Elastic beams lab report

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1 Summary

Within this report, a beam is loaded by a point-load of 50N in order to understand the effects of bending on the curvature of a simply supported beam. The beam is kept within its elastic region so no plastic deformation occurs. The curvature values were gathered using a digital curvature gauge, which was calibrated on a larger wheel of known diameter. There was a "sanity check" measurement of the midpoint's vertical displacement also taken to compare against numerically integrated values. The results showed that the gathered data was consistent with theory, and that the curvature linearly decreases from the centre until the supports, then beyond the supports there is no curvature as there is no capacity to carry moment.

2 Introduction

The aim of this report is to investigate the nature of beams bending under a point load within the elastic region. These deflections are so small that it is necessary to set up equipment to measure them precisely. A device will be used to measure the local curvature of the beam rather than placing multiple displacement gauges along the length.

The way in which curvative is investigated is using a simply supported aluminium beam loaded in the midpoint.

3 Laboratory setup

3.1 Apparatus

Within this section, the equipment used within the laboratory will be listed with reasoning why it was chosen and a short summary of its functionality.
The beam itself was an aluminium beam with notches every 5 centimetres, with notch 1 and 11 to be placed on a roller and pin joint respectively. This were to be placed into a loading frame.
A curvature gauge was used in order to measure the local curvature at each of the 14 points in the beam. The curvature gauge was datumed on a known flat beam in order to remove zero error from the experiment. They were already previously calibrated on a large wheel of known curvature close to where they are stored.
In order to measure the central displacement another separate displacement gauge was also required. There was no need to zero this as the measurement was simply a difference rather than an absolute with refenced to zero.
A load stalk and 5 kilogram masses were required in order to achieve the desired load on the midpoint of the beam.

3.2 Testing methodology

Prior to adding any load to the setup, the initial curvature must be measured. The empty load stalk was added and the digital curvature gauge was placed at each of the 14 points along the beam and a measurement was taken and recorded into a table.

After recording the pre-load curvature, 5 kilogram masses were added to the load stalk. The curvature was once again recorded at each of the 14 measurement points along the beam. Finally, once the midpoint displacement was measured with the load added using the displacement gauge, then the setup was unloaded and a second reading was taken. The difference between these two measurements was recorded - this is mainly to have a comparison for the calculation later.

4 Results, observation and calculations

The results were collated into a spreadsheet and python to graph the different quantities associated with the experiment. The calibration factor of the curvature found by comparing the measured to known curvature on a reference wheel. The gauge measured 0.01685 cm^{-1} and the actual is 0.01666 cm^{-1}. This means that the gauge is off by a factor of 1.1%, which is relatively accurate for what is required to be measured. The measured curvature against station number is shown below in figure 1. The curvature is a linear increase until the maximum at the point where the load is applied, and rising back up to zero by time it the gauge is over the second support. There is no curvature past the support which is as expected as there is no continued bending moment.

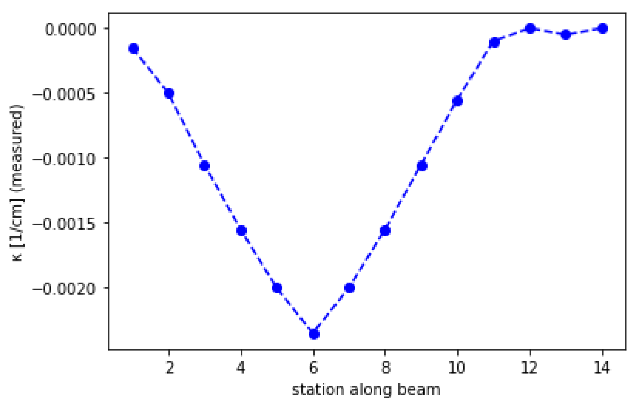

Figure 1: Graph to show measured curvature values against station number

In order to calculate the moment in the beam it is possible to use a simple free body diagram as shown below.

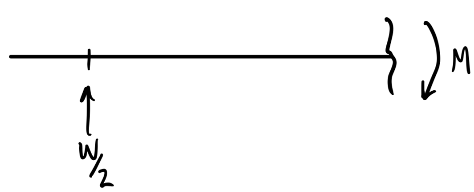

Figure 2: Moment FBD

$$M = \frac{W}{2}.x \tag{1}$$

Where actual horizontal distance across the beam is approximated to just x ($S \approx X$). From this, it is possible to calculate M/B where B is obtained from the gradient of the experimental data shown in the figure below 3. B was calculated to $\approx 2.4\text{x}10^5$ Nm^{-2}.

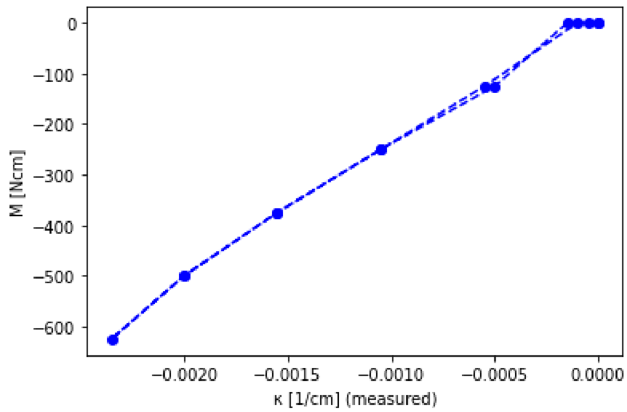

Figure 3: Graph to show measured curvature values against calculated moment

The graphs of M and κ are both linearly increasing until station 11, so it is expected that the graph of one against the other would form another linear graph. Using this calculated value of B, it is possible to use numerical integration in order to calculate the midpoint displacement. The value of B is relatively close to reality as the graph of measured and simplfied kappa are close.

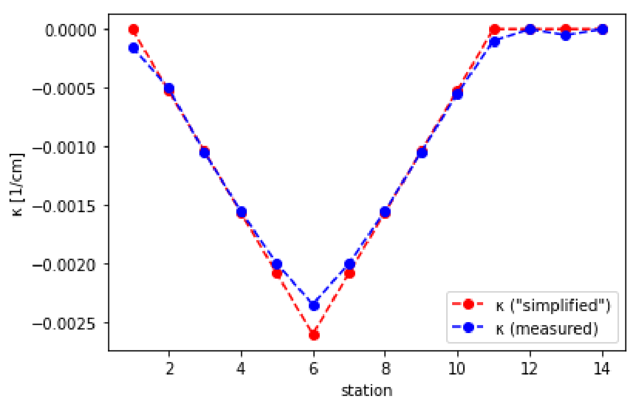

Figure 4: Graph to show measured curvature values against simplfied moment

Integrating up twice and accounting for boundary conditions ($+c$ in integration) allows for a graph of y(vertical displacement) against stations to be formed. This can be compared to the measured value of midpoint displacement as during the laboratory session a midpoint measurement was gathered using the displacement gauge. The plot in figure 5 has the measured point plotted along with the calculated displacement.

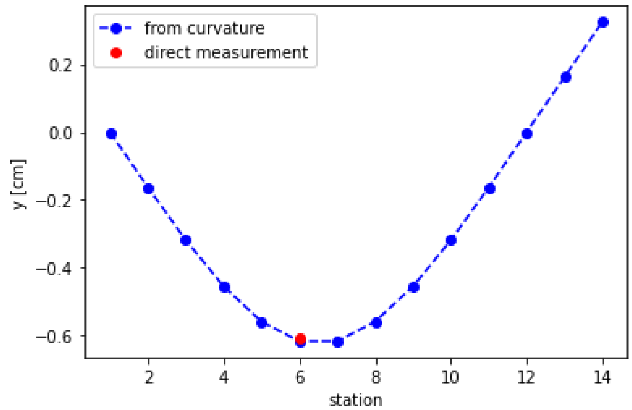

Figure 5: Graph to show displacement values(y), with a measured reference at station 6

The curvature due to bending moment between stations 1-10 is as expected, and the linear region after 11 is expected as displacement $= R.\theta$ where theta is the angle at the support. The gradient of the displacement is 0 at the midpoint, which is expected as this is where the load was applied. It would still be possible to integrate given one of these boundary conditions was not available, such as under an asymmetrical deflections.

5 Conclusion

The calculations of deflection using flexural stiffness yield comparable results to those found in the experiment within this report. The curvature also matches theory as it increases linearly until a maximum, then decreases steadily to zero from the support and beyond. This supports the original hypothesis that a point load produces curvature between two simply supported points, but as there is nothing to counteract moments beyond the support - this region will only have linear deflection and no curvature.

6 Discussion

The measured deflection is close to the calculated deflection (around 3% error), so the calculations are relatively accurate for this one point. Measuring just one point is not enough to justify saying that the deflection is valid throughout the whole beam, as there may be greater error within the other points along the beam. If the experiment were to be repeated, a minimum of 4 points of recorded displacement (excluding supports) would help to understand if the calculations made are valid for multiple sections of the beam. The calibration factor on the curvature gauge would have little effect on the outcome of this experiment as the error was small.

One source of error within the experiment could be the assumption of g = 10 ms^{-2} rather than the actual numerical value which could slightly chance the moments calculated, this in turn would chance the calculated value of flexural stiffness.