8.4 The Equations of Sinusoidal Functions (pt 2)

Today's Focus: Identify characteristics of the equations of sinusoidal functions

KEY IDEAS!!!

- Any sinusoidal function can be expressed as either a cosine function or a sine function.

THINGS TO REMEMBER!!!

- A sinusoidal function of the form

$$y = a \sin b(x - c) + d \text{ or}$$
$$y = a \cos b(x - c) + d$$

has the following characteristics:

- The value of a is the **amplitude**:

$$a = \frac{\text{maximum value} - \text{minimum value}}{2}$$

- The value of b is the number of cycles in $360°$ or 2π. The **period** is $\frac{360°}{b}$ or $\frac{2\pi}{b}$.
- The value of c indicates the horizontal translation that has been applied to the graph of $y = \sin x$ or $y = \cos x$.
- The **equation of the midline** is

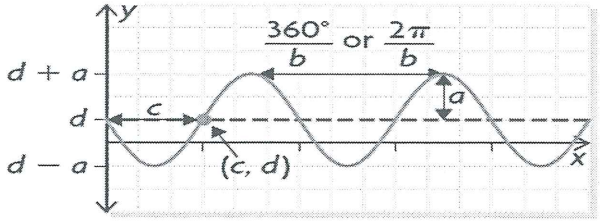
$$y = d$$

where

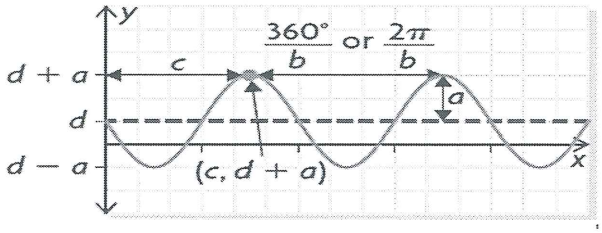
$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- The **maximum value** is $d + a$, and the **minimum value** is $d - a$.

- In the graph of a sine function, c is the distance from the vertical axis to the first midline point where the function is increasing.



- In the graph of a cosine function, c is the distance from the vertical axis to the first maximum point.



$y = a \sin b(x - c) + d$

Example 1: Determine the characteristics from each equation.

a) Given $y = 5\sin(3x) + 2$

a = 5 so the amplitude is 5

b = 3 so the period is $\frac{2\pi}{3}$

c = 0 so No H.T

d = 2 so the midline is $y=2$ / Vertical Translation 2 up

max = $d + |a| = 2 + 5 = 7$ min = $|d| - |a| = 2 - 5 = 3$ Range $3 \leq y \leq 7$

b) Given $y = 3\cos 3x + 1$

a = 3 so amplitude is 3

b = 3 so Period is $\frac{2\pi}{3}$

c = 0 so no Horizontal translation

d = 1 so Vertical translation 1 up

max = $1 + 3 = 4$ min = $1 - 3 = -2$ Range: $-2 \leq y \leq 4$

c) Given $y = 2\sin 4(x - 45°)$

a = 2 so amp is 2

b = 4 so period is $\frac{2\pi}{4} = \frac{\pi}{2}$

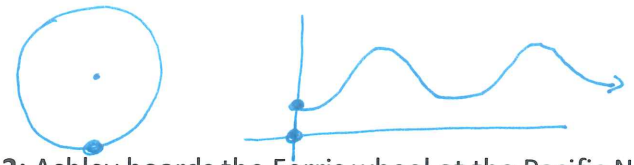
c = 45 so H.T 45° right

d = 0 so No V.T.

max = $d + a = 0 + 2 = 2$

Range $-2 \leq y \leq 2$

min = $d - a = 0 - 2 = -2$



Example 2: Ashley boards the Ferris wheel at the Pacific National Exhibition. When the ride begins, her position can be modelled by the function

$$y = 43 \sin 3.5(x - 0.9) + 47$$

Where y represents the height in feet and x represents the time in minutes.

a) Determine the diameter of the Ferris wheel. diameter = 2(amplitude)

$2(43) = 86$ ft

b) How long does it take for the Ferris wheel to complete one revolution? period

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{3.5} \doteq 1.8$ min

c) How high above the ground is Ashley at the lowest point? min

$\text{min} = d - a = 47 - 43 = 4$ ft