8.4 The Equations of Sinusoidal Functions (pt 1)

Today's Focus: Identify characteristics of the equations of sinusoidal functions

INVESTIGATION - ~~$y = a \sin(bx + c) + d$~~ $y = a \sin b(x - c) + d$

Window Settings:

[-2π, 2π, π/2]
[-2, 2, 1]

Remember, you must be in **Radian** mode

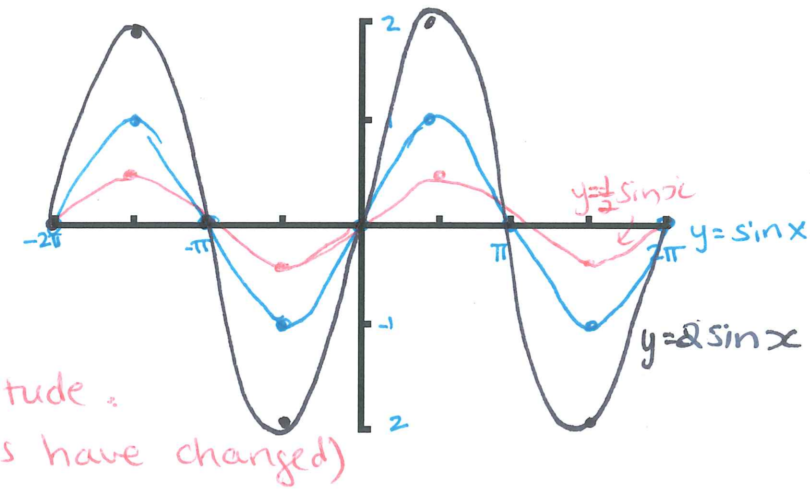
AMPLITUDE

Investigating the "a" value:

Graph y = sin (x)

y = 2 sin (x)

y = 0.5 sin (x)



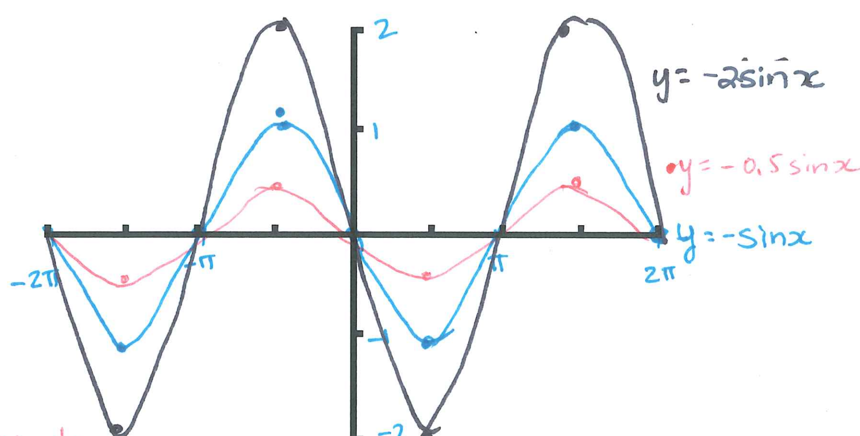
What has changed?

'a' changes the amplitude.
(min & max values have changed)

Graph y = -sin (x)

y = -2 sin (x)

y = -0.5 sin (x)



What has changed?

- "flipped"/Reflected in x-axis
- max/min values ⟶ amplitude

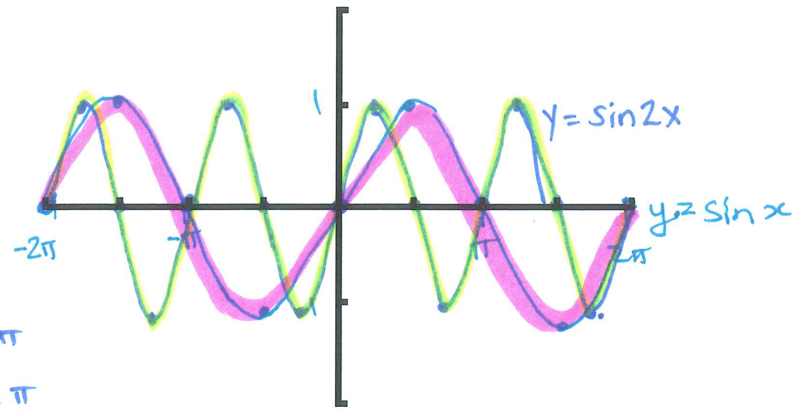
- The a-value of the graph is equivalent to the amplitude of the graph.
- If "a" is negative the graph of y = a sin (x) is flipped about the midline.

RELATED TO PERIOD

Investigating the "b" value: y = sin bx

Graph y = sin (x)

y = sin (2x)



What has changed?

x-intercepts; Period y = sin x → period = 2π

y = sin 2x → period = π

the 'b' value tells you how many complete rotations in 2π there are

For each equation, calculate $\frac{2\pi}{b}$.

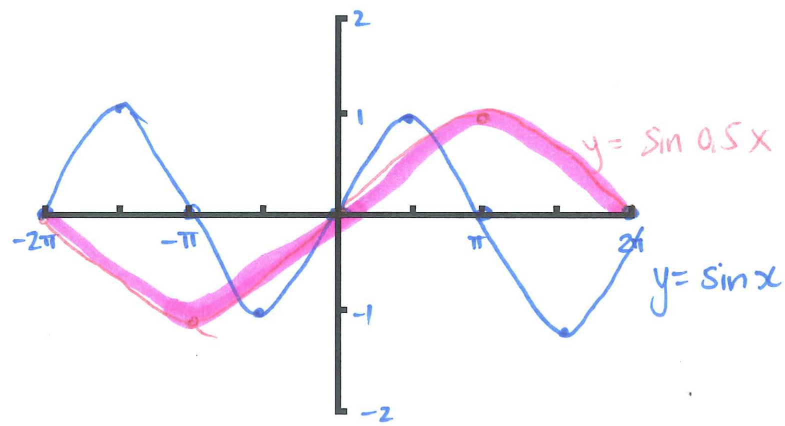
$\sin x \rightarrow \frac{2\pi}{1} = 2\pi$ = period

$\sin 2x \rightarrow \frac{2\pi}{2} = \pi$ = period

∴ period = $\frac{2\pi}{b}$ or b = $\frac{2\pi}{\text{period}}$

Graph y = sin (x)

y = sin (0.5x)



What has changed?

- x-intercepts
- Period.

For each equation, calculate $\frac{2\pi}{b}$.

$y = \sin x \rightarrow \frac{2\pi}{1} = 2\pi \rightarrow$ period

$y = \sin 0.5x = \frac{2\pi}{0.5} = 4\pi \rightarrow$ period.

- If |b| is bigger than 1 the period of the y = sin (bx) compressed.
  - We can say the graph has been horizontally compressed.
- If |b| is between 0 and 1 the period of the y = sin (bx) expanded.
  - We can say the graph has been horizontally expanded.
- If b is negative then y = sin (bx) is reflected in the y-axis.

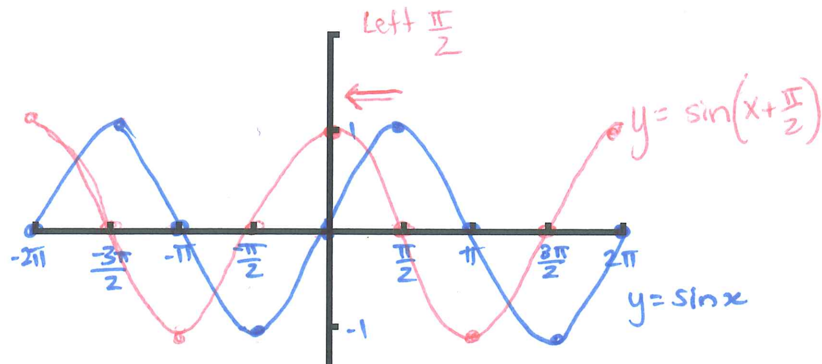
$y = \sin(x - c)$

"PHASE SHIFT"

Investigating the "c" value:

Graph $y = \sin(x)$

$y = \sin(x + \pi/2)$



What has changed? x-ints; y-int

The graph was shifted $\frac{\pi}{2}$ to the left

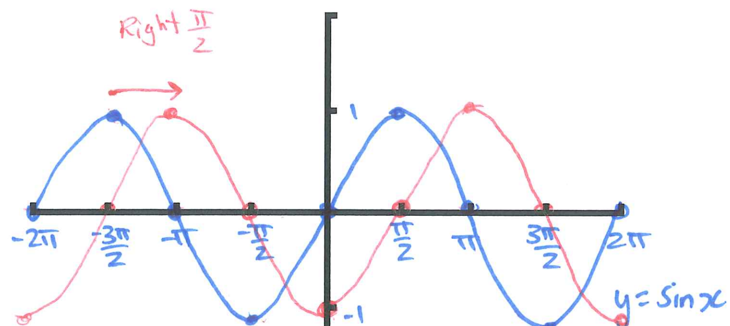
What is the new starting point for the function?

(0,1)

Graph $y = \sin(x)$

$y = \sin(x + (-\pi/2))$

or $y = \sin(x - \frac{\pi}{2})$



What has changed? x-int; y-int

the graph has shifted to the right by $\frac{\pi}{2}$.

What is the new starting point for the function?

(0,−1)

OMIT

- If c is bigger than zero (positive), then the graph of $y = \sin(x + c)$ is horizontally shifted to the ___left___.
- If c is smaller than zero (negative), then the graph $y = \sin(x + c)$ is horizontally shifted to the ___right___.

$y = \sin(x - c)$ ex. $y = \sin(x-3) \rightarrow 3$ right

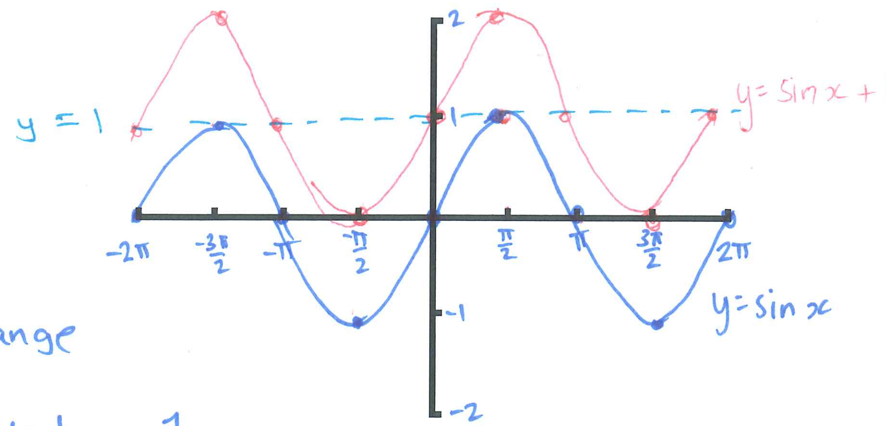
$y = \sin(x + \frac{\pi}{3}) \rightarrow \frac{\pi}{3}$ left

MIDLINE

<u>Investigating the "d" value:</u>

Graph $y = \sin(x)$

$y = \sin(x) + 1$



What has changed?
max/min, x-int; y-int; range

- The graph was shifted up 1.

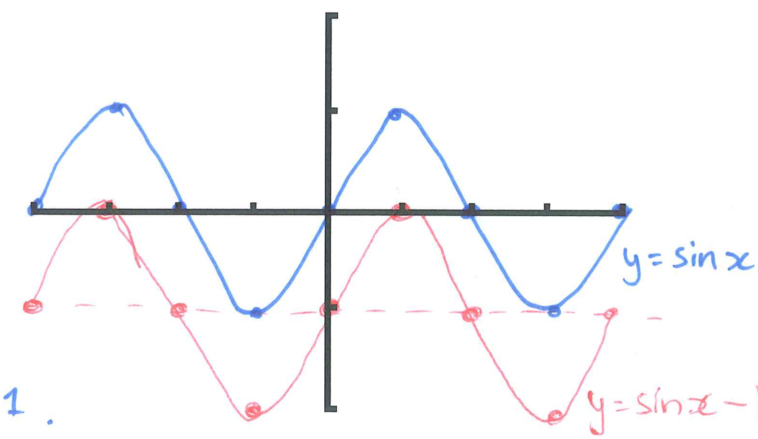
What is the midline value for each function?

$y = \sin x \rightarrow \boxed{y = 0}$

$y = \sin x + 1 \rightarrow \boxed{y = 1}$

Graph $y = \sin(x)$

$y = \sin(x) - 1$



What has changed?
max/min; x-int; y-int; range

- Graph was shifted down 1.

What is the median value for each function?

$y = \sin x \rightarrow \boxed{y = 0}$

$y = \sin x - 1 \rightarrow \boxed{y = -1}$

• If d is larger than 0 (positive), then the graph of y = sin(x) + d <u>Shifts up</u>.   • If d is smaller than 0 (negative), then the graph of y = sin(x) +d <u>Shifts down</u>.

$y = a \sin b(x-c) + d$

$y = \sin x$

Period $\Rightarrow 2\pi$

amp = 1

$x: [-2\pi, 2\pi, \frac{\pi}{6}]$

$y: [-1, 1, 0.5]$

max = 1

min = -1

midline $y = 0$

Range:

min $\leq y \leq$ max

$y = a \sin b(x-c) + d$

amplitude

$\text{amp} = \frac{\text{max} - \text{min}}{2}$

$b = \frac{2\pi}{\text{period}}$

or

$\text{Period} = \frac{2\pi}{b}$

horizontal

phase shift

$(x+c) \rightarrow c$ left

$(x-c) \rightarrow c$ right

vertical

phase shift / midline

$\text{mid} = \frac{\text{max} + \text{min}}{2}$

Note: max = $d + a$

min = $d - a$