International Spillovers of Forward Guidance Shocks*

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Abstract

We estimate a two-country model of the US and Canada over the post 2009 sample to study the cross-country spillovers of forward guidance shocks. To do so, we propose a method to identify forward guidance shocks during the fixed interest rate regime. US forward guidance shocks have a larger impact than conventional monetary policy shocks. A 2 quarter expansionary forward guidance shock decreases Canadian output by about 0.2% to 0.4% on impact. The effect of US forward guidance shocks on Canadian output, unlike conventional policy shocks, depends crucially on the state of the US risk premium shock. The estimated forward guidance shocks coincide with significant US monetary policy announcements such as the introduction of calendar based guidance.

Keywords: Forward Guidance Shocks, Identification, Spillovers, Zero Lower Bound.

JEL classifications: E2, E4, E5, F4.

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1 Introduction

In the aftermath of the 2008 financial crisis, the Federal Reserve cut its policy interest rate to near zero and turned to unconventional monetary policies, including forward guidance. While it is widely acknowledged that these unprecedented monetary actions supported economic growth in the US, there there is less agreement about their effect on other countries. On the one hand, stronger economic conditions in the US supported global demand and the exports of other economies. On the other hand, as emphasized by policymakers in small open economies which saw their currencies appreciate against the US dollar, expansionary US monetary policy also had an expenditure-switching effect, which likely dampened demand for the output of small open economies.[1]

In this paper, we use a medium-scale small open economy model to quantify the international spillovers of forward guidance by the Federal Reserve after 2009. This model, like other open economy models, has both expenditure-augmenting and expenditure-switching forces. These two competing forces are also highlighted in studies of optimal policy and policy rules in open economies (Bodenstein et al., 2017; Fujiwara et al., 2013; Haberis and Lipinska, 2012). In theory, the net effect of international monetary policy spillovers on domestic output is ambiguous and depends crucially on the trade elasticity of substitution between domestic and foreign goods. When domestic goods easily substitute for foreign goods, the exchange rate appreciation caused by an expansionary monetary policy shock abroad causes a decline in the demand for domestic goods that can outweigh any increase due to an expanding foreign economy – a 'beggar-thy-neighbor' effect. It is therefore an empirical issue to determine the relative strength of expenditure-augmenting and expenditure-switching forces and the net effect of monetary policy spillovers.

An important contribution of this paper, in our view, is to propose a method to identify forward guidance shocks during the fixed interest rate regime. To do so, we first estimate expected durations of the fixed interest rate regime and the structural shocks using the method outlined in Kulish, Morley and Robinson (2017). This gives us an *estimated duration* for each

[1] See, for example, Rajan (2015).

quarter of the fixed interest rate regime. Then, we use the solution for models with occasionally binding constraints of Guerrieri and Iacoviello (2015) and Jones (2017) to compute the expected duration implied by the structural shocks and the constraint. We call this duration the *lower bound duration*.

Just as traditional monetary policy shocks are identified as unanticipated changes in the policy rate that are orthogonal to the state of the economy, we identify forward guidance shocks as unanticipated changes in the expected path of the policy rate that are orthogonal to the state of the economy. Forward guidance shocks are unanticipated changes in the *estimated duration* orthogonal to the *lower bound duration*. Identification is achieved because unanticipated changes in the fixed interest rate duration brought about by structural shocks have different implications for observable economic variables than those which are not; output and inflation fall after contractionary shocks that makes the constraint bind for longer, but output and inflation increase after a policy announcement that extends the duration beyond what the constraint already implied. Section 4 discusses the identification of forward guidance shocks in detail.

The literature gives mixed evidence on the net effect of monetary policy spillovers prior to 2009.[2] For the period since 2009, evidence from structural forward looking models on the spillovers of forward guidance shocks has so far been absent from the literature. Some mixed evidence on the effects of US quantitative easing policies on emerging markets comes from a variety of studies using reduced form methods.[3] For example, Inoue and Rossi (2019) use a functional VAR approach to assess the effect of monetary policy on various exchange rates during conventional and unconventional times. Their approach for identifying shocks relies on changes of the whole yield curve. Our approach also relates to changes in the yield curve as it involves unanticipated changes of the expected path of the policy rate, but differs from Innoue and Rossi because our model is forward looking and allows us to trace out the broader economic implications of forward guidance, while they focus attention on the behaviour of the exchange rate.[4]

[2]See Ammer et al. (2016), Dedola et al. (2017), Bernanke (2017), and Kim (2001).

[3]See Anaya et al. (2017), Chen et al. (2016) and Tillmann (2016) for evidence that relies on vector autoregressions, and Chen et al. (2014) for regression evidence using an event-study type identification for monetary policy shocks.

[4]In general, Inoue and Rossi (2019) find that a monetary policy tightening (easing) leads to an appreciation

Although there is a considerable literature exploring the effects of interest rate shocks with linear models, these results do not carry over to the effect of forward guidance shocks because, as we discuss below, these shocks propagate non-linearly and their impact is state-dependent.[5] Given the current low interest rate environment, the Federal Reserve is likely to rely on forward guidance at some point in the future. For this reason it is important for policymakers in open economies to understand how US forward guidance shocks propagate across borders.

We apply our model to US and Canadian data and focus on the period after 2009. We choose Canada for two reasons. First, Canada is closely linked financially and through trade in goods and services with the US – around three quarters of Canada's merchandise exports go to the US, accounting for close to 20 per cent of its GDP. Second, the Bank of Canada closely followed US monetary policy after the crisis and responded with calendar-based forward guidance of its own in 2009 (see Murray, 2013). This provides an opportunity to quantify the spillovers when the small open economy is also in a fixed interest rate regime, a situation which is likely to become increasingly common.[6]

Our main quantitative findings are as follows. The average size of a US forward guidance shock is around 2 quarters, with the degree of expansionary forward guidance strongest between 2011 and 2013, consistent with the introduction of the calendar-based announcements and contractionary forward guidance shocks in the second quarter of 2013, when the Federal Reserve announced its tapering of asset purchases. In terms of magnitudes, a 2 quarter expansionary US forward guidance shock leads to a fall in Canadian output of around 0.4 to 1.0 percentage points about 4 to 5 quarters after the shock. It also triggers an appreciation of the Canadian real exchange rate of between 0.5 to 2 percentage points and reduces Canadian inflation by 0.1 percentage points. The size of these effects, however, depends on the state of the US risk premium. When the US risk premium shock is more negative, a 2 quarter forward guidance shock in the US has a smaller expansionary effect on US output and smaller contractionary

(depreciation) of the US dollar, consistent with our findings. During the unconventional period, however, they do not find a statistically significant impact on the Canadian exchange rate following an expansionary US policy shock. This result, however, is at odds with Ferrari et al. (2017) and Glick and Leduc (2018) who find that the exchange rates respond more to unconventional monetary policy shocks than conventional shocks.

[5] See Bernanke and Reinhart (2004) and Eggertsson and Woodford (2003).

[6] Following the classification of Campbell et al. (2012), our focus is on calendar-based 'Odyssean' forward guidance policies.

effect on Canadian output. Comparing policy shocks after 2009 with those before 2009, the average forward guidance shock has a larger effect than the average standard monetary policy shock. A 2 quarter US forward guidance shock lowers US output by as much as a 100 basis point conventional US monetary policy shock would. In these terms, the average forward guidance shock has a similar impact to a 2.5 standard deviation conventional monetary policy shock. In counterfactual simulations removing forward guidance shocks, we find that in aggregate the US and Canada are both better off by jointly responding with expansionary monetary policy to the large contractionary shocks that took place during the Great Recession.

McKay, Nakamura and Steinsson (2016) argue that if agents face uninsurable income risk and borrowing constraints, their responses to changes in future interest rates are muted and so forward guidance would be less powerful. In our model, the power of forward guidance depends on the state of the risk premium shock. As in Jones, Midrigan and Philippon (2018) borrowing constraints drive a wedge in the Euler consumption equation just as risk premium shocks do in our model. So our findings can be thought to be consistent with the view that borrowing constraints mute the impact of forward guidance. Although we emphasize the state dependence of forward guidance, this does not imply that our model is immune to the forward guidance puzzle. So to address this we also estimate a version with discounting in the Euler equation. In this case we find larger durations and forward guidance shocks.

The rest of the paper is structured as follows. Section 2 outlines the model and discusses the transmission of monetary policy shocks in a simplified version. Section 3 discusses the solution and estimation method and Section 4 discusses identification of forward guidance shocks. Section 5 presents estimation results while Section 6 presents the international spillovers of forward guidance from the estimated model and computes counterfactual paths for the US and Canada absent forward guidance shocks. Section 7 concludes.

2 Model

We conduct our analysis using a small open economy model along the lines of Galí and Monacelli (2005). The model features two economies: a large (foreign) economy and a small (domestic) economy. Economic developments in the large economy affect the small economy, but the reverse is not true. As in De Paoli (2009a), the model is the limiting case of a two-country model where the relative size of one of the economies goes to zero. We extend this otherwise standard framework in four dimensions, including: (i) imperfect exchange rate pass-through; (ii) wage stickiness; (iii) trend inflation; (iv) interest rates of longer maturities; and (v) habits in the utility function.[7]

2.1 The Large Economy

Households The large economy contains a representative household composed of a continuum of workers, each specialised in a particular labour type, indexed by $j \in [0, 1]$. The household's intertemporal welfare function is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t^*} \left[\log(C_t^* - h^* C_{t-1}^*) - \frac{\nu}{1+\varphi} \int_0^1 (N_t^*(j))^{1+\varphi} dj \right]$$

where $N_t^*(j)$ is labor supply of type j and ξ_t^* denotes an intertemporal preference shock that follows an AR(1) process. C_t^* is a composite consumption index given by $C_t^* = \left[\int_0^1 C_t^*(i)^{\frac{\epsilon_p^*-1}{\epsilon_p^*}} di \right]^{\frac{\epsilon_p^*}{\epsilon_p^*-1}}$, where $\epsilon_p^* > 1$ is the elasticity of substitution between types of differentiated goods. Households face the flow budget constraint:

$$P_t^* C_t^* + \frac{B_{t+1}^*}{R_t^*} \leq B_t^* + \int_0^1 W_t^*(j) N_t^*(j) dj + T_t^*$$

where P_t^* is the large economy's CPI, $W_t^*(j)$ is the nominal wage rate of labour type j and T_t^* are lump sum taxes and transfers. B_{t+1}^* is a one period risk-free nominal bond and R_t^* is the interest rate on the bond.

[7] The full set of log-linear equations used in estimation is given in the Appendix.

Labour unions A continuum of perfectly competitive labour aggregating firms combine the specialised labour types according to the technology:

$$N_t^* = \left[\int_0^1 N_t^*(j)^{\frac{\epsilon_w^*-1}{\epsilon_w^*}} dj\right]^{\frac{\epsilon_w^*}{\epsilon_w^*-1}}$$

Competition between the labour aggregating firms ensures that the nominal wage paid to aggregate labour and demand for individual varieties are given by:

$$W_t^* = \left[\int_0^1 W_t^*(j)^{1-\epsilon_w^*} dj\right]^{\frac{1}{1-\epsilon_w^*}} \quad \text{and } N_{t+s}^*(j) = \left(\frac{W_t^*(j)\Omega_{t,t+s}^{w*}}{W_{t+s}^*}\right)^{-\epsilon_w^*} N_{t+s}^*$$

Workers of type j unionise in order take exploit their monopoly power. Like Erceg et al. (2000) we assume that these unions are subject to a Calvo-style friction such that each quarter only a fraction of unions, $1-\theta_w^*$, are able to set wages optimally. Unions that do *not* re-optimise follow an indexation rule that links wages growth to a weighted average of lagged wage inflation and steady-state wages growth:

$$\breve{W}_t^*(j) = \left(\Pi_{t-1}^{w*}\right)^{\chi_w^*} \left(\bar{\Pi}^*\mathcal{M}\right)^{1-\chi_w^*} W_{t-1}^*(j)$$

where $\breve{W}_t^*(j)$ is union j's wage conditional on *not* re-optimising in period t, $\Pi_t^{w*} = W_t^*/W_{t-1}^*$ is aggregate wage inflation and $\bar{\Pi}^*\mathcal{M}$ is steady state nominal wages growth, equal to the product of the central bank's inflation target, $\bar{\Pi}^*$, and steady state labour productivity growth, $\mathcal{M}$.

The wage setting problem for a union that is able to reset its wages at time t is:

$$\max_{W_t^*(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_w^*)^s \left[(1+\iota_w^*)\frac{\Lambda_{t+s}^*}{P_{t+s}^*} W_{t+s}^*(j)\Omega_{t,t+s}^{w*} N_{t+s}^*(j) - \frac{\nu}{1+\varphi} N_{t+s}(j)^{*1+\varphi}\right]$$

subject to the labour demand constraint given above. $\Omega_{t,t+s}^{w*}$ is the cumulative wage growth between period t and $t+s$ for a union that does not re-optimise, Λ_{t+s}^* is the shadow price of consumption in period $t+s$ and ι_w^* is a wage subsidy calibrated to offset the steady state distortion associated with monopolistic competition in the labour market.[8]

[8]Given the indexing rule, $\Omega_{t,t+s}^{w*} = \left(\bar{\Pi}^*\mathcal{M}\right)^{(1-\chi_w^*)s} \prod_{k=t}^{t+s-1} \left(\Pi_k^{w*}\right)^{\chi_w^*}$

Firms The economy's final good is produced by a representative firm that aggregates individual varieties according to the production function:

$$Y_t^* = \left[\int_0^1 Y_t^*(i)^{\frac{\epsilon_p^*-1}{\epsilon_p^*}} di\right]^{\frac{\epsilon_p^*}{\epsilon_p^*-1}}$$

Perfect competition implies that the aggregate price index and demand functions for individual varieties are:

$$P_t^* = \left[\int_0^1 P_t^*(i)^{1-\epsilon_p^*} di\right]^{\frac{1}{1-\epsilon_p^*}} \text{ and } Y_{t+s}^*(i) = \left(\frac{P_t^*(i)\Omega_{t,t+s}^*}{P_{t+s}^*}\right)^{-\epsilon_p^*} Y_{t+s}^*$$

Intermediate goods are produced by monopolistically-competitive firms with the technology:

$$Y_t^*(i) = Z_t L_t^*(i)$$

where $Y_t^*(i)$ is the production and $L_t^*(i)$ is firm i's input of aggregate labour.[9] Z_t is the trend component of productivity, which in logs follows a random walk with drift that grows at the rate $\mathcal{M}$.

Intermediate firms face Calvo-style pricing frictions. Each quarter, a fraction of firms, $1-\theta^*$, sets prices optimally. The remaining firms that do not re-optimise their prices follow an indexation rule that links prices growth to a weighted average of lagged CPI inflation and the central bank's inflation target:

$$\breve{P}_t^*(i) = \left(\Pi_{t-1}^*\right)^{\chi_p^*} \left(\bar{\Pi}^*\right)^{1-\chi_p^*} P_{t-1}^*(i)$$

where $\breve{P}_t^*(i)$ is firm i's price condition on not re-optimising in period t and $\Pi_t^* = P_t^*/P_{t-1}^*$ is CPI inflation.

[9] Market clearing in the labour market requires that the amount of aggregate labour supplied by households equals the amount of aggregate labour demanded by firms, i.e. that $N_t^* = \int_0^1 L_t^*(i)di$.

The pricing problem for a firm that re-optimises at time t is:

$$\max_{\{P_t^*(i)\}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_p^*)^s \left\{ \Lambda_{t+s}^* \left[P_t^*(i)\Omega_{t,t+s}^* Y_{t+s}^*(i) - \frac{1}{1+\iota_p^*} \frac{W_{t+s}^*(i)}{P_{t+s}^* Z_{t+s}} Y_{t+s}^*(i) \right] \right\}$$

subject to the demand constraint given above. The term ι_p^* is a production subsidy that offsets the distortionary effects of monopolistic competition on the steady state and $\Omega_{t,t+s}^*$ is the cumulative price growth between t and $t+s$ is the firm does not re-optimise.[10].

Fiscal policy and market clearing The government's period budget constraint is:

$$P_t^* G_t^* + B_t^* \leq T_t^* + \frac{B_{t+1}^*}{R_t^*}$$

where G_t^* is real government consumption of goods and services, which evolves according to:

$$\frac{G_t^*}{Z_t} = \left(\frac{\bar{G}^*}{Z}\right)^{\rho_g^*} \left(\frac{G_{t-1}^*}{Z_{t-1}}\right)^{1-\rho_g^*} \exp(\varepsilon_{G,t}^*)$$

Goods market clearing in the large economy requires that:

$$Y_t^* = C_t^* + G_t^*$$

Monetary Policy When the interest rate is not fixed, monetary policy follows an interest rate rate rule that responds to inflation, output growth and the deviation of the level of output from trend:

$$\frac{R_t^*}{R^*} = \left[\frac{R_{t-1}^*}{R^*}\right]^{\rho_R^*} \left[\left(\frac{\Pi_t^*}{\bar{\Pi}^*}\right)^{\phi_\pi^*} \left(\frac{Y_t^*}{Z_t}\right)^{\phi_y^*}\right]^{1-\rho_R^*} \left(\frac{Y_t^*}{Y_{t-1}^*\mu}\right)^{\phi_g^*} \exp(\varepsilon_{R,t}^*) \tag{1}$$

where R_t^* is the policy rate in the large economy, $R^* = \mathcal{M}\bar{\Pi}^*/\beta$ is the steady-state policy rate and $\varepsilon_{R,t}^*$ is a monetary policy shock. Between 2009Q1 and 2015Q4, the central bank fixes the level of the interest rate at $\bar{R}^*$, that is

$$\frac{R_t^*}{R^*} = \bar{R}^* \tag{2}$$

[10] $\Omega_{t,t+s}^* = \left(\bar{\Pi}^*\right)^{(1-\chi_p^*)s} \prod_{k=t}^{t+s-1} \left(\Pi_k^*\right)^{\chi_p^*}$

where $\bar{R}^*$ need not be the effective lower bound. For the US we set $\bar{R}^*$ to the mid point of the target range for the Federal Funds rate, to 0.125 %. Forward guidance, as we define it below, is the difference between the number of periods the policy rate is expected to be fixed at $\bar{R}^*$ and the number of periods that the policy rate is expected to be at the lower bound, given the shocks and state of the economy. The latter is the duration which would be implied by the shocks and the central bank adhering to a rule like $\max(\bar{R}^*, \text{Taylor rule})$. We refer to this duration as the *lower bound* duration. Forward guidance allows the central bank to alter the duration relative to what the constraint and Taylor rule would imply and therefore drives a wedge between the expected duration of the fixed policy rate regime and the lower bound duration. We discuss the lower bound and forward guidance durations in more detail below.

Longer-term interest rates are determined via the expectations hypothesis. We link model longer-term interest rates to observed longer-term interest rates following Graeve et al. (2009). For any maturity $m > 1$:

$$R_{m,t}^{*,\text{obs}} = R_{m,t}^* \exp\left(c_m^* \eta_t^* \varepsilon_{m,t}^*\right) \tag{3}$$

where $R_{m,t}^*$ is the interest rate on a bond that pays one unit of the large economy's currency in m quarters as determined by the expectations hypothesis, c_m^* is a constant risk premia on the m quarter interest rate, η_t^* is shock, common to all interest rates in the large economy, that follows an exogenous autoregressive process and $\varepsilon_{m,t}^*$ is an idiosyncratic shock to the m quarter interest rate in the large economy. Because the expectations hypothesis holds, longer-term nominal interest rates in the model, $R_{m,t}$, are also subject to the lower bound of R_t^*.

2.2 The Small Economy

The structure of the small economy is similar to that of the large economy, except that households consume goods and services produced abroad and firms sell their output overseas as well as domestically.

Households The small economy is populated by a representative household that maximizes the expected present discounted value of lifetime utility, given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left[\log(C_t - hC_{t-1}) - \frac{\nu}{1+\varphi} \int_0^1 (N_t(j))^{1+\varphi} dj \right]$$

where ξ_t is an intertemporal preference shock that follows an AR(1) process and C_t is a composite of domestically-produced and imported goods and services (defined below). Households face the budget constraint:

$$P_t C_t + \frac{B_{t+1}}{R_t} + \frac{S_t B^F_{t+1}}{R^F_t} \leq B_t + S_t B^F_t + \int_0^1 W_t(j) N_t(j) dj + T_t. \tag{4}$$

Households in the small economy have access to two financial assets. These are domestic one-period risk free bonds, B_{t+1}, denominated in the domestic currency, and overseas one-period risk free bonds, B^F_{t+1}, denominated in the currency of the large economy. We assume that domestic bonds are not traded internationally and are in zero net supply. The variable S_t represents the small economy's nominal exchange rate, defined as the number of units of the small economy's currency required to purchase one unit of the large economy's currency. The interest rate on overseas bonds depends on the large economy's interest rate and the net foreign asset position of the small economy:

$$R^F_t = R^*_t \exp\left[-\psi_H \left(\frac{S_t B^F_{t+1}}{P_t Y_t} - b_F \right) + \psi_t \right]$$

where the term ψ_t is a risk premium shock that follows an AR(1) process in logs and b_F is the steady state net foreign assets to GDP ratio.

Labour Unions As in the large economy, perfectly competitive labour aggregators combine the specialised labour types into a labour aggregate that they supply to firms. Individual labour types unionise to exploit their monopoly power subject to a Calvo-type friction that prevents them from re-optimising their wage each period. Unions that do not re-optimise index wages growth to a weighted average of lagged and steady-state wage inflation.

Domestic Final Goods Retailers The domestically-produced final good, $Y_{H,t}$ is assembled by a perfectly competitive retailer that combines domestically-produced intermediate goods using the technology:

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di\right]^{\frac{\epsilon_p}{\epsilon_p - 1}}, \tag{5}$$

where ϵ_p is the elasticity of substitution between varieties of domestic intermediate goods. The price of the domestic final good and demand for individual varieties are given by:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\epsilon_p} di\right]^{\frac{1}{1-\epsilon_p}} \text{ and } Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p} Y_{H,t}$$

Domestic Intermediate Goods Producers Domestic intermediate goods producers manufacture heterogeneous goods using the technology:

$$Y_{H,t}(i) = Z_t L_t(i), \tag{6}$$

where $L_t(i)$ is firm i's labour input.[11]

As for the large economy, firms in the small economy face Calvo-style pricing frictions such that only a fraction, $1-\theta_p$, of firms are able to adjust their prices freely each quarter. Firms that do *not* re-optimise index their prices to a weighted average of lagged inflation and the central bank's inflation target, $\bar{\Pi}$, with weights given by χ_p and $1-\chi_p$. The pricing problem for a firm i that does re-optimise is

$$\max_{P_{H,t}(i)} \sum_{s=0}^{\infty} (\beta\theta_p)^s \, \mathbb{E}_t \left\{ \Lambda_{t+s} \left[\frac{P_{H,t}(i)\Omega_{t,t+s}\Gamma_{H,t+s}}{P_{H,t+s}} Y_{H,t+s}(i) - \frac{1}{1+\iota_p} \frac{W_t}{P_{H,t} Z_t} \Gamma_{H,t+s} Y_{H,t+s}(i) \right] \right\} \tag{7}$$

subject to the domestic final goods demand condition given above. $\Gamma_{H,t} = P_{H,t}/P_t$ is the relative price of domestically-produced goods, $\Omega_{t,t+s}$ is the cumulative increase in prices for a firm that does not re-optimise between t and $t+s$ and ι_p is a production subsidy calibrated to offsets the distortionary effects of monopolistic competition on steady-state output.

[11] As for the large economy, labour market clearing requires that $N_t = \int_0^1 L(i)di$.

Exporters Exporters purchase the domestic final good at price $P_{H,t}$ and differentiate it through branding for sale in the foreign economy. As in Burgess et al. (2013), and consistent with the prevalence of a dominant currency documented in Boz et al. (2018), all export contracts and prices are specified in the currency of the large economy. An export retailer bundles these goods before selling them overseas according to the technology:

$$X_t = \left[\int_0^1 X_t(i)^{\frac{\epsilon_x - 1}{\epsilon_x}} di \right]^{\frac{\epsilon_x}{\epsilon_x - 1}}, \tag{8}$$

where $\epsilon_x > 1$ is the elasticity of substitution between different varieties for export. The corresponding price index, in foreign currency terms, and demand function for total exports are given by:

$$P^*_{X,t} = \left[\int_0^1 P^*_{X,t}(i)^{1-\epsilon_x} di \right]^{\frac{1}{1-\epsilon_x}} \text{ and } X_t = \alpha_X \left(\frac{P^*_{X,t}}{P^*_t} \right)^{-\tau} Y^*_t$$

As in Justiniano and Preston (2010b) the elasticity of export demand, τ, is the same as the elasticity of substitution between domestic and foreign goods in the domestic final goods basket (described below). This implies that household's preferences in the small and large economies are the same, which for the US and Canada is a reasonable assumption.

Exporters face Calvo-style pricing frictions, with only a fraction, $1 - \theta_x$, of firms able to adjust their prices each quarter. Firms that do not re-optimise index their prices to steady-state US inflation. The resulting pricing problem for firm i is:

$$\max_{P^*_{X,t}(i)} \sum_{s=0}^{\infty} (\beta\theta_x)^s \, \mathbb{E}_t \left\{ \Lambda_{t+s} \left[\frac{P^*_{X,t}(i)\Omega^x_{t,t+s}\Gamma_{x,t+s}}{P^*_{X,t+s}} X_{t+s}(i) - \frac{1}{1+\iota_x} \Gamma_{H,t+s} X_{t+s}(i) \right] \right\} \tag{9}$$

subject to the usual demand constraint. $\Gamma_{x,t+s} = S_t P^*_{X,t}/P_t$ is the relative price between exports (in domestic currency terms) and the domestic CPI and ι_x is a production subsidy calibrated to offset the effect of imperfect competition on steady-state exports.

Importers Importers bring in homogeneous products from abroad at price $S_t P^*_t$ and differentiate them through branding. Importers then sell the differentiated goods to a retailer that combines them into the final imported good using the technology:

$$Y_{F,t} = \left[\int_0^1 Y_{F,t}(i)^{\frac{\epsilon_f - 1}{\epsilon_f}} di \right]^{\frac{\epsilon_f}{\epsilon_f - 1}}, \tag{10}$$

where $Y_{F,t}$ is the total volume of imports, $Y_{F,t}(i)$ is the quantity of the imported good of variety i used in the production of the final imported good and ϵ_f is the elasticity of substitution between different varieties of imported goods. The price index for the imported final good and demand curve for individual varieties are:

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\epsilon_f} di \right]^{\frac{1}{1-\epsilon_f}} \text{ and } Y_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\epsilon_f} Y_{F,t} \tag{11}$$

Importers face Calvo-style pricing frictions. Each quarter, a fraction, $1 - \theta_f$ sets prices optimally, while the remainder adjusts their prices by the small economy's steady-state inflation rate, $\bar{\Pi}$. The pricing problem for a representative firm i is:

$$\max_{P_{F,t}(i)} \sum_{s=0}^{\infty} (\beta\theta_f)^s \, \mathbb{E}_t \left\{ \Lambda_{t+s} \left[\frac{P_{F,t}(i)\Omega^F_{t,t+s}\Gamma_{F,t+s}}{P_{F,t+s}} Y_{F,t+s}(i) - \frac{1}{1+\iota_f} \frac{S_{t+s}P^*_{t+s}}{P_{t+s}} Y_{F,t+s}(i) \right] \right\}, \tag{12}$$

subject to the demand constraint above. $\Gamma_{F,t} = P_{F,t}/P_t$ is the price of imports (in domestic currency terms) relative to the domestic CPI, $\Omega^F_{t,t+s}$ is the cumulative price change for an importer that does not adjust its prices between t and $t+s$ and ι_f is a subsidy calibrated to offset the effect of imperfect competition on import volumes on steady state,

Fiscal policy The government's period budget constraint is:

$$P_t G_t + B_t \leq T_t + \frac{B_{t+1}}{R_t}$$

where G_t is real government consumption of goods and services, which evolves according to:

$$\frac{G_t}{Z_t} = \left(\frac{\bar{G}}{Z} \right)^{1-\rho_g} \left(\frac{G_{t-1}}{Z_{t-1}} \right)^{\rho_g} e^{\varepsilon_{G,t}}$$

Domestic Final Demand and Market Clearing The final good consumed domestically is assembled by perfectly competitive retailers using the technology:

$$DFD_t = \left[(1-\alpha)^{\frac{1}{\tau}}\left(Y_{H,t}^{H}\right)^{\frac{\tau-1}{\tau}} + \alpha^{\frac{1}{\tau}}\left(Y_{F,t}\right)^{\frac{\tau-1}{\tau}}\right]^{\frac{\tau}{\tau-1}}, \tag{13}$$

where $Y_{H,t}^{H}$ is the domestic consumption of the domestically-produced final good and $Y_{F,t}$ is the volume of imports. The parameter τ is the elasticity of substitution between domestic- and foreign-produced goods. The price index corresponding to this bundle is:

$$P_t = \left[(1-\alpha)\left(P_{H,t}\right)^{1-\tau} + \alpha\left(P_{F,t}\right)^{1-\tau}\right]^{\frac{1}{1-\tau}}, \tag{14}$$

where $P_{H,t}$ is the price of the domestic composite good and $P_{F,t}$ is the price of the imported composite good, both expressed in domestic currency.

We assume that domestic households and the government have identical preferences for domestically-produced and imported goods. This implies that the household sectors demand for domestically-produced goods and services is $C_{H,t} = (1-\alpha)\Gamma_{H,t}^{-\tau}C_t$ and for imports is $C_{F,t} = \alpha\Gamma_{F,t}^{-\tau}C_t$. Analogous expressions hold for the government's demand for domestically-produced and imported goods and services.

Goods market clearing requires that all domestic production is either consumed domestically by households, by the government, or exported:

$$Y_{H,t} = C_{H,t} + G_{H,t} + X_t \tag{15}$$

Monetary Policy Before 2009Q2, when R_t is not fixed, the Canadian central bank follows the feedback rule:

$$\frac{R_t}{R} = \left[\frac{R_{t-1}}{R}\right]^{\rho_R}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_{H,t}}{Z_t}\right)^{\phi_y}\right]^{1-\rho_R}\left(\frac{Y_{H,t}}{Y_{H,t-1}\mathcal{M}_t}\right)^{\phi_g} exp(\varepsilon_{R,t}). \tag{16}$$

As for the US, we allow for an arbitrary level of the fixed nominal interest rate. Canada fixed its policy rate at 0.25 % for a year and then at 1 % for about 5 years in our sample so, from

2009Q2, policy follows

$$\frac{R_t}{R} = \bar{R} \tag{17}$$

and

$$\frac{R_t}{R} = \bar{\bar{R}} \tag{18}$$

from 2010Q3 until lift-off.[12]

The term structure of interest rates in the small economy is determined in a similar manner to the large economy. For any $m > 1$:

$$R_{m,t}^{\text{obs}} = R_{m,t} \exp\left(c_m \eta_t \varepsilon_{m,t}\right), \tag{19}$$

where $R_{m,t}$ is the interest rate on a bond that pays one unit of domestic currency in m quarters as determined by the expectations hypothesis, c_m is a constant risk premia on the m quarter interest rate, η_t is shock, common to all small economy interest rates, that follows an exogenous autoregressive process and $\varepsilon_{m,t}$ is an idiosyncratic shock to the m quarter interest rate in the small economy.

2.3 The Transmission of Foreign Monetary Policy

In the empirical implementation, we use the model that we describe above. But to understand the key forces that determine the strength of foreign monetary policy shocks in the larger model it is useful to consider the impact of foreign shocks in a simpler context. With some simplifying assumptions – no habits in consumption, complete markets, flexible wages and full exchange rate pass through at home and abroad – our economy model collapses to that of De Paoli (2009b)

[12] The change from $\bar{R}$ to $\bar{\bar{R}}$ is modeled as an unanticipated policy change in 2010Q3.

which can be characterized by the set of log-linear equations:

$$\begin{aligned}
\pi_{H,t} &= \kappa \left(c_t + \varphi y_{H,t} + \frac{\alpha}{1-\alpha} q_t \right) + \beta \mathbb{E}_t \{\pi_{H,t+1}\} \\
\pi_t &= \pi_{H,t} + \frac{\alpha}{1-\alpha} \Delta q_t \\
y_{H,t} &= (1-\alpha) c_t + \alpha y_t^* + \gamma q_t \\
c_t &= y_t^* + q_t \\
c_t &= \mathbb{E}_t \{c_{t+1}\} - (r_t - \mathbb{E}_t \pi_{t+1})
\end{aligned}$$

and a monetary policy reaction function. In the equations above, variables represent percentage deviations from steady state, $\gamma = \frac{\tau\alpha(2-\alpha)}{(1-\alpha)}$ and $\kappa = \frac{(1-\theta\beta)(1-\theta)}{(1+\varphi)\theta}$. The only exogenous shock is y_t^*. Although this system of equations does not feature foreign interest rates, it can nonetheless be used to examine foreign monetary policy spillovers following foreign monetary policy shocks, conditional on the response of foreign output, y_t^*.

To assess the response of domestic output, first assume that domestic monetary policy follows a rule of the form:

$$r_t = \phi_\pi \mathbb{E}_t \pi_{t+1} \tag{20}$$

where $\phi_\pi > 1$. This reaction function implies that the small economy's central bank stabilizes the *ex-ante* real interest rate. One can think of this exercise as illustrating the direct effects of a foreign monetary policy shock, holding the stance of domestic monetary policy fixed.

The monetary policy rule and Euler equation imply that the level of consumption in the small economy is stable at its steady state level. The UIP condition then ensures that the small economy's real exchange rate appreciates one-for-one with increases in foreign output. Substituting these results into the resource constraint, we can express output in the small economy as a function of foreign output:

$$y_{H,t} = \left[\alpha - \frac{\tau\alpha(2-\alpha)}{1-\alpha} \right] y_t^* \tag{21}$$

The expression in brackets summarizes the two forces that govern the response of domestic output. The first term, αy_t^*, is the income effect – the direct increase in exports from the small economy resulting from expansionary monetary policy abroad that increases demand in the large economy. This term is increasing in the openness of the domestic economy, reflected in the parameter α. The second term, $\frac{\tau\alpha(2-\alpha)}{1-\alpha}y_t^*$, is the substitution effect – the reduction in demand for goods produced in the small economy resulting from appreciation of the small economy's real exchange rate triggered by expansionary monetary policy abroad. The strength of this term is increasing in the openness of the domestic economy, as well as in the substitutability between domestic and foreign products, reflected in the parameter τ. In the case where the domestic central bank perfectly stabilizes the real interest rate, foreign monetary policy is beggar-thy-neighbour if $\tau > \frac{1-\alpha}{2-\alpha}$.

The mechanism described above also summarizes the effect of foreign forward guidance shocks on the domestic economy. With a constant *ex-ante* real interest rate, Equation (21) describes the relationship between domestic and foreign output in each period. An announcement about the future path of interest rates in the foreign economy triggers a sequence of foreign output realizations. These realizations will expand or contract domestic output over time according to the path of foreign output and the relative strength of the income and substitution effects.

If the domestic central bank follows a reaction function that responds to current realizations of domestic inflation:

$$r_t = \phi_\pi \pi_t \tag{22}$$

then the real interest rate is no longer constant and the relationship between foreign and domestic output depends also on the flexibility of domestic prices, on the response of domestic marginal costs to demand conditions, and on the response of monetary policy to inflation, summarized by the following equation:[13]

$$y_{H,t} = a\left[\alpha - (1-\alpha+\gamma)\phi_\pi b - \frac{\tau\alpha(2-\alpha)}{1-\alpha}\right]y_t^* \tag{23}$$

Although neat analytical expressions are not available in this case, Equation (23) reveals

[13] Where $a = \frac{1-\alpha+\phi(\kappa+\alpha)}{1-\alpha+\phi(\kappa+\alpha)-\kappa\varphi(1-\alpha)}$, $b = -\frac{(1+\kappa)\alpha}{1-\alpha+\phi(\kappa+\alpha)}$ and $\gamma = \frac{\tau\alpha(2-\alpha)}{1-\alpha}$.

which additional parameters – in particular, the elasticity of labor supply, the slope of the Phillips curve and the strength of the monetary policy response to inflation – also matter in determining the extent to which foreign monetary policy is beggar-thy-neighbour.

A lower wage elasticity of labor supply (a lower value of φ) implies that wages, and hence marginal costs, are more responsive to changes in labor demand. Hours worked, in contrast, are less responsive. Foreign monetary policy is less beggar-thy-neighbour when the elasticity of labor supply is low.

When prices are more flexible (a larger value of κ) a given change in marginal costs translates more into changes in prices and less into changes in quantities. In the case where expansionary foreign monetary policy triggers a fall in domestic output, more flexible prices dampen the expenditure switching channel. This is because the fall in labor demand and marginal costs translates into a larger reduction in the prices of domestically-produced goods. This cushions the fall in the consumption of domestic goods.

A central bank that responds to inflation will cut interest rates following an expansionary foreign monetary policy shock that lowers domestic production, wages and inflation. In doing so, the central bank reduces the interest rate differential between the domestic and foreign economies and diminishes the initial exchange rate appreciation of the domestic currency. This too will reduce the extent to which expansionary foreign monetary policy is beggar-thy-neighbour.

The mechanisms described above are also at work in our larger model. In estimation, we place priors over the parameters that govern the response of domestic output to foreign monetary policy disturbances, in particular the trade elasticity, τ, that are wide enough to allow the data to explain whether expansionary foreign monetary policy is also expansionary for the domestic economy, or beggar-thy-neighbour. We now turn to how the full model is solved and estimated with periods of fixed interest rates.

3 Solution and Estimation

The model is solved and estimated following the approach in Kulish, Morley and Robinson (2017). Here, for completeness and to introduce the necessary notation for defining forward guidance shocks, we describe the solution in the case of a single fixed interest rate regime. Details, including how the method is extended to the two country case with two fixed interest rate regimes, are presented in the online appendix.

The system of linearized equations in which monetary policy follows a Taylor type rule is given by

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}\varepsilon_t. \tag{24}$$

where x_t is the state vector and ε_t is the vector of structural shocks, which we take to be i.i.d. without loss of generality. If it exists and is unique, the standard rational expectations solution to (24) is

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t \tag{25}$$

During the fixed interest rate regime, however, the structural equations are given by

$$\bar{\mathbf{A}}x_t = \bar{\mathbf{C}} + \bar{\mathbf{B}}x_{t-1} + \bar{\mathbf{D}}\mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}\varepsilon_t. \tag{26}$$

where the only equation that changes is the one corresponding to the monetary policy rule which now fixes R_t at some level. Assume that at $t = 1$ monetary policy fixes the interest rate and is expected to revert back to the Taylor rule in $\bar{T} + 1$. For periods, $t = 1, 2, ..., \bar{T}$, Kulish and Pagan (2017) show that the solution for x_t is a time-varying coefficient VAR of the form

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{27}$$

where the reduced form matrices satisfy the following recursions:

$$\mathbf{J}_t = \left[\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{Q}_{t+1}\right]^{-1}\left(\bar{\mathbf{C}} + \bar{\mathbf{D}}\mathbf{J}_{t+1}\right) \tag{28}$$

$$\mathbf{Q}_t = \left[\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{Q}_{t+1}\right]^{-1}\bar{\mathbf{B}} \tag{29}$$

$$\mathbf{G}_t = \left[\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{Q}_{t+1}\right]^{-1}\bar{\mathbf{F}}. \tag{30}$$

Starting from the terminal condition $\mathbf{Q}_{\bar{T}+1} = \mathbf{Q}$, the sequence of reduced form matrices $\{\mathbf{Q}_t\}_{t=1}^{\bar{T}}$

can be solved for via the backward recursion implied by (29). With $\{\mathbf{Q}_t\}_{t=1}^{\bar{T}}$ in hand, it is straightforward to find $\{\mathbf{J}_t\}_{t=1}^{\bar{T}}$ and $\{\mathbf{G}_t\}_{t=1}^{\bar{T}}$. Note that in the sequence, $\{\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_{\bar{T}}\}$ each reduced form matrix has associated with it an expected duration of the fixed interest rate regime: $\mathbf{Q}_1$ is associated with an expected duration of $\bar{T}$ quarters, $\mathbf{Q}_2$ with an expected duration of $\bar{T}-1$ quarters and so on.

In estimation, we treat agents' expected duration at time t, $\mathbf{d}_t$, as a time-varying parameter over the fixed interest rate regimes. To keep track of the reduced form that prevails at each point in the sample, we allow $\bar{T}$ to be an arbitrary large upper bound on the duration and re-label the sequence of reduced forms, $\{\mathbf{J}_d\}_{d=1}^{\bar{T}}$, $\{\mathbf{Q}_d\}_{d=1}^{\bar{T}}$ and $\{\mathbf{G}_d\}_{d=1}^{\bar{T}}$, so that the reduced-form on a given quarter t can be written as $\mathbf{J}_t = \mathbf{J}_{\mathbf{d}_t}$, $\mathbf{Q}_t = \mathbf{Q}_{\mathbf{d}_t}$ and $\mathbf{G}_t = \mathbf{G}_{\mathbf{d}_t}$. Adopting the convention that $\mathbf{d}_t = 0$ in periods where the Taylor rule is in operation $\mathbf{J}_0 = \mathbf{J}$, $\mathbf{Q}_0 = \mathbf{Q}$ and $\mathbf{G}_0 = \mathbf{G}$, we may write the reduced-form over the entire sample as in (27).

The likelihood, $\mathcal{L}(\vartheta, \mathbf{d}|Y^{\text{obs}})$ is a function of both the structural parameters, ϑ, and sequences of durations, $\mathbf{d} = \{\mathbf{d}_t\}_{t=1}^{T}$. We put priors over structural parameters and independent priors over durations to construct the posterior.[14] Details, including the Kalman filter and smoother can be found in the online appendix. Next, we turn to discuss how we use the estimated model and the occasionally binding constraint solution to identify forward guidance.

4 Identifying Forward Guidance

The very nature of an expected duration, $\mathbf{d}_t$, is that, in the absence of future shocks, it is expected to shrink by one period in every period. That is,

$$\mathbb{E}_t \mathbf{d}_{t+1} = \mathbf{d}_t - 1 \text{ for } \mathbf{d}_t > 0 \tag{31}$$

Importantly, in estimation we do not require the estimated expected durations, $\{\mathbf{d}_t\}_{t=1}^{T}$ to equal the duration implied by the constraint and structural shocks. Thus, the estimation allows for the possibility that central banks adopted the optimal policy prescription of Eggertsson and

[14]See Kulish et al. (2017) for details on how the sampler is set up in two blocks, one for the structural parameters which have continuous support and one for the durations which are integers.

Woodford (2003) by extending the duration of the fixed interest rate policy beyond the horizon implied by the lower bound constraint itself.

4.1 Lower Bound Durations

With the occasionally binding constraint algorithm of Jones (2017) we find at each period t, the expected duration which is implied by the estimated state x_{t-1}, the estimated structural shocks ε_t, and a given lower bound.[15] We denote this expected duration at t by $\mathbf{d}_t^{\text{lb}}$ and refer to it as the *lower bound* duration. In the absence of future shocks, as the effect of current shocks ε_t unwind, the lower bound duration is also expected to fall by one period in every period. In other words,

$$\mathbb{E}_t \mathbf{d}_{t+1}^{\text{lb}} = \mathbf{d}_t^{\text{lb}} - 1 \;\text{ for }\; \mathbf{d}_t^{\text{lb}} > 0 \tag{32}$$

Shocks that unfold in $t+1$ may make the constraint bind for a longer or a shorter period of time, and hence the lower bound duration may expand or contract in period $t+1$. With the estimated structural shocks and state in hand, we compute the lower bound durations that prevail in each quarter of the fixed interest rate regime, that is $\{\mathbf{d}_t^{\text{lb}}\}_{t=1}^{T}$.

4.2 Forward Guidance Shocks

For each period t of the fixed interest rate, we define the *forward guidance* duration, $\mathbf{d}_t^{\text{fg}}$, as the difference between the *estimated* duration, $\mathbf{d}_t$ and the *lower bound* duration, $\mathbf{d}_t^{\text{lb}}$, that is

$$\mathbf{d}_t^{\text{fg}} \equiv \mathbf{d}_t - \mathbf{d}_t^{\text{lb}} \tag{33}$$

The *forward guidance* duration, $\mathbf{d}_t^{\text{fg}}$, captures announcements or other factors that can change the duration beyond what the structural shocks and the constraint imply, that is beyond $\mathbf{d}_t^{\text{lb}}$.[16] This decomposition of the estimated duration characterizes monetary policy when the policy rate

[15] Jones (2017) shows that the occasionally binding constraint solution constitutes a good approximation to the non-linear economy. Details of the occasionally binding constraint solution are given in full in the online appendix. For each country we take the level of the fixed interest rate as a lower bound constraint.

[16] Other factors beyond explicit central bank communication can influence the duration. As is well known, markets reassessed the time of lift-off as the Federal Reserve was tapering the rate of bond purchases.

is fixed and is useful since the level of the nominal interest rate itself is an insufficient statistic of the stance of monetary policy.

Forward guidance shocks are unexpected changes of the forward guidance duration, that is

$$\varepsilon_t^{\text{fg}} = \mathbf{d}_t^{\text{fg}} - \mathbb{E}_{t-1}\mathbf{d}_t^{\text{fg}} \tag{34}$$

The expectation of the forward guidance duration, $\mathbb{E}_{t-1}\mathbf{d}_t^{\text{fg}}$, however, depends on whether the lower bound is binding. Equations (31) and (32) imply that

$$\mathbb{E}_{t-1}\mathbf{d}_t^{\text{fg}} = \begin{cases} \mathbf{d}_{t-1}^{\text{fg}} - 1, & \text{if } \mathbf{d}_t^{\text{lb}} = 0 \\ \mathbf{d}_{t-1}^{\text{fg}}, & \text{otherwise} \end{cases} \tag{35}$$

If the lower bound binds, so that $\mathbf{d}_t^{\text{lb}} > 0$, the forward guidance shock is the change in the forward guidance duration, that is $\Delta\mathbf{d}_t^{\text{fg}}$. But when the lower bound constraint does not bind, $\mathbf{d}_t^{\text{lb}} = 0$, so that $\mathbf{d}_t = \mathbf{d}_t^{\text{fg}}$, the forward guidance duration is expected to fall by one period in every period as dictated by Equation (31). Therefore, the expected one quarter reduction in $\mathbf{d}_t^{\text{fg}}$ is not a forward guidance shock but a continuation of policy. Forward guidance shocks in this case are defined as increases or decreases in the expected duration of the fixed interest rate policy beyond this expected one quarter change. Formally, the *forward guidance shock* in period t is given by:

$$\varepsilon_t^{\text{fg}} = \begin{cases} \mathbf{d}_t^{\text{fg}} - \mathbf{d}_{t-1}^{\text{fg}}, & \text{if } \mathbf{d}_t^{\text{lb}} \geq 1 \\ \mathbf{d}_t^{\text{fg}} - \mathbf{d}_{t-1}^{\text{fg}} + 1, & \text{if } \mathbf{d}_t^{\text{lb}} = 0 \end{cases} \tag{36}$$

Figure 1 illustrates a forward guidance shock at t with an example. At period $t-1$ the estimated duration, $\mathbf{d}_{t-1}$ is 5 quarters: the interest rate is expected to be fixed from period $t-1$ to period $t+3$, with lift-off in $t+4$. In the absence of structural and forward guidance shocks after $t-1$, in the forecast horizon, the duration falls by one quarter every quarter. In the middle panel of Figure 1, $\mathbb{E}_{t-1}\{\mathbf{d}_{t-1}, \mathbf{d}_t, ..., \mathbf{d}_{t+3}\} = \{5, 4, 3, 2, 1\}$ and its associated interest rate path is labeled the "*forecast path at $t-1$*" in the top panel.[17]

[17] Our forward guidance shock is specified in terms of a duration and it is therefore measured in quarters

In period $t-1$, the structural shocks, ε_{t-1}, imply an evolution of the lower bound durations. In this case, $\mathbb{E}_{t-1}\{\mathbf{d}^{\text{lb}}_{t-1}, \mathbf{d}^{\text{lb}}_{t}, \ldots, \mathbf{d}^{\text{lb}}_{t+3}\} = \{3, 2, 1, 0, 0\}$. The associated path for the interest rate is shown in the top panel of Figure 1 and is labeled the "*lower bound forecast*". These lower bound durations are represented by the blue bars in the middle panel of Figure 1. The difference between the fixed interest rate duration and the lower bound duration, illustrated by the black bars in the middle panel of Figure 1, is the forward guidance duration. At $t-1$, the forward guidance duration is expected to evolve as follows: $\mathbb{E}_{t-1}\{\mathbf{d}^{\text{fg}}_{t-1}, \mathbf{d}^{\text{fg}}_{t}, \ldots, \mathbf{d}^{\text{fg}}_{t+3}\} = \{2, 2, 2, 2, 1\}$.

The bottom panel of Figure 1 illustrates a one-quarter expansionary forward guidance shock in period t. We continue to assume that there are no structural shocks after $t-1$. Following the forward guidance shock at t, the lower bound and the forward guidance durations are expected to evolve in the forecast at t as shown by the bottom panel of Figure 1. At $t-1$, the estimated duration was expected to fall to 4 quarters by t, that is $\mathbb{E}_{t-1}\mathbf{d}_t = 4$. However, a one quarter forward guidance shock increases $\mathbf{d}_t$ to 5 quarters. Although the total duration is the same in $t-1$ as it is in t, that is $\mathbf{d}_t = \mathbf{d}_{t-1}$, the decomposition of the duration is different because of the forward guidance shock.

We achieve identification because unexpected changes in $\mathbf{d}_t$ which stem from unexpected changes in $\mathbf{d}^{\text{lb}}_t$ have different implications for the observable variables than those which stem from unexpected changes in $\mathbf{d}^{\text{fg}}_t$. Figure 2 shows a simulation with the large economy (i.e. a standard closed economy model) to illustrate the impact on observables of a given increase in duration. The figure shows paths for output growth, inflation and the interest rate in two cases: the first is a negative preference shock that depresses output growth and inflation and in doing so makes the constraint bind for 4 quarters (i.e. it increases $\mathbf{d}^{\text{lb}}_t$), while the second case considers a credibly announcement by the central bank to keep rates at zero for 4 quarters (i.e. it increases $\mathbf{d}^{\text{fg}}_t$). Thus, we compare a negative shock that makes the constraint bind for 4 quarters to a 4 quarter forward guidance shock. In both cases we start from $\mathbf{d}_{t-1} = 0$, implying $\mathbb{E}_{t-1}\mathbf{d}_t = 0$ so the unexpected change in the duration is the same in both case.

Output and inflation fall following the negative preference shock, but rise following expan-

but as Figure 1 shows it has implications for the forecast of the policy rate and other macro variables as well. Our definition is therefore complementary to the one used for example by Del Negro et al. (2015) who consider changes in the forecast of a variable at various horizons h caused by a forward guidance announcement.

sionary forward guidance. Note also that at the time of lift-off the policy rate increases faster following the forward guidance shock because the monetary authority is faced with a stronger economy with higher inflation and output growth. The different expected paths for the policy rate have, in turn, implications for longer maturity rates which, though not shown in Figure 2, are used as observables in estimation.

5 Estimation

5.1 Data

For estimation, we use US and Canadian nominal interest rates, output and consumption growth, inflation, nominal wage inflation, Canadian imports and export volumes growth and changes in the US/Canadian nominal exchange rate. We also use 2-year nominal interest rates for both the US and Canada in helping identify the fixed interest rate durations. The series start in 1991Q1 to coincide with start of inflation targeting in Canada and end in 2019Q2. All details of the data are in the online appendix.

5.2 Assigned and Calibrated Parameters

We calibrate the following structural parameters. We set the inflation target in both economies equal to 2% in annualised terms. Trend per capita growth is calibrated to 0.43% in annualised terms which corresponds to the mean across the two economies for the pre-2009 subsample. Given trend growth and the inflation target, we set the quarterly discount factor β, common to both countries, to 0.9985 to get a steady nominal interest rate of 4.3% in annualised terms which is the average of nominal interest rates for the two economies for the pre-2009 subsample, 4.1% for the US and 4.5% for Canada. The inverse of the Frisch elasticity of labor supply, ψ, is calibrated to 2 in both countries, close to the values estimated in Smets and Wouters (2007) and Justiniano and Preston (2010b).[18] The import share of the Canadian consumption basket, α, is

[18] In preliminary attempts, we found that ψ was poorly identified. This is related to the fact that we do not use hours data. There is an hours gap between Canada and the US which opened up during our sample and for which the model is not well-suited to explain. We also found that estimating the parameters that pin down the

set to 0.25 to match the average share of exports and imports to Canadian GDP, while the steady state values of $\bar{G}^*$ and $\bar{G}$ are set to ensure that we match the average share of consumption to GDP in the US and Canada, equal to 65% and 57.5% respectively.

5.3 Fixed Interest Rate Regimes

We measure expected durations in the US from the time at which the Federal Funds rate reached 0.125% in 2009Q1 until lift-off in 2015Q4. Similarly, we measure these durations in Canada when the policy rate was at 0.25% between 2009Q2 and 2010Q2, and then when the rate was at 1% between 2010Q3 and 2015Q1. This modeling choice is motivated by the clear communication of the Bank of Canada providing forward guidance of its fixed interest rate policy at these values.[19]

5.4 Estimation Results

Structural Parameters The moments of the prior and posterior distribution for each parameter that we estimate are reported in Tables 1 and 2. We use the same prior distributions for parameters common to both countries. The prior of the trade elasticity, τ, is centered at 1, and allows for a wide range of possible outcomes for the sign and magnitude of spillovers of US monetary policy to Canada. Our priors for the slopes of the price and wage equations are centered around conventional values of the frequency of price changes of about once every four quarters, while our priors imply a reasonably small degree of wage indexation.

The monetary policy feedback rules respond to the growth rate of GDP and to the detrended level of output, given the Fed's emphasis on the subdued labor market outcomes over the 2009-2015 period. We use standard priors implying relatively stronger responses to inflation deviations. The estimates of the monetary policy rule will be important for estimating forward guidance durations, as it is the monetary policy regime that the economy reverts to following a

steady state of the nominal interest rate led to less precise estimates of the durations. This is to be expected as the duration and steady state of the policy rate are key determinants of the expected path of the policy rate.

[19] As an example, in the May 2013 press release, the Bank of Canada said that '*With continued slack in the Canadian economy, the muted outlook for inflation, and the constructive evolution of imbalances in the household sector, the considerable monetary policy stimulus currently in place will likely remain appropriate for a period of time, after which some modest withdrawal will likely be required, consistent with achieving the 2 per cent inflation target.*'

period of time at a fixed interest rate which is important for governing expectations about the path of the interest rate, and therefore in determining the stimulatory effect of forward guidance. Finally, we use wide priors for the standard deviations of the autoregressive shocks for both the US and Canada.

Most of the estimated structural parameters are similar for the two economies. The estimated Calvo parameters imply a frequency of price and wage changes of once every four quarters, with insignificant differences between the frequency of changes for domestic goods, imports and exports. The degree of price and wage indexation in both economies is estimated to be small.

The posterior estimate of the trade elasticity, τ, is centered around 3.0. An estimate of above 1 is typical for estimated small open economy models, although Justiniano and Preston (2010b) report an estimate of 0.8 using Canadian data.[20] As this parameter exerts an important influence on the response of domestic output to foreign monetary policy disturbances, we describe in more detail how it is identified in estimation.

The trade elasticity, τ, influences how much domestic output responds to changes in relative prices between the domestic and foreign economy. A lower trade elasticity makes domestic output less responsive to changes in relative prices. Less responsive output implies smaller changes in labor demand which in turn imply smaller changes in marginal cost and inflation. Through the policy rule this means the nominal interest rate is also less responsive. Following an expansionary monetary policy abroad, the interest rate differential between foreign and domestic interest rates would be larger the lower the trade elasticity. Through uncovered interest rate parity, this implies larger changes of the nominal exchange rate.

The estimation procedure chooses a value of τ that helps the model to jointly match the behaviour of domestic output and the nominal exchange rate. While a lower value of τ would imply that expansionary monetary policy in the US is also expansionary for Canada, a lower value of τ reduces the variance of domestic output, exports and imports but increases the variance of the nominal exchange rate.

In Figure 3, we compare the responses *on impact* of Canadian output to an expansionary US monetary policy shock at different values of τ, the trade elasticity. We draw parameters from the

[20] For example Adolfson et al. (2013) report an estimate of 1.41 for Sweden.

prior and posterior distributions and solve the model for each draw. Figure 3 illustrates that our priors are wide enough to allow an expansionary US monetary policy shock to be expansionary or contractionary for Canadian output. For the entire posterior distribution, however, we find that the output responses are negative. The important quantitative role of the trade elasticity, τ, in determining the domestic output response is highlighted in Figure 4 which shows responses of Canadian variables to an expansionary US monetary policy shock for a high and low value of τ, with all other parameters set to the mode of the posterior distribution.

Turning to the estimates of the shock processes, relative to the priors, the data prefer relatively more persistent preference and markup shocks in the US, consistent with the relative differences in persistence found in Kulish and Rees (2011). For Canada, risk premium and import and export markup shocks are highly persistent, indicative of substantial deviations from UIP and incomplete exchange rate pass through. The estimation also points to larger markup shocks in Canada than the US, although the estimated standard deviations of preference and monetary policy shocks are similar. To understand the implications of these parameter estimates, we report in Table 3 the forecast error variance decomposition of both US and Canadian variables into the structural shocks of the model.[21] US preference shocks account for about 38% of the variation in detrended US output growth in the long run, and 64% of the variation in the Fed Funds rate. Consistent with the findings of Smets and Wouters (2007), US monetary policy shocks drive a small percentage of US output and inflation. US preference shocks account for about 2% of the forecast error of Canadian output growth, while US monetary policy shocks account for little of the long-run variation in Canadian output. Instead, most of the variation in Canadian output is driven by Canadian shocks, consistent with the findings in Justiniano and Preston (2010a).

Expected Durations For the sequence of US expected durations, we use an informative prior as in Kulish, Morley and Robinson (2017), using survey data from Blue Chip from 2009 to 2010, and from the NY Fed survey of primary dealers from 2011 to 2015. For Canada we use an uninformative prior as equivalent survey measures are not available.[22] The posterior

[21] The shocks to the 2Y yields capture only deviations of the 2Y interest rates from the expectations hypothesis, and so do not affect the model's other variables.

[22] The priors and posterior distributions for the sequence of expected durations are plotted in the Appendix.

distributions for the durations of the US Fed Funds rate at 0.125% between 2009Q1 and 2015Q4, and the Bank of Canada's Bank Rate at 0.25% between 2009Q2 and 2010Q2 and 1% between 2010Q3 and 2015Q1 are in shown in Figure 5.

For the US, the central value of each posterior density lies between 3 and 10 quarters. The posterior densities noticeably shift towards longer durations over 2011, with the mode of the posteriors increasing from 4 quarters in 2011Q1 to 7 quarters in 2011Q3 and 10 quarters in 2012Q1, and staying around 8 quarters until the start of 2014Q1, after which the mode of the posteriors declines to around 2 quarters by 2014Q4. These values are consistent with the results of Swanson and Williams (2014), who find expected durations of around 7 quarters for each quarter from 2011, and aligns with the explicit adoption of forward guidance from 2011Q3, when the Federal Reserve announced that it would maintain the interest rate at 0.125% until mid-2013. Subsequently, the Fed repeatedly extended the explicit liftoff date back, which is consistent with our estimated posterior distributions remaining centered around 6 to 7 quarters.

For Canada, the estimated posterior distributions are centered around lower fixed interest rate durations, with mean durations of between 2 and 7 quarters over the estimation sample. The mass of the posterior distribution of the fixed interest rate regimes shrinks towards smaller durations for the first three quarters that the Canadian Bank Rate is fixed at 0.25% in 2009, which is consistent with the Bank of Canada raising its policy interest rate to 1% in third quarter of 2010, and keeping it fixed thereafter.

6 Spillovers of US Monetary Policy

We next use the estimated model to study the spillovers of US monetary policy shocks on Canada. We first use our solution methods to measure forward guidance shocks in each country. We then compare the size and magnitude of the spillovers of conventional US monetary policy shocks when interest rates in the US and Canada follow a standard reaction function, to the spillovers that arise in response to a typical US forward guidance shock. Next, we show that the extent of these spillovers depends on the state of the US economy. Finally, we compute counterfactuals and use the model to quantify the spillovers from US forward guidance shocks between 2009 and

2014.

6.1 Identified Forward Guidance in the US and Canada

We first report our estimated measure of forward guidance in the US and Canada. To measure forward guidance, we repeatedly draw from the posterior distribution of parameters and durations. For each draw, we use the observables and the Kalman smoother to estimate the model's structural shocks. We then follow the methods discussed in Section 4 and use the structural shocks to identify lower bound durations for each country.

Figure 5 plots the mean of the set of draws of fixed interest rate durations and lower bound durations for the US and Canada. For the US, the mean across durations is initially around 4 quarters to 6 quarters, from 2009 to early 2011, of which the mean across lower bound durations is roughly half the total duration, between 2 quarters and 3 quarters. We estimate therefore that US forward guidance was responsible for about half of the expected duration of fixed interest rate policy between 2009 and 2011, or between 2 quarters and 3 quarters in duration.

From early to mid-2011, to late 2013, we find that forward guidance shocks in the US expanded markedly. Following the Fed's explicit calendar-based commitment to holding the Fed Funds rate fixed from early to mid-2011, we estimate that the lower bound durations stayed roughly constant, between 2 quarters and 4 quarters, so that the forward guidance announcements were about 4 quarters to 5 quarters. As discussed below in the counterfactual simulations, we find that these announcements were stimulatory for the US and, owing to the estimated elastic demand by the US for Canadian exports, were contractionary in Canada.

In Panel B of Figure 5, we plot the corresponding decomposition of the estimated total expected durations of fixed interest rate policy in Canada. These durations are for fixed interest rate policy at 0.25% between 2009Q2 and 2010Q1, and then at 1% from 2010Q2 to 2014Q4, the end of the sample. The estimated lower bound duration is roughly constant over the 2009Q2 to 2014Q4 period, mostly being between 2 quarters and 4 quarters in duration.

Next, we assess the external validity of the estimated US forward guidance shocks in two ways. First, we compare the estimated forward guidance shocks to the historical record of

FOMC announcements. We examine two salient quarters: 2011Q2 when the Federal Reserve introduced explicit calendar-based forward guidance in its statement and 2013Q2 when the Federal Reserve announced a reduction in the pace of government bond purchases, which was interpreted by markets as corresponding to an earlier lift-off, prompting the so called 'taper tantrum'.[23] Figure 6 plots the posterior distributions of the forward guidance shocks in the US for 2011Q2, in Panel A, and 2013Q2, in Panel B. For 2011Q2, the forward guidance shock is 4 quarters at the mean with the bulk of the posterior distribution on positive durations. For 2013Q2, however, when the Federal Reserve announced the tapering of its asset purchases, the forward guidance shock is -2 quarters at the mean with the bulk of the posterior on negative durations.

Out second exercise confirms our model-based results using an event study of how financial market pricing changed in the immediate aftermath of Federal Reserve announcements that conveyed information about the likely duration of the constant interest rate policy. For this exercise, we use the series of US forward guidance shocks derived by Bundick and Smith (forthcoming). They use the change in futures contract pricing around FOMC board meetings to infer changes in forward guidance policy. Although we would not expect this series to align exactly with the estimated forward guidance shocks from our model, as our shocks represent the sum of all forward guidance announcements over an entire quarter, the two series have a correlation of 0.4, suggesting substantial overlap between them.

We estimate the effect of changes in US forward guidance on Canadian financial market pricing by estimating the model:

$$\Delta y_t^{Can} = \alpha + \beta FG_t^{US} + \epsilon_t \tag{37}$$

where Δy_t^{Can} is the change in a Canadian asset price between the end of date t and the end of date $t-1$ and FG_t^{US} is the US forward guidance shock on date t. We estimate the model between 2009Q1 and 2015Q4, representing the period of fixed interest rate policy in the US. We consider six measures of interest rates: 3-month, 6-month, 1-year, 2-year 5-year and 10-year

[23] See http://www.federalreserve.gov/newsevents/press/monetary/20110809a.htm

Canadian government bond yields, as well as the bilateral exchange rate between the Canadian and US dollars.

Table 4 shows the results of the exercise. A lengthening of forward guidance in the US (an increase in FG_t^{US}) leads to a decline in Canadian bond yields and an appreciation of the Canadian dollar. The effects are larger for longer-maturity interest rates, although this in part reflects the fact that, because the Bank of Canada maintained its own fixed interest rate policy, the short end of the Canadian yield curve showed almost no variability for much of the sample. These results are consistent with our finding in the main text on expansionary forward guidance shocks in the US.

6.2 Conventional and Unconventional Policy Shocks

The non-linearity of the solution during the period of fixed interest rates, as we showed above, implies a time-varying VAR of the form $x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t$. A forward guidance shock, as we explained above, is an unanticipated change in duration; as a result it changes the reduced-form matrices, $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$, that prevail at the time of the shock as well as those which are expected to prevail in the future. Its impact depends on the state of the economy x_{t-1} and current shocks, ε_t.

We use generalized impulse responses as proposed by Koop et al. (1996). In principle, the impulse response to a forward guidance shock can be averaged over different signs of the shock, from different starting (or base) durations, quarters in which the forward guidance shock occurs and even different histories. We select a base duration, a quarter of the fixed interest rate regime and compute generalised impulse responses conditional on the history of the observed variables. These are the difference between the forecast paths of variables with and without the forward guidance shock, that is

$$\text{GIRF}(x_{t+n}) = \mathbb{E}(x_{t+n}|\varepsilon_t^{\text{fg}}, \hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) - \mathbb{E}(x_{t+n}|\hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T})$$

where $\hat{x}_{t-1|T}$ and $\hat{\varepsilon}_{t|T}$ are smoothed estimates of the state and shocks. Notice that an expansionary forward guidance shock implies a fall in the GIRF of the policy rate because the forecast for the policy rate under the forward guidance shock is lower than otherwise. In other words, $\mathbb{E}(r_{t+n}|\varepsilon_t^{\text{fg}}, \hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) \leq \mathbb{E}(r_{t+n}|\hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T})$. Until there is lift-off, $\mathbb{E}(r_{t+n}|\varepsilon_t^{\text{fg}}, \hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) = \mathbb{E}(r_{t+n}|\hat{x}_{t-1|T}, \hat{\varepsilon}_{t|T}) = \bar{r}$, and an expansionary forward guidance shock has no impact on the policy rate.

We explore state dependence in the next section, but to get a sense of the magnitudes and dynamics involved, we first compare impulse responses to a conventional policy shock with those to a forward guidance shock. We do so by taking draws from the posterior and keeping those draws for which the duration in 2011Q3 is 5 quarters. From that base duration of 5 quarters, we consider a 2 quarter forward guidance shock in 2011Q3. The forward guidance shock amounts to an extension of 2 quarters to the estimated duration, which at the median draw is 7 quarters. We choose 2 a quarter forward guidance shock because it is the mean across the sample for forward guidance shocks in the US. The online appendix contains additional GIRFs assessing the sensitivity to different base durations and quarters.

Figure 7 compares impulse responses of a one standard deviation conventional expansionary monetary policy (about 50 annual basis points) with the impulse response to a 2 quarter forward guidance shock.

An expansionary forward guidance shock implies the same qualitative responses as a conventional shock: it increases output and inflation in the US and appreciates the Canadian real exchange rate which leads to a decrease in domestic output and inflation in Canada. At the mean of the draws notice that a forward guidance shock implies responses which are between 2 or 3 times larger than a conventional shock and, in the case of output, more persistent responses.[24]

6.3 State Dependence of Forward Guidance Shocks

Figure 7 also highlights that there is considerably more variability in the magnitude of the responses to forward guidance shocks. We find that the state of the US preference shock, ξ_t^*, is an important determinant of the size of the responses; it can amplify or diminish the impact of a given forward guidance shock. To see this, take a simpler version of the Euler equation for the

[24] Consistent with our findings, Gertler and Karadi (2015) provide VAR evidence on forward guidance shocks which also points to forward guidance shocks having larger effects than conventional monetary policy shocks.

US, which is the only equation apart from the foreign monetary policy rule where r_t^* enters

$$y_t^* = \mathbb{E}_t y_{t+1}^* - \left(r_t^* - \mathbb{E}_t \pi_{t+1}^* - (1-\rho_\xi^*)\xi_t^*\right) \tag{38}$$

Iterating this equation forward reveals that it is the expected path of the last term on the right hand side, $(r_t^* - \mathbb{E}_t \pi_{t+1}^* - (1-\rho_\xi^*)\xi_t^*)$, that matters for output. This explains why the impact of a change in duration depends on the state of the risk premium shock. A large, negative and persistent ξ_t^* offsets the potentially expansionary impact of extending the duration. Conversely, a small, positive and persistent ξ_t^* can amplify the expansionary impact of an extension of the expected duration. This explains why one may find estimates of the duration consistent with survey measures which do not give rise to implausibly large responses of aggregate variables as for example is the case in Figure 1 of Carlstrom et al. (2015) for which the natural rate shock process is fixed.

It is important to recognize, however, that the state dependence of forward guidance shocks does not imply that further extensions in the duration will not eventually lead to implausibly large responses of aggregate variables, consistent with the *forward guidance puzzle* identified by Carlstrom et al. (2015) and Del Negro et al. (2012).

To assess the sensitivity of our results, we also estimate a version of the model with discounting in the Euler equation, as in McKay et al. (2017), a version immune to the forward guidance puzzle. We find that adding discounting in the Euler equation leads to somewhat larger durations and forward guidance shocks. This is perhaps not surprising as in estimation the data is fixed across specifications, and so muting the impact of forward guidance leads to larger estimated forward guidance shocks. The estimates under discounting in the Euler equation can be found in the online appendix.

Figure 8 plots the impulse responses for the US and Canada to an expansionary forward guidance shock of 2 quarters for two sets of different draws. In one case, we take those draws for which the standard deviation of the US preference shock, ξ_t^*, is smaller than 0.35. And in the other case, we take only those draws for which the standard deviation of the US preference shock, ξ_t^*, is higher than 0.55.[25] We further condition on draws for which the duration in 2012Q1 is 8 quarters.

For each draw we obtained smoothed estimates of the shocks and states and use these to compute generalized impulse responses for output, inflation and the real exchange rate. A 2 quarter forward guidance shock in the US leads to mild spillovers on Canadian output (a peak response of -0.15 % at mean) when the state of the US preference shock is very negative. On the other hand, when the state of the US preference shock is benign, the spillovers from US forward guidance can be up to two times as large (a peak response of -0.3 % at the mean).

6.4 Counterfactual Scenarios

Next, we use the estimated model to construct a counterfactual in which we remove forward guidance in both the US and Canada, to see the impact of forward guidance policies. To construct the counterfactual, we set $\mathbf{d}_t^{\text{fg}} = 0$ and thus effectively solve the model under the occasionally binding lower bound constraint given the estimated structural shocks.

In Figure 9, we plot, for the US and Canada, the change in output and inflation using 20 randomly chosen draws from the posterior distribution, and compare it to observed output and inflation. We find that, absent forward guidance in the US and Canada, the average cumulative decline in output in the US would be about 37%, suggesting that forward guidance was stimulatory. Forward guidance in Canada was also stimulatory for most draws. We find that removing forward guidance in both the US and Canada would, on average, reduce Canadian output by about 29% in cumulative terms. For inflation, in the US we find that forward guidance increased the rate of inflation. In the case of Canada, we find that forward guidance also increased inflation for most draws, with larger variation around the observed path. As is clear from our estimates, the joint monetary stimulus was on average positive for both countries.

[25] The mode of the standard deviation of ξ_t^* is 0.42.

7 Conclusions

In this paper, we estimate a two-country small open economy model on US and Canadian data accounting for fixed interest rates from 2009Q1 onwards. We propose an identification of forward guidance using the estimated model and an occasionally binding constraint solution. In estimation, we use a sufficiently wide prior over the trade elasticity to accommodate both signs of the responses of domestic output to foreign monetary policy. According to our estimated model expenditure-switching effects are stronger than expenditure-augmenting effects across the posterior range, so expansionary US monetary policy is found to be mildly contractionary for Canada. But, as in Mundell-Fleming type models, we find using counterfactuals that although expansionary US monetary policy is ‘beggar-thy-neighbor’, in aggregate the US and Canada are both better off by jointly responding with expansionary monetary policy to the large contractionary shocks that took place during the Great Recession.

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Table 1: Estimated Structural Parameters

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
				US				
h^*	B	0.7	0.6	0.8	0.74	0.73	0.66	0.79
θ_p^*	B	0.7	0.7	0.8	0.76	0.76	0.73	0.79
θ_w^*	B	0.7	0.7	0.8	0.78	0.78	0.75	0.81
ρ_r^*	B	0.5	0.2	0.8	0.89	0.89	0.86	0.91
ϕ_π^*	N	2.0	1.7	2.3	1.76	1.75	1.49	2.04
ϕ_g^*	G	0.5	0.3	0.7	0.09	0.10	0.08	0.12
ϕ_y^*	G	0.5	0.3	0.7	0.08	0.09	0.07	0.11
c_8^*	N	0.3	0.1	0.8	0.08	0.10	0.05	0.15
χ_p^*	B	0.1	0.04	0.2	0.03	0.04	0.02	0.07
χ_w^*	B	0.1	0.04	0.2	0.08	0.11	0.05	0.18
				Canada				
h	B	0.7	0.6	0.8	0.81	0.81	0.78	0.85
τ	N	1.0	0.4	1.6	3.06	3.04	2.74	3.36
θ_p	B	0.7	0.7	0.8	0.79	0.79	0.76	0.82
θ_w	B	0.7	0.7	0.8	0.78	0.78	0.74	0.81
θ_x	B	0.7	0.7	0.8	0.78	0.78	0.75	0.80
θ_F	B	0.7	0.7	0.8	0.77	0.77	0.74	0.80
ρ_r	B	0.5	0.2	0.8	0.91	0.91	0.89	0.93
ϕ_π	N	2.0	1.7	2.3	2.20	2.17	1.88	2.46
ϕ_g	G	0.5	0.3	0.7	0.11	0.11	0.09	0.14
ϕ_y	G	0.5	0.3	0.7	0.17	0.17	0.13	0.24
c_8	N	0.3	0.1	0.9	0.12	0.12	0.06	0.18
χ_p	B	0.1	0.04	0.2	0.04	0.06	0.03	0.10
χ_w	B	0.1	0.04	0.2	0.07	0.08	0.03	0.14

Table 2: Estimated Parameters, Exogenous Processes

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
				US				
ρ_{ξ}^{*}	B	0.5	0.2	0.8	0.95	0.95	0.93	0.96
ρ_{g}^{*}	B	0.5	0.2	0.8	0.95	0.95	0.93	0.97
$\rho_{\xi_p}^{*}$	B	0.5	0.2	0.8	0.99	0.98	0.97	0.99
$\rho_{\xi_w}^{*}$	B	0.5	0.2	0.8	0.80	0.78	0.67	0.86
ρ_{tp}^{*}	B	0.5	0.2	0.8	0.73	0.73	0.64	0.82
$100 \times \sigma_z$	IG	0.3	0.1	2.6	0.11	0.11	0.09	0.14
$100 \times \sigma_r^{*}$	IG	0.3	0.1	0.6	0.11	0.11	0.10	0.12
$100 \times \sigma_{\xi}^{*}$	IG	0.3	0.1	0.7	0.25	0.28	0.21	0.41
$10 \times \sigma_g^{*}$	IG	0.3	0.1	0.7	0.13	0.13	0.12	0.14
$100 \times \sigma_{\xi_p}^{*}$	IG	0.1	0.1	0.3	0.15	0.15	0.13	0.17
$100 \times \sigma_{\xi_w}^{*}$	IG	0.3	0.1	0.9	0.11	0.11	0.08	0.13
$100 \times \sigma_{r,8}^{*}$	IG	0.3	0.1	0.9	0.09	0.09	0.09	0.10
				Canada				
ρ_{rp}	B	0.5	0.2	0.8	0.96	0.95	0.91	0.97
ρ_{ξ}	B	0.5	0.2	0.8	0.64	0.65	0.52	0.79
ρ_{g}	B	0.5	0.2	0.8	0.93	0.93	0.89	0.95
ρ_{ξ_H}	B	0.5	0.2	0.8	0.85	0.83	0.73	0.90
ρ_{ξ_w}	B	0.5	0.2	0.8	0.29	0.27	0.16	0.38
ρ_{ξ_X}	B	0.5	0.2	0.8	0.91	0.91	0.87	0.93
ρ_{ξ_F}	B	0.5	0.2	0.8	0.96	0.95	0.92	0.98
ρ_{tp}	B	0.5	0.2	0.8	0.74	0.72	0.62	0.82
$100 \times \sigma_r$	IG	0.3	0.1	0.7	0.17	0.17	0.15	0.19
$100 \times \sigma_{rp}$	IG	0.1	0.1	0.2	0.31	0.32	0.26	0.43
$10 \times \sigma_g$	IG	0.3	0.1	0.9	0.14	0.14	0.13	0.15
$100 \times \sigma_{\xi}$	IG	0.3	0.1	0.9	0.24	0.25	0.22	0.29
$100 \times \sigma_{\xi_H}$	IG	0.2	0.1	0.3	0.37	0.36	0.32	0.40
$100 \times \sigma_{\xi_w}$	IG	0.3	0.1	0.8	0.50	0.50	0.45	0.55
$100 \times \sigma_{\xi_X}$	IG	0.3	0.1	0.9	1.29	1.32	1.15	1.53
$100 \times \sigma_{\xi_F}$	IG	0.1	0.1	0.2	1.04	1.05	0.89	1.24
$100 \times \sigma_{r,8}$	IG	0.3	0.1	0.7	0.12	0.12	0.11	0.13

Table 3: Variance Decomposition Due to Shocks, %

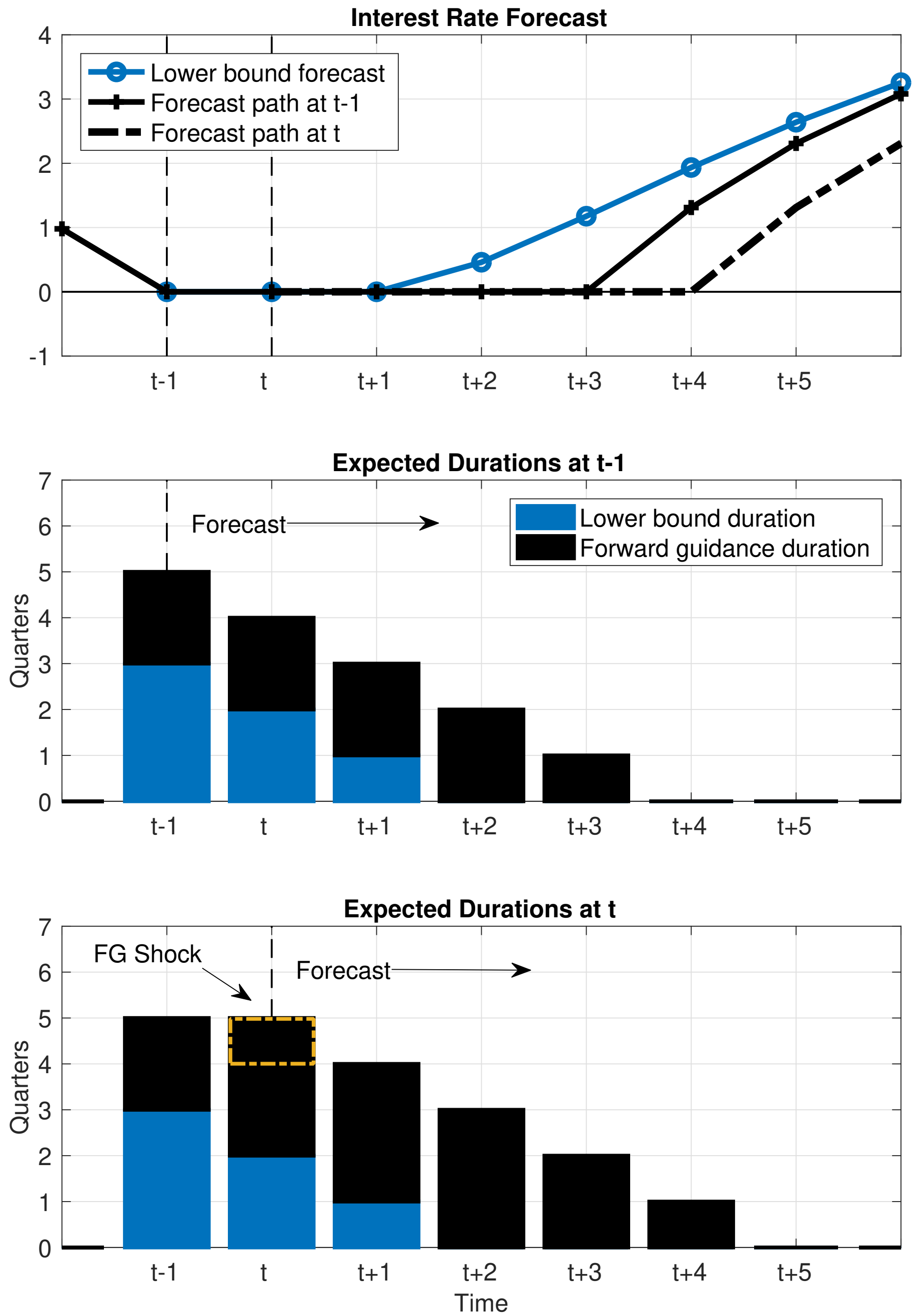
Variable \ Shock	Common	US Shocks					Canadian Shocks							
	Prod.	Pref.	Policy	Demand	Price	Wage	Pref.	Policy	Demand	Risk Pr.	Price	Wage	Exports	Imports
A. US Variables														
Policy Rate	0.1	64.1	14.6	4.4	8.9	7.9								
2Y Interest Rate	0.1	65.3	4.5	2.8	8.3	7.2								
Output Growth	1.4	37.9	3.3	52.9	2.1	2.4								
Consumption Growth	0.8	78.1	6.8	5.1	4.3	4.9								
Inflation	1.8	10.5	4.7	0.2	44.3	38.5								
Wage Growth	17.6	0.1	0.4	0.0	66.1	15.8								
B. Canadian Variables														
Policy Rate	0.0	4.8	0.5	0.4	0.3	0.5	2.5	10.1	0.8	58.9	2.0	1.1	8.0	10.2
2Y Interest Rate	0.0	4.9	0.3	0.2	0.2	0.6	1.7	2.4	0.6	59.0	1.0	0.8	7.6	10.0
Output Growth	0.5	2.2	0.7	2.0	0.2	0.2	10.1	8.8	8.2	25.7	12.4	2.1	26.7	0.3
Consumption Growth	0.3	2.5	0.1	0.1	0.1	0.3	49.8	2.7	2.0	33.8	1.9	0.6	4.1	1.9
Inflation	0.4	0.6	0.2	0.0	0.0	0.1	0.0	5.0	0.1	12.7	45.7	10.5	4.5	20.1
Wage Growth	2.4	0.1	0.1	0.0	0.0	0.0	0.1	0.1	0.0	4.0	32.4	48.1	4.7	8.0
Imports Growth	0.0	0.9	0.5	0.0	0.1	0.1	4.3	0.3	21.5	25.8	3.6	0.4	11.2	31.3
Exports Growth	0.0	2.1	0.1	1.4	0.2	0.1	0.1	1.0	0.0	14.0	0.4	0.1	64.5	16.0
Nominal Ex Rate, Δ	0.0	0.6	2.6	0.1	0.4	0.3	0.5	6.2	0.0	43.1	0.9	0.3	22.4	22.5

Table 4: Response of Canadian Asset Prices to US Forward Guidance Announcements

	Canadian Government Bond Yields						Exchange Rate
	3-month	6-month	1-year	2-year	5-year	10-year	
Change in Forward Guidance	-0.06*** (0.02)	-0.08*** (0.03)	-0.14*** (0.04)	-0.33*** (0.06)	-0.61*** (0.08)	-0.60*** (0.10)	-0.06** (0.03)
Regression R^2	0.13	0.13	0.23	0.43	0.63	0.57	0.18

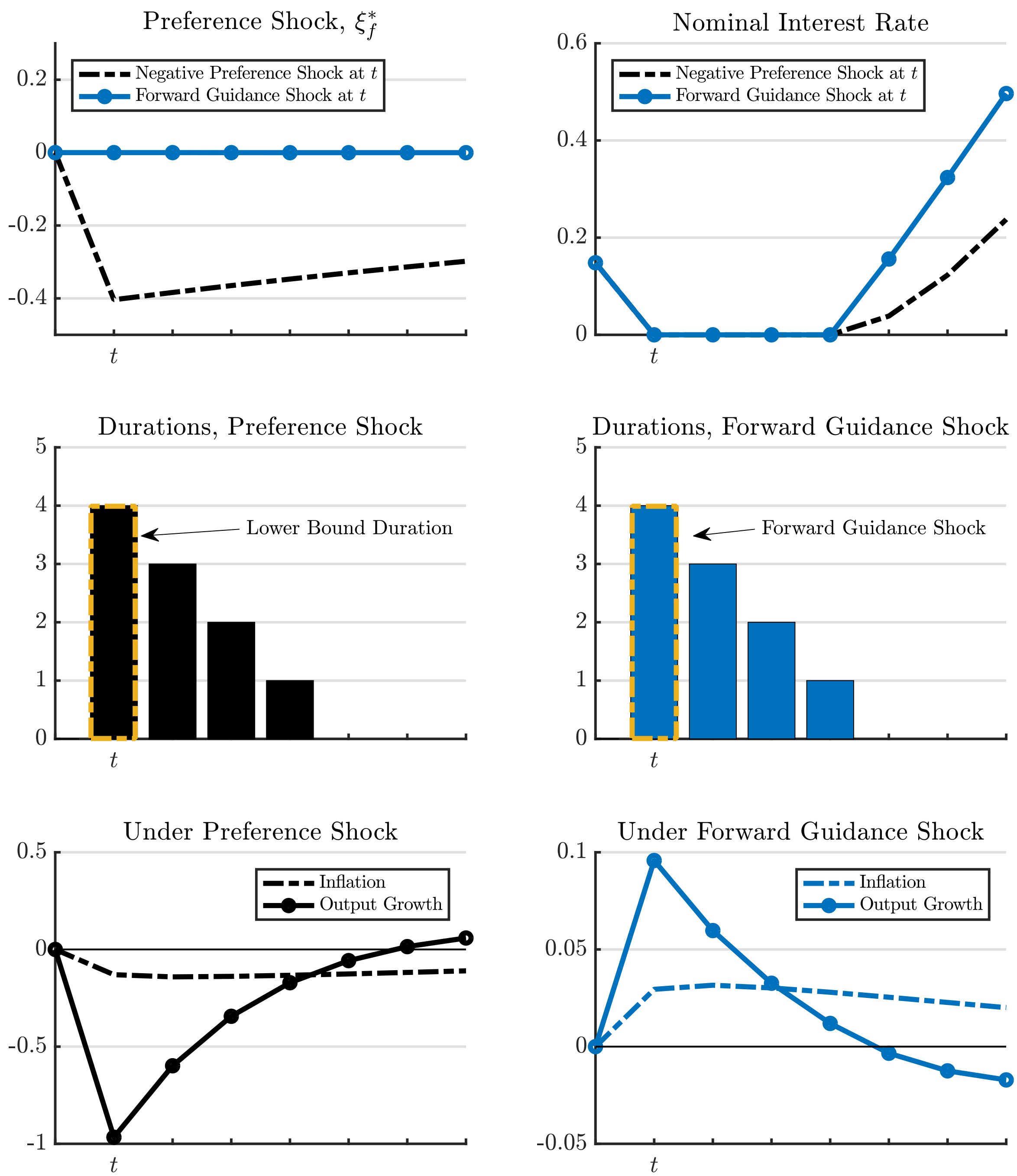
Note: Coefficients β from regressions $\Delta y_t = \alpha + \beta FG_t^{US} + \varepsilon_t$. Huber-White heteroskedasticity-consistent standard errors in parentheses. ***, ** and * denote statistical significance at the 1% 5% and 10% levels. See text for details.

Figure 1: Example of a Forward Guidance Shock at t



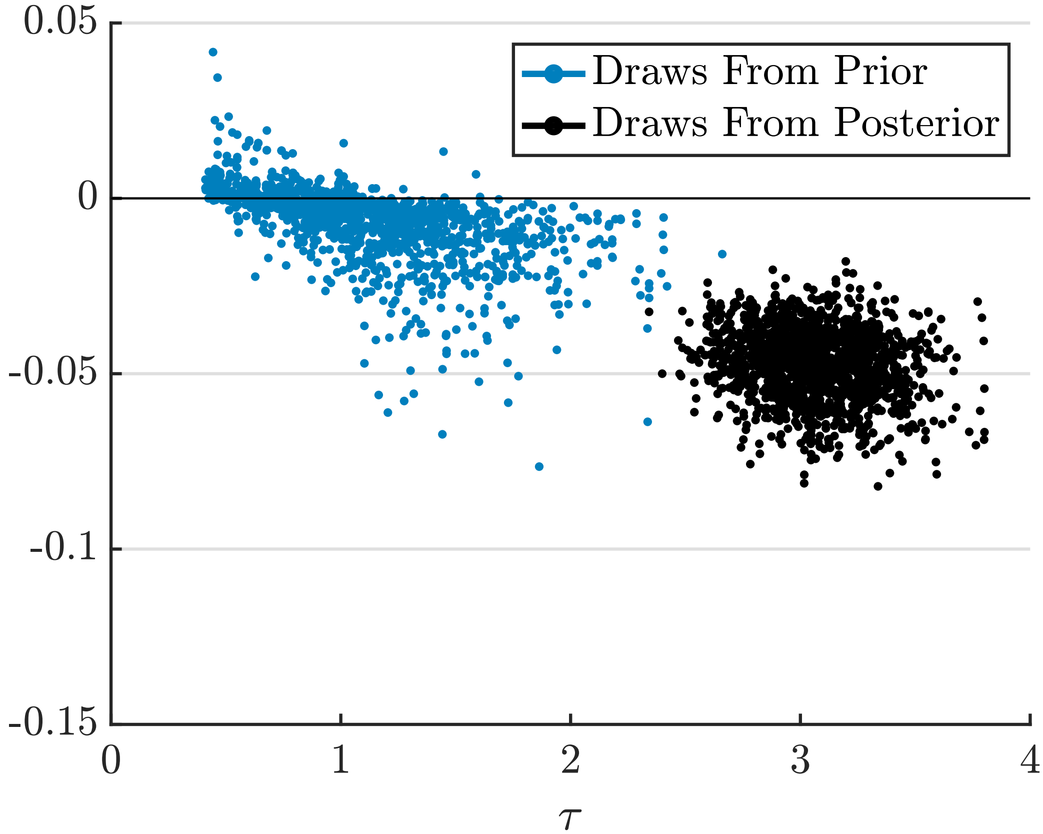
Note: This figure shows a stylized example of a forward guidance shock at period t. At period $t-1$, the expected duration of the fixed interest rate from period t is 4 quarters. Assume at period t that no further structural shocks arrive but policy unexpectedly commits to holding the interest rate fixed for an additional quarter. The total duration of the fixed interest rate regime increases to 5 quarters in period t, with the increase due to the positive forward guidance shock.

Figure 2: Identification of Forward Guidance Shocks



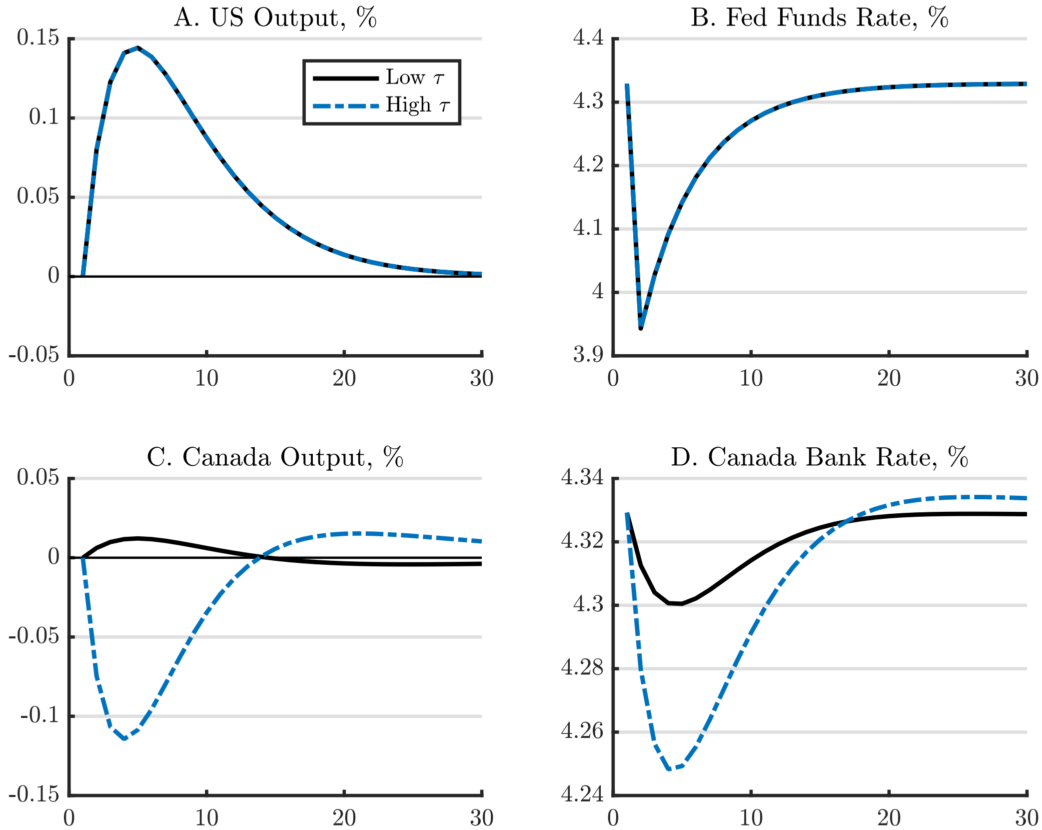
Note: This figure illustrates how forward guidance shocks are identified. Two paths are constructed using the estimated model. In the path in black, the economy is hit with a negative preference shock at period t which causes inflation and output growth to fall and the policy rate to be fixed at its lower bound for four quarters before liftoff. In the blue path, the economy is subject to a forward guidance shock of four quarters which causes inflation and output growth to increase. In both cases, the interest rate is expected to be fixed for four quarters, and aggregate data is used to identify forward guidance shocks from structural shocks that cause the lower bound to bind.

Figure 3: Canada Output Response to Expansionary US Monetary Policy Shock, %



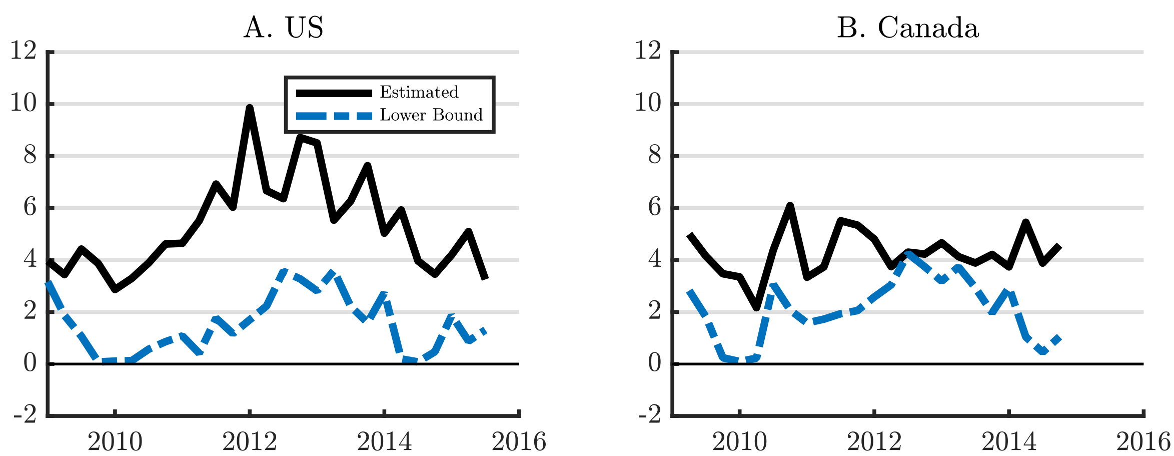
Note: This figure shows, on the vertical axis, the initial response of Canadian output to a one standard-deviation expansionary US monetary policy shock for random draws from the prior and posterior distributions. The horizontal axis plots the value of τ, the trade elasticity, for each draw.

Figure 4: Impulse Response to Expansionary US Monetary Policy Shock



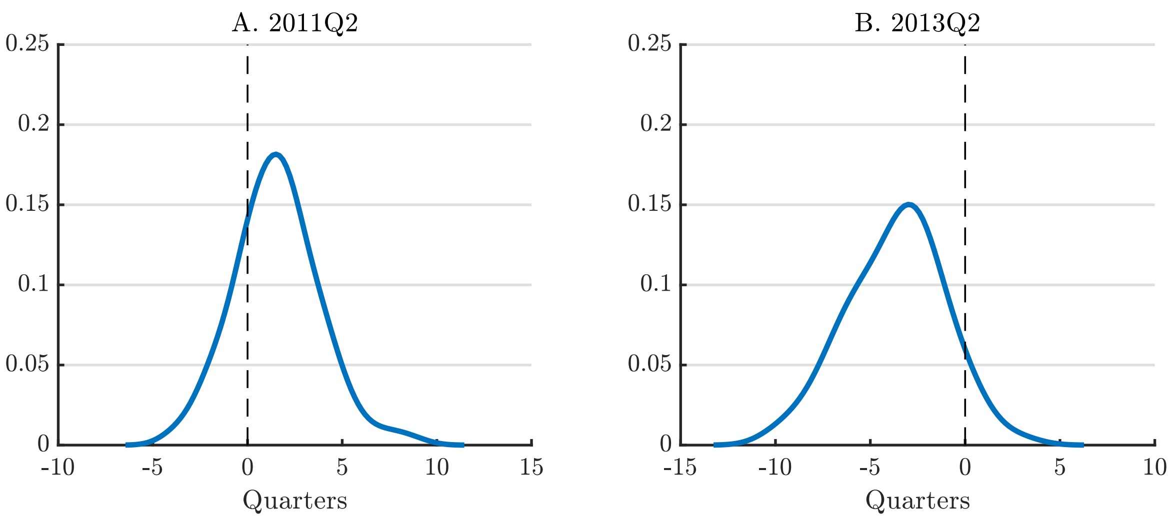
Note: This figure shows the response of Canadian variables to an expansionary US monetary policy shock for a high and low value of τ, with all other parameters set to the mode of the posterior distribution.

Figure 5: Fixed Interest Rate Duration and Forward Guidance, Mean Across Draws



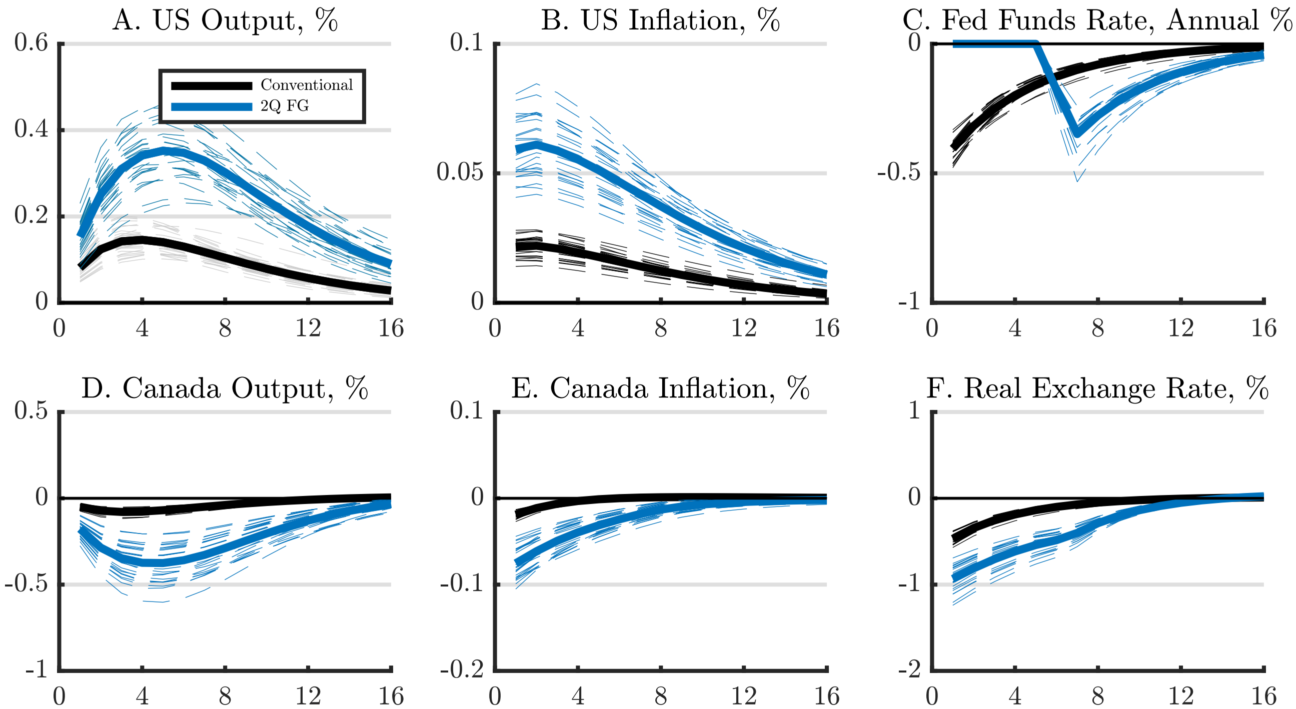
Note: This figure plots the mean of the posterior distribution of fixed interest rate durations in black. The dashed blue line plots the mean of the lower bound durations following the decomposition of the fixed interest rate durations.

Figure 6: Posterior Distributions of US Forward Guidance Shocks



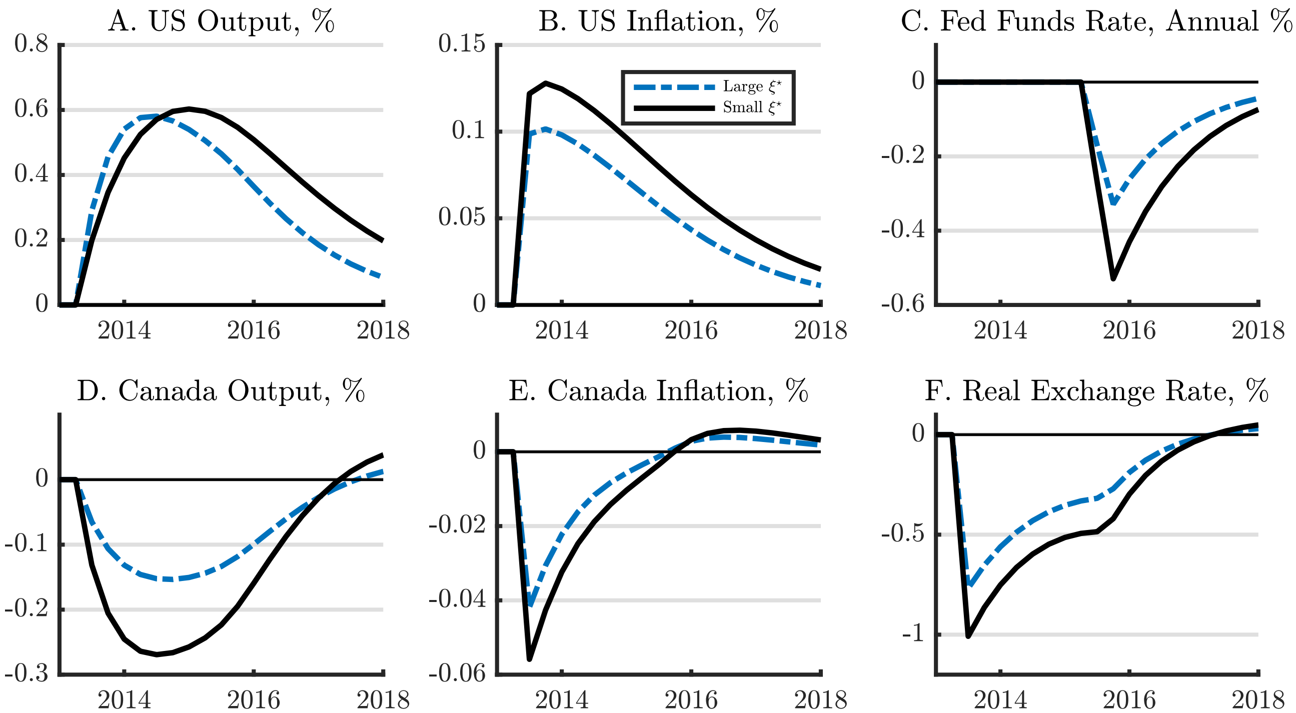
Note: This figure plots the distribution of our estimated forward guidance shocks for two dates. In the left panel, forward guidance shocks are on average 2 quarters for 2011Q2, the period when the Federal Reserve announced explicit calendar-based forward guidance. In the right panel, the distribution of forward guidance shocks is on average -3 quarters for 2013Q2, the period of the 'taper tantrum' following the Federal Reserve's announced slowdown of the quantitative easing program.

Figure 7: IRF of Conventional US Policy Shock and GIRF of 2Q US Forward Guidance Shock



Note: This figure plots the impulse response function of a one-standard deviation conventional monetary policy shock and the generalized impulse response to a two quarter forward guidance shock computed at randomly chosen posterior draws in 2011Q3. A two quarter forward guidance shock is chosen as it is close to the average forward guidance shock across the period of fixed interest rates. The forward guidance shock lowers the path of the policy interest rate, so that the response of the federal funds rate is negative beyond the period of fixed interest rates.

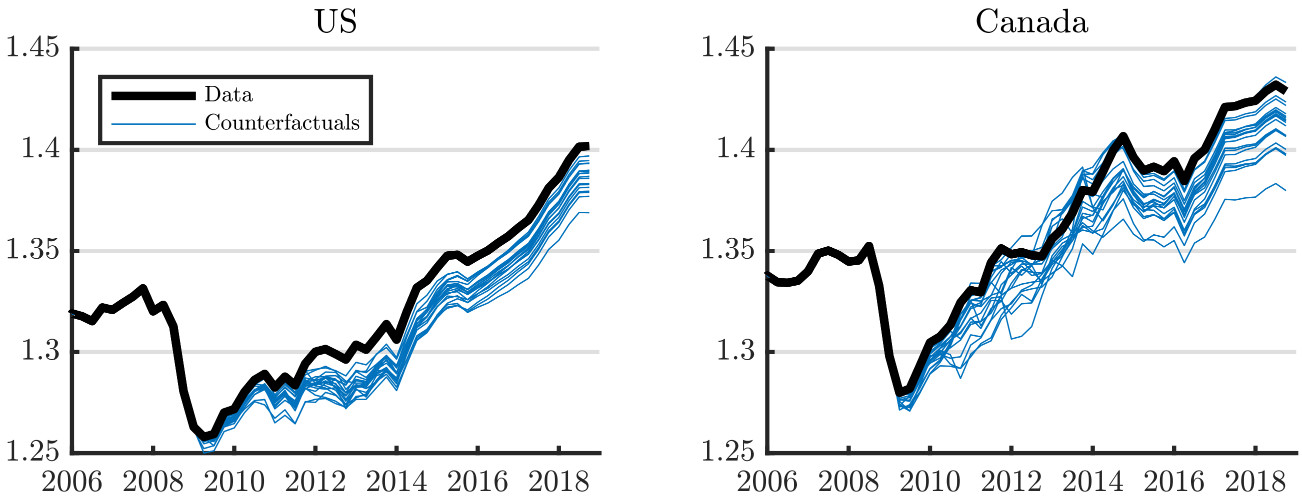
Figure 8: State Dependent Spillovers of US Forward Guidance Shock



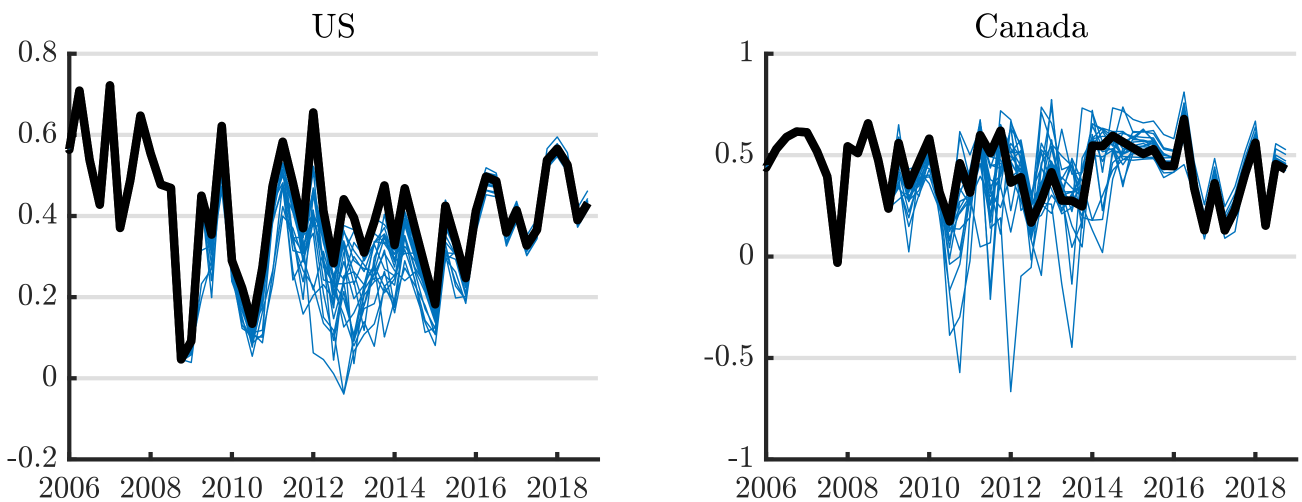
Note: This figure shows generalized impulse responses in 2013Q3 to a three quarter forward guidance shock in the US for draws of the posterior distribution that have high variance of the US preference shock, in blue, and draws that have low variance of the US preference shock, in black. The posterior draws are restricted to those with an expected duration in the US of 8 quarters. When the US demand process is small, a US forward guidance shock causes a more persistent output boom and higher inflation in the US, and generates larger spillovers on the Canadian economy.

Figure 9: Counterfactual Paths With No Forward Guidance in US or Canada

(a) Output



(b) Inflation



Note: This figure plots the path of output and inflation in the US and Canada in the data (in black) and in counterfactuals computed using 20 randomly chosen draws from the posterior distribution and where forward guidance shocks in both countries are removed (in blue).