# 8.5 Modelling Data with Sinusoidal Functions

**Today's Focus:** Determine the sinusoidal function that best models a set of data, and use your model to solve a problem.

## KEY IDEAS!!!

- Regression is a process where a curve or line of best fit is fitted to a set of data using a "least squares process". This process is programmed into the TI-83 calculators so that the calculator can find the best-fit equation for a set of given data.

## How to calculate regression equations:

1. Turn your diagnostics on [2$^{nd}$ catalog, "d", scroll until you find *diagnostics on*, enter] The diagnostics give the r-values/correlation coefficients.
2. enter each set of data into your lists, $L_1$ and $L_2$
3. turn your stat plots on [2$^{nd}$ y=]
4. adjust your windows to appropriate window settings [use the lists to choose the best windows]
5. calculate the appropriate regression equation
6. graph the equation, type it or enter it into y= using the vars calculator key [vars, 5: statistics, >>Eq, 1: RegEQ]

An r-value/correlation coefficient indicates how good a match a regression equation is for the given data. i.e. An r-value of 1 is a perfect match. An r-value of 0.5 would not be a very good match.

**Example 1:** A cuckoo clock is hanging on a wall. The table below gives the height of the pendulum above the ground as the clock ticks.

| Time (s) | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|---|---|---|---|---|---|---|---|
| Height (in.) | 65.0 | 66.5 | 68.0 | 66.5 | 65.0 | 66.5 | 68.0 |

a) Plot the data.
b) Determine the equation of a sinusoidal regression function that models the movement of the pendulum.

$y = 1.5\sin\left(\pi x - \frac{\pi}{2}\right) + 66.5$

**Example 2:** The table below gives the time, in minutes, versus the height, in meters, of a Ferris wheel. The diameter of the wheel is 76 m

| Time ($t$) minutes | Height ($h$) meters |
|---|---|
| 0 | 8 |
| 2.25 | 46 |
| 4.5 | 84 |
| 6.75 | 46 |
| 9 | 8 |

a) Enter the data above into the lists of the calculator and perform a sin regression. Round the parameters to the nearest hundredth if necessary. Set an appropriate window and graph the data on your calculator. Write the equation below and sketch the graph produced.

*[0, 18, 1][0, 90, 10]*

$y = 38\sin(0.698x + (-1.57079)) + 46$

$y = 38\sin(0.698x - 1.57079) + 46$

b) Determine the period of the graph and describe what it represents in this scenario.

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{0.698} = 9$

type in exact value $(0.698131\cancel{7}008)$

c) What is the relationship between the amplitude and the diameter of the wheel?

$\text{diameter} = 2(\text{amplitude})$

$= 2(38)$

$= 76 \text{ m}$

d) How high above the ground did people board the Ferris wheel in this scenario? minimum

$\min = d - a = 46 - 38 = 8\text{m}$

e) How far above the ground would a person be after three minutes?

$t = 3$ → 2nd, trace, 1:Value, 3, enter

$h = 65$ m

f) At what point(s) is a person 15 m above the ground in one full revolution?

$y_2 = 15$m, → 2nd, trace, 5: Intersect

$t = 0.88$, 8.12 min

g) How long, to the nearest tenth of a minute, is a person in one revolution when he is at least 15 m above the ground?

h) Write the equation of the median line and illustrate this on your graph.

$y_2 = 46$

i) A new Ferris wheel is constructed that is one meter higher when boarding than the original wheel. Describe the effect the new height of the wheel would have one:

- Amplitude of the graph

  amplitude → same, since diameter hasn't change

- Period of the graph

  same → since it will still rotate in the same time.

- Median of the graph

  shift vertically 1 up → $46+1=47$

- Height of the graph

  shifted vertically 1 up.

j) Describe a method to determine the new function of the graph for the new wheel.

$y = 38\sin(0.698x - 1.57079) + \underline{\underline{47}}$

k) Which parameters in the equation can be used to determine the maximum height the wheel reaches in the air?

$\boxed{max = d + a} = 46 + 38 = 84$ min in the original wheel

**Example 3:** The Bay of Fundy, in the Maritimes, has the highest tides in the world. The height of the water, in metres above the seabed, is shown for one point over 36 h.

| Hour | Height (m) | Hour | Height (m) | Hour | Height (m) | Hour | Height (m) |
|---|---|---|---|---|---|---|---|
| 1 | 3.5 | 10 | 7.0 | 19 | 4.6 | 28 | 1.6 |
| 2 | 2.4 | 11 | 6.4 | 20 | 5.9 | 29 | 1.9 |
| 3 | 1.8 | 12 | 5.3 | 21 | 6.9 | 30 | 2.7 |
| 4 | 1.9 | 13 | 4.0 | 22 | 7.2 | 31 | 4.0 |
| 5 | 2.7 | 14 | 2.7 | 23 | 6.8 | 32 | 5.4 |
| 6 | 3.8 | 15 | 1.8 | 24 | 5.9 | 33 | 6.5 |
| 7 | 5.2 | 16 | 1.7 | 25 | 4.6 | 34 | 7.1 |
| 8 | 6.3 | 17 | 2.2 | 26 | 3.2 | 35 | 7.0 |
| 9 | 7.0 | 18 | 3.3 | 27 | 2.1 | 36 | 6.3 |

a) Graph the data. Determine the equation of a sinusoidal regression function that models the height of the water. Does the regression equation match the data closely?

$y = 2.73\sin(0.506x + 3.02) + 4.43$

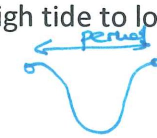
yes, it does.

b) How high is the water at high tide, to the nearest tenth of a metre? How high is the water at low tide?

max: $d + a = 4.43 + 2.73 = \underline{7.15\text{ m}}$

min: $d - a = \underline{1.70}$ m

c) How long, to the nearest minute, does it take for the tide to cycle from high tide to low tide and back again?




period $= \frac{2\pi}{b} = \boxed{12.4 \text{ min}}$

$= \frac{2\pi}{0.50597}$

d) Simon plans to go fishing at hour *50.* How high, to the nearest tenth of a metre, will the tide be when he begins fishing?

$x = 50$, 2nd, trace, 1:Value, 50, enter

$\boxed{4.3\text{ m}}$