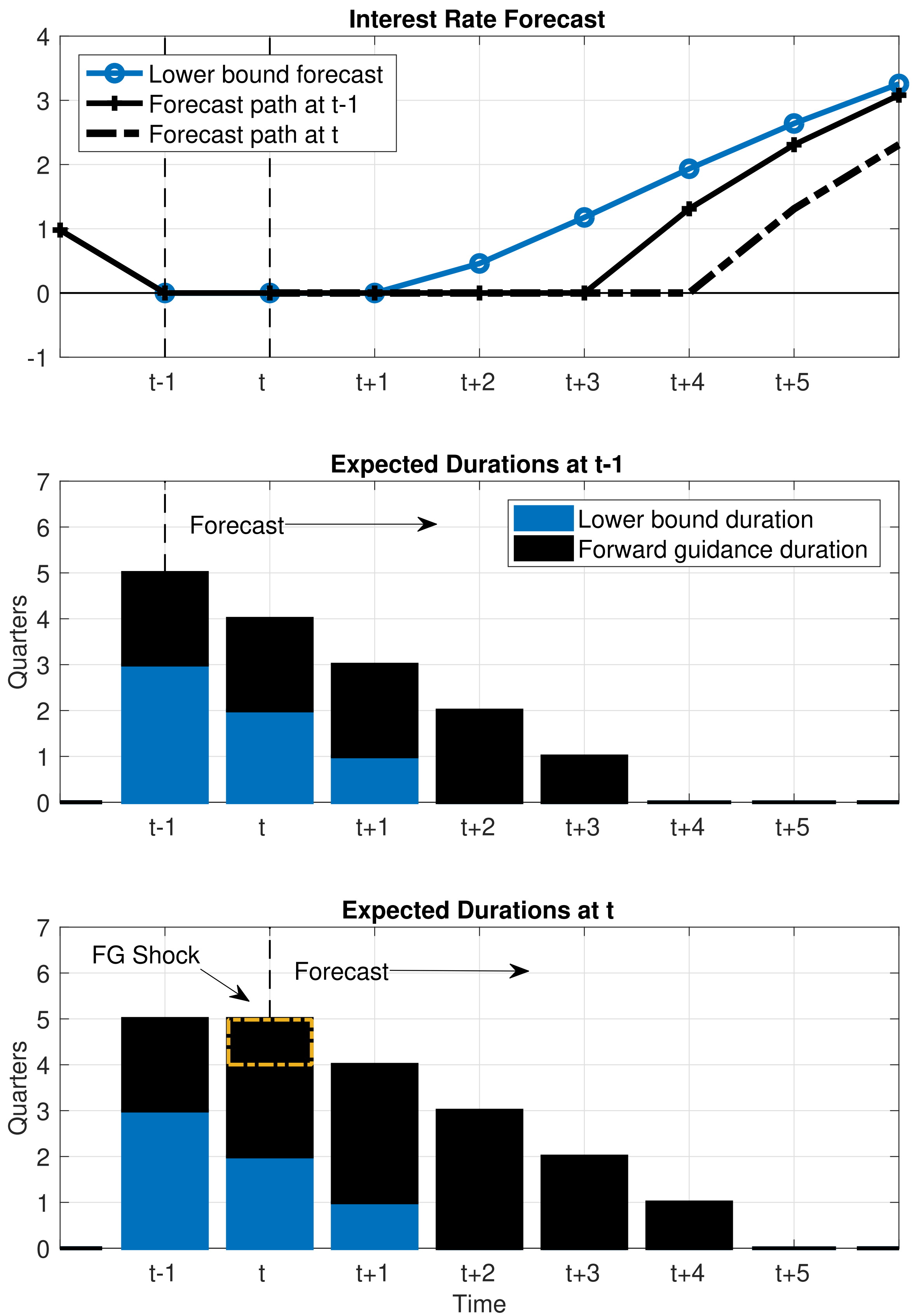

Figure 3: Quarterly Data Used in Estimation

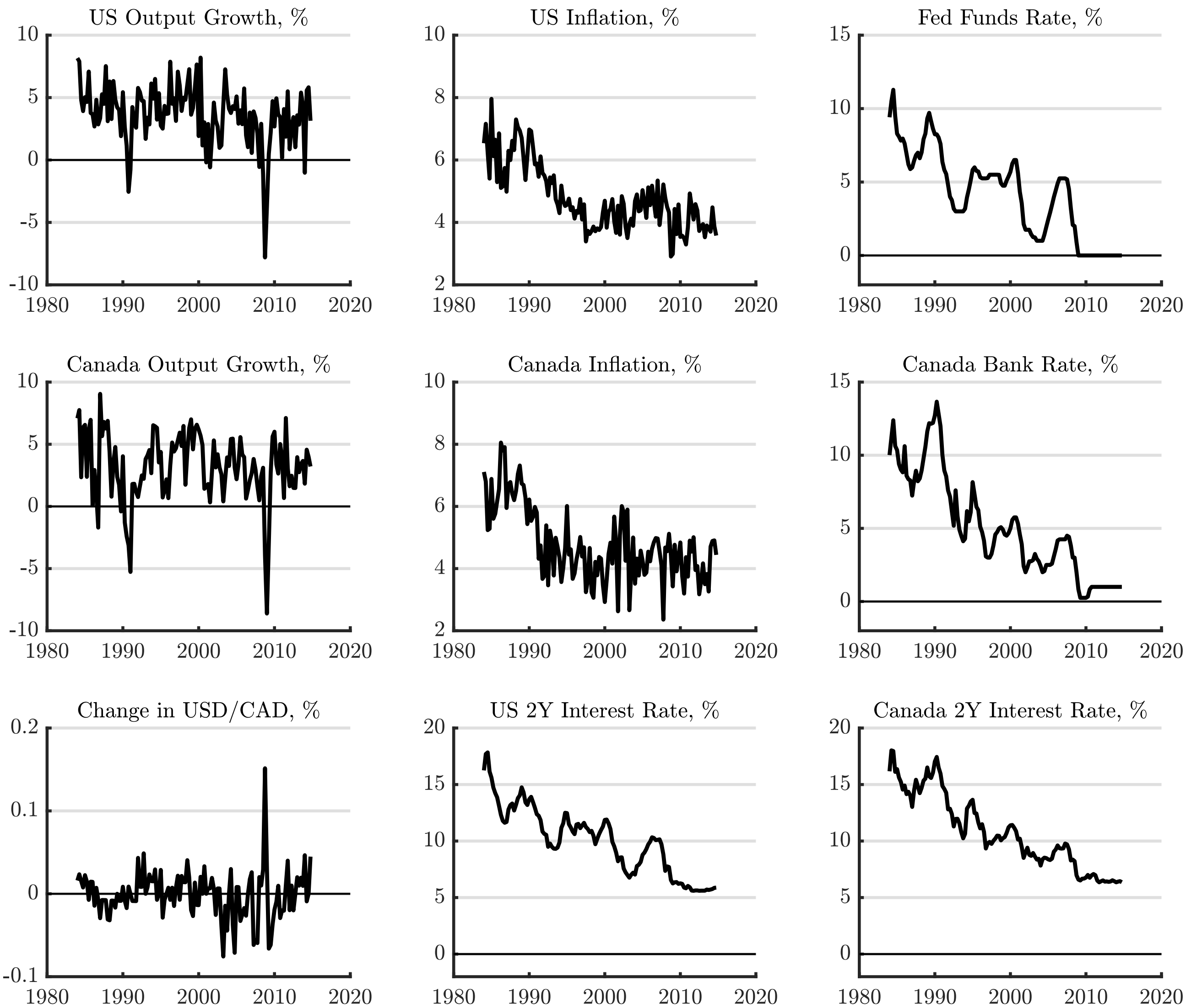

Figure 4: Canada Output Response to Expansionary US Monetary Policy Shock, %

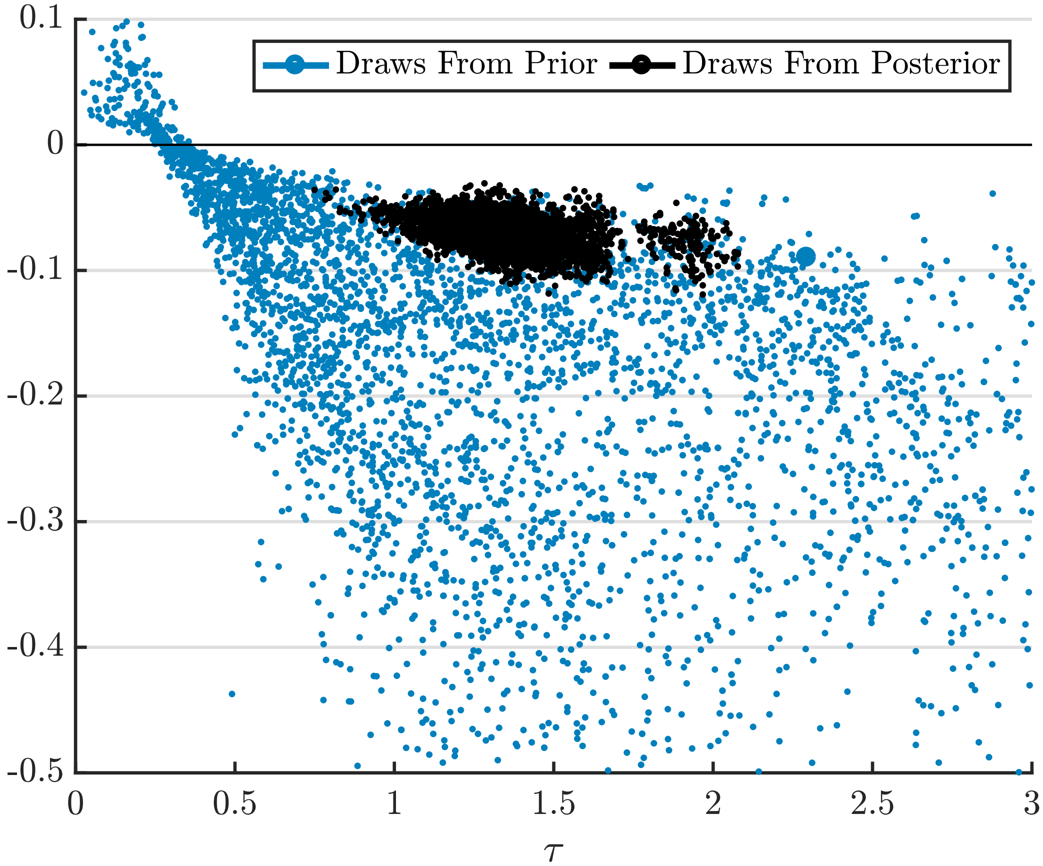

Figure 5: Impulse Response to Expansionary US Monetary Policy Shock

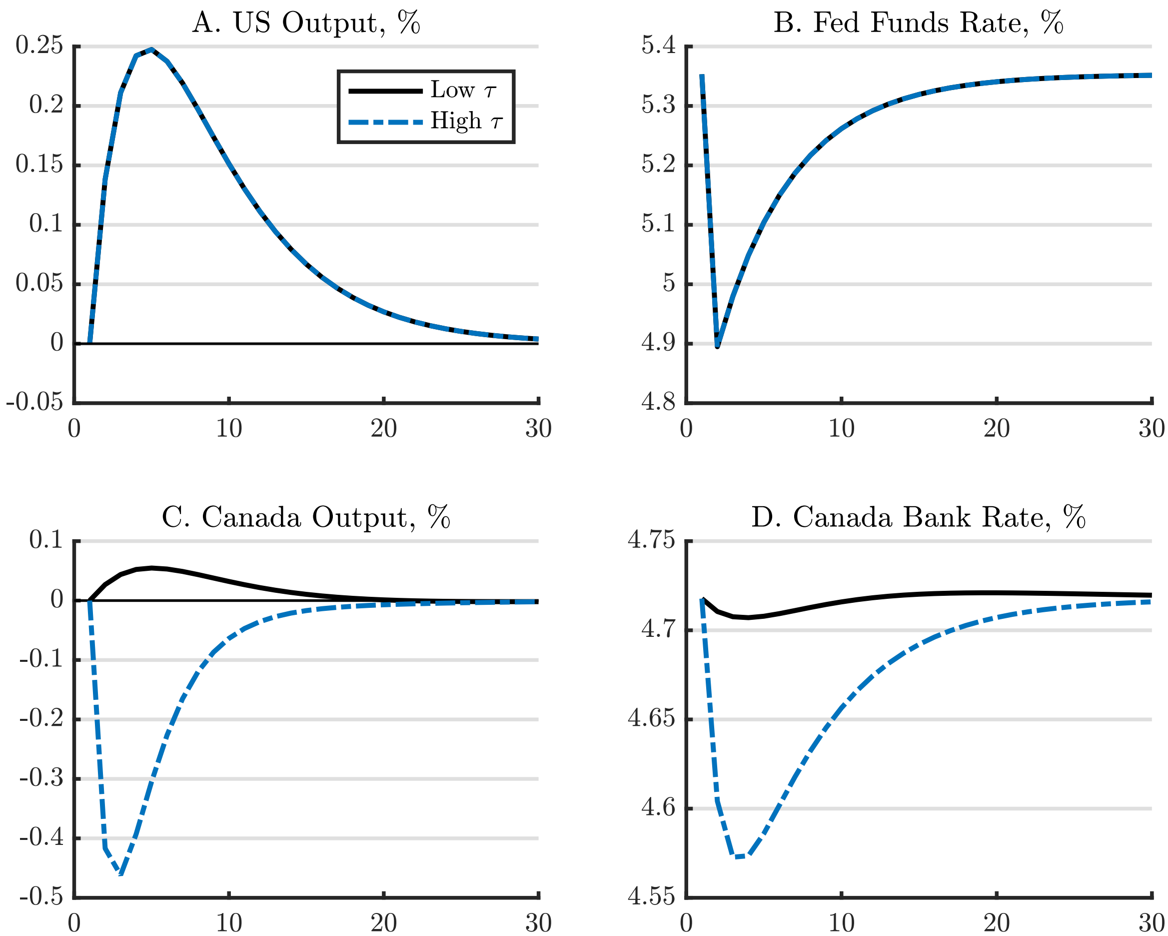

Figure 6: Fixed Interest Rate Duration and Forward Guidance, Mean Across Draws

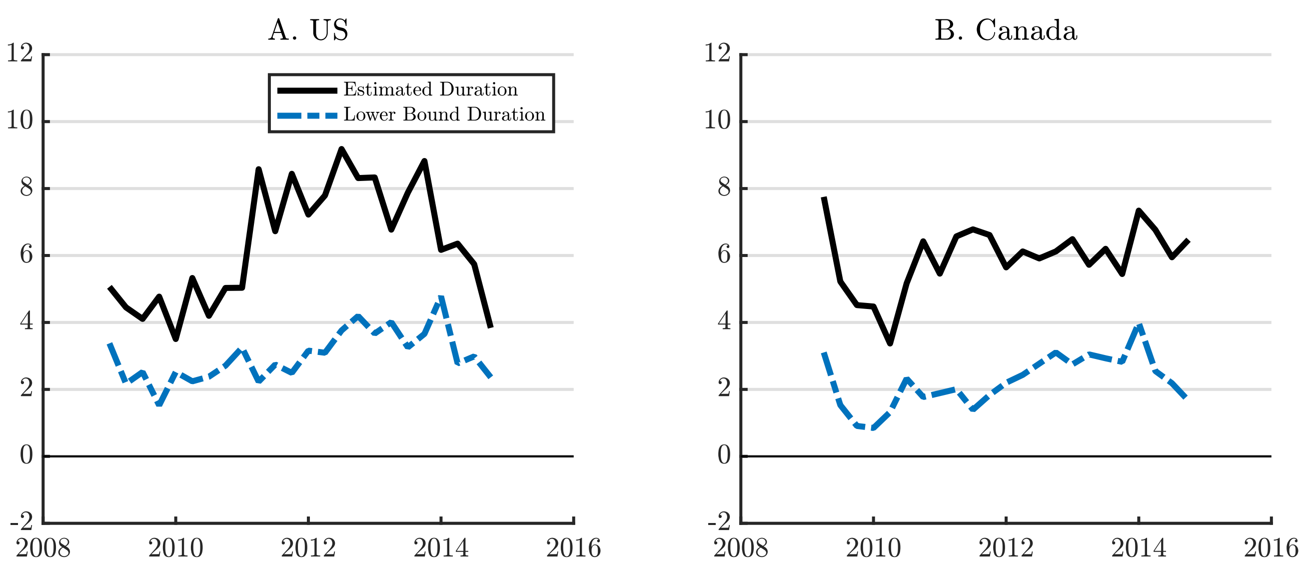

Figure 7: Posterior Distributions of US Forward Guidance Shocks

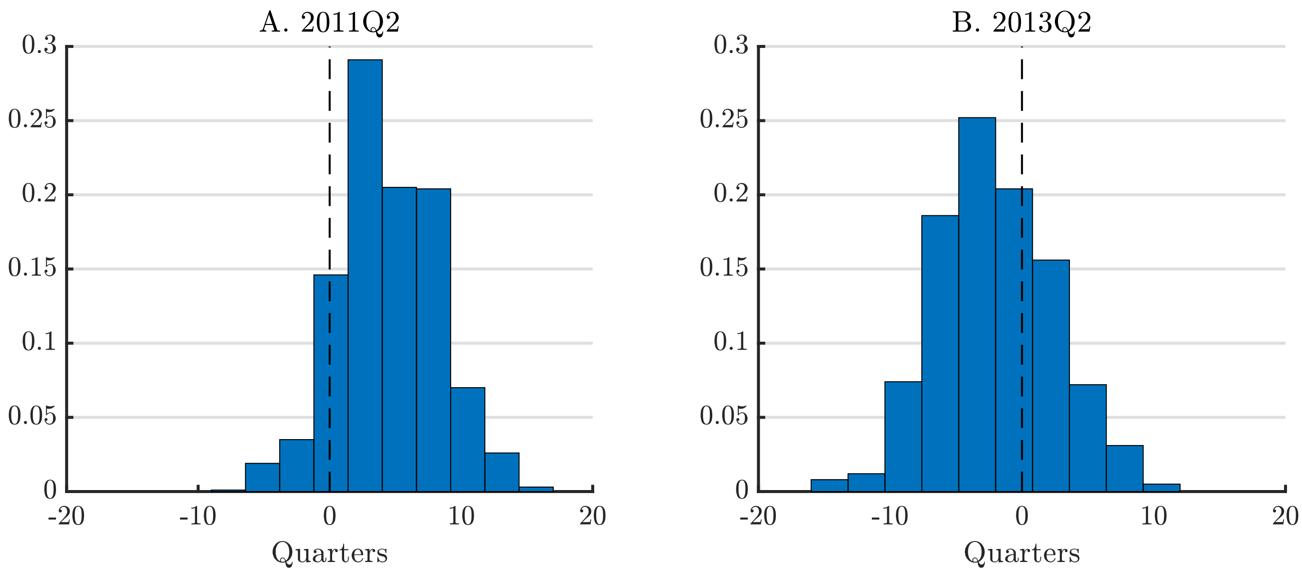

Figure 8: US Shadow Policy Rates, Annual

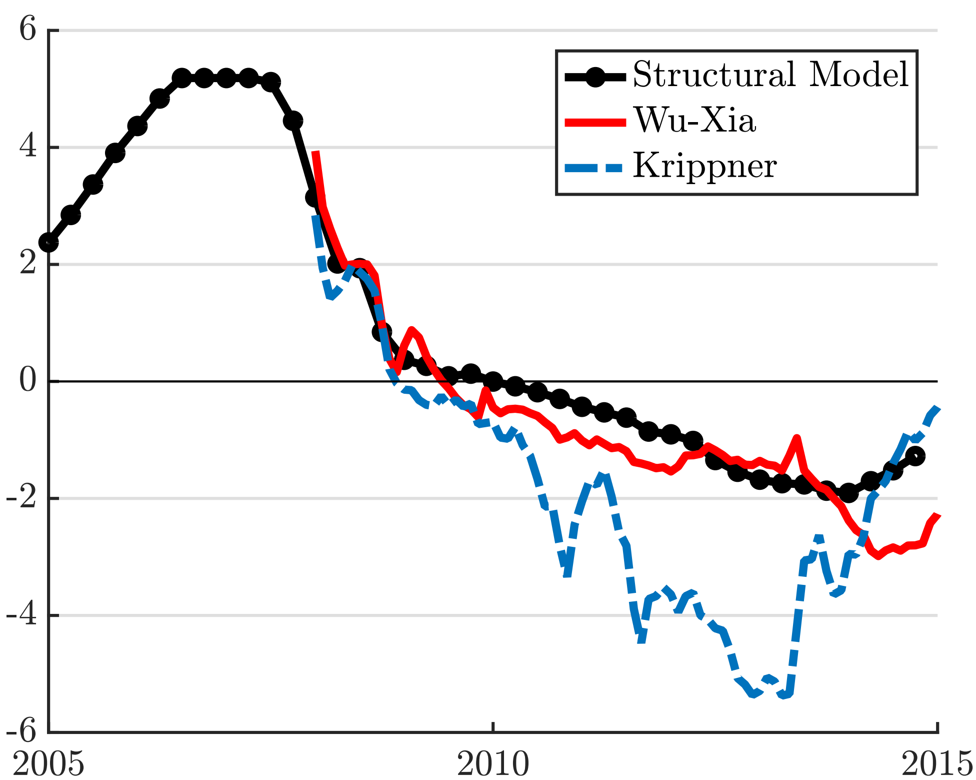

Figure 9: Comparison of Conventional US Policy Shock and US Forward Guidance Shock

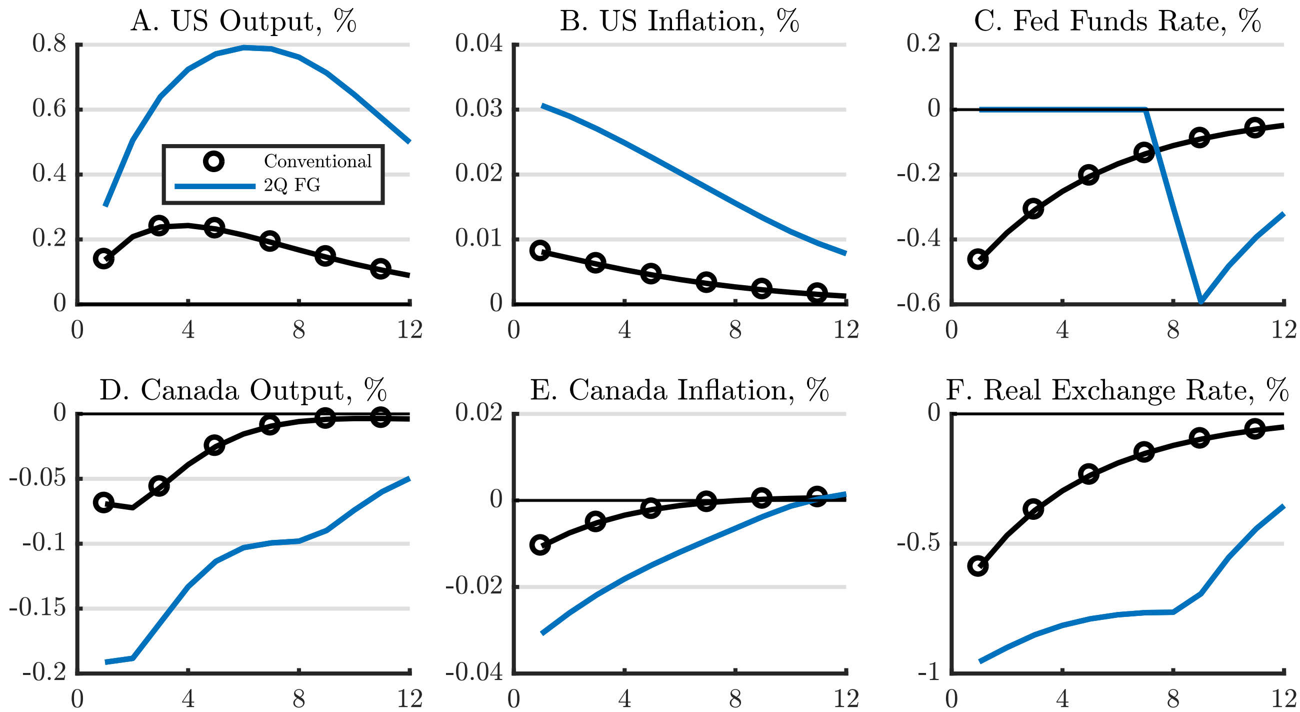

Figure 10: State Dependent Spillovers of US Forward Guidance Shock

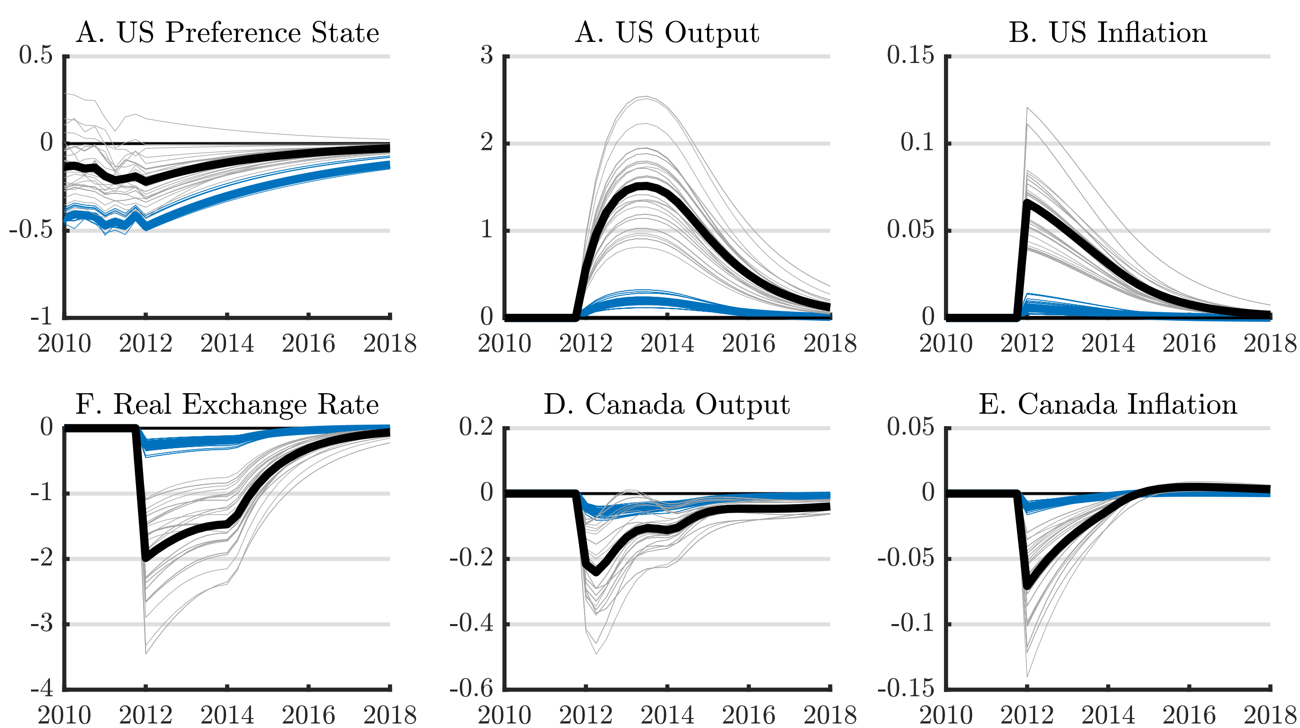

Figure 11: Percentage Change in Output With No Forward Guidance in US and Canada

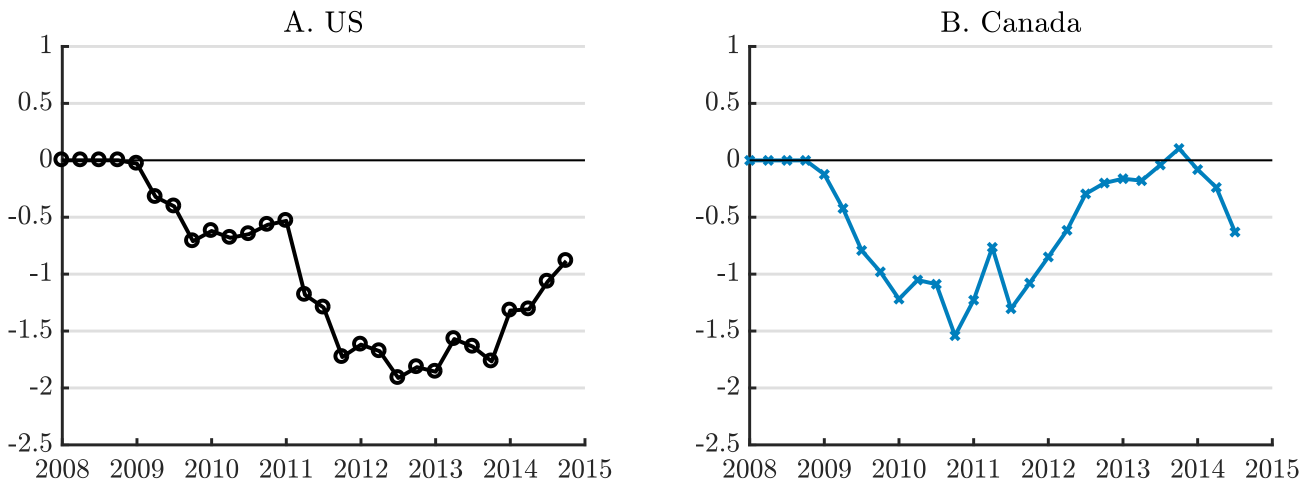

Figure 12: Percentage Change in Output With No Forward Guidance in US Only