International Spillovers of Forward Guidance Shocks*

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Abstract

We estimate a two-country model of the US and Canada over the post 2009 sample to study the cross-country spillovers of forward guidance shocks. To do so, we propose a method to identify forward guidance shocks during the fixed interest rate regime. The estimated forward guidance shocks coincide with significant US monetary policy announcements such as the introduction of calendar based guidance in 2011. While a 2 quarter expansionary forward guidance shock decreases Canadian output by about 0.2% to 0.4% on impact, we find that the US and Canada were both better off by responding with expansionary monetary policy to the large contractionary shocks that took place during the Great Recession. The central message of our paper is that the focus on whether monetary policy spillovers are expansionary or contractionary is incomplete. What matters is whether the combined monetary policy response is stabilizing in aggregate.

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1 Introduction

In the aftermath of the 2008 financial crisis, the Federal Reserve cut its policy interest rate to near zero and turned to unconventional monetary policies, including forward guidance. The Covid crisis has again pushed policy interest rates in the US and many other economies to their lower bound. With policy rates expected to remain fixed for some time, forward guidance about the future path of interest rates is likely to be an important tool of monetary policy. It is therefore critical for policymakers in open economies to understand how forward guidance shocks propagate across borders and the effects of their own forward guidance.

While it is widely acknowledged that the Federal Reserve's unprecedented post-2009 monetary actions supported economic growth in the US, there is less agreement about their effect on other countries. Expansionary US monetary policy that depreciates the U.S. dollar has an expenditure-switching effect, dampening demand for the output of small open economies.[1] But stronger economic conditions in the US support global demand and the exports of other economies.

In this paper, we use an open economy model to quantify the international spillovers of forward guidance by the Federal Reserve after 2009. Our model, like other open economy models, has both expenditure-augmenting and expenditure-switching forces. These competing forces imply that the net effect of international monetary policy on domestic output is ambiguous.[2] Furthermore, the domestic monetary authority can react to foreign forward guidance with stabilizing forward guidance of its own; a point emphasized by Ammer et al. (2016). It is thus an empirical issue to determine the net effect that monetary policy abroad has at home.

Our paper is related to three strands of the literature which tackle the empirical issue of monetary policy spillovers. First, a large number of papers have investigated the international output spillovers of conventional US monetary policy. Typically, these papers work with structural VAR models, often estimated on a pre-2009 sample (see Dedola et al. (2017), Kim (2001) and the references therein). Second, there is a literature estimating the spillovers of US unconventional monetary policy on financial market variables, most often using VARs and event

[1] See, for example, Rajan (2015).

[2] See Bodenstein et al. (2017), Fujiwara et al. (2013), and Haberis and Lipinska (2012) who make this point theoretically.

studies (see Miranda-Agrippino et al. (2020), Alpanda and Kabaca (2020), Neely et al. (2010), and Moessner (2015)).[3] Third, a smaller literature has explored the macroeconomic spillovers of US unconventional monetary policy using structural models. Examples include Kolasa and Wesołowski (2020) who find that quantitative easing overseas reduces an economy's international competitiveness and depresses domestic output in the context of a calibrated model, Cook and Devereux (2016) who use a stylized two-country New Keynesian model to assess the exchange rate regime under the zero lower bound, or Hernández (2017) who estimates a DSGE model of the Mexican economy to study the possible consequences of US monetary policy normalization, but does so without accounting for the period of fixed interest rates in estimation.

But no studies have estimated the impact of US forward guidance shocks on overseas macroeconomic variables with a structural forward-looking open economy model that is estimated over periods of fixed interest rates. Working with a structural model is important for two reasons. First, as discussed by Chari et al. (2004), Fernández-Villaverde et al. (2007) and Ravenna (2007), the identification of monetary policy shocks in VARs can be problematic. Second, the data samples of many economies now include a substantial period in which policy interest rates have been fixed. Failing to account for the period of fixed interest rates with linear VARs or DSGEs that ignore fixed interest rate regimes will deliver biased estimates of the underlying policy rule parameters and shocks.[4]

At the heart of our study is the need to measure *forward guidance shocks*. A contribution of our paper is to propose a new way to identify these shocks when the policy interest rate is fixed. To do so, we first apply the method outlined in Kulish, Morley and Robinson (2017) to estimate durations of the fixed interest rate regime and structural shocks using aggregate data, yield curve data, and survey measures of the expected duration of the fixed interest rate regime. This

[3]See Anaya et al. (2017), Chen et al. (2016) and Tillmann (2016) for evidence on the effects of US quantitative easing policies that relies on vector autoregressions, and Chen et al. (2014) for regression evidence using an event-study type identification for monetary policy shocks.

[4]To address this limitation, Inoue and Rossi (2019) use a functional VAR approach to assess the effect of monetary policy. Their focus, however, is on exchange rates during conventional and unconventional times. Our focus is on the broader economic implications of forward guidance. Inoue and Rossi (2019) find that a monetary policy tightening (easing) leads to an appreciation (depreciation) of the US dollar, consistent with our findings. During the unconventional period, however, they do not find a statistically significant impact on the Canadian exchange rate following an expansionary US policy shock. This result, however, is at odds with Ferrari et al. (2017) and Glick and Leduc (2018) who find that the exchange rates respond more to unconventional monetary policy shocks than conventional shocks.

gives us the actual expected duration of the fixed interest rate regime in each quarter. Then, we use the solution for models with occasionally binding constraints of Guerrieri and Iacoviello (2015) and Jones (2017) to compute in each quarter the duration of fixed interest rate policy that would be implied by a strict application of the underlying policy rule. This is a non-trivial extension of the existing literature as there are no forward guidance shocks in Kulish, Morley and Robinson (2017), Guerrieri and Iacoviello (2015), or Jones (2017).

In our setup, the duration of the fixed interest rate regime has two dimensions: (i) the actual duration communicated by the central bank and expected by households, firms and financial markets, and (ii) the duration prescribed by the underlying policy rule that applies away from the fixed interest rate regime. A forward guidance shock takes place if unanticipated changes in (i) cannot be accounted for by unanticipated changes in (ii). Identification is achieved because unanticipated changes in the fixed interest rate duration brought about by structural shocks have different implications for observable variables than changes in the duration caused by forward guidance shocks. For example, output and inflation fall after a contractionary shock that makes the constraint bind for longer. But output and inflation increase after a policy announcement that extends the fixed interest rate duration beyond what the constraint already implied.[5] In Section 4, we discuss the identification of forward guidance shocks in detail.

We apply the method to an estimated model of the US and Canada and focus on the period after 2009. We choose Canada for two reasons. First, Canada is closely linked financially and through trade in goods and services with the US – around three quarters of Canada's merchandise exports go to the US, accounting for close to 20 per cent of its GDP. Second, the Bank of Canada closely followed US monetary policy after the crisis and responded with calendar-based forward guidance of its own in 2009 (see Murray, 2013). This provides an opportunity to quantify the spillovers when the small open economy is also in a fixed interest rate regime, a situation which is likely to become increasingly common.[6]

Our main quantitative findings are as follows. The average size of a US forward guidance shock

[5]We use 2-year yields in our estimation. We show, however, that our results are broadly similar in an estimation without long-rates. This suggests that our estimates of forward guidance are largely identified off survey measures of the duration and their implications for other macroeconomic aggregates like output and inflation.

[6]Following the classification of Campbell et al. (2012), our focus is on calendar-based forward guidance policies.

is around 2 quarters, with the degree of expansionary forward guidance strongest between 2011 and 2013. This spans the introduction of calendar-based forward guidance announcements and the contractionary forward guidance shocks in the second quarter of 2013 when the Federal Reserve announced its tapering of asset purchases. In terms of magnitudes, a 2 quarter expansionary US forward guidance shock lowers Canadian output by 0.3 to 0.4% after four quarters. It also triggers an appreciation of the Canadian dollar of 0.5 to 0.7% and reduces Canadian inflation by a bit under 0.07 percentage points. However, while the spillovers of US forward guidance for Canadian output were mildly contractionary, Canadian monetary policymakers were able to offset these effects through forward guidance of their own. In fact, we find that the combined forward guidance response supported output in the US and Canada. Without forward guidance, the average cumulative decline in output in the US would be 37%, and in Canada 29%, post 2009.

The central message of our paper is that the focus on whether monetary policy spillovers are expansionary or contractionary is incomplete. What matters is whether the combined monetary policy response is stabilizing in aggregate. We find that this is indeed the case for the US and Canada as the expansionary forward guidance response by both countries mitigated the large contractionary shocks that took place during the Great Recession. As in Ammer et al. (2016), monetary policy in small economies with flexible exchange rates can rely on a key equilibrating instrument: their own independent monetary policy.

The rest of the paper is structured as follows. Section 2 outlines the model and discusses the transmission of monetary policy shocks in a simplified version. Section 3 discusses the solution and estimation method and Section 4 discusses identification of forward guidance shocks. Section 5 presents estimation results while Section 6 shows the spillovers of forward guidance from the estimated model. Section 7 concludes.

2 Model

We conduct our analysis using a small open economy model along the lines of Galí and Monacelli (2005). The model features two economies: a large (foreign) economy and a small (domestic) economy. Economic developments in the large economy affect the small economy, but the reverse is not true. As in De Paoli (2009a), the model is the limiting case of a two-country model where the relative size of one of the economies goes to zero. We extend this otherwise standard framework in a number of dimensions, including: (i) imperfect exchange rate pass-through, (ii) wage stickiness, (iii) non-zero steady state inflation,[7] (iv) interest rates of longer maturities, and (v) habits in the utility function.

2.1 The Large Economy

Households The large economy contains a representative household composed of a continuum of workers, each specialized in a particular labor type, indexed by $j \in [0, 1]$. The household's intertemporal welfare function is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t^*} \left[\log(C_t^* - h^* C_{t-1}^*) - \frac{1}{1+\varphi} \int_0^1 (N_t^*(j))^{1+\varphi} \, \mathrm{d}j \right],$$

where $N_t^*(j)$ is labor supply of type j and ξ_t^* denotes an intertemporal preference shock that follows an AR(1) process. C_t^* is a composite consumption index given by $C_t^* = \left[\int_0^1 C_t^*(i)^{\frac{\epsilon_p^*-1}{\epsilon_p^*}} \, \mathrm{d}i \right]^{\frac{\epsilon_p^*}{\epsilon_p^*-1}}$, where $\epsilon_p^* > 1$ is the elasticity of substitution between types of differentiated goods. Households face the flow budget constraint:

$$P_t^* C_t^* + \frac{B_{t+1}^*}{R_t^*} \leq B_t^* + \int_0^1 W_t^*(j) N_t^*(j) \, \mathrm{d}j + T_t^*,$$

where P_t^* is the large economy's CPI, $W_t^*(j)$ is the nominal wage rate of labor type j and T_t^* are lump sum taxes and transfers. B_{t+1}^* is a one period risk-free nominal bond and R_t^* is the interest rate on the bond.

Nominal Frictions There are nominal frictions on the setting of wages and prices in the large economy. The formulation of these frictions in our model is standard, and so a full exposition is provided in the Online Appendix.

[7] Our model has non-zero inflation but no price dispersion in steady state. For models of trend inflation see Ascari and Sbordone (2014).

To introduce frictions on wage setting, the labor variety from each union is aggregated into a labor input used in production. The wages of workers in each union are subject to Calvo-style frictions, such that a fraction $1-\theta_w^*$ are able to set wages optimally. A union that does not re-optimize its wage follows an indexation rule that links wages growth to a weighted average of lagged wage inflation (with weight χ_w^*) and steady-state wages growth.

Pricing frictions are introduced through Calvo-style restrictions on the setting of the prices of intermediate goods, which themselves are aggregated into the large economy's final good. Each quarter, a fraction $1-\theta^*$ of intermediate producers can reset their prices optimally. Firms that do not reset their prices index price growth to lagged CPI inflation (with weight χ_p^*) and the central bank's target inflation rate $\bar{\Pi}^*$.

Fiscal policy and market clearing The government's period budget constraint is:

$$P_t^* G_t^* + B_t^* \leq T_t^* + \frac{B_{t+1}^*}{R_t^*},$$

where G_t^* is real government consumption of goods and services, which evolves according to:

$$\frac{G_t^*}{Z_t} = \left(\bar{G}^*\right)^{\rho_g^*} \left(\frac{G_{t-1}^*}{Z_{t-1}}\right)^{1-\rho_g^*} \exp(\varepsilon_{G,t}^*),$$

where Z_t is labor augmenting productivity, which is common to both economies and evolves according to a random walk with drift at the rate $\mathcal{M}$. Goods market clearing in the large economy requires that:

$$Y_t^* = C_t^* + G_t^*.$$

Monetary Policy When the interest rate is not fixed, monetary policy follows an interest rate rule that responds to inflation, output growth and the deviation of the level of output from its trend:

$$\frac{R_t^*}{R^*} = \left[\frac{R_{t-1}^*}{R^*}\right]^{\rho_R^*} \left[\left(\frac{\Pi_t^*}{\bar{\Pi}^*}\right)^{\phi_\pi^*} \left(\frac{Y_t^*}{Z_t}\right)^{\phi_y^*}\right]^{1-\rho_R^*} \left(\frac{Y_t^*}{Y_{t-1}^* \mathcal{M}_t}\right)^{\phi_g^*} \exp(\varepsilon_{R,t}^*), \tag{1}$$

where R_t^* is the policy rate in the large economy, $R^* = \mathcal{M}\bar{\Pi}^*/\beta$ is the steady-state policy rate and $\varepsilon_{R,t}^*$ is a monetary policy shock. Between 2009Q1 and 2015Q4, the central bank fixes the level of the interest rate at $\bar{R}^*$, that is

$$\frac{R_t^*}{R^*} = \bar{R}^*, \tag{2}$$

where $\bar{R}^*$ is taken as the lower bound. For the US we set $\bar{R}^*$ to the mid point of the target range for the Federal Funds rate, to 0.125%. We define forward guidance as the difference between the number of periods the policy rate is expected to be fixed at $\bar{R}^*$ and the number of quarters for which the lower bound is expected to be a binding constraint, given the shocks and the state of the economy. The latter is the duration which would be implied by the shocks and the central bank adhering to a rule like $\max(\bar{R}^*, \text{Taylor rule})$. We refer to this duration as the *lower bound* duration. Forward guidance allows the central bank to alter the duration relative to what the constraint and policy rule would imply and therefore drives a wedge between the expected duration of the fixed policy rate regime and the lower bound duration. We discuss the lower bound and forward guidance durations in more detail below.

Longer-term interest rates are determined by the expectations hypothesis. We link longer-term interest rates in the model to observed longer-term interest rates following De Graeve et al. (2009). For any maturity $m > 1$:

$$R_{m,t}^{*,\text{obs}} = R_{m,t}^* \exp\left(c_m^* \eta_{m,t}^*\right), \tag{3}$$

where $R_{m,t}^*$ is the interest rate on a bond that pays one unit of the large economy's currency in m quarters as determined by the expectations hypothesis, c_m^* is a constant risk premia on the m quarter interest rate, and $\eta_{m,t}^*$ is AR(1) idiosyncratic shock to the m quarter interest rate in the large economy. Because the expectations hypothesis holds, longer-term nominal interest rates in the model, $R_{m,t}^*$, are also subject to the lower bound of R_t^*.

It is well known that the expectations hypothesis cannot account for the volatility of long rates. Shocks to $\eta_{m,t}^*$ in equation (3) soak up deviations from the expectations hypothesis. In our case the other structural shocks account for 80 per cent of the variance of the 2 year rate, so one may wonder to what extent our inferences are affected by relying on the expectations hypothesis. To assess this we estimate a version of the model removing long-rates from the set of observable variables. Our inferences on forward guidance shocks are similar, suggesting that market economist surveys and other macroeconomic aggregates also contain information about the expected duration of the fixed interest rate regime.[8]

2.2 The Small Economy

The structure of the small economy is similar to that of the large economy, except that households consume goods and services produced abroad and firms sell their output overseas as well as domestically.

Households The small economy is populated by a representative household that maximizes the expected present discounted value of lifetime utility, given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left[\log(C_t - hC_{t-1}) - \frac{1}{1+\varphi} \int_0^1 (N_t(j))^{1+\varphi} \, \mathrm{d}j \right],$$

where ξ_t is an intertemporal preference shock that follows an AR(1) process and C_t is a composite of domestically-produced and imported goods and services (defined below). Households face the budget constraint:

$$P_t C_t + \frac{B_{t+1}}{R_t} + \frac{S_t B_{t+1}^F}{R_t^F} \leq B_t + S_t B_t^F + \int_0^1 W_t(j) N_t(j) \, \mathrm{d}j + T_t. \tag{4}$$

Households in the small economy have access to two financial assets. These are domestic one-period risk free bonds, B_{t+1}, denominated in the domestic currency, and overseas one-period risk free bonds, B_{t+1}^F, denominated in the currency of the large economy. We assume that domestic bonds are not traded internationally and are in zero net supply. The variable S_t represents the small economy's nominal exchange rate, defined as the number of units of the small economy's

[8]See the Online Appendix for these additional results.

currency required to purchase one unit of the large economy's currency. The interest rate on overseas bonds depends on the large economy's interest rate and the net foreign asset position of the small economy:

$$R_t^F = R_t^* \exp\left[-\psi_H\left(\frac{S_t B_{t+1}^F}{P_t Y_t} - b_F\right) + \psi_t\right],$$

where the term ψ_t is a risk premium shock that follows an AR(1) process in logs and b_F is the steady state net foreign assets to GDP ratio.

Labor Unions As in the large economy, perfectly competitive labor aggregators combine the specialized labor types into a labor aggregate that they supply to firms. Individual labor types unionize to exploit their monopoly power subject to a Calvo-type friction that prevents them from re-optimizing their wage each period, with a fraction of unions $1-\theta_p$ that are able to set wages optimally. Unions that do not re-optimize index wages growth to a weighted average of lagged wage inflation (with weight χ_w) and steady-state wage inflation.

Domestic Intermediate Goods Producers Domestic intermediate goods producers manufacture heterogeneous goods using the technology:

$$Y_{H,t}(i) = Z_t L_t(i), \tag{5}$$

where $L_t(i)$ is firm i's labor input.[9] These varieties $Y_{H,t}(i)$ are aggregated into a domestically-produced final good by perfectly competitive retailers with a CES technology with elasticity of substitution ϵ_p.

As for the large economy, intermediate producers in the small economy face Calvo-style pricing frictions such that only a fraction $1-\theta_p$ of firms are able to adjust their prices freely each quarter. Firms that do *not* re-optimize index their prices to a weighted average of lagged inflation and the central bank's inflation target, $\bar{\Pi}$, with weights given by χ_p and $1-\chi_p$.

[9] As for the large economy, labor market clearing requires that $N_t = \int_0^1 L(i)\,\mathrm{d}i$.

Exporters Exporters purchase the domestic final good at price $P_{H,t}$ and differentiate it through branding for sale in the foreign economy. As in Burgess et al. (2013), and consistent with the prevalence of a dominant currency documented in Boz et al. (2018), all export contracts and prices are specified in the currency of the large economy. An export retailer bundles these goods with a CES aggregator and elasticity of substitution ϵ_x. The demand function that exporters face given by:

$$X_t = \alpha \left(\frac{P^*_{X,t}}{P^*_t} \right)^{-\tau} Y^*_t,$$

where $P^*_{X,t}$ is the price of the exported good in foreign currency terms, α is the degree of openness and τ is the price elasticity of export demand. As in Justiniano and Preston (2010b) the elasticity of export demand is the same as the elasticity of substitution between domestic and foreign goods in the domestic final goods basket (described below). This implies that household's preferences in the small and large economies are the same, which for the US and Canada is a reasonable assumption.

Exporters face Calvo-style pricing frictions, with only a fraction, $1 - \theta_x$, of firms able to adjust their prices each quarter.

Importers Importers bring in homogeneous products from abroad at price $S_t P^*_t$ and differentiate them through branding. Importers then sell the differentiated goods to a retailer that combines them into the final imported good using the technology:

$$Y_{F,t} = \left[\int_0^1 Y_{F,t}(i)^{\frac{\epsilon_f - 1}{\epsilon_f}} \, \mathrm{d}i \right]^{\frac{\epsilon_f}{\epsilon_f - 1}}, \tag{6}$$

where $Y_{F,t}$ is the total volume of imports, $Y_{F,t}(i)$ is the quantity of the imported good of variety i used in the production of the final imported good and ϵ_f is the elasticity of substitution between different varieties of imported goods. The price index for the imported final good and demand curve for individual varieties are:

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\epsilon_f} \, \mathrm{d}i \right]^{\frac{1}{1-\epsilon_f}}, \quad \text{and} \quad Y_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\epsilon_f} Y_{F,t}. \tag{7}$$

Importers face Calvo-style pricing frictions. Each quarter, a fraction, $1-\theta_f$ sets prices optimally, while the remainder adjusts their prices by the small economy's steady-state inflation rate, $\bar{\Pi}$.

Fiscal policy The government's period budget constraint is:

$$P_t G_t + B_t \leq T_t + \frac{B_{t+1}}{R_t},$$

where G_t is real government consumption of goods and services, which evolves according to:

$$\frac{G_t}{Z_t} = \left(\bar{G}\right)^{1-\rho_g} \left(\frac{G_{t-1}}{Z_{t-1}}\right)^{\rho_g} e^{\varepsilon_{G,t}}.$$

Domestic Final Demand and Market Clearing The final good consumed domestically is assembled by perfectly competitive retailers using the technology:

$$DFD_t = \left[(1-\alpha)^{\frac{1}{\tau}} \left(Y_{H,t}^H\right)^{\frac{\tau-1}{\tau}} + \alpha^{\frac{1}{\tau}} \left(Y_{F,t}\right)^{\frac{\tau-1}{\tau}}\right]^{\frac{\tau}{\tau-1}}, \tag{8}$$

where $Y_{H,t}^H$ is the domestic consumption of the domestically-produced final good and $Y_{F,t}$ is the volume of imports. The parameter τ is the elasticity of substitution between domestic- and foreign-produced goods. The price index corresponding to this bundle is:

$$P_t = \left[(1-\alpha)\left(P_{H,t}\right)^{1-\tau} + \alpha \left(P_{F,t}\right)^{1-\tau}\right]^{\frac{1}{1-\tau}}, \tag{9}$$

where $P_{H,t}$ is the price of the domestic composite good and $P_{F,t}$ is the price of the imported composite good, both expressed in domestic currency.

We assume that domestic households and the government have identical preferences for domestically-produced and imported goods. This implies that the household sector's demand for domestically-produced goods and services is $C_{H,t} = (1-\alpha)\Gamma_{H,t}^{-\tau} C_t$ and for imports is $C_{F,t} = \alpha \Gamma_{F,t}^{-\tau} C_t$, where $\Gamma_{H,t}$ and $\Gamma_{F,t}$ are the relative prices of domestically-produced and imported goods. Analogous expressions hold for the government's demand for domestically-produced and imported goods and services.

Goods market clearing requires that all domestic production is either consumed domestically by households, by the government, or exported:

$$Y_{H,t} = C_{H,t} + G_{H,t} + X_t. \tag{10}$$

Monetary Policy Before 2009Q2, when R_t is not fixed, the Canadian central bank follows the feedback rule:

$$\frac{R_t}{R} = \left[\frac{R_{t-1}}{R}\right]^{\rho_R} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_{H,t}}{Z_t}\right)^{\phi_y}\right]^{1-\rho_R} \left(\frac{Y_{H,t}}{Y_{H,t-1}\mathcal{M}_t}\right)^{\phi_g} exp(\varepsilon_{R,t}). \tag{11}$$

As for the US, we allow for an arbitrary level of the fixed nominal interest rate. Canada fixed its policy rate at 0.25% for a year and then at 1% for about 5 years in our sample so, from 2009Q2, policy follows

$$\frac{R_t}{R} = \bar{R}, \tag{12}$$

and

$$\frac{R_t}{R} = \bar{\bar{R}}, \tag{13}$$

from 2010Q3 until the peg is abandoned. In the case of Canada we allow for fixed interest rates at two different lower bounds consistent with their policy rate and communications over our sample. We model the change from $\bar{R}$ to $\bar{\bar{R}}$ as an unanticipated policy change in 2010Q3.

The term structure of interest rates in the small economy is determined in a similar manner to the large economy. For any $m > 1$:

$$R^{\text{obs}}_{m,t} = R_{m,t} \exp\left(c_m \varepsilon_{m,t}\right), \tag{14}$$

where $R_{m,t}$ is the interest rate on a bond that pays one unit of domestic currency in m quarters as determined by the expectations hypothesis, c_m is a constant risk premia on the m quarter interest rate, and $\varepsilon_{m,t}$ is an idiosyncratic shock to the m quarter interest rate in the small economy.

2.3 The Transmission of Foreign Monetary Policy

In the empirical implementation, we use the model that we describe above. But to understand the key forces that determine the strength of foreign monetary policy shocks in our model it is useful to consider the impact of foreign shocks in a framework with fewer frictions and i.i.d shocks, detailed in Appendix A.

To assess the response of domestic output, first assume that domestic monetary policy follows a rule of the form:

$$r_t = \phi_\pi \mathbb{E}_t \pi_{t+1},$$

where $\phi_\pi > 1$. This reaction function implies that the small economy's central bank stabilizes the *ex-ante* real interest rate. One can think of this exercise as illustrating the direct effects of a foreign monetary policy shock, holding the stance of domestic monetary policy fixed.

The monetary policy rule and Euler equation imply that the level of consumption in the small economy is stable at its steady state level. The UIP condition then ensures that the small economy's real exchange rate appreciates one-for-one with increases in foreign output. Under these conditions, we can express output in the small economy as a function of foreign output:

$$y_{H,t} = \left[\alpha - \frac{\tau\alpha(2-\alpha)}{1-\alpha}\right] y_t^*. \tag{15}$$

The expression in brackets summarizes the two forces that govern the response of domestic output. The first term, αy_t^*, is the income effect – the direct increase in exports from the small economy resulting from expansionary monetary policy abroad that increases demand in the large economy. This term is increasing in the openness of the domestic economy, reflected in the parameter α. The second term, $\frac{\tau\alpha(2-\alpha)}{1-\alpha} y_t^*$, is the substitution effect – the reduction in demand for goods produced in the small economy resulting from appreciation of the small economy's real exchange rate triggered by expansionary monetary policy abroad. The strength of this term is increasing in the openness of the domestic economy, as well as in the substitutability between domestic and foreign products, reflected in the parameter τ. In the case where the domestic central bank perfectly stabilizes the real interest rate, foreign monetary policy is beggar-thy-neighbor if $\tau > \frac{1-\alpha}{2-\alpha}$.

The mechanism described above also summarizes the effect of foreign forward guidance shocks on the domestic economy. With a constant *ex-ante* real interest rate, Equation (15) describes the relationship between domestic and foreign output in each period. An announcement about the future path of interest rates in the foreign economy triggers a sequence of foreign output realizations. These realizations will expand or contract domestic output over time according to the path of foreign output and the relative strength of the income and substitution effects.

If the domestic central bank follows a reaction function that responds to current realizations of domestic inflation:

$$r_t = \phi_\pi \pi_t,$$

then the real interest rate is no longer constant and the relationship between foreign and domestic output depends also on the flexibility of domestic prices, on the response of domestic marginal costs to demand conditions, and on the response of monetary policy to inflation, summarized by the following equation:[10]

$$y_{H,t} = a \left[\alpha + (1 - \alpha + \gamma)\phi_\pi b - \frac{\tau\alpha(2-\alpha)}{1-\alpha} \right] y_t^* \tag{16}$$

Although neat analytical expressions are not available in this case, Equation (16) reveals which additional parameters – in particular, the elasticity of labor supply, the slope of the Phillips curve and the strength of the monetary policy response to inflation – also matter in determining the response of domestic output. This expression shows that the domestic monetary policy response, ϕ_π, is an important determinant of the direction and magnitude of the response of domestic output to fluctuations in foreign output. The intuition is that in response to expansionary monetary policy abroad, a more aggressive domestic response to inflation offsets the contractionary impact of the exchange rate appreciation.

A lower wage elasticity of labor supply (a lower value of $1/\varphi$) implies that wages, and hence marginal costs, are more responsive to changes in labor demand. Hours worked, in contrast, are

[10] Where $a = \frac{1-\alpha+\phi_\pi(\kappa+\alpha)}{1-\alpha+\phi_\pi(\kappa+\alpha)-\kappa\varphi(1-\alpha)}$, $b = \frac{(1+\kappa)\alpha}{1-\alpha+\phi_\pi(\kappa+\alpha)}$ and $\gamma = \frac{\tau\alpha(2-\alpha)}{1-\alpha}$.

less responsive. Foreign monetary policy is less beggar-thy-neighbor when the elasticity of labor supply is low.

When prices are more flexible (a larger value of κ) a given change in marginal costs translates more into changes in prices and less into changes in quantities. In the case where expansionary foreign monetary policy triggers a fall in domestic output, more flexible prices dampen the expenditure switching channel. This is because the fall in labor demand and marginal costs translates into a larger reduction in the prices of domestically-produced goods. This cushions the fall in the consumption of domestic goods.

A central bank that responds to inflation will cut interest rates following an expansionary foreign monetary policy shock that lowers domestic production, wages and inflation. In doing so, the central bank reduces the interest rate differential between the domestic and foreign economies and diminishes the initial exchange rate appreciation of the domestic currency. This too will reduce the extent to which expansionary foreign monetary policy is beggar-thy-neighbor.

The mechanisms described above are also at work in our larger model. In estimation, we place priors over the parameters that govern the response of domestic output to foreign monetary policy disturbances, in particular the trade elasticity, τ, that are wide enough to capture both directions for domestic output. We now turn to how the full model is solved and estimated with periods of fixed interest rates.

3 Solution and Estimation

The model is solved and estimated following the approach in Kulish, Morley and Robinson (2017). Here, for completeness and to introduce the necessary notation for defining forward guidance shocks, we describe the solution in the case of a single fixed interest rate regime. Details, including how the method is extended to the two country case with two fixed interest rate regimes, are presented in the Online Appendix.

The system of linearized equations in which monetary policy follows a Taylor type rule is given by

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}\varepsilon_t, \tag{17}$$

where x_t is the state vector and ε_t is the vector of structural shocks, which we take to be i.i.d. without loss of generality. If it exists and is unique, the standard rational expectations solution to (17) is

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t. \tag{18}$$

During the fixed interest rate regime, however, the structural equations are given by

$$\bar{\mathbf{A}}x_t = \bar{\mathbf{C}} + \bar{\mathbf{B}}x_{t-1} + \bar{\mathbf{D}}\mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}\varepsilon_t, \tag{19}$$

where the only equation that changes is the one corresponding to the monetary policy rule which now fixes R_t at some level.[11] Assume that at $t = 1$ monetary policy fixes the interest rate and is expected to revert back to the Taylor rule in $\bar{T} + 1$. For periods, $t = 1, 2, \ldots, \bar{T}$, Kulish and Pagan (2017) show that the solution for x_t is a time-varying coefficient VAR of the form

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t\varepsilon_t, \tag{20}$$

where the reduced form matrices are determined recursively in equations that we present in Appendix B. We show that an expected duration of the fixed interest rate regime has associated with it a reduced form.

In estimation, we treat agents' expectations of the duration at time t, $\mathbf{d}_t$ as a time-varying parameter. To keep track of the reduced form that prevails at each point in the sample, we allow $\bar{T}$ to be an arbitrary large upper bound on the duration and re-label the sequence of reduced forms, $\{\mathbf{J}_d\}_{d=1}^{\bar{T}}$, $\{\mathbf{Q}_d\}_{d=1}^{\bar{T}}$ and $\{\mathbf{G}_d\}_{d=1}^{\bar{T}}$, so that the reduced-form on a given quarter t can be written as $\mathbf{J}_t = \mathbf{J}_{\mathbf{d}_t}$, $\mathbf{Q}_t = \mathbf{Q}_{\mathbf{d}_t}$ and $\mathbf{G}_t = \mathbf{G}_{\mathbf{d}_t}$. Adopting the convention that $\mathbf{d}_t = 0$ in periods where the Taylor rule is in operation $\mathbf{J}_0 = \mathbf{J}$, $\mathbf{Q}_0 = \mathbf{Q}$ and $\mathbf{G}_0 = \mathbf{G}$, we may write the reduced-form over the entire sample as in (20).

[11] Bianchi and Melosi (2017) capture the ZLB through changes in the policy rule coefficients using a Markov-switching specification. In the Markov-switching case the duration of the period of fixed interest rates is determined by the transition probabilities. In our case, the expected duration is estimated and can be different from the duration implied by shocks. This difference may capture changes in the central bank response during the zero lower bound or it can be thought to capture additional monetary stimulus analogous to a monetary policy shock in a Taylor rule. For more discussion comparing our methodology with the Markov-switching one see Kulish and Pagan (2017).

The likelihood, $\mathcal{L}(\vartheta, \mathbf{d}|Y^{\text{obs}})$ is a function of both the structural parameters, ϑ, and sequences of durations, $\mathbf{d} = \{\mathbf{d}_t\}_{t=1}^{T}$. We put priors over structural parameters and independent priors over durations to construct the posterior.[12] Details, including the Kalman filter and smoother can be found in the Online Appendix. Next, we turn to discuss how we use the estimated model and the occasionally binding constraint solution to identify forward guidance.

4 Identifying Forward Guidance

The duration of a fixed interest rate regime has two components: (i) the actual duration which is communicated by policymakers or is expected by market participants, and (ii) the duration prescribed by the underlying policy rule the central bank follows when the interest rate is not fixed. A forward guidance shock is an unanticipated change in (i) that cannot be accounted for by an unanticipated change in (ii). We elaborate on this below.

4.1 Actual Durations

We denote by $\mathbf{d}_t$ the *actual duration* of a fixed interest rate regime. This is the duration that is expected by agents in the model each period. In the absence of future shocks, $\mathbf{d}_t$ is expected to shrink by one period in every period. That is,

$$\mathbb{E}_t \mathbf{d}_{t+1} = \mathbf{d}_t - 1, \quad \text{for} \quad \mathbf{d}_t > 0. \tag{21}$$

In estimation we do not place any restrictions on the values of the actual durations $\{\mathbf{d}_t\}_{t=1}^{T}$.

4.2 Lower Bound Durations

With the occasionally binding constraint algorithm of Jones (2017) we find in each period t, the duration prescribed by the rule, i.e. the expected duration which is implied by the estimated

[12] See Kulish, Morley and Robinson (2017) for further details on how the sampler is set up in two blocks, one for the structural parameters which have continuous support and one for the durations which are integers.

state x_{t-1}, the estimated structural shocks ε_t, and a given lower bound.[13] This is the duration that corresponds to monetary policy following exactly $R = \max(\text{Lower Bound}, \text{Taylor Rule})$. We denote this expected duration at t by $\mathbf{d}_t^{\text{lb}}$ and refer to it as the *lower bound* duration. In the absence of future shocks, the lower bound duration is also expected to fall by one period in every period as the effect of current shocks ε_t unwind. In other words,

$$\mathbb{E}_t \mathbf{d}_{t+1}^{\text{lb}} = \mathbf{d}_t^{\text{lb}} - 1, \quad \text{for} \quad \mathbf{d}_t^{\text{lb}} > 0. \tag{22}$$

Shocks that unfold in $t+1$ may make the constraint bind for a longer or a shorter period of time, and hence the lower bound duration may expand or contract in period $t+1$. With the estimated structural shocks and state variables in hand obtained using the actual durations, we compute the lower bound durations that prevail in each quarter of the fixed interest rate regime, that is $\{\mathbf{d}_t^{\text{lb}}\}_{t=1}^{T}$.

Our definition of the lower bound duration implies that these durations must be interpreted relative to the interest rate peg chosen by the central bank. Intuitively, the higher the peg the tighter the constraint and the longer the lower bound duration for given shocks.[14]

4.3 Forward Guidance Shocks

For each period t of the fixed interest rate, we define the *forward guidance* duration, $\mathbf{d}_t^{\text{fg}}$, as the difference between the actual duration, $\mathbf{d}_t$ and the lower bound duration, $\mathbf{d}_t^{\text{lb}}$, that is

$$\mathbf{d}_t^{\text{fg}} \equiv \mathbf{d}_t - \mathbf{d}_t^{\text{lb}}. \tag{23}$$

The forward guidance duration captures announcements or other factors that can change the duration beyond what the structural shocks and the constraint imply, that is beyond $\mathbf{d}_t^{\text{lb}}$. Other factors beyond explicit central bank communication can influence the duration. As is well known,

[13] Jones (2017) shows that the occasionally binding constraint solution constitutes a good approximation to the non-linear economy. Details of the occasionally binding constraint solution are given in full in the Online Appendix.

[14] In the Online Appendix we provide an example that shows how the level of the fixed interest rate affects the lower bound duration.

markets reassessed the time of lift-off as the Federal Reserve was tapering the rate of bond purchases.

The decomposition in equation (23) of the actual duration characterizes monetary policy when the policy rate is fixed and thus the level of the nominal interest rate itself is an insufficient statistic of the stance of monetary policy. This is desirable if the peg is intended to continue after the bound is no longer binding (i.e., the policy rate is held "lower for longer") or if the peg is implemented at a level different to that of the effective lower bound as is the case in Canada.[15] Thus, the estimation allows for the possibility that central banks adopted the optimal policy prescription of Eggertsson and Woodford (2003) by extending the duration of the fixed interest rate policy beyond the horizon implied by the lower bound constraint itself.

Forward guidance shocks are unexpected changes of the forward guidance duration, that is

$$\varepsilon_t^{\mathrm{fg}} = \mathbf{d}_t^{\mathrm{fg}} - \mathbb{E}_{t-1}\mathbf{d}_t^{\mathrm{fg}}. \tag{24}$$

The expectation of the forward guidance duration, $\mathbb{E}_{t-1}\mathbf{d}_t^{\mathrm{fg}}$, however, depends on whether the lower bound is binding. Equations (21) and (22) imply that

$$\mathbb{E}_{t-1}\mathbf{d}_t^{\mathrm{fg}} = \begin{cases} \mathbf{d}_{t-1}^{\mathrm{fg}} - 1, & \text{if } \mathbf{d}_t^{\mathrm{lb}} = 0 \\ \mathbf{d}_{t-1}^{\mathrm{fg}}, & \text{otherwise .} \end{cases} \tag{25}$$

If the lower bound binds, so that $\mathbf{d}_t^{\mathrm{lb}} > 0$, the forward guidance shock is the change in the forward guidance duration, that is $\Delta\mathbf{d}_t^{\mathrm{fg}}$. But when the lower bound constraint does not bind, $\mathbf{d}_t^{\mathrm{lb}} = 0$, so that $\mathbf{d}_t = \mathbf{d}_t^{\mathrm{fg}}$, the forward guidance duration is expected to fall by one period in every period as dictated by Equation (21). Therefore, the expected one quarter reduction in $\mathbf{d}_t^{\mathrm{fg}}$ is not a forward guidance shock but a continuation of policy. Forward guidance shocks in

[15] For each country we take the level of the fixed interest rate as a lower bound constraint. In decomposing the duration we treat the level of the peg as if it were a *lower bound* in the occasionally binding constraint calculations. The level of the peg need not be the *effective lower bound.* This is useful when the peg is not the effective lower bound as is the case in Canada for part of the sample. Say the peg is at 1% but the effective lower bound is 0%. Assume also that the lower bound duration implied by the shocks and max(1%, Taylor Rule) is 4 quarters but the actual duration is 3 quarters. When the rule takes over in 3 quarters, the policy rate is expected to fall because 1% was expected to be binding for 4 quarters. Agents in the model can therefore forecast states of the world in which policy rates are below the current level of the peg.

this case are defined as increases or decreases in the expected duration of the fixed interest rate policy beyond this expected one quarter change. Formally, the *forward guidance shock* in period t is given by:

$$\varepsilon_t^{\text{fg}} = \begin{cases} \mathbf{d}_t^{\text{fg}} - \mathbf{d}_{t-1}^{\text{fg}}, & \text{if } \mathbf{d}_t^{\text{lb}} \geq 1 \\ \mathbf{d}_t^{\text{fg}} - \mathbf{d}_{t-1}^{\text{fg}} + 1, & \text{if } \mathbf{d}_t^{\text{lb}} = 0. \end{cases} \tag{26}$$

Figure 1 illustrates a forward guidance shock at t with an example. At period $t-1$ the actual duration, $\mathbf{d}_{t-1}$ is 5 quarters: the interest rate is expected to be fixed from period $t-1$ to period $t+3$, with lift-off in $t+4$. In the absence of structural and forward guidance shocks after $t-1$, in the forecast horizon, the duration falls by one quarter every quarter. In the middle panel of Figure 1, $\mathbb{E}_{t-1}\{\mathbf{d}_{t-1}, \mathbf{d}_t, ..., \mathbf{d}_{t+3}\} = \{5, 4, 3, 2, 1\}$ and its associated interest rate path is labeled the "*forecast path at* $t-1$" in the top panel.[16]

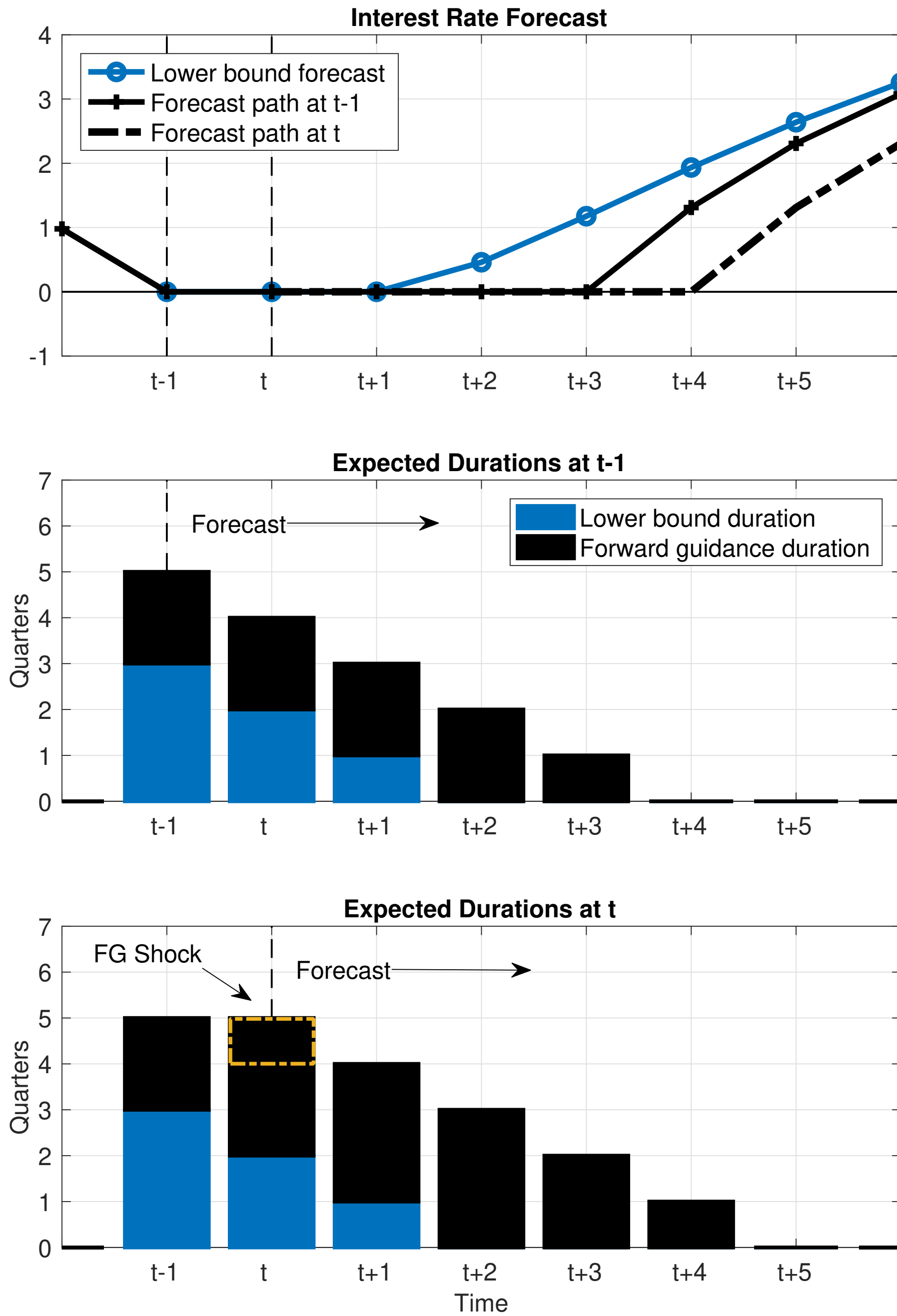
In period $t-1$, the structural shocks, ε_{t-1}, imply an evolution of the lower bound durations. In this case, $\mathbb{E}_{t-1}\{\mathbf{d}_{t-1}^{\text{lb}}, \mathbf{d}_t^{\text{lb}}, \ldots, \mathbf{d}_{t+3}^{\text{lb}}\} = \{3, 2, 1, 0, 0\}$. The associated path for the interest rate is shown in the top panel of Figure 1 and is labeled the "*lower bound forecast*". These lower bound durations are represented by the blue bars in the middle panel of Figure 1. The difference between the fixed interest rate duration and the lower bound duration, illustrated by the black bars in the middle panel of Figure 1, is the forward guidance duration. At $t-1$, the forward guidance duration is expected to evolve as follows: $\mathbb{E}_{t-1}\{\mathbf{d}_{t-1}^{\text{fg}}, \mathbf{d}_t^{\text{fg}}, \ldots, \mathbf{d}_{t+3}^{\text{fg}}\} = \{2, 2, 2, 2, 1\}$.

The bottom panel of Figure 1 illustrates a one-quarter expansionary forward guidance shock in period t. There are no structural shocks after $t-1$. Following the forward guidance shock at t, the lower bound and the forward guidance durations are expected to evolve in the forecast at t as shown by the bottom panel. At $t-1$, the actual duration was expected to fall to 4 quarters by t, that is $\mathbb{E}_{t-1}\mathbf{d}_t = 4$. However, a one quarter forward guidance shock increases $\mathbf{d}_t$ to 5 quarters.

We achieve identification because changes in the actual duration due to the endogenous response of policy ($\mathbf{d}_t^{\text{lb}}$) have a different effect on macroeconomic variables than exogenous

[16] Our forward guidance shock is specified in terms of a duration and it is therefore measured in quarters but as Figure 1 shows it has implications for the forecast of the policy rate, long rates, and other macro variables as well. To compare forward guidance shocks to conventional monetary policy shocks we normalize by matching the impact response of the 2 year rate in both cases. Our definition of forward guidance is complementary to the one used by Del Negro et al. (2015) who consider changes in the forecast of a variable at various horizons h caused by a forward guidance announcement.

Figure 1: Example of a Forward Guidance Shock at t



Note: This figure shows a stylized example of a forward guidance shock at period t. At period $t-1$, the expected duration of the fixed interest rate from period t is 4 quarters. Assume at period t that no further structural shocks arrive but policy unexpectedly commits to holding the interest rate fixed for an additional quarter. The total duration of the fixed interest rate regime increases to 5 quarters in period t, with the increase due to the positive forward guidance shock.

changes in the policy rate ($\mathbf{d}_t^{\mathrm{fg}}$). Figure 2 shows a simulation with the large economy to illustrate the impact on observables of a given increase in duration. The figure shows paths for output growth, inflation and the interest rate in two cases: the first is a negative preference shock that depresses output growth and inflation and in doing so makes the constraint bind for 4 quarters (i.e. it increases $\mathbf{d}_t^{\mathrm{lb}}$), while the second is a credible announcement by the central bank to keep rates at zero for 4 quarters (i.e. it increases $\mathbf{d}_t^{\mathrm{fg}}$). Thus, we compare a negative shock that makes the constraint bind for 4 quarters to a 4 quarter forward guidance shock. In both cases we start from $\mathbf{d}_{t-1} = 0$, implying $\mathbb{E}_{t-1}\mathbf{d}_t = 0$ so the unexpected change in the duration is the same in both cases.

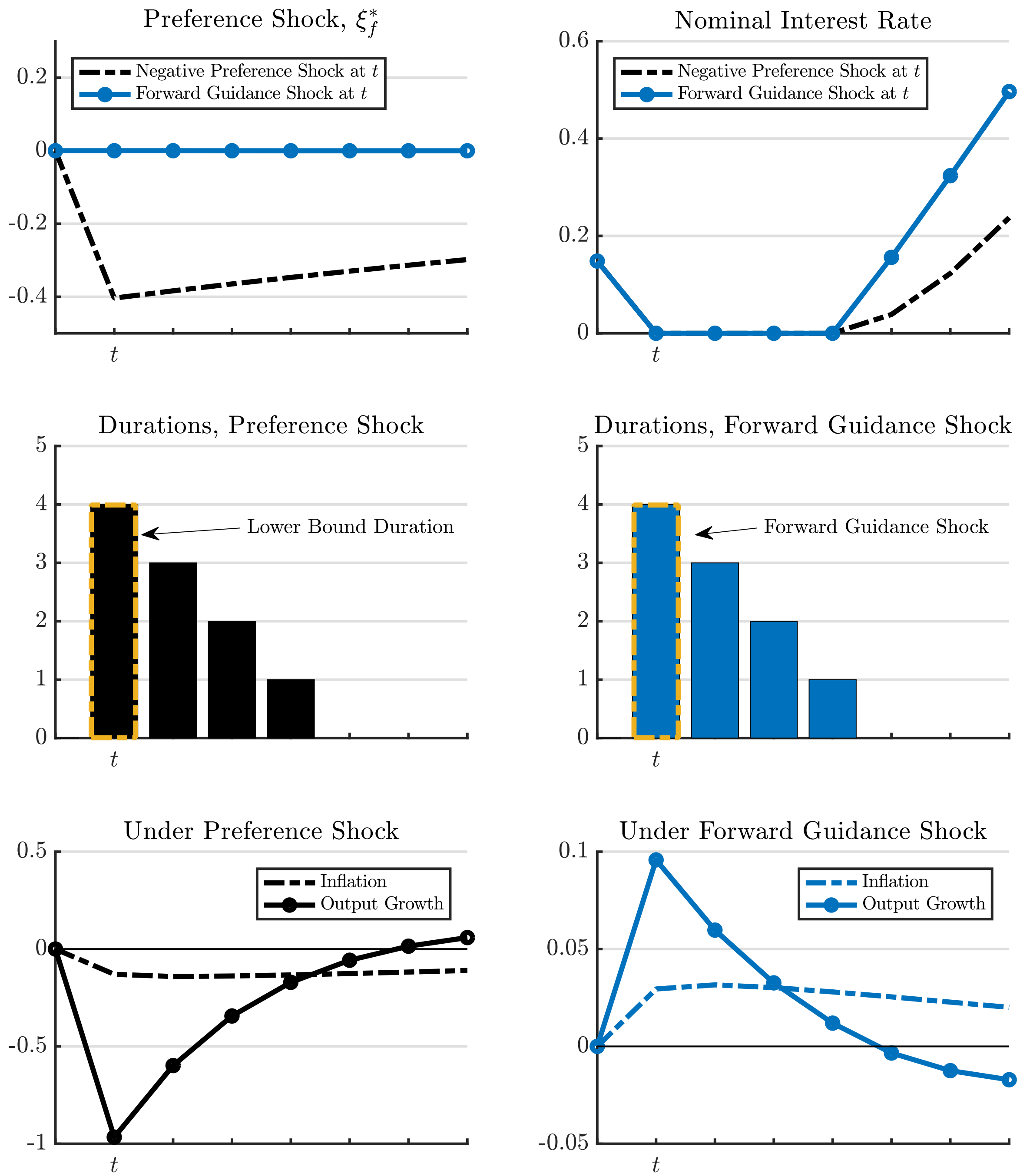
Output and inflation fall following the negative preference shock, but rise following expansionary forward guidance. Note also that at the time of lift-off the policy rate increases faster following the forward guidance shock because the monetary authority is faced with a stronger economy with higher inflation and output growth. The different expected paths for the policy rate have, in turn, implications for longer maturity rates which, though not shown in Figure 2, are used as observable variables in estimation.

5 Estimation

5.1 Data

For estimation, we use US and Canadian nominal interest rates, output growth, consumption growth, inflation, nominal wage inflation, Canadian import and export volumes growth, and changes in the US/Canadian nominal exchange rate. We also use 2-year nominal interest rates for both the US and Canada to help identify the fixed interest rate durations. The series start in 1991Q1 to coincide with start of inflation targeting in Canada and end in 2019Q2. All details of the data are in the Online Appendix.

Figure 2: Identification of Forward Guidance Shocks



Note: This figure illustrates how forward guidance shocks are identified. Two paths are constructed using the estimated model. In the path in black, the economy is hit with a negative preference shock at period t which causes inflation and output growth to fall and the policy rate to be fixed at its lower bound for four quarters before liftoff. In the blue path, the economy is subject to a forward guidance shock of four quarters which causes inflation and output growth to increase. In both cases, the interest rate is expected to be fixed for four quarters, and aggregate data is used to identify forward guidance shocks from structural shocks that cause the lower bound to bind.

5.2 Assigned and Calibrated Parameters

We calibrate the following structural parameters. We set the inflation target in both economies to 2% in annualized terms. Trend per capita growth is calibrated to 0.43% in annualized terms, which corresponds to the mean across the two economies for the pre-2009 subsample. Given trend growth and the inflation target, we set the quarterly discount factor β, common to both countries, to 0.9985 to get a steady nominal interest rate of 4.3% in annualized terms. This is the average of nominal interest rates for the two economies for the pre-2009 subsample, 4.1% for the US and 4.5% for Canada. The inverse of the Frisch elasticity of labor supply, ψ, is calibrated to 2 in both countries, close to the values estimated in Smets and Wouters (2007) and Justiniano and Preston (2010b).[17] The import share of the Canadian consumption basket, α, is set to 0.25 to match the average share of exports and imports to Canadian GDP, while the steady state values of $\bar{G}^*$ and $\bar{G}$ are set to ensure that we match the average share of consumption to GDP in the US and Canada, equal to 65% and 57.5% respectively.

5.3 Fixed Interest Rate Regimes

We measure expected durations in the US from the time at which the Federal Funds rate reached 0.125% in 2009Q1 until lift-off in 2015Q4. Similarly, we measure these durations in Canada when the policy rate was at 0.25% between 2009Q2 and 2010Q2, and then when the rate was at 1% between 2010Q3 and 2015Q1. This modeling choice is motivated by the clear communication of the Bank of Canada providing forward guidance of its fixed interest rate policy at these values.[18]

[17] In preliminary attempts, we found that ψ was poorly identified. This is related to the fact that we do not use hours data. There is an hours gap between Canada and the US which opened up during our sample and for which the model is not well-suited to explain. We also found that estimating the parameters that pin down the steady state of the nominal interest rate led to less precise estimates of the durations. This is to be expected as the duration and steady state of the policy rate are key determinants of the expected path of the policy rate.

[18] As an example, in the May 2013 press release, the Bank of Canada said that '*With continued slack in the Canadian economy, the muted outlook for inflation, and the constructive evolution of imbalances in the household sector, the considerable monetary policy stimulus currently in place will likely remain appropriate for a period of time, after which some modest withdrawal will likely be required, consistent with achieving the 2 per cent inflation target.*'

5.4 Estimation Results

Structural Parameters The moments of the prior and posterior distribution for each parameter that we estimate are reported in Tables 1 and 2. We use the same prior distributions for parameters that are common to both countries. The prior of the trade elasticity, τ, is centered at 1, and allows for a wide range of possible outcomes for the sign and magnitude of spillovers of US monetary policy to Canada. Our priors for the slopes of the price and wage equations are centered around conventional values of the frequency of price changes of about once every four quarters, while our priors imply a reasonably small degree of wage indexation.

The monetary policy feedback rules respond to the growth rate of GDP and to the detrended level of output, given the Fed's emphasis on the subdued labor market outcomes over the 2009-2015 period. We use standard priors implying relatively stronger responses to inflation deviations. The estimates of the monetary policy rule will be important for estimating forward guidance durations, as it is the monetary policy regime that the economy reverts to following a period of time at a fixed interest rate which is important for governing expectations about the path of the interest rate, and therefore in determining the stimulatory effect of forward guidance. Finally, we use wide priors for the standard deviations of the autoregressive shocks for both the US and Canada.

Most of the estimated structural parameters are similar for the two economies. The estimated Calvo parameters imply a frequency of price and wage changes of once every four quarters, with insignificant differences between the frequency of changes for domestic goods, imports and exports. The degree of price and wage indexation in both economies is estimated to be small.

The posterior estimate of the trade elasticity, τ, is centered around 3.0. An estimate of above 1 is typical for estimated small open economy models, although Justiniano and Preston (2010b) report an estimate of 0.8 using Canadian data.[19] As this parameter exerts an important influence on the response of domestic output to foreign monetary policy disturbances, we describe in more detail how it is identified in estimation.

The trade elasticity, τ, influences how much domestic output responds to changes in relative

[19] For example Adolfson et al. (2013) report an estimate of 1.41 for Sweden.

Table 1: Estimated Structural Parameters

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
				US				
h^*	B	0.7	0.6	0.8	0.74	0.73	0.66	0.79
θ_p^*	B	0.7	0.7	0.8	0.76	0.76	0.73	0.79
θ_w^*	B	0.7	0.7	0.8	0.78	0.78	0.75	0.81
ρ_r^*	B	0.5	0.2	0.8	0.89	0.89	0.86	0.91
ϕ_π^*	N	2.0	1.7	2.3	1.76	1.75	1.49	2.04
ϕ_g^*	G	0.5	0.3	0.7	0.09	0.10	0.08	0.12
ϕ_y^*	G	0.5	0.3	0.7	0.08	0.09	0.07	0.11
c_8^*	N	0.3	0.1	0.8	0.08	0.10	0.05	0.15
χ_p^*	B	0.1	0.04	0.2	0.03	0.04	0.02	0.07
χ_w^*	B	0.1	0.04	0.2	0.08	0.11	0.05	0.18
				Canada				
h	B	0.7	0.6	0.8	0.81	0.81	0.78	0.85
τ	N	1.0	0.4	1.6	3.06	3.04	2.74	3.36
θ_p	B	0.7	0.7	0.8	0.79	0.79	0.76	0.82
θ_w	B	0.7	0.7	0.8	0.78	0.78	0.74	0.81
θ_x	B	0.7	0.7	0.8	0.78	0.78	0.75	0.80
θ_F	B	0.7	0.7	0.8	0.77	0.77	0.74	0.80
ρ_r	B	0.5	0.2	0.8	0.91	0.91	0.89	0.93
ϕ_π	N	2.0	1.7	2.3	2.20	2.17	1.88	2.46
ϕ_g	G	0.5	0.3	0.7	0.11	0.11	0.09	0.14
ϕ_y	G	0.5	0.3	0.7	0.17	0.17	0.13	0.24
c_8	N	0.3	0.1	0.9	0.12	0.12	0.06	0.18
χ_p	B	0.1	0.04	0.2	0.04	0.06	0.03	0.10
χ_w	B	0.1	0.04	0.2	0.07	0.08	0.03	0.14

Table 2: Estimated Parameters, Exogenous Processes

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
				US				
ρ^*_ξ	B	0.5	0.2	0.8	0.95	0.95	0.93	0.96
ρ^*_g	B	0.5	0.2	0.8	0.95	0.95	0.93	0.97
$\rho^*_{\xi_p}$	B	0.5	0.2	0.8	0.99	0.98	0.97	0.99
$\rho^*_{\xi_w}$	B	0.5	0.2	0.8	0.80	0.78	0.67	0.86
ρ^*_{tp}	B	0.5	0.2	0.8	0.73	0.73	0.64	0.82
$100 \times \sigma_z$	IG	0.3	0.1	2.6	0.11	0.11	0.09	0.14
$100 \times \sigma^*_r$	IG	0.3	0.1	0.6	0.11	0.11	0.10	0.12
$100 \times \sigma^*_\xi$	IG	0.3	0.1	0.7	0.25	0.28	0.21	0.41
$10 \times \sigma^*_g$	IG	0.3	0.1	0.7	0.13	0.13	0.12	0.14
$100 \times \sigma^*_{\xi_p}$	IG	0.1	0.1	0.3	0.15	0.15	0.13	0.17
$100 \times \sigma^*_{\xi_w}$	IG	0.3	0.1	0.9	0.11	0.11	0.08	0.13
$100 \times \sigma^*_{r,8}$	IG	0.3	0.1	0.9	0.09	0.09	0.09	0.10
				Canada				
ρ_{rp}	B	0.5	0.2	0.8	0.96	0.95	0.91	0.97
ρ_ξ	B	0.5	0.2	0.8	0.64	0.65	0.52	0.79
ρ_g	B	0.5	0.2	0.8	0.93	0.93	0.89	0.95
ρ_{ξ_H}	B	0.5	0.2	0.8	0.85	0.83	0.73	0.90
ρ_{ξ_w}	B	0.5	0.2	0.8	0.29	0.27	0.16	0.38
ρ_{ξ_X}	B	0.5	0.2	0.8	0.91	0.91	0.87	0.93
ρ_{ξ_F}	B	0.5	0.2	0.8	0.96	0.95	0.92	0.98
ρ_{tp}	B	0.5	0.2	0.8	0.74	0.72	0.62	0.82
$100 \times \sigma_r$	IG	0.3	0.1	0.7	0.17	0.17	0.15	0.19
$100 \times \sigma_{rp}$	IG	0.1	0.1	0.2	0.31	0.32	0.26	0.43
$10 \times \sigma_g$	IG	0.3	0.1	0.9	0.14	0.14	0.13	0.15
$100 \times \sigma_\xi$	IG	0.3	0.1	0.9	0.24	0.25	0.22	0.29
$100 \times \sigma_{\xi_H}$	IG	0.2	0.1	0.3	0.37	0.36	0.32	0.40
$100 \times \sigma_{\xi_w}$	IG	0.3	0.1	0.8	0.50	0.50	0.45	0.55
$100 \times \sigma_{\xi_X}$	IG	0.3	0.1	0.9	1.29	1.32	1.15	1.53
$100 \times \sigma_{\xi_F}$	IG	0.1	0.1	0.2	1.04	1.05	0.89	1.24
$100 \times \sigma_{r,8}$	IG	0.3	0.1	0.7	0.12	0.12	0.11	0.13

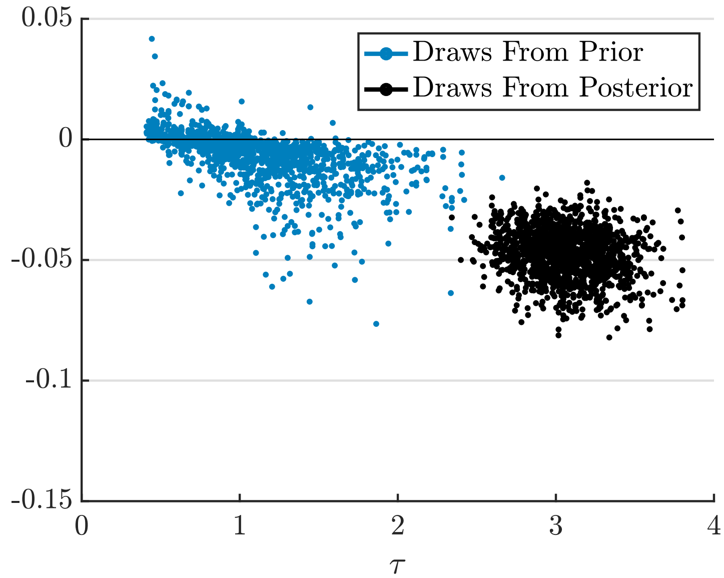
prices between the domestic and foreign economy. A lower trade elasticity makes domestic output less responsive to changes in relative prices. Less responsive output implies smaller changes in labor demand which in turn imply smaller changes in marginal cost and inflation. Through the policy rule this means the nominal interest rate is also less responsive. Following an expansionary monetary policy abroad, the interest rate differential between foreign and domestic interest rates would be larger the lower the trade elasticity. Through uncovered interest rate parity, this implies larger changes of the nominal exchange rate.

The estimation procedure chooses a value of τ that helps the model to jointly match the behavior of domestic output and the nominal exchange rate. While a lower value of τ would imply that expansionary monetary policy in the US is also expansionary for Canada, a lower value of τ reduces the variance of domestic output, exports and imports but increases the variance of the nominal exchange rate.

In Figure 3, we compare the responses *on impact* of Canadian output to an expansionary US monetary policy shock at different values of τ, the trade elasticity. We draw parameters from the prior and posterior distributions and solve the model for each draw. Figure 3 illustrates that our priors are wide enough to allow an expansionary US monetary policy shock to be expansionary or contractionary for Canadian output. For the entire posterior distribution, however, we find that the output responses are negative. The important quantitative role of the trade elasticity, τ, in determining the domestic output response is highlighted in Figure 4 which shows responses of Canadian variables to an expansionary US monetary policy shock for a high and low value of τ, with all other parameters set to the mode of the posterior distribution.

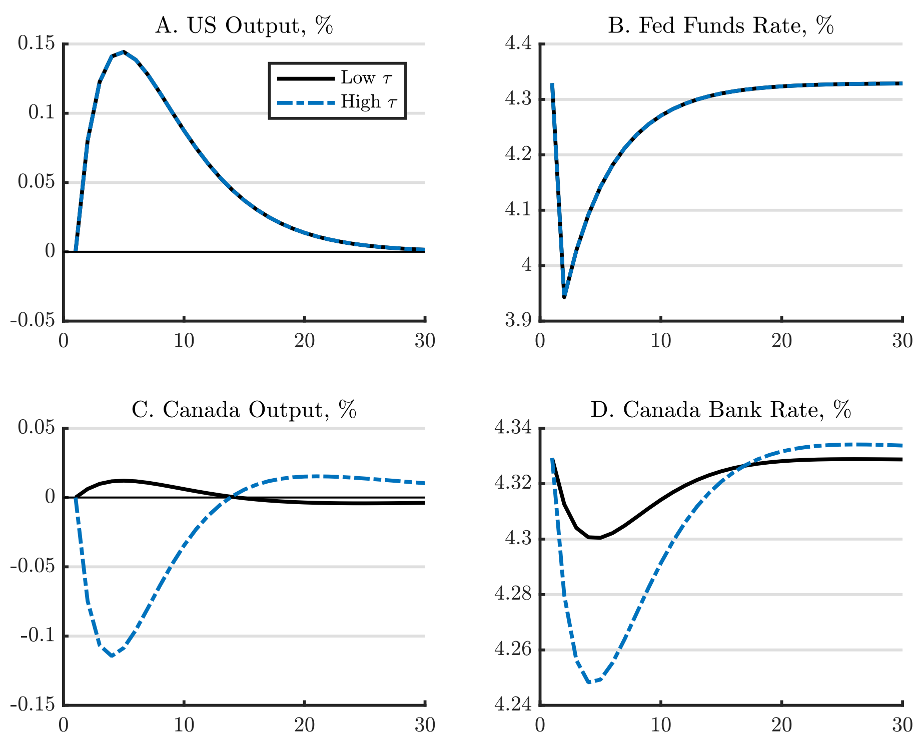
Turning to the estimates of the shock processes, relative to the priors, the data prefer relatively more persistent preference and markup shocks in the US, consistent with the relative differences in persistence found in Kulish and Rees (2011). For Canada, risk premium and import and export markup shocks are highly persistent, indicative of substantial deviations from UIP and incomplete exchange rate pass through. The estimation also points to larger markup shocks in Canada than the US, although the estimated standard deviations of preference and monetary policy shocks are similar. To understand the implications of our parameter estimates, we report

Figure 3: Canada Output Response to Expansionary US Monetary Policy Shock, %



Note: This figure shows, on the vertical axis, the initial response of Canadian output to a one standard-deviation expansionary US monetary policy shock for draws from the prior and posterior distributions. The horizontal axis plots the value of τ, the trade elasticity, for each draw.

Figure 4: Impulse Response to Expansionary US Monetary Policy Shock



Note: This figure shows the response of Canadian variables to an expansionary US monetary policy shock for a high and low value of τ, with all other parameters set to the mode of the posterior distribution.

in Table 3 the forecast error variance decomposition of both US and Canadian variables into the structural shocks of the model.[20] US preference shocks account for about 38% of the variation in detrended US output growth in the long run, and 64% of the variation in the Fed Funds rate. Consistent with the findings of Smets and Wouters (2007), US monetary policy shocks drive a small percentage of US output and inflation. US preference shocks account for about 2% of the forecast error of Canadian output growth, while US monetary policy shocks account for little of the long-run variation in Canadian output. Instead, most of the variation in Canadian output is driven by Canadian shocks, consistent with the findings in Justiniano and Preston (2010a).

As documented by Justiniano and Preston (2010a), open economy models like ours do not fully account for the reduced-form co-movement of macroeconomic variables across countries. However, it is worth noting that they measure co-movement using linearly detrended output, which has a correlation of 0.7, while we use output growth, whose correlation is somewhat lower at 0.5. In addition to these differences in measurement, Justiniano and Preston (2010a) focus on the unconditional correlation, while our focus is on monetary policy shocks which do not account for much of the unconditional variance of most macroeconomic variables. Our baseline model generates a correlation of around 0.2 between output growth across countries. As an additional check of the importance of co-movement in driving our results, we estimated a modification of our model suggested by Justiniano and Preston (2010a) to improve co-movement by allowing foreign shocks to be correlated with domestic shocks. This specification increases the estimated co-movement between Canadian and US output growth to 0.4. But in spite of this, our inferences about the spillovers of forward guidance do not change. The Online Appendix contains these additional results.

Expected Durations For the sequence of US expected durations, we use an informative prior as in Kulish, Morley and Robinson (2017), an average of the durations in the survey data from Blue Chip from 2009 to 2010, and from the NY Fed survey of primary dealers from 2011 to 2015 and a uniform distribution over all durations ranging from 1 to 24. We use an upper of 24

[20] The shocks to the 2Y yields capture only deviations of the 2Y interest rates from the expectations hypothesis, and so do not affect the model's other variables.

Table 3: Variance Decomposition Due to Shocks, %

	Common	US Shocks					Canadian Shocks							
Shock / Variable	Prod.	Pref.	Policy	Gov.	Price	Wage	Pref.	Policy	Gov.	Risk Pr.	Price	Wage	Exports	Imports
A. US Variables														
Policy Rate	0.1	64.1	14.6	4.4	8.9	7.9	-	-	-	-	-	-	-	-
2Y Interest Rate	0.1	65.3	4.5	2.8	8.3	7.2	-	-	-	-	-	-	-	-
Output Growth	1.4	37.9	3.3	52.9	2.1	2.4	-	-	-	-	-	-	-	-
Consumption Growth	0.8	78.1	6.8	5.1	4.3	4.9	-	-	-	-	-	-	-	-
Inflation	1.8	10.5	4.7	0.2	44.3	38.5	-	-	-	-	-	-	-	-
Wage Growth	17.6	0.1	0.4	0.0	66.1	15.8	-	-	-	-	-	-	-	-
B. Canadian Variables														
Policy Rate	0.0	4.8	0.5	0.4	0.3	0.5	2.5	10.1	0.8	58.9	2.0	1.1	8.0	10.2
2Y Interest Rate	0.0	4.9	0.3	0.2	0.2	0.6	1.7	2.4	0.6	59.0	1.0	0.8	7.6	10.0
Output Growth	0.5	2.2	0.7	2.0	0.2	0.2	10.1	8.8	8.2	25.7	12.4	2.1	26.7	0.3
Consumption Growth	0.3	2.5	0.1	0.1	0.1	0.3	49.8	2.7	2.0	33.8	1.9	0.6	4.1	1.9
Inflation	0.4	0.6	0.2	0.0	0.0	0.1	0.0	5.0	0.1	12.7	45.7	10.5	4.5	20.1
Wage Growth	2.4	0.1	0.1	0.0	0.0	0.0	0.1	0.1	0.0	4.0	32.4	48.1	4.7	8.0
Imports Growth	0.0	0.9	0.5	0.0	0.1	0.1	4.3	0.3	21.5	25.8	3.6	0.4	11.2	31.3
Exports Growth	0.0	2.1	0.1	1.4	0.2	0.1	0.1	1.0	0.0	14.0	0.4	0.1	64.5	16.0
Nominal Ex Rate, Δ	0.0	0.6	2.6	0.1	0.4	0.3	0.5	6.2	0.0	43.1	0.9	0.3	22.4	22.5

quarters as there is zero probability in the survey for higher durations. For Canada we use an uninformative prior as equivalent survey measures are not available. The posterior distributions for the durations of the US Fed Funds rate at 0.125% between 2009Q1 and 2015Q4, and the Bank of Canada's Bank Rate at 0.25% between 2009Q2 and 2010Q2 and 1% between 2010Q3 and 2015Q1 are in shown in Figure 5.

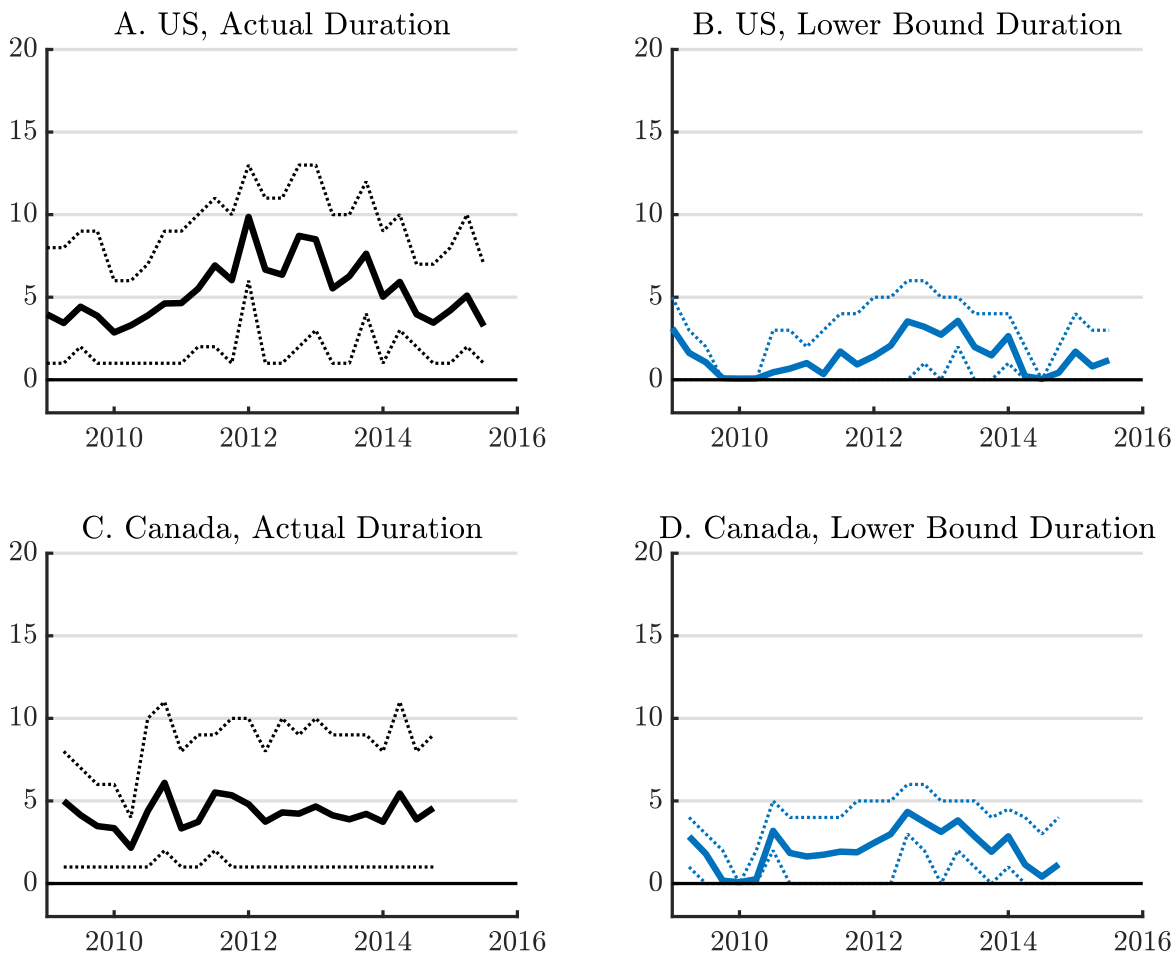
For the US, the central value of each posterior density lies between 3 and 10 quarters. The posterior densities noticeably shift towards longer durations over 2011, with the mode of the posteriors increasing from 4 quarters in 2011Q1 to 7 quarters in 2011Q3 and 10 quarters in 2012Q1, and staying around 8 quarters until the start of 2014Q1, after which the mode of the posteriors declines to around 2 quarters by 2014Q4. These values are consistent with the results of Swanson and Williams (2014), who find expected durations of around 7 quarters for each quarter from 2011, and aligns with the explicit adoption of forward guidance from 2011Q3, when the Federal Reserve announced that it would maintain the interest rate at 0.125% until mid-2013. Subsequently, the Fed repeatedly extended the explicit liftoff date back, which is consistent with our estimated posterior distributions remaining centered around 6 to 7 quarters.

For Canada, the estimated posterior distributions are centered around lower fixed interest rate durations, with mean durations of between 2 and 7 quarters over the estimation sample. The mass of the posterior distribution of the fixed interest rate regimes shrinks towards smaller durations for the first three quarters that the Canadian Bank Rate is fixed at 0.25% in 2009, which is consistent with the Bank of Canada raising its policy interest rate to 1% in third quarter of 2010, and keeping it fixed thereafter.

6 Spillovers of US Monetary Policy

We next use the estimated model to study the spillovers of US monetary policy shocks on Canada. We first use our solution methods to measure forward guidance shocks in each country. We then compare the size and magnitude of the spillovers of conventional US monetary policy shocks when interest rates in the US and Canada follow a standard reaction function, to the spillovers that arise in response to a typical US forward guidance shock. Next, we show that the extent of

Figure 5: Fixed Interest Rate Duration and Forward Guidance



Note: This figure plots the mean and 90 percent credible bands of the fixed interest rate durations in black, and the lower bound durations in blue.

these spillovers depends on the state of the US economy. We finally show that the joint monetary policy stimulus over the period of fixed interest rate regimes was on average positive for both the US and Canada by constructing a counterfactual in which forward guidance is removed from both countries.

6.1 Identified Forward Guidance in the US and Canada

We first report our estimated measure of forward guidance in the US and Canada. To measure forward guidance, we repeatedly draw from the posterior distribution of parameters and durations. For each draw, we use the observables and the Kalman smoother to estimate the model's structural shocks. We then follow the methods discussed in Section 4 and use the structural shocks to identify lower bound durations for each country.

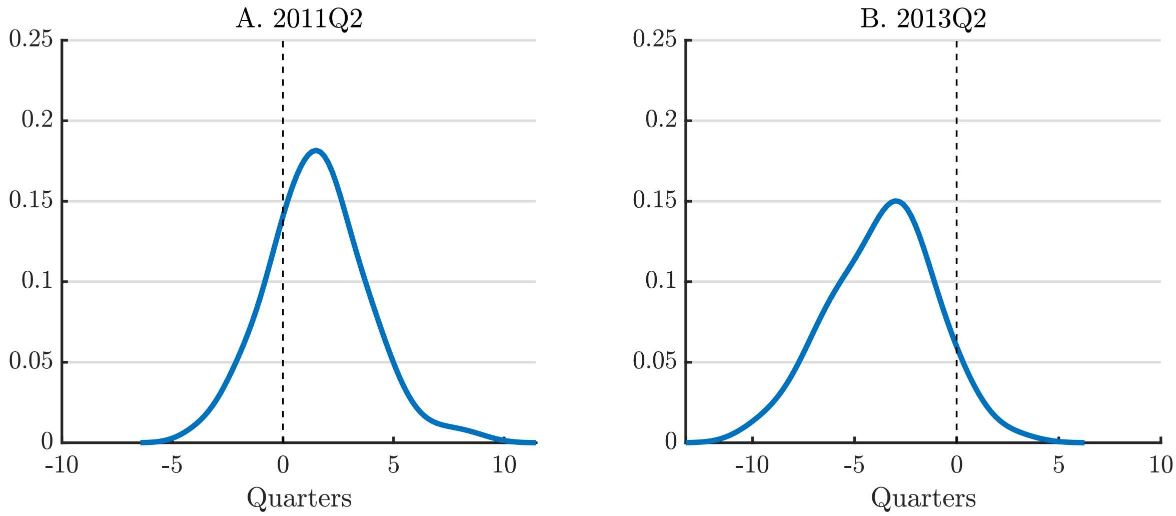
Figure 5 plots the mean of the set of draws of actual interest rate durations and lower bound durations for the US and Canada. For the US, the mean across durations is initially around 4 quarters to 6 quarters, from 2009 to early 2011, of which the mean across lower bound durations is roughly half the total actual duration, between 2 quarters and 3 quarters. We estimate therefore that US forward guidance was responsible for about half of the actual duration of fixed interest rate policy between 2009 and 2011, or between 2 quarters and 3 quarters in duration.

From early to mid-2011, to late 2013, we find that forward guidance shocks in the US expanded markedly. Following the Fed's explicit calendar-based commitment to holding the Fed Funds rate fixed from early to mid-2011, we estimate that the lower bound durations stayed roughly constant, between 2 quarters and 4 quarters, so that the forward guidance announcements were about 4 quarters to 5 quarters.

In Panel B of Figure 5, we plot the corresponding decomposition of the estimated total expected durations of fixed interest rate policy in Canada. These durations are for fixed interest rate policy at 0.25% between 2009Q2 and 2010Q1, and then at 1% from 2010Q2 to 2014Q4, the end of the sample. The estimated lower bound duration is roughly constant over the 2009Q2 to 2014Q4 period, mostly being between 2 quarters and 4 quarters in duration.

Next, we assess the external validity of the estimated US forward guidance shocks in two

Figure 6: Posterior Distributions of US Forward Guidance Shocks



Note: This figure plots the distribution of our estimated forward guidance shocks for two dates. In the left panel, forward guidance shocks are on average 2 quarters for 2011Q2, the period when the Federal Reserve announced explicit calendar-based forward guidance. In the right panel, the distribution of forward guidance shocks is on average -3 quarters for 2013Q2, the period of the 'taper tantrum' following the Federal Reserve's announced slowdown of the quantitative easing program.

ways. First, we compare the estimated forward guidance shocks to the historical record of FOMC announcements. We examine two salient quarters: 2011Q2 when the Federal Reserve introduced explicit calendar-based forward guidance in its statement and 2013Q2 when the Federal Reserve announced a reduction in the pace of government bond purchases, which was interpreted by markets as corresponding to an earlier lift-off, prompting the so called 'taper tantrum'.[21] Figure 6 plots the posterior distributions of the forward guidance shocks in the US for 2011Q2, in Panel A, and 2013Q2, in Panel B. For 2011Q2, the forward guidance shock is 4 quarters at the mean with the bulk of the posterior distribution on positive durations. For 2013Q2, however, when the Federal Reserve announced the tapering of its asset purchases, the forward guidance shock is -2 quarters at the mean with the bulk of the posterior on negative durations.

Out second exercise confirms our quantitative model-based results using an event study of how financial market pricing changed in the immediate aftermath of Federal Reserve announcements that conveyed information about the likely duration of the constant interest rate policy. For

[21] See http://www.federalreserve.gov/newsevents/press/monetary/20110809a.htm

this exercise, we use the series of US forward guidance shocks derived by Bundick and Smith (forthcoming). They use the change in futures contract pricing around FOMC board meetings to infer changes in forward guidance policy. Although we would not expect this series to align exactly with the estimated forward guidance shocks from our model, as our shocks represent the sum of all forward guidance announcements over an entire quarter, the two series have a correlation of 0.4, which suggests some overlap between them.

We estimate the effect of changes in US forward guidance on Canadian financial market pricing by estimating the model:

$$\Delta p_t = \alpha + \beta FG_t^{US} + \epsilon_t \tag{27}$$

where Δp_t is the change in an asset price between the end of date t and the end of date $t-1$ and FG_t^{US} is the US forward guidance shock on date t. We estimate the model between 2009Q1 and 2015Q4, representing the period of fixed interest rate policy in the US. As dependent variables we use the US 2-year bond yield, the 3-month, 1-year, 2-year 5-year and 10-year Canadian government bond yields, as well as the bilateral exchange rate between the Canadian and US dollars.

Table 4 shows the results of the exercise. A one standard deviation lengthening of forward guidance in the US (an increase in FG_t^{US}) leads to a decline in US 2-year government bond yields of 68 basis points. It also prompts a decrease in Canadian bond yields and an appreciation of the Canadian dollar. The effects are larger for longer-maturity interest rates, although this in part reflects the fact that, because the Bank of Canada maintained its own fixed interest rate policy, the short end of the Canadian yield curve showed almost no variability for much of the sample. These results are qualitatively consistent with our finding in the main text on expansionary forward guidance shocks in the US.

Comparing the event study and model-based results quantitatively is complicated because the units of the forward guidance shock in the model – quarters – differ from those in the event study. However, we can get some sense of the relative magnitudes by comparing the change in Canadian asset prices *relative* to the change in US bond yields in the two exercises. In the

Table 4: Response of Asset Prices to Expansionary US Forward Guidance Announcements

	US Government Bond Yields	Canadian Government Bond Yields					Exchange Rate
	2-year	3-month	1-year	2-year	5-year	10-year	
Change in Forward Guidance	-0.68** (0.05)	-0.06** (0.02)	-0.14** (0.04)	-0.33** (0.06)	-0.61** (0.08)	-0.60** (0.09)	-5.03* (2.22)
Regression R^2	0.86	0.13	0.23	0.43	0.63	0.57	0.18

Note: Coefficients β from regressions $\Delta y_t = \alpha + \beta FG_t^{US} + \varepsilon_t$. Huber-White heteroskedasticity-consistent standard errors in parentheses. ** and * denote statistical significance at the 1% and 5% levels. Decrease in the exchange rate represents an appreciation of the Canadian dollar. See text for details.

structural model, a one standard deviation expansion in US forward guidance prompts a 12 basis point decrease in US 2-year government bond yields, a 4 basis point decrease in Canadian 2-year yields and a 0.8 percentage point exchange rate appreciation. The event study results, after scaling the change in US forward guidance to deliver a 12 basis point decrease in US 2-year bond yields, imply a 6 basis point decrease in Canadian 2-year yields and a 0.9 percentage point exchange rate appreciation. Both estimates lie comfortably within the confidence intervals of the model's impulse response to a forward guidance shock plotted in Figure 7 (discussed below), indicating that the forward guidance shocks in our structural model successfully replicate the patterns of correlations between US and Canadian asset prices observed in the data.

6.2 Conventional and Unconventional Policy Shocks

The non-linearity of the solution during the period of fixed interest rates, as we showed above, implies a time-varying VAR of the form $x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t$. A forward guidance shock, as we explained above, is an unanticipated change in duration; as a result it changes the reduced-form matrices, $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$, that prevail at the time of the shock as well as those which are expected to prevail in the future. Its impact depends on the state of the economy x_{t-1} and current shocks, ε_t.

To study the response of the economy to forward guidance shocks, we use generalized impulse response functions (GIRFs) as proposed by Koop et al. (1996). In principle, the impulse response to a forward guidance shock can be averaged over different signs of the shock, from different starting (or base) durations, quarters in which the forward guidance shock occurs and even different histories. We select a base duration, a quarter of the fixed interest rate regime and compute GIRFs conditional on the history of the observed variables. These are the difference between the forecast paths of variables with and without the forward guidance shock, that is

$$\text{GIRF}(x_{t+n}) = \mathbb{E}(x_{t+n}|\varepsilon_t^{\text{fg}}, \hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) - \mathbb{E}(x_{t+n}|\hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}),$$

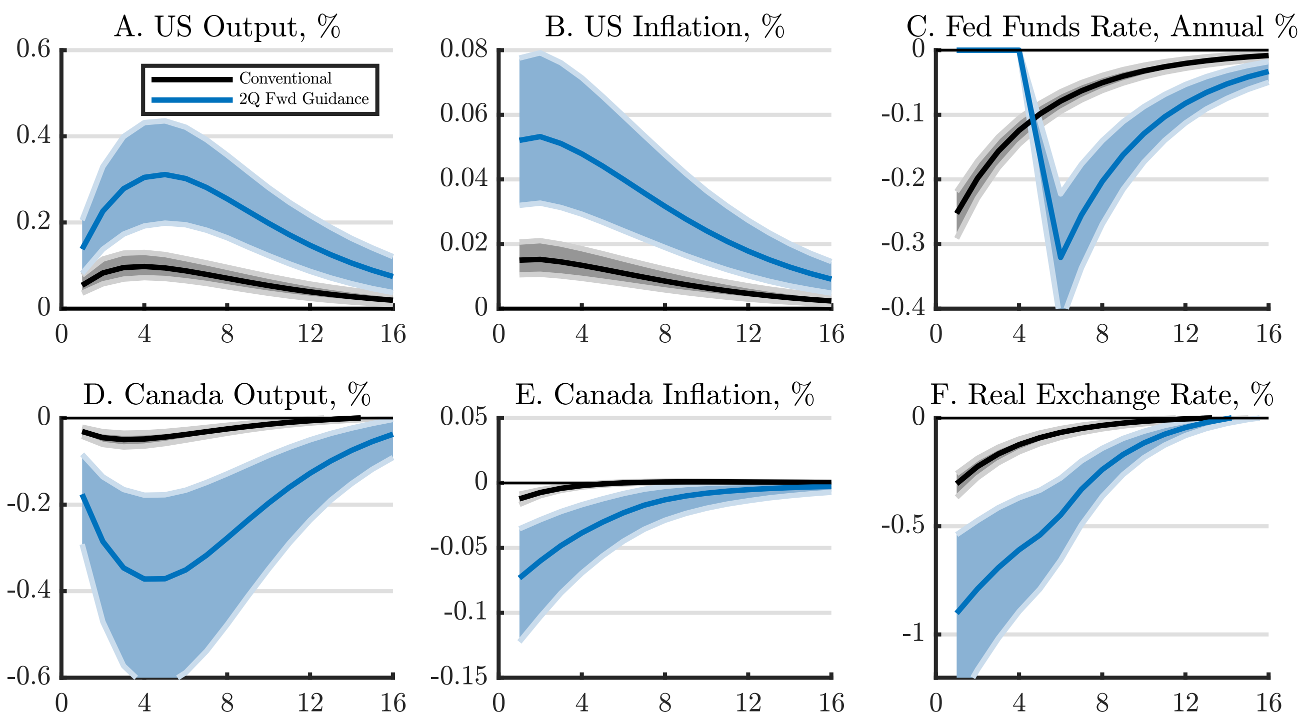
where $\hat{x}_{t-1|T}$ and $\hat{\varepsilon}_{t|T}$ are smoothed estimates of the state and shocks. Notice that an expansionary forward guidance shock implies a fall in the GIRF of the policy rate because the forecast for the policy rate under the forward guidance shock is lower than otherwise. In other words, $\mathbb{E}(r_{t+n}|\varepsilon_t^{\text{fg}}, \hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) \leq \mathbb{E}(r_{t+n}|\hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T})$. Until there is lift-off, $\mathbb{E}(r_{t+n}|\varepsilon_t^{\text{fg}}, \hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) = \mathbb{E}(r_{t+n}|\hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) = \bar{r}$, and an expansionary forward guidance shock has no impact on the policy rate.

We explore state dependence in the next section, but to get a sense of the magnitudes and dynamics involved, we first compare impulse responses to a conventional policy shock with those to a forward guidance shock. We do so by taking draws from the posterior and keeping those draws for which the duration in 2011Q3 is 4 quarters. From that base duration of 4 quarters, we consider a 2 quarter forward guidance shock in 2011Q3. The forward guidance shock amounts to an extension of 2 quarters to the actual duration, which at the median draw is 7 quarters. We choose 2 a quarter forward guidance shock because it is the mean across the sample for forward guidance shocks in the US. The Online Appendix contains additional GIRFs assessing the sensitivity to different base durations and quarters.

Figure 7 compares impulse responses of a 2 quarter forward guidance shock to a conventional monetary policy shock, whose size is chosen so that the decline in 2 year long-rates in the U.S. is the same as that following the forward guidance shock.

An expansionary forward guidance shock implies the same qualitative responses as a conventional shock: it increases output and inflation in the US and appreciates the Canadian real exchange rate which leads to a decrease in domestic output and inflation in Canada. At the mean of the draws notice that a forward guidance shock implies responses which are between 2 or

Figure 7: IRF of Conventional US Policy Shock and GIRF of 2Q US Forward Guidance Shock



Note: This figure plots the mean and 90 percent credible bands of the generalized impulse response to a two quarter forward guidance shock computed at posterior draws in 2011Q3 and the impulse response to a policy shock chosen to generate the same decline in US 2 year rates on impact. A two quarter forward guidance shock is chosen as it is close to the average forward guidance shock across the period of fixed interest rates. The forward guidance shock lowers the path of the policy interest rate, so that the response of the federal funds rate is negative beyond the period of fixed interest rates.

3 times larger than a conventional shock and, in the case of output, more persistent responses.[22]

6.3 Forward Guidance Shocks and the State of the Economy

Figure 7 also highlights that there is considerably more variability in the magnitude of the responses to forward guidance shocks. We find that the state of the US preference shock, ξ_t^*, is an important determinant of the size of the responses; it can amplify or diminish the impact of a given forward guidance shock. To see this, take a simpler version of the Euler equation for the US, which is the only equation apart from the foreign monetary policy rule where r_t^* enters

$$y_t^* = \mathbb{E}_t y_{t+1}^* - \left(r_t^* - \mathbb{E}_t \pi_{t+1}^* - (1-\rho_\xi^*)\xi_t^*\right). \tag{28}$$

Iterating this equation forward reveals that it is the expected path of the last term on the right hand side, $(r_t^* - \mathbb{E}_t \pi_{t+1}^* - (1-\rho_\xi^*)\xi_t^*)$, that matters for output. This explains why the impact of a change in duration depends on the state of the risk premium shock. A large, negative and persistent ξ_t^* offsets the potentially expansionary impact of extending the duration. Conversely, a small, positive and persistent ξ_t^* can amplify the expansionary impact of an extension of the expected duration. This explains why one may find estimates of the duration consistent with survey measures which do not give rise to implausibly large responses of aggregate variables as for example is the case in Figure 1 of Carlstrom et al. (2015) for which the natural rate shock process is fixed.

In this thought experiment we start from some fixed actual duration and considering a forward guidance shock that extends that duration for two states of the preference shock, ξ_t^*. The state of ξ_t^* interacts with the expected path of policy rate as described above, but it also determines the decomposition of the initial actual duration into its lower bound and forward guidance components. The impact of a forward guidance shock would be smaller when more of the actual duration is accounted for by the lower bound rule (i.e. the larger is the lower bound duration component).

[22]Consistent with our findings, Gertler and Karadi (2015) provide VAR evidence on forward guidance shocks which also points to forward guidance shocks having larger effects than conventional monetary policy shocks.

It is important to recognize, however, that the state dependence of forward guidance shocks does not imply that further extensions in the duration will not eventually lead to implausibly large responses of aggregate variables, consistent with the *forward guidance puzzle* identified by Carlstrom et al. (2015) and Del Negro et al. (2012).

McKay, Nakamura and Steinsson (2016) argue that if agents face uninsurable income risk and borrowing constraints, their responses to forward guidance are muted. In our model, the impact of forward guidance depends on the preference shock. As in Jones, Midrigan and Philippon (2018) borrowing constraints drive a wedge in the Euler consumption equation just as preference shocks do in our model. So our findings can be thought to be consistent with the view that borrowing constraints mute the impact of forward guidance. This is not to say that our model is immune to the forward guidance puzzle.

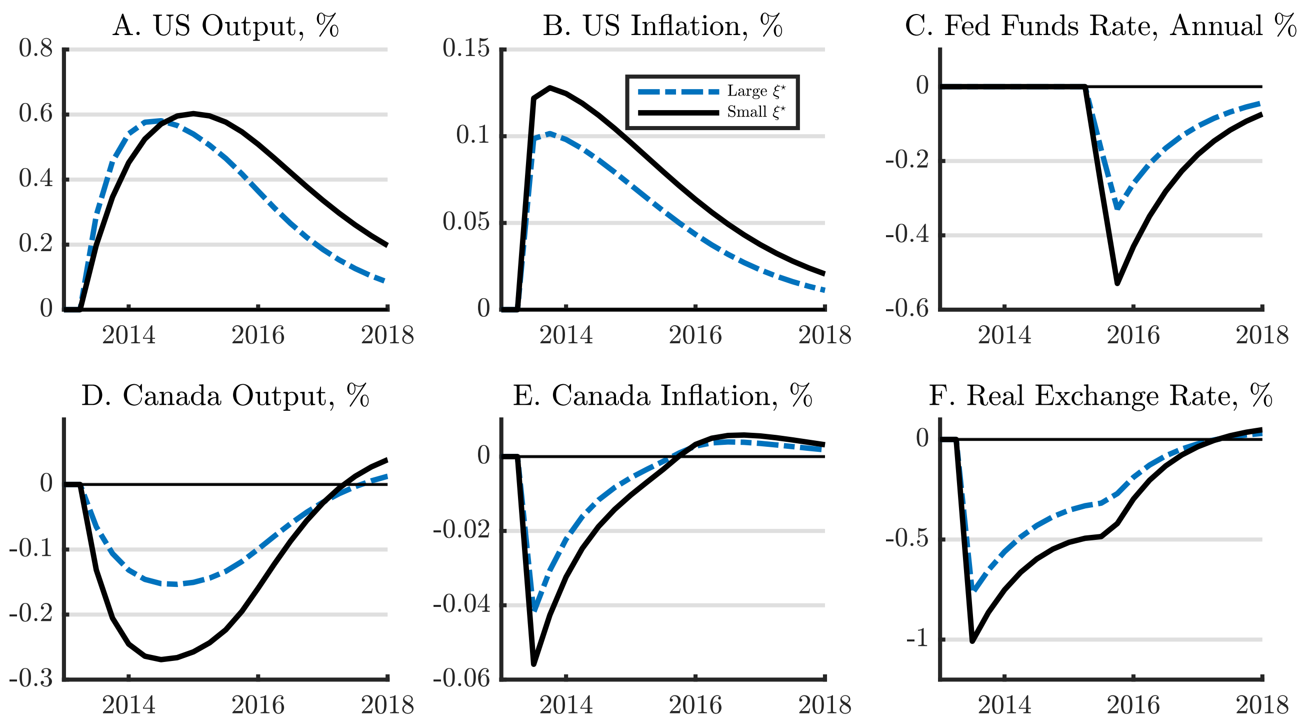
To assess the sensitivity of our results, we also estimate a version of the model with discounting in the Euler equation, as in McKay et al. (2017), a version immune to the forward guidance puzzle. We find that adding discounting in the Euler equation leads to somewhat larger durations and forward guidance shocks. This is perhaps not surprising as in estimation the data is fixed across specifications, and so muting the impact of forward guidance leads to larger estimated forward guidance shocks. The estimates under discounting in the Euler equation can be found in the Online Appendix.

Figure 8 plots the impulse responses for the US and Canada to an expansionary forward guidance shock of 2 quarters for two sets of different draws. In one case, we take those draws for which the standard deviation of the US preference shock, ξ_t^*, is smaller than 0.35. And in the other case, we take only those draws for which the standard deviation of the US preference shock, ξ_t^*, is higher than 0.55.[23] We further condition on draws for which the duration in 2012Q1 is 8 quarters.

For each draw we obtained smoothed estimates of the shocks and states and use these to compute generalized impulse responses for output, inflation and the real exchange rate. A 2 quarter forward guidance shock in the US leads to mild spillovers on Canadian output (a peak response of -0.15 % at mean) when the state of the US preference shock is very negative. On

[23] The mode of the standard deviation of ξ_t^* is 0.42.

Figure 8: State Dependent Spillovers of US Forward Guidance Shock



Note: This figure shows generalized impulse responses in 2013Q3 to a two quarter forward guidance shock in the US for draws of the posterior distribution that have high variance of the US preference shock, in blue, and draws that have low variance of the US preference shock, in black. The posterior draws are restricted to those with an expected duration in the US of 8 quarters. When the US demand process is small, a US forward guidance shock causes a more persistent output boom and higher inflation in the US, and generates larger spillovers on the Canadian economy.

the other hand, when the state of the US preference shock is less negative, the spillovers from US forward guidance can be up to two times as large (a peak response of -0.3 % at the mean).

Canadian output declines by more when the US preference shock is smaller because of a larger real exchange rate appreciation. Real rigidities (eg habits) mean that initial response of US output to the forward guidance shock – and hence the increase in demand for Canadian exports through a positive income effect – is similar regardless of the state of the US preference shock. However, through the mechanism described above, the total response of US output and inflation to the forward guidance shock is larger when the US preference shock is less negative. In conjunction with the forward guidance extension, this ensures that the expected path of the US real interest rates is lower and the initial appreciation of the Canadian dollar – which depends on all future real interest rate differentials – is larger. The resulting increase in the relative price of Canadian goods and services induces a negative substitution effect that accounts for the larger decline in Canadian output when the US preference shock is less negative.

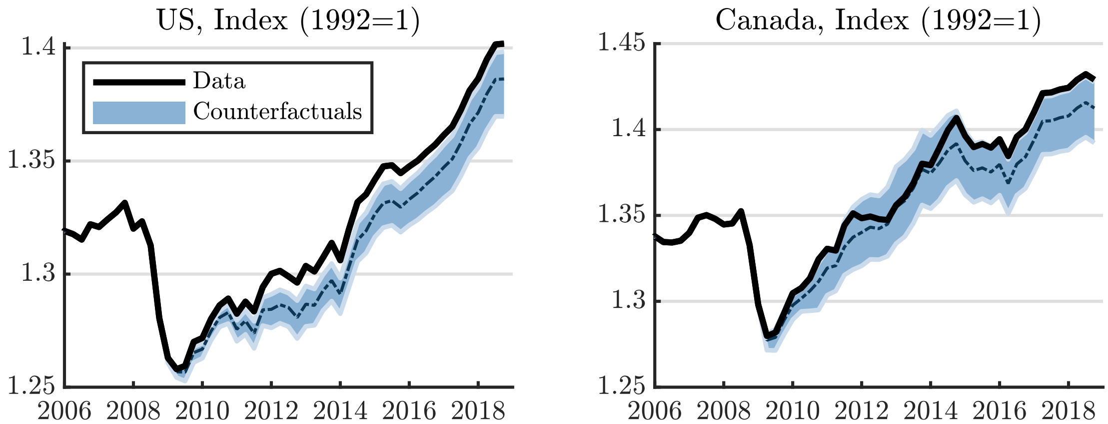
6.4 The Joint Monetary Policy Response

Up to this point we have assessed the estimated responses to US monetary policy shocks. Behind the impact on the Canadian economy is the Canadian monetary policy response. To assess the combined monetary policy stimulus, we use the estimated model over the period of fixed interest rate regimes to construct two counterfactuals. In the first, forward guidance is removed from both the US and Canada, and in the second, forward guidance is removed from the US only.

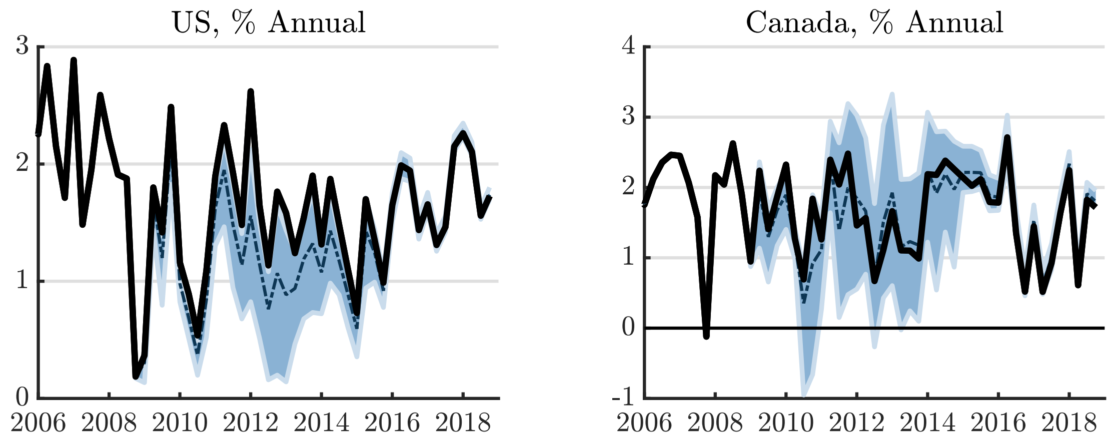
At the mean we find a correlation between forward guidance shocks in the US and Canada of 0.5. This correlation suggests that the Bank of Canada engaged in their own forward guidance in response to forward guidance in the US. Motivated by this correlation, in the first counterfactual, we remove forward guidance from both countries, that is we set $\mathbf{d}_t^{\text{fg}} = 0$ for both countries and thus effectively solve the model under the occasionally binding lower bound constraint given the estimated structural shocks. In Figure 9, we plot, for the US and Canada, the change in output and inflation using draws from the posterior distribution, and compare it to observed output and inflation. We find that, absent forward guidance in the US and Canada, the average cumulative

Figure 9: Counterfactual Paths With No Forward Guidance in US or Canada

(a) Log Output



(b) Inflation



Note: This figure plots the path of output and inflation in the US and Canada in the data (in black), and the 90 percent posterior probability bands of counterfactuals where forward guidance shocks in both countries are removed (in blue, with the mean in dashed).

decline in output in the US would be about 37%, and in Canada would be about 29%. Forward guidance increased inflation in the US. In the case of Canada, we find that forward guidance also increased inflation for most draws, with larger variation around the observed path. As is clear from our estimates, the combined monetary stimulus was on average expansionary for both countries.

Our second counterfactual measures the consequences of removing US forward guidance only, but keeping Canadian forward guidance. To construct this counterfactual, we set $\mathbf{d}_t^{\text{fg}} = 0$ for the US only and keep the expected fixed interest rate durations in Canada $\mathbf{d}_t$ at their posterior draws. We plot the change in output and inflation in Figure 10. As one would expect, the decline in US output in Panel A is the same as the decline in the counterfactual removing forward guidance in both countries. The experience of Canada is, however, different. Instead, between 2011 and 2014, output in Canada is higher, with output being around 0.7% higher in 2012Q1 when US forward guidance shocks are removed. Inflation in Canada is also higher between 2011 and 2016. Over the period that the US was at the lower bound, 2009Q1 to 2015Q3, Canadian inflation would have been at the mean almost 0.3% higher on an annualized basis with the credible upper band reaching 4% had the US not conducted forward guidance but Canada had. As US monetary policy is less expansionary in this counterfactual, the US dollar is stronger and the Canadian dollar weaker. The weaker domestic currency increases inflation as it raises the domestic price of foreign goods. This counterfactual provides a direct measure of the spillovers of US forward guidance and the expansionary policy actions of the Bank of Canada.

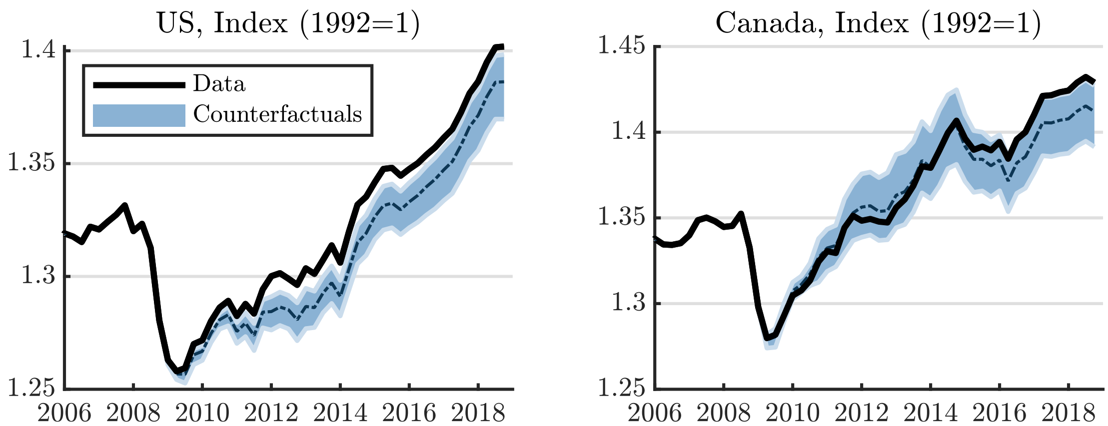
Comparing the domestic inflation response in Figure 9 with that in Figure 10 shows that, given the stance of US monetary policy, the actual monetary policy response of the Bank of Canada was appropriate to keep inflation close to the 2 per cent midpoint of its target range of 1 to 3 per cent.

7 Conclusions

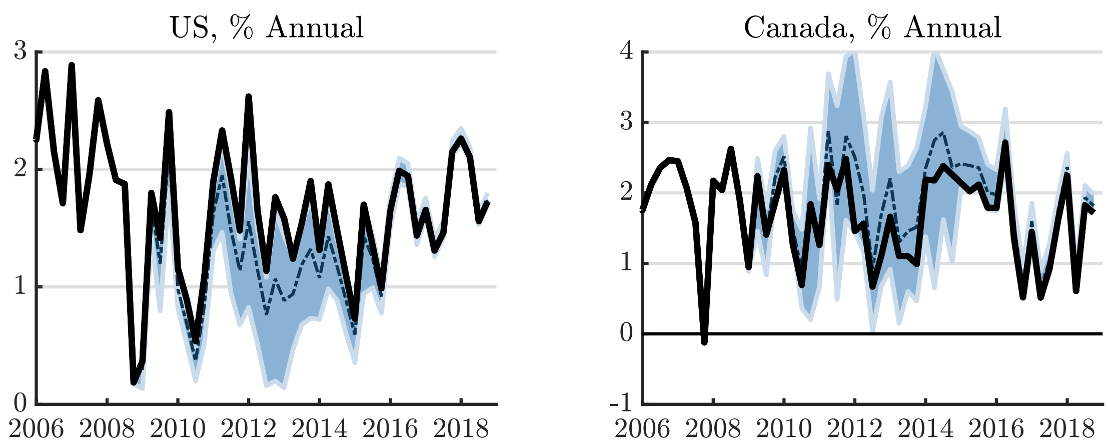
In this paper, we estimate a two-country small open economy model on US and Canadian data accounting for fixed interest rates from 2009Q1 onwards. We propose an identification of forward

Figure 10: Counterfactual Paths With Fixed Durations in Canada Only

(a) Log Output



(b) Inflation



Note: This figure plots the path of output and inflation in the US and Canada in the data (in black), and the 90 percent credible bands of counterfactuals where only forward guidance shocks in the US are removed and the durations in Canada are at their estimated draws (in blue, with the mean in dashed).

guidance shocks using the estimated model and an occasionally binding constraint solution, to study the spillovers of US forward guidance shocks on Canada.

We argue that a focus on whether monetary policy spillovers are ‘beggar-thy-neighbor’ is less relevant. Measuring spillovers is important, whatever their direction may be, for calibrating the domestic monetary policy response. What ultimately matters is whether the domestic monetary authority is able to set policy appropriately to meet its objectives. We find that the combined monetary policy response in both the US and Canada post-2009 was stabilizing for both countries.

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Appendix (For Publication)

A Simplified Model for Section 2.3

Under the following simplifying assumptions to our model – no habits in consumption, complete markets, flexible wages, and full exchange rate pass through at home and abroad – our economy model is that of De Paoli (2009b) which can be characterized by the set of log-linear equations:

$$\begin{aligned}
\pi_{H,t} &= \kappa \left(c_t + \varphi y_{H,t} + \frac{\alpha}{1-\alpha} q_t \right) + \beta \mathbb{E}_t \{\pi_{H,t+1}\} \\
\pi_t &= \pi_{H,t} + \frac{\alpha}{1-\alpha} \Delta q_t \\
y_{H,t} &= (1-\alpha) c_t + \alpha y_t^* + \gamma q_t \\
c_t &= y_t^* + q_t \\
c_t &= \mathbb{E}_t \{c_{t+1}\} - (r_t - \mathbb{E}_t \pi_{t+1})
\end{aligned}$$

and a monetary policy reaction function. In the equations above, variables represent percentage deviations from steady state, $\gamma = \frac{\tau\alpha(2-\alpha)}{(1-\alpha)}$ and $\kappa = \frac{(1-\theta\beta)(1-\theta)}{(1+\varphi)\theta}$. The only exogenous shock is y_t^* assumed to be i.i.d.

B Solution Method

Our model has the following time-varying VAR solution:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t,$$

where the time-varying matrices are given by the following recursion:

$$\mathbf{J}_t = \left[\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{Q}_{t+1}\right]^{-1} \left(\bar{\mathbf{C}} + \bar{\mathbf{D}}\mathbf{J}_{t+1}\right) \tag{29}$$

$$\mathbf{Q}_t = \left[\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{Q}_{t+1}\right]^{-1} \bar{\mathbf{B}} \tag{30}$$

$$\mathbf{G}_t = \left[\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{Q}_{t+1}\right]^{-1} \bar{\mathbf{F}}. \tag{31}$$

Starting from the terminal condition $\mathbf{Q}_{\bar{T}+1} = \mathbf{Q}$ (the solution in the case when monetary policy is following the Taylor rule) the sequence of reduced form matrices $\{\mathbf{Q}_t\}_{t=1}^{\bar{T}}$ can be solved for via the backward recursion implied by (30). With $\{\mathbf{Q}_t\}_{t=1}^{\bar{T}}$ in hand, it is straightforward to find $\{\mathbf{J}_t\}_{t=1}^{\bar{T}}$ and $\{\mathbf{G}_t\}_{t=1}^{\bar{T}}$. Note that in the sequence, $\{\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_{\bar{T}}\}$ each reduced form matrix has associated with it an expected duration of the fixed interest rate regime: $\mathbf{Q}_1$ is associated with an expected duration of $\bar{T}$ quarters, $\mathbf{Q}_2$ with an expected duration of $\bar{T}-1$ quarters and so on.