Unanticipated Shocks and Forward Guidance at the Zero Lower Bound*

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Abstract

The expected duration of a zero lower bound episode can change due to shocks or deliberate policy. The cause of the extension can have very different economic effects. In this paper, I propose a method to identify whether shocks or policy have caused interest rates to be at zero. In a New Keynesian model of the US estimated from 1984 to 2015, I find that forward guidance policy has extended the ZLB for up to a year between mid-2011 and 2013. I also show that a forward guidance rule can reduce output and inflation volatility, and that government spending multipliers at the ZLB fall under a forward guidance rule.

Keywords: Zero Lower Bound Duration, Forward Guidance.

JEL classifications: E5.

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1 Introduction

Around the world, central banks have set policy interest rates close to the zero lower bound (ZLB) since 2009. Eggertsson and Woodford (2003) show that, when faced with a binding ZLB, the optimal policy for a central bank is to credibly extend the duration that the policy rate is expected to be at zero beyond the duration that is implied by shocks. More recently, Kulish et al. (2017) show how to estimate the expected durations of the ZLB, and Guerrieri and Iacoviello (2015) develop a toolkit to compute the expected duration of a ZLB episode which is consistent with exogenous shocks.

Changes in the expected duration of the ZLB can have sharply different implications for an economy if they are brought about by shocks as opposed to being the result of a deliberate policy action. In particular, an increase in the expected duration caused by shocks which make the constraint bind for longer can be contractionary, while a deliberate *forward guidance* policy increase in the expected duration can instead be expansionary.

In this paper, I show how to identify whether changes in the expected duration are due to shocks or to policy by combining an approach to estimating ZLB durations with a method I develop to efficiently approximate models with occasionally binding constraints like the ZLB. I apply this decomposition to an estimated model of the US in which I allow for a break in trend growth (Fernald, 2015; Gordon, 2016). Allowing for a break in trend growth lowers the steady-state nominal interest rate, thereby interacting in a highly nonlinear way with the ZLB to cause it to bind more often and for longer.

I use Bayesian likelihood methods to estimate the parameters and ZLB durations of a New Keynesian model of the US, on aggregate data between 1984 and 2015. I estimate that annual trend growth falls from about 2.2% to 1.4% in the early 2000s. Furthermore, I estimate that the ZLB was expected to bind for durations ranging between three quarters and two years from 2009 to mid-2011. Between mid-2011 and the end of 2013, I find that these durations increase to between two years and three years, after which I estimate that they fall to about one year. When my algorithm for identifying the policy component of these ZLB durations is applied, I find that policy extended the length of time that the ZLB was expected to bind for between one quarter and one year between mid-2011 and the start of 2013. This aligns with the calendar-based forward guidance *announcements* by the Federal Reserve over this period. Using the structural model, I quantify the macroeconomic implications of these extensions, finding that, absent forward guidance, across the ZLB period output and consumption would have dropped a further 2.5%, and about 7% for investment.

The estimated expansionary benefits of the policy extensions of the ZLB are quite modest in light of the forward guidance puzzle studied by Del Negro et al. (2012), in which small changes in the expected path of real interest rates can have implausibly large aggregate responses. The reason for this in my model is twofold. First, to match the behavior of inflation since the recession, I estimate that prices are very inflexible, which reduces the responsiveness of the real interest rate to announcements about the path of nominal interest rates. Furthermore, the estimated decline in trend growth implies that the long-run level of the nominal interest rate falls. Forward guidance at the ZLB has more bite when the change in the expected path is more pronounced. The estimated decline in trend growth flattens the expected path of nominal interest rates so that announcements become less expansionary.

After estimating and decomposing the durations of the ZLB in the US since the recession, I then analyze calendar based forward guidance in the estimated model by formulating a forward guidance rule under which the Federal Reserve extends an expected ZLB episode by an amount which is increasing in the output gap that would have arisen in the absence of such an extension, with that counterfactual output gap computed using the method developed in this paper. I show that calendar based forward guidance policies of the kind studied in theory by Eggertsson and Woodford (2003), Laseen and Svensson (2011) and Werning (2012), when formalized as a policy rule, can reduce inflation and output volatility, even if the economy is hit with additional shocks during the period that the Federal Reserve has committed to holding the policy interest rate constant.

I then explore how government spending multipliers can fall when the ZLB duration is manipulated deliberately in response to shocks, with the size of fiscal policy multipliers at the ZLB falling to be around the size of the multiplier that arises when there is no ZLB constraint. This is because the stimulatory effect of an unexpected positive government spending shock at the ZLB relies on an inactive central bank (see Christiano et al., 2011), so that if monetary policy follows a forward guidance rule of the kind that I study, the central bank offsets the government spending shock. This conflicting policy behavior, which is typical in equilibrium models outside the ZLB, means that monetary policy can lower the fiscal multiplier, even though nominal interest rates are fixed.

The model I use is a modified version of Smets and Wouters (2007) and to study it with the ZLB, I develop a piecewise-linear approximation of dynamic general equilibrium models with occasionally binding constraints in which monetary policy switches between a regime in which the ZLB binds and one in which the policy rate is positive. In a rational expectations solution, applying the ZLB is equivalent to changing the structure of the model in a way which is anticipated by workers and firms. The algorithm iterates on the path of the nominal interest rate until it aligns with agents' expectations. I use the method to impose calendar based forward guidance policies, in which the central bank announces and commits to an interest rate path. I compare the approximation developed to a nonlinear approximation for a simple model and show that it is accurate.

In work that is contemporaneous and independent to my own, Guerrieri and Iacoviello (2015) develop a method to solve models with occasionally binding constraints. Their approach and mine give the same answer, provided that the guess and verify iterations find that the ZLB binds in the same periods. There is, however, a difference in the methodologies which is worth highlighting. Because my method is an iterative application of the solution for anticipated structural changes of Cagliarini and Kulish (2013), it has the advantage that the conditions of uniqueness can be checked. This allows me to show, for the ZLB, that the conditions of determinacy hold for *any* finite expected duration of fixed interest rates.

Other papers have studied forward guidance policies at the ZLB and have suggested that a commitment to calendar based forward guidance raises the risk of undesirable aggregate outcomes during an announcement period (Campbell et al., 2012; Coenen and Warne, 2014; Boneva et al., 2015). Boneva et al. (2015) show in a simple model that an alternative forward guidance policy, threshold-based guidance policy, can be an effective hedge against adverse outcomes during an announcement period. Under threshold-based guidance, a central bank announces it will hold its policy interest rate at zero so long as pre-announced thresholds (like numerical unemployment or inflation targets) are not breached.

Furthermore, the optimal calendar based forward guidance policies of Eggertsson and Woodford (2003) and Werning (2012) are derived in tractable models with few state variables. The piecewise linear algorithm I use instead allows me to analyze these policies in models with rich price ridigity and a number of sources of persistence, and with many shocks that hit every period, such as the Smets and Wouters (2007) model. I show that the optimal theoretical policies are largely replicated in this model, and that, because of the nonlinearities induced by the ZLB, the dynamic behavior of an economy under forward guidance is substantially different to the behavior of an economy that is not faced with the ZLB constraint.

The paper is organized as follows. Section 2 discusses the model used in the estimation and the algorithm and solution method. Next, Section 3 discusses the estimation results. Section 4 examines (i) the policy decomposition in the estimated model, (ii) how forward guidance as set by a simple rule linked to the output gap can mitigate the effect of shocks, and (iii) how government spending multipliers change under such a rule. Section 5 concludes.

2 Model and solution method

2.1 Model

The model I use is that of Smets and Wouters (2007) (SW) with two changes: to monetary policy and to trend growth. The model is well-known, so the details of the linearized equations are left to the Appendix.

The first, and main, difference between the SW model and the model used in this paper is the specification of monetary policy. Monetary policy operates in one of two possible regimes. In the first regime, the nominal interest rate can be set according to a standard Taylor rule, as in SW:

$$r_t - r = \rho(r_{t-1} - r) + (1-\rho)\left[r_\pi \pi_t + r_Y(y_t - y_t^p)\right] + r_{\Delta y}\left[\Delta(y_t - y_t^p)\right]. \tag{1}$$

The nominal interest rate r_t responds to deviations in inflation from a target rate π_t, deviations in output y_t from its potential level y_t^p, and the growth rate of potential output.[1]

In the second regime, the nominal interest rate is at zero and monetary policy enters the ZLB regime:

$$r_t = 0. \tag{2}$$

It can be in the ZLB regime in two ways: first, if the Taylor rule calls for negative nominal interest rates, and second, if the Federal Reserve has announced, or has previously announced, an extension of the ZLB beyond that implied by the constraint.

The second difference of the model I use to that of SW is that I allow for the trend growth parameter to change exogenously and unexpectedly. That is, there is a permanent, one-time, unanticipated shock to trend growth. The date and magnitude of that change will be estimated, as described below.

2.2 Solution method

In this section, I describe the algorithm that is developed to approximate the model subject to the ZLB.[2]

2.2.1 Notation and initialization

Let x_t be the $n \times 1$ vector of state and jump variables of the model at t, one of which is the nominal interest rate r_t, and w_t be a vector of exogenous variables at t. The nonlinear

[1] Unless otherwise stated, variables are expressed in log-deviations from their steady-state values.

[2] An implementation of the algorithm integrated with Dynare is available at *https://callumjones.github.io*

rational expecations model is a system of n equations $x_t = \Psi(x_{t-1}, \mathbb{E}_t x_{t+1}, w_t)$. The known variables at period t are the shock that hits at period t, w_t, and the initial vector of variables x_{t-1}.

Linearize the model around its non-stochastic steady-state and denote the resulting n equations as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}w_t. \tag{3}$$

Standard methods are used to obtain the reduced form:

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}w_t. \tag{4}$$

If the ZLB binds at t^*, denote the model system as:

$$\mathbf{A}^* x_{t^*} = \mathbf{C}^* + \mathbf{B}^* x_{t^*-1} + \mathbf{D}^*\mathbb{E}_{t^*} x_{t^*+1} + \mathbf{F}^* w_{t^*}. \tag{5}$$

The only difference between (3) and (5) is that the Taylor rule in (3) is replaced with $r_t = 0$. In general, the system (5) will not have a solution in the form of (4) because the Blanchard-Kahn conditions are not satisfied if the nominal interest rate is fixed (see Davig and Leeper, 2007).

2.2.2 The ZLB algorithm

Given the exogenous variables w_t and a vector of state variables x_{t-1}, the first step of the algorithm is to forecast the path of all variables under a linear approximation when all future uncertainty is ignored and where the ZLB constraint is also ignored. The second step is to check the computed path of the nominal interest rate, and in the time periods that the ZLB does bind, if at all, impose the ZLB so that it is anticipated by agents in the period the shock hits. An anticipated change in the future structure of the economy affects all endogenous variables before that change. In turn, changes to the endogenous variables affect the path of the nominal interest rate. To ensure the realized path is consistent with agents' expectations, an iterative step is needed to update the endogenous variables period-by-period.

Formally, the steps of the algorithm are:

0. Linearize the model around the non-stochastic steady state, ignoring the ZLB, to obtain (3) and (4).

1. For each period t:

   (a) Solve for the path $\{x_\tau\}_{\tau=t}^T$ with T large, using (4), given w_t and x_{t-1}, and assuming no future uncertainty $\{w_\tau\}_{\tau=t+1}^T = 0$, so that $x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}w_t$, and $x_{t+1} = \mathbf{J} + \mathbf{Q}x_t$, up to $x_T = \mathbf{J} + \mathbf{Q}x_{T-1}$. This step gives a path $\mathbf{r}_t^k = \{r_\tau\}_{\tau=t}^T$.

(b) Examine the path $\mathbf{r}_t^k = \{r_\tau\}_{\tau=t}^T$. If $\mathbf{r}_t^k \geq 0$, then the ZLB does not bind, and move to step (2). If $\mathbf{r}_t^k < 0$, then move to step (1c). That is:

- If $r_\tau \geq 0$ for all $t \leq \tau < T$, accept $\{x_\tau\}_{\tau=t}^T$. The path of the nominal interest rate does not violate the ZLB today or in future.
- If $r_\tau < 0$ for any $t \leq \tau < T$, move to step (1c).

(c) Update the path of $\{r_\tau\}_{\tau=t}^T$ for the ZLB. Denote by t^* the *first* time period where $\mathbf{r}_t^k < 0$, and set the nominal interest rate in that period to zero. This changes the anticipated structure of the economy.

Computing the new path $\{r_\tau\}_{\tau=t}^T$ under the structural change involves computing $\{x_\tau\}_{\tau=t}^{t^*}$ and $\{x_\tau\}_{\tau=t^*+1}^T$. At t^*, $\mathbb{E}_{t^*} x_{t^*+1}$ is computed using the the reduced form solution (4) and $w_{t^*+1} = 0$. This expresses x_{t^*} as a function of x_{t^*-1}. Proceeding in this way with the correct structural matrices (either (3) or (5) at each time period), compute the path $\mathbf{r}_t^{k+1} = \{r_\tau\}_{\tau=t}^T$.

A convenient way to compute the new path $\mathbf{r}_t^{k+1} = \{r_\tau\}_{\tau=t}^T$ is to form the time varying matrices $\{\mathbf{J}_\tau, \mathbf{Q}_\tau, \mathbf{G}_\tau\}_{\tau=t}^T$ which satisfy the recursion:

$$
\begin{aligned}
\mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t \\
\mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} (\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}) \\
\mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{F}_t,
\end{aligned}
$$

with the final set of reduced form matrices for the recursion being the non-ZLB matrices $\mathbf{J}$, $\mathbf{Q}$, $\mathbf{G}$.

These time-varying matrices are then used to compute the path $\{x_\tau\}_{\tau=t}^T$ by calculating $x_\tau = \mathbf{J}_\tau + \mathbf{Q}_\tau x_{\tau-1} + \mathbf{G}_\tau w_\tau$. For more details on this particular recursion, see Kulish and Pagan (2017).

Iterate on steps 2 and 3 until convergence of $\mathbf{r}_t^{k+1}$ and $\mathbf{r}_t^k$.

2. Increment t. The initial vector of variables becomes x_t, which was solved for in step 1. Draw a new vector of unanticipated shocks w_{t+1} and return to step 1.

The algorithm yields a set of time-varying structural matrices:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t, \tag{6}$$

from which we get the path $\{x_\tau\}_{\tau=t}^\infty$ where the nominal interest rate is subject to the ZLB. Furthermore, if the ZLB does bind under the sequence of exogenous variables, the algorithm produces, at each period t, a sequence of expected durations of the ZLB.

2.2.3 Existence and uniqueness

In general, the path of the economy under this approximation will exist and be unique, as summarized in the following result.

Result 1. *If there are n distinct equations for n variables in the linearized model (3) under the non-ZLB regime and the Blanchard-Kahn conditions are satisfied for the linearized model under the non-ZLB regime, then the path during the ZLB period exists and is unique.*

The full reasoning is in the Appendix. Intuitively, the result says that if the linearized model has a unique solution (4) when the ZLB does not bind, that solution pins down agents' expectations of when the ZLB regime ends. With those expectations pinned down, the path of the economy is uniquely described by the structural equations (5). For a discussion of issues of existence and uniqueness in the presence of foreseen structural changes, see Cagliarini and Kulish (2013).[3]

2.3 Forward guidance decomposition of a ZLB duration

Incorporating calendar based forward guidance involves setting, at step (1b) of the algorithm, the nominal interest rate regime to the ZLB regime in the periods that the central bank announces that it will hold interest rates at zero. The path $\{x_\tau\}_{\tau=t}^{\infty}$ under forward guidance announcements will differ from the endogenous ZLB path to the extent that credible announcements change agents' expectations of the periods that the ZLB will bind. As recognized by Eggertsson and Woodford (2003), Jung et al. (2005), and Werning (2012), this manipulation of agents' expectations can be stimulatory if it lowers the path of real rates.

The total ZLB duration can, in principle, be any length and can be decomposed into two components. The first component is one which is consistent with the structural shocks that prevail at each point in time. I call this duration the *endogenous duration.* The second component is the difference between the total expected duration and the endogenous duration. This second component can be interpreted as a calendar based forward guidance duration. If the endogenous duration is expected by all agents in the economy, the central bank is acting passively in response to the structural shocks and expects to raise the nominal interest rate off the ZLB as soon as the policy rule prescribes. If, instead, the expected ZLB duration is longer than the endogenous duration, then agents in the economy believe the monetary authority is making a commitment to hold its policy rate at zero beyond the period implied

[3] A worked example applying the Sims (2002) solution concept and setting up the structural matrices is given in the Appendix.

by the endogenous duration.

The algorithm of this section can be used to compute this decomposition of the sequence of ZLB durations so as to uncover the fraction of the duration which can be assigned to a forward guidance announcement. To make this clear, suppose, at period t, the ZLB binds and we have in hand:

1. the duration T_t of the ZLB at t, so that the interest rate is expected to stay at zero until period $t + T_t$, and
2. the history of the states $\{x_\tau\}_{\tau=0}^{t-1}$ and the structural shocks $\{w_\tau\}_{\tau=1}^{t}$. Estimates of these can be obtained using the Kalman filter and smoother.

The ZLB durations and shocks recover the observed series. To decompose the estimated duration into a component due to the smoothed structural shocks and a forward guidance component, at each point of time, we use the initial state x_0 and the sequence of exogenous shocks $\{w_\tau\}_{\tau=1}^{t}$ to compute, using the algorithm, a counterfactual series for the ZLB durations. This counterfactual sequence of ZLB durations defines the endogenous durations, with the difference between this series and the T_t series being forward guidance.

3 Estimation

I estimate the expanded SW model on a sample from 1984Q1 to 2015Q3. The ZLB first binds in 2009Q1. The data series are constructed in the same way as in SW, and are plotted in Figure 1. The prior distributions and posterior estimates for each parameter are given in Table 1. The priors used for the structural parameters are the same as those used in SW except that I set a wider prior on the trend growth rate (Table 2). For the trend growth parameter, I set a normal prior, centered at 0.5% a quarter, with a standard deviation of 0.25%. For the estimated break date in the trend rate of growth, I use a uniform prior between the 20th and 80th percentiles of the sample, which restricts the trend growth break to occur between 1990Q1 and 2009Q2. I follow Kulish et al. (2017) in setting priors on the expected ZLB durations, which are based on the New York Federal Reserve's Survey of Primary Dealers. In that survey, primary dealers were asked when they expected the Federal Reserve would first lift the Federal Funds rate off the ZLB. The range of responses to those surveys are used to assign probabilities to liftoff dates. For the Bayesian Markov Chain Monte Carlo estimation, I construct four chains of half a million, and discard the first quarter million draws of each chain.

The posterior estimates of the structural parameters are broadly similar to the estimates of SW. To highlight some key differences, the persistence of the risk premia shock is substantially higher, with the posterior mode of ρ_b being 0.938 as compared to the SW value of 0.18. Offsetting this, the posterior mode of the variance of risk premia shocks σ_b is estimated here to be 0.062, as compared to a higher value of 0.2 in SW. Taken together, the unconditional variance of risk premia shocks in my estimates is around three times the size of the unconditional variance calculated from SW estimates. Accordingly, risk premia shocks play a much larger role in explaining the forecast error variance of output. These differences are most likely due to the large and persistent contraction in output, consumption, and hours associated with the financial crisis.

The estimate of the annual inflation target is about 2.25%, which is lower than the estimate of SW. This is because SW use a longer sample, starting in 1948, covering the period of elevated inflation in the 1970s and early 1980s. Wages and prices are also much less flexible than the estimates of SW, with the average duration of price contracts lasting about 4 years, significantly longer than the 3Q estimate in SW. Estimating less flexible prices, particularly using data from more recent samples, is consistent with other studies documenting the flattening of the Philips curve (see, for example Del Negro et al., 2015).

The Taylor rule parameters are important because they determine the expected behavior of the Federal Funds rate outside of the ZLB, and therefore when nominal interest rates can be expected to be lifted off the ZLB. For example, observing long durations at the ZLB together with large output gaps is more indicative of a strong response of the nominal interest rates to the output gap outside of the ZLB. The posterior estimates suggest a relatively weak long-run reaction r_π to deviations of inflation from target $\bar{\pi}$ (1.31) as compared to the estimate of SW (2.03). In contrast, I estimate the long-run response to output gap deviations to be 0.185, which is much higher than the SW estimate of 0.08, and which are consistent with the narrative that the Federal Reserve has been more focused on employment outcomes than deviations in inflation from a 2% target (see also Rudebusch and Williams, 2016). One way to shed light on how these parameters are identified is to compare (i) a counterfactual series for the Federal Funds rate computed using the parameter values of SW, and (ii) the observed Federal Funds rate. That comparison suggests that, under the SW estimates, the Federal Funds rate would have been lifted off the ZLB as early as 2010.

Table 2 summarizes the prior and posterior distributions for the trend rate of growth before and after its break on the 1984Q1 to 2015Q3 sample. The data prefer an estimate of trend growth before the structural break of around 0.53% a quarter, and around 0.34%

a quarter after the break. The posterior distribution of the date that trend growth falls is centered around 2001Q2. The posterior distribution of the break date is shown in Figure 2, illustrating how the data are supportive of the change in trend growth occuring in the early 2000s. This is consistent with a number of studies of trend growth and with Federal Reserve and Congressional Budget Office projections of potential output (also see Luo and Startz, 2014; Fernald, 2015).

At the posterior mode, the implied annual steady-state real and nominal interest rates are 2.63% and 4.85% before 2001Q2, and fall to 1.91% and 4.11% after 2001Q2. This steady-state real rate is close to the values estimated by Johannsen and Mertens (2016), while the steady-state nominal rate is higher than the 3% median long-run Federal Funds rate projected by the Federal Reserve. The implied posterior distribution for the real interest rate places very little mass on the real interest rate being below 1%. This estimate is important, because of the potential for very low and negative real interest rates to give rise to a persistent output gap when the ZLB binds (Eggertsson and Mehrotra, 2014).

Table 3 presents the estimates of the posterior distributions of the ZLB durations each quarter from 2009Q1 to the end of the sample in 2015Q3. The data are quite informative about the estimates of the duration, with the posterior distributions having a much smaller variance than the prior distributions. For the periods between 2009 and 2010, and again between 2014 and 2015, the posterior distributions exhibit thick right tails, with some probability mass assigned to quite long durations. The mode of the ZLB durations suggests that agents expected a relatively rapid return to a positive Federal Funds rate between 2009 and 2010. The estimated durations then increase notably in 2011Q3, coinciding with Federal Reserve announcements about the future path of the Federal Funds rate. The estimated ZLB durations stay high at around 2 to 2.5 years throughout 2012 and into 2013, before gradually declining to expected durations of one year in 2015. The last column of Table 3 shows a data series of the sequence of expected durations computed by Morgan Stanley using Federal Funds futures data. This series displays the same pattern as the mode of the posterior distribution, but is consistently shorter by about two to three quarters, likely reflecting positive risk premia in Federal Funds futures prices.

Next, I extract the smoothed estimates of the structural model by setting the parameters to the mode of their respective posterior kernels, by setting the sequence of ZLB durations to the mode of the distribution implied by the New York Survey of Primary Dealers, and by setting the date that trend growth changes to its modal estimate. The smoothed estimates of the structural shocks are presented in Figure 3. To fit the 2008-09 recession and the persistent deviation in employment from its steady-state, the model requires a large negative risk-premia shock. When the ZLB binds, the monetary policy shock is zero.

With the structural shocks, we can now examine a number of counterfactual paths of the economy (Figure 4). We first look at what the model says would have occurred had trend per capita growth not fallen by an estimated 0.75% annual percentage points. Strikingly, log per capita output would be about 13 percentage points higher in 2015, while the recovery of the economy's pre-recession output level would have been swift, occurring by mid-2010. This compares to a similar period of time it took for output to recover from the early 1990s recession (two years).

Furthermore, while the change in trend output growth also causes the steady-state nominal interest rate to rise, it was not enough for the economy to avoid the ZLB. On the other hand, the increase in the steady-state nominal interest rate results in shorter counterfactual ZLB durations–instead of being around 12 quarters in 2013, in the counterfactual with higher trend growth, they are, at most, 6 quarters in duration. The nonlinearities associated with the ZLB are also apparent when comparing the ZLB durations under the counterfactual with high trend growth to the ZLB durations used to compute the shocks. Focusing in particular on the period between 2011 and 2014 when the expected duration the ZLB binds doubles from about 6 quarters to 12 quarters in a year, in the counterfactual with higher trend growth, they increase much more modestly from around 4 to 6 quarters.

The model is well suited to quantifying the extent that the ZLB is a binding constraint, by examining the path of the economy absent the ZLB. In this case the Federal Reserve acts according to its Taylor rule at its estimated parameter values, and when faced with the same exogenous shocks. Absent the ZLB, the annual Federal Funds rate would have declined to about -2 per cent in 2009 and stayed there through to 2015. Since inflation is largely unchanged when the ZLB constraint is removed, output and consumption recover faster as a lower real interest rate follows from slacker monetary policy.

The essentially unchanged path of inflation when the ZLB constraint is removed is consistent with the flat Philips curve in the estimated model, and with the results of Kulish et al. (2017) and the analysis of Del Negro et al. (2015). These papers discuss how, in New Keynesian models with a forward looking Philips curve, inflation is a function of the future path of marginal costs. Following risk premia shocks of the kind that drive the 2008-09 recession in the SW model, forecasted marginal costs recover sufficiently fast so that inflation does not fall much. This force is particularly strong when prices react less to current marginal costs so that the Philips curve is essentially horizontal.

To make this point, I consider a counterfactual path of the economy where prices are more flexible. In particular, I compute, using the structural shocks, a counterfactual path of the economy where the average duration of a price contract is about half as long as implied by the baseline estimation results (8 quarters versus the estimated 15 quarters). In this counterfactual, I remove forward guidance stimulus. Inflation reacts more to low marginal costs caused by the recessionary shocks. At the ZLB, the response of the real interest is particularly acute, rising notably from the observed real interest rate. As a result, output contracts. An implication of this observation is that guidance made about the path of the Federal Funds rate will have more power when prices are more flexible. The flattening of the Philips curve therefore tempers the severity of the forward guidance puzzle highlighted by Del Negro et al. (2012).

4 Calendar based forward guidance at the ZLB

Calendar based forward guidance – the explicit announcement of the length of time for which the policy rate is to be held at zero – is a tool that has been argued can provide monetary stimulus when the ZLB binds (Werning, 2012). By credibly lowering expectations about the path of interest rates, the monetary authority encourages households to intertemporally shift consumption stimulating output and inflation. In this section, I analyze the quantitative implications of calendar based forward guidance policies in the estimated model.

4.1 Decomposition of the ZLB durations

I first decompose the estimated ZLB durations into the endogneous component due to the shocks interacting with the ZLB, and a component which cannot be explained by the shocks alone, and is therefore attributed to an announcement by the Federal Reserve to hold interest rates at zero. The decomposition follows the procedure laid out in section 2.3.

The result of this decomposition of the ZLB durations is shown in the bottom left panel of Figure 4. The durations expected by all agents in the economy are the same as those that are consistent with the structural shocks of the model from 2009Q1 to 2011Q3, after which forward guidance policy becomes more apparent. From 2011Q3 to 2013Q1, the decomposition reveals that the ZLB duration expected is between 1Q and 4Q longer than the ZLB duration implied by the structural shocks, with the largest forward guidance duration occurring in 2012Q4. These results are consistent with the Federal Reserve making explicit calendar-based announcements about when the Federal Funds rate would be raised. For example, in August

2011, the Federal Open Market Committee announced in its statement that the Federal Funds rate would be held at 'exceptionally low' levels 'at least through mid-2013' – consistent with the 9Q duration expected in 2011Q3. In October 2012, the FOMC further committed to hold the Federal Funds rate low until at least mid-2015, which is also consistent with the 12Q duration used to extract the model's shocks.

The decomposition suggests that announcements did affect expectations by lengthening the expected ZLB period by one year. Table 4 documents the difference in macroeconomic aggregates due to these announcements. The macroeconomic implications of calendar based forward guidance in the estimated model are fairly small because of the largely unchanged response of inflation. At its largest, an announced 4Q increase in the ZLB duration over the endogenous duration due to structural shocks causes inflation to increase by 0.01 log points, while output and consumption increase by about 0.5 log points. Investment increases by a little more–about 1.2 log points. Cumulatively, absent forward guidance the reduction in output was 2.9%, for consumption the reduction was 2.7%, and 8.0% for investment.

4.2 A forward guidance policy rule

In principle, the central bank can announce any sequence of ZLB durations. I consider a simple rule for a forward guidance commitment, where the duration announced is explicitly linked to the output gap that would prevail in the absence of active policy. The central bank, when the policy interest rate is unconstrained, follows a Taylor rule. However, when the ZLB binds or is expected to bind, I consider a rule where the central bank forecasts output under the prevailing shocks, and also forecasts the anticipated length of time the ZLB will bind. Under these forecasts, it announces an extension of the time period for which it expects to hold the nominal interest rate at zero. Explicitly, if the Federal Funds rate is expected to hit or go below zero, the duration T_t expected follows:

$$T_t = \max\left\{T_t^* + \lfloor -\gamma_y \left(y_t - y_t^p\right)^* \rfloor, T_t^* + T_{t-1} - T_{t-1}^* - 1\right\}, \tag{7}$$

where T_t^* represents, at time t, the endogenous duration (the length of time that the ZLB is expected to bind under the shocks only), and $\left(y_t - y_t^p\right)^*$ represents the output gap which would prevail under no forward guidance policy. The parameter γ_y is a positive number, so that any announced duration is increasing in the output gap. The value $\lfloor -\gamma_y \left(y_t - y_t^p\right)^* \rfloor$ represents the floor of $-\gamma_y \left(y_t - y_t^p\right)^*$, so that, for example, an additional reaction of 2.5 quarters becomes 2 quarters.[4] A rule such as this says that if the output gap is significantly

[4]This is needed because of the discrete nature of an announcement period.

negative, then the central bank will keep the interest rate at zero for longer than is expected under the sequence of shocks causing the ZLB to bind. This forward guidance policy is stimulatory at time t.

To commit the central bank to a prior forward guidance announcement, the second element of equation (7) says that the policy component of a ZLB duration can contract by, at most, one period. For example, suppose that at $t-1$ the central bank reacts to a shock that would ordinarily cause the ZLB to bind for 4Q (so that $T^*_{t-1} = 4$) by announcing an extension of the ZLB by an additional 2Q (so that $T_{t-1} = 6$). Suppose that in the next period, shocks would call for an endogenous duration of 2Q (so that $T^* = 2$). To commit the central bank at t to the policy announcement made at $t-1$, the shortest duration agents can expect in that period is 3Q, which is the endogenous duration plus one quarter less than the duration caused by the $t-1$ policy action.

To illustrate the operation of the rule, I parameterize $\gamma_y = 0.85$. The initial responses of inflation to risk premia shocks of different sizes in the estimated model are shown in Figure 5. The length of additional forward guidance is given by the gap between the two lines of the fourth panel of the figure. The stimulus of the forward guidance policy can be seen by comparing the initial reactions of inflation and output growth with and without the forward guidance rule. The additional duration under the rule can be substantial. For example, for a large risk premia shock that would cause the ZLB to bind for 20 quarters, the forward guidance rule calls for the interest rate to remain at zero for an additional 11 quarters. Under this policy, output growth is about 4.5 percentage points higher, and the quarterly inflation rate is about 0.2 percentage points higher. Also, under the forward guidance rule, the initial reaction of the interest rate can be zero when the interest rate is positive without a rule. The reason for this is that, due to persistence in the policy rule, the interest rate is not forecasted to hit zero immediately but after a lag, and therefore attracts forward guidance immediately.

Under a calibration of $\gamma_y = 1.2$, Figure 6 plots the impulse response to a large negative risk premia shock when the central bank follows (7). The calendar-based forward guidance policy causes inflation to not fall as much, while output is subject to a less pronounced decline when the shock hits, followed by a relatively rapid recovery, eventually overshooting relative to steady-state. The overshooting of output mimics the optimal forward guidance policy discussed by Werning (2012). The impulse responses show that a rule of the form (7) generates paths for inflation and output growth that are substantially different to the paths of the economy without the ZLB. In particular, the drop in output under the forward guidance rule is similar to the drop in output in the economy without the ZLB, although the drop in inflation under the forward guidance rule is about half the drop in inflation without the ZLB. This illustrates how nonlinear the dynamics can become when the economy is at the ZLB. The relatively strong response of inflation occurs because the forecasted path of marginal costs recovers relatively quickly along with the recovery in output and employment.

4.3 The calendar based forward guidance tradeoff

The previous section showed, in the estimated SW model featuring a range of pricing frictions, that the central bank can use credible announcements about the policy rate to react against shocks that might drive the Federal Funds rate to the ZLB, and mitigate potentially large fluctuations in inflation and output growth.

For forward guidance to be a time-consistent policy, a central bank must commit to holding the policy rate at zero for the duration of the announcement. A commitment to holding the interest rate constant for a period of time exposes the economy to volatility from unexpected shocks that would ordinarily call for an increase in the policy rate. There is, then, a trade-off between stabilizing contemporaneous shocks with an announcement rule and exposing the economy to excess volatility during the commitment period. This trade-off raises a number of questions including: how strong should a central bank following (7) tie its announcements to the realized shocks and do these policies improve overall macroeconomic stability?

In this section, I examine these questions by simulating 100 paths of the estimated SW model for 1,000 periods under two policy regimes: when monetary policy is using the forward guidance rule (7) and when it is inactive at the ZLB. I compare the volatility of inflation and of output under the two policies for the same set of shocks but under different γ_y values.

For each simulation, I compute the following loss function, which is motivated by a central bank which maximizes households' utility function over time:

$$\text{Loss} = \text{var}(\pi_t) + \lambda \text{var}(y_t).$$

The value of λ determines the relative weight between the variance of inflation and the variance of output.

Figures 7 plots the mean, across simulated paths of the economy, of $\text{var}(\pi_t)$, $\text{var}(y_t)$ and the loss when $\lambda = 0.1$, normalized by their values for $\gamma_y = 0$. The forward guidance rule changes the dynamic behavior of the economy substantially. First, the mean length of the ZLB duration is increasing in γ_y, as expected under the policy rule (7). Second, for the estimated parameters, forward guidance reduces the variance of inflation and output, because it tempers large negative inflation and output responses when the ZLB binds. The loss function is minimized for values of γ_y between 1 and 1.5. For these values, the ZLB can be expected to be at zero twice as often as compared to an economy without forward guidance.

For large values of γ_y, however, the variance of inflation across the forecast horizon rises significantly. This reflects two factors: first, the central bank may be holding the interest rate at zero for much longer than is optimal, which significantly raises initial inflation and output, and second, the volatility caused by shocks while constrained at the ZLB. The first force is related to the forward guidance puzzle (see Del Negro et al., 2012; McKay et al., 2015), whereby announcements about the future path of the Federal Funds can have large aggregate consequences in the period of the announcement.

4.4 The government spending multiplier under announcements

The previous section showed that at the ZLB, a forward guidance rule like (7) can be stimulatory. A key prediction of the New Keynesian model when monetary policy is inactive is that the impact of an unanticipated increase in government spending is larger at the ZLB, driven by the decrease in the real interest rate that occurs when the nominal interest rate is constant (Christiano et al., 2011). These multipliers are derived when the central bank has no policy options at the ZLB. In this section, I investigate how an endogenous rule of the kind (7) can change the magnitude of the fiscal multiplier at the ZLB.

To illustrate this, I use the ZLB algorithm and the forward guidance rule calibrated at $\gamma_y = 0.85$ and compute the path of the economy under a large negative risk premia shock sufficient to cause the Federal Funds rate to hit the ZLB, both with and without a government spending shock which arrives at the same time as the risk premia shock. With these paths in hand, I compute the multiplier:

$$\frac{y_t^g - y_t}{g_1},$$

as in Fernández-Villaverde et al. (2012) where y_t^g is the path of output under the risk premia and government spending shock, y_t is the path of output under the risk premia shock only, and g_1 is the size of the government spending shock in the first period.

Figure 8 plots the path of the economy both with and without the government spending shock. First, consider how the economy reacts when the ZLB does not bind. The government spending shock causes output and inflation to rise. The Federal Reserve responds by raising the Federal Funds rate, increasing the real interest rate and dampening the rise in output. The government spending multiplier is well below 1 both when the government spending shock hits and in subsequent periods. Next, consider how the economy behaves when the ZLB binds. For large negative risk premia shocks, the real interest rate rises significantly when the ZLB binds, causing a substantial decline in output. The shock to government spending leads to a decline in the real interest rate, driven by an increase in the rate of inflation under the government spending shock. This generates a relatively larger government spending multiplier, consistent with the theory, of above 0.8 and remaining quite high across the forecast horizon. The increase in government spending at the ZLB also affects the date when the ZLB is expected to rise. This endogenous response of the ZLB reduces the government spending multiplier and explains why the multiplier remains below one, as compared to the multipliers above one documented in Christiano et al. (2011).

Under the forward guidance rule, the forward guidance channel lowers real interest rates compared to the endogenous durations case, and narrows the output gap. Adding the government spending shock under the forward guidance rule, does not change the the output gap much. This is because the government spending shock causes the output gap to narrow. As a result, the central bank refrains from announcing a longer ZLB duration. This can be seen by observing the Federal Funds rate lifts off the ZLB two quarters earlier under the government spending shock. The resulting multiplier is comparable to the multiplier that arises without the ZLB, reinforcing the observation that in New Keynesian models, higher government spending multipliers rely on the Federal Funds rate remaining fixed, at the same time as the government spending shock, and in the date the ZLB is expected to stop binding. Under a forward guidance rule, the exit date from the ZLB is endogenous and affected by government spending. This interaction has been discussed by policymakers following the 2016 election, after which expectations for stronger fiscal policy went hand in hand with a steepening yield curve.[5]

5 Conclusion

This paper introduced a method for solving an identification problem that arises at the ZLB, answering how stimulatory monetary policy is at the ZLB by decomposing the duration that the ZLB is expected to bind into a comoponent due to shocks and a component due to monetary policy extensions of the ZLB. The method relies on an efficient algorithm for implementing the ZLB which piecewise-linear combines approximations of a model economy in a recursive way so that the periods that the ZLB is expected to bind aligns with the expectations of agents in the model. I show how a simple extension of the algorithm easily

[5]For a discussion of these observations, see Bernanke (2017).

accommodates calendar based forward guidance policies.

I estimated a version of the Smets and Wouters (2007) model, modified to accommodate forward guidance policy and a one-time but permanent change in trend growth. In the results, I find that there was a substantial decline in trend growth occurring around the start of the 2000s, and that the ZLB was a substantial constraint on monetary policy. Applying the decomposition to the estimated model, I find evidence for forward guidance policy between 2011Q3 and 2013Q1, which in turn stimulated the economy.

With the estimated model and the ZLB algorithm, I studied a simple inflation-linked forward guidance policy rule, and showed that it replicated the features of the optimal forward guidance policies studied in the literature. Exploiting the computational efficiency of the algorithm, the paper then asked how well the policy rule performs when the economy is hit by stochastic shocks and found that a forward guidance rule does reduce the ex-post volatility of inflation and output growth when monetary policy uses forward guidance to stabilize large shocks. Furthermore, I showed that because a forward guidance rule formalizes a way for monetary policy to stimulate the economy at the ZLB, the stimulatory effect of government spending which occurs at the ZLB can be undone by the response of monetary policy to those shocks because of shorter forward guidance.

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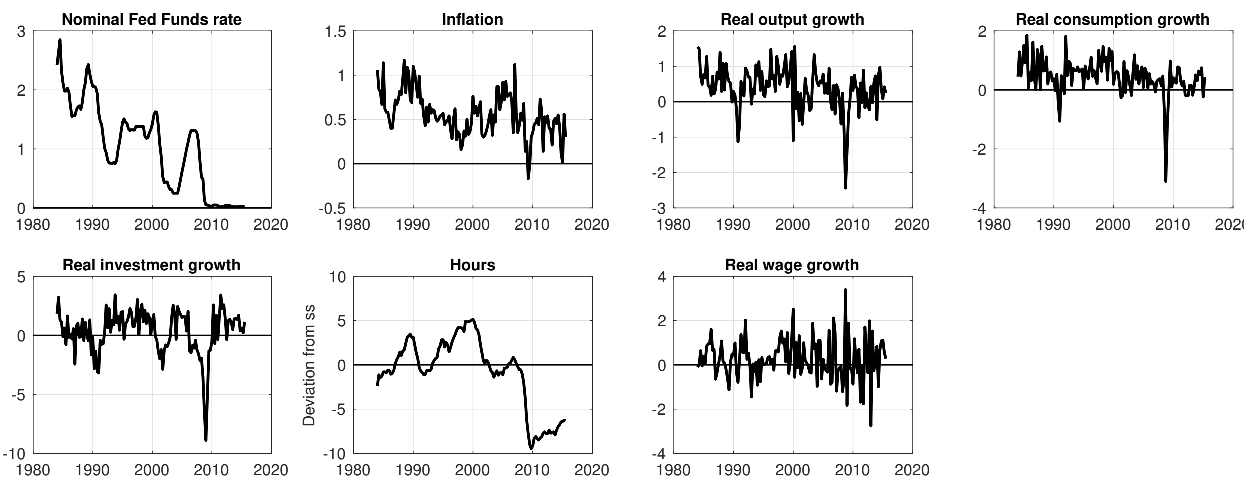

Figure 1: Quarterly data.

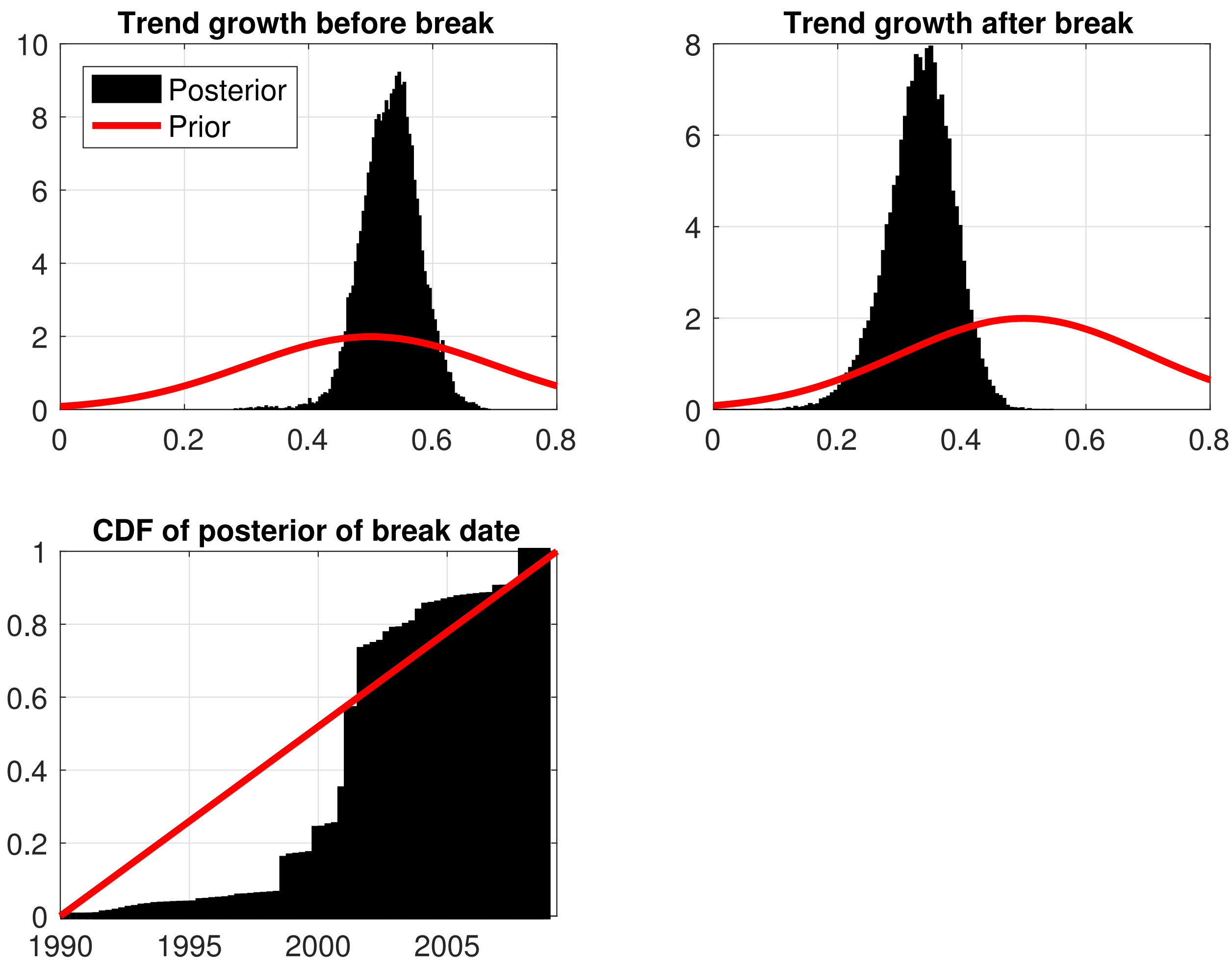

Figure 2: Quarterly trend growth posterior distributions.

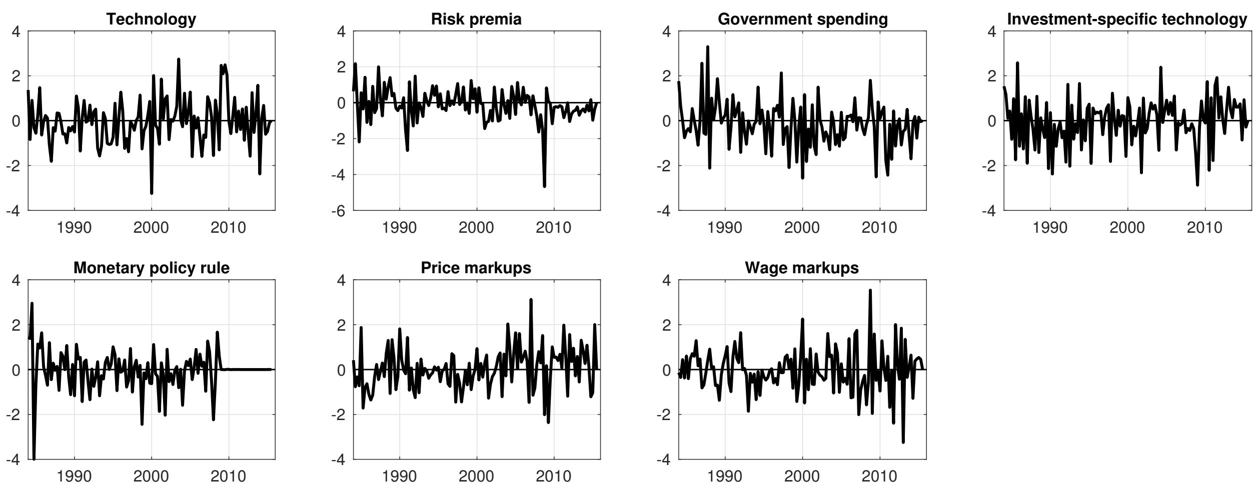

Figure 3: Structural shocks.

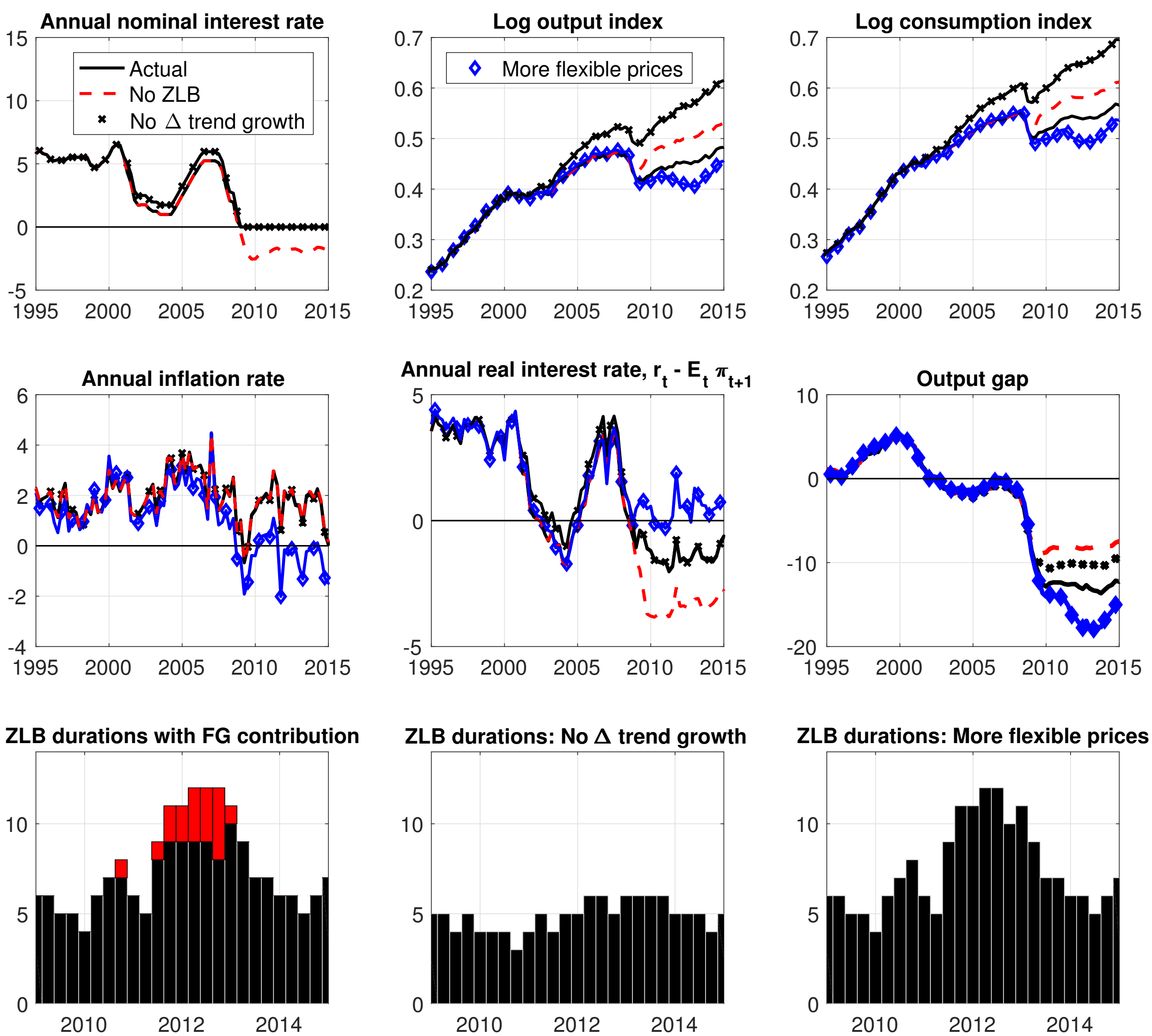

Figure 4: The ZLB and forward guidance. In computing the counterfactual with more flexible prices, the contribution of forward guidance is removed.

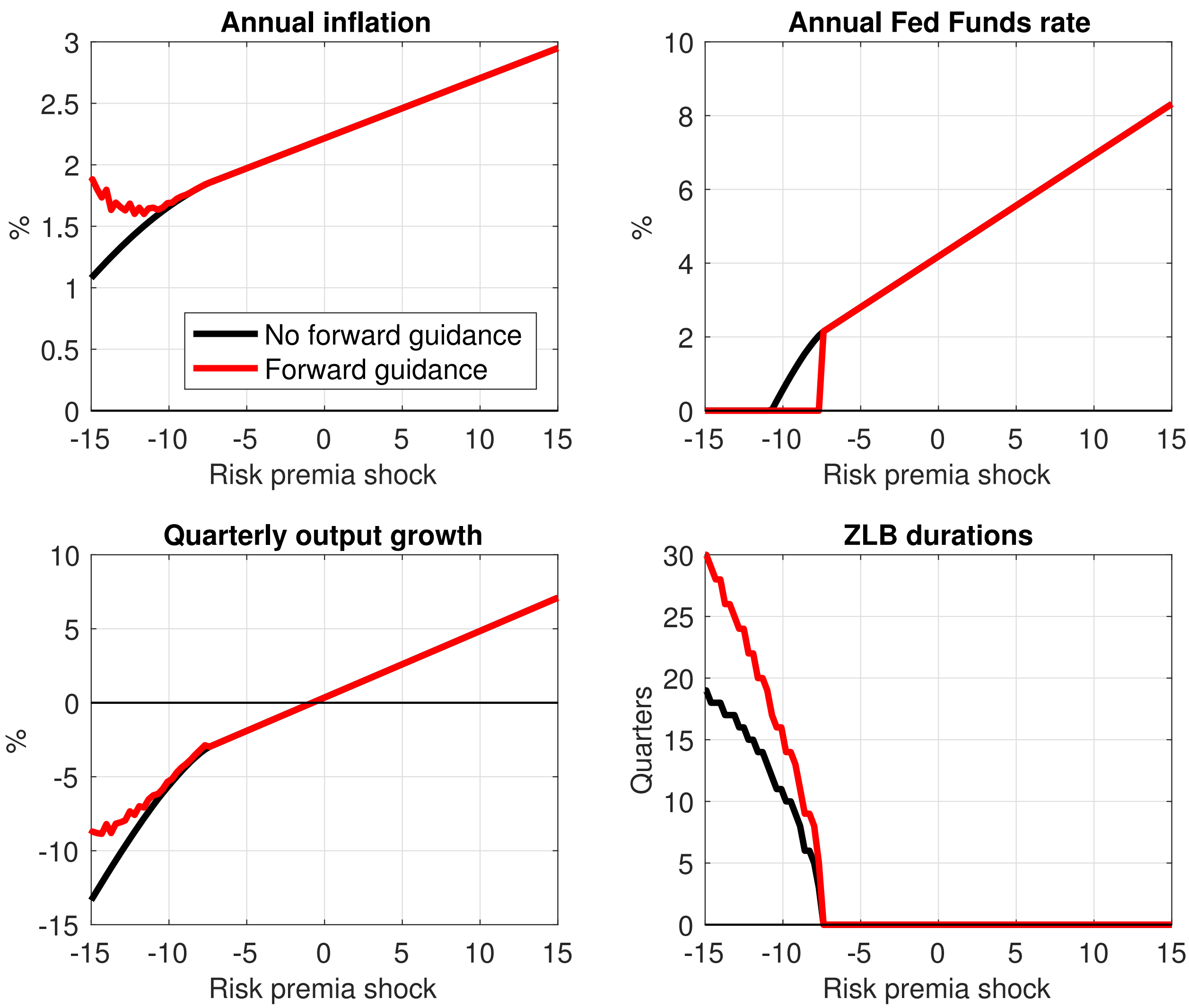

Figure 5: Initial response of variables to risk premia shocks with forward guidance.

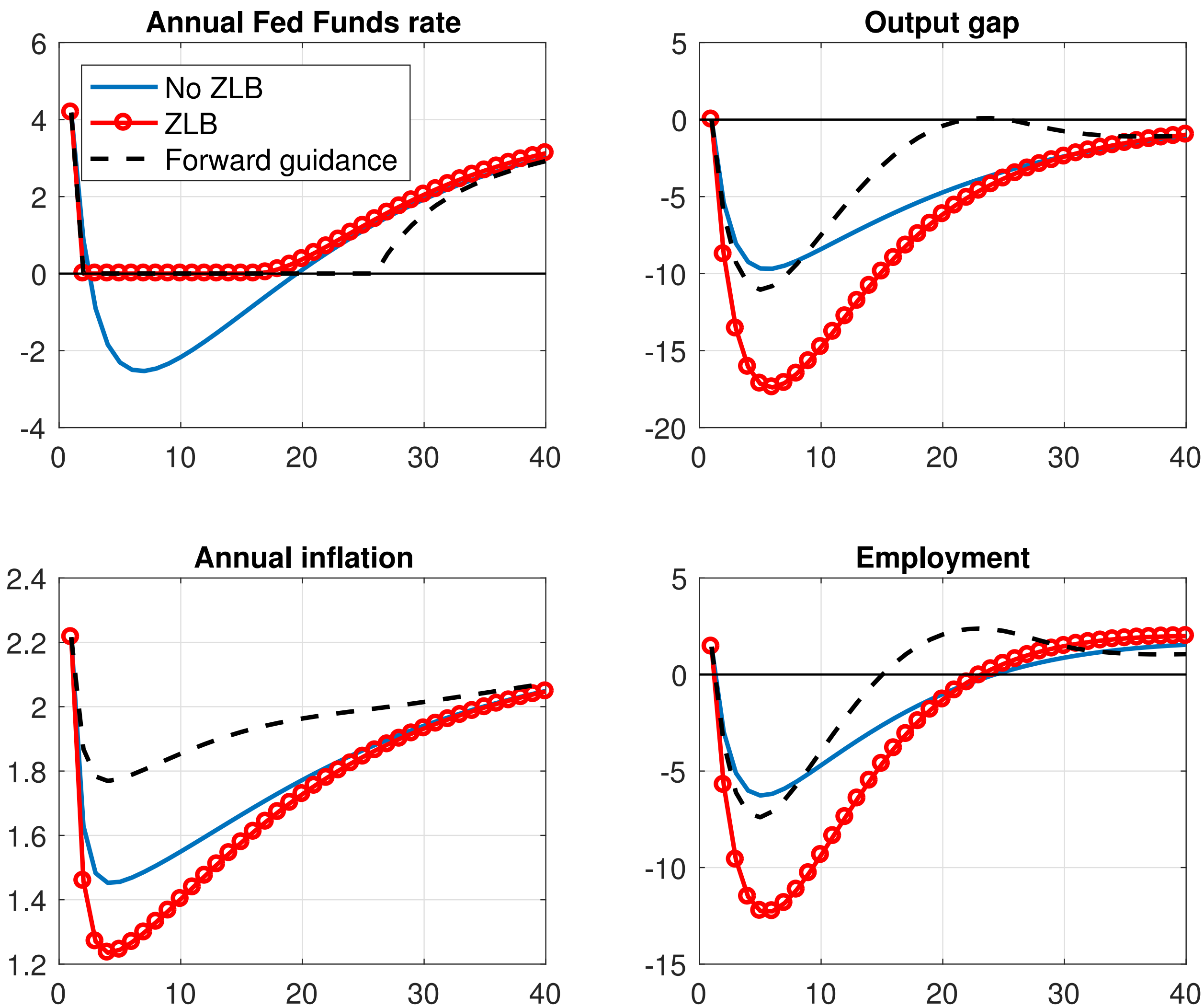

Figure 6: Impulse response to negative risk premia shock

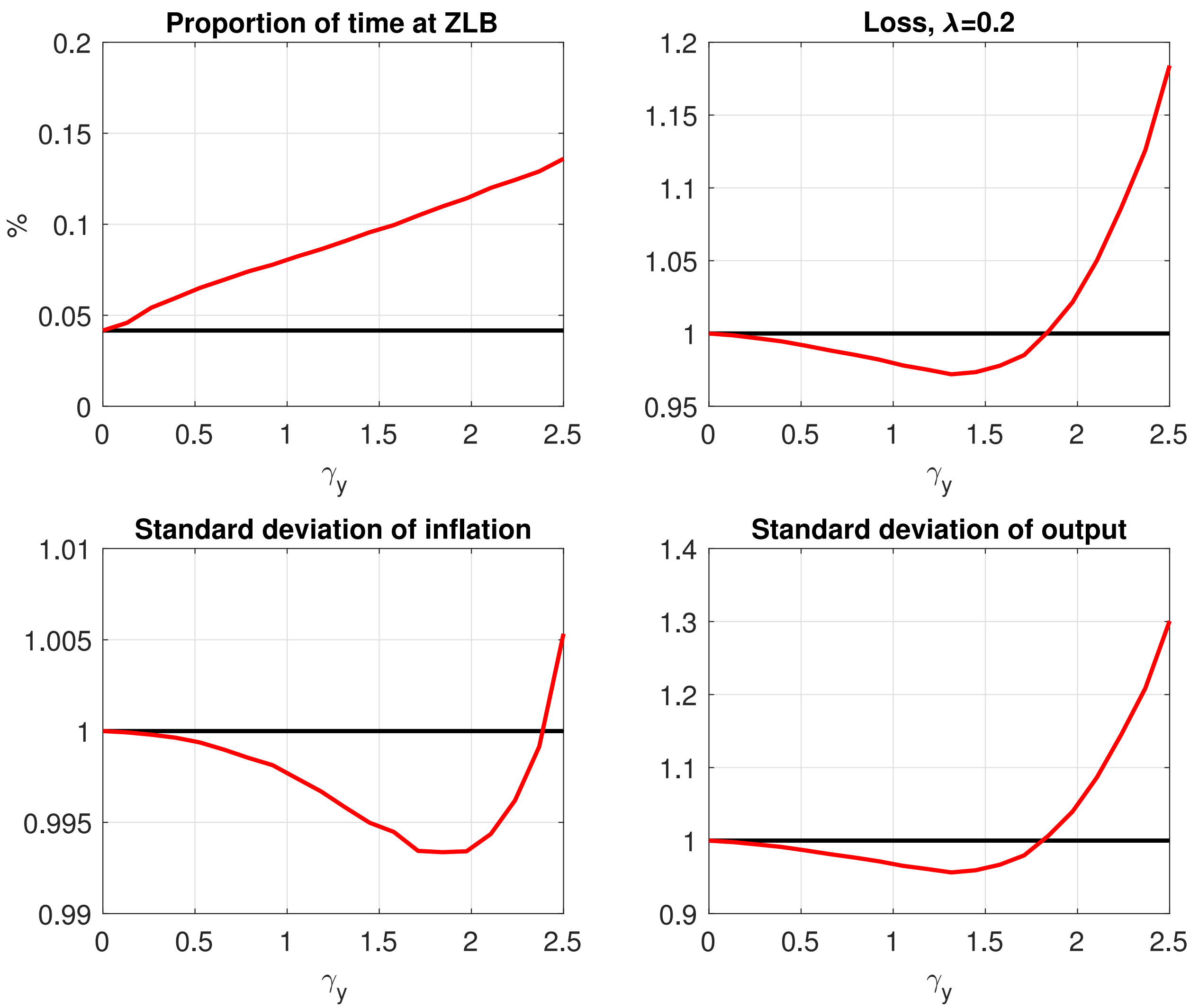

Figure 7: Volatility of inflation and output. The standard deviation for each series is normalized to the corresponding standard deviation when no forward guidance rule is in use.

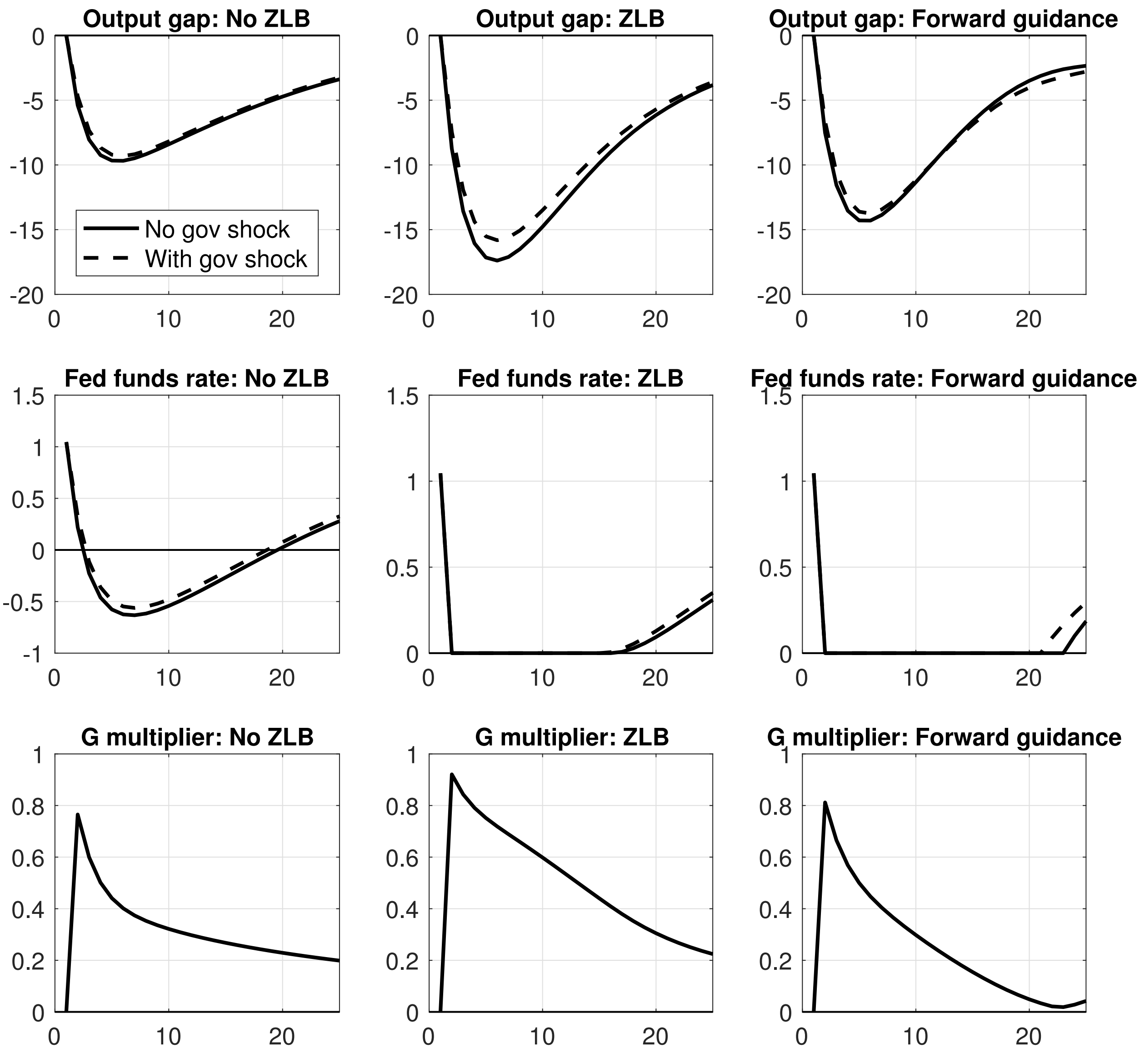

Figure 8: Impulse response to negative risk premia shock

Table 1: Estimated parameters

Parameter	Prior			Posterior			
	Distribution	Mean	St Dev	Mode	Mean	10%	90%
σ_a	IG	0.100	2.000	0.417	0.426	0.390	0.464
σ_b	IG	0.100	2.000	0.062	0.063	0.053	0.073
σ_g	IG	0.100	2.000	0.380	0.381	0.350	0.413
σ_i	IG	0.100	2.000	0.300	0.315	0.255	0.376
σ_r	IG	0.100	2.000	0.116	0.118	0.105	0.132
σ_p	IG	0.100	2.000	0.112	0.115	0.100	0.131
σ_w	IG	0.100	2.000	0.441	0.445	0.397	0.495
ρ_a	B	0.500	0.200	0.964	0.955	0.931	0.976
ρ_b	B	0.500	0.200	0.938	0.938	0.920	0.955
ρ_g	B	0.500	0.200	0.978	0.973	0.958	0.987
ρ_i	B	0.500	0.200	0.678	0.685	0.591	0.784
ρ_r	B	0.500	0.200	0.500	0.485	0.390	0.580
ρ_p	B	0.500	0.200	0.712	0.672	0.531	0.810
ρ_w	B	0.500	0.200	0.931	0.792	0.585	0.935
ρ_{ga}	N	0.500	0.250	0.380	0.407	0.300	0.513
$\bar{l}$	N	0.000	1.500	1.373	1.394	1.215	1.635
$\bar{\pi}$	N	0.500	0.100	0.549	0.546	0.482	0.610
$100 * (\beta^{-1} - 1)$	G	0.250	0.100	0.161	0.168	0.091	0.255
μ_w	B	0.500	0.200	0.900	0.748	0.531	0.907
μ_p	B	0.500	0.200	0.624	0.577	0.397	0.751
α	N	0.300	0.050	0.148	0.152	0.128	0.176
ψ	B	0.500	0.150	0.801	0.784	0.671	0.884
φ	N	4.000	1.500	6.034	6.194	5.065	7.687
σ_c	N	1.500	0.375	0.918	0.983	0.850	1.135
h	B	0.700	0.100	0.603	0.590	0.519	0.659
Φ	N	1.250	0.125	1.375	1.399	1.299	1.500
ι_w	B	0.500	0.150	0.306	0.387	0.213	0.596
ξ_w	B	0.500	0.100	0.886	0.861	0.798	0.916
ι_p	B	0.500	0.150	0.217	0.255	0.148	0.372
ξ_p	B	0.500	0.100	0.939	0.940	0.927	0.953
σ_l	N	2.000	0.750	2.687	2.790	2.001	3.629
r_π	N	1.500	0.250	1.308	1.308	1.009	1.599
$r_{\Delta y}$	N	0.125	0.050	0.116	0.120	0.088	0.153
r_y	N	0.125	0.050	0.182	0.185	0.146	0.226
ρ	B	0.750	0.100	0.819	0.811	0.769	0.850

Table 2: Quarterly trend growth estimates

	Prior				Posterior		
	Dist	Median	10%	90%	Median	10%	90%
γ after break	N	0.5	0.25	0.76	0.533	0.476	0.586
γ after break	N	0.5	0.25	0.76	0.336	0.262	0.398
Date of break*					2001Q2	1998Q4	2008Q1

*: Note, break date prior is uniform between 1990Q1 and 2009Q2.

Table 3: Estimated ZLB durations

	Prior			Posterior				
Date	Median	10%	90%	Mode	Median	10%	90%	Futures Data
2009Q1	5	1	18	4	7	4	13	2
2009Q2	5	1	18	4	8	4	14	1
2009Q3	3	1	18	4	9	3	16	1
2009Q4	3	1	18	5	8	3	14	1
2010Q1	3	1	18	3	9	3	14	2
2010Q2	4	1	18	4	10	3	16	2
2010Q3	4	2	18	5	7	4	12	4
2010Q4	5	2	18	5	8	4	15	3
2011Q1	6	2	18	7	7	4	12	2
2011Q2	5	2	18	6	7	4	13	3
2011Q3	9	3	18	9	9	6	14	6
2011Q4	9	3	18	9	10	6	14	6
2012Q1	10	3	19	10	10	6	15	5
2012Q2	10	3	19	9	10	6	15	7
2012Q3	11	4	18	10	10	7	14	8
2012Q4	11	3	19	12	11	7	15	7
2013Q1	10	3	18	10	10	6	14	6
2013Q2	7	2	18	8	9	5	14	4
2013Q3	8	3	18	9	9	6	13	5
2013Q4	8	3	18	8	9	6	14	4
2014Q1	7	3	18	7	8	5	13	4
2014Q2	6	2	18	6	8	4	14	3
2014Q3	4	1	18	4	7	4	14	2
2014Q4	3	1	18	4	8	3	15	2
2015Q1	3	1	18	4	8	3	14	2
2015Q2	3	1	18	4	8	3	14	2
2015Q3	2	1	18	9	9	3	15	–

Table 4: Percentage change due to forward guidance

Date	Inflation	Output	Consumption	Investment
2011Q2	0.002	0.042	0.062	0.148
2011Q3	0.003	0.116	0.138	0.301
2012Q1	0.004	0.168	0.181	0.442
2012Q2	0.005	0.257	0.261	0.671
2012Q3	0.005	0.299	0.286	0.833
2012Q4	0.009	0.478	0.451	1.263
2013Q1	0.005	0.378	0.319	1.203
2013Q2	0.004	0.289	0.216	1.083
2013Q3	0.003	0.228	0.156	0.964
2013Q4	0.003	0.187	0.122	0.850
2014Q1	0.003	0.156	0.102	0.742
2014Q2	0.002	0.133	0.091	0.640