Aging, Secular Stagnation and the Business Cycle

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ECB

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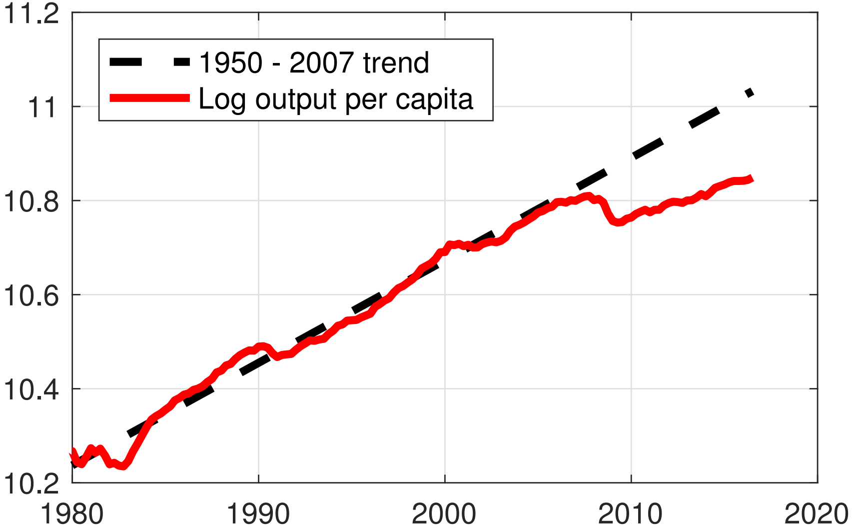
US Output Per Capita and its Long-Run Trend



Longer View

EU GDP

I Study Two Explanations For The Gap

1. Aging of the population

2. Constrained monetary policy

Explanation 1: Aging of the US Population

- Labor force participation declines with age
  - Lower employment growth
- Savings rates decline with age
  - Lower investment growth
- Older workers accumulate human capital slower
  - Lower productivity growth

Data

I Study Two Explanations For The Gap

1. Aging of the population
2. Constrained monetary policy

Explanation 2: Constrained Monetary Policy

- Financial factors reduced the equilibrium interest rate
- Fed can offset this by reducing the nominal rate
- But the Fed Funds rate was at the ZLB from 2009
- Zero lower bound, shocks and nominal frictions $\Rightarrow$ output falls

This Paper

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- Develop and estimate a model featuring:
  - Overlapping generations of workers/retirees
  - Nominal rigidities and the zero lower bound
  - Shocks to productivity, discounts, investment, markups, policy

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- Ask, quantitatively, what explains the gap in output from trend
- Develop and estimate a model featuring:
  - Overlapping generations of workers/retirees
  - Nominal rigidities and the zero lower bound
  - Shocks to productivity, discounts, investment, markups, policy
- ZLB causes nonlinearities
  - Aging causes interest rates to decline
  - Declining interest rate: ZLB more likely
  - Develop a solution method to handle this

Main Findings

- 12% gap between output per capita and its long-run trend in 2015
  - 4% due to aging of the population alone
- In addition, without aging from 1984:
  - ZLB would not have been a binding constraint
- Fed enacted stimulatory forward guidance policy
  - Without it, output falls by a further 2% between 2011 - 2013

Outline

- New Keynesian lifecycle model with ZLB
- Solution
- Estimation
- Results

New Keynesian Lifecycle Model with ZLB

Overview

1. Overlapping generations
    - Individuals of age s consume, work, and save
2. Representative firm
    - Firm produces with capital and labor
3. Real and nominal rigidities $\Rightarrow$ monetary policy + ZLB
4. Aggregate shocks

Demographics

- n_t^s: number alive at period t of age $s \in [0, S]$
- Fertility: n_t^0 new people born at t
- Mortality: fraction γ_t^s of those age s die from t to $t+1$

$$
\begin{aligned}
n_{t+1}^{1} &= (1-\gamma_t^0)n_t^0 \\
&\vdots \\
n_{t+1}^{s+1} &= (1-\gamma_t^s)n_t^s \\
&\vdots \\
n_{t+1}^{S+1} &= 0
\end{aligned}
$$

- Individuals' unintentional bequests redistributed to surviving peers

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Individuals

- An individual of age s chooses consumption c_t^s, labor ℓ_t^s, capital k_t^s, bonds b_t^s:

$$\max_{\{c_\tau^s, \ell_\tau^s, a_\tau^s\}} \sum_{\tau=0}^{S} \chi_t \beta^s \prod_{j=0}^{\tau} (1 - \gamma_j^s) \left[\frac{(c_t^s)^{1-\sigma}}{1-\sigma} + \upsilon^s \frac{(\ell_t^s)^{1+\varphi}}{1+\varphi} \right]$$

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subject to:

$$c_t^s + k_t^s + \frac{\phi_k}{2}\left(\frac{k_t^s}{k_{t-1}^{s-1}}\kappa_t - 1\right)^2 k_{t-1}^{s-1} + \frac{b_t^s}{p_t R_t} =$$

$$z^s w_t \ell_t^s (1-\tau_t) + \xi_t^s + \frac{b_{t-1}^{s-1}}{p_t} + (r_t + 1 - \delta)k_{t-1}^{s-1} + d_t^s$$

Last period of life:

$$c_{t+S}^s = (1+r_{t+S})k_{t+S-1}^{s-1} + \frac{1}{p_t} b_{t+S-1}^{s-1} + d_{t+S}^s$$

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- Born with zero wealth:

$$k_t^0 = 0$$

Firms

- Intermediate goods-producing firms:
  - Produce differentiated good $y_t(i)$ with elasticity of substitution ξ_t

$$y_t(i) = \mu_t \left(k_{t-1}(i)\right)^{\alpha} \left(\ell_t(i)\right)^{1-\alpha}$$

  - Aggregates: $k_t = \sum_s^S n_t^s k_t^s$, and $\ell_t = \sum_s^S z^s n_t^s \ell_t^s$
  - Quadratic costs of price adjustment, parameterized by ϕ_p

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- Price-setting gives rise to a Philips curve:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(\xi - 1)}{\phi_p} \widehat{\text{mc}}_t + \hat{\xi}_t$$

- Cost-minimization gives factor prices:

$$w_t = \text{mc}_t (1 - \alpha) \frac{y_t(i)}{\ell_t(i)}, \qquad r_t = \text{mc}_t \alpha \frac{y_t(i)}{k_{t-1}(i)}$$

Monetary Policy

- Monetary policy operates in two possible regimes:

  1. Standard Taylor rule regime

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_r} \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{y_t}{y_t^{\mathrm{F}}}\right)^{\phi_y} \left(\frac{y_t/y_{t-1}}{y_t^{\mathrm{F}}/y_{t-1}^{\mathrm{F}}}\right)^{\phi_g} \varepsilon_{R,t}$$

  2. Zero lower bound regime:

$$\log R_t = 0$$

- ZLB can bind because of shocks or forward guidance

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  2. Zero lower bound regime:

$$\log R_t = 0$$

- ZLB can bind because of shocks or forward guidance
  - Forward guidance: extension of the duration of zero interest rates
  - Account for forward guidance by using observed ZLB durations

Government

- Taxes labor income to fund a PAYG social security system
- Tax rate adjusts to balance the government budget
- Levies a lump-sum tax to pay for exogenous expenditures g_t
- Resource constraint:

$$y_t = c_t + g_t + k_t - (1-\delta)k_{t-1} + \text{Adjustment Costs}_t$$

Six Aggregate Shocks

- Productivity
- Discount factor
- Markups
- Investment adjustment costs
- Exogenous government spending
- Monetary policy rule

Solution and Approximation

An Accurate Approximation

- The model's size makes it difficult to solve, impossible to estimate
- Argue that dynamics are well approximated by an alternative setup
- Alternative setup: representative agent derived from:
  - Households are born at time 0 and can trade
  - They value consumption and leisure only when alive
- Can aggregate alternative to representative household problem
- Much quicker to solve / allows estimation

Aggregation

Why Is This Aggregation Possible?

- Unintentional bequests redistributed to the same generation
- This eliminates mortality risk
- And so in the decentralized equilibrium, the individual i sets:

$$\lambda_t^i = \beta(1 + r_{t+1})\lambda_{t+1}^i$$

- So, for two individuals i, j over periods t, t':

$$\frac{\lambda_t^i}{\lambda_t^j} = \frac{\lambda_{t'}^i}{\lambda_{t'}^j}$$

Alternative Model ⇒ A Representative HH

- Representative agent utility function:

$$\max_{c_t,h_t,k_t} \sum_t \beta^t \left(\phi_t \frac{c_t^{1-\sigma}}{1-\sigma} - v_t \frac{h_t^{1+\varphi}}{1+\varphi} \right)$$

- Production function:

$$y_t = \theta_t k_{t-1}^{\alpha} h_t^{1-\alpha}$$

- Where ϕ_t, v_t and θ_t are:

$$\phi_t = \left[\sum_{s=0}^{S} n_t^s (\mu^s)^{\frac{1}{\sigma}} \right]^{\sigma}, \quad v_t = \left[\sum_{s=0}^{S} n_t^s (\hat{z}^s)^{\frac{1}{\varphi}+1} (v^s \mu^s)^{-\frac{1}{\varphi}} \right]^{-\varphi}, \quad \theta_t = \sum_{s=0}^{S} n_t^s z^s$$

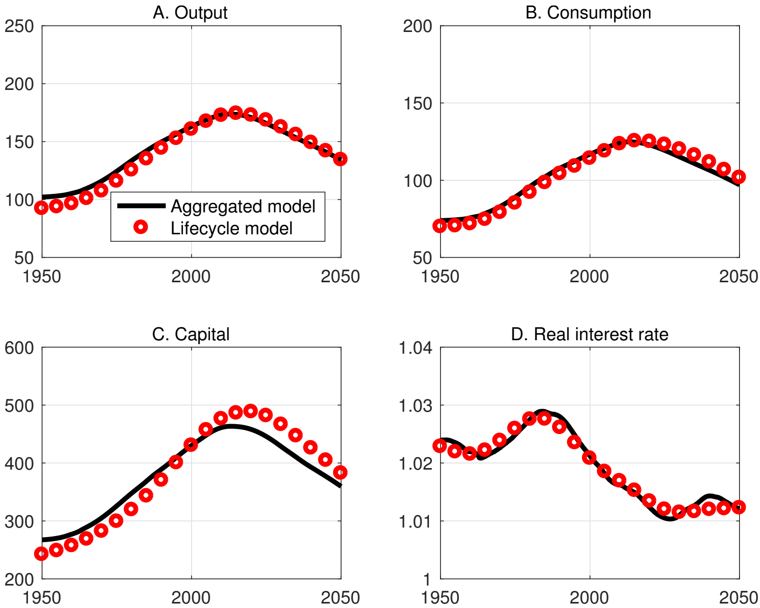
Overview of Aggregated Model

- Representative household
  - Demographics appear as time-varying parameters
- Firms:
  - Demographics affect labor productivity
- Monetary policy:
  - Monetary policy rule subject to zero lower bound

How Good Is The Approximation?

- Calibrate and assign parameters and demographic trends
- Compute perfect foresight solution under second-order approximation
- Compare trends in both models

Trends With Perfect Foresight



Estimation and Solution

Assigned/Calibrated Parameters

Table: Time period is quarterly

δ	10.6% pa	$\frac{\xi}{\xi-1}$	14%
α	1/3	π^*	2.2% pa
φ	2	ϕ_k	40
σ	2	S	80 years
ϕ_r	0.81	ϕ_y	0.08
ϕ_π	2.03	ϕ_g	0.22
v^s	*Lifecycle LFP rate*	γ_t^s	*Mortality profiles*
z^s	*Lifecycle earnings*	n_t^0	*Population data*

- $\beta = 0.99875$ calibrated to match capital-output ratio

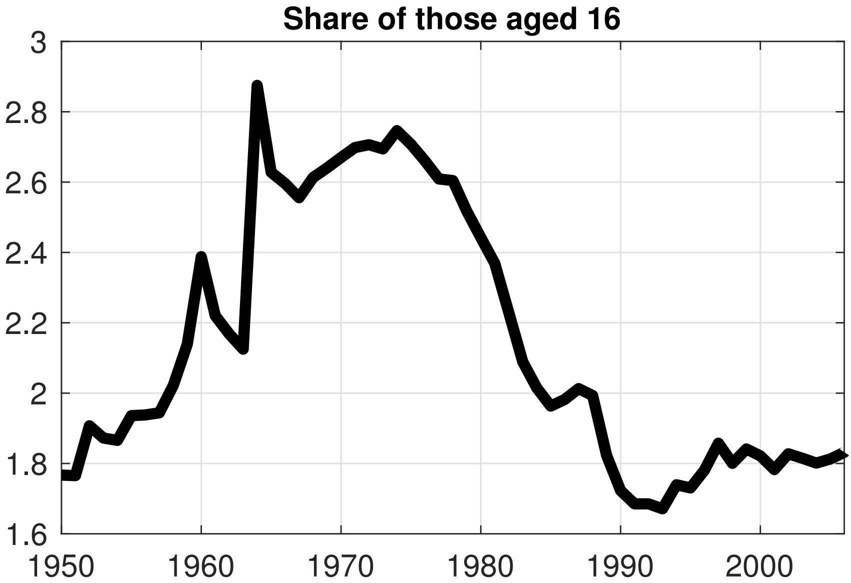
Calibrating Demographic Changes

1. Fertility: Entering cohort size n_t^0:
   - Chosen to match 16-year old share of population

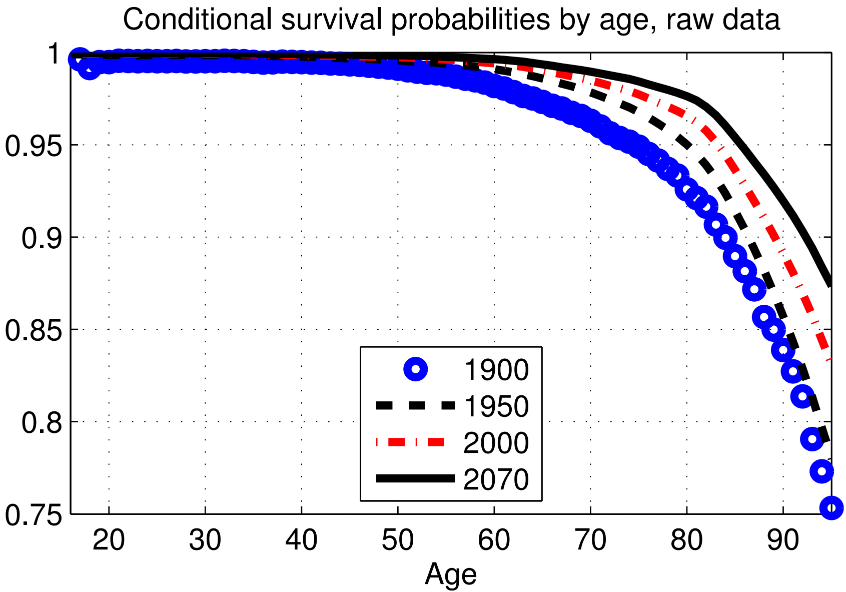
2. Mortality: Conditional mortality rates γ_t^s:
   - Actuarial life tables, 1900 to 2100. Dept of Social Security
   - Cohort-specific mortality rates

Time-Varying Demographic Parameters

Fertility: n_t^0 Matches 16 y/o Share of Population



Mortality: Survival $(1 - \gamma_t^s)$ Profile by Birth Year



Lifecycle Parameters

1. Age-productivity parameters z^s:

- Match experience-earnings curves from Census / ACS

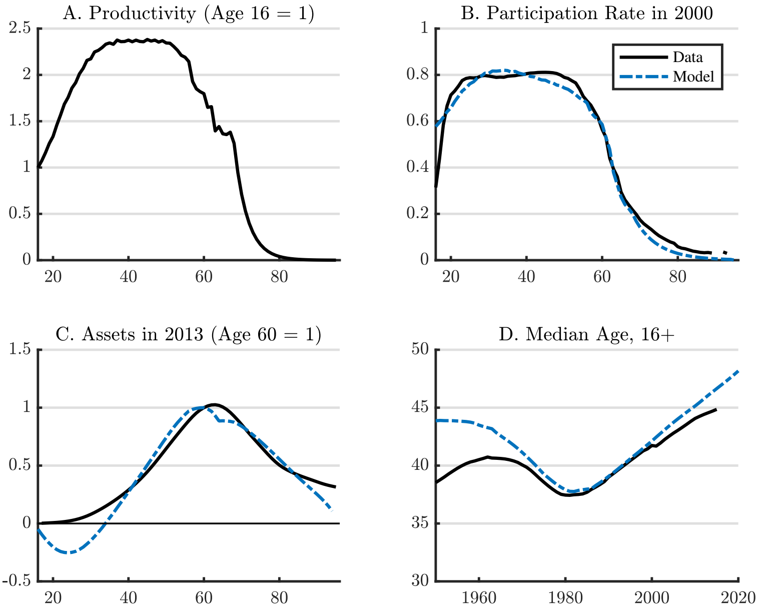
Productivity Profile

2. Labor disutility v^s:

- Match labor force participation by age in 2000

Labor Disutility Profile

Lifecycle Calibration: Model v US Data



Solution Method

- Two features in the computation:
  1. Anticipated changes to the model's parameters from demographics
  2. Zero lower bound
- Use piecewise-linear procedure:
  - Anticipated changes to parameters for demographic wedges
  - Regime-switching method for zero interest rates

Methodology: Piecewise-linear Procedure

- Approximation: time-varying parameters
- Linearize the model at each point in time:

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \mathbb{E}_t x_{t+1} + \mathbf{F}_t \varepsilon_t$$

- x_t: state vector, ε_t: vector of structural shocks
- Agents know the structure of the economy to T
- After T, the economic structure is time-invariant:

$$x_t = \mathbf{J} + \mathbf{Q} x_{t-1} + \mathbf{G} \varepsilon_t$$

Methodology: Anticipated Structural Changes

- I obtain a time-varying solution:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t \tag{1}$$

- Under rational expectations, get the recursion from t to T:

$$\mathbf{Q}_t = \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t$$

- With (1), a state space representation $\Rightarrow$ likelihood

The Zero Lower Bound

- Suppose at period t, the ZLB expected to bind at $\tau > t$
- Replace Taylor rule with $R_\tau = 0$ in linearized model:

$$\mathbf{A}_\tau^* x_\tau = \mathbf{C}_\tau^* + \mathbf{B}_\tau^* x_{\tau-1} + \mathbf{D}_\tau^* \mathbb{E}_t x_{\tau+1} + \mathbf{F}_\tau^* \varepsilon_\tau$$

- Use ZLB starred system in recursion to compute $\mathbf{J}_t, \mathbf{Q}_t, \mathbf{G}_t$

Example

Data: Quarterly Aggregate Time Series

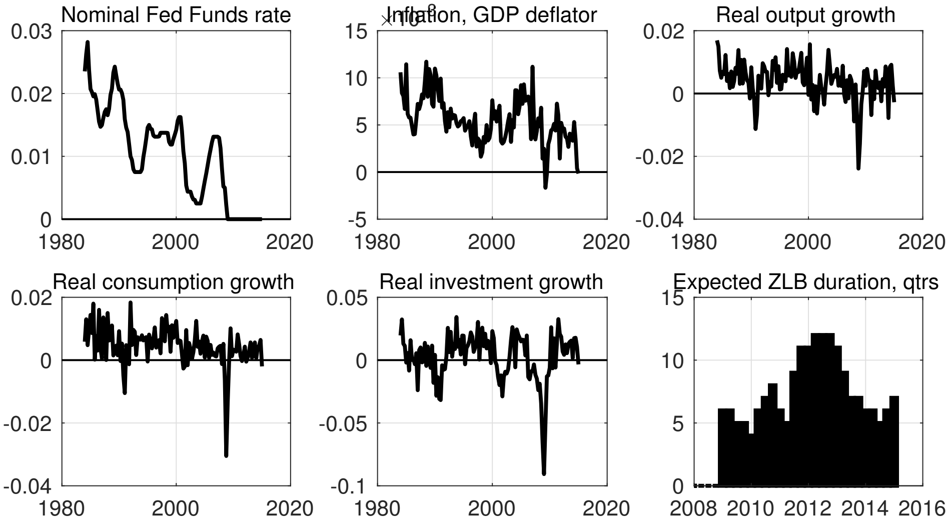


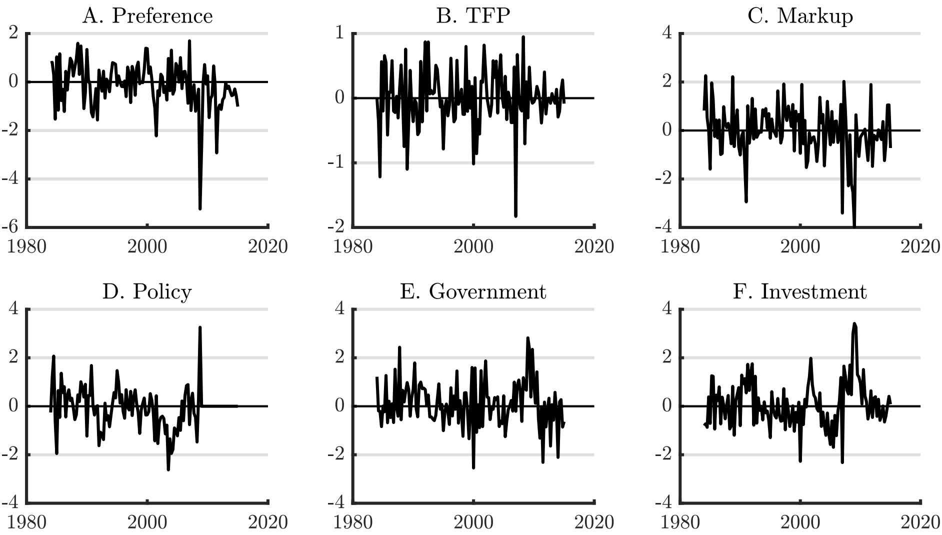
Table: Estimated Parameters

	Prior				Posterior			
Parameter	Dist	Median	10%	90%	Mode	Median	10%	90%
ϵ_p	U	0.1	0.0	0.2	0.01	0.01	0.01	0.02
$400 \times (z-1)$	N	2.0	1.1	3.0	1.34	1.35	1.24	1.46
ρ_χ	B	0.5	0.3	0.7	0.94	0.94	0.93	0.95
ρ_μ	B	0.5	0.3	0.7	0.76	0.73	0.55	0.83
ρ_θ	B	0.5	0.3	0.7	0.96	0.96	0.95	0.97
ρ_g	B	0.5	0.3	0.7	0.95	0.95	0.94	0.97
ρ_κ	B	0.5	0.3	0.7	0.94	0.94	0.93	0.95
$100 \times \sigma_\chi$	IG	1.2	0.5	3.7	2.15	2.18	1.95	2.47
$100 \times \sigma_\mu$	IG	1.2	0.5	3.7	0.48	0.52	0.33	0.73
$100 \times \sigma_\theta$	IG	1.2	0.5	3.7	3.51	3.56	3.20	4.03
$100 \times \sigma_i$	IG	1.2	0.5	3.7	0.16	0.16	0.15	0.18
$100 \times \sigma_g$	IG	1.2	0.5	3.7	1.05	1.06	0.98	1.15
$100 \times \sigma_\kappa$	IG	1.2	0.5	3.7	1.00	1.04	0.86	1.25

Table: Variance Decomposition Due to Shocks, %

Variable \ Shock	Pref	Tech	Markup	Policy	Gov	Inv
A. Conditional, 4 Quarter Ahead						
Fed Funds Rate	28.5	7.8	5.8	22.9	8.3	26.8
Inflation	21.9	2.4	42.1	8.9	6.4	18.4
Wages	6.8	2.6	74.8	10.5	0.6	4.8
Output	0.8	0.3	47.7	4.3	19.6	27.4
Consumption	28.0	0.7	26.7	6.8	26.7	11.2
Investment	7.2	0.1	38.8	2.3	0.8	50.8

Estimated Shocks



Results

Path of Variables, Observed

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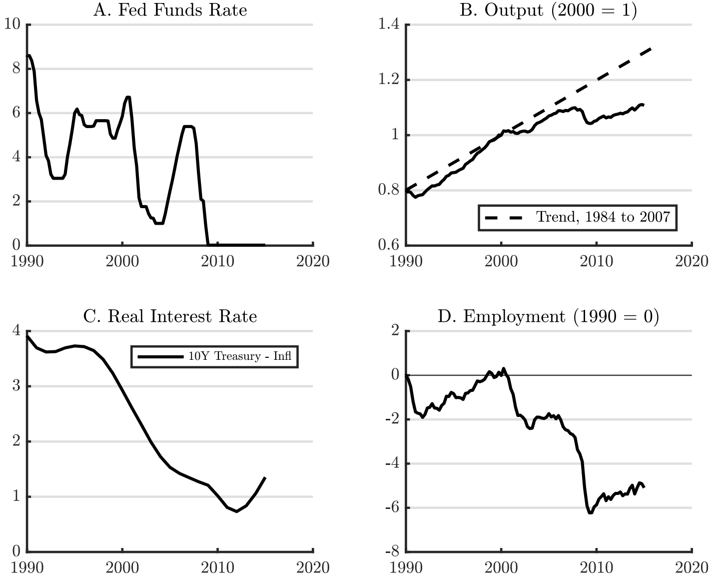

Path of Variables, Demographics Only

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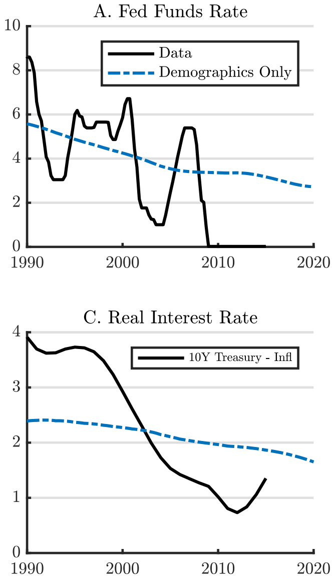


Demographic Trends

- Output / Employment: changes in k_{t-1} and labor

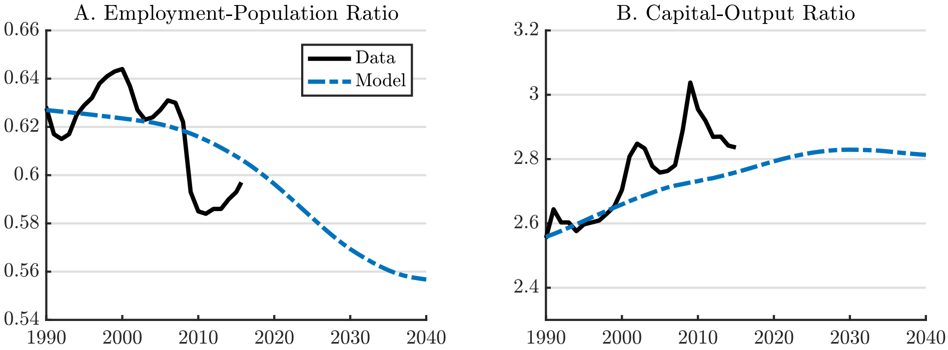
$$y_t = k_{t-1}^{\alpha} l_t^{1-\alpha}$$

- MPK:

$$r_t = \mathrm{mc}_t \alpha \frac{y_t}{k_{t-1}} = \mathrm{mc}_t \alpha \left(\frac{l_t}{k_{t-1}} \right)^{1-\alpha}$$

- Both size and composition aspects important
- Increase in k/y due to increased longevity over time
- Increase in l/k and then decrease, due to fertility

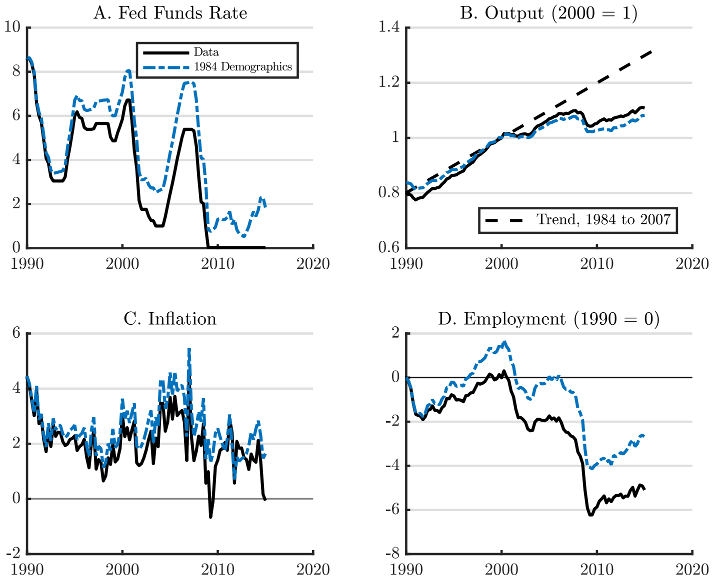
Demographic Trends



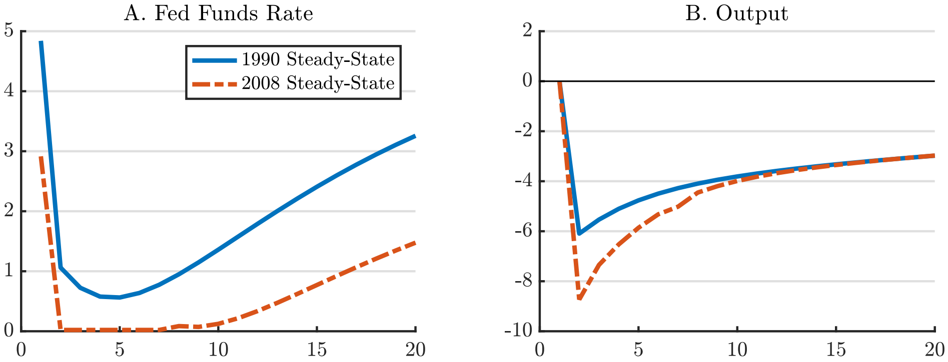
Demographics and the ZLB

- Were demographic trends responsible for ZLB binding?
- Hold fixed demographics in 1984 onwards
- Fix time-varying demographic parameters
- Use filtered shocks in a simulation

Path of Variables, Demographics Fixed in 1984



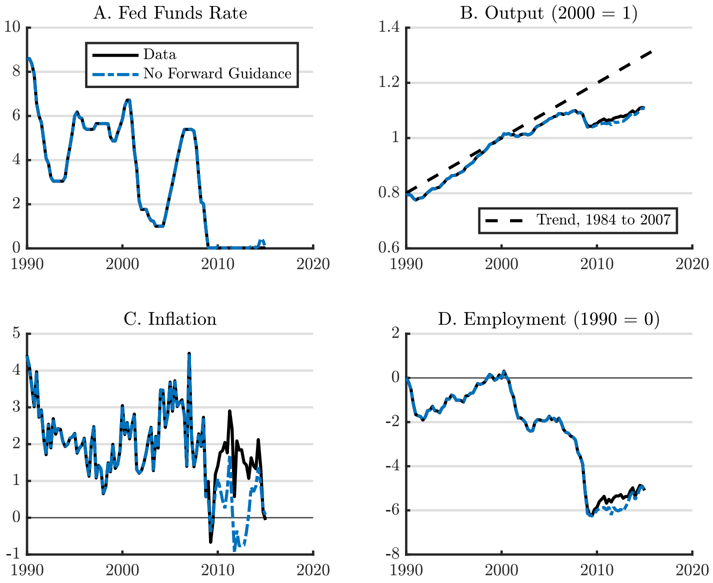
Nonlinearities Make This Important



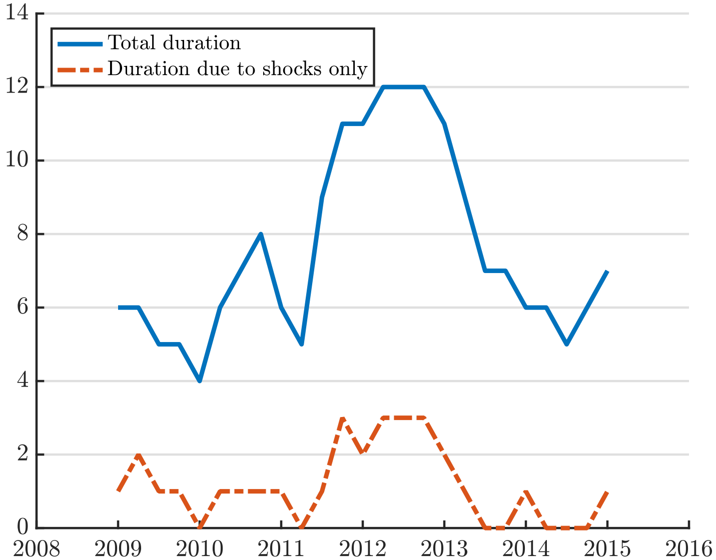
Monetary Policy During the ZLB

- Role of monetary stimulus during ZLB?
- Take structural shocks, compute counterfactual
- Allow the expected duration of the ZLB to vary in response to shocks

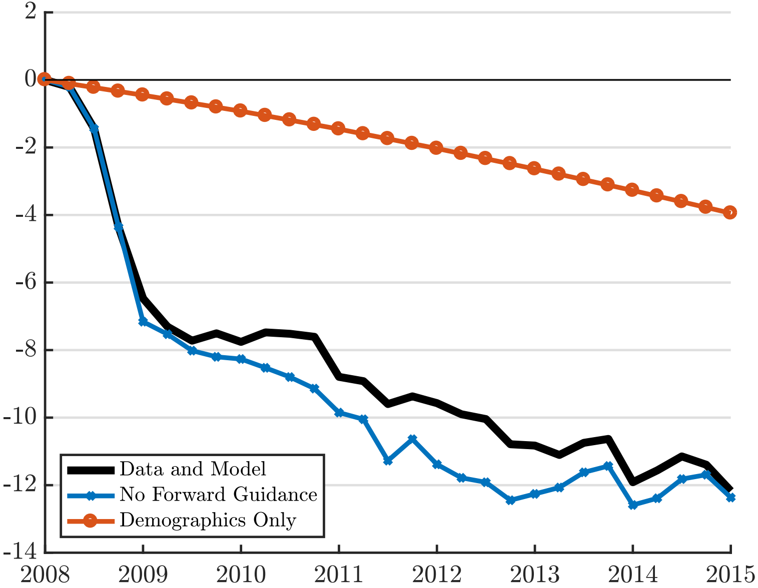
Path of Variables, No Forward Guidance



Forward Guidance Decomposition, Quarters



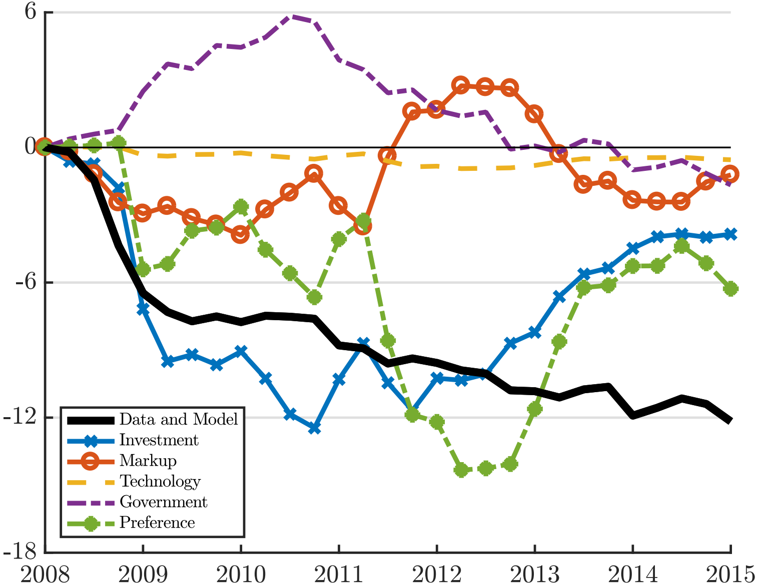
Log Output per Capita Relative to Trend



What Shocks Drive Decline in Output?

- Answer not obvious because of the ZLB
- Removing a shock might mean that the ZLB does not bind
- If so, then the Fed can respond more easily to other shocks
- One way to answer is to hold fixed ZLB durations at those observed
- Remove each shock, conditional on those fixed ZLB durations

Contribution of Each Shock to Output



Related Literature

1. Demographic trends. Gagnon, Johannsen and Lopez-Salido (2016). Eggertsson, Mehrotra and Robbins (2016). Cooley and Henriksen (2016). Carvalho, Ferrero and Nechio (2015). Aaronson et al (2014). Jimeno and Rodriguez-Palenzuela (2002). Rios-Rull (1996)

- New Keynesian model with monetary policy

2. Secular stagnation. Gordon (2016). Rogoff (2015). Hamilton et al (2015). Eggertsson and Mehrotra (2014). Summers (2014). Antolin et al (2014). Fernald (2014). Gomme, Ravikumar and Rupert (2012).

- Joint study of long-run trends, business cycle, and ZLB

3. Structural change. Canova, Feroni and Matthes (2016). Kulish and Pagan (2015). Jaimovich and Sui (2009). Fernandez-Villaverde et al (2007).

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2. Secular stagnation. Gordon (2016). Rogoff (2015). Hamilton et al (2015). Eggertsson and Mehrotra (2014). Summers (2014). Antolin et al (2014). Fernald (2014). Gomme, Ravikumar and Rupert (2012).

- Joint study of long-run trends, business cycle, and ZLB

3. Structural change. Canova, Feroni and Matthes (2016). Kulish and Pagan (2015). Jaimovich and Sui (2009). Fernandez-Villaverde et al (2007).

- Demographics as structural changes

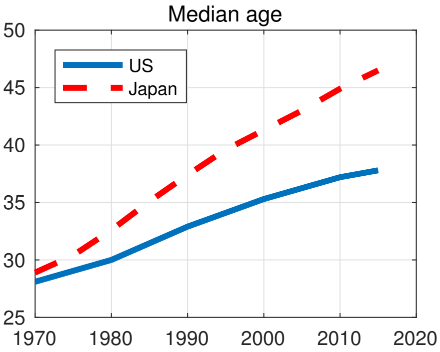
Conclusions

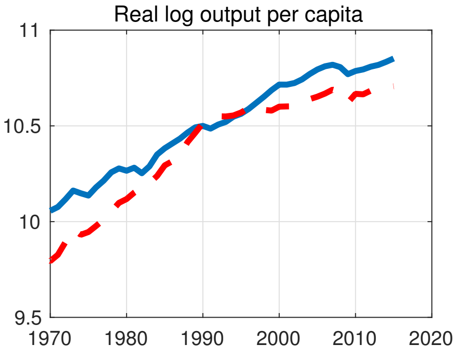
- Studied the gap between output per capita and its trend
- Combined demographic trends, ZLB in an estimated model
- Key role for demographic trends, financial factors
- ZLB bites, but the effects offset by forward guidance

Thanks!

Extra slides

Does the Story Fit Japan?

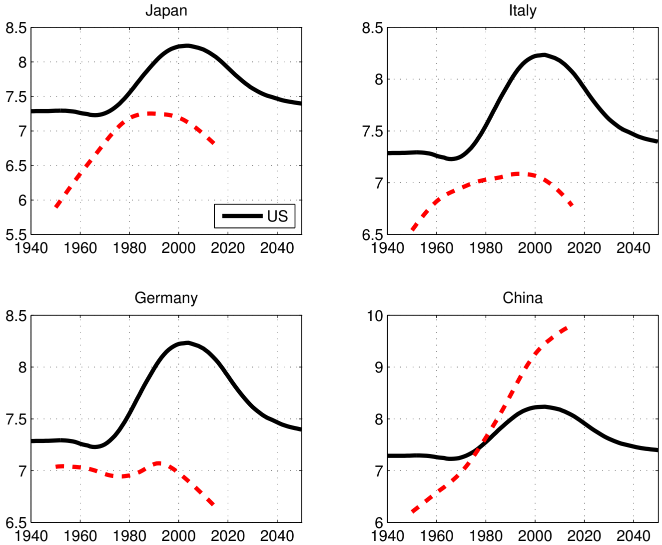




- And Japan entered its ZLB in 1996

Japan Wedges

Country Productivity Wedge



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What Features of Data Suggest Aging is Key?

- Compositional analysis of labor-force participation rate:

$$\mathrm{PR}_t = \sum_{s=16}^{95} \mathrm{PR}_\tau^s \frac{n_t^s}{n_t}$$

	Change, percentage points	
PR_τ^s profile	1996 to 2007	2008 to 2015
$\tau = 1990$	-0.35	-1.83
Data	-0.80	-3.40

- Cross-country regressions of output per capita on age-structure

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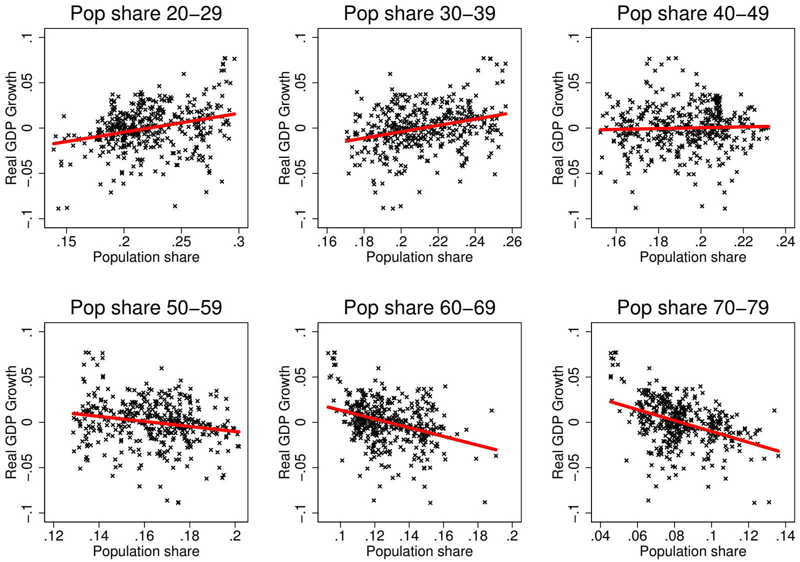
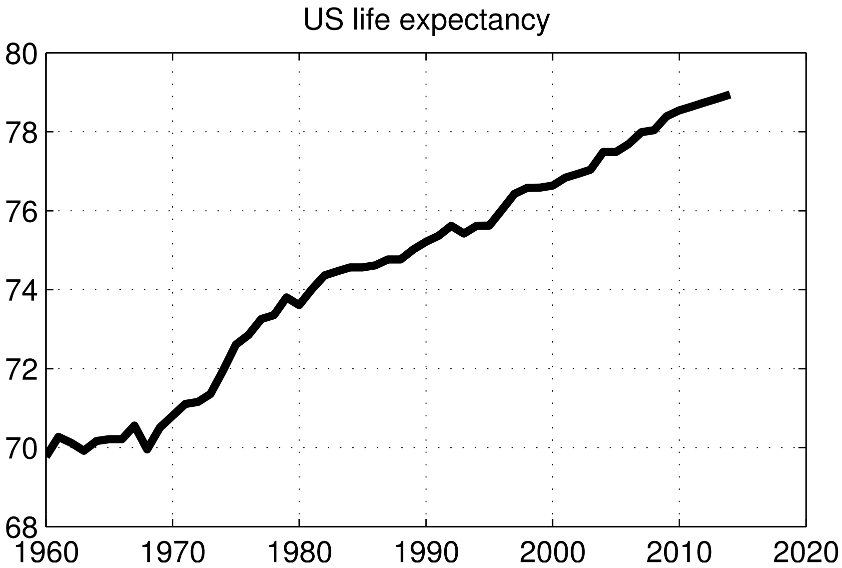
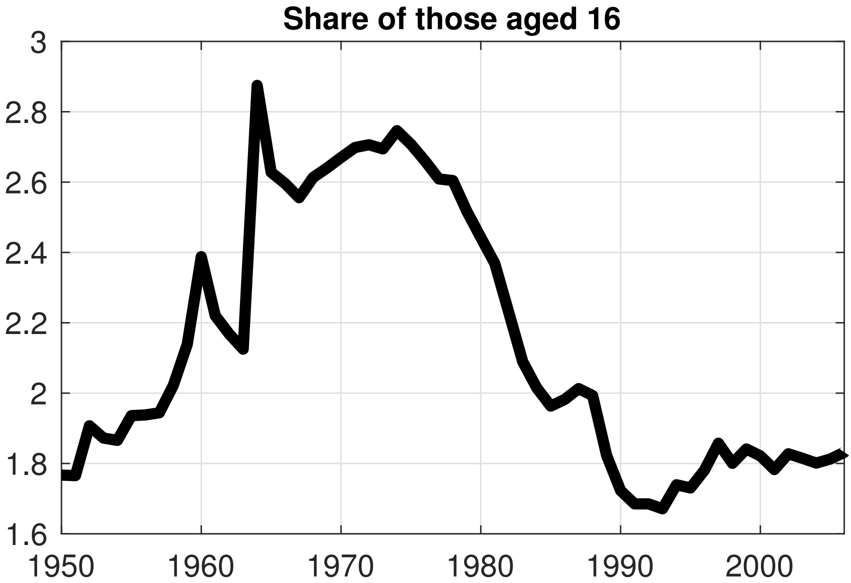
Output per Capita and Age Shares



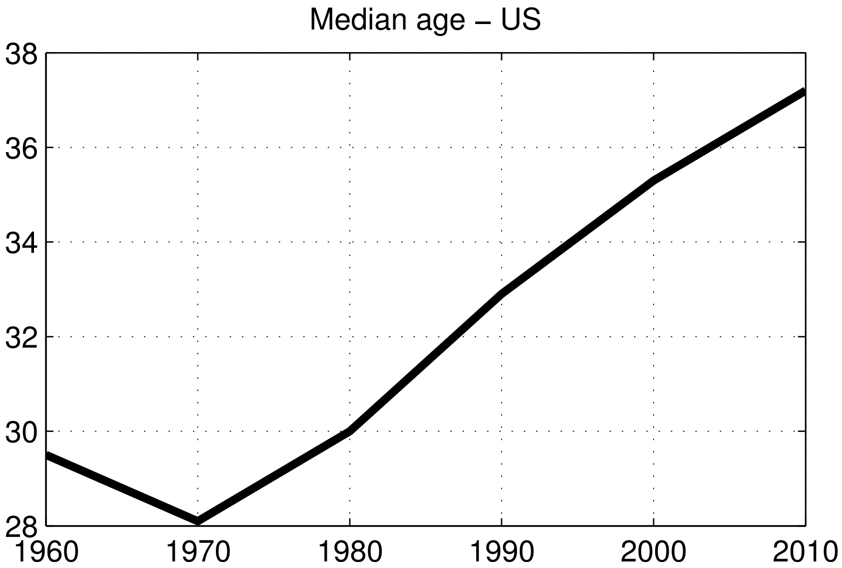
Increased Longevity



Fertility Spike – Baby Boomer Wave



Median Age is Increasing



Alternative Model Solution

- Aggregate alternative problem with a Negishi approach

1. Planner's period-by-period allocation:

$$U(c_t, \ell_t) \equiv \max_{\{c_t^s, \ell_t^s\}_s} \int \mu^i u(c_t^i, \ell_t^i) \, \mathrm{d}i$$

subject to $c_t = \int c_t^i \, \mathrm{d}i$ and $\ell_t = \int \ell_t^i \, \mathrm{d}i$

2. Planner optimizes c_t, ℓ_t and savings over time:

$$\max_{\{c_t, k_t, b_t, \ell_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t)$$

subject to the economy's resource constraint

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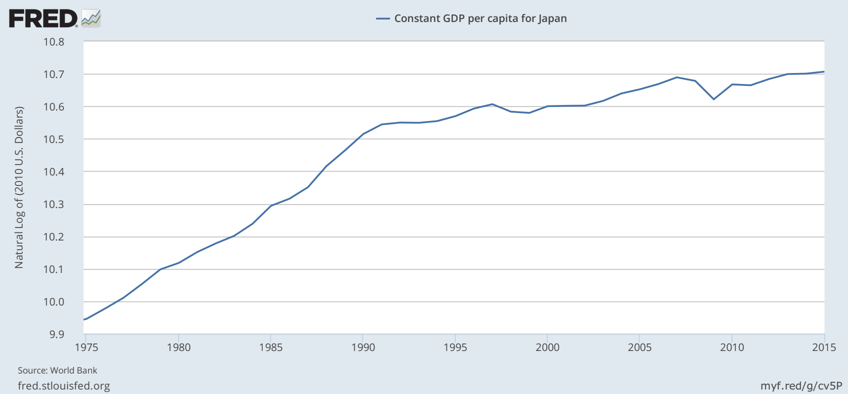
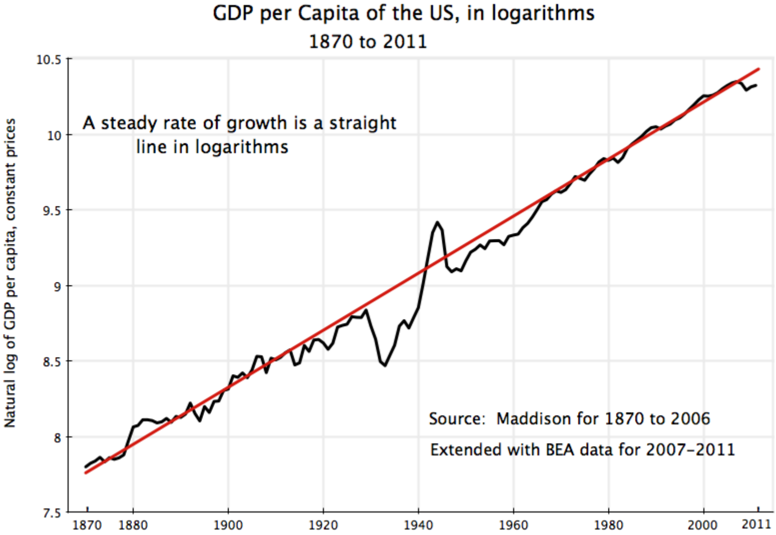
Japan output per capita



US output per capita in long-run



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Demographics and employment-population ratio

Alternative model: Social planner's problem

- Aggregate this problem with a social planner's two-step problem

1. Planner's period-by-period allocation:

$$U(c_t, \ell_t) \equiv \max_{\{c_t^s, \ell_t^s\}_s} \int \mu^i u(c_t^i, \ell_t^i) \; \mathrm{d}i$$

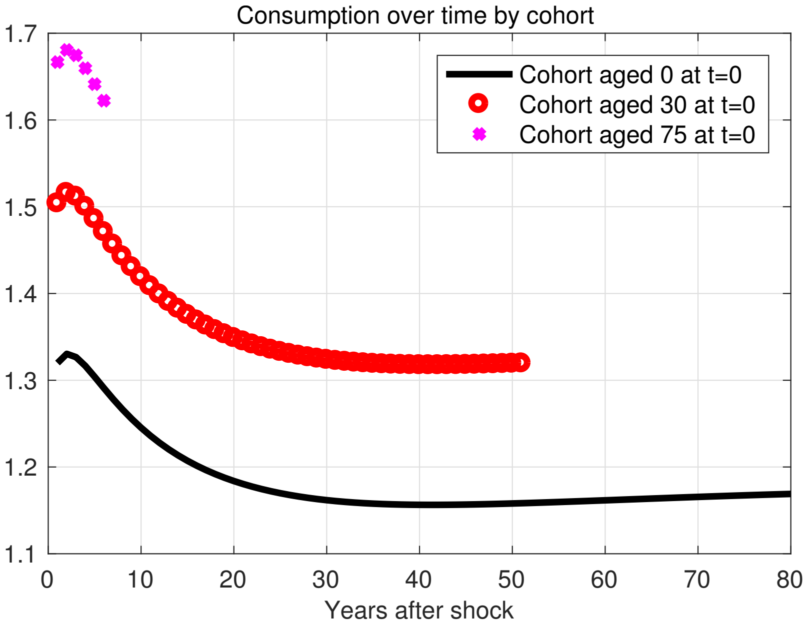
subject to $c_t = \int c_t^i \; \mathrm{d}i$ and $\ell_t = \int \ell_t^i \; \mathrm{d}i$

2. Planner optimizes c_t, ℓ_t and k_t over time:

$$\max_{\{c_t, k_t, \ell_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t)$$

subject to the economy's resource constraint

IRF to TFP shock by age, linear approximation



Why welfare weights do not matter

- Take a IRF to technology
- Consumption in each cohort is approximately proportional to each other
- Consumption changes by age are a fraction of the aggregate
- In the alternative model, consumption at each age is a constant fraction of the aggregate, where that fraction depends on the Pareto weights

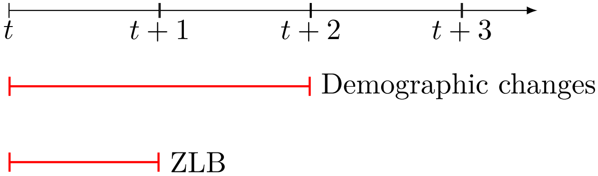
Measuring capital

Capital input data–service-flows of equipment, structures, intellectual property products, inventories, and land.

BLS measures of capital service inputs are prepared using NIPA data on real gross investment in depreciable assets and inventories.

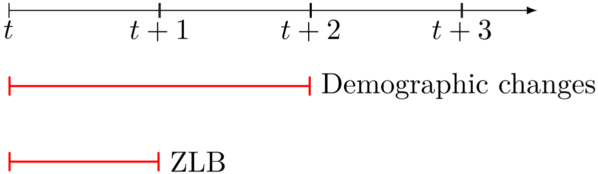
Example

- From t, expected evolution of economy is:



Example

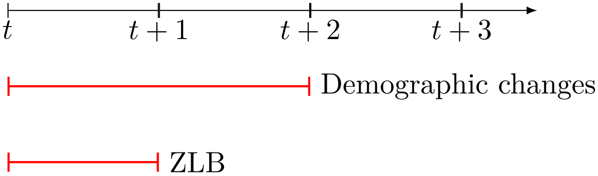
- From t, expected evolution of economy is:



- From $t+2$, parameters are fixed: $\mathbf{Q}_{t+2} = \mathbf{Q}$

Example

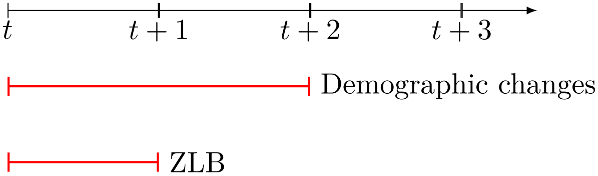
- From t, expected evolution of economy is:



- From $t+2$, parameters are fixed: $\mathbf{Q}_{t+2} = \mathbf{Q}$
- From $t+1$: demographics: $\mathbf{Q}_{t+1} = \left[\mathbf{A}_{t+1} - \mathbf{D}_{t+1}\mathbf{Q}_{t+2}\right]^{-1}\mathbf{B}_{t+1}$

Example

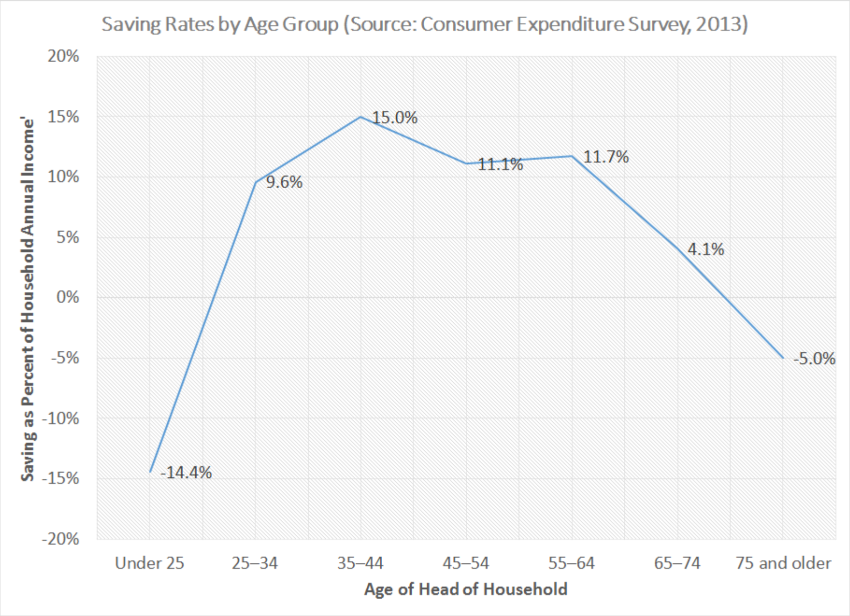
- From t, expected evolution of economy is:



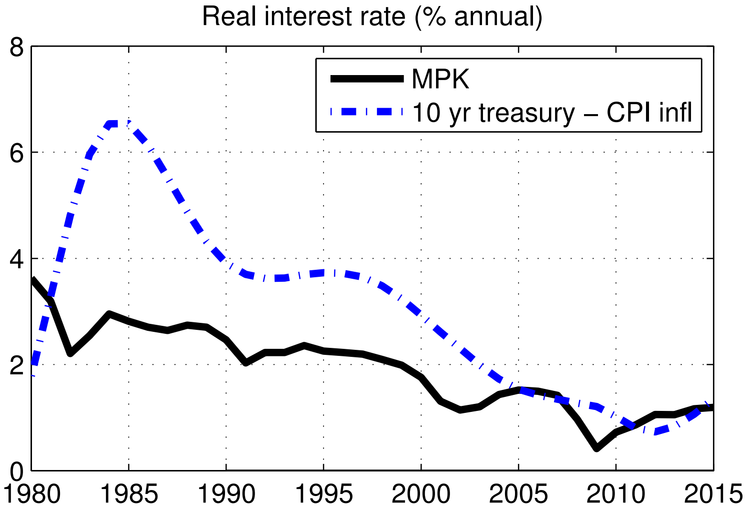
- From $t+2$, parameters are fixed: $\mathbf{Q}_{t+2} = \mathbf{Q}$
- From $t+1$: demographics: $\mathbf{Q}_{t+1} = \left[\mathbf{A}_{t+1} - \mathbf{D}_{t+1}\mathbf{Q}_{t+2}\right]^{-1}\mathbf{B}_{t+1}$
- From t: ZLB + demographics: $\mathbf{Q}_{t} = \left[\mathbf{A}^*_{t} - \mathbf{D}^*_{t}\mathbf{Q}_{t+1}\right]^{-1}\mathbf{B}^*_{t}$

Back

Savings rate by age



Real interest rate declining since the 1980s



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Unintentional bequests

Receive:

$$n_{t-1}^{s-1}\gamma_{t-1}^{s-1}a_{t-1}^{s-1}R_t$$

Divide between remaining members:

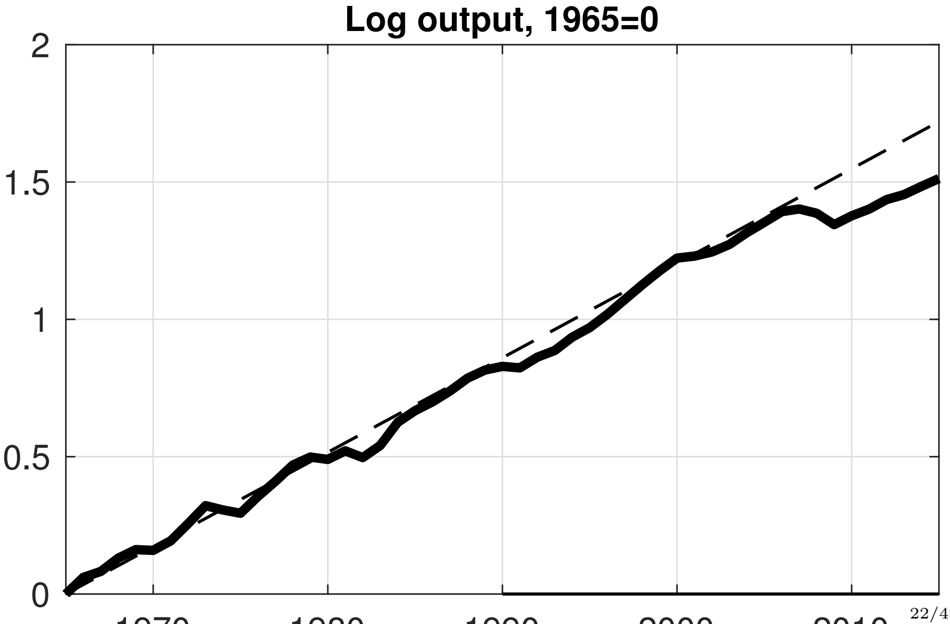
$$\frac{n_{t-1}^{s-1}}{n_{t-1}^{s-1}}\frac{\gamma_{t-1}^{s-1}}{1-\gamma_{t-1}^{s-1}}a_{t-1}^{s-1}R_t$$

So have:

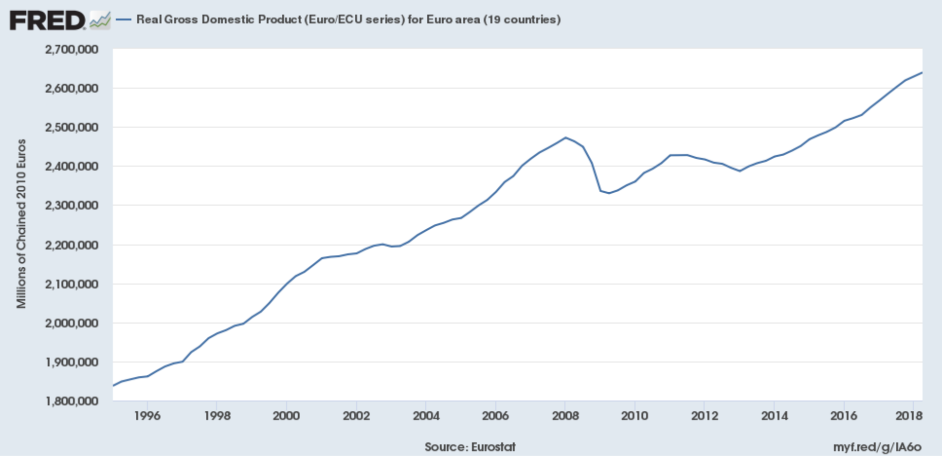
$$\left(1+\frac{\gamma_{t-1}^{s-1}}{1-\gamma_{t-1}^{s-1}}\right)a_{t-1}^{s-1}R_t$$

back

Output and its pre-crisis trend



Euro Area Output from 1996



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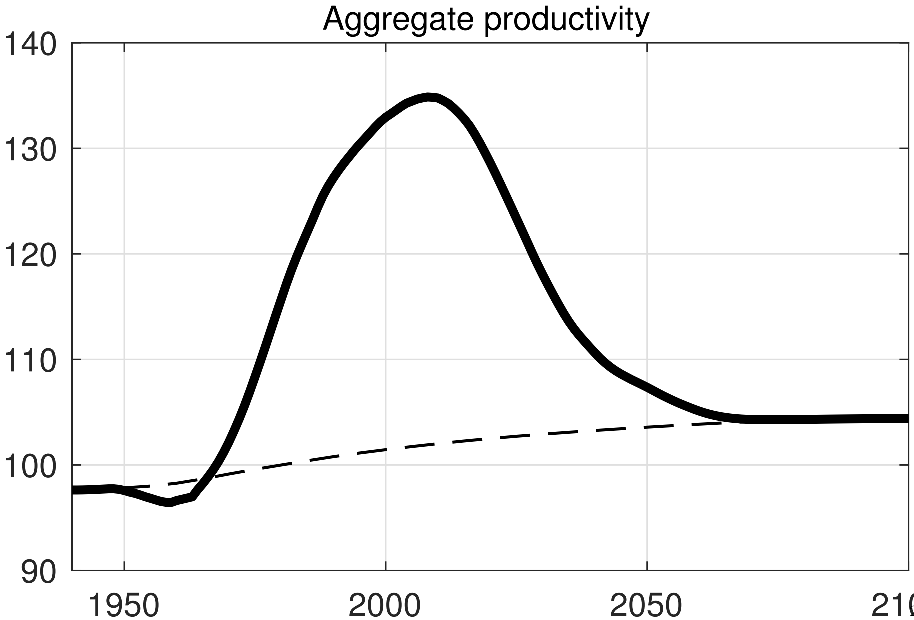
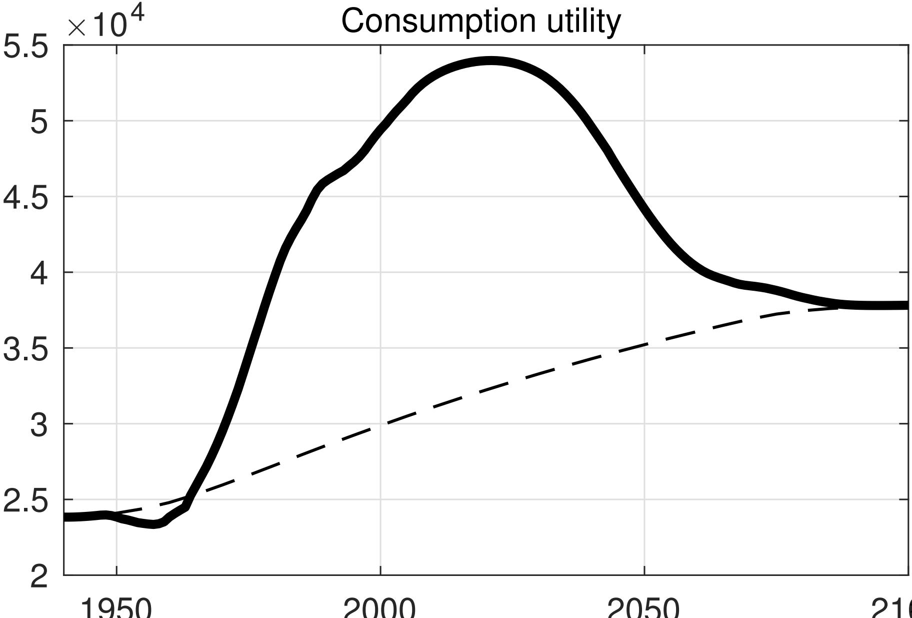
Calibrated productivity profile

back

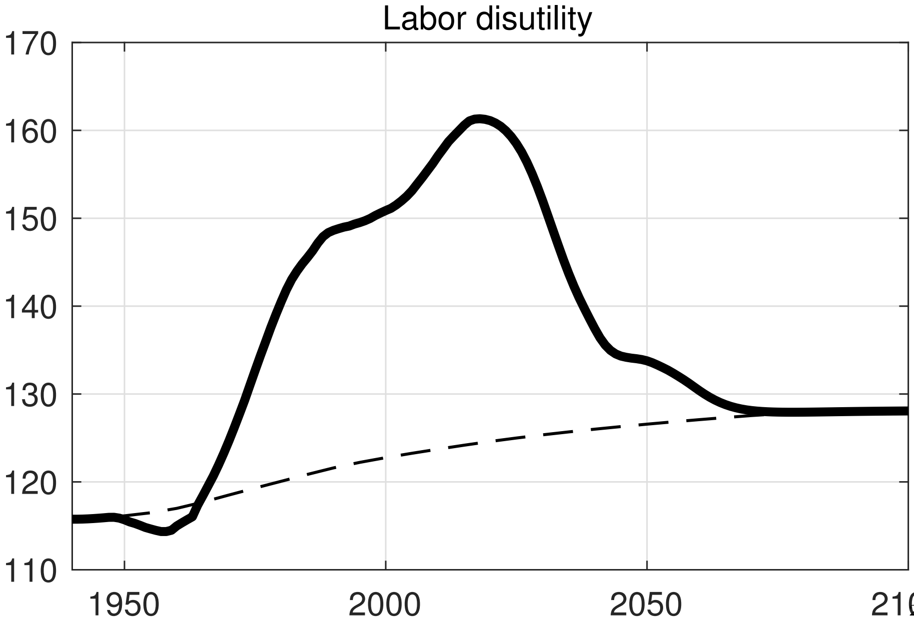
Total productivity, $y_t = \theta_t k_t^{\alpha} \ell_t^{1-\alpha}$



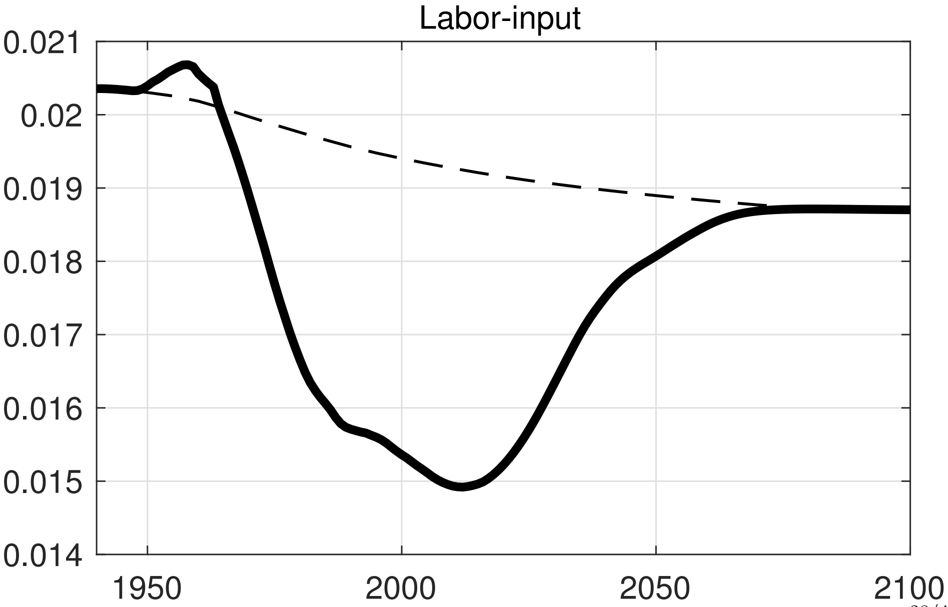
Marginal utility of consumption, $\phi_t u_1(c_t, \ell_t) = \lambda_t$



Marginal disutility of labor, $v_t u_2(c_t, \ell_t) = w_t \lambda_t$



Labor-input shock, $\ell_t = A_t h_t$



Average percentage growth rates

	Output		Capital		Productivity	
Period	Model	Data	Model	Data	Model	Data
1985 to 2000	3.52	3.82	3.94	4.05	2.74	2.10
2001 to 2015	3.00	1.97	3.55	2.41	2.39	2.39
2010 to 2015	2.75	2.84	3.28	1.64	2.30	1.15
Model predictions						
2016 to 2025	2.31	2.58	2.58	2.17		
2026 to 2035	2.13	1.91	2.42	2.10		

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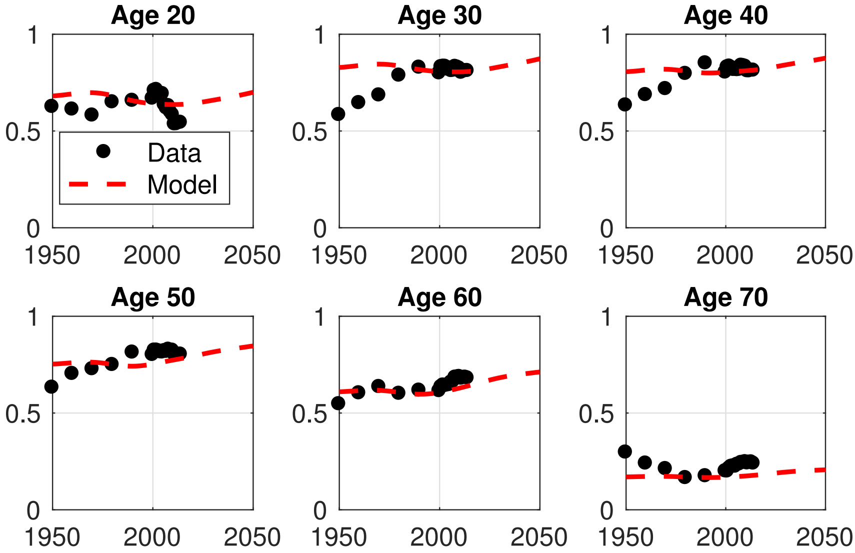
Labor disutility

back

Labor force participation rates by age over time



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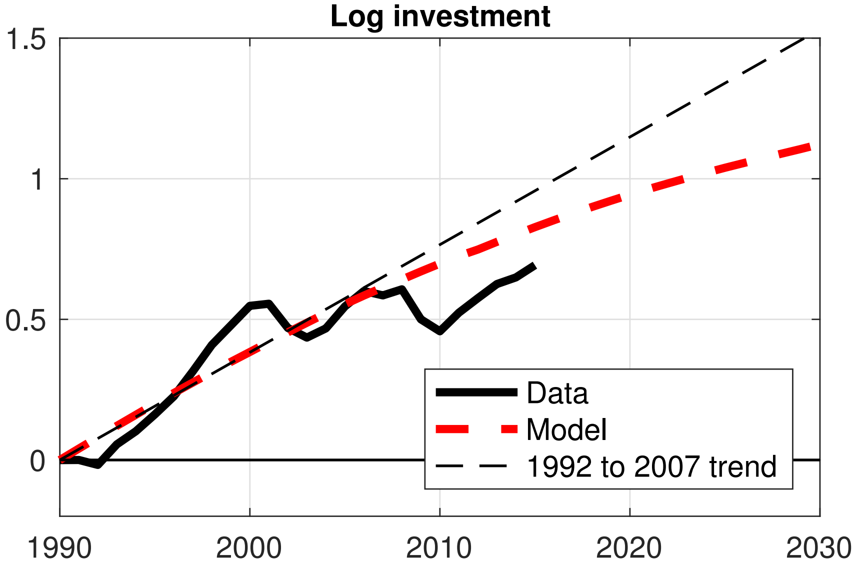
Investment index with TFP growth



What drives productivity growth?

Model-implied output per unit of labor:

$$\frac{y_t}{\ell_t} = k_t^{\alpha} \ell_t^{-\alpha}$$

Labor input can be written as labor quality and hours:

$$\ell_t = \mathrm{LQ}_t \times h_t$$

So: decompose observed output per hour worked into:

$$\frac{\mathrm{d}\frac{y_t}{\ell_t}}{\frac{y_t}{\ell_t}} = \alpha \frac{\mathrm{d}k_t}{k_t} - \alpha \frac{dh_t}{h_t} - \alpha \frac{\mathrm{dLQ}_t}{\mathrm{LQ}_t}$$

back

Social security. PAYG system: workers fund retiree benefits

Above age T^*, workers get paid a fraction of lifetime earnings:

$$\xi_t^s = \lambda \frac{W_t^s}{T^* - 1}.$$

λ is the replacement rate of average earnings and W_t^s is:

$$W_t^s = \begin{cases} w_t z^s \ell_t^s + W_{t-1}^{s-1}, & \text{if } s < T^* \\ W_{t-1}^{s-1}, & \text{if } s \geq T^*. \end{cases}$$

A common lump-sum tax τ_t adjusts to balance budget

Social security replacement rate of earnings, λ set to 46%

back

Steady-state changes in longevity

	γ^s fixed profiles				
	1940	1970	2000	2030	2070
K/Y	2.564	2.597	2.627	2.657	2.686
C/Y	0.724	0.721	0.717	0.714	0.711
L/N	0.619	0.621	0.620	0.618	0.615
$1+r-\delta$	1.024	1.023	1.021	1.019	1.018

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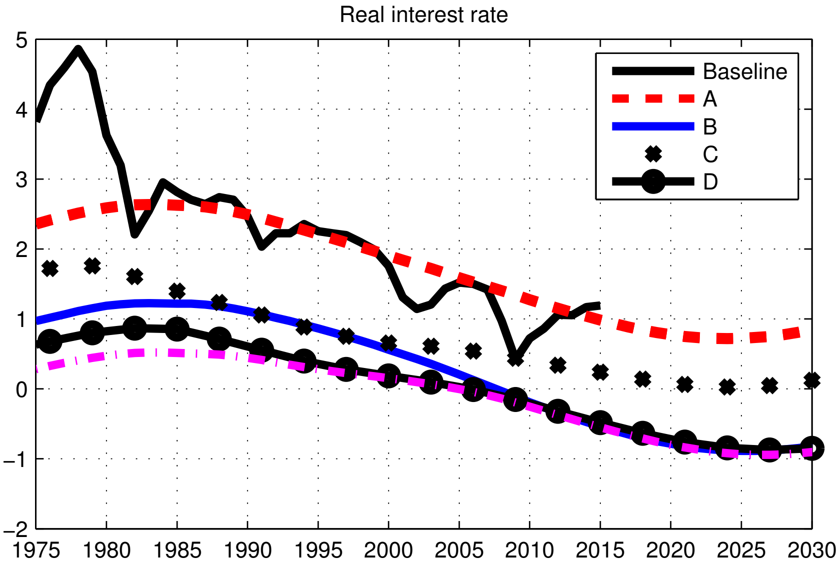
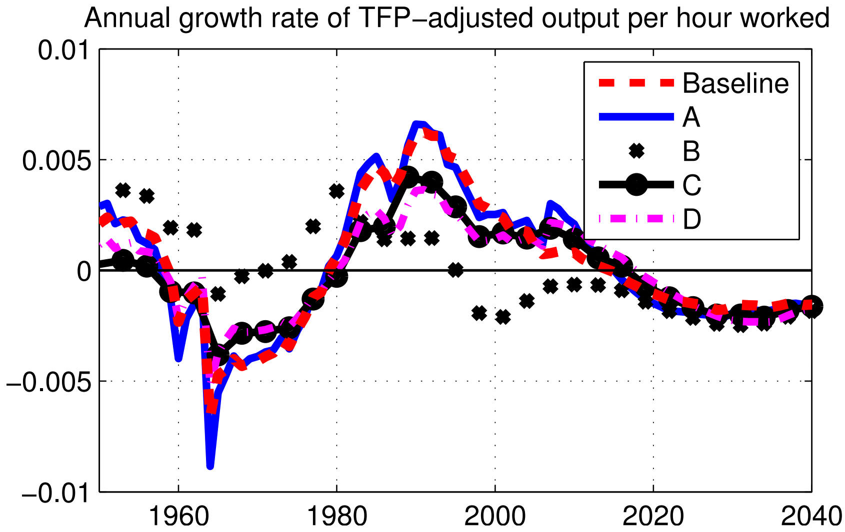
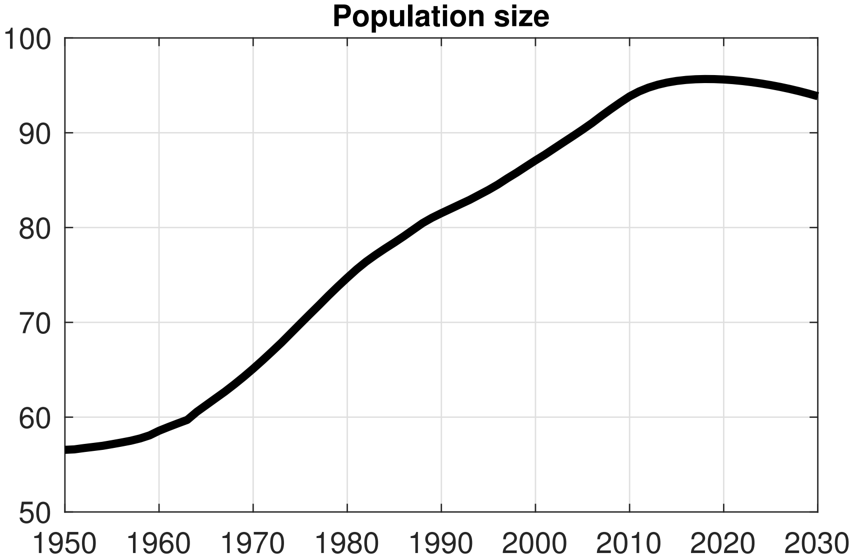
Robustness: real interest rate



Robustness: growth in output per worker



Model population size



Real interest rate (MPK) falls under demographic transition

$$R_t = \alpha \frac{y_t}{k_{t-1}} - (1 - \delta)$$

Longevity:

- Implies k_t/y_t increases

Fertility:

- 1960s to 1980s: increase in labor supply
- 1980s to 2020s: increase in savings for retirement
- 2010s to 2030s: exit from the labor market and dissaving

Average percentage growth rates

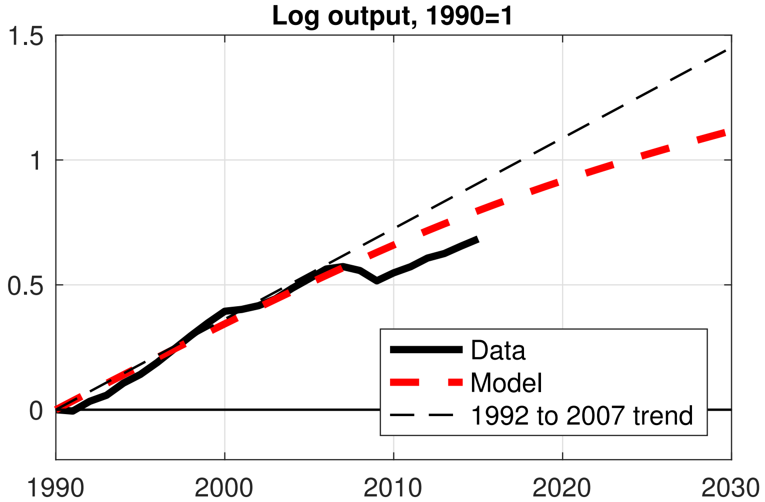
	Output		Labor Productivity	
Period	Model	Data	Model	Data
1985 to 2000	1.00	1.00	1.00	1.00
2001 to 2015	0.85	0.52	0.87	1.14
2010 to 2015	0.78	0.74	0.84	0.54
Model predictions				
2016 to 2025	0.65		0.79	
2026 to 2035	0.61		0.77	

Note: values are normalized to 1985 to 2000 average

In growth rates

Steady-states

Log output with TFP growth + pre-crisis trend



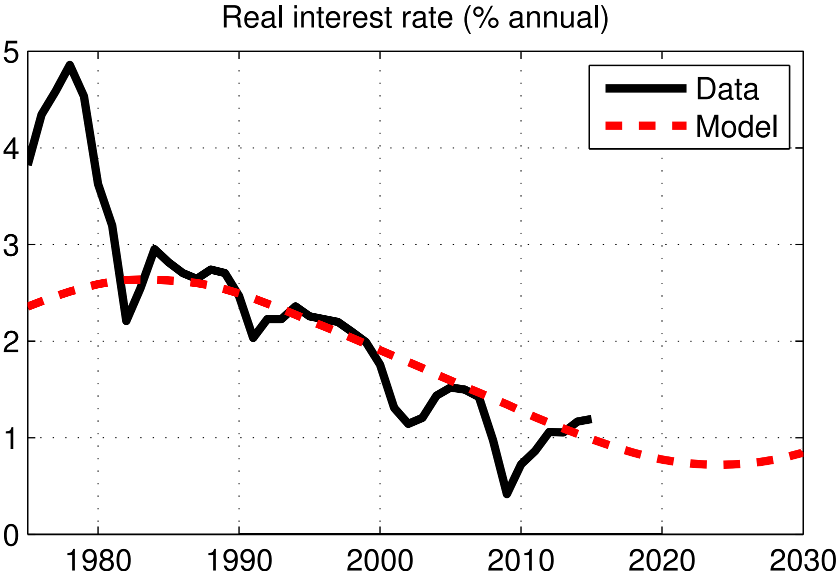
Employment-population ratio

How Well Does Aging Explain Trends in Data?

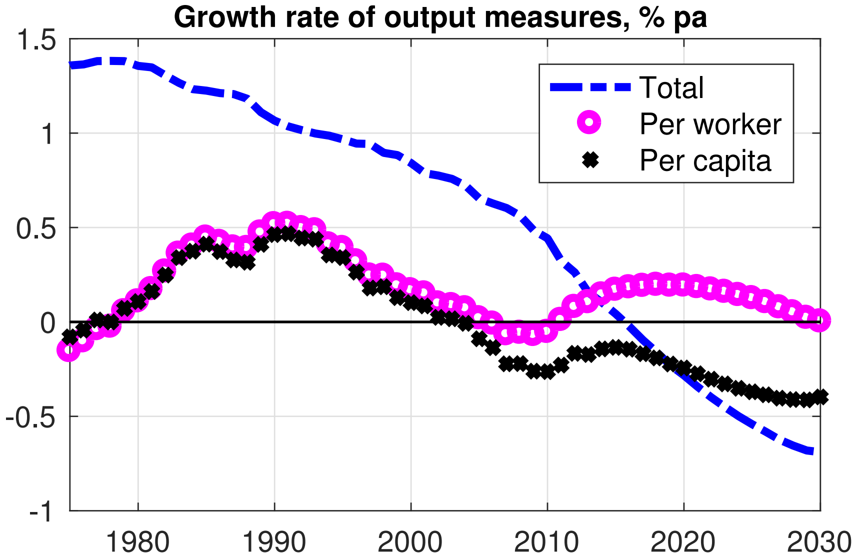
- Exercise: turn off all shocks and compare data v model
- **Real interest rate** from 1985 to 2015:
  - Data: 1.9 pp decline
  - Model: 1.5 pp decline
- **Employment-population ratio** from 1990 to 2015:
  - Data: 2 pp decline
  - Model: 1.2 pp decline
- **Labor productivity growth** from 1990 to 2015:
  - Data: 1.2 pp decline
  - Model: 0.5 pp decline

Note: Trends in the data computed from HP-filtered series

Real Interest Rate



Model's Output Growth Measures



Decomposition of Model's Output per Worker