Aging, Secular Stagnation and the Business Cycle*

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Abstract

As of 2015, US log output per capita was 12% below what its pre-2008 trend would predict. To understand why, I develop and estimate a model of the US with demographics, real and monetary shocks, and the occasionally binding zero lower bound on nominal rates. Demographic changes generate slow-moving trends in the real interest rate, employment, and productivity. I find that demographics alone can explain one-third of the gap between log output per capita and its linear trend in 2015. Demographics also lowered real rates, causing the zero lower bound to bind between 2009 and 2015, contributing to the slow recovery from the Great Recession.

Keywords: Great Recession, Demographics, Zero Lower Bound, Forward Guidance.

JEL classifications: E3, E4, E5, J11.

1 Introduction

Much attention has been given to the *secular stagnation* trends that have emerged in the US since the Great Recession. By 2015, log output per capita was 12% lower than its 1950 to 2007 linear trend would predict. The Federal Reserve has held the Fed Funds rate at or near its lower bound since 2009, while the real interest rate declined from about 4% in 1995 to 1% in 2015, and the employment-population ratio has fallen from 65% in 2000 to just above 60% in 2018. Figure 1 illustrates these series.

Economists have proposed several theories for these observations. Summers (2014), for example, argues that the US has entered a period of secular stagnation characterized by low real interest rates and

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constrained monetary policy. Gordon (2016) suggests that supply-side factors are key, as productivity growth has slowed over time. Rogoff (2015) argues that sluggish output growth is driven by financial factors and a protracted period of deleveraging, particularly when monetary policy is constrained (Jones et al., 2018).

The aging of the US population can explain, in theory, why real interest rates are low and therefore why the zero lower bound binds – an individual's savings changes with age, and an economy with a higher fraction of older people has more savings and lower interest rates (Eggertsson et al., 2019). An aging population can also rationalize why productivity growth and the employment-population ratio are low, since younger workers face a steeper human capital profile and older workers work fewer hours.

This paper develops a tractable model of the US economy to quantitatively account for the gap in output from its pre-crisis trend. The model has a rich demographic structure where individuals in overlapping generations can live for up to 80 years. They choose their labor supply and savings each period, borrow when young, and consume their savings when old. These demographic features generate slow-moving trends over many decades. Aggregate shocks generate business cycle fluctuations around the demographic trend, and nominal rigidities make monetary policy relevant, with the Fed Funds rate set by a monetary policy rule, and is subject to the zero lower bound.

It is important to model the aging of the population alongside the occasionally binding zero lower bound constraint because the aging of the population can cause real and nominal interest rates to decline, so that the zero lower bound is more likely to bind following a shock. There are therefore highly nonlinear interactions between demographic trends and constrained monetary policy that I am able to capture in my estimated model. To handle foreseen demographic changes together with the nonlinearities associated with the zero lower bound, I combine the piecewise-linear solution methods developed by Kulish and Pagan (2016), Guerrieri and Iacoviello (2015), and Jones (2017).

My key contribution is to quantify how demographic trends affected the recovery from the financial crisis along two dimensions: first, directly, as demographic changes affect aggregate labor and capital, and second, indirectly, as demographic changes cause interest rates to fall making the zero lower bound a more binding constraint between 2009 and 2015. I find that the direct effects are responsible for about one-third of the decline in output relative to trend by 2015. I also find that, had demographics not changed between 1984 and 2015, the zero lower bound would not have been a binding constraint. Mitigating the effect of the zero lower bound, I find that forward guidance policy kept output from falling by a further 2 percentage points between 2011 and mid-2013.

My second contribution is methodological. I show how to parsimoniously model anticipated demographic trends alongside the zero lower bound and a standard set of business cycle shocks such that it becomes feasible to apply likelihood methods to estimate the model's parameters. In particular, I show that my model is approximated well with an aggregate representation with time-varying and anticipated changes to the parameters of the model that are exogenous functions of demographic variables. I can exploit the resulting computational advantages to filter quarterly output and consumption for the shocks that generate those series, while also accounting for the trends caused by demographics, and for the stimulatory effects of forward guidance by disciplining, with survey data, expectations of how long the zero lower bound is expected to last each quarter between 2009 and 2015.

I calibrate the demographic trends to those observed from 1940 and projected out to 2070, and use my methodology to estimate the model's business cycle shocks from 1984 to 2015. I find that between 1990 and 2015, changes in the composition of the workforce caused by the aging of the population explain a decline of around 2 percentage points in the labor force participation rate, with a further decline of 4 percentage points expected by 2035. This contraction of the labor force together with a decline in savings and investment as fewer workers save for retirement drag down output growth. Furthermore, demographic changes are responsible for about a 1 percentage point decline in real interest rates and about a 2 percentage point decline in the nominal interest rate between 1990 and 2015. Demographic trends, therefore, reproduce well the long-run trends observed in the US economy, consistent with the findings of Gagnon et al. (2016); Eggertsson et al. (2019); and Aksoy et al. (2019).

The nonlinearities associated with the zero lower bound and forward guidance policy mean that it is a nontrivial problem to assess which shocks, other than demographics, were responsible for the decline in output following the Great Recession. To shed light on this, I hold fixed the expected duration of the zero lower bound each quarter between 2009 and 2015, and find that the contribution of aggregate shocks that capture financial distresses depressed output, but were partly offset by positive government spending shocks between 2009 and 2012.

My model has a rich demographic structure. Workers accumulate human capital over the lifecycle and gradually withdraw from the labor market as they age. Workers have age-specific labor productivities, which I calibrate using US Census and survey data. There are two exogenous demographic trends in the model. First, workers have age-specific and time-varying mortality rates, which are matched to observed and projected mortality profiles. Second, the size of the initial cohort each period is chosen to match observed changes in the relative size of the the 16-year-old cohort. The model's lifecycle parameters are calibrated to individual-level data and the demographic trends are fully anticipated. I show that the demographics in my model closely track those that are observed in the US.

The unintentional bequests of those who die between periods are redistributed to their living cohort members, as in Yaari (1965) and Blanchard (1985). This insurance against mortality risk means that the consequences of demographic changes are approximated very well by slow-moving, anticipated changes to parameters of the preferences of a representative agent and to aggregate productivity. These parameters vary over time with changes in the composition of the population and can be readily constructed from population data and lifecycle parameters – in particular, the age-productivity profile and the disutility of labor supply by age. Approximating the model with these time-varying parameters reduces the model's state-space substantially, and makes it feasible to conduct a Bayesian estimation with quarterly data and the zero lower bound.

Using this efficient representation, I estimate the parameters of the six aggregate shocks to the discount factor, productivity, markups, investment adjustment costs, monetary policy, and government spending, that drive fluctuations around the model's demographic trend, using quarterly data on output, consumption, investment, the Fed Funds rate, and inflation between 1984 and 2015. During the zero lower bound period, I discipline the durations that the Fed Funds rate are expected to remain at zero with survey data from Blue Chip and the New York Fed.

Estimating the model with demographic trends affects the posterior estimates of the model's structural parameters and the interpretation of which shocks generate business cycles. The estimate of annual trend growth is 0.4% lower when demographic trends are accounted for, as the changing composition of the workforce implied by those trends is favorable to growth across the estimation sample. Accounting for slow-moving demographic trends also lowers the importance of productivity shocks and raises the role of markup shocks for driving output and consumption.

Using the model, I explore in detail the extent to which demographic trends alone are responsible for the observed trends in aggregate variables. The gradual fall in mortality rates causes a compositional shift towards an older population: longer expected lifetimes means an increased need for savings and capital for retirement consumption, contributing about a 1 percentage point decline in the real interest rate. Fertility shocks cause a slow-moving increase and then decrease in the real interest rate around the path implied by the fall in mortality rates because they change, over time, the relative size of labor and capital, affecting the marginal product of capital. I find that demographic trends, and the resulting change in the composition of the workforce, can explain a decline in the employment-population ratio of 1.9 percentage points from 1990 to 2015. This compares to the observed decline of 3.5 percentage points from 1990 to 2015. I also find that my model predicts that the *growth rate* of labor productivity – output per hour worked – has declined by about 0.5 percentage points from 1990 to 2011. This compares to the observed average decline of about 1 percentage point over the same period (Fernald, 2015). While much of the change is driven by variation in aggregate labor and capital, I find that changes in *labor quality* have been an important factor since the 1990s. In particular, when the baby boomer generation enters the workforce, there is an increase, and subsequent decrease, in the growth rates of human capital and thus productivity.

I model the US as a closed economy. However, conclusions about the fall in real interest rates can, in theory, be sensitive to international factors. As argued by Gagnon et al. (2016), the relationship between an aging population and a falling real interest rate is arguably robust to open-economy considerations since, first, most advanced economies are experiencing similar demographic changes, and second, the observed magnitude and direction of international capital flows were opposite to the outflows that the aging of the US population should imply. Furthermore, Eggertsson et al. (2019) consider open-economy factors in their study of the real interest rate from the 1970s, finding that a decline in the real interest rate caused by capital inflows were roughly the same as an increase caused by the growth in US public debt – both factors are absent in my model.

Related literature My paper relates to the secular stagnation debate by quantifying the direct and indirect channels between demographics and growth (see also Summers, 2014; Fernald, 2015; Hamilton et al., 2015; Rogoff, 2015; Bernanke, 2015). My emphasis on the importance of demographics in explaining the decline in the real interest rate is shared by the quantitative work of Carvalho et al. (2016), Gagnon et al. (2016), and Eggertsson et al. (2019).

The paper also relates to research that studies more broadly the macroeconomic implications of demographic trends (Aksoy et al., 2019; Backus et al., 2014; Bloom et al., 2010; Auerbach and Kotlikoff, 1987) and builds on the insights of the empirical work connecting demographic trends to productivity growth (Feyrer, 2007) and the labor force participation rate (Aaronson et al., 2014). Finally, my paper relates to studies of how the propagation of shocks change as an economy undergoes structural changes, either anticipated or unanticipated (Kulish and Pagan, 2016; Canova et al., 2015; Wong, 2015; Jaimovich and Siu, 2012; Fernández-Villaverde et al., 2007). I build on this work by modeling demographic changes with anticipated time-varying parameters.

2 Model

In this section, I develop a model in which trends in key macroeconomic variables are driven by population aging. The model features individuals of different ages, monopolistically competitive firms that produce with capital and labor and face price adjustment costs, aggregate shocks, monetary policy, and the zero lower bound on nominal interest rates.

2.1 Households

Demographics Individuals are organized into overlapping generations. Each generation lives for a maximum of T periods, so that the age range of an individual is 0 to $T-1$. Generation s is of mass n_t^s and comprised of a continuum of identical members of age s, measured at the start of period t. I abstract from trend population growth, and normalize the initial population size to $n_t^0 = 1$. The total size of the population at time t is:

$$n_t = \sum_{s=0}^{T-1} n_t^s. \tag{1}$$

Each period, a fraction of each cohort dies with the exogenous age-specific mortality rate γ_t^s:

$$n_{t+1}^{s+1} = (1-\gamma_t^s) n_t^s. \tag{2}$$

These mortality rates are time-varying; for example, permanent decreases in mortality rates imply increases in longevity. A maximum lifespan of T implies $\gamma_t^{T-1} = 1$.

Household problem An individual of age s has the period utility function $u(c_t^s, \ell_t^s)$ and chooses consumption c_t^s, labor supply ℓ_t^s, capital k_t^s, and one-period risk-free bonds b_t^s, to maximize lifetime utility. The value function of an individual of age s at period t is, in recursive form:

$$V_t^s = \max_{\{c_t^s,\ \ell_t^s,\ b_t^s,\ k_t^s\}} \left\{ \chi_t u(c_t^s, \ell_t^s) + \beta(1-\gamma_t^s)\mathbb{E}_t V_{t+1}^{s+1} \right\}, \tag{3}$$

where the expectation is taken with respect to the aggregate stochastic shocks, β is the discount factor common to all individuals, and χ_t is an aggregate autoregressive preference process subject to shocks:

$$\ln \chi_t = (1-\rho_\chi)\ln\chi + \rho_\chi \ln \chi_{t-1} + \sigma_\chi \varepsilon_{\chi,t}. \tag{4}$$

Because of mortality risk, individuals have time-varying and age-specific discount factors $\beta(1-\gamma_t^s)$.

The unintentional bequests made by individuals of a household who die between periods are aggregated and redistributed to the remaining living households of the same cohort. Individuals have age-specific productivities z^s, receive a transfer from the government ξ_t^s (described in more detail below), earn a return R_t on last period's bond holdings b_{t-1}^{s-1}, earn the rental rate on capital r_t on last period's capital holdings k_{t-1}^{s-1}, receive $\frac{1}{n_t^s}\frac{d_t}{p_t}$ dividends from firms, and ψ_t^s for the redistributed unintentional bequest.[1] Capital adjustment requires paying a quadratic cost parameterized by ϕ_k. These adjustment costs are subject to an aggregate, exogenous autoregressive process κ_t, which captures changes in the efficiency of investment adjustment:

$$\ln\kappa_t = (1-\rho_\kappa)\ln\kappa + \rho_\kappa \ln\kappa_{t-1} + \sigma_\kappa\varepsilon_{\kappa,t}. \tag{5}$$

These features imply that the period budget constraint of the individual of age s is:

$$c_t^s + k_t^s + \frac{\phi_k}{2}\left(\frac{k_t^s}{k_{t-1}^{s-1}}\kappa_t - 1\right)^2 k_{t-1}^{s-1} + \frac{b_t^s}{p_t R_t} \leq z^s w_t \ell_t^s(1-\tau_t) + \frac{b_{t-1}^{s-1}}{p_t} + (r_t+1-\delta)k_{t-1}^{s-1} + \tau_t^s. \tag{6}$$

where $\tau_t^s = \xi_t^s + \psi_t^s + \frac{1}{n_t^s}\frac{d_t}{p_t} - T_t^g$ collects various transfers and a lump-sum tax T_t^g, and p_t is the price level. In the last period of life, the budget constraint is:

$$c_t^T \leq (r_t+1-\delta)k_{t-1}^{T-1} + \frac{b_{t-1}^{T-1}}{p_t} + \tau_t^T. \tag{7}$$

By assumption, an individual retires fully from the labor market in her last period of life. Individuals are born with zero wealth, so that $k_t^0 = 0$ and $b_t^0 = 0$ for all t, and nominal bonds are in net zero supply $b_t = \sum_s n_t^s b_t^s = 0$. Substituting in for the unintentional bequests $\psi_t^s = \frac{n_{t-1}^{s-1}}{n_t^s}\left[\frac{b_{t-1}^{s-1}}{p_t} + (r_t+1-\delta)k_{t-1}^{s-1}\right]$, and denoting the marginal utility of wealth of an individual of age s in time t by λ_t^s, the optimal choice of risk-free bonds implies the standard Euler equation:

$$\mathbb{E}_t\left[\frac{\lambda_{t+1}^{s+1}}{\lambda_t^s}\right] = \frac{1}{\beta}\mathbb{E}_t\left[\frac{\Pi_{t+1}}{R_t}\right], \tag{8}$$

where $\Pi_t = p_t/p_{t-1}$ is the rate of inflation.

[1] The redistribution scheme for unintentional bequests scales the return on savings k_{t-1}^{s-1} and b_{t-1}^{s-1} by $1/(1-\gamma_{t-1}^{s-1})$.

2.2 Firms

There are a continuum of intermediate goods firms who hire capital and labor from households to supply a substitutable good $y_t(i)$ at price $p_t(i)$ to final goods producers, who in turn use a CES technology with elasticity of substitution ξ_t to aggregate the intermediate goods into a final good which it sells to consumers at the price p_t. Aggregate capital is the sum of each cohort's capital savings: $k_t = \sum_s n_t^s k_t^s$, while aggregate labor hired by the firm is in efficiency units of labor $\ell_t = \sum_s z^s n_t^s \ell_t^s$. I assume that firms use the capital of the deceased in production. The production function of firm i is:

$$y_t(i) = \mu_t^{1-\alpha} \left(k_{t-1}(i)\right)^{\alpha} \left(Z_t \ell_t(i)\right)^{1-\alpha}, \tag{9}$$

where $\frac{Z_t}{Z_{t-1}} = z$ generates trend growth, and μ_t is an aggregate autoregressive TFP process:

$$\ln \mu_t = (1-\rho_\mu)\ln \mu + \rho_\mu \ln \mu_{t-1} + \sigma_\mu \varepsilon_{\mu,t}, \tag{10}$$

where $\varepsilon_{\mu,t}$ is a standard normal innovation. Denoting $\mathrm{mc}_t(i)$ as the Lagrange multiplier on the firm's cost minimization problem $\min_{\{k_{t-1}(i),\ \ell_t(i)\}} r_t k_{t-1}(i) + w_t \ell_t(i)$, the rental rate on capital is $r_t = \alpha \mathrm{mc}_t(i) \frac{y_t(i)}{k_{t-1}(i)}$, and the wage is $w_t = (1-\alpha)\mathrm{mc}_t(i)\frac{y_t(i)}{\ell_t(i)}$. Intermediate goods-producing firms also face a Rotemberg quadratic cost of adjusting prices, parameterized by ϕ_p. The problem of the firm i is to choose its price $p_t(i)$ to maximize firm value:

$$\max_{p_t(i)} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left(\frac{d_t(i)}{p_t}\right), \tag{11}$$

where λ_t is an average of households' shadow values of wealth,[2] and where real dividends are:

$$\frac{d_t(i)}{p_t} = \left(\frac{p_t(i)}{p_t}\right)^{1-\xi_t} y_t - \mathrm{mc}_t(i)\left(\frac{p_t(i)}{p_t}\right)^{-\xi_t} y_t - \frac{\phi_p}{2}\left[\frac{1}{\Pi^*}\frac{p_t(i)}{p_{t-1}(i)} - 1\right]^2 y_t, \tag{12}$$

where the elasticity of substitution between intermediate goods ξ_t is subject to stochastic shocks, which generates time-varying markups over marginal costs:

$$\ln \xi_t = (1-\rho_\xi)\ln \xi + \rho_\xi \ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t}. \tag{13}$$

[2] Under the model's approximation, discussed below, the choice of λ_t is unimportant.

The first order condition on the optimal choice of price resetting is:

$$\beta\phi_p\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}\frac{y_{t+1}}{y_t}\left[\frac{\Pi_{t+1}}{\Pi^*}-1\right]\left[\frac{\Pi_{t+1}}{\Pi^*}\right]=\xi_t-1-\xi_t\mathrm{mc}_t+\phi_p\left[\frac{\Pi_t}{\Pi^*}-1\right]\left[\frac{\Pi_t}{\Pi^*}\right], \tag{14}$$

Log-linearizing (14) gives rise to a forward-looking New Keynesian Phillips curve. I denote the slope of the log-linearized Phillips curve by ϵ_p, which is a function of the steady-state elasticity of substitution ξ and the parameter governing the capital adjustment cost ϕ_p.

2.3 Monetary Policy

Monetary policy operates in one of two possible regimes. In the first regime, the nominal interest rate is set according to a Taylor rule, as in Smets and Wouters (2007):

$$\frac{R_t}{R}=\left(\frac{R_{t-1}}{R}\right)^{\phi_r}\left(\frac{\Pi_t}{\Pi^*}\right)^{(1-\phi_r)\phi_\pi}\left(\frac{y_t}{y_t^{\mathrm{F}}}\right)^{(1-\phi_r)\phi_y}\left(\frac{y_t/y_{t-1}}{y_t^{\mathrm{F}}/y_{t-1}^{\mathrm{F}}}\right)^{\phi_g}\exp(\sigma_R\varepsilon_{R,t}). \tag{15}$$

The nominal interest rate R_t responds to deviations in inflation Π_t from a target rate Π^*, deviations in output y_t from its flexible-price level y_t^F, and the growth rate of output relative to the growth rate of potential output, and is subject to stochastic shocks $\varepsilon_{R,t}$.

In the second regime, the nominal interest rate is at the zero lower bound:

$$\log(R_t)=0. \tag{16}$$

Monetary policy can enter the zero lower bound regime in two ways: first, if the Taylor rule calls for negative nominal interest rates, and second, if the Fed has announced, or has previously announced, an extension of the zero lower bound beyond that implied by the constraint and the Taylor rule. I assume that the Fed can manipulate expectations of how the path of interest rates evolves, as in Eggertsson and Woodford (2003) and Werning (2012). In estimation, I use survey data from the New York Fed to discipline the expected duration of the zero interest rate regime during the 2009 to 2015 period.

2.4 Government

The government taxes labor income at the rate τ_t^w to fund a pay-as-you-go social security system. The period-by-period transfer paid to individuals above an eligibility age T^* depends on the accumulated pre-tax labor income of the worker, and a parameter ω governing the replacement rate of past earnings.

Denote by W_t^s accumulated gross lifetime earnings, defined recursively as:

$$W_t^s = \begin{cases} w_t z^s \ell_t^s + W_{t-1}^{s-1}, & \text{if } s < T^* \\ W_{t-1}^{s-1}, & \text{if } s \geq T^*. \end{cases} \tag{17}$$

The amount ξ_t^s redistributed to an agent of age $s \geq T^*$ depends on W_t^s:

$$\xi_t^s = \omega \frac{W_t^s}{(T^* - 1)}, \tag{18}$$

where the denominator reflects the amount of time that W_t^s is accumulated over. For those younger than the eligibility age T^*, the transfer $\xi_t^s = 0$.[3] The budget constraint of the social security system is:

$$\sum_s n_t^s \xi_t^s = \sum_s n_t^s z^s w_t \ell_t^s \tau_t^w. \tag{19}$$

The tax rate τ_t^w adjusts to equalize social security outlays and tax revenues.

Finally, the government levies a lump-sum tax on households to pay for government expenditures g_t, which are assumed to be autoregressive and subject to stochastic shocks:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}, \tag{20}$$

where the budget constraint of exogenous government expenditures is $g_t = n_t T_t^g$.

3 Model Approximation

My objective is to study the relationship between demographic changes and aggregate outcomes, including nonlinear interactions between the aging population, declining natural interest rate, and monetary policy. To do so, the model is set to a quarterly frequency and the shocks driving the business cycle are estimated. Solving for a full rational expectations equilibrium in this model would require characterizing the distribution of wealth **across** individuals. The rich sources of persistence and aggregate shocks makes solving the model this way computationally infeasible. To address these computational challenges, I argue in this section that the model can be approximated by a representative agent framework with time-varying parameters that are functions of exogenous demographic variables.

[3] I abstract from questions about the sustainability of pension systems in an aging society and do not allow pension funds to accumulate assets or liabilities (see Attanasio, Kitao, and Violante, 2007).

Furthermore, because I assume that demographic changes are perfectly foreseen, the path of these time-varying parameters are also assumed to be fully anticipated. I then show how this anticipated path of time-varying parameters gives rise to a VAR representation appropriate for estimation.

3.1 Derivation

To derive the approximation, we will proceed in two steps. First, we will consider a similar problem to the decentralized overlapping generations model described in Section 2 but which differs only in the timing assumption about when individuals can trade. In the second step, we will show that this alternative problem has an aggregate representation that is tractable for estimation.

Timing Assumption In the model's overlapping generations setup, individuals are born with no wealth and make period-by-period asset trades. Assume, instead, that each generation is born at $t = 0$ with positive wealth and can trade claims to future consumption, and write the preferences of an individual j of age s at time t as:

$$\sum_{t=0}^{\infty} \beta^t \left[\prod_{r=0}^{s} (1 - \gamma_t^r) \right] \phi_t^{j,s+t} \sum_{\sigma^t} \Pr\left[\sigma^t | \sigma^{t-1}\right] u \left[c_t^{j,s+t}(\sigma^t), \ell_t^{j,s+t}(\sigma^t) \right], \tag{21}$$

where the term $\phi_\tau^{j,s} = 1$ if $0 \leq s \leq T-1$, and $\phi_\tau^{j,s} = 0$ otherwise, indicating that individuals value utility only in the periods when they are between the ages of 0 and $T - 1$. The term $\Pr\left[\sigma^t | \sigma^{t-1}\right]$ denotes the transition probability from state σ^{t-1} to σ^t. By assumption, unintentional bequests are redistributed to members of the same generation, so that individuals are insured against the idiosyncratic uncertainty associated with mortality risk. The Euler equation arising from the choice of savings in the problem where individual's maximize (21) subject to the lifetime budget constraint is:

$$\lambda_t^{j,s}(\sigma^t) = \beta \sum_{\sigma^{t+1}} \Pr\left[\sigma^{t+1} | \sigma^t\right] \lambda_{t+1}^{j,s+1}(\sigma^{t+1}) \frac{R_t(\sigma^t)}{\Pi_{t+1}(\sigma^{t+1})}, \tag{22}$$

where $\lambda_t^{j,s}(\sigma^t)$ is the Lagrange multiplier on the budget constraint. If, between any two individuals i and j, the Lagrange multipliers are identically linear in the state variables, then across two periods t and t', the ratio of the multipliers between individuals is constant:

$$\frac{\lambda_t^{j,s}(\sigma^t)}{\lambda_t^{i,s'}(\sigma^t)} = \frac{\lambda_{t'}^{j,s+t'}(\sigma^{t'})}{\lambda_{t'}^{i,s'+t'}(\sigma^{t'})} = \frac{\lambda^{i,s'}}{\lambda^{j,s}}, \tag{23}$$

where $\lambda^{j,s} = \frac{\lambda_t(\sigma^t)}{\lambda_t^{j,s}(\sigma^t)}$. As a result, under Equation (23), the ratio of marginal utilities of wealth across individuals is the same at any point in time. Note, the condition in Equation (23) is what arises when there are complete asset markets.

Aggregate Representation If Equation (23) holds, the economy's equilibrium can be found by solving the problem of a social planner that maximizes a weighted sum of individuals' utility functions. In the social planner's problem, the planner first determines how to allocate, period-by-period, aggregate consumption and aggregate labor supply between individuals. Given the optimal allocation, the planner then solves its intertemporal problem and maximizes aggregate consumption, capital, and labor supply subject to the economy's resource constraint. This approach reflects the aggregation arguments made in Constantinides (1982) and Maliar and Maliar (2003).

Assume that an individual of age s has a separable period utility function over consumption c_t^s and hours ℓ_t^s of the type $\frac{(c_t^s)^{1-\sigma}}{1-\sigma} - \upsilon^s \frac{(\ell_t^s)^{1+\varphi}}{1+\varphi}$. Under these preferences, in the Appendix, I show that there is a representative agent with preferences over aggregate consumption c_t and aggregate efficiency units of labor ℓ_t that take the form:

$$U(c_t, \ell_t) = \phi_t \frac{c_t^{1-\sigma}}{1-\sigma} - \upsilon_t \frac{\ell_t^{1+\varphi}}{1+\varphi}. \tag{24}$$

The representative agent's problem is to maximize (24) over time by choosing c_t, ℓ_t, and aggregate capital k_t subject to the economy's resource constraint and its production function $y_t = \theta_t^{1-\alpha} k_t^{\alpha} \ell_t^{1-\alpha}$. The relationship between the efficiency units of labor and aggregate hours is $\ell_t = A_t h_t$.

In the Appendix, I show that θ_t and A_t are the following time-varying parameters:

$$\theta_t = \sum_s n_t^s z^s, \qquad \text{and} \qquad A_t = \frac{\sum_s n_t^s (\hat{z}^s)^{1+1/\varphi} (\upsilon^s \lambda^s)^{-1/\varphi}}{\sum_s n_t^s (\hat{z}^s)^{1/\varphi} (\upsilon^s \lambda^s)^{-1/\varphi}}, \tag{25}$$

where the value $\hat{z}^s = z^s/\theta_t$ denotes individual s's relative productivity and the λ^s parameters are the Pareto weights attached to an individual of age s. The shock θ_t encodes changes in output caused by variation in the size of the workforce and its composition over idiosyncratic skill levels. In particular, larger populations imply a higher θ_t, as do populations with more productive workers (or more z^s workers). The time-varying parameter A_t affects the hours needed to obtain an effective unit of labor, and is a population-weighted average of relative productivity and the disutility of providing labor. When labor supply is inelastic, θ_t and A_t together affect the labor input by $\frac{\sum_s n_t^s z^s}{\sum_s n_t^s}$: the labor input reflects the population's composition only.

The term ϕ_t inversely affects the marginal utility of consumption, and has a simple expression mapping to the size of the population at each point in time:

$$\phi_t = \left[\sum_s n_t^s \left(\lambda^s\right)^{\frac{1}{\sigma}} \right]^{\sigma}. \tag{26}$$

This term scales the marginal value of wealth in the economy.

The term v_t is a time-varying parameter affecting the marginal disutility of labor:

$$v_t = \left[\sum_s n_t^s (\hat{z}^s)^{\frac{1}{\varphi}+1} \left(v^s \lambda^s\right)^{-\frac{1}{\varphi}} \right]^{-\varphi}, \tag{27}$$

so that v_t is a population-weighted average of age-specific disutilities of providing labor. The greater the relative size of the population with high disutilities of providing labor, the higher is v_t. Equating the marginal utility of consumption and the marginal disutility of labor, and substituting in for hours worked gives the labor wedge as a function of demographic changes ${}^{w_t}/({\ell_t^{\varphi}}/{c_t^{-\sigma}}) = {}^{v_t}/{\phi_t}$.

Two additional trends are needed in the computations to ensure the aggregate representation approximates well the aggregate dynamics of the full lifecycle solution. The first is a gradual trend in the aggregate resource constraint to account for variation in the average mortality rate over time. The second trend is to proportional taxes used to finance the social security system. I take the path of labor income taxes from the non-stochastic perfect foresight path of the economy.

In the aggregate representation of the time-zero trading problem, demographics therefore affects the aggregate economy through time-varying parameters which are functions of observable population dynamics (n_t^s), the age-specific parameters of the model (z^s and v^s), and the Pareto weights that the planner attaches to each generation (λ^s). Assuming the planner equally weights each generation, the trends are straightforward to compute and do not depend on endogenous variables. In the Appendix, I verify that the aggregate approximation recovers closely the paths of the aggregate variables by comparing them to the paths computed using a second order approximation of the decentralized lifecycle model under perfect foresight paths of fertility and mortality rates. These results are consistent with Ríos-Rull (1996), who finds that the business cycle properties of large-scale, stochastic, overlapping-generations economies are similar to the properties of representative agent real business cycle models. The contribution here is to additionally describe an approximation that makes likelihood estimation computationally feasible.

3.2 Solution Method

Under the approximation of the model, demographic trends imply that the economy can be written with time-varying parameters. Furthermore, since demographic changes are anticipated, this path of time-varying parameters is also anticipated. In this section, I describe the methodology used to solve for the path of the aggregated model under the path of time-varying parameters. Moreover, I describe how the methodology handles the zero lower bound through a regime-switching procedure.

Time-Varying Demographic Trends Let x_t be the vector of model variables (state and jump), and ε_t a vector that collects the exogenous unanticipated shocks. The linearized rational-expectations approximation of the model under time-varying parameters is:

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \mathbb{E}_t x_{t+1} + \mathbf{F}_t \varepsilon_t, \tag{28}$$

where $\mathbf{A}_t$, $\mathbf{B}_t$, $\mathbf{C}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$ are time-varying matrices that encode the structural equations of the model linearized at each point in time around the steady-state corresponding to the time t structural parameters.[4] A solution to the problem with anticipated time-varying parameters exists if agents expect the structural matrices to be fixed at some point in the future at values which are consistent with a time-invariant equilibrium (Kulish and Pagan, 2016). In this case, the solution has a time-varying VAR representation:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{29}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are conformable matrices which are functions of the evolution of beliefs about the time-varying structural matrices $\mathbf{A}_t$, $\mathbf{B}_t$, $\mathbf{C}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$

$$\begin{aligned} \mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t \\ \mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}\right) \\ \mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{E}_t. \end{aligned} \tag{30}$$

This iteration is obtained by noting that, from (29), $\mathbb{E}_t x_{t+1} = \mathbf{J}_{t+1} + \mathbf{Q}_{t+1} x_t$, which is substituted into (28) and rearranged for x_t. The law of motion for the model's state variables at a time period

[4] One can instead linearize the model around its original steady-state, the steady-state associated with the time-varying system's final structure, or the steady-state implied by the structure at each point in time. Given the somewhat large movements in the steady-state induced by demographic changes, I use the latter approach, linearizing each set of structural matrices around the steady-state implied by that structure.

t therefore depends on the full anticipated path of the structural matrices. The final structure of the economy needed for the iteration (30) is the one that arises at the expected completion of the demographic transition and under Taylor-rule policy. Under my calibration, this final demographic structure applies from the year 2070 onwards.[5]

Zero Lower Bound To implement the occasionally-binding zero lower bound in the solution (29), we follow the approach of Guerrieri and Iacoviello (2015) and Jones (2017) and define two additional regimes in (28), one for when the zero lower bound does not bind, and one for when the zero lower bound binds, given the demographic parameters. If the zero lower bound binds, we assume that agents believe no shocks will occur in the future and iterate backwards through our model's equilibrium conditions from the date that the zero lower bound is conjectured to stop binding. We then iterate on the periods that the interest rate is conjectured to be in effect until it converges, after which the solution is that of (29).[6]

4 Estimation

Next, in this section I discuss how the parameters of the model are set, including the calibration of the lifecycle parameters of the model, the transitory and permanent demographic changes which drive the trends in the model, and the estimation of the transitory shocks that govern the business cycle around the model's demographic trends.

4.1 Assigned and Calibrated Parameters

Before estimation, I calibrate a subset of the parameters and the demographic shocks.

4.1.1 Lifecycle Parameters

The model is quarterly. Individuals begin life at 16 years of age and live for at most 80 more years, up to age 95. Full retirement is only imposed in the last period of life.[7]

[5] This approach resembles that of Fernández-Villaverde et al. (2007) but instead of innovations driving parameter drift, the time-varying parameters are known and perfectly foreseen functions of exogenous demographic variables.

[6] See also Canova et al. (2015), Kulish and Pagan (2016), and Fernández-Villaverde et al. (2007). More generally, Jones (2017) discusses how the zero lower bound is a change in the structural parameters of the monetary policy rule that applies for a state-contingent period, or to stimulate the economy with forward guidance.

[7] Given the low choice of labor supply at older ages, this choice is not too important. Kulish et al. (2010) show how unanticipated changes in life expectancy can change labor supply for retired workers, as older workers with few assets return to the workforce to fund consumption during their unanticipated increase in the lifespan.

I calibrate the disutility of providing labor v^s with a scaled cumulative density function of a normal distribution, so that v^s increasing in s. This specification is motivated by studies which link the disutility of work to deteriorating health.[8] The parameters of the function for v^s are chosen so that the labor force participation rates by age broadly match those observed in 2000. For the social security system, I set the replacement ratio of accumulated earnings λ to 46.7%, the same value that is used in Attanasio et al. (2007). Retirement benefits are received from age 65 on ($T^* = 49$).

I calibrate the age-productivity parameters z^s to the age-experience earnings profile. I follow Elsby and Shapiro (2012) in constructing the log experience-earnings profile using deflated data on full-time, full-year workers. The data is decennial Census data from 1960 to 2000, and annual American Community Survey data from 2001 to 2007.[9] To minimize cohort effects, I pool, across years, high school dropouts, high school graduates, those with some college education, and those who have completed college or higher education.[10] Panel A of Figure 2 plots the earnings-profile over age. The estimates imply a peak increase in wages of about 134% at age 45, before gradually declining around the age of 50: in line with the estimate of Guvenen et al. (2015) who find an increase in the earnings of the mean worker of 127%. There is less reliable data on the earnings of older workers, so after age 65, I calibrate the productivity of workers to decay by 20% a year.

I assess the calibration of the lifecycle parameters in Figure 2 by plotting, in Panel B, the labor force participation rate by age, in 2000, in the model and the data, and in Panel C, the age-profile of assets normalized to asset holdings at age 60, in 2013, in the model and in the Survey of Consumer Finances. The calibrated age-productivity profile and the disutility of labor parameter at each age implies labor force participation rates that rise when young, flatten out during an individual's prime working life, and decline rapidly around retirement ages. The lifecycle asset-profile is hump-shaped and peaks around 60 years old. Individuals borrow when young in the model and begin accumulating assets around the age of 35.

[8]Kulish et al. (2010) use this specification and also make the function time-varying with increases in life-expectancy, with the result that the disutility of labor from employment declines in the gap between age and life-expectancy (see also, Bloom et al., 2010). I keep it constant to ensure age-participation rates do not vary significantly over time.

[9]Computed off IPUMS-USA extracts. A full description is given in the Appendix.

[10]A finer calibration could distinguish between workers of different education levels. In robustness exercises reported in the Appendix, I distinguish between these education groups and analyze how anticipated changes in the earnings profile map to labor supply decisions. The robustness results indicate that the patterns of the aggregate variables are similar, suggesting that compositional changes in the population drive the aggregate dynamics.

4.1.2 Mortality Profiles

Next, I calibrate the mortality probabilities of each generation during the 80 years they could possibly live, γ_t^s, to the actuarial probabilities reported by the Social Security Administration.[11] By calibrating to these probabilities, I also match changes in the life expectancy of each generation over time, conditional on an individual reaching 16 years of age. The values used are the cohort-specific survival rates computed for the cohort year of birth. These profiles include both observed survival rates of cohorts up to their current age, and extrapolated survival rates based on the Social Security Administration's forecasts of life expectancy. I assume that all changes to these actuarial probabilities are exogenous and perfectly foreseen. For the initial γ_t^s profile, I use the survival probabilities reported for those born in 1900 onwards. For those cohorts born before 1900 but who are alive in 1940, I use extrapolated values of the survival probabilities.[12] Under the calibration, between 1950 and 2020, life expectancy for 16 year olds increases from about 77 years to about 85 years.

4.1.3 Incoming Cohort Size Shocks

I choose anticipated shocks to the size of the incoming cohort so that the change in the observed cohort share is the same as the change in the model cohort share.[13] This ensures that the model captures the baby-boomer generation and imperfectly captures changes in the population distribution due to, for example, immigration. I assume that changes to the incoming population beyond 2015 decay to zero, so that the population distribution converges to the steady-state implied by the mortality profile that is constant from 2070.

I plot in Panel D of Figure 2 the median age of the population above 16 years of age implied by the calibrated mortality profiles and the incoming cohort size shocks. The profile tracks well the corresponding median age of those above 16 years of age in the data, declining from around the 1960s to around the 1980s to about 37 years of age, before steadily increasing as the baby-boomer population ages and longevity continues to rise.

[11] These probabilities were sourced from Table 7 from the Cohort Life Tables for the Social Security Area by Calendar Year, in Actuarial Study No. 120 by Felicitie C. Bell and Michael L. Miller, available at: `https://www.ssa.gov/oact/STATS/table4c6.html`. A full description is given in the Appendix.

[12] Because the survival probabilities are low for those years, the results are robust to alternative specifications and are not important for the model outcomes beyond 1970.

[13] Choosing initial population shocks to matching the *changes* is necessary because the model is initialized at the 1940 steady-state and matching the actual cohort sizes would imply very large and counterfactual initial population shocks.

4.1.4 Preference and Nominal Parameters

I calibrate the remaining parameters to values which imply steady-state capital-output ratios that align with those in the Bureau of Labor Studies' Multifactor Productivity (BLS-MFP) program (see Fernald, 2015).[14] I set capital to depreciate by $\delta = 10.6\%$ a year. The capital share α is set to 1/3, the average of the capital share reported by Fernald (2015) over 1948 to 2015. The intertemporal elasticity of substitution σ is set to 1, and the inverse Frisch elasticity of labor supply φ is set to 2, in line with the estimates of Reichling and Whalen (2012) and with the analysis of Rios-Rull et al. (2012). Both values are also in the range considered by Auerbach and Kotlikoff (1987) in a computational overlapping generations model. The quarterly discount factor β is set to 0.99875. Together, these parameters imply a capital-output ratio in 2000 of about 2.7, which is the observed capital-output ratio in the BLS-MFP in 2000.

I calibrate a small set of the parameters describing the nominal side of the economy to values commonly used in the literature. The steady-state value of ξ is set at 8, which implies a steady-state markup over marginal costs of $\xi/(\xi - 1) = 14\%$. The parameter governing the quadratic cost of capital adjustment ϕ_k is set to 40. The slope of the Phillips curve, ϵ_p, and therefore the quadratic cost of price adjustment ϕ_p is estimated in the next section. I estimate this parameter because it is key for determining the behavior of inflation and therefore the real interest rate, particularly in response to changes in the expected zero lower bound duration which are due to forward guidance announcements. The annual inflation target Π^* is set to 2.2%. Finally, I use the values that Smets and Wouters (2007) estimate for the Taylor rule parameters, which at their posterior mode are: $\phi_r = 0.81$, $\phi_\pi = 2.03$, $\phi_y = 0.08$, and $\phi_g = 0.22$.

4.2 Bayesian Estimation

I next use Bayesian likelihood techniques to estimate the slope of the Phillips curve, trend growth, and the model's shocks that drive business cycle fluctuations around the demographic trend.

[14]These observed capital-output ratios vary between 2 and 2.7 over the period 1950 to 2013. A description of the BLS-MFP dataset used is given in the Appendix.

4.2.1 Quarterly Data

The solution expressed in equation (29) has a state-space representation, allowing me to adapt Bayesian likelihood methods to estimate the remaining parameters of the model. The quarterly data used are:

$$\text{Data} = \left\{ \log\left(\frac{y_t}{y_{t-1}}\right),\ \log\left(\frac{c_t}{c_{t-1}}\right), \log\left(\frac{i_t}{i_{t-1}}\right), \log\Pi_t,\ \log\tilde{R}_t,\ T_t \right\}_t, \tag{31}$$

over the time period 1984Q1 to 2015Q1. I use, as observables, the growth rate of output per capita, of consumption per capita, of investment per capita, the GDP deflator, and the Fed Funds rate, and follow Smets and Wouters (2007) in constructing these series.[15] The nominal interest rate is no longer an observable when the zero lower bound binds between 2009Q1 and 2015Q1. I implement this with a time-varying observation equation in the state-space representation of the model (see Kulish et al., 2017). The sequence of expected durations of the zero lower bound, $\tilde{T}_t$, between 2009 and 2015 are taken from the Blue Chip Financial Forecasts survey from 2009 to 2010 and the New York Federal Reserve's Survey of Primary Dealers from 2011 to 2015.

4.2.2 Parameter Estimates

Table 1 reports moments of the prior and posterior distributions. The priors are diffuse. The prior distribution over the slope of the Phillips curve is wide and allows for a high degree of price flexibility.[16] The prior distribution for the trend rate of growth z is also wide, with the 10th (90th) percentiles implying annual trend growth of about 1% (3%). I use a Markov Chain Monte Carlo approach to characterize the parameters' posterior distributions, computing two independent chains of 150,000 draws. The Appendix provides additional details of the estimation and an analysis of the convergence of the chains.

First, I find that prices are quite inflexible, with the posterior estimate of the slope of the Phillips curve ϵ_p centered tightly around 0.01. At the posterior mode, this value translates into a Calvo probability of price adjustment every quarter of about 10%, consistent with the estimates in Smets and Wouters (2007), but a little more flexible than the estimates of Del Negro et al. (2015) and Aruoba et al. (2017).

Next, I find that the modal estimate of trend growth implies an annual growth rate of around

[15] The Appendix provides more details of the data series used in estimation.

[16] Translating the quadratic price adjustment cost into a Calvo price-reset probability, the 10th percentile of the prior distribution for the slope implies a quarterly Calvo reset probability of 13%, while the 90th percentile of the prior distribution of the slope implies a Calvo reset probability of 34%.

1.35%. This is about 0.4% annual percentage points less than the estimate implied by Smets and Wouters (2007), in part because my estimation sample includes a period of lower average growth, but also because demographic trends alone account for some of the growth over this period, as discussed in the next section.

The remaining parameters are those governing the persistence and size of the shock processes. To interpret these, I report, in Table 2, the unconditional and one-year forecast error variance decomposition of the observable variables (and wages). These decompositions reveal how important each shock is in driving the observable variables around the demographic trend. At the one-year horizon, monetary policy shocks, exogenous government spending shocks, and investment shocks together make up about half of the forecast error variance for output, while at a longer horizon, about 70 percent of the forecast error variance is caused by markup shocks. About one-third of the forecast error variance of the Fed Funds rate is driven by preference shocks. As discussed in the next section under counterfactual simulations, these shocks are largely accomodated by monetary policy, but can be very contractionary when monetary policy is constrained.

Monetary policy shocks do not affect output, consumption, and investment much, comprising around 5% of the forecast error variance at the one-year horizon, and less than 1% at the infinite horizon, which is close to the fraction reported by Smets and Wouters (2007). Monetary policy shocks instead account for about 23% of the forecast error variance of the Fed Funds rate at short horizons. Finally, consistent with the results in Justiniano et al. (2011), investment shocks are very important for output and investment at both the one-year and infinite horizons, explaining 27% and 50% of the forecast error variance of output and investment at the one-year horizon, and 18% and 30% of the forecast error variance of output and investment at the infinite horizon.

I gauge the importance of including demographic trends in the estimation by reporting, in the Appendix, the posterior distributions of the estimated parameters and the forecast error variance decomposition of the observables in an estimation where demographics are held constant from 1984 on. I find that the estimate of annual trend growth is higher by 0.4%, since demographic trends contribute positively to growth, as discussed in more detail in the next section. I also find a much more important role for technology shocks in explaining output and consumption when demographics are not explicitly taken into account. This suggests that demographic trends manifest themselves in ways that reflect highly persistent changes in productivity.

5 Demographic Trends and the Business Cycle

I next study, using the estimated model, the role that demographic trends have had in explaining the decline in log output relative to its pre-crisis trend. I first show that, absent all other shocks, demographic trends alone are responsible for about one-third of the decline in output. On top of this, demographic changes have an additional nonlinear effect by causing the zero lower bound to bind between 2009 and 2015. I show this by extracting the model's shocks that generate the data and compute a counterfactual holding demographics constant from 1984. After removing the forward guidance response of the Fed to a binding zero lower bound, I find that output would have fallen by an additional 2% relative to trend, primarily between 2011 and mid-2013.

5.1 Effect of Demographics

Effect of Demographics Alone First, I discuss how demographic trends alone affect the economy's key variables, plotted in Figure 3. Starting in 1984, I turn off all shocks except for the expected demographic trends. Panel A shows that, between 1990 and 2015, the Fed Funds rate declines by about 2 percentage points. Panel B plots an index of log output and illustrates how demographic changes cause a slowdown in output growth relative to log output's 1984 to 2007 trend. In Panel C of Figure 3, I show that the real interest rate is expected to fall by about 1 percentage point – driven by changes in the capital-output ratio – while in Panel D, demographic trends cause a decline in employment of about 2.5% between 1990 and 2015.

Regarding the implications of demographics alone for growth, from 1980 to 2015, my model predicts growth falls by about 1.25 percentage points. There are three main channels through which output growth can change over time because of changing demographics. Workers can supply more hours, affecting both output and aggregate labor. There are also changes in physical capital, as individuals save and consume out of accumulated savings in retirement. Third, the *quality* of labor can change; namely, changes in the distribution of workers resulting from demographic changes alters the average skill-level of the workforce, which shows up in a decomposition of productivity growth as fluctuations in the average productivity of labor (Fernald, 2015).

I decompose the model's predictions for output growth and labor productivity growth into their component parts and show that accelerating capital accumulation increases the growth rate of both labor productivity and total output up to 1995, after which the growth rate starts to decline.[17] The

[17] This decomposition is presented in more detail in the Appendix.

change in labor supply has a large negative effect on productivity growth, but a positive effect on total growth, when the baby boomer cohorts enter the labor force around 1960. A key component of both labor productivity and total growth is the change in the average skill level of the workforce caused by the interaction of a changing composition of the workforce with the age-productivity profile. The decomposition implies that the contribution of the change in average labor quality to the growth rate of output and output per worker peaks around 1990, adding roughly 0.3 percentage points to total growth and productivity growth. The contribution of labor quality becomes a drag on productivity growth in 2000 as a large fraction of workers reach the peak of the age-productivity profile, exhausting the potential for further growth in average human capital across the workforce. This force is forecast to depress productivity growth until 2030. In total, I find that demographic trends will be a drag on output growth through to 2070.

In Figure 4, I explore the trends generated by demographics only in more detail by plotting first, in Panel A, the employment population ratio in the model and the data. Demographics alone capture the dynamics of the aggregate employment-population ratio well under the calibration of the lifecycle parameters, which generated age-specific labor force participation rates that are consistent with those observed. The labor force participation rate declines in the model at a pace that is roughly as fast as that observed and is predicted to continue to fall by a further 4 percentage points from 2020 to 2040. This result is driven by the compositional changes in the workforce towards workers with lower participation rates (as shown in Panel B of Figure 2), a point supported empirically by Aaronson et al. (2014).[18]

Demographic trends have important implications for the capital-output ratio, plotted in the second panel of Figure 4. As life expectancy rises and mortality rates fall, aggregate savings increases to fund longer expected retirements. As a result, the capital-output ratio increases and the marginal product of capital and real interest rate fall. In addition, the aging of the baby boomer cohorts generates an increase, and then decrease, in the path for the real interest rate around the secular decline implied by increasing longevity. The oscillation is driven by changes in the relative size and composition of the workforce. The workforce is relatively young as the baby-boomers enter the labor market in the 1960s to 1980s, so that aggregate hours supplied is high relative to capital, thereby increasing the marginal

[18]Furthermore, to decompose the contribution of the mechanical effect of an aging population to the decline in the labor input, we can compare (i) the labor input predicted by the model in our baseline exercise where labor endogenously responds to wages, to (ii) the labor input predicted by the model when labor supply is inelastic ($\varphi \to \infty$). In this comparison, almost all of the forecasted decline in the labor input is due to the mechanical effect of demographic changes; the endogenous response of labor in my model to demographic changes mitigates the 8 percent decline (relative to 2015) by only 2 percentage points.

return to capital. As the baby-boomer cohort ages and accumulate savings for retirement, the marginal return to capital and the real interest rate decline. This decline is then reinforced by the withdrawal of the baby-boomer cohort from the labor market, depressing the marginal return to capital, which stays low beyond 2030 (see also Carvalho et al., 2016).

These results on the implications of demographics for macroeconomic trends are consistent with the findings of other studies. Using a calibrated overlapping generations model, Gagnon et al. (2016) find that demographic trends generate a decline in the growth rate from around 2.3 percentage points in 1980 to just under 0.5 percentage points by 2030. They find, similar to what is predicted by my model, that much of the decline is due to declining fertility and the associated exit of the baby boomer generations from the labor force.[19] In an empirical study, Aksoy et al. (2019) predict an average decline in annual output growth rates across OECD countries of 1¼ percentage points between 2010 and 2030. Regarding real interest rates, Gagnon et al. (2016) find a peak in the real rate between 1975 and 1985 of around 1.7%, and a decline of about 1.4 to 1.5 percentage points by 2030, close to my model's predictions. In an empirical study, Johannsen and Mertens (2016) find a decline in the real rate of about ¾ percentage points between 1985 and 2015, while Eggertsson et al. (2019) forecast a larger decline in their model implied interest rate due to demographics of around 3 percentage points between the 1980s and 2030.

Decline in Real Interest Rate and the Zero Lower Bound Next, I examine the nonlinear interactions between the decline in the real interest rate and the zero lower bound on the Fed Funds rate. In Figure 5, I plot the response of the Fed Funds rate and output to a large negative investment shock under two different demographic states – one initialized at the steady-state associated with the 1990 demographic profile, and the other for the steady-state associated with the 2008 demographic profile. The Fed Funds rate is higher in steady-state for the 1990 demographics, when the population is younger the supply of savings is relatively lower. The two responses illustrate how the same shock can have very different implications for the economy because of the binding zero lower bound.[20] Initialized at the 2008 steady-state, the shock is large enough to cause the zero lower bound to bind for about two years, with output falling by an additional 2 percentage points on impact. These impulse responses show that the decline in the real interest rate can be quantitatively important in the presence of the

[19] I present decompositions of these contributing factors in the Appendix.

[20] Due to the time-varying structural changes induced by demographics, there are also small differences in the propagation of shocks unrelated to the binding lower bound. I present additional results of such differences in the Appendix.

zero lower bound, as discussed next.

Holding Demographic Trends Constant from 1984 Here, I examine whether demographic trends were responsible for the Fed Funds rate hitting the zero lower bound between 2009 and 2015. To answer this, I hold the demographic profile constant at its 1984 state, which was the first year that quarterly data is used in the Bayesian estimation, and construct a counterfactual using the estimated structural shocks. As illustrated in Panel A of Figure 6, had the population not aged between 1984 and 2015, the Fed Funds rate would have remained above zero in the aftermath of the Great Recession, falling to, at its lowest point, 0.5% in annual terms in 2012Q4.[21] The declining Fed Funds rate caused by demographic changes implies that, going forward, the zero lower bound is likely to be visited more frequently. One potential policy response would be to raise the inflation target, thereby raising the steady-state nominal interest rate. In simulations, I find that the annual inflation target would need to be raised to about 3.5% to obtain approximately the same ex-ante distribution of expected zero lower bound episodes in 2015 as would arise under an inflation target of 2% and when demographics are set to their 1984 state.

Panel B shows the effect of unchanged demographics for output. Interestingly, the counterfactual response shows that output growth would have been lower. The reason for this is that the composition of the workforce in 1984 is skewed towards younger workers and is therefore favorable to productivity growth between 1984 and the mid-2000s, as younger cohorts move up the age-productivity profile, generating productivity growth. Panel D shows that holding the demographic profile fixed at its 1984 value, the employment gap would have been closed just prior to the recession, and in 2009 would have been about 2 percentage points narrower, consistent with the results presented in Figure 3 that demographic trends have caused a secular decline in the employment-population ratio.

5.2 Effect of Forward Guidance

The previous section showed how demographic trends over the past 25 years caused a decline in real and nominal interest rates, thereby making the zero lower bound a constraint on monetary policy. I explore how much of a constraint the zero lower bound was, quantitatively, by constructing a counterfactual simulation of the economy in which the Fed acts passively in response to shocks that cause the zero lower bound to bind. In this simulation, the expected zero lower bound duration adjusts in response

[21] This period corresponds to the time of the lowest 10-year Treasury yields, which were around 1.6%.

to the shock only. By contrast, in the estimation, I fix the expected zero lower bound duration to those observed in survey data, which allows the Fed to extend the zero lower bound duration beyond the duration implied by the shocks themselves (see also Campbell et al., 2012; Jones, 2017). The counterfactual simulation therefore provides a measure of the degree to which the zero lower bound was a binding constraint, absent explicit forward guidance policies.

Figure 7 illustrates the counterfactual path of the economy without forward guidance. Panel C shows that inflation would have been lower by about 1 to 2 percentage points between 2009 and 2015, and Panel D illustrates that employment would have been lower by a further 0.7 percentage points in 2012Q2. Cumulatively, over the 2009 to 2015 period, employment would have been lower by 7.6%.

Next, I compute the zero lower bound durations implied by the shocks alone, which provides a measure of how stimulatory forward guidance is.[22] I find some degree of forward guidance stimulus every quarter between 2009 and 2015, but the strongest forward guidance announcements occur between 2011Q3 and 2013Q3, when the forward guidance component of the total duration is estimated to be between 8 and 9 quarters. This period corresponds to low yields on long-term Treasuries and the explicit calendar-based targets announced by the Fed. These results are also consistent with the findings in Swanson and Williams (2014), who show that between 2009 and 2011, long-term yields were relatively unconstrained, and that after 2011, long-term yields tightened significantly towards their lower bounds; consistent with the Fed announcing expansive unconventional monetary policies. In particular, around 2011, the Fed announced its "to mid-2013" guidance announcement, the first of many subsequent calendar-based extensions of the lower bound regime.[23]

5.3 Output Since the Great Recession

In this section, I put together the results from the previous two sections to study how the model decomposes the decline in log output relative to its long-run linear trend. First, Figure 8 plots the difference between output and its long-run trend relative to 2008. The data show the severity of the slowdown in output, with the gap between the data and its long-run trend widening between 2008 and 2015 to 12% by 2015. Demographics alone account for a third – 4 percentage points – of the gap in

[22] I calculate the zero lower bound durations implied by the structural shocks, using a method described in Jones (2017). The difference between those computed endogenous durations and the durations used in the estimation is the contribution of forward guidance, or the extension of the zero lower bound regime that, together with the structural shocks, will generate the observed series. The decomposed zero lower bound durations are plotted in the Appendix.

[23] For example, in the FOMC press release, August 9 2011, the FOMC announced: "The Committee currently anticipates that economic conditions – including low rates of resource utilization and a subdued outlook for inflation over the medium run – are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013."

2015. Without expansionary forward guidance policy, the gap would have been larger by, at most, 2 percentage points, primarily between 2011 and 2013.

Because of forward guidance policy and the nonlinearities associated with the zero lower bound, there is no linear decomposition of the remaining gap between output and its demographic path into the contribution of individual shocks.[24] For this reason, I explore how each estimated shock affected output in two cases: first, where the zero lower bound duration adjusts in response to changes in shocks, and second, where the sequence of zero lower bound durations are held constant at their observed values.

Figure 9a plots the contribution of each shock to output, computed in the following way. First, I compute counterfactual paths of output when each shock is set to zero, respectively, and the Fed Funds rate responds endogenously. I then evaluate the contribution of each shock as the difference between the observed output series and the counterfactual output series. The results suggest that investment efficiency shocks primarily explain why output is low relative to trend, and have a persistent negative effect on output through to 2015. These shocks, as emphasized by Justiniano et al. (2011), capture disruptions in the intermediation between savings and investment. They therefore capture the impact that financial factors have on aggregate variables which, as shown by Jermann and Quadrini (2012) and Del Negro et al. (2015), are key to explaining the decline in output around 2008-09.[25] The results also suggest that government spending shocks kept output from falling even further between mid-2009 and 2012. Discount factor shocks have a small effect on output when monetary policy is unconstrained and able to respond. This observation is consistent with the variance decompositions of Table 2, where preference shocks explain about a third of the forecast error variance of the Fed Funds rate, but a small fraction of the variance of output, consumption, and investment.

Finally, in Figure 9b, I compute the contribution of each shock holding fixed the sequence of zero lower bound durations between 2009 and 2015. This exercise magnifies the role of each shock, since the Fed does not endogenously react when that shock is removed. Government spending shocks have a larger marginal impact because these shocks stimulate inflation and lower real interest rates more when nominal interest rates are fixed (see also Christiano et al., 2011). Similarly, markup shocks have a positive effect on output when the interest rate is fixed in 2012, because they raise inflation and lower real interest rates. The estimated preference shocks have a substantial contractionary effect on output when the interest rate is fixed. These shocks have a large effect on the real interest rate when

[24] The shocks are presented in the Appendix.

[25] The shocks to the efficiency of investment – shocks to κ_t in (6) – correlate with the spread between the yields of Baa Corporate debt and 10 year Treasury bonds, and spike at the same time as the spike in the spread in 2008.

they are not accommodated by monetary policy, thereby affecting consumption and output.

5.4 Robustness

I conduct a number of experiments to verify that the aggregate trend predictions from the model under the expected demographic changes are robust to alternative specifications of the model. The results of each experiment are presented in the Appendix but discussed briefly here.

Borrowing Constraints I first check that the model's predictions hold when individuals face a constraint restricting their borrowing early in life. With borrowing constraints, there are more savings, pushing up the capital-output ratio. As a consequence, the real interest rate is lower than in the baseline model. The magnitude of the fluctuations of the real interest rate, the participation rate and output growth are very similar to the baseline model.

Time-Varying Productivity Profile The second robustness check is to adopt time-varying productivity profiles to account for a possible flattening of productivity profiles over time. Such a flattening can affect the accumulation of human capital and can impact aggregate productivity measures in two ways: first, by a growth effect, by lowering the potential for new workers to accumulate human capital, and second, by a level effect, by affecting the productivity level that individuals enter the workforce on. I calibrate the age-productivity profiles by recomputing for each cross-sectional sample, the profile and then interpolating between those points in time. The overall pattern of aggregate labor productivity is much the same as the baseline model, although the magnitude of the amplitude of the change in labor productivity growth is smaller, with demographics contributing the most to labor productivity growth in 1980 rather than in 1990 (as in the baseline results).

Female Labor Force Participation and Multiple Skill Types From 1985 on, the baseline predictions for the participation rate, aggregate labor productivity growth and the real interest rate are largely unaffected when the age-productivity and labor disutility profiles are calibrated to match female age-earnings profiles and female labor force participation rates from the 1940s to 1990s, after which female labor force participation is roughly constant. As a final point of comparison, I verify that the directions of the aggregate predictions are robust to a calibration where an additional source of heterogeneity is modeled—where there are two types of workers, low or high skilled, where low skill workers are those with less than college education. These robustness exercises emphasize how the important demographic dynamics are captured primarily through changes in the size of the population.

6 Conclusions

This paper studies why the level of US output per capita was 12% below its pre-crisis trend in 2015. I use a New Keynesian model with demographic trends and the zero lower bound to show that declining mortality rates and changes to the age population composition can generate trends that match the low frequency movement of output growth, productivity, the real interest rate, and the employment-population ratio.

I estimate the transitory shocks of the model using Bayesian likelihood methods and accounting for zero lower bound, forward guidance, and the demographic transition. With the estimated model, I find that the zero lower bound would not have been a binding constraint between 2009 and 2015 had the population not aged. I find that the aging of the population alone is responsible for a sizeable proportion of the decline in output per capita relative to its pre-crisis trend by 2015 – about one-third. Furthermore, using the estimated shocks, my results suggest that absent any forward guidance policy used by the Fed, the zero lower bound would have caused output to fall by an additional 2 percentage points between 2011 and 2013.

The shocks themselves can have highly nonlinear effects on output and consumption, depending on whether the zero lower bound adjusts in response to those shocks. I assess the contribution of each of the estimated shocks to the decline in output since the Great Recession, and find an important role for investment shocks in causing output to fall. These shocks proxy for financial shocks as they capture disruptions in the intermediation between savings and investment. The results are consistent with the view that financial shocks contributed to the Great Recession and subsequent slow recovery.

The results illustrate the importance of demographic trends as a major driver of macroeconomic trends over time. Further research could focus on how demographic trends interact with the housing market or with the efficacy of fiscal policy. It would also be interesting to model a more detailed financial sector to study the interactions between demographic changes and financial frictions, particularly as borrowing constraints may bind differently across cohorts.

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Table 1: Estimated Parameters

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
ϵ_p	U	0.1	0.0	0.2	0.01	0.01	0.01	0.02
$400 \times (z-1)$	N	2.0	1.1	3.0	1.34	1.35	1.24	1.46
ρ_χ	B	0.5	0.3	0.7	0.94	0.94	0.93	0.95
ρ_μ	B	0.5	0.3	0.7	0.76	0.73	0.55	0.83
ρ_θ	B	0.5	0.3	0.7	0.96	0.96	0.95	0.97
ρ_g	B	0.5	0.3	0.7	0.95	0.95	0.94	0.97
ρ_κ	B	0.5	0.3	0.7	0.94	0.94	0.93	0.95
$100 \times \sigma_\chi$	IG	1.2	0.5	3.7	2.15	2.18	1.95	2.47
$100 \times \sigma_\mu$	IG	1.2	0.5	3.7	0.48	0.52	0.33	0.73
$100 \times \sigma_\theta$	IG	1.2	0.5	3.7	3.51	3.56	3.20	4.03
$100 \times \sigma_i$	IG	1.2	0.5	3.7	0.16	0.16	0.15	0.18
$100 \times \sigma_g$	IG	1.2	0.5	3.7	1.05	1.06	0.98	1.15
$100 \times \sigma_\kappa$	IG	1.2	0.5	3.7	1.00	1.04	0.86	1.25

Table 2: Variance Decomposition Due to Shocks, %

Shock / Variable	Preference	Technology	Markup	Policy	Government	Investment
A. Conditional, 4 Quarter Ahead						
Fed Funds Rate	28.5	7.8	5.8	22.9	8.3	26.8
Inflation	21.9	2.4	42.1	8.9	6.4	18.4
Wages	6.8	2.6	74.8	10.5	0.6	4.8
Output	0.8	0.3	47.7	4.3	19.6	27.4
Consumption	28.0	0.7	26.7	6.8	26.7	11.2
Investment	7.2	0.1	38.8	2.3	0.8	50.8
B. Unconditional						
Fed Funds Rate	34.6	1.8	28.7	5.6	9.7	19.6
Inflation	20.5	1.1	55.7	4.2	5.8	12.7
Wages	1.7	0.4	93.0	2.0	0.2	2.7
Output	4.2	0.0	70.9	0.7	6.2	18.0
Consumption	7.0	0.1	70.0	1.0	13.6	8.4
Investment	8.6	0.0	59.1	0.5	1.2	30.5

Figure 1: Macroeconomic Trends

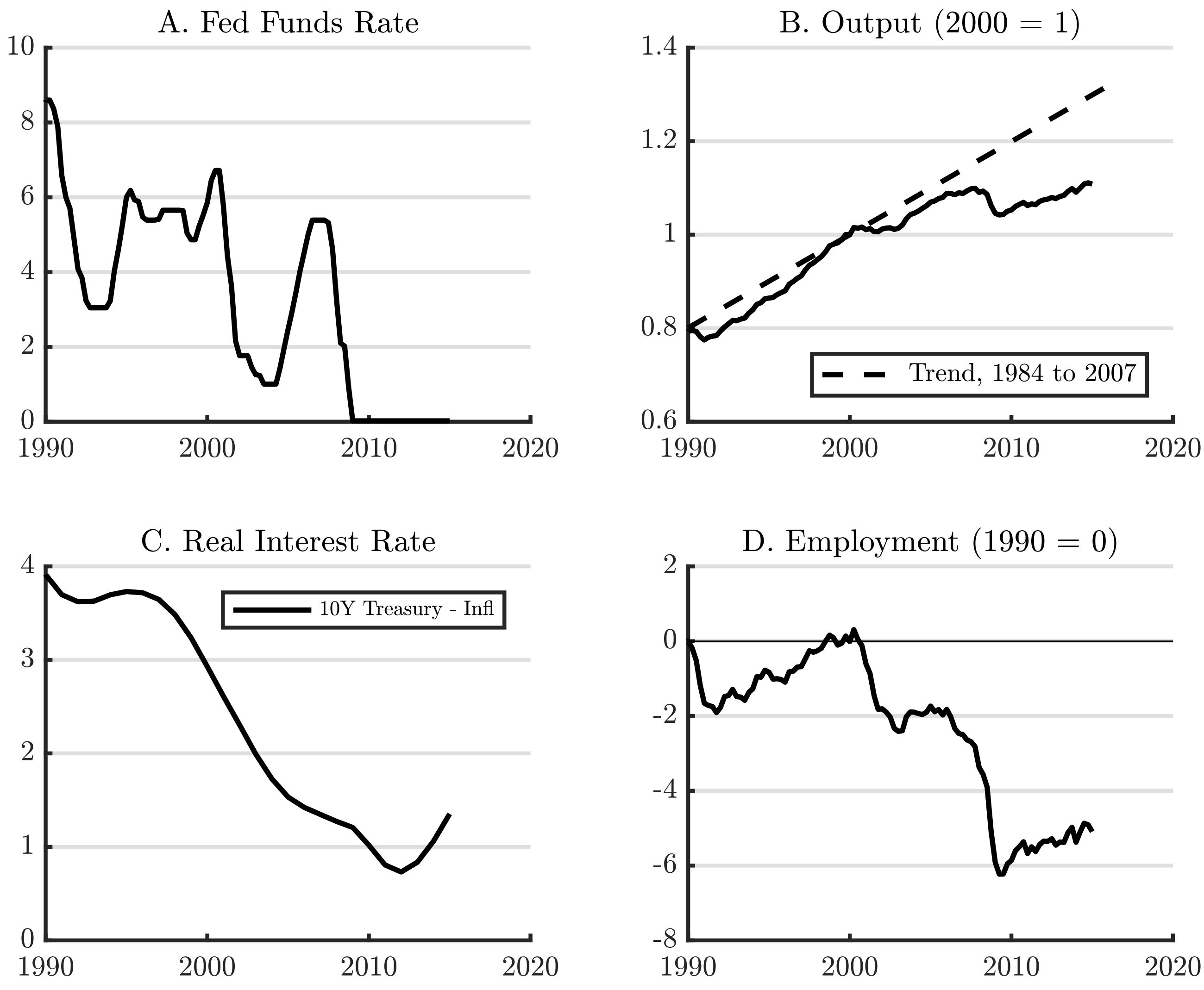
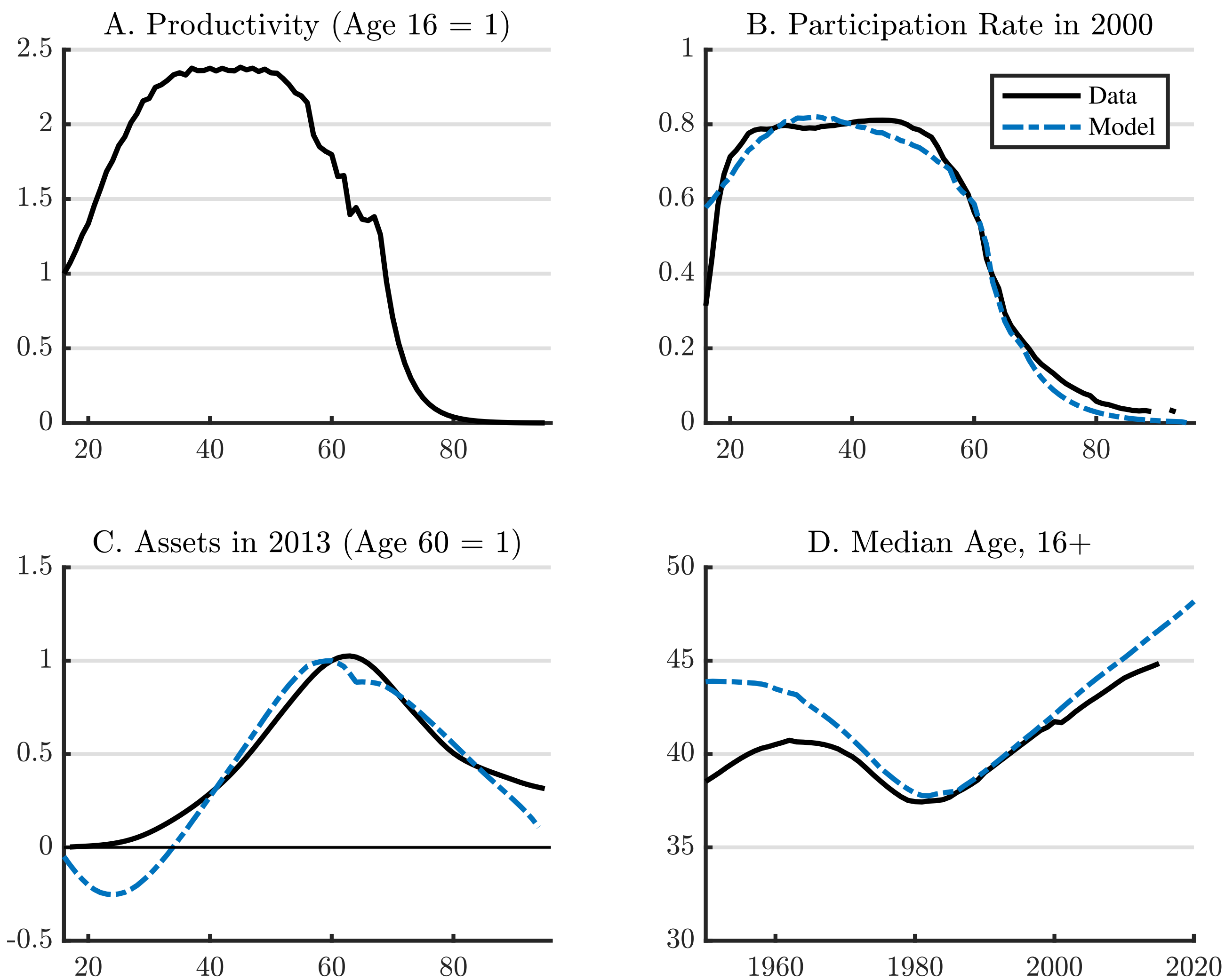


Figure 2: Individual Profiles



Notes: The data for the asset profile is taken from the Survey of Consumer Finances for 2013, HP-filtered, and normalized to the values of assets when 60. The productivity profile is computed from pooled Census and American Community Survey datasets. All details are in the Appendix.

Figure 3: Path of Variables, Demographics Only

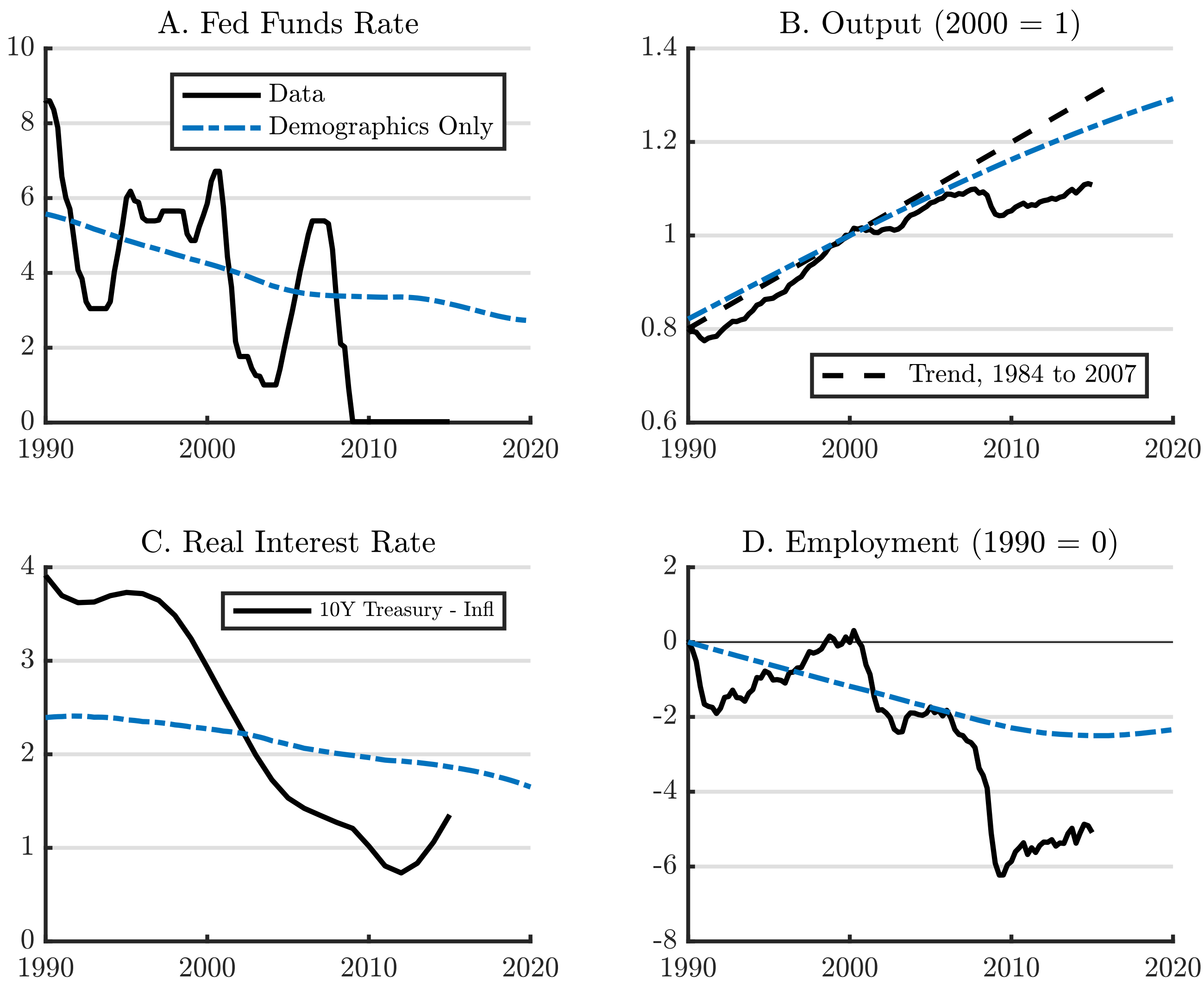
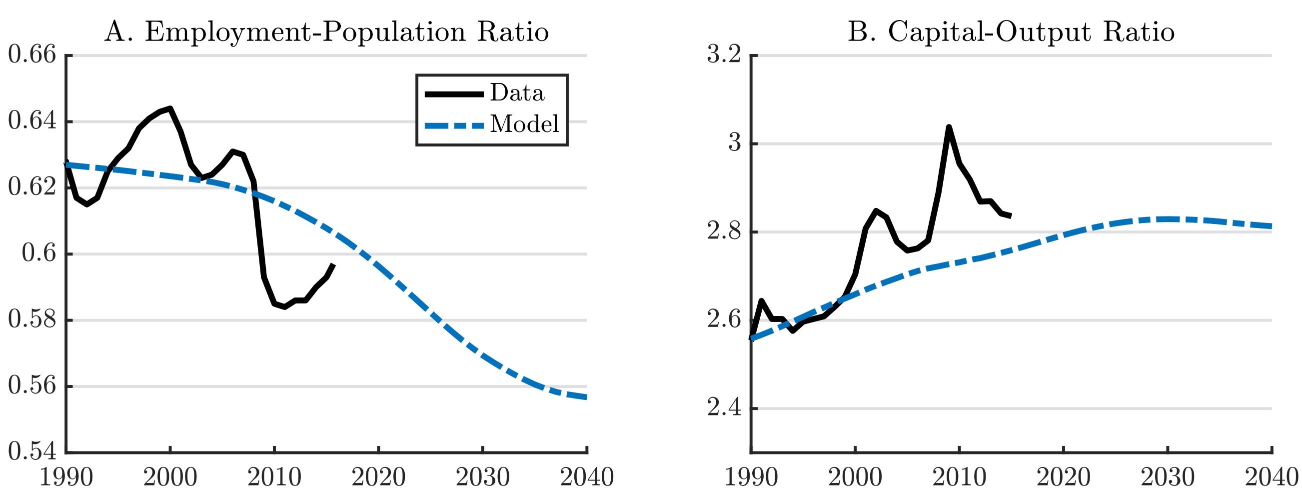


Figure 4: Model Trends



Notes: The data for the capital-output ratio is extracted from the BLS's Multifactor Productivity program. All details are in the Appendix.

Figure 5: Impulse Response to Large Investment Shock

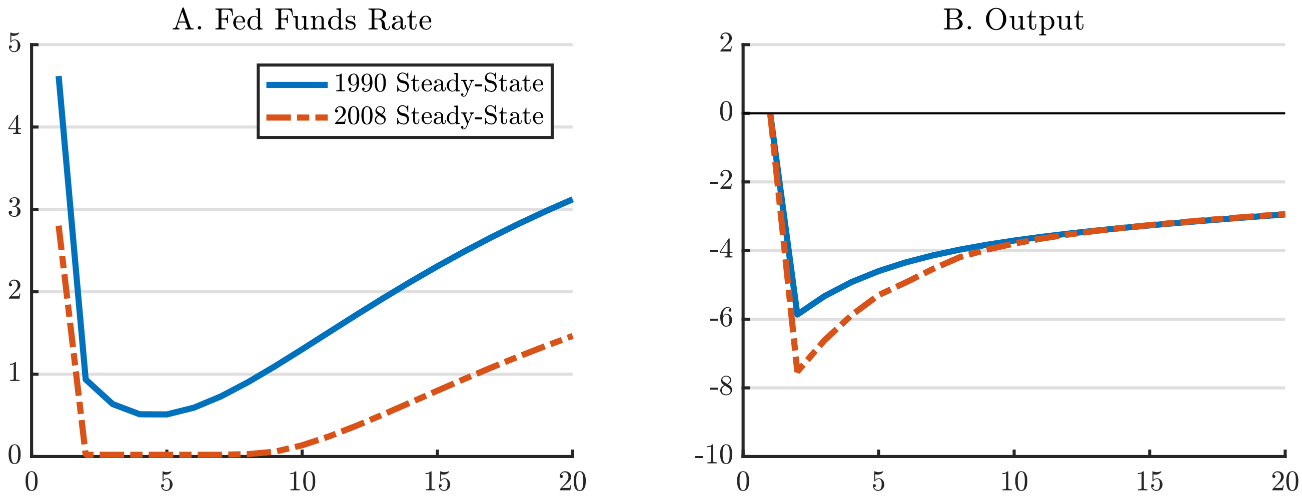


Figure 6: Path of Variables, Demographics Fixed from 1984

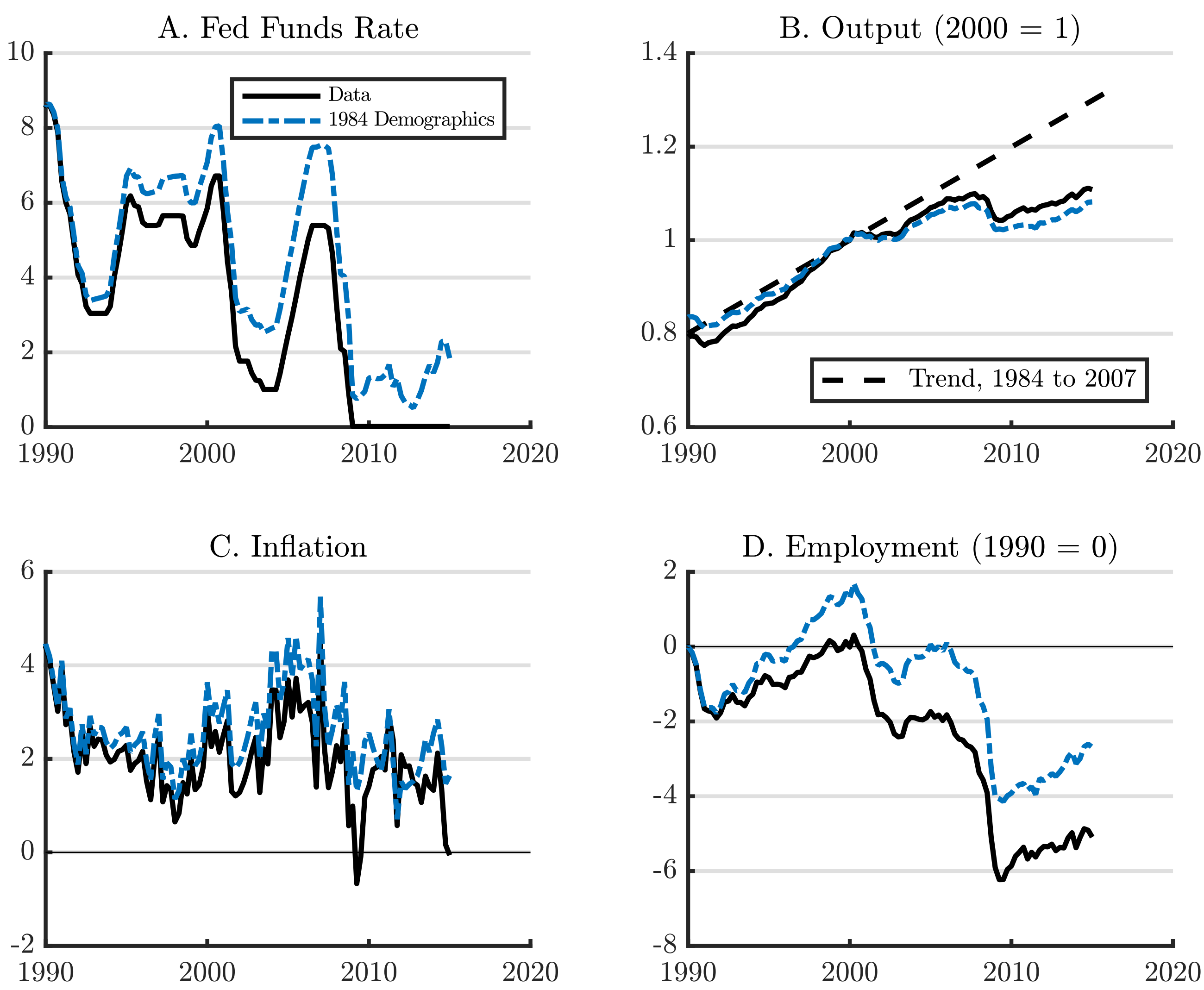


Figure 7: Path of Variables, No Forward Guidance

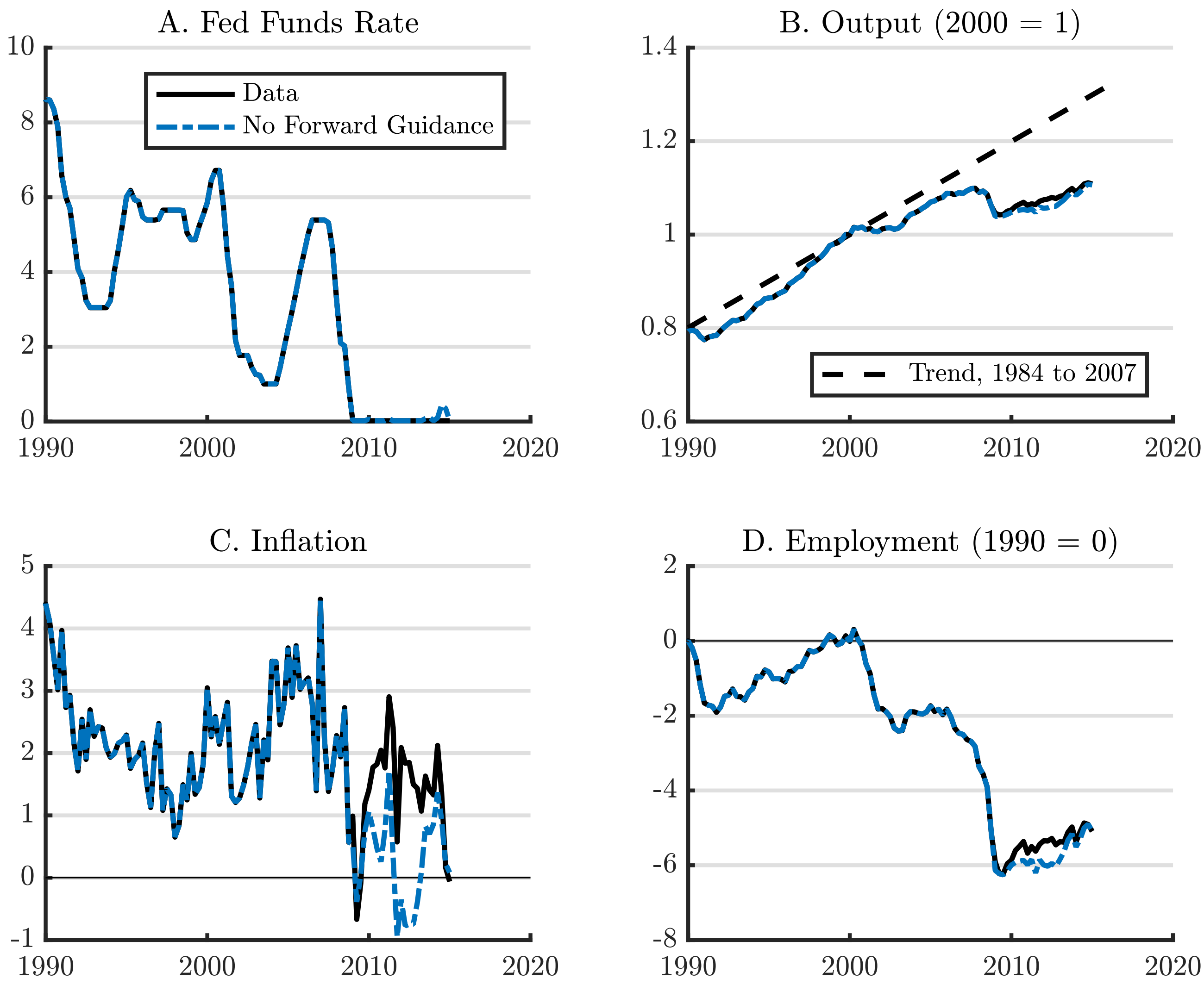


Figure 8: Output Relative to Trend

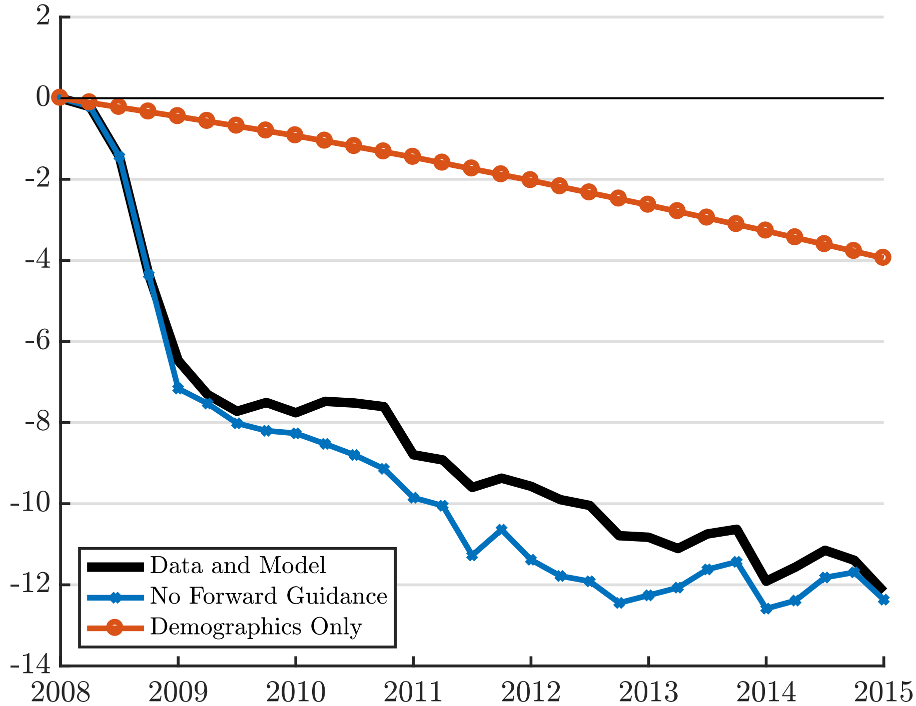
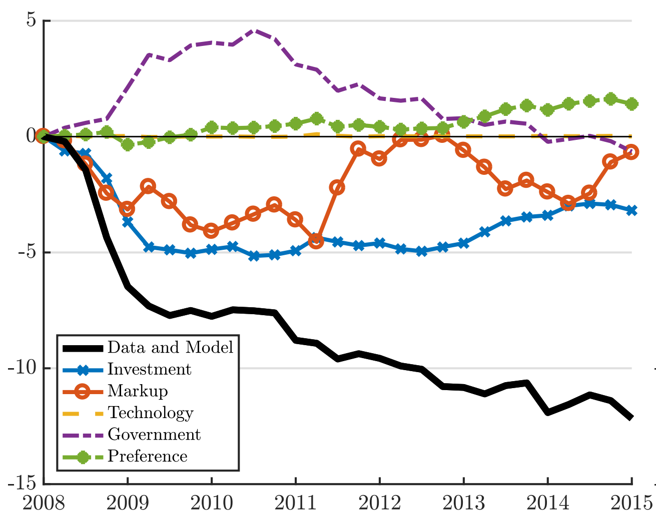


Figure 9: Output Relative to Trend, Contribution of Shocks

(a) Endogenous Zero Lower Bound Durations



(b) Fixed Zero Lower Bound Durations