International Spillovers of Unconventional Monetary Policy*

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Abstract

We measure how forward guidance in the United States at its zero lower bound affects a small open economy. Using piecewise linear methods, we jointly estimate the structural parameters of a two-country model and the expected durations of the fixed interest rate regimes in the US and Canada. We decompose the durations into a component implied by the constraint itself and calendar-based forward guidance. We conduct counterfactuals and find that without forward guidance in the US, monetary policy in Canada would not have been as constrained by its effective lower bound because its exchange rate would have appreciated significantly less.

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1 Introduction

In response to the financial crisis which originated in the United States, the Federal Reserve cut its policy interest rate to near zero. As the financial crisis spread, a number of small open economies followed suit, also cutting their policy rates to near zero. The Bank of Canada, for example, reduced its policy rate to 0.25 per cent in April of 2009 and the Bank of England set its official Bank Rate to 0.5 per cent in March of 2009. Since 2009, Denmark, Sweden and Switzerland have also lowered their policy interest rates to zero and, more recently, to just below zero.[1]

Soon after running up against their effective lower bounds, central banks turned to unconventional policies. One unconventional policy which has been used and has received considerable attention is forward guidance. Forward guidance refers to announcements made by the central bank about the future path of its policy rate with the explicit aim of influencing expectations so as to increase the degree of policy accommodation.[2]

There is a good argument in theory for why forward guidance can alleviate the contractionary impact of the zero lower bound (ZLB). In forward-looking *closed economy* models the current stance of monetary policy depends on the expected path of the nominal interest rate. Therefore forward guidance can, in principle, stimulate aggregate demand to the extent it lowers private agents' forecasts of future nominal interest rates. So, a credible commitment to maintain interest rates at zero for longer than would have otherwise been implied by the zero bound itself represents an additional channel of monetary stimulus. Eggertsson and Woodford (2003), Jung et al. (2005) and more recently Werning (2012) all make this point: monetary policy can stimulate an economy by creating the right kind of expectations about the way the policy rate will be used once the constraint ceases to bind.[3]

Forward guidance in a *small open economy* also works to provide monetary stimulus. Its impact,

[1]It does not matter for our purposes if this bound is zero or some other number. In any case it is clear that with physical cash whose rate or return is zero, there must exist some lower bound on the nominal interest rate that a central bank can set. No equilibria exist for nominal interest rates beyond that bound given predominant current legal arrangements that establish that balances at the central bank may be converted into notes at an exchange rate of unity. In the paper, we use the more familiar term 'zero lower bound' and the more precise term 'effective lower bound' interchangeably.

[2]Following the classification of Campbell et al. (2012), forward guidance that commits the central bank to a particular path of its policy rate beyond that implied by a policy interest rate rule is *Odyssean* guidance. This is different than *Delphic* forward guidance, which is an announcement that is informative about the state of the economy. Odyssean announcements can be calendar-based, with the announcement ending at a specified time period, or threshold-based, with the policy rate held at zero so long as specified thresholds are not breached. Our focus is on calendar-based Odyssean forward guidance policies near the zero lower bound.

[3]Krugman (1998) was the first to recast the liquidity trap as an expectations-driven phenomenon.

however, is not independent of the stance of monetary policy abroad. It therefore matters when calibrating unconventional policies in an open economy to be able to measure if foreign forces as opposed to domestic ones are responsible for driving the open economy to, or keeping it at, its ZLB. For a small open economy there are two relevant bounds highlighted in the literature: the bound that applies to the foreign economy's policy rate and the one that applies to its own policy rate.

Forward guidance abroad at the foreign ZLB lowers the expected future path of foreign policy rates inducing an appreciation of the home currency. Because the appreciation reduces imported goods inflation it has the potential to drive an inflation targeting central bank to its lower bound. The appreciation induces a substitution towards goods produced in the foreign economy, as they become relatively cheaper. But foreign forward guidance also operates through a foreign demand channel for the home economy's output, a channel emphasized by Bodenstein et al. (2009), Haberis and Lipinska (2012) and Fujiwara et al. (2013). The net effect of forward guidance spillovers on the demand for domestic output is determined by the *trade elasticity* of substitution. When domestic goods easily substitute for foreign goods, the decline in foreign demand for domestic goods caused by the exchange rate appreciation outweighs the effect of stronger demand for domestic goods from a stimulated foreign economy – a 'beggar-thy-neighbor' effect. But when domestic and foreign goods are complements, higher demand from the foreign economy can outweigh the substitution effect caused by the exchange rate appreciation, so that the home economy can benefit from forward guidance abroad. Because of the net effect is uncertain, it is an empirical issue, that we tackle in this paper, to establish the sign and magnitudes of the responses of domestic variables to foreign forward guidance.

Cook and Devereux (2014) highlight another reason why the ZLB and forward guidance may work differently in open economies. In an open economy at the ZLB, the exchange rate can exacerbate the adverse effects of domestic shocks. In normal times, i.e. away from the lower bound, floating exchange rates adjust to buffer the effects of these shocks. At the ZLB, however, domestic monetary policy cannot respond to adverse shocks and so the exchange rate appreciates more than it would otherwise. The lack of a domestic monetary policy response reinforces the contraction.[4]

The papers highlighted are part of a literature that studies *in theory* the ZLB and forward guidance in open economies. Taken together, they illustrate that there are possibly competing

[4]More generally, the unresponsiveness of the policy interest rate when combined with unanticipated shocks can change not only the magnitude of the response of the system, as pointed out by (Christiano et al., 2011), but also the sign of as shown by (Wieland, 2014; Eggertsson, 2010).

mechanisms which govern the strength and direction of cross-country spillovers of monetary policy at the ZLB. To the best of our knowledge there are no empirical studies including the period of unconventional monetary policy that attempts to quantify these mechanisms in the context of a structural open economy model.[5] This paper fills this gap.

Our objective is to measure the spillovers of forward guidance from large to small economies as well as assessing the impact of domestic forward guidance itself.[6] To do so, we set up a two-country small open economy model based on Galí and Monacelli (2005) and estimate jointly the parameters of the model and the expected durations of the fixed interest rate regimes of the US and Canada for each quarter since 2009Q1.[7] Estimating forward-looking structural models at the ZLB requires accounting for a regime shift in policy. In estimation we follow Kulish et al. (2014) and allow, but do not require, central banks to engage in forward guidance that extends the expected duration of the fixed interest rate policy beyond that implied by the bound itself.

A methodological innovation of our analysis is to combine estimation at the ZLB with the piecewise linear method to handle occasionally binding constraints proposed by Jones (2015). We handle two occasionally binding interest rate constraints by exploiting the block-exogenous structure of the two-country small open economy model. We identify, first, foreign ZLB durations, and with those expected durations in hand, we then identify domestic ZLB durations.[8] The method for computing equilibria subject to occasionally binding constraints allows us to decompose the estimated durations of the fixed interest rate regime into a component due to structural shocks and one which has the interpretation of an extension of the fixed interest rate duration. We use this decomposition to study the contribution of US forward guidance to fluctuations in output, inflation and the exchange rate in Canada.

As the theory emphasizes that the impact of foreign forward guidance on a small open economy depends on the trade elasticity of substitution, we set a wide prior over this parameter. We find a posterior mode of the trade elasticity of substitution in line with the estimates of Justiniano and Preston (2010). We then assess the impact of foreign forward guidance across the entire posterior distribution and find that it exerts a contractionary effect on Canada. So while it is possible in

[5] Ammer et al. (2016) study monetary policy spillovers of conventional policy. See also Eggertsson et al. (2016) for an analysis of spillovers under conditions of secular stagnation at the zero lower bound.

[6] We capture other unconventional policies such as quantitative easing but only to the extent suggested by Bernanke and Reinhart (2004) that they may influence expectations of the future path of policy rates.

[7] In the case of Canada we estimate forward guidance at one per cent from 2010Q3 onwards.

[8] The method for handling two constraints can be generalized further to models that lack block exogeneity as well as to models like that of Philippon and Midrigan (2011) that have more than two occasionally binding constraints.

theory for foreign forward guidance to be expansionary for the domestic economy, this does not seem to be an empirically relevant case for Canada. We estimate that forward guidance by the US and Canada during our ZLB sample raised the rate of inflation and the level of output of both economies. In a counterfactual exercise, we remove Canadian forward guidance and find that it would have taken 2 years longer for Canadian output to reach its pre-crisis level and that annualized Canadian inflation would have been on average 1.5 percentage points lower.

We estimate that the expected durations of the fixed interest rate policy were longer for the US than for Canada. But decomposing these durations reveals that the fraction of the duration that is due to forward guidance – the duration which is not explained by shocks – is relatively larger in Canada. In a counterfactual exercise, we remove from the estimated durations the component due to forward guidance and find that the Canadian exchange rate depreciates. This suggests forward guidance policy in the US has been relatively more expansionary than that of Canada.

We make the following additional contributions to the open-economy literature. For samples that incorporate the period of unconventional monetary policy, that go beyond 2009, we show how solution and estimation methods may be extended and applied efficiently to understand monetary policy and the impact of foreign and domestic shocks in small open economies. A contribution of the paper is to measure monetary policy, not via the level of interest rate, but through the decomposition of the durations. We decompose the duration of the ZLB in each quarter after 2009 into a duration which is implied by the lower bound itself and an additional duration which we identify as forward guidance. This additional duration captures other factors that may influence the expected duration like explicit announcements or balance sheet expansions made by the monetary authority that change the expected path of policy rates. By decomposing the duration we are able to address the issues highlighted by Fratto and Uhlig (2014) regarding the importance of measuring expectations of future policy at the ZLB.

Forward guidance may well become a part of the monetary policy toolkit, in which case these methods are useful. But even if monetary policy were not to run up against its lower bound ever again, these methods are useful because future samples of data will still contain a long spell of low and fixed interest rates.

The rest of the paper is structured as follows. Section 2 outlines the model. Section 3 discusses the solution and estimation strategy used. Section 4 discusses through the estimation results while Section 5 presents the cross-country effects of forward guidance in our estimated model. Section 6 uses counterfactuals to measure the impact of forward guidance and lower bound constraints. Section 7 concludes.

2 The model

We conduct our analysis using a small open economy model along the lines of Galí and Monacelli (2005). The model features two economies: a large (foreign) economy and a small (domestic) economy. Economic developments in the large economy affect the small economy, but the reverse is not true. As in De Paoli (2009), the model can be thought of as the limiting case of a two-country model as the relative size of one of the economies goes to zero. We extend this otherwise standard framework in four dimensions: (i) we include imperfect exchange rate pass-through; (ii) we allow for trend inflation; (iii) we incorporate interest rates of longer maturities; and (iv) we include habits in the utility function. The final set of log-linear equations used in estimation is given in the appendix.

2.1 The Large Economy

Variables with a star superscript ($*$) correspond to the large economy. This economy is populated by a large number of households who maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi_t^* \left(\log(C_t^* - hC_{t-1}^*) - \frac{N_t^{*\ 1+\varphi}}{1+\varphi} \right) \right],$$

where N_t^* is labor supply and ξ_t^* denotes intertemporal preference shocks that follow an AR(1) process in logs. C_t^* is a composite consumption index given by $C_t^* = \left[\int_0^1 C_t^*(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}$, where ν is the elasticity of substitution between types of differentiated goods. Households face in every period t the flow budget constraint:

$$P_t^* C_t^* + \mathbb{E}_t \left\{ Q_{t,t+1}^* D_{t+1}^* \right\} \leq D_t^* + W_t^* N_t^* + T_t^*,$$

for all $t > 0$, where P_t^* is the large economy's CPI, W_t^* is the nominal wage rate and T_t^* denotes taxes and transfers. D_{t+1}^* is the nominal payoff in period $t+1$ of the portfolio held at the end of period t and $Q_{t,t+1}^*$ is the stochastic discount factor for one-period ahead nominal payoffs. Households have access to a complete set of internationally traded contingent claims.

Firms produce differentiated goods with the technology:

$$Y_t^*(i) = Z_t N_t^*(i),$$

where $Y_t^*(i)$ is the production and $N_t^*(i)$ the labor input of firm i. Z_t is total factor productivity, which follows a random walk with drift, μ, in logs. Real marginal costs are common across firms and given by:

$$MC_t^* = \frac{W_t^*}{P_t^* A_t^*}.$$

Firms face Calvo-style pricing frictions. Each quarter, a fraction of firms, $1-\theta^*$, sets prices optimally, while the remainder adjusts their prices by the steady-state inflation rate, Π^*. The pricing problem for a representative firm i is:

$$\max_{P_t^*(i)} \sum_{k=0}^{\infty} (\beta\theta^*)^k \mathbb{E}_t \left\{ \frac{\Lambda_{t+k}^* P_t^*}{\Lambda_t^*} \left[\frac{P_t^*(i)\,(\Pi^*)^k}{P_{t+k}^*} Y_{t+k}^*(i) - MC_{t+k}^* Y_{t+k}^*(i) \right] \right\},$$

subject to the demand constraint $Y_{t+k}^*(i) = \left[\frac{P_t^*(i)(\Pi^*)^k}{P_{t+k}^*}\right]^{-\chi} Y_{t+k}^*$, where Y_{t+k}^* is aggregate output. In the expression above Λ_t^* is the shadow value of an additional unit of income in period t. Goods market clearing in the large economy requires that all production is consumed, that is $Y_t^* = C_t^*$.

Monetary policy follows an interest rate rate rule that responds to inflation, output growth and the deviation of the level of output from trend, subject to a lower bound:

$$\frac{R_t^*}{R^*} = \max \left\{ \bar{R}^*, \left[\frac{R_{t-1}^*}{R^*}\right]^{\rho_R^*} \left[\left(\frac{\Pi_t^*}{\Pi^*}\right)^{\phi_\pi^*} \left(\frac{Y_t^*}{Y_{t-1}^*\mu}\right)^{\phi_g^*} \left(\frac{Y_t^*}{Z_t}\right)^{\phi_y^*} \right]^{1-\rho_R^*} \exp(\varepsilon_{R,t}^*) \right\}, \tag{1}$$

where R_t^* is the policy rate in the large economy, $R^* = \mu\Pi^*/\beta$ is the steady-state policy rate and $\varepsilon_{R,t}^*$ is a monetary policy disturbance. The monetary policy rule, (1), lets the lower bound of the nominal interest rate, $\bar{R}^*$, to be different from zero. This specification is useful to handle fixed interest rates regimes where the effective lower bound on the policy rate is not zero.

Longer-term interest rates are determined via the expectations hypothesis. We link model longer-term interest rates to observed longer-term interest rates following Graeve et al. (2009). For any maturity $m > 1$:

$$R_{m,t}^{*,\text{obs}} = R_{m,t}^* \exp\left(c_m^* \eta_t^* \varepsilon_{m,t}^*\right),$$

where $R^*_{m,t}$ is the interest rate on a bond that pays one unit of the large economy's currency in m quarters as determined by the expectations hypothesis, c^*_m is a constant risk premia on the m quarter interest rate, η^*_t is shock, common to all interest rates in the large economy, that follows exogenous autoregressive process and $\varepsilon^*_{m,t}$ is an idiosyncratic shock to the m quarter interest rate in the large economy. Because the expectations hypothesis holds, longer-term nominal interest rates in the model, $R_{m,t}$, are also subject to the lower bound of the short-rate R^*_t.

2.2 The Small Economy

The structure of the small economy is similar to that of the large economy, except for the fact that households can consume goods and services produced abroad and firms can sell their output overseas as well as domestically. Unless stated otherwise, variables have the same interpretation as in the large economy.

2.2.1 Households

The small economy is populated by a representative household that maximizes the expected present discounted value of lifetime utility, given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi_t \left(\log(C_t - hC_{t-1}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right],$$

subject to the flow budget constraint in every period t:

$$P_t C_t + \mathbb{E}_t \left\{ Q_{t,t+1} D_{t+1} \right\} \leq D_t + W_t N_t + T_t.$$

As in the large economy, households have access to a complete set of internationally traded contingent claims. Complete markets and household optimization implies the familiar risk-sharing condition linking the marginal utilities of consumption in the large and small economies, $\Lambda_t = \frac{\Lambda^*_t}{Q_t}$ where Λ_t is the marginal value of an additional unit of domestic income to households in the small economy, $Q_t = S_t P^*_t / P_t$ is the real exchange rate between the small and large economy and S_t is the nominal exchange rate between the small and large economy. The exchange rate is defined as the number of units of the small economy's currency required to purchase one unit of the large economy's currency. According to this definition, an increase in S_t corresponds to a depreciation of the domestic currency. To account for deviations from uncovered interest rate parity in estimation, we add a risk premium shock to the risk-sharing condition that follows an AR(1) process in logs.

2.2.2 Consumption Retailers

The final consumption good is assembled by perfectly competitive retailers using the technology:

$$C_t = \left[(1-\alpha)^{\frac{1}{\tau}} \left(C_{H,t}\right)^{\frac{\tau-1}{\tau}} + \alpha^{\frac{1}{\tau}} \left(C_{F,t}\right)^{\frac{\tau-1}{\tau}}\right]^{\frac{\tau}{\tau-1}},$$

where $C_{H,t}$ and $C_{F,t}$ are composite consumption indices of domestically- and foreign- produced final goods. The parameter τ is the elasticity of substitution between domestic- and foreign-produced goods. The price index corresponding to this bundle is:

$$P_t = \left[(1-\alpha)\left(P_{H,t}\right)^{1-\tau} + \alpha \left(P_{F,t}\right)^{1-\tau}\right]^{\frac{1}{1-\tau}},$$

where $P_{H,t}$ is the price of the domestic composite good and $P_{F,t}$ is the price of the imported composite good, both expressed in domestic currency.

2.2.3 Domestic Final Goods Retailers

The domestically-produced final good, $Y_{H,t}$ is assembled by a perfectly competitive final good retailer that combines domestically-produced intermediate goods using the technology:

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(i)^{\frac{\chi-1}{\chi}} di\right]^{\frac{\chi}{\chi-1}},$$

where χ is the elasticity of substitution between varieties of domestic intermediate goods. The price of the domestic final good is:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\chi} di\right]^{\frac{1}{1-\chi}}.$$

It follows that the final goods firm's demand for each variety is given by:

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\chi} Y_{H,t}.$$

2.2.4 Domestic Intermediate Goods Producers

Domestic intermediate goods producers manufacture heterogeneous goods using the technology:

$$Y_t(i) = Z_t A_t N_t(i),$$

where A_t is a stationary technology process that follows an AR(1) process in logs. Real marginal costs are equal across firms and given by:

$$MC_t = \frac{W_t}{P_t A_t}.$$

Firms face Calvo-style pricing frictions. Each quarter a fraction, $1-\theta$, of firms are able to adjust their prices freely. The remaining firms index their prices to the steady-state inflation rate, Π. The resulting pricing problem for firm i is

$$\max_{P_t(i)} \sum_{k=0}^{\infty} (\beta\theta)^k \, \mathbb{E}_t \left\{ \frac{\Lambda_{t+k} P_t}{\Lambda_t} \left[\frac{P_{H,t}(i) Y_{t+k}(i) \Pi^k}{P_{t+k}} - MC_{t+k} Y_{t+k}(i) \right] \right\},$$

subject to the domestic final goods demand condition given above.

2.2.5 Exporters

Exporters purchase the domestic final good at price $P_{H,t}$ and differentiate it through branding for sale in the foreign economy. The exporters are owned by domestic households. However, all export contracts and prices are specified in the currency of the large economy. An export retailer bundles these goods before selling them overseas according to the technology:

$$X_t = \left[\int_0^1 X_t(i)^{\frac{\nu_X - 1}{\nu_X}} (i) \right]^{\frac{\nu_X}{\nu_X - 1}},$$

where ν_X is the elasticity of substitution between different varieties for export. The corresponding price index, in foreign currency terms, is:

$$P^*_{X,t} = \left[\int_0^1 P^*_{X,t}(i)^{1-\nu_X} di \right]^{\frac{1}{1-\nu_X}}.$$

The export retailer faces the demand function:

$$X_t = \left(\frac{P^*_{X,t}}{P^*_t}\right)^{-\tau} Y^*_t,$$

As in Justiniano and Preston (2010) the elasticity of export demand, τ, is the same as the elasticity of substitution between domestic and foreign goods in the consumption basket. This assumes that household's preferences in the small and large economies are the same, which for the US and Canada is a reasonable assumption. It follows that the demand for each exporter's goods are given by:

$$X_t(i) = \left(\frac{P^*_{X,t}(i)}{P^*_{X,t}}\right)^{-\nu_X} X_t.$$

Exporters face Calvo-style pricing frictions, with $1-\theta_x$ governing the share of firms that are able to adjust their prices each quarter. The resulting pricing problem for firm i is:

$$\max_{P^*_{X,t}(i)} \sum_{k=0}^{\infty} (\beta\theta_x)^k \, \mathbb{E}_t \left\{ \frac{\Lambda_{t+1} P_t}{\Lambda_t} \left[\frac{P^*_{X,t}(i)\Pi^k X_{t+k}(i)}{S_{t+k}P_{t+k}} - \frac{P_{H,t+k}X_{t+k}(i)}{P_{t+k}} \right] \right\},$$

subject to the demand constraint given above.

Finally, market clearing for the domestic final good requires that all production is sold, either domestically or overseas. That is:

$$Y_{H,t} = C_{H,t} + X_t.$$

2.2.6 Importers

Importers bring in homogeneous products from abroad at price $S_t P^*_t$ and differentiate them through branding. Importers then sell the differentiated goods to a retailer that combines them into the final imported good using the technology:

$$C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\nu_F-1}{\nu_F}} di\right]^{\frac{\nu_F}{\nu_F-1}},$$

where $C_{F,t}(i)$ is the quantity of the imported good of variety i used in the production of the final imported good and ν_F is the elasticity of substitution between different imported good varieties.

The price index corresponding to the imported final good is:

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\nu_F} di \right]^{\frac{1}{1-\nu_F}}.$$

Consequently, each importer faces the demand curve:

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\nu_F} C_{F,t}.$$

Importers face Calvo-style pricing frictions. Each quarter, a fraction, $1 - \theta_m$ sets prices optimally, while the remainder adjusts their prices by the small economy's steady-state inflation rate, Π. The pricing problem for a representative firm i is

$$\max_{P_{F,t}(i)} \sum_{k=0}^{\infty} (\beta\theta_m)^k \, \mathbb{E}_t \left\{ \frac{\Lambda_{t+1} P_t}{\Lambda_t} \left[\frac{P_{F,t}(i) C_{F,t}(i) \Pi^k}{P_{t+k}} - \frac{S_{t+k} P^*_{t+k} C_{F,t+k}(i)}{P_{t+k}} \right] \right\},$$

subject to the demand constraint above.

2.2.7 Monetary Policy

The domestic central bank follows a Taylor Rule and is subject to a lower bound parameterized by $\bar{R}$:

$$\frac{R_t}{R} = \max \left\{ \bar{R}, \left[\frac{R_{t-1}}{R} \right]^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}\mu} \right)^{\phi_g} \left(\frac{Y_t}{Z_t} \right)^{\phi_y} \right] e^{\varepsilon_{R,t}} \right\}. \tag{2}$$

As in (1) we allow for the lower bound of the nominal interest rate, $\bar{R}$, to be different from zero. This is useful in Canada which has fixed its policy rate at 1% for about 5 years in our sample.

The term structure of interest rates in the small economy is determined in a similar manner to the large economy. For any $m > 1$:

$$R^{\text{obs}}_{m,t} = R_{m,t} \exp\left(c_m \eta_t \varepsilon_{m,t} \right),$$

where $R_{m,t}$ is the interest rate on a bond that pays one unit of domestic currency in m quarters as determined by the expectations hypothesis, c_m is a constant risk premia on the m quarter interest rate, η_t is shock, common to all small economy interest rates, that follows an exogenous autoregressive process and $\varepsilon_{m,t}$ is an idiosyncratic shock to the m quarter interest rate in the small economy.

3 Solution and estimation methods

3.1 Solution for a given ZLB duration

We use the solution methods proposed in Cagliarini and Kulish (2013), Kulish and Pagan (forthcoming) and Jones (2015) and the estimation method of Kulish et al. (2014). Because these methods have more general application than the context we are considering, we discuss how the methods apply to our case. The appendix provides additional details.[9]

To estimate the model we take a sample of data of size T. At a given point in the sample, the system can be in one of the following four possible regimes: i) lower bounds are non-binding, ii) only the lower bound of the large economy binds, iii) only the lower bound of the small economy binds and, iv) both lower bounds bind. Figure 1 illustrates one possibility, in which in an initial sub-sample conventional policy applies to both economies, then there is a period of time for which the ZLB binds only in the large economy. After that the ZLB binds in both economies eventually there is a return to conventional policy which takes place out-of-sample.

We first linearize the model around the steady state for which the ZLBs do not bind and write the resulting system of equations in matrix form as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}w_t. \tag{3}$$

where x_t is the state vector and w_t is the vector of structural shocks, which we take to be *iid* without loss of generality. If it exists and is unique, the standard rational expectations solution to (3) is $x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}w_t$.

When only the *foreign* ZLB binds the structural equations are given by:

$$\mathbf{A}^* x_t = \mathbf{C}^* + \mathbf{B}^* x_{t-1} + \mathbf{D}^*\mathbb{E}_t x_{t+1} + \mathbf{F}^* w_t. \tag{4}$$

where the only equation that has changed in the starred system relative to (3) is the equation defining the *foreign* policy interest rate rule, which is now specified such that the nominal interest rate is fixed at its bound.[10]

[9] The approach can be applied to a range of problems in which one requires to solve a model subject to occasionally binding constraints or anticipated changes in the structural parameters efficiently.

[10] Notice that the notation can accommodate additional structural changes which have to be accounted for if the expansion point of the approximation changes. In our application we work around the intended steady state.

When only the *domestic* ZLB binds the structural equations are given by:

$$\bar{\mathbf{A}}x_t = \bar{\mathbf{C}} + \bar{\mathbf{B}}x_{t-1} + \bar{\mathbf{D}}\mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}w_t. \tag{5}$$

where the only equation that has changed relative to (3) is the equation defining the *domestic* policy interest rate rule, which is now specified such that the nominal interest rate is fixed at its bound.

And when both *foreign* and *domestic* lower bounds bind the structural equations are given by:

$$\bar{\mathbf{A}}^* x_t = \bar{\mathbf{C}}^* + \bar{\mathbf{B}}^* x_{t-1} + \bar{\mathbf{D}}^*\mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}^* w_t. \tag{6}$$

Because at least one nominal interest rate is fixed in regimes (4), (5) or (6), if any of these regimes were expected to prevail indefinitely in our model, they would be found to be inconsistent with a unique rational expectations solution.[11] However, provided both nominal interest rates are expected to be governed by policy rules consistent with a unique solution, there is a unique path under temporarily binding constraints (Cagliarini and Kulish, 2013).

In general, the structural equations in-sample may be written:

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \mathbb{E}_t x_{t+1} + \mathbf{F}_t w_t. \tag{7}$$

where $\{\mathbf{A}_t, \mathbf{C}_t, \mathbf{B}_t, \mathbf{D}_t, \mathbf{F}_t\}_{t=1}^{T}$ is the sequence of structural matrices over the sample. For example, if ZLBs do not bind at time t, then $\mathbf{A}_t = \mathbf{A}$, $\mathbf{C}_t = \mathbf{C}$, $\mathbf{B}_t = \mathbf{B}$, and so. If both constraints were to bind then $\mathbf{A}_t = \bar{\mathbf{A}}^*$, $\mathbf{C}_t = \bar{\mathbf{C}}^*$, $\mathbf{B}_t = \bar{\mathbf{B}}^*$, and so on.

Now suppose that at period t' ZLB constraints are not expected to bind from some future period T' onwards. Agents anticipate how the structural matrices will evolve until then. Say agents foresee that the structural equations will evolve as follows $\{\mathbf{A}_t, \mathbf{C}_t, \mathbf{B}_t, \mathbf{D}_t, \mathbf{F}_t\}_{t=t'}^{T'}$. Following Kulish and Pagan (forthcoming), the solution may be written as a time-varying VAR of the form:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t, \tag{8}$$

given perfect foresight of the evolution of the structure of the economy. For this to be a rational expectations solution, expectations must satisfy (7) in all periods t. Equation (8) implies that

[11] This is the case in our model where monetary policy is 'active' in the terminology of Leeper (1991). As emphasized recently by Cochrane (2014), a fixed interest rate need not imply indeterminacy in models with 'active' fiscal policy.

expectations are $\mathbb{E}_t x_{t+1} = \mathbf{J}_{t+1} + \mathbf{Q}_{t+1} x_t$. Substituting expectations into (7) implies that at period t:

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \left(\mathbf{J}_{t+1} + \mathbf{Q}_{t+1} x_t \right) + \mathbf{F}_t w_t.$$

Which implies by undetermined coefficients the following recursions:

$$\begin{aligned}
\mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1} \right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1} \right) \\
\mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1} \right]^{-1} \mathbf{B}_t \\
\mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1} \right]^{-1} \mathbf{F}_t.
\end{aligned}$$

The backward recursion for the sequence of time-varying reduced form matrices, $\{\mathbf{Q}_t\}_{t=t'}^{T'-1}$ starts from the terminal condition $\mathbf{Q}_{T'} = \mathbf{Q}$ and works its way back to period t', yielding the system of time-varying reduced form matrices corresponding to (8). With the sequence for $\mathbf{Q}_t$ in hand, the sequence for $\mathbf{G}_t$ may computed as well as the sequence for $\mathbf{J}_t$ given the terminal condition $\mathbf{J}_{T'} = \mathbf{J}$. Note that the terminal conditions, $\mathbf{J}$ and $\mathbf{Q}$, correspond to the solution matrices to the model around the steady state for which the ZLBs do not bind, consistent with the assumption that agents do not expect constraints to bind from period T' onwards.

3.2 The ZLB algorithm: endogenous durations

Equation (8) is the solution for the case in which agents anticipate given durations at time period t'. These durations, however, may or may not be those implied by the constraint itself and the structural shocks that have hit the economy up to t'. Jones (2015) proposes an efficient algorithm to find at each point in time the duration which is consistent with the constraint itself by using the solution above iteratively under the assumption of no future shocks. Indeed, unanticipated shocks at time period $t' + 1$ could imply anticipated ZLB durations at $t' + 1$ that are different to those that were expected at t'.

We refer to this duration as the *endogenous* duration and compute it at time period t' as follows. We describe the algorithm as it applies to the large economy. We know the state $x_{t'-1}$ and the shocks $w_{t'}$. We first compute a forecast under the assumption that the ZLB constraint does not bind. We then examine the forecast path of the nominal rate for violations of the constraint. If the path of the nominal interest rate contains values below the bound, we set the nominal interest rate to its bound but only for the *first* period that the bound is violated. In that first period the nominal interest rate can be thought to be governed by a different policy rule and so from the perspective of t' it corresponds to a given ZLB duration. This means the solution we discussed above can be applied. That solution implies a new forecast path for the nominal interest rate that must be examined for additional violations. Iterating in this way we arrive at a duration of the ZLB at t' which is consistent with state $x_{t'-1}$ and shocks $w_{t'}$. As shocks unfolds in $t'+1$ one recomputes the duration. The solution satisfies the constraint and captures, at each point in time, agents' forecasts of when the constraint is expected to bind. The appendix describes the algorithm in full. Jones (2015) shows that this solution constitutes a good approximation to the non-linear economy.

The block-exogeneity of the two-country model allows us to find the endogenous durations of the large economy first and then take those as given when computing the durations for the small economy. This means that we must apply the algorithm twice.

3.3 The Likelihood function

Since (8) is a linear system, the Kalman filter can be used to construct the likelihood. At the ZLB, implicit in the evolution of the structural matrices are expected sequences of ZLB durations. This means that the likelihood is a function both of the structural parameters and the sequences of expected durations. Following Kulish et al. (2014) we estimate jointly the structural parameters and anticipated durations of the fixed interest rate regimes in the large and small economies.[12] Importantly, as in Kulish et al. (2014), we do not impose in estimation the restriction that the anticipated durations must match the duration implied by shocks themselves. It is important to not impose those constraints in estimation in light of the optimal policy prescription of Eggertsson and Woodford (2003) of extending the duration of the zero interest rate policy beyond the horizon implied by the constraint itself.

3.4 Identification of forward guidance

We identify forward guidance as follows. We first estimate the model to get estimates of the durations and of the shocks. We then use the method in subsection 3.2 to compute the *endogenous durations* that correspond with the estimated shocks. For each quarter of the ZLB we decompose the total estimated duration into a duration implied by the constraint itself (the endogenous duration) and an additional duration. This additional duration captures other factors such as announcements

[12] Details of the Kalman filter and Kalman smoother are relegated to the appendix.

intended to extend the duration of the fixed interest rate regime beyond the period implied by the constraint itself.[13] We therefore identify forward guidance as the difference between the total estimated duration and the endogenous duration. This decomposition characterizes monetary policy at the ZLB and is useful since the level of the nominal interest rate at the ZLB is an insufficient statistic of the stance of monetary policy.

Figure 2 illustrates how the decomposition is computed at each point in time. Say that at period t' the estimated duration of the ZLB is 5 quarters, so that the ZLB is expected to bind from period t' to period $t'+4$. We use the Kalman smoother to obtain an estimate of the structural shocks $w_{t'}$ prevailing at time period t'. Using those shocks, we use the ZLB algorithm described above to obtain a forecast of the nominal interest rate. In Figure 2, we call this the 'endogenous forecast'. Associated with this forecast is an endogenous duration of the ZLB, illustrated by the $\{3, 2, 1\}$ sequence of durations, the black bars in the bottom panel of figure 2. Comparing this sequence with the sequence which would be expected given the estimated duration at t', in this case $\{5, 4, 3, 2, 1\}$, we identify forward guidance components of $\{2, 2, 2, 2, 1\}$. So at t' the total duration of 5 quarters is 3 quarters shocks and 2 quarters forward guidance. Unanticipated shocks that hit in $t'+1$ do not influence the decomposition at t', but may well change the decomposition relative to the $t'+1$ decomposition that was predicted at t'. So the decomposition is recomputed at each point in time.

4 Estimation

For estimation, we use US and Canadian nominal interest rates, output growth, and inflation, and changes in the US/Canadian nominal exchange rate. These seven quarterly data series are plotted in Figure 3. We also use yield curve data, using 6-month, 1-year, and 2-year nominal interest rates for both the US and Canada. The series start in 1984Q1 and end in 2014Q4. Prior to estimation, the data are demeaned by their model implied means.

4.1 Calibrated parameters

We calibrate a number of structural parameters in line with values from the literature. The quarterly discount factor β, common to both countries, is set at 0.995. The annual trend growth rate μ is

[13] Other factors beyond explicit central bank communication can influence the duration. As is well known, markets reassessed the time of lift-off as the Federal Reserve was tapering the rate of bond purchases (taper-tantrum). So, to the extent that quantitative easing, as emphasized by Bernanke and Reinhart (2004), or other factors change the expected duration, these are soaked up by the additional duration in our setup.

calibrated to 1.4 per cent. Both central banks are assumed to target an annual inflation rate of 2.0 per cent. These three parameters imply a steady-state annual nominal interest rate in both countries of 5.5 per cent. The consumption habit formation parameter h is set at 0.71, the posterior mode of the estimated habit parameter in Smets and Wouters (2007), while the inverse of the Frisch elasticity of labor supply ϕ, is calibrated to 1.5 in both countries, close to the values estimated in Smets and Wouters (2007) and Justiniano and Preston (2010). The export share of Canadian consumption, α is calibrated to 0.29 to match the average share of exports and imports to Canadian GDP.

The calibration of the slopes of the pricing equations, κ^* for the US, and κ, κ_f and κ_x for Canada, are important for the pass-through of foreign shocks to domestic inflation and output. We calibrate κ^* and κ to 0.02, which is close to the calibration of Ireland (2004) and in line with estimates of the Phillips curves for domestically produced goods in the US (Del Negro et al., 2015). We set κ_x to 1, which implies that roughly 60% of domestic exporters can change prices optimally each quarter, in line with the estimates of Justiniano and Preston (2010). Finally, we calibrate κ_f to 0.1, which says that roughly 25% of domestic importers can change prices optimally each quarter.

4.2 Effective lower bounds

We measure anticipated interest rate durations in the US from the time at which the Federal Funds rate reached 0.125% in 2009Q1 until the end of our sample 2014Q4. Similarly, we measure these durations in Canada when the policy rate was at 0.5% between 2009Q2 and 2010Q2, and then when the rate was at 1% between 2010Q3 and the end of the sample. This modeling choice is motivated by the clear communication of the Bank of Canada to providing forward guidance of its fixed interest rate policy during this period.[14] Strictly speaking, the methodology we use allows the Canadian central bank to lower the nominal interest rate below 1% following the conclusion of any anticipated lower bound duration and for agents to anticipate this to happen.

[14] In the May 2013 press release, the Bank of Canada said that '*With continued slack in the Canadian economy, the muted outlook for inflation, and the constructive evolution of imbalances in the household sector, the considerable monetary policy stimulus currently in place will likely remain appropriate for a period of time, after which some modest withdrawal will likely be required, consistent with achieving the 2 per cent inflation target.*'

4.3 Estimation results

4.3.1 Structural parameters

The prior distribution for each estimated parameter is shown in Table 1. We set equal prior distributions for parameters common to both countries. We use loose priors for the standard deviations of the shock processes for each country.[15] The estimates of the monetary policy rule will be important for accurately estimating anticipated forward guidance durations, as it is the monetary policy regime that the economy reverts to following a period of time at the interest rate lower bound which is important for governing expectations about the path of the interest rate, and therefore in determining the stimulatory effect of forward guidance. Given the Federal Reserve's emphasis on the subdued labor market outcomes over the 2009-2015 period, we allow for a rule that responds to the growth rate of GDP and to the detrended level of output. We put loose priors over these coefficients.

The estimation results are presented in Table 1. Figures C.2 and C.3 plot prior and posterior distributions for the structural parameters. Relative to the priors, the data finds persistent demand shocks in the US, and relatively less persistent demand and TFP shocks in Canada, consistent with the estimates on US-Australian data of Kulish and Rees (2011). US demand shocks are estimated to be roughly three times as large as permanent technology shocks, in line with the relative sizes of comparable shocks found in Ireland (2004). The posterior estimate of the trade elasticity τ is centered around 0.84, which is close to the estimate obtained by Justiniano and Preston (2010).

4.3.2 Expected durations

The posterior distributions for the anticipated durations of the US Fed Funds rate at 0.125% between 2009Q1 and 2014Q4, and the Bank of Canada's Bank Rate at 0.25% between 2009Q2 and 2010Q2 and 1% between 2010Q3 and 2014Q4 are in Table 2.[16]

The posteriors show that the anticipated durations are well identified. Interestingly, the data puts little weight on short durations in the US throughout the sample period, with the modal value of each posterior density around 6 to 7 quarters. These values are consistent with the results of Swanson and Williams (2014), who find anticipated durations of around 7 quarters for each quarter from 2011. The mass of the posterior density shifts up in 2011Q3, with a modal anticipated duration

[15] Due to numerical considerations we choose to scale the standard deviation of demand shocks by less.

[16] Posterior distributions for the expected durations of the US Fed Funds rate at 0.125% are shown in Figure C.4. The posteriors for Canadian durations are shown in Figure C.5.

of 9 quarters. This shift aligns with the explicit adoption of forward guidance from 2011Q3, when the Federal Reserve announced that it would maintain the interest rate at 0.125% until mid-2013. Subsequently, the Federal Reserve repeatedly pushed the explicit liftoff date back, which is consistent with our estimated posterior distributions remaining centered around 6-7 quarters. For Canada, it is worth noticing that the mass of the posterior distribution of the durations shrinks towards smaller durations for the first two quarters of 2010, which is consistent with the fact that the Bank of Canada raised its policy interest rate to 1% in third quarter of 2010.

5 Spillovers of foreign forward guidance

The trade elasticity, τ, as emphasized by Haberis and Lipinska (2012), is a key determinant of the effect of foreign forward guidance on the domestic economy. US forward guidance that extends the expected duration of the ZLB regime leads to a depreciation of the US dollar and to an increase in US output. Other things equal, the expansion in US economic activity increases the exports of open economies that trade with the US. The increase demand for exports leads to an increase in domestic output. And as domestic producers shift supply towards the US market, the inflation rate of domestically-produced goods tends to rise. The depreciation of the US dollar, on the other hand, gives rise to an expenditure switching effect away from domestic goods. The magnitude of the expenditure switching effect is larger the closer substitutes home and foreign goods are. The appreciation of the domestic currency also leads to lower imported goods inflation. The overall impact of foreign forward guidance on domestic output and inflation therefore depends crucially on the trade elasticity. When the trade elasticity, τ, is high, US forward guidance lowers domestic output and inflation in small open economies and may potentially drive domestic monetary policy to its lower bound. When the trade elasticity is low, US forward guidance may be expansionary for small open economies. So it is an empirical issue whether US forward guidance is expansionary or contractionary for the domestic economy. To let the data speak we put a wide prior over the trade elasticity. With then use the estimated model to assess the impact of US forward guidance on Canada.

In Figure 4, we compare the responses *on impact* of Canadian variables to stimulatory US forward guidance. We draw a set of parameters from the prior distribution and another set from the posterior distribution and solve the model for each draw. We then draw large negative US shocks, which for

Table 1: Estimated parameters

	Prior			Posterior				Prior			Posterior		
Parameter	Dist	Mean	Std Dev	Mode	Mean	Std Dev	Parameter	Dist	Mean	Std Dev	Mode	Mean	Std Dev
ρ_ξ^*	B	0.600	0.200	0.938	0.934	0.010	$10 \times \sigma_p$	IG	1.000	2.000	0.467	0.461	0.043
ρ_r^*	B	0.600	0.200	0.863	0.860	0.019	τ	G	1.000	1.500	0.836	0.740	0.179
ϕ_π^*	N	2.000	0.300	2.251	2.111	0.243	ρ_η^*	B	0.600	0.200	0.888	0.902	0.038
ϕ_g^*	N	0.200	0.100	0.354	0.278	0.087	$100 \times \sigma_\eta^*$	IG	0.071	0.025	0.084	0.090	0.007
ϕ_y^*	G	0.250	0.130	0.091	0.129	0.051	ρ_η	B	0.600	0.200	0.690	0.661	0.084
$10 \times \sigma_\xi^*$	IG	0.429	1.414	0.277	0.301	0.033	$100 \times \sigma_\eta$	IG	0.071	0.025	0.071	0.069	0.006
$100 \times \sigma_z$	IG	0.429	1.414	1.511	1.556	0.106	$100 \times \sigma_2$	IG	0.071	0.025	0.048	0.044	0.005
$100 \times \sigma_r^*$	IG	0.429	1.414	0.157	0.163	0.015	$100 \times \sigma_4$	IG	0.071	0.025	0.029	0.033	0.005
ρ_ξ	B	0.600	0.200	0.847	0.846	0.015	$100 \times \sigma_8$	IG	0.071	0.025	0.078	0.079	0.008
ρ_a	B	0.600	0.200	0.766	0.763	0.045	$100 \times c_2$	N	0.000	2.000	0.020	0.019	0.023
ρ_p	B	0.600	0.200	0.970	0.969	0.012	$100 \times c_4$	N	0.000	2.000	0.041	0.052	0.024
ρ_r	B	0.600	0.200	0.912	0.908	0.010	$100 \times c_8$	N	0.000	2.000	0.061	0.058	0.028
ϕ_π	N	2.000	0.300	2.533	2.569	0.238	$100 \times \sigma_2^*$	IG	0.143	0.050	0.259	0.265	0.019
ϕ_g	N	0.200	0.100	0.380	0.372	0.082	$100 \times \sigma_4^*$	IG	0.071	0.025	0.035	0.037	0.005
ϕ_y	G	0.250	0.130	0.057	0.058	0.023	$100 \times \sigma_8^*$	IG	0.071	0.025	0.055	0.054	0.006
$10 \times \sigma_\xi$	IG	0.429	1.414	0.495	0.500	0.049	$100 \times c_2^*$	N	0.000	2.000	−0.114	−0.118	0.181
$100 \times \sigma_a$	IG	0.429	1.414	3.300	3.315	0.428	$100 \times c_4^*$	N	0.000	2.000	−0.047	−0.073	0.179
$100 \times \sigma_r$	IG	0.429	1.414	0.196	0.197	0.016	$100 \times c_8^*$	N	0.000	2.000	0.028	−0.001	0.179

comparison, we use both for the prior and posterior draws. We keep those cases where the demand shocks cause the US to hit its ZLB. For those cases we add US forward guidance by extending the length of the ZLB duration implied by the constraint by two quarters. Figure 4 plots the response on impact of Canadian inflation and output for different values of the trade elasticity. Notice that the prior puts some mass on positive Canadian output and inflation responses following foreign forward guidance. For the entire posterior distribution, however, the reponses are negative.

Next, we use the estimated model to assess the impact of foreign forward guidance on the dynamics of the domestic economy. The structural parameters are set at the mode. The ZLB introduces a non-linearity which implies that conventional impulse responses are not valid. The ZLB magnifies the effect of negative shocks. So to assess the response of the domestic economy to US forward guidance, we hit the US economy with a negative demand shock large enough so that the US ZLB binds. At the same time, we hit the Canadian economy with a negative demand shock. But these shocks are such that the Canadian interest rate gets close to, but remains above, the ZLB. The negative US demand shock is sufficiently large to cause the US nominal interest rate to be at the ZLB for 6 quarters. Because of interest rate persistence in the policy rule, the US reaches the ZLB two quarters after the shock hits. When US monetary policy is constrained by the ZLB, the US real interest rate increases, causing US inflation and output to decline.

Monetary policy, however, can resort to forward guidance to alleviate the contractionary effect of the ZLB. Suppose then that after observing the shock, US monetary authorities announce that they will immediately lower the policy interest rate to zero and keep it there for 12 quarters. This announcement corresponds to an additional 6 quarters over the *endogenous duration* caused by the shock. US monetary authorities can use forward guidance policy to approximate what the behavior of the economy would had been had there been no ZLB on the policy rate: the policy stimulates US output, raises US inflation and depreciates the US dollar (i.e.appreciates the Canadian dollar). This, in turn, causes Canadian inflation to fall and Canadian monetary policy to hit its ZLB.

Figure 5 compares the responses of Canadian variables with and without US forward guidance. The Canadian exchange rate appreciates significantly more when the US conducts forward guidance. This causes a fall in Canadian inflation. At the estimated trade elasticity of 0.84, Canadian exports respond little to US forward guidance because the effect of stronger US demand is offset by the appreciation of the Canadian dollar. Estimated forward guidance policy spillovers from the US to Canada are strong and lead to a fall in inflation and a contraction of domestic activity: the export stimulus due to increased foreign demand is not strong enough to overcome the expenditure switching triggered by the appreciation of the exchange rate.

So, at the mode of our estimation results, we see a notable endogenous effect on the level of the Canadian interest rate from US forward guidance. The appreciation of the Canadian dollar lowers Canadian inflation through a decline in import price inflation. To offset an increase in the real interest rate, the Canadian central bank eases by more and so US forward guidance leads to lower interest rates abroad.

It is possible for US forward guidance policy to be strong enough to drive the Canadian policy interest rate to its ZLB. When the Canadian central bank is constrained, it cannot lower the real interest rate further, so that Canadian inflation and output may fall relative to the path without US forward guidance. As a result of the relatively higher real interest rate, the Canadian dollar appreciates, reinforcing the decline in Canadian inflation and leading to a decline in Canadian exports. This illustrates the channel highlighted by Cook and Devereux (2014) that the exchange rate exacerbates shocks when nominal interest rates are at their lower bounds.

The additional appreciation of the Canadian dollar as a result of the Canadian monetary authority being constrained by the ZLB provides an incentive for the Bank of Canada to engage in forward guidance policies of its own. Canadian forward guidance would lower the path of the Canadian real interest rate, undoing some of the negative effect of US forward guidance. This illustrates the 'beggar thy neighbor' tension inherent in competing forward guidance policies across countries in our estimated model: forward guidance policy by large countries can, through the exchange rate, lead small open economies to conduct forward guidance policies which compete in their effect on exchange rates. Overall, these policies can have little net effect on the exchange rate and imply substantial periods of time at each country's ZLB. But even then, forward guidance policies in both economies weaken the contractionary impacts of binding constraints and are stimulatory on the whole. Ammer et al. (2016) find that in the case of conventional monetary policies easing actions by the Federal Reserve and other central banks are likely to be stabilizing for the global economy. We find that this is the case as well for the recent period of unconventional monetary policies in the US and Canada.

Table 2: Forward guidance decomposition

	United States duration (quarters)						Canadian duration (quarters)					
	Posterior draw			Forward guidance			Posterior draw			Forward guidance		
Date	Median	10%	90%	Median	10%	90%	Median	10%	90%	Median	10%	90%
2009Q1	6	6	7	6	5	7						
2009Q2	7	6	9	6	4	7	4	3	5	4	3	5
2009Q3	7	6	9	4	4	6	3	2	5	3	2	5
2009Q4	7	6	9	5	4	6	4	3	5	4	3	5
2010Q1	7	6	9	4	4	4	3	2	5	3	2	4
2010Q2	7	6	9	4	4	4	2	1	3	2	1	3
2010Q3	7	6	9	4	4	4	5	3	6	4	3	5
2010Q4	7	6	9	4	3	4	5	3	7	4	3	5
2011Q1	8	6	10	4	4	4	4	3	6	4	3	4
2011Q2	8	6	10	5	4	6	5	3	7	4	3	5
2011Q3	9	7	11	4	4	5	6	4	7	5	4	5
2011Q4	8	6	9	4	4	5	6	4	8	4	4	5
2012Q1	9	7	11	4	4	5	6	4	8	4	4	5
2012Q2	9	7	11	4	4	5	6	5	8	4	4	5
2012Q3	8	7	10	4	4	4	6	4	8	4	3	4
2012Q4	9	7	11	4	4	4	6	4	8	4	3	5
2013Q1	8	7	10	4	4	4	6	5	8	4	4	5
2013Q2	8	6	10	4	3	4	6	4	8	4	3	4
2013Q3	8	6	10	4	4	4	6	4	8	4	3	4
2013Q4	8	6	10	4	4	4	6	4	7	4	3	4
2014Q1	8	6	10	4	3	4	6	5	8	4	4	5
2014Q2	8	6	10	4	4	5	7	5	8	4	4	5
2014Q3	7	6	9	4	4	4	6	4	8	4	4	5
2014Q4	6	5	8	4	3	4	6	4	8	4	4	4

Note: From 2010Q3, Canadian forward guidance is at 1%

6 Counterfactual scenarios

In this section, we use the decomposition of the estimated durations described above to quantify, through counterfactual scenarios, the impact of forward guidance and that of the constraints. The estimation yields posterior distributions for the structural parameters and durations. To calculate counterfactuals we select a draw from the distribution, which we describe next.

6.1 The median-target draw

One might assume that the most likely sequence of durations is the median or the mode of the posterior marginal distributions. That particular sequence, however, might not exist in the posterior. Because of this, we define the *median-target* draw as the draw that minimizes the sum of the distances between the draw's durations and the median of the posterior durations. That is, letting $\mathbf{T}$ and $\mathbf{T}^*$ be the posterior durations for Canada and the US respectively, we choose the draw i that finds $\min\{|\mathbf{T}_i - \text{median}(\mathbf{T})| + |\mathbf{T}_i^* - \text{median}(\mathbf{T}^*)|\}$. That draw is associated with a posterior draw of the parameters so we select that parameter draw for the decomposition as well.

Figure 6 illustrates the output of the decomposition algorithm at the median-target draw. The decomposition shows that the durations for the US followed roughly a humped shaped pattern increasing to 2011Q3, which is consistent with policy announcements by the Federal Reserve in that quarter, when the Fed explicitly committed to maintaining the interest rate at 0 to 0.25 basis points until mid-2013.[17] The decomposition, however, shows that these explicit announcements were not indicative of any additional policy easing by the Federal Reserve, but were instead primarily due to structural shocks which implied extended periods of time at the ZLB.

The second panel of Figure 6 shows the same decomposition of durations for Canada. The pattern of durations in Canada over the 2009 to 2015 period is different to that of the US. Instead of a hump shape, the Canadian lower bound durations initially start quite low, at around 3 to 4 quarters, before increasing to about 6 quarters around 2013. Since then, the estimated anticipated durations have declined to about 4 quarters in early 2015. The decomposition indicates that, unlike for the US, a strict adherence to the estimated Canadian Taylor rule would have implied that the Bank of Canada would have used policy interest rates above the bound from 2009Q2 to 2010Q2. This period coincides with the year that the Canadian overnight policy rate was at 0.25%. From 2010Q3 to the end of our sample 2014Q4, the policy interest rate was at 1%, and the decomposition computed at that lower bound suggests that, for most periods, the majority of the estimated lower bound durations are due to forward guidance although the contribution of structural shocks is not negligible.

To assess the extent to which the decomposition is sensitive to the particular draw chosen in computing Figure 6, we take 1,000 random draws from the posterior distributions of durations and parameters and recompute the decomposition. Figure C.1 plots the joint distribution of those 1,000 draws of the duration for the US and Canada against the decomposed forward guidance duration. An increased forward guidance component is represented by more mass of the joint density being closer to the diagonal: if, for example, the posterior duration is 6 quarters, and the decomposed forward guidance duration is also 6 quarters, that point would lie on the diagonal. In contrast, durations which are explained more by structural shocks have a joint density with mass far from the diagonal, so that the forward guidance component is closer to zero quarters.

Under this representation, Figure C.1 shows that the forward guidance durations are stronger in

[17] FOMC statement, August 9, 2011.

Canada than the US, as is the case at the median-target draw.[18] In the US, most of the mass of the joint density of the total duration and forward guidance duration lies around 8 quarters and 4 quarters respectively, while for Canada, most of the mass of the joint density lies around 4 to 6 quarters for the total duration and 4 to 6 quarters for the forward guidance duration.

6.2 Removing forward guidance policies

Our first counterfactual simulation explores the role of forward guidance policies.[19] Figure 7 plots the actual path of observable variables and the path that would prevail removing the forward guidance component for both countries. With the estimated model, we extract the structural shocks and feed those shocks through the model using the ZLB algorithm, so that the durations that prevail in every quarter are those implied by the constraints and shocks, the endogenous durations in Figure 6. We find that without forward guidance policy in either the US and Canada, inflation and the level of output would have been substantially lower in both countries, in line with the theoretical predictions of forward guidance policies. In particular, the shocks imply a rate of US deflation of around -2% in annual terms and the level of US output would have recovered significantly slower, remaining below its pre-crises level, for all of the 2009 to 2014 period. Canadian inflation would also have fallen compared to the observed rate, although not to the same extent. In part this is because removing US and Canadian forward guidance appreciates the US dollar (i.e. depreciates the Canadian dollar) which increases inflation through import prices.[20]

The counterfactual behavior of the nominal exchange rate can be taken as an indicator of the relative stimulus of US and Canadian monetary policies. When removing forward guidance policies, the US dollar appreciates. This suggests that the combined forward guidance policies lowered the US real interest rate relative to the Canadian one, which can be partly seen in the relative movements of inflation. Under forward guidance policies, inflation in both the US and Canada increase, but more so in the US; with nominal interest rates in the US and Canada fixed at their lower bounds, this relatively larger increase in US inflation leads to an appreciation of the Canadian real exchange rate.

[18]We have experimented using other draws of the posterior for the counterfactuals–our results are not particularly sensitive to this choice.

[19]Our results are under the assumption that the monetary authority is perfectly credible. The impact of forward guidance is smaller under imperfect credibility. Recently, McKay et al. (2015) also argue that the impact of forward guidance policies depends on heterogeneity in borrowing limits. Such a formulation gives rise to a discounted Euler equation which mutes the effect of forward guidance. From these perspectives, our results may be seen as providing an upper bound of the effects of forward guidance.

[20]See Figure 11 for the exchange rate response associated with each counterfactual scenario.

6.3 Removing lower bound constraints

In the second counterfactual scenario, we remove lower bound constraints on either the US or Canadian policy interest rates, so that both central banks follow their Taylor rules without restrictions.[21] This counterfactual exercise highlights the strong deflationary forces faced by the Federal Reserve in the US economy after 2009. We find forward guidance policies to be stimulatory even relative to the no lower bound constraint–a result that aligns with Wu and Xia (2014), who find that monetary policy in the US since 2009 was stimulatory relative to the policy behavior of the Federal Reserve prior to 2009. The lower bound of zero would not have been a binding constraint in Canada, although the lower bound of 1% at which Canada conducted most of its forward guidance would have been binding in many quarters.

6.4 Removing Canadian forward guidance

This counterfactual experiment empirically documents the importance of identifying forward guidance durations across countries. To illustrate the spillover effects of US forward guidance on the Canadian economy, in Figure 9 we plot the counterfactual where the US follows its estimated forward guidance policy, while Canada reacts passively in response, and does not respond with its own forward guidance policies. Canada's 1% lower bound would have endogenously been visited every period from late 2010 on. Both inflation and output would have been notably lower.

The response of the exchange rate crucially determines how forward guidance spillovers operate. As the US undergoes expansionary forward guidance policy and Canada acts passively in response, the US dollar depreciates (i.e. Canadian dollar appreciates) significanlty more (Figure 11). In Canada, the rate of imported goods inflation falls, which pushes down aggregate inflation. Because the Canadian nominal interest rate is constrained, this decline in inflation causes an increase in the real interest rate in Canada, further reducing current Canadian consumption and output. In addition, the appreciation of the Canadian dollar causes Canadian exports to fall.

6.5 Removing US forward guidance

Another way to quantify the spillover effects of US forward guidance is by removing only US forward guidance, so that the Federal Reserve is passively constrained by the monetary policy rule. Figure 10

[21] Note that the interest rate in this exercise is not a shadow policy interest rate in the way Wu and Xia (2014) define it, a measure of the stance of policy, but instead is the series that would have arisen given the structural shocks and in the absence of lower bounds.

plots the counterfactual response. Clearly, in the absence of US forward guidance, Canadian monetary policy settings would have been too expansionary with a higher level of output but with inflation persistently above the Bank of Canada's target of 2%. The Canadian dollar significantly depreciates which stimulates exports and increases imported goods inflation.

6.6 A fixed exchange rate policy

As the goal of the paper is to study the response of small open economies to foreign unconventional policy, in this section we ask what happens if, instead of a nominal interest rate policy, Canada were to implement a fixed exchange rate policy ($\Delta e = 0$), as is effectively the case for the 19 EU member states of the euro area. In the counterfactual we pegged exchange rate at the level that prevailed when the US was first constrained by the ZLB in 2009Q1. To implement this policy, we simply replace the Canadian Taylor rule with $\Delta e = 0$, remove the risk premia shock and allow the nominal interest rate to be determined endogenously, by uncovered interest rate parity. Assuming $\Delta e = 0$ is sustained, Canada simply inherits US monetary policy settings, both in terms of the level of policy rates and the expected durations that prevail in every quarter.

In Figure 12, we compare the actual path with path under the fixed exchange rate policy. US monetary policy settings would clearly not have been appropriate for Canada. It would have implied an unnecessarily large degree of monetary stimulus. Following the depreciation of the Canadian dollar in early 2009, a significant appreciation of the Canadian dollar follows which would have been prevented by the fixed exchange rate policy.

To keep the exchange rate constant over the US ZLB period, Canadian monetary policy is required to be to extremely loose relative to US policy, corresponding to high counterfactual inflation and a rapid recovery in output. This scenario highlights the importance of a flexible exchange rate for monetary policy to respond appropriately to domestic conditions.

7 Conclusion

In this paper, we examine empirically how forward guidance of the US Federal Funds rate at its lower bound affects monetary policy and the dynamics of Canada, a small open economy. To this end, we set up and estimate a two-country small open economy model on US and Canadian data accounting for lower bounds from 2009Q1 onwards. In estimating these durations we allow, but do not require, that the durations be those which are implied by the lower bounds themselves. This is important given the optimal policy prescription of prolonging the duration of the fixed interest rate period to alleaviate the contractionary impact of the constraints.

We find anticipated lower bound durations of around 6 to 8 quarters, of which roughly half in the US is explained by shocks. The Canadian durations seem mostly the results of deliverate forward guidance. Our results suggest that both countries made use of forward guidance successfully to stimulate output and inflation. In counterfactual simulations, we find that the estimated durations represent a stronger monetary stimulus than the policy that would have arisen in the absence of the lower bound constraints. If neither country used the expectations channel of policy, output and inflation in both countries would have been substantially lower. In doing so, we identify notable spillovers of US forward guidance policy on Canada. We find US forward guidance to be contractionary for Canada, but we find that US and Canadian forward guidance were stabilizing on the whole.

A The log-linear equations

This appendix describes the model's log-linear equations. For a variable X_t with steady state $\bar{X}_t$ we write $x_t = \ln X_t - \bar{X}_t$.

A.1 Large country

Log-linearized, the large country's problem can be expressed as the following four equations. From the large country's household's optimal choice of consumption:

$$(\mu - \beta h)(\mu - h)\lambda_t^* = \mu h y_{t-1}^* + \mu\beta h \mathbb{E}_t\{y_{t+1}^*\} - \left(\mu^2 + \beta h^2\right) y_t^* - \mu h z_t + (\mu - h)\left(\mu - \beta h \rho_\xi^*\right)\xi_t^*.$$

The household's intertemporal condition:

$$0 = \lambda_t^* - \mathbb{E}_t\left\{\lambda_{t+1}^*\right\} - \left(r_t^* - \mathbb{E}_t\left\{\pi_{t+1}^*\right\}\right).$$

Firms' pricing condition:

$$\pi_t^* = \beta \mathbb{E}_t \pi_{t+1}^* + \kappa^*\left[\xi_t^* + \varphi y_t^* - \lambda_t^*\right].$$

And a monetary policy rule:

$$r_t^* = \max\left\{0, \rho_R^* r_{t-1}^* + (1 - \rho_R^*)\left(\phi_\pi^* \pi_t^* + \phi_g^*(y_t^* - y_{t-1}^* + z_t) + \phi_y^* y_t^*\right) + \varepsilon_{R,t}^*\right\}.$$

When habits are turned off ($h = 0$) these equations reduce to the typical three-equation New Keynesian model.

A.2 Small country

The small country's system of equations comprises of the small country's household's optimal choice of consumption:

$$(\mu - \beta h)(\mu - h)\lambda_t = \mu h c_{t-1} + \mu\beta h \mathbb{E}_t\{c_{t+1}\} - (\mu^2 + \beta h^2)c_t - \mu h z_t + (\mu - h)(\mu - \beta h \rho_\xi)\xi_t.$$

An intertemporal condition:

$$0 = \lambda_t - \mathbb{E}_t\left\{\lambda_{t+1}\right\} - (r_t - \mathbb{E}_t\{\pi_{t+1}\}),$$

a risk-sharing condition:

$$-\lambda_t = -\lambda_t^* + q_t + rp_t,$$

three pricing equations for the prices of exports, imports and domestic goods:

$$\begin{aligned}
\pi_{F,t} &= \kappa_f(q_t - \gamma_{F,t}) + \beta\mathbb{E}_t\{\pi_{F,t+1}\} \\
\pi_{X,t} &= \kappa_z(\gamma_{H,t} - \gamma_{X,t}) + \beta\mathbb{E}_t\{\pi_{X,t+1}\} \\
\pi_{H,t} &= \kappa(\xi_t - \lambda_t + \varphi y_t - (1+\varphi)a_t - \gamma_{H,t}) + \beta\mathbb{E}_t[\pi_{H,t+1}].
\end{aligned}$$

Relative prices for exports, imports and domestic goods:

$$\begin{aligned}
\gamma_{F,t} &= \gamma_{F,t-1} + \pi_{F,t} - \pi_t \\
\gamma_{X,t} &= \gamma_{X,t-1} + \pi_{X,t} - \pi_t + \Delta e_t \\
\gamma_{H,t} &= \gamma_{H,t-1} + \pi_{H,t} - \pi_t.
\end{aligned}$$

Market clearing conditions:

$$\begin{aligned}
\pi_t &= (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \\
y_{H,t} &= \frac{C_H}{Y_H}\left(c_t - \tau\gamma_{H,t}\right) + \frac{X}{Y_H}\left(y_t^* + \tau(q_t - \gamma_{X,t})\right)
\end{aligned}$$

The definition of the change in the real exchange rate:

$$q_t = q_{t-1} + \Delta e_t + \pi_t^* - \pi_t.$$

And the small country's monetary policy rule:

$$r_t = \max\left\{0, \rho_r r_{t-1} + (1-\rho_r)\left[\phi_\pi\pi_t + \phi_g(y_t - y_{t-1} + z_t) + \phi_y y_t\right] + \varepsilon_{r,t}\right\}.$$

If $\alpha = 0$, then $\frac{X}{Y_H} = 0$ and $\gamma_{H,t} = 0$, so that the small country is closed and separate from the large country.

A.3 Long-Rates

To include a long-rate of maturity m, we need to include all previous $m-1$ maturity interest rates and relate them to each other by the expectations hypothesis:

$$m r_{m,t} = r_{1,t} + (m-1)\mathbb{E}_t r_{m-1,t+1}. \tag{9}$$

This is computationally more efficient than including m leads of $r_{1,t}$. We relate the observed rates $r_{j,t}^{\text{obs}}$ to the model implied rates as per Kulish et al. (2014),

$$r_{j,t}^{\text{obs}} - r = r_{j,t} + c_j + \eta_t + \varepsilon_{j,t}, \tag{10}$$

where c_j is a constant and estimated component of the term premia, $\varepsilon_{j,t}$ is a idiosyncratic and time-varying component of the term premia and η_t is an additional and persistent component of the time-varying term premia which is common to all maturities:

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t}. \tag{11}$$

There is one set of equations (9) to (11) for each of the foreign and domestic countries.

A.4 Observation Equations

US output growth:

$$\Delta y_t^{*,obs} = y_t^* - y_{t-1}^* + z_t.$$

US inflation:

$$\pi_t^{*,obs} = \pi_t^* + \pi^*.$$

US policy rate:

$$r_t^{*,obs} = r_t^* + r^*.$$

Canada output growth:

$$\Delta y_t^{obs} = y_t - y_{t-1} + z_t.$$

Canada inflation rate:

$$\pi_t^{obs} = \pi_t + \pi.$$

Canada policy rate:

$$r_t^{obs} = r_t + r.$$

Canada exchange rate:

$$\Delta e_t^{obs} = \Delta e_t.$$

B Implementation

There are two occasionally binding lower bound constraints to impose in this model, one to the large country's nominal interest rate, and one to the small country's nominal interest rate. A flexible algorithm is developed which can handle cases where shocks in the large country endogenously push the large country's interest rate to its ZLB (perhaps many time periods after a shock hits) and which subsequently causes the small country's interest rate to endogenously fall to the ZLB (perhaps many time periods periods after the large country has hit its ZLB). The algorithm relies on constructing a perfect foresight path of the nominal interest rate in both countries, and piecing together linear systems in a step-by-step way. These methods are based on the solution concepts developed in Cagliarini and Kulish (2013); Kulish and Pagan (forthcoming). As shown by Guerrieri and Iacoviello (2015) and Jones (2015), the approximation does a good job at capturing the non-linear effects induced by the occasionally binding constraints.

B.1 Notation

Denote by x_t^* the vector of endogenous variables for the large country at time period t, one of which is the nominal interest rate in the large country, and x_t the vector of endogenous variables for the small country at time period t, one of which is the nominal interest rate R_t. The initial conditions are $[x_{t-1}^*{}' \; x_{t-1}{}']'$ and the initial vector of unanticipated exogenous variables, denoted by ε_t. The model is a system of n equations.

B.2 Initialization at time period t

We know, at period t:

- The shock that hits at period t: ε_t.
- The initial vector of variables x_{t-1}.

B.3 The algorithm

The steps of the algorithm are:

0. Linearize the model around the non-stochastic steady state, ignoring the ZLBs in both countries.

1. For each period t:

   For the large country:

   (a) Solve for the path $\{x_\tau\}_{\tau=t}^{T}$ with T large, using the solution of the linearized economy from step (0), given ε_t and the initial vector of variables x_{t-1}, and assuming no future uncertainty. This gives a path for the nominal interest rate, $\mathbf{i}_t^k = \{i_\tau^k\}_{\tau=t}^{T}$.

   (b) Examine the path $\mathbf{i}_t^k$. If $\mathbf{i}_t^k \geq 0$, then the ZLB does not bind, so move onto step (2). If $\mathbf{i}_t^k < 0$, then move onto step (1c).

   (c) For the *first* time period where $\mathbf{i}_t^k < 0$, set the nominal interest rate in that period to zero. This changes the anticipated structure of the economy. Under this new structure, calculate the path of all variables, including the new path for the nominal interest rate $\mathbf{i}_t^{k+1} = \{i_\tau^{k+1}\}_{\tau=t}^{T}$.

   Iterate on steps 1a and 1c until convergence of $\mathbf{i}_t^*$.

   Repeat steps 1a to 1c for the small country.

2. Increment t by one. The initial vector of variables now becomes x_t, which was solved for in step 1. Draw a new vector of unanticipated shocks ε_{t+1} and return to step 1.

To compute the path $\{x_\tau\}_{\tau=t}^{T}$ under forward guidance, compute step (1c) first, imposing the sequence of structural matrices corresponding to the ZLB and non-ZLB periods. Then examine the path $\{i_\tau\}_{\tau=t}^{T}$ for subsequent violations of the ZLB.

B.4 Details of each step

At the following steps:

0. Write the n equations of the linearized structural model at t as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}w_t, \tag{SM}$$

where x_t is a $n \times 1$ vector of state and jump variables and w_t is a $l \times 1$ vector of exogenous variables. Use standard methods to obtain the reduced form:

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}w_t. \tag{RF}$$

1. For each period t:

   (a) Using (RF), obtain the path $\{x_\tau\}_{\tau=t}^{T}$ given w_t. Set T to be large. Assume $\{w_\tau\}_{\tau=t+1}^{T} = 0$ (no future uncertainty), so that $x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}w_t$, and $x_{t+1} = \mathbf{J} + \mathbf{Q}x_t$, up to $x_T = \mathbf{J} + \mathbf{Q}x_{T-1}$.

   This step gives a path $\mathbf{i}_t = \{i_\tau\}_{\tau=t}^{T}$.

   (b) Examine the path $\{i_\tau\}_{\tau=t}^{T}$.

   - If $i_\tau \geq 0$ for all $t \leq \tau < T$, accept $\{x_\tau\}_{\tau=t}^{T}$. The i_t path does not violate ZLB today or in future.
   - If $i_\tau < 0$ for any $t \leq \tau < T$, move to step (1c).

   (c) Update the path of $\{i_\tau\}_{\tau=t}^{T}$ for the ZLB. For the first time period t^* where $i_{t^*} < 0$, set $i_{t^*} = 0$. The model system at t^* therefore becomes:

$$\mathbf{A}^* x_{t^*} = \mathbf{C}^* + \mathbf{B}^* x_{t^*-1} + \mathbf{D}^* \mathbb{E}_{t^*} x_{t^*+1} + \mathbf{F}^* w_{t^*},$$

   Compute the new path $\{i_\tau\}_{\tau=t}^{T}$. This involves computing $\{x_\tau\}_{\tau=t}^{t^*}$ and $\{x_\tau\}_{\tau=t^*+1}^{T}$. At t^*, $\mathbb{E}_{t^*} x_{t^*+1}$ is computed using the the reduced form solution (RF) and $w_{t^*+1} = 0$. This expresses x_{t^*} as a function of x_{t^*-1}. Proceeding in this way with the correct structural matrices (either ZLB $*$ or no ZLB at each time period), compute the path $\{i_\tau\}_{\tau=t}^{T}$.

   A convenient way to compute the new path $\{i_\tau\}_{\tau=t}^{T}$ is to form the time varying matrices

$\{\mathbf{J}_\tau, \mathbf{Q}_\tau, \mathbf{G}_\tau\}_{\tau=t}^{T}$ which satisfy the recursion:

$$\begin{aligned}
\mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t \\
\mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t\mathbf{J}_{t+1}\right) \\
\mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1} \mathbf{F}_t,
\end{aligned}$$

with the final set of reduced form matrices for the recursion being the non-ZLB matrices $\mathbf{J}$, $\mathbf{Q}$, $\mathbf{G}$ from (RF).

These time-varying matrices are then used to compute the path $\{x_\tau\}_{\tau=t}^{T}$ by calculating $x_\tau = \mathbf{J}_\tau + \mathbf{Q}_\tau x_{\tau-1} + \mathbf{G}_\tau w_\tau$.

B.5 Output of the algorithm

The algorithm yields a set of time-varying structural matrices:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t,$$

from which we get the path of $\{x_\tau\}_{\tau=t}^{\infty}$ where the nominal interest rate is subject to the ZLB. Both the current value of the nominal interest rate, and expectations of the ZLB binding, affect current values of state variables.

B.6 Identifying forward guidance

Here, we explain how to use the algorithm in Appendix B.3 to decompose an anticipated duration of the ZLB into a component due to structural shocks, and a component due to Odyssean or commitment forward guidance. Assume that at period t, the ZLB binds and we have used procedures to estimate the model parameters and the anticipated length of the ZLB at period t. We have in hand at period t:

1. An estimated duration $\widetilde{T}$ of the ZLB at t, so that the interest rate is expected to stay at zero until time period $t + \widetilde{T}$.

2. An estimate of the history of the states $\{x_\tau\}_{\tau=0}^{t-1}$ and an estimate of the structural shocks $\{w_\tau\}_{\tau=1}^{t}$, computed using the Kalman smoother.

The estimated parameters, durations and shocks recover the observed series and give an estimate of the model's state variables x_t. To decompose the proportion of the estimated duration due to structural shocks, so that the remainder is due to forward guidance policies, at each point of time:

1. Use the state x_{t-1} and the structural shock w_t to compute, using the ZLB algorithm of Appendix B.3, the endogenous duration of the ZLB.

2. If the computed endogenous duration is less than the estimated duration, then the additional time is assigned to commitment forward guidance policy.

The endogenous duration is the duration that would have occured had the central bank simply set the nominal interest rate to zero in periods where the policy rule would have specified that it be negative, and set the interest rate to its positive value when the policy rule specifies that it be positive.

B.7 Kalman filter

The model in state space representation is:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t \qquad \text{(State Eqn)}$$

$$z_t = \mathbf{H}_t x_t. \qquad \text{(Obs Eqn)}$$

The structural shocks are Gaussian, so that $w_t \sim N(0, \mathbf{Q})$, where $\mathbf{Q}$ is the covariance matrix of w_t. There is no observation error by assumption. The Kalman filter recursion is given by the following equations. The state of the system is the state vector and its covariance matrix $(\hat{x}_t, P_{t-1})$. The predict step involves using the structural matrices $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$:

$$\hat{x}_{t|t-1} = \mathbf{J}_t + \mathbf{Q}_t \hat{x}_t$$

$$\mathbf{P}_{t|t-1} = \mathbf{Q}_t \mathbf{P}_{t-1} \mathbf{Q}_{t|t-1}^{\top} + \mathbf{G}_t \mathbf{Q} \mathbf{G}_t^{\top}.$$

This formulation differs from the time-invariant Kalman filter step because in the forecast stage the structural matrices $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$ can vary over time. We update these forecasts with imperfect observations of the state vector. Also note that $\mathbf{H}_t$ is time-varying, reflecting that when the nominal interest rate is at its lower bound, we lose it as an observable variable. The update step involves computing forecast errors $\widetilde{y}_t$ and its associated covariance matrix $\mathbf{S}_t$:

$$\widetilde{y}_t = z_t - \mathbf{H}_t \hat{x}_{t|t-1}$$
$$\mathbf{S}_t = \mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^\top.$$

The Kalman gain matrix is given by:

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^\top \mathbf{S}_t^{-1}.$$

With $\widetilde{y}_t$, $\mathbf{S}_t$ and $\mathbf{K}_t$ in hand, the optimal update of the state x_t and its associated covariance matrix is:

$$\hat{x}_t = \hat{x}_{t|t-1} + \mathbf{K}_t \widetilde{y}_t$$
$$\mathbf{P}_t = (I - \mathbf{K}_t \mathbf{H}_t)\, \mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with x_0 and $\mathbf{P}_0$ computed from their unconditional moments. The recursion is computed until the final time period T of data.

B.8 Kalman smoother

With the estimates of the parameters and durations in hand at time period T, the Kalman smoother gives an estimate of $x_{t|T}$, or an estimate of the state vector at each point in time given all available information. With $\hat{x}_{t|t-1}$, $\mathbf{P}_{t|t-1}$, $\mathbf{K}_t$ and $\mathbf{S}_t$ in hand from the filter, the vector $x_{t|T}$ is computed by:

$$x_{t|T} = \hat{x}_{t|t-1} + \mathbf{P}_{t|t-1} r_{t|T},$$

where the vector $r_{T+1|T} = 0$ and is updated with the recursion:

$$r_{t|T} = \mathbf{H}_t^\top \mathbf{S}_t^{-1} \left(z_t - \mathbf{H}_t \hat{x}_{t|t-1} \right) + (I - \mathbf{K}_t \mathbf{H}_t)^\top \mathbf{P}_{t|t-1}^\top r_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable, denoted by e_t, we compute:

$$e_t = \mathbf{G}_t w_t = \mathbf{G}_t r_{t|T}.$$

B.9 Sampler

This section describes the sampler used to obtain the posterior distribution of interest. Denote by ϑ the vector of parameters to be estimated and $\mathbf{T}$ the vector of durations to be estimated. Contained in $\mathbf{T}$ are a set of durations for both the foreign and domestic countries. Denote by $Z = \{z_\tau\}_{\tau=1}^{T}$ the sequence of observable vectors. The posterior $\mathcal{P}(\vartheta, \mathbf{T} \mid Z)$ satisfies:

$$\mathcal{P}(\vartheta, \mathbf{T} \mid Z) \propto \mathcal{L}(Z \mid \vartheta, \mathbf{T}) \times \mathcal{P}(\vartheta, \mathbf{T}).$$

With Gaussian errors, the likelihood function $\mathcal{L}(Z \mid \vartheta, \mathbf{T})$ is computed using the appropriate sequence of structural matrices and the Kalman filter:

$$\log \mathcal{L}(Z \mid \vartheta, \mathbf{T}) = -\left(\frac{N_z T}{2}\right) \log 2\pi - \frac{1}{2}\sum_{t=1}^{T} \log\det \mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top - \frac{1}{2}\sum_{t=1}^{T} \widetilde{y}_t^\top \left(\mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top\right)^{-1} \widetilde{y}_t.$$

The prior is simply computed using priors over ϑ which are consistent with the literature, and with flat priors over $\mathbf{T}$.[22]

The Markov Chain Monte Carlo posterior sampler has two blocks, corresponding to ϑ and $\mathbf{T}$. Initialize the sampler at step j with the last accepted draw of the structural parameters, the period of the breaking parameters and durations, denoted by ϑ_{j-1} and $\mathbf{T}_{j-1}$ respectively. The blocks are, in order of computation:

1. In the first block, randomly choose up to $\bar{T}$ durations to test in each country, corresponding to up to $\bar{T}$ time periods that each economy is at the ZLB. For each of those time periods, randomly choose a duration in the interval $[1, T^*]$ for each country to generate a new $\mathbf{T}_j$ proposal. Recompute the sequence of structural matrices associated with $(\vartheta_{j-1}, \mathbf{T}_j)$, compute the posterior $\mathcal{P}(\vartheta_{j-1}, \mathbf{T}_{j-1} \mid Z)$, and accept the proposal $(\vartheta_{j-1}, \mathbf{T}_j)$ with probability $\frac{\mathcal{P}(\vartheta_{j-1}, \mathbf{T}_j \mid Z)}{\mathcal{P}(\vartheta_{j-1}, \mathbf{T}_{j-1} \mid Z)}$. If $(\vartheta_{j-1}, \mathbf{T}_j)$ is accepted, then set $\mathbf{T}_{j-1} = \mathbf{T}_j$.

2. The second block is a more standard Metropolis-Hastings random walk step. Start by selecting which structural parameters to propose a new value for. For those parameters, draw a new proposal ϑ_j from a proposal density centered at ϑ_{j-1} and with thick tails to ensure sufficient coverage of the parameter space and an acceptance rate of roughly 20%. The proposal ϑ_j is

[22] For practical convenience, we require that each estimated duration lies below some maximum value T^* which, in practice, is rarely visited by the sampler.

accepted with probability $\frac{\mathcal{P}(\vartheta_j, \mathbf{T}_{j-1}|Z)}{\mathcal{P}(\vartheta_{j-1}, \mathbf{T}_{j-1}|Z)}$. If $(\vartheta_j, \mathbf{T}_{j-1})$ is accepted, then set $\vartheta_{j-1} = \vartheta_j$.

C Additional figures

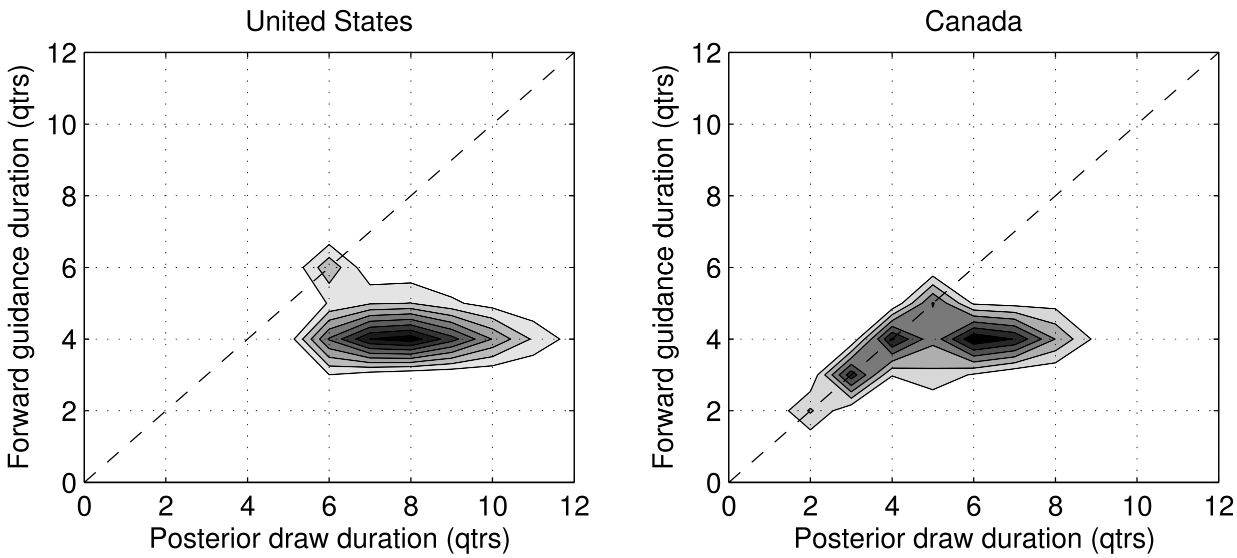

Figure C.1: ZLB duration decomposition. This figure shows the joint distribution of draws from the estimated posterior distribution of ZLB durations against the decomposed forward guidance duration associated with that draw. The draws and decomposed durations are aggregated over the 2009-2015 time period. A flatter density suggests ZLB durations are associated less with forward guidance policy and more with structural shocks. A density that lies closer to the diagonal suggests ZLB durations are associated more with forward guidance commitments.

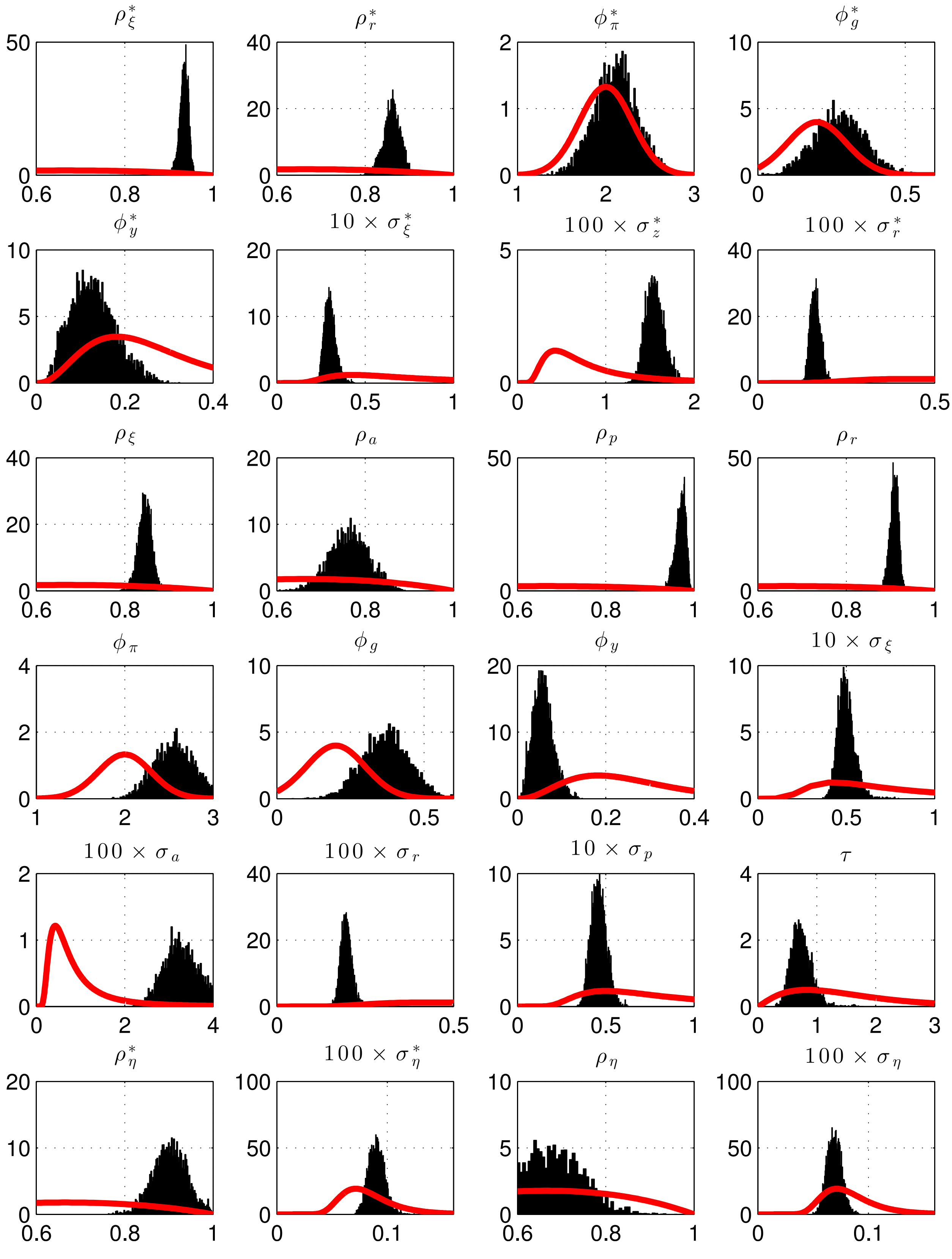

Figure C.2: Prior and posterior distributions. Structural parameters.

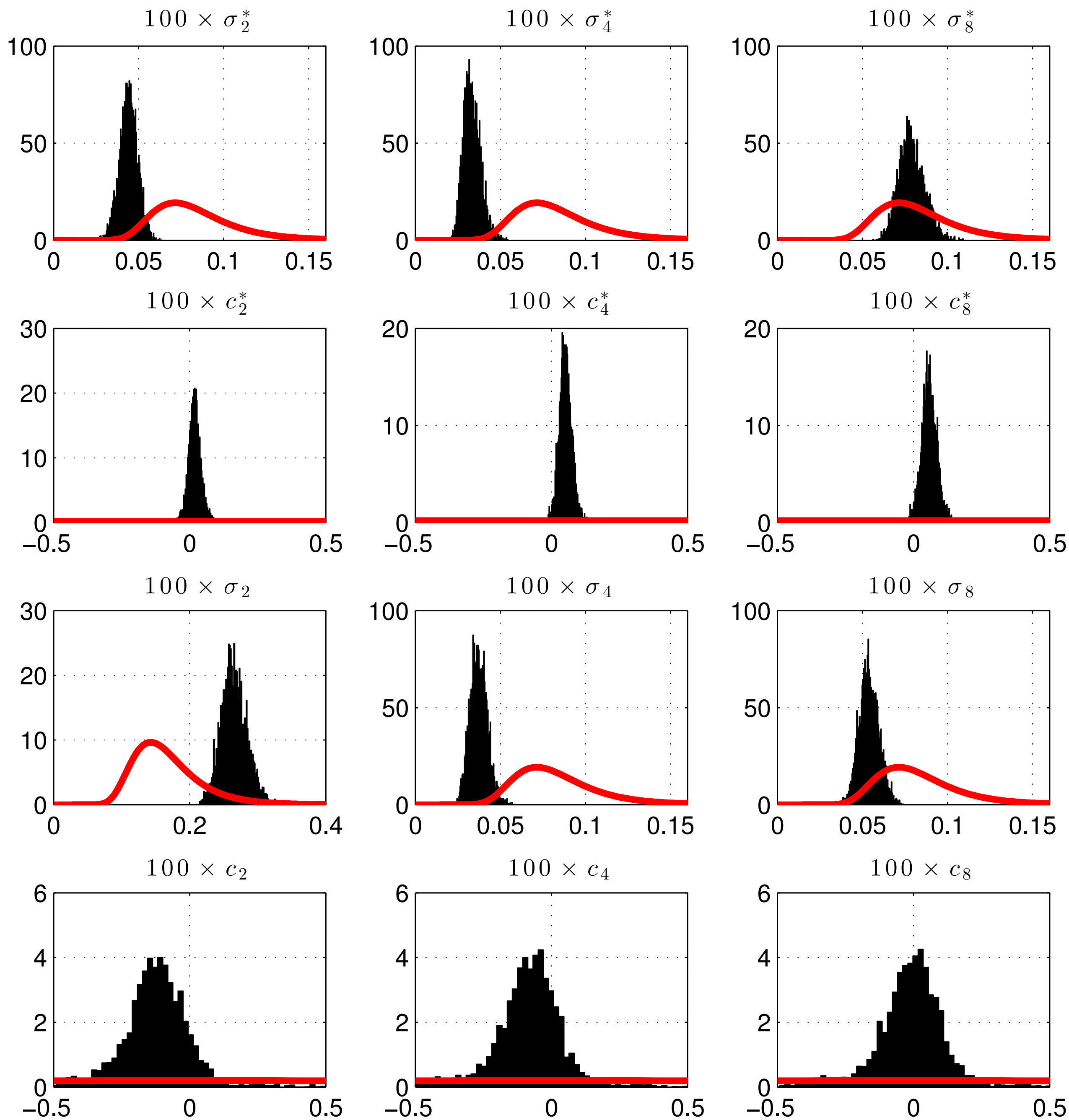

Figure C.3: Prior and posterior distributions. Structural parameters for long-rates.

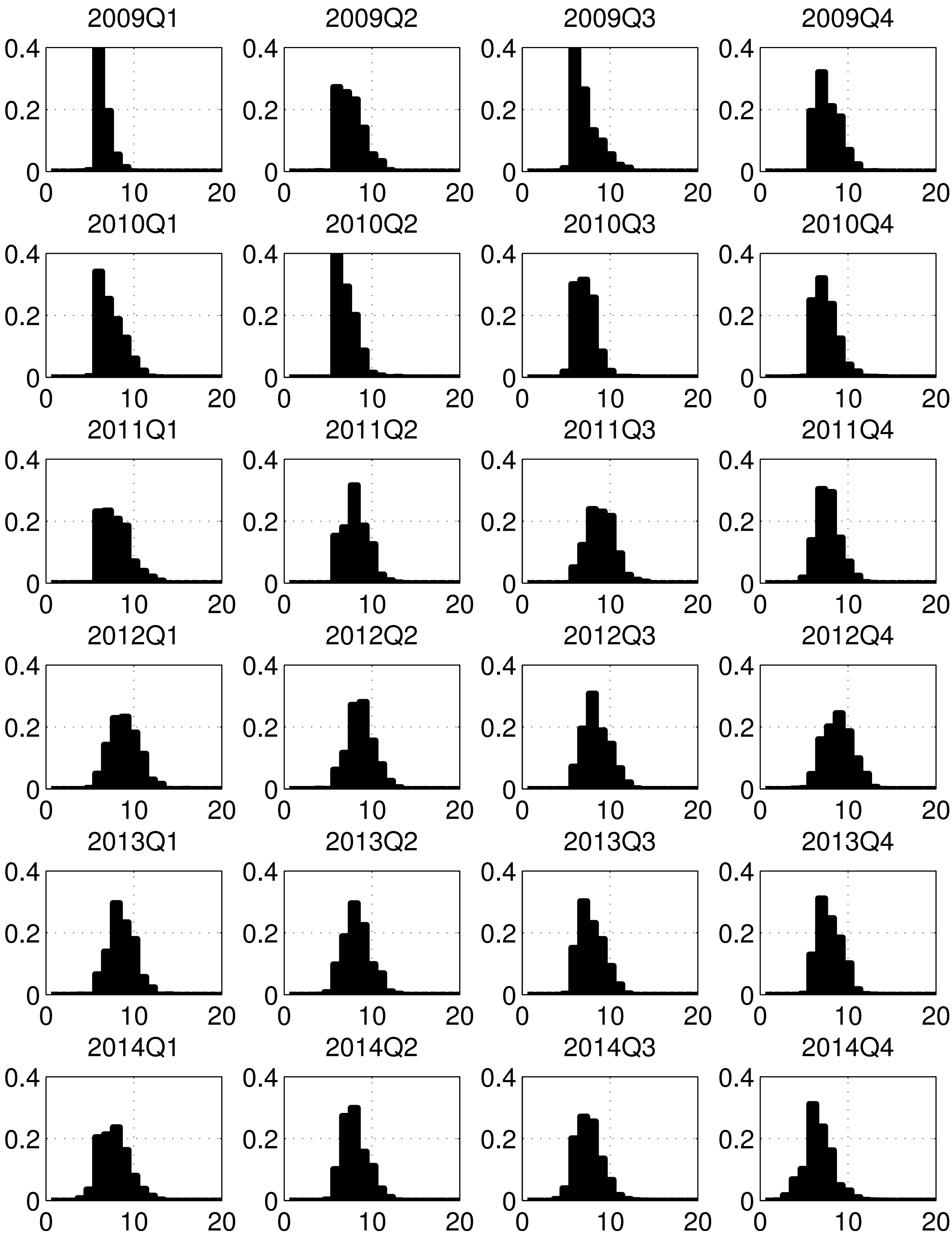

Figure C.4: Posterior of expected durations in quarters, US.

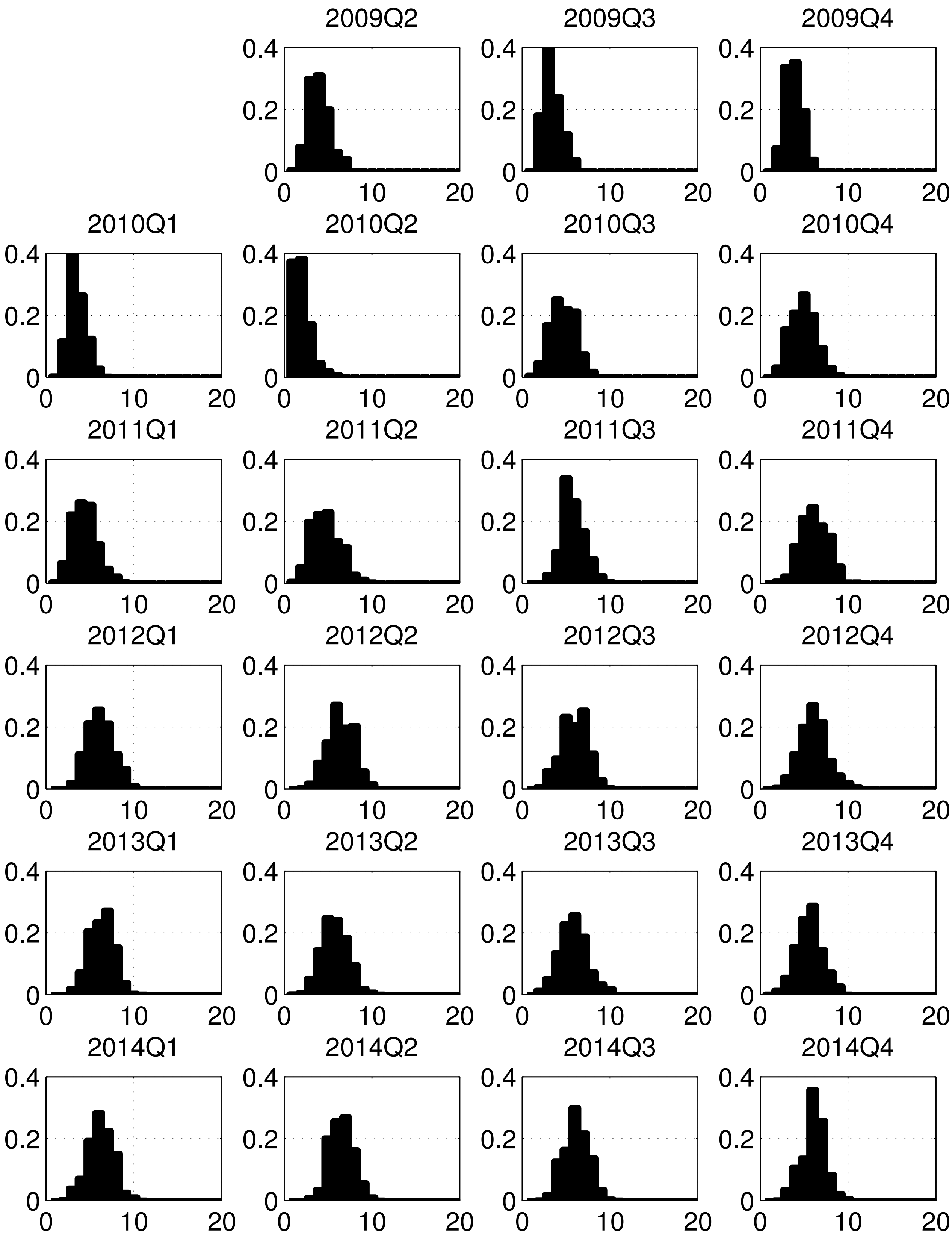

Figure C.5: Posterior of expected durations in quarters, Canada.

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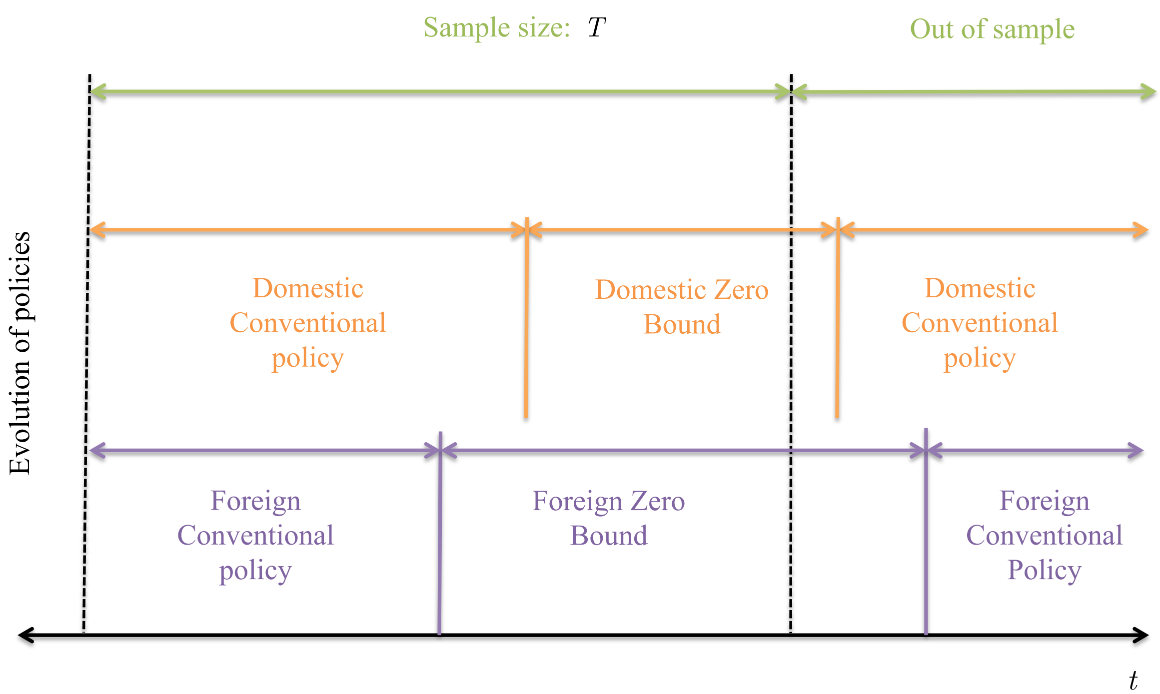

Figure 1: Timing. This figure represents the four possible regimes which can arise in our model. During our sample of size T, the ZLB is observed to bind in both the domestic and foreign countries. The foreign country hits the ZLB first, following by both countries being at their ZLBs. Out-of-sample, the domestic country exits its ZLB first, followed by the foreign country.

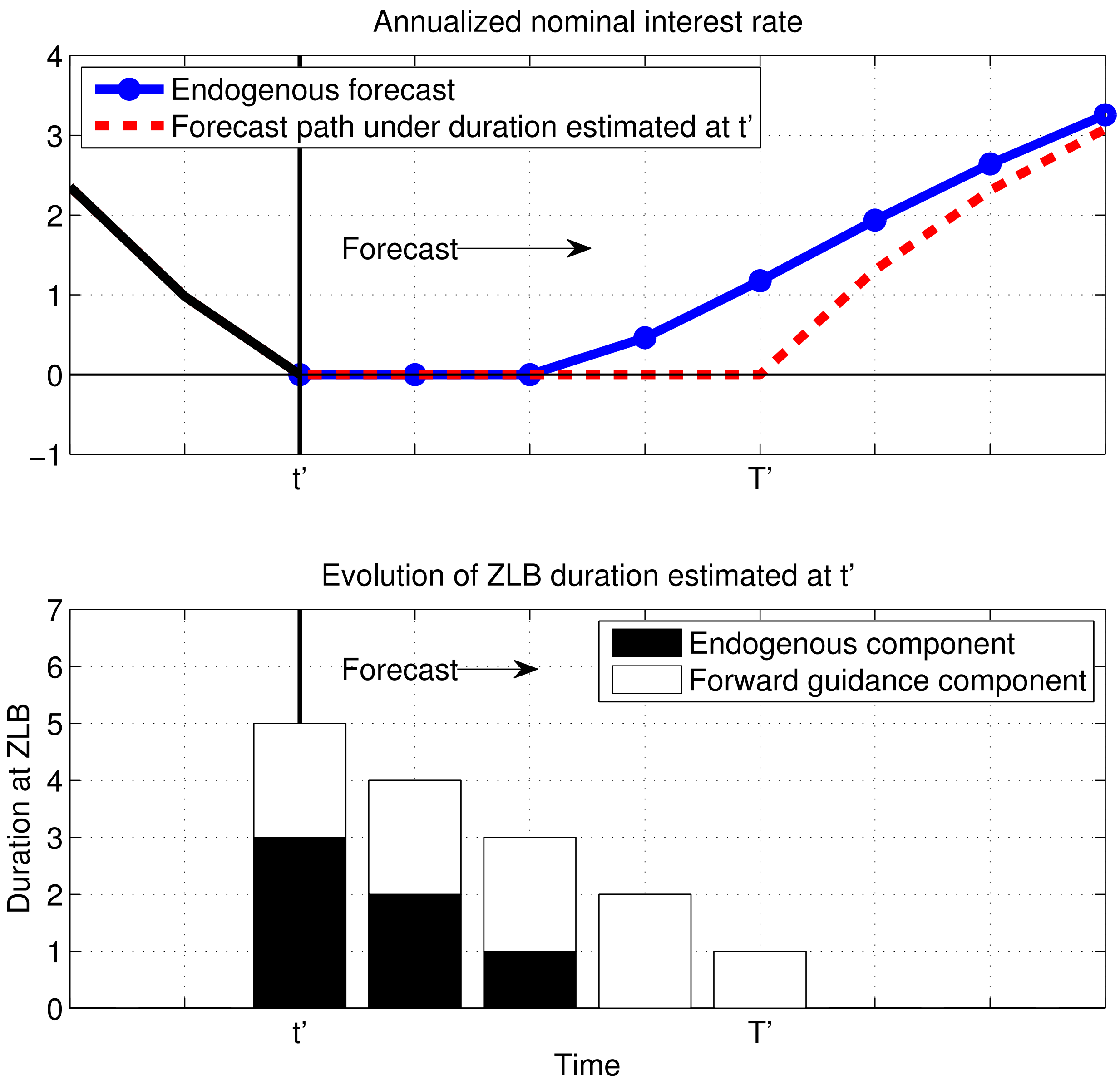

Figure 2: Decomposition example. At period t' the estimated duration is 5, so that the ZLB is expected by agents in the model to last until period $t' + 4$, after which the nominal interest rate rises above zero (red dashed line). The Kalman smoother is used to obtain an estimate of the structural shock $w_{t'}$. Using that shock, the ZLB algorithm we use gives us a forecast of the nominal interest rate which implies a ZLB duration of 3 quarters (blue line). The forward guidance component at period t' is the difference between the estimated duration and the forecasted duration, which is 2 quarters.

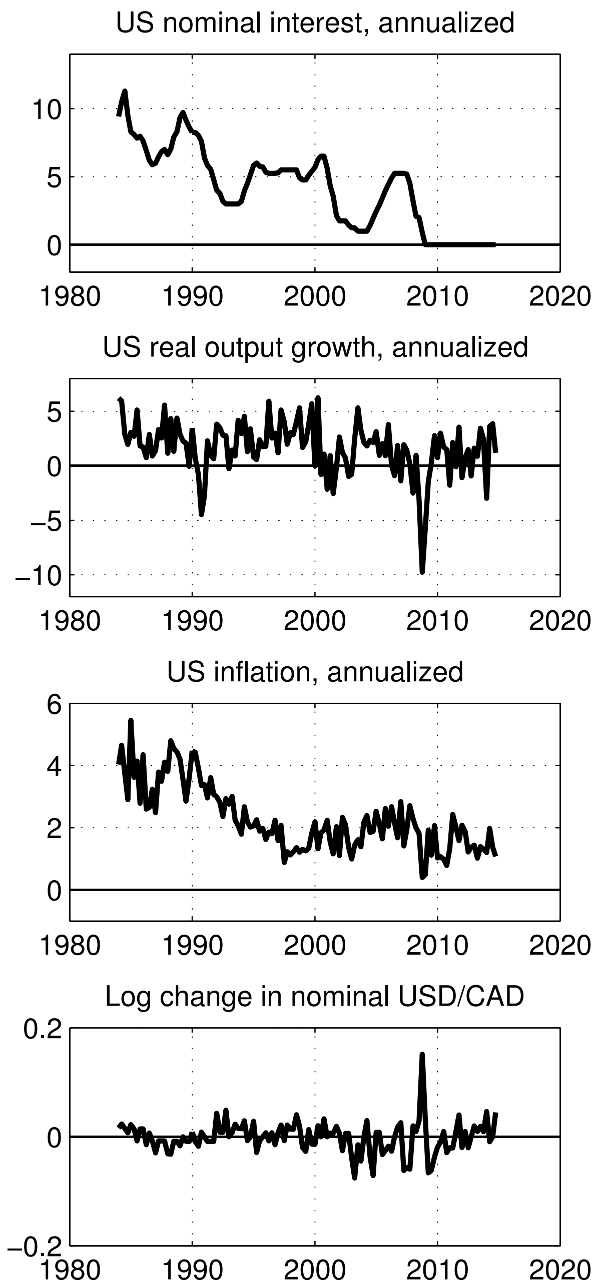

Figure 3: Quarterly data used in estimation. We also use 2Q, 1Y and 2Y Treasury yields in estimation.

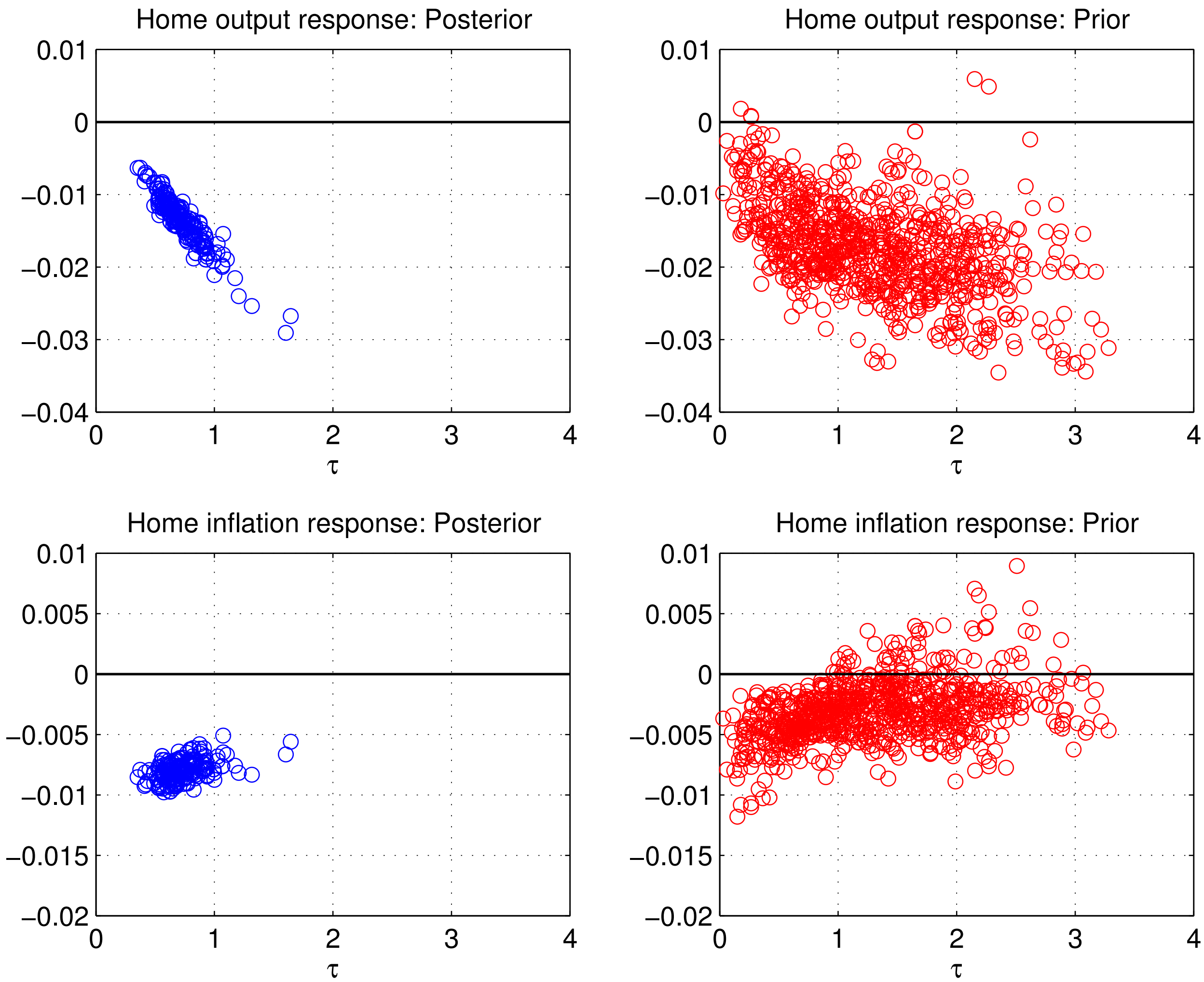

Figure 4: Effect of forward guidance. This figure plots the initial response of Canadian inflation and output against the trade elasticity τ across parameter draws from the prior and posterior following a large negative US demand shock sufficient to cause the US interest rate to hit its ZLB and which induces the US to impose two additional quarters of forward guidance policy. The shocks are scaled so that they are the same across the prior and posterior draws. The figure shows that the prior places some mass on a positive response of Canadian output and inflation to US forward guidance whereas the posterior places all weight on a negative response of Canadian inflation and output to US forward guidance.

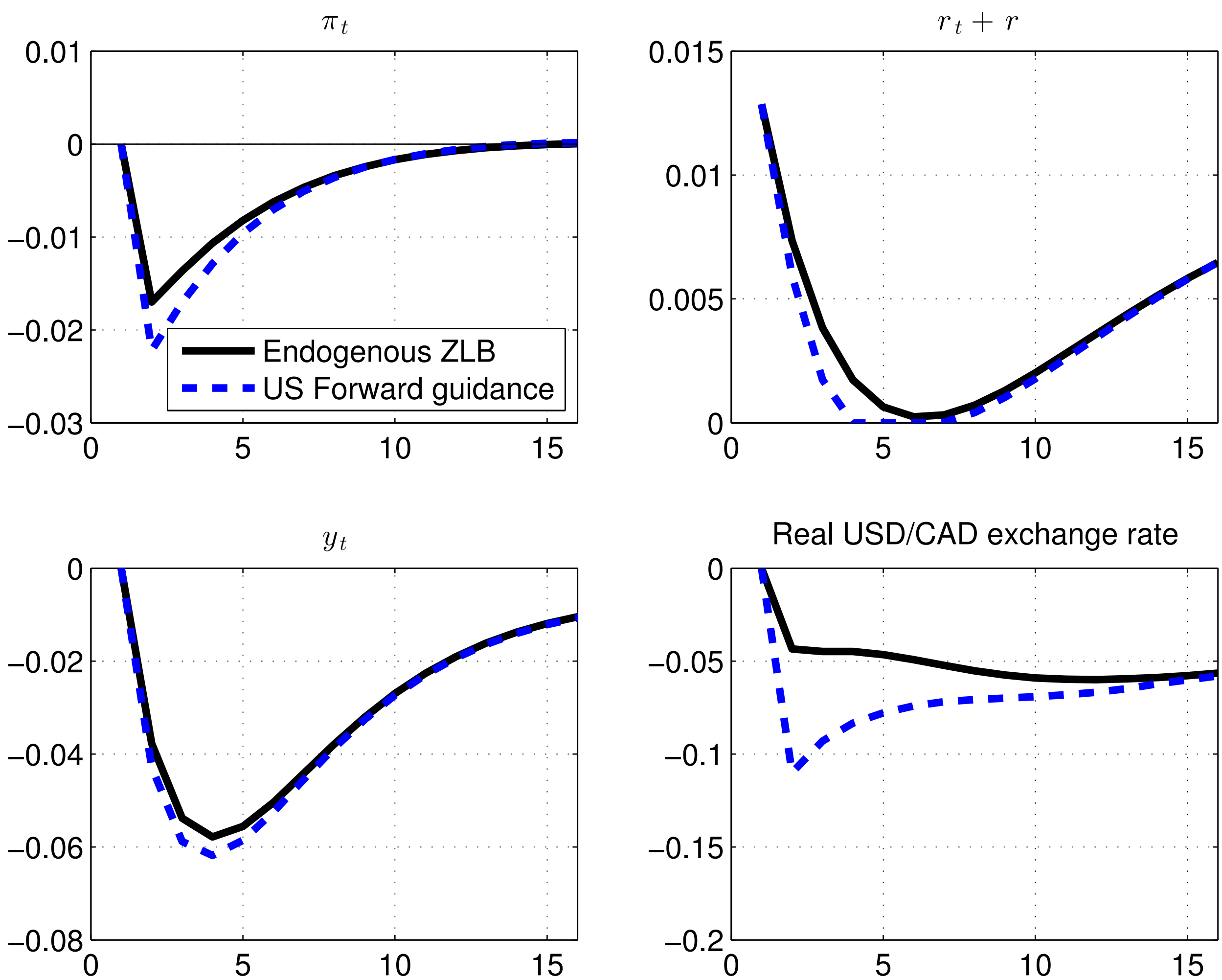

Figure 5: Cross-country spillovers of forward guidance. This figure plots an impulse response of the small open economy following a negative US demand shock and a negative Canadian demand shock. The dashed blue line plots the impulse responses of Canadian variables following these shocks when the US also announces 6 additional quarters at the ZLB over the ZLB duration induced by the US demand shock. Following this policy, the Canadian interest rate hits its ZLB, with a large appreciation of the real CAD/USD pair.

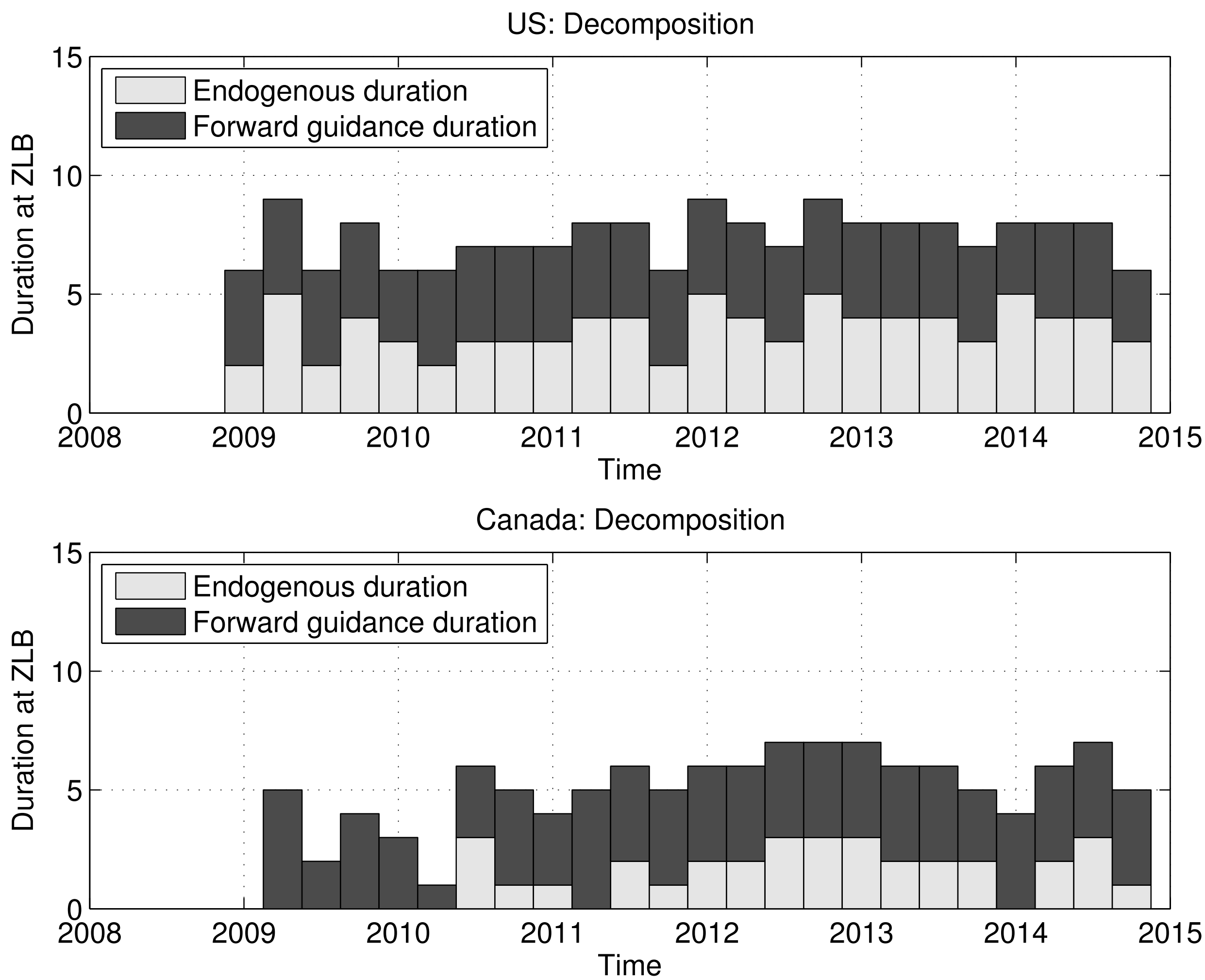

Figure 6: ZLB duration decomposition at the target median draw. This figure shows the decomposition of lower bound durations for the target median draw of the posterior distribution. That draw is chosen as the one which minimizes the sum of distances from the median draw of the posterior of durations for the US and Canada.

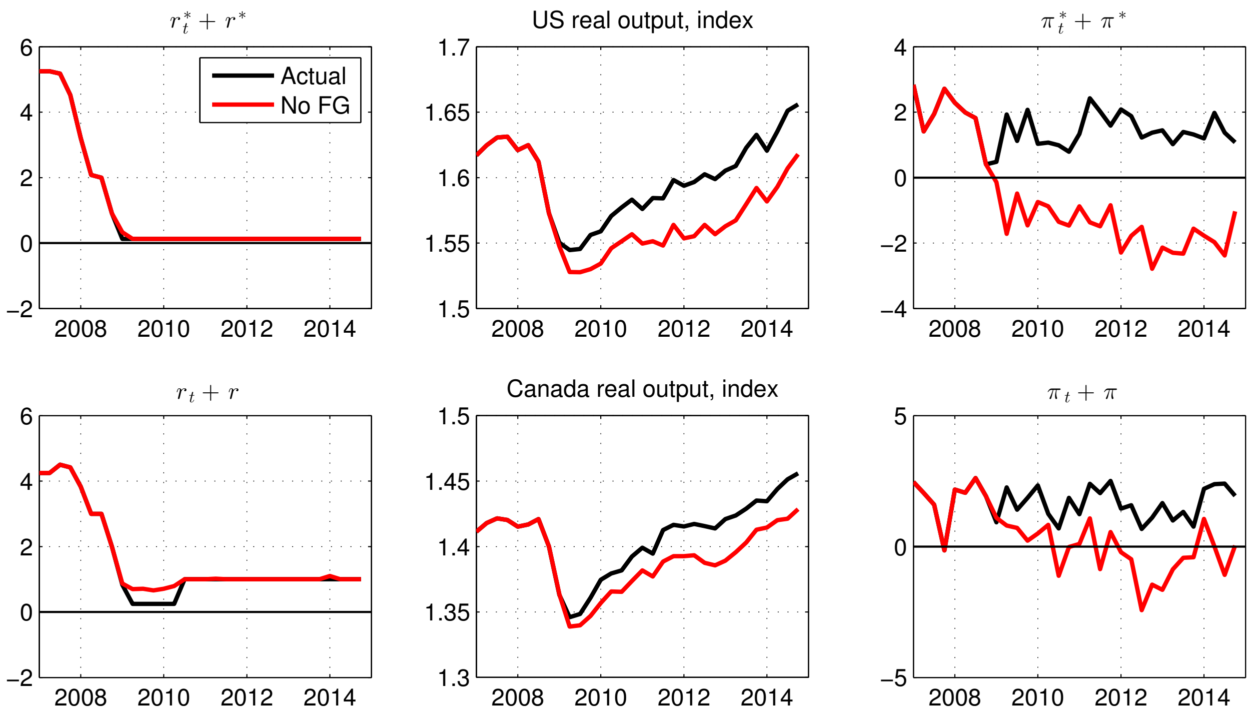

Figure 7: Removing forward guidance policies. This figure shows counterfactual series when both the US and Canada do not use forward guidance policies and instead act passively in response to the estimated structural shocks. Inflation and the nominal interest rate are annualized.

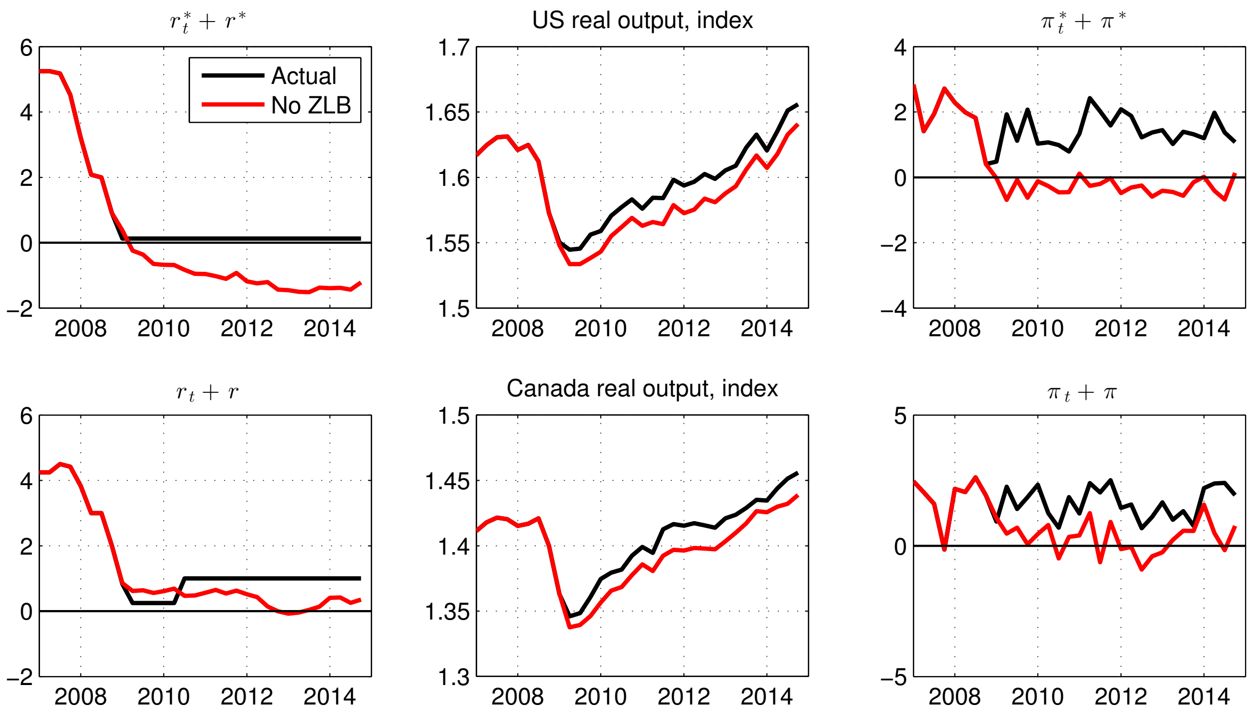

Figure 8: Removing lower-bound constraints. This figure shows counterfactual series when the ZLB is removed as a constraint on both US and Canadian policy interest rates. Inflation and the nominal interest rate are annualized.

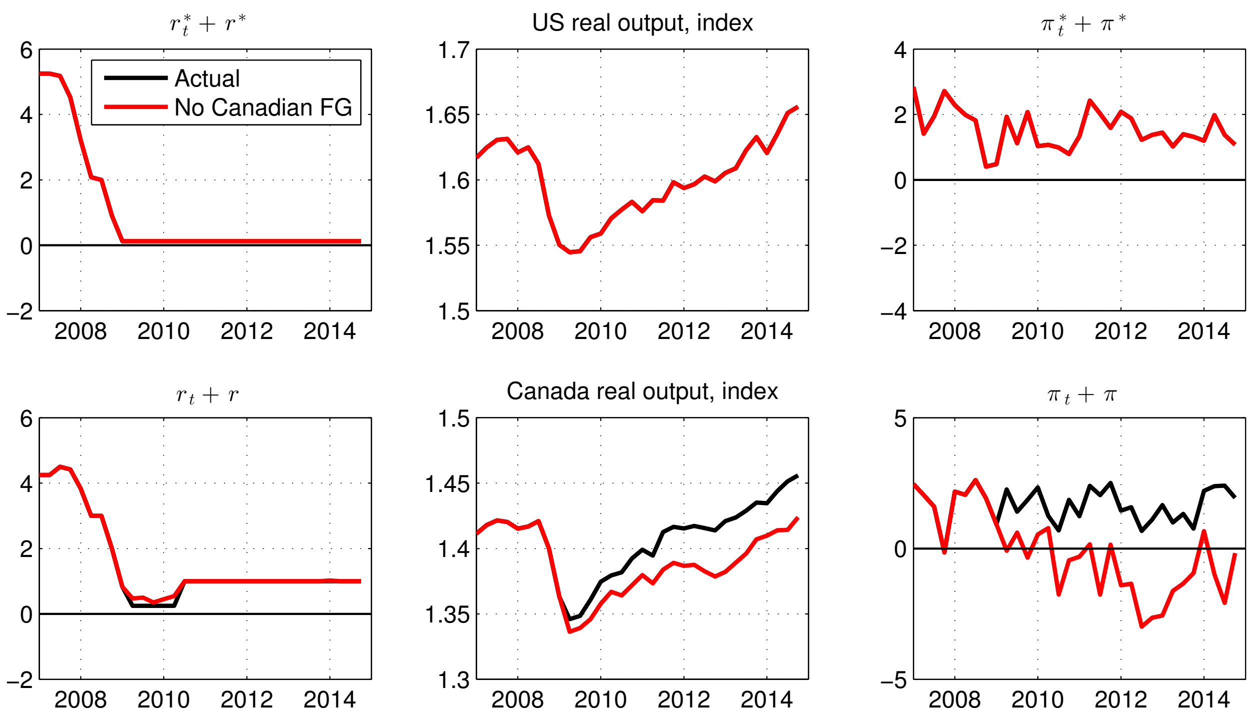

Figure 9: Removing Canadian forward guidance. This figure shows an additional counterfactual experiment, imposing estimated forward guidance in the US but not in Canada, so that the ZLB durations arising in Canada are those which are implied by the structural shocks with the Bank of Canada acting passively in response to those shocks. Inflation and the nominal interest rate are annualized.

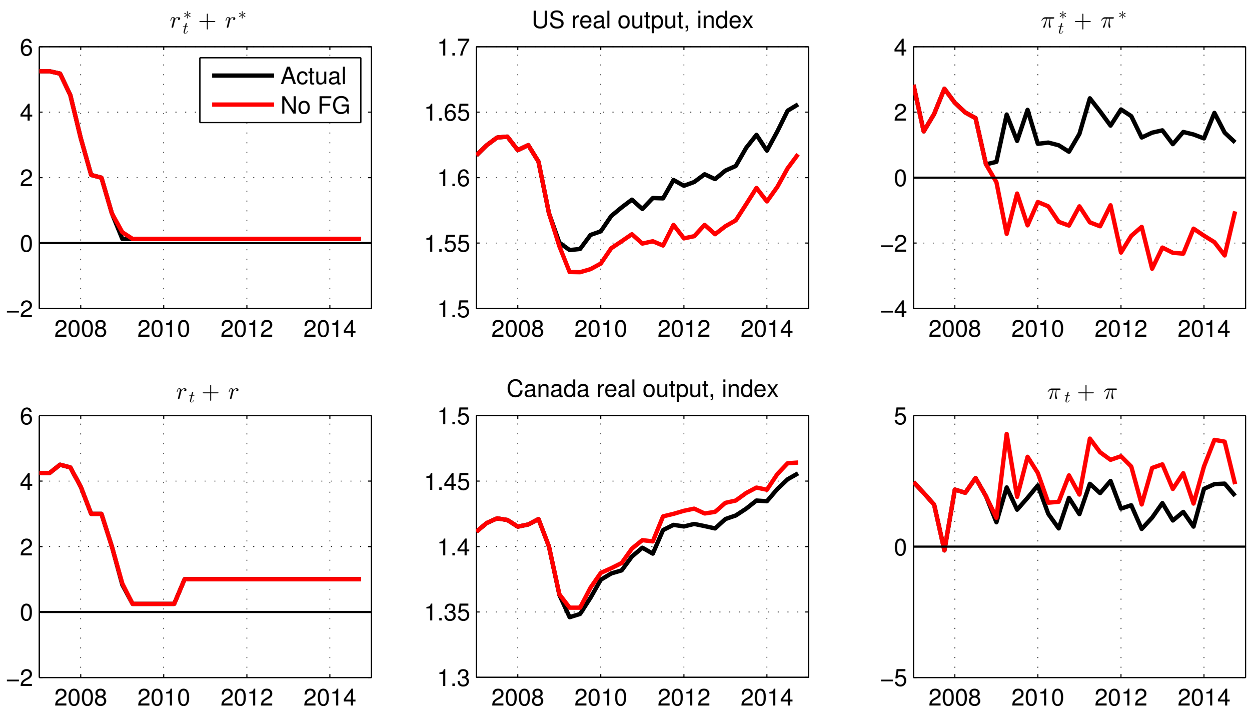

Figure 10: Removing US forward guidance. This figure shows counterfactual series when only the US does not use forward guidance policies and instead act passively in response to the estimated structural shocks. Inflation and the nominal interest rate are annualized.

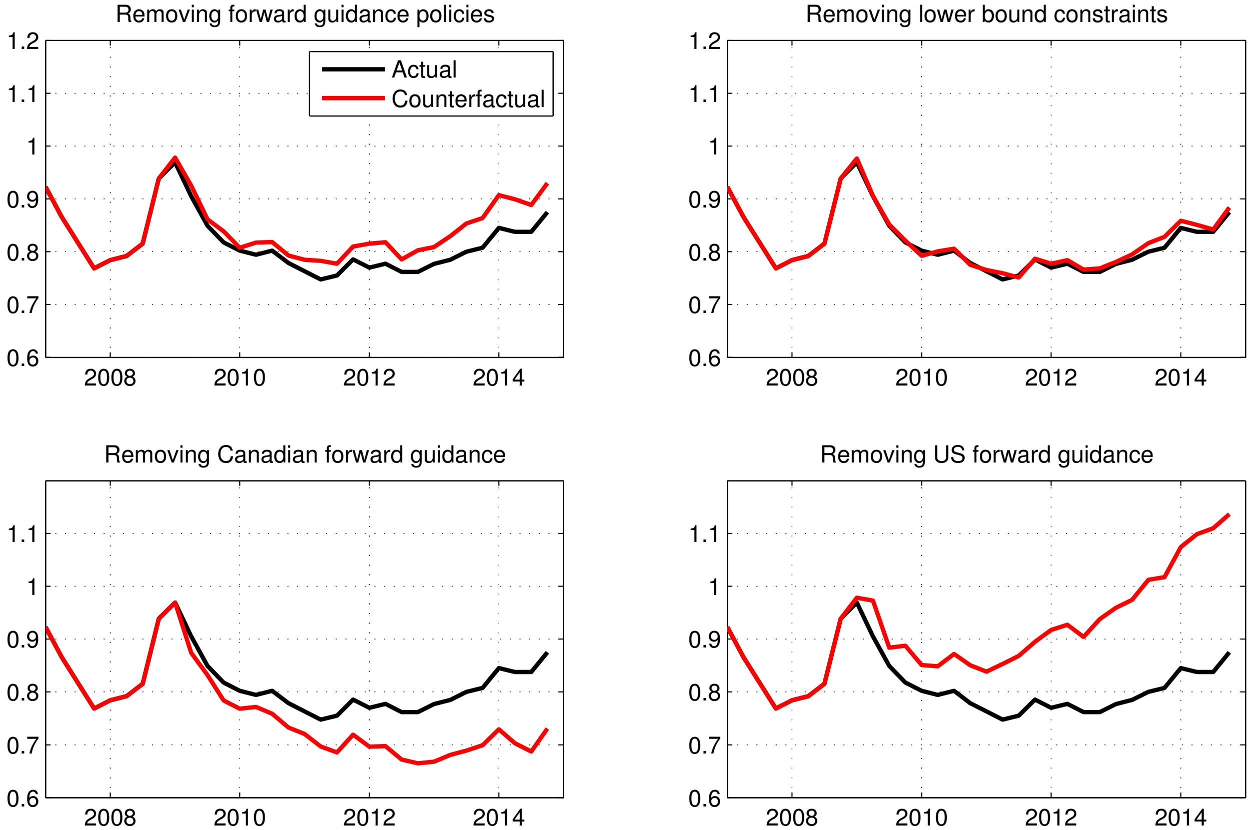

Figure 11: Index of nominal exchange rate under counterfactuals. The black line is the observed nominal interest rate. Removing US forward guidance and maintaining Canadian forward guidance generates a large appreciation of the US dollar against the Canadian dollar.

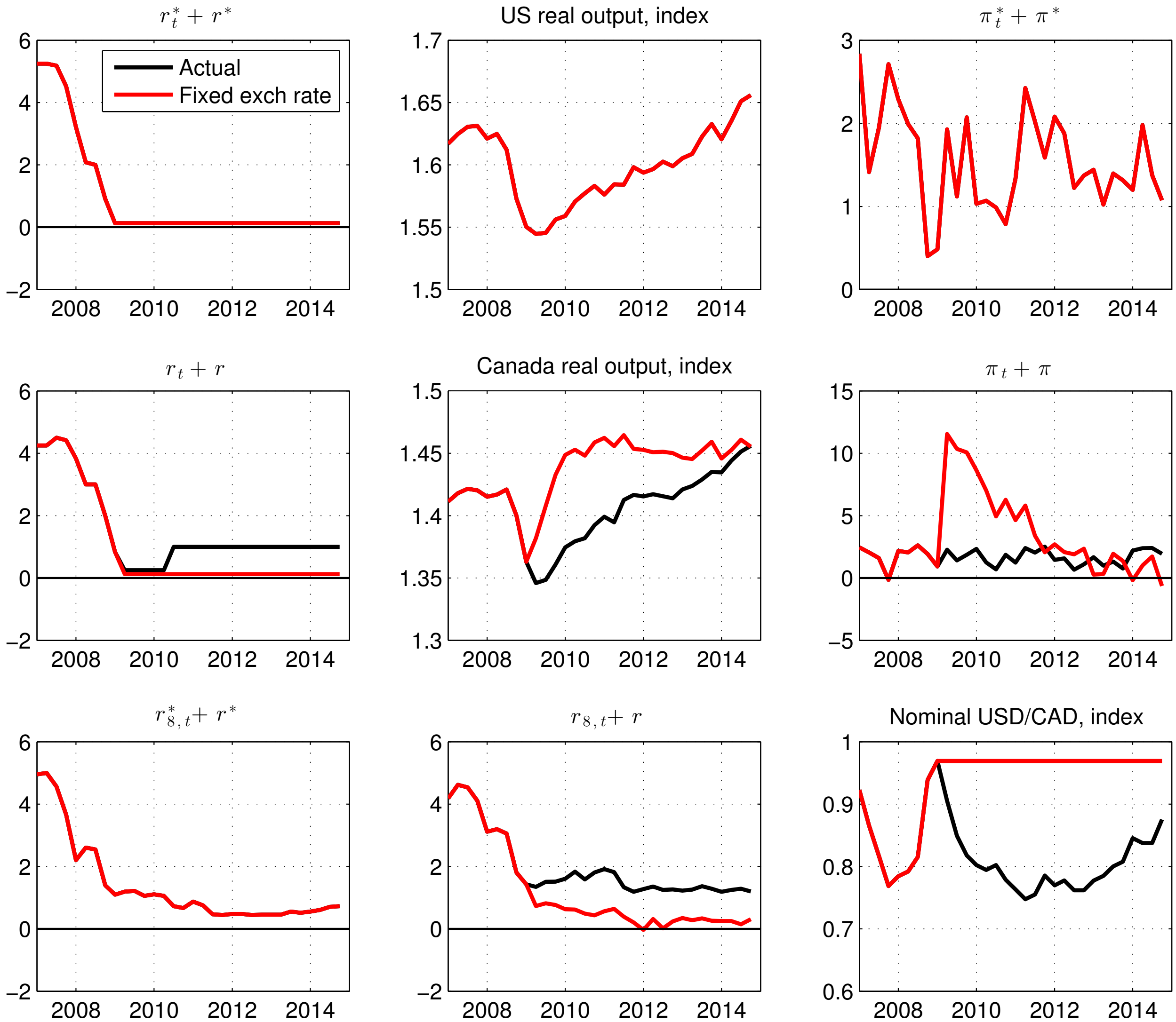

Figure 12: Fixed nominal exchange rate counterfactual. In this figure we compute a counterfactual where Canadian monetary policy is aimed at keeping the nominal exchange rate constant. We shut off the risk premium shock..