

# A graphical representation of an estimated DSGE model

Callum Jones<sup>a</sup> and Mariano Kulish<sup>b</sup>

<sup>a</sup>Department of Economics, New York University, New York, NY, USA; <sup>b</sup>School of Economics, University of New South Wales, Sydney, Australia

#### **ABSTRACT**

We write a New Keynesian model as an aggregate demand curve and an aggregate supply curve, relating inflation to output growth. The graphical representation shows how structural shocks move aggregate demand and supply simultaneously. We estimate the curves on US data from 1948 to 2010 and study two recessions: the 2001 recession and the Great Recession of 2008–2009. The Great Recession is explained by a collapse of aggregate demand driven by adverse preference and permanent technology shocks, and expectations of low inflation.

#### **KEYWORDS**

DSGE estimation; graphical analysis; New Keynesian model: shock decomposition

**JEL CLASSIFICATION** E27; E32; E58

### I. Introduction

From month to month, discussions of monetary policy focus on understanding recent fluctuations in the data. Formal models contribute to this understanding to varying degrees across central banks. But as put by Alvarez-Lois et al. (2008), while there is a large literature that discusses the theory and estimation of dynamic stochastic general equilibrium (DSGE) models, there is relatively little that examines the communication of model results to policymakers. This issue is important and here we contribute to it by developing, for estimated DSGE models, a graphical representation that, because of its simplicity, helps formal models contribute even more to policy deliberations. The key innovation of our analysis is to obtain a graphical representation in the space of observable variables, the space of inflation and output growth. Because the data are given, the graphical analysis reveals how a given parametrized model decomposes the underlying sources of movements from one quarter to the next into what can be labelled as aggregate demand and aggregate supply fluctuations.

In particular, we represent the New Keynesian model of Ireland (2004) graphically, characterizing it in terms of an aggregate demand curve and aggregate supply curve. Each curve links inflation to output growth and a structural shift factor. This is not the first graphical representation of a dynamic macroeconomic model, but is, to the best of our knowledge, the first graphical representation of an estimated DSGE model for which aggregate demand and supply curves live in the observable space of output growth and inflation. In recent work - that is independent to our own - Benigno (2009) develops a graphical representation of a two-period New Keynesian framework to illustrate, among other things, the impact of productivity and mark-up shocks and the role of fiscal multipliers.<sup>2</sup> Although these graphical representations yield valuable insights, they cannot be taken to the data, either because they rely on unobservable quantities, like the output gap, or because they make strong simplifying assumptions, like a two-period economy.

The graphical analysis of the structural model yields insights that are likely to be novel to many. For example, the graphical representation makes it clear that with forward-looking agents any shock, be it a 'supply' shock or a 'demand' shock, shifts both the aggregate demand and aggregate supply curves through its impact on expectations. It also shows how increases in inflation expectations simultaneously expand aggregate demand but

contract aggregate supply. The degree to which inflation expectations exercise this joint influence depends crucially on the parameters that govern the slope of the curves: the degree of nominal rigidities, which governs the slope of the aggregate supply curve, and the parameters of the monetary policy rule, which, in turn, govern the slope of the aggregate demand curve.

As an application of the graphical representation, we show how the aggregate demand and aggregate supply curves behaved over two recessions in the United States: the 2001 recession and the Great Recession of 2007 through 2009.

The article proceeds as follows.<sup>3</sup> Section II describes the specific model we illustrate the method with, showing how to relate the equations of the model to aggregate demand and aggregate supply curves. Section III then uses the graphical representation to analyse two recessions, the 2001 recession and the Great Recession of 2008-2009. Section IV discusses the feasibility of graphical representations in larger DSGE models and concludes.

# II. The graphical representation

# The model

The details of the New Keynesian model we use can be found in Ireland (2004). The eight linearized equations which characterize the economy's equilibrium are4

$$\hat{x}_t = E_t \hat{x}_{t+1} - (\hat{r}_t - E_t \hat{\pi}_{t+1}) + (1 - \omega)(1 - \rho_a)\hat{a}_t$$
(1)

$$\pi_t = \pi + \beta E_t \hat{\pi}_{t+1} + \psi \hat{x}_t - \hat{e}_t \tag{2}$$

$$\hat{r}_t = \hat{r}_{t-1} + \rho_{\pi}(\pi_t - \pi) + \rho_{g}(g_t - g) + \rho_{x}\hat{x}_t + \varepsilon_{r,t}$$
 (3)

$$\hat{x}_t = \hat{y}_t - \omega \, \hat{a}_t \tag{4}$$

$$g_t = g + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{5}$$

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t} \tag{6}$$

$$\hat{e}_t = \rho_{e} \hat{e}_{t-1} + \varepsilon_{e,t} \tag{7}$$

$$\hat{z}_t = \varepsilon_{z,t} \tag{8}$$

where hat-variables represent deviations from steady state values with  $\hat{y}_t$  denoting the deviation of detrended output from its steady-state,  $\hat{x}_t$  the deviation of the output gap from its steady-state and  $\hat{r}_t$  is the deviation of the one-period interest rate from its steady-state,  $\pi_t$  the one-period inflation rate and  $\hat{\pi}_t = \pi_t - \pi$ ,  $a_t$  a preference shock,  $e_t$  a negative cost-push shock and  $z_t$  a total factor productivity shock which follows a random walk with drift. Note that output growth  $g_t$  and inflation  $\pi_t$  are expressed in levels.

Equation 1 is derived from the representative household's Euler equation, relating the output gap negative to the real interest rate. Equation 2 represents the economy's Phillip's curve, positively relating inflation to the output gap. The parameter  $\psi$  is decreasing in the output cost that intermediate goods producing firms face when changing prices. The monetary policy Taylor rule is given by Equation 3 with the central bank adjusting nominal rates in response to inflation, output growth and the output gap. Equation 4 defines the output gap, and reflects that only preference shocks move the detrended level of efficient output relative to detrended actual output. Equation 5 defines output growth, while Equations 6-8 define the exogenous processes.

# **Graphical representation**

The equations of the model can be manipulated to form aggregate supply and aggregate demand schedules relating inflation to output growth. To find the aggregate supply schedule, substitute Equations 4 and 5 in Equation 2 to get

$$\pi_t = \psi \, g_t + \hat{s}_t + (\pi - \psi \, g) \tag{9}$$

where  $\hat{s}_t = \beta E_t \hat{\pi}_{t+1} + \psi \hat{y}_{t-1} - \psi \hat{z}_t - \omega \psi \hat{a}_t - \hat{e}_t$ . In the space of output growth and inflation  $(g_t, \pi_t)$ , Equation 9 expresses inflation as a linear function of output growth, with slope  $\psi$  and intercept  $\hat{s}_t + (\pi - \psi g)$ . Note that the time-varying intercept  $\hat{s}_t$  is zero when the economy is on its balanced growth path. Also note that the slope of the curve depends on the degree of nominal price rigidities. That is, as  $\psi \to \infty$ , prices become fully flexible, which implies a vertical aggregate supply curve. Conversely as the

<sup>&</sup>lt;sup>3</sup>MATLAB files for this article are provided with the Supplementary material.

<sup>&</sup>lt;sup>4</sup>Detailed notes and MATLAB code accompanies Ireland's paper at http://www2.bc.edu/irelandp/programs.html.

cost of price adjustment rises,  $\psi \to 0$ , implying that the aggregate supply curve flattens.

To obtain the aggregate demand schedule, substitute Equations 3-5 in 1 to get

$$\pi_t = -\left(\frac{1 + \rho_g + \rho_x}{\rho_\pi}\right) g_t + \hat{d}_t + \left(\pi + \frac{1 + \rho_g + \rho_x}{\rho_\pi} g\right)$$
(10)

where

$$\begin{split} \hat{d}_{t} &= -\frac{1}{\rho_{\pi}} \hat{r}_{t-1} + \frac{1}{\rho_{\pi}} E_{t} \hat{x}_{t+1} + \frac{1}{\rho_{\pi}} E_{t} \hat{\pi}_{t+1} \\ &- \left( \frac{1 + \rho_{x}}{\rho_{\pi}} \right) \hat{y}_{t-1} + \left( \frac{1 + \rho_{x}}{\rho_{\pi}} \right) \hat{z}_{t} \\ &+ \frac{\omega (1 + \rho_{x}) + (1 - \omega)(1 - \rho_{a})}{\rho_{\pi}} \hat{a}_{t} - \frac{1}{\rho_{\pi}} \varepsilon_{r,t}. \end{split}$$

Note that, as for the time-varying intercept in the aggregate supply curve, when the economy is on its balanced growth path,  $d_t$  is zero. The slope of the curve (Equation 10) depends on the parameters of the policy rule. A greater response to deviations of inflation from target,  $\rho_{\pi}$ , flattens the curve. Vice versa, stronger responses to output growth,  $\rho_{g}$ , and the output gap,  $\rho_x$ , steepen the aggregate demand curve.

With the values of  $\hat{s}_t$  and  $\hat{d}_t$  in hand, the aggregate supply curve (Equation 9) and the aggregate demand curve (Equation 10) can be written as a system of two equations in two variables  $g_t$  and  $\pi_t$ . In matrix notation:

$$\begin{bmatrix} 1 & -\psi \\ 1 & \frac{1+\rho_g+\rho_x}{\rho_\pi} \end{bmatrix} \begin{bmatrix} \pi_t \\ g_t \end{bmatrix} = \begin{bmatrix} \hat{s}_t + \pi - \psi g \\ \hat{d}_t + \pi + \left(\frac{1+\rho_g+\rho_x}{\rho_\pi}\right)g \end{bmatrix}$$

The square matrix must be nonsingular because the condition  $\psi + \frac{1}{\rho_{\pi}} + \frac{\rho_{g}}{\rho_{\pi}} + \frac{\rho_{x}}{\rho_{\pi}} \neq 0$  is always satisfied for reasonable values of the policy parameters and  $\psi > 0$ . Inverting the matrix,  $\pi_t$  and  $g_t$  can be solved by

$$\begin{bmatrix} \pi_{t} \\ g_{t} \end{bmatrix} = \frac{1}{1 + \rho_{g} + \psi \rho_{\pi} + \rho_{x}}$$

$$\begin{bmatrix} \pi (1 + \rho_{x} + \rho_{g} + \psi \rho_{\pi}) + \psi \rho_{\pi} \hat{d}_{t} + (1 + \rho_{g} + \rho_{x}) \hat{s}_{t} \\ g (1 + \rho_{x} + \rho_{g} + \psi \rho_{\pi}) + \rho_{\pi} \hat{d}_{t} - \rho_{\pi} \hat{s}_{t} \end{bmatrix}.$$
(11)

At any point in time, the economy can be described in the output growth-inflation space, by the intersection of the aggregate supply curve (Equation 9), and the aggregate demand curve (Equation 10). Figure 1 shows the  $(g_t, \pi_t)$  space populated by quarterly US observations from 1948Q1 to 2010Q3. If drawn, the linear curves (Equations 9 and 10) would meet on these observations. But importantly, these observations are the same for every model.

We need estimates of the model parameters to generate the estimated aggregate demand and aggregate supply curves. We follow Ireland (2004) with updated data. We calibrate  $\beta$  to 0.99,  $\psi$  to 0.1 and  $\omega$  to 0.06. The remaining parameter estimates and SEs are given in Table 1. They are much the same as Ireland's. Because of the strong estimated response of the short rate to output growth and the output gap relative to the response to inflation, the slope of the aggregate demand curve is estimated to be steep:  $-\frac{1+\rho_g+\rho_x}{\rho_\pi}$ -4.4. The aggregate supply curve is, owing to the low value of  $\psi = 0.1$ , relatively flat. This explains why the model attributes an important role to aggregate demand shocks.

As noted, the evolution of the economy, in output growth-inflation space, can be described by the intersection of the aggregate supply and demand schedules at each point in time. A shock will shift the time-varying intercepts of the curves,  $\hat{s}_t$  and  $d_t$ . Over time, these curves move back toward the steady state. So, for example, on the impact of a 1 SD shock to  $\varepsilon_{r,t}$ ,  $\hat{s}_t = -0.001$  and  $d_t = -0.026$ . Simultaneously, both curves move. To see this,

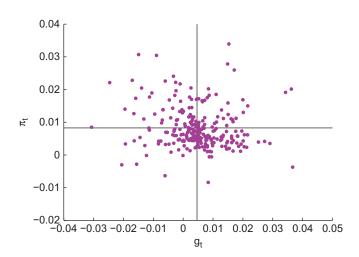


Figure 1. United States, 1948Q1-2010Q3.

Table 1. Maximum likelihood estimates.

| Parameter    | Estimate | SE     |  |  |
|--------------|----------|--------|--|--|
| $\rho_a$     | 0.9489   | 0.0224 |  |  |
| $\rho_e$     | 0.9470   | 0.0393 |  |  |
| $\rho_{\pi}$ | 0.2885   | 0.0404 |  |  |
| $\rho_q$     | 0.2090   | 0.0451 |  |  |
| $\rho_{x}$   | 0.0679   | 0.0177 |  |  |
| $\sigma_a$   | 0.0365   | 0.0122 |  |  |
| $\sigma_e$   | 0.0008   | 0.0002 |  |  |
| $\sigma_z$   | 0.0120   | 0.0015 |  |  |
| $\sigma_r$   | 0.0026   | 0.0003 |  |  |

return to the expression of the system in matrix notation (Equation 11) and substitute in the estimated parameter values to get

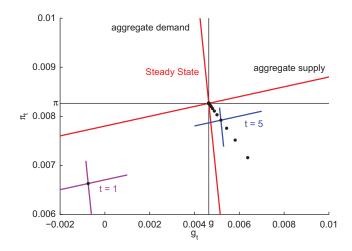
$$\begin{bmatrix} \pi_t \\ g_t \end{bmatrix} = 0.766 \begin{bmatrix} 1.317 + 0.029 \ \hat{d}_t + 1.277 \ \hat{s}_t \\ 1.312 + 0.289 \ \hat{d}_t - 0.289 \ \hat{s}_t \end{bmatrix}$$

This illustrates an important feature of the analysis, that in general equilibrium, shocks simultaneously act on the shifting components of aggregate demand and aggregate supply,  $d_t$  and  $\hat{s}_t$ , through the expectations channel. In the case of the monetary policy shock, in the period of the shock, the increase in  $\hat{r}_t$  directly leads to a contraction in aggregate demand as agents adjust consumption. Simultaneously, agents expect the output gap to be negative in the next period and inflation to be below steady state which, through the Euler condition, contracts aggregate demand further. For the supply side, a fall in inflation expectations acts to expand supply: because it is costly for firms to adjust prices, they have an incentive, for any given level of output, to reduce prices today.

We illustrate the curves and their dynamics following a monetary policy shock in output growth-inflation space. Figure 2 draws the aggregate demand and aggregate supply curves at steady state, the period of the monetary policy shock t=1, and four periods later at t=5. It shows the contraction in aggregate demand and the expansion in aggregate supply from steady state following the shock, and the movements in the curves as the economy returns to steady state.

## The zero lower bound

In regions of inflation and output growth where the zero lower bound binds, the shape of the aggregate



**Figure 2.** Aggregate demand and aggregate supply curves:  $\varepsilon_r$  shock.

demand curve changes in an economically interesting way. To see this, first rewrite the Taylor rule under the zero lower bound constraint as

$$\hat{r}_{t} = \max(-r, \hat{r}_{t-1} + \rho_{\pi}(\pi_{t} - \pi) + \rho_{g}(g_{t} - g) + \rho_{x}\hat{x}_{t} + \epsilon_{r,t})$$

so that  $\hat{r}_t = r_t - r = -r$  or  $r_t = 0$  at the zero lower bound, where r is the steady-state level of the nominal interest rate. Substitute  $\hat{r}_t = -r$ , Equations 4 and 5 to 1 to get an aggregate demand schedule:

$$g_t = \tilde{d}_t + g, \tag{12}$$

where  $\tilde{d}_t = r + \mathbb{E}_t \hat{x}_{t+1} + \mathbb{E}_t \hat{\pi}_{t+1} - \hat{y}_{t-1} + \hat{z}_t + (\omega + 1)$  $(1-\omega)\rho_a$ ) $\hat{a}_t$ . Equation 12 says that, in the space of output growth and inflation, the aggregate demand curve becomes vertical in regions where the zero lower bound binds. Intuitively, a vertical aggregate demand curve says that the central bank cannot engineer an expansion of output by lowering the nominal interest rate, so that time t growth is determined solely by structural shocks and expectations. The derivation of the aggregate supply curve is unchanged when the nominal interest rate is zero, so the shape of the aggregate supply curve is the same in the region where the zero lower bound binds. Taken together, we can use the graphical representation with the zero lower bound to see, for example, how a given shock affects inflation and output growth differently depending on whether the equilibrium lies on the sloped (nonzero lower bound) or vertical (binding zero lower bound) segment of the aggregate demand curve. For a discussion of the effects of shocks which hit the economy at the zero lower bound and the role forward guidance has in response to those shocks, see Jones (2015).

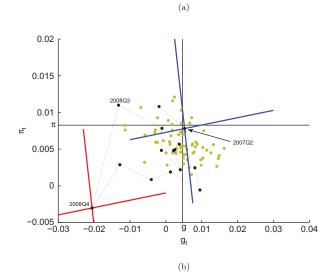
#### III. Two recessions

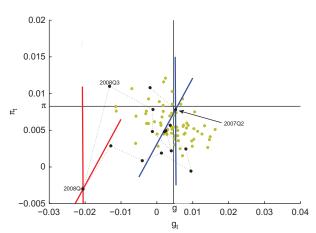
We estimated the model on US data from 1948Q1 to 2010Q3. This allows us to analyse the behaviour of aggregate demand and aggregate supply over the period of the Great Recession (Ireland 2011). As per Ireland (2004), the observed quarterly series were the growth rate in US GDP per person, inflation of the US GDP deflator and the 3-month Treasury bill rate. The advantage of drawing the curves in output growth-inflation space is that the analysis is done relative to fixed and observable points and that through estimation we can understand the forces moving the aggregate demand and aggregate supply curves from the observed points.

We focus on the more recent period of estimation. Figure 3 gives the position of the US economy for each quarter from 1990Q1 to 2010Q3, with the quarters from 2007Q1 highlighted in black. The aggregate demand and supply curves are drawn for 2007Q2 and for 2008Q4 when the curves were farthest from the steady state. The dashed lines can be followed to give the path of the economy over this period.

The figure makes it clear the extent of the recessionary forces on the US economy in late 2008, with both inflation and output growth falling well below steady state. At the estimated parameter values, the shift of the economy from early 2007 to late 2008 reflected a large decline in aggregate demand and a significant increase in aggregate supply, as illustrated by the relative positions of the curves. Following the trough in 2008Q4, aggregate demand was estimated to recover to above its steady-state position by 2009Q4, while aggregate supply remained away from steady state.

To understand why these curves moved, Table 2 shows the components of aggregate demand and aggregate supply as per Equations 9 and 10. A few observations are notable. Over the period, expected inflation below steady-state inflation held back demand which, according to the model, led households to hold back consumption. Further, a sequence





**Figure 3.** United States, 1990Q1–2010Q3. (a)  $\psi = 0.1$ ; (b)  $\psi = 0.9.$ 

of relatively large negative preference shocks and permanent technology shocks restrained aggregate demand. On the aggregate supply curve, expected inflation below steady state in 2008Q4 accounts for much of the movement in the supply curve from 2008Q3. A series of estimated positive cost push shocks also pushed the aggregate supply curve down: a positive cost push shock temporarily increases the elasticity of demand for firms' output and reduces their markup over marginal costs, decreasing prices.

The graphical analysis highlights the importance of  $\psi$ , the degree of nominal rigidities, in interpreting the data. If we were to set this parameter differently and re-estimate the model, we would find, not surprisingly, different shocks that match the data. This seems clear enough, but is

4

Table 2. Decomposition of US aggregate demand and aggregate supply, 2008Q1-2009Q4.

| Component   | 2008Q1  | 2008Q2           | 2008Q3  | 2008Q4  | 2009Q1    | 2009Q2  | 2009Q3  | 2009Q4   |  |  |  |
|---|---------|------------------|---------|---------|-----------|---------|---------|----------|--|--|--|
|   |         | Aggregate demand |         |         |           |         |         |          |  |  |  |
| $\hat{d}_t$   | -0.0295 | -0.0258          | -0.0761 | -0.1228 | -0.0827   | -0.0460 | -0.0212 | 0.0127   |  |  |  |
| $\frac{1}{\rho_{\pi}}E_{t}\hat{X}_{t+1}$                        | 0.0818  | 0.0981           | 0.1008  | 0.0863  | 0.0889    | 0.0807  | 0.0796  | 0.0802   |  |  |  |
| $\frac{1}{\rho_{\pi}}E_{t}\hat{\pi}_{t+1}$                      | -0.0115 | -0.0042          | 0.0040  | -0.0317 | -0.0164   | -0.0214 | -0.0186 | - 0.0250 |  |  |  |
| $-\left(\frac{1+\rho_x}{\rho_x}\right)\hat{y}_{t-1}$            | -0.0500 | -0.0854          | -0.1119 | -0.1211 | -0.0510   | -0.0667 | -0.0487 | -0.0491  |  |  |  |
| $-\frac{1}{\rho_n}\hat{r}_{t-1}$                                | 0.0105  | 0.0221           | 0.0257  | 0.0269  | 0.0372    | 0.0379  | 0.0383  | 0.0384   |  |  |  |
| $\left(\frac{1+\rho_x}{\rho_n}\right)\hat{Z}_t$                 | -0.0572 | -0.0477          | -0.0751 | -0.0232 | -0.0804   | -0.0142 | -0.0129 | 0.0239   |  |  |  |
| $\frac{\omega(1+\rho_x)+(1-\omega)(1-\rho_a)}{\rho_n}\hat{a}_t$ | -0.0130 | -0.0154          | -0.0190 | -0.0458 | -0.0499   | -0.0540 | -0.0554 | -0.0560  |  |  |  |
| $-\frac{1}{\rho_n} \varepsilon_{r,t}$                           | 0.0098  | 0.0066           | -0.0006 | -0.0143 | -0.0112   | -0.0083 | -0.0036 | 0.0004   |  |  |  |
| <i>r n</i>  |         | ,                |         | Aggrega | te supply |         |         |          |  |  |  |
| $\hat{s}_t$   | -0.0028 | 0.0001           | 0.0045  | -0.0088 | -0.0036   | -0.0065 | -0.0060 | -0.0093  |  |  |  |
| $\beta E_t \hat{\pi}_{t+1}$                                     | -0.0033 | -0.0012          | 0.0011  | -0.0091 | -0.0047   | -0.0061 | -0.0053 | -0.0071  |  |  |  |
| $\psi \hat{y}_{t-1}$  | 0.0014  | 0.0023           | 0.0030  | 0.0033  | 0.0014    | 0.0018  | 0.0013  | 0.0013   |  |  |  |
| $-\psi \hat{z}_t$   | 0.0015  | 0.0013           | 0.0020  | 0.0006  | 0.0022    | 0.0004  | 0.0003  | -0.0006  |  |  |  |
| $-\omega\psi\hat{a}_t$  | 0.0002  | 0.0002           | 0.0003  | 0.0007  | 0.0008    | 0.0008  | 0.0009  | 0.0009   |  |  |  |
| $-\hat{e}_t$  | -0.0027 | -0.0025          | -0.0020 | -0.0043 | -0.0033   | -0.0035 | -0.0033 | - 0.0037 |  |  |  |

emphasized by observing that in Fig. 3, the data points are fixed. Different parameter values would change the slopes of the two curves. As such, different shocks will be needed to move those curves from observation to observation, that is, the contents of Table 2 will change. For example, if we instead calibrate a steep aggregate supply curve, the interpretation of the movement in aggregate supply in moving from 2007Q1 to 2008Q4 is different. In Fig. 3(b),  $\psi$  is calibrated to be equal to 0.9, which steepens the aggregate supply curve. With this calibration, the interpretation of the movement in aggregate supply from 2007Q2 to 2008Q4 is reversed: aggregate supply contracted. Much of difference is due to the magnitude of the estimated technology and preference shocks outweighing the expansionary effect of falling inflation expectations.

As a point of comparison to the Great Recession, Fig. 4 draws the aggregate demand and aggregate supply curves for 2001Q2 and 2001Q3, estimated using the original calibration of Ireland (2004). Also highlighted are the inflation and output growth observations between 2001Q1 and 2002Q1. The movement of the aggregate demand and supply curves between 2001Q2 and 2001Q3 mirror those of the Great Recession: aggregate demand contracted, while aggregate supply increased. Aggregate demand then gradually rebounded in the subsequent quarters.

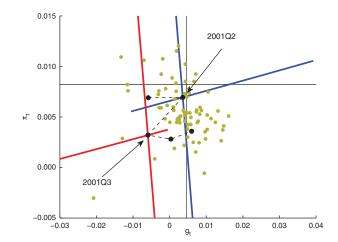


Figure 4. United States, 2001 recession.

#### IV. Conclusion

In this article, we develop a graphical representation of an estimated DSGE model. We use the version of the New Keynesian model of Ireland (2004) estimated on US aggregate data to illustrate aggregate demand and aggregate supply fluctuations. The graphical analysis is useful because it makes it easier for formal models to contribute to regular policy deliberations. Because the representation is achieved in the space of observable data, inflation and output growth, the analysis reveals how a given parametrized model decomposes the underlying sources of movements from one quarter to the next. We apply the graphical analysis to two recessions: the

recession of the early 2000s and the Great Recession of 2008-2009.

One may wonder, however, if the graphical representation is possible in larger models, like that of Smets and Wouters (2007), or in different circumstances, as that which arises when monetary policy reaches the zero lower bound on nominal interest rates.

As we discussed earlier, the aggregate supply curve is implied by the Phillips curve which relates the output gap to inflation. A relation between output growth and inflation can then be easily obtained. The aggregate demand curve, however, is derived by substituting out the nominal interest rate from the investment/saving (IS) equation by using the monetary policy rule. In this case, the specification of the monetary policy rule matters. In particular, the existence of an aggregate demand curve in the space of inflation and output growth relies on a contemporaneous response of the nominal interest rate to inflation. But since macroeconomic models are estimated at a quarterly frequency and inflation data are typically available at a monthly frequency, the assumption that monetary policy responds contemporaneously to inflation seems reasonable. So, provided monetary policy responds with the interest rate to current inflation and the interest rate enters contemporaneously into the IS equation, larger models could also be represented graphically. The difference would then lie on the 'shift factors' of aggregate demand and aggregate supply, what we labelled  $d_t$ and  $\hat{s}_t$ . At the zero lower bound, the aggregate demand curve would become vertical as we discussed earlier and the model may be solved and estimated following the methods outlined in Cagliarini and Kulish (2013), Kulish et al. (2014) and Jones (2015).

# **Disclosure statement**

No potential conflict of interest was reported by the authors.

#### References

- Alvarez-Lois, P., R. Harrison, L. Piscitelli, and A. Scott. 2008. "On the Application and Use of DSGE Models." Journal of Economic Dynamics and Control 32: 2428-2452. doi:10.1016/j.jedc.2007.09.003.
- Benigno P.2009. " New Keynesian Economics: An AS-AD View." Working Paper 14824, National Bureau of Economic Research.
- Jones C. 2015. "Unanticipated Shocks and Forward Guidance at the Zero Lower Bound." Mimeograph, New York University.
- Cagliarini A. and M. Kulish. 2013. "Solving Linear Rational Expectations Models with Predictable Structural Changes." Review of Economics and Statistics 95: 328-336.
- Carlin, W. and D. Soskice. 2005. "The 3-Equation New Graphical Exposition." Keynesian Model – A Contributions to Macroeconomics 5 (1): Article 13.
- Ireland, P. N. 2004. "Technology Shocks in the New Keynesian Model." The Review of Economics and Statistics 86 (4): 923-936. doi:10.1162/0034653043125158.
- Ireland, P. N.. 2011. "A New Keynesian Perspective on the Great Recession." Journal of Money, Credit and Banking 43 (1): 31-54. doi:10.1111/jmcb.2011.43.issue-1.
- King, R. G. 2000. "The New IS-LM Model: Language, Logic, and Limits." Federal Reserve Bank of Richmond Economic Quarterly 86 (3): 45-103.
- Mankiw, G. 2009. "A Dynamic Model of Aggregate Demand and Aggregate Supply." chapter 14 in Macroeconomics. Worth Publishers.
- Kulish, M., J. Morley, and T. Robinson. 2014. "Estimating DSGE Models with Forward Guidance." Working Paper 32A, UNSW, December.
- Smets, F. and R. Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." American Economic Review 97: 586-606.